Contents

Things to Remember

Trig Identities:

Pythagorean Identities:

$$\cos^2 x + \sin^2 x = 1 \qquad 1 + \tan^2 x = \sec^2 x \qquad 1 + \cot^2 x = \csc^2 x$$

Half Angle Formulas:

$$\sin^2 x = \frac{1 - \cos 2x}{2} \qquad \cos^2 x = \frac{1 + \cos 2x}{2}$$

Double Angle Formulas:

$$\sin 2x = 2 \sin x \cos x \qquad \cos 2x = \cos^2 x - \sin^2 x$$

Product-to-Sum Formulas:

$$\sin x \cos y = \frac{1}{2}[\sin(x+y) + \sin(x-y)]$$

$$\cos x \cos y = \frac{1}{2}[\cos(x+y) + \cos(x-y)]$$

$$\sin x \sin y = -\frac{1}{2}[\cos(x+y) - \cos(x-y)]$$

Sum-Difference Formulas:

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$

Even-Odd Identities:

$$\sin(-x) = -\sin x \qquad \cos(-x) = \cos x \qquad \tan(-x) = -\tan x$$

$$\csc(-x) = -\csc x \qquad \sec(-x) = \sec x \qquad \cot(-x) = -\cot x$$

Co-Function Identities:

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x \qquad \cos\left(\frac{\pi}{2} - x\right) = \sin x \qquad \tan\left(\frac{\pi}{2} - x\right) = \cot x$$

$$\csc\left(\frac{\pi}{2} - x\right) = \sec x \qquad \sec\left(\frac{\pi}{2} - x\right) = \csc x \qquad \cot\left(\frac{\pi}{2} - x\right) = \tan x$$

Ch 1: Limits and Fundamentals

1.1 General Hints and Rules of Limits

Let c and a be scalar constants.

Trig Limits

$$\lim_{x\to c} \sin x = \sin c$$

$$\lim_{x\to c} \cos x = \cos c$$

$$\lim_{x\to 0} \tan c = \tan c$$

$$\lim_{x\to 0} \sin x = 0$$

$$\lim_{x\to 0} \cos x = 1$$

$$\lim_{x\to 0} \tan x = 0$$

$$\lim_{x\to 0} \frac{\sin kx}{cx} = \frac{k}{c}$$

$$\lim_{x\to 0} \frac{1-\cos x}{x} = 0$$

Limit Rules

1. $\lim_{x\to a}[f(x) \pm g(x)] = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x)$
2. $\lim_{x\to a} f(x)g(x) = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x)$
3. $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$ if $\lim_{x\to a} g(x) \neq 0$
4. $\lim_{x\to a} kf(x) = k \lim_{x\to a} f(x)$
5. $\lim_{x\to a}[f(x)]^n = \left[\lim_{x\to a} f(x)\right]^n$ where n is a positive integer.
6. If $f(x) \leq g(x)$ on an interval which contains a, then $\lim_{x\to a} f(x) \leq \lim_{x\to a} g(x)$

1.2 Limits

DEF'N

Let $f(x)$ be a function, and a and L real numbers. As x approaches a, if $f(x)$ approaches L, then we say: L is the limit of $f(x)$ as x approaches a.
We denote this by:

$$\lim_{x \to a} f(x) = L$$

NOTE:

- x does not necessarily ever need to be equal to a, but merely approach it.
- $f(x)$ does not necessarily need to be defined at $x = a$, but at values close to a.

Identifying Clues:

- Question explicitly asks for evaluation of a limit.

Procedure:

1. If the function is continuous and defined at a, evaluate.
2. If discontinuous, evaluate as $f(x)$ approaches a.
3. If necessary, use one of the tools presented later in this booklet.

BYC's Concept Clarifier:

Find the value of $\lim_{x \to 0} f(x)$, where

$$f(x) = \begin{cases} x & x \neq 0 \\ 1 & x = 0 \end{cases}$$

1.3 One-Sided Limits

DEF'N
The left-hand limit of $f(x)$ at $x = a$ is calculated by approaching the point a only from the left, denoted as

$$\lim_{x \to a^-} f(x) = L$$

The right-hand limit of $f(x)$ at $x = a$ is calculated by approaching the point a only from the right, denoted as

$$\lim_{x \to a^+} f(x) = L$$

NOTE:
If these two limits are equal, then the limit of $f(x)$ as x approaches a exists and is equal to L:

$$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = L \leftrightarrow \lim_{x \to a} f(x) = L$$

Identifying Clues:

- Function has a discontinuity or is piece-wise.
- Limit is written in one-sided form.

Procedure:

1. Identify any jumps, steps, or discontinuities.
2. Evaluate limit from left or right, as asked.

BYC's Concept Clarifier:

The function f is defined by:

$$f(x) = \begin{cases} -3x + 2 & x \leq 1 \\ 2x^3 & x > 1 \end{cases}$$

Evaluate: $\lim_{x \to 1^-} f(x)$, $\lim_{x \to 1^+} f(x)$, and $\lim_{x \to 1} f(x)$.