Question 1 Let V be the set of 2×2 matrices whose entries are elements in $\mathbb{Q}$.

(a) Write V as a set using mathematical symbols.

(b) Define the $\underset{\sim}{0}$ element, the additive inverse $V \to V$, and two operations, addition $V \times V \to V$ and scalar multiplication $\mathbb{Q} \times V \to V$, which make V a vector space over $\mathbb{Q}$.

(c) Using your definitions from (b), show that axiom S3: $(a + b)\underset{\sim}{u} = a\underset{\sim}{u} + b\underset{\sim}{u}$ for all $a, b \in \mathbb{Q}$ and $\underset{\sim}{u} \in V$, holds. You may use that $\mathbb{Q}$ is a field.

Solution:

(a)
$$V = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \,\middle|\, a, b, c, d \in \mathbb{Q} \right\}.$$