Data Plan Parameter Estimation

Haoxiang Gao

October 16, 2017

1 Model Assumptions

Assume that weight at each day of week is w_i, where $i = 1, 2, ..7$. According to deterministic mdoel's conclusion:

$$a_j^* = \begin{cases} (w_j/\phi_i)^{\frac{1}{\alpha_i}} & \textbf{light user} \\ (w_j/(\pi + \phi_i))^{\frac{1}{\alpha_i}} & \textbf{heavy user} \end{cases} \tag{1}$$

2 Parameters Estimation

We have user's monthly data usage, then we can estimate the parameters by minimizing the sum of squared errors between actural usages and theoretical usages:

$$\arg\min_{w,\phi_i,\alpha_i} \sum_{j=1}^{n} (a_j^* - a_j)^2$$

3 Least Squares Solver

The gradiant of the objective function in light user case is:

$$\frac{\partial F}{\partial w_j} = \sum_{Day\,of\,Week=j} 2(a_j^* - a_j)(\frac{1}{\alpha_i \phi_i}(w_j/\phi_i)^{\frac{1}{\alpha_i}-1})$$

$$\frac{\partial F}{\partial \phi_i} = -\sum_{j=1} 2(a_j^* - a_j)\frac{w_j}{\alpha_i \phi_i^2}(w_j/\phi_i)^{\frac{1}{\alpha_i}-1}$$

$$\frac{\partial F}{\partial \alpha_i} = -\sum_{j=1} 2(a_j^* - a_j)a_j^* \frac{1}{\alpha_i^2} \ln(w_j/\phi_i)$$

The gradiant of the objective function in heavy user case is:

$$\frac{\partial F}{\partial w_j} = \sum_{Day\,of\,Week=j} 2(a_j^* - a_j)(\frac{1}{\alpha_i(\phi_i + \pi)}(w_j/(\phi_i + \pi))^{\frac{1}{\alpha_i}-1})$$

$$\frac{\partial F}{\partial \phi_i} = -\sum_{j=1} 2(a_j^* - a_j)\frac{w_j}{\alpha_i(\phi_i+\pi)^2}(w_j/(\phi_i+\pi))^{\frac{1}{\alpha_i}-1}$$

$$\frac{\partial F}{\partial \alpha_i} = -\sum_{j=1} 2(a_j^* - a_j)a_j^*\frac{1}{\alpha_i^2}\ln(w_j/(\phi_i+\pi))$$

We can use Levenberg Marquardt Optimizer to solve the optimal parameters (Reference is here).