Introduction.

This document is used to explain the results obtained from modifying a Variational Autoencoding algorithm to use complex manifolds instead of real ones.

Why bother?

The purpose of this experiment was to see if using complex manifolds can reduce training time on this particular type of algorithm.

How does it work?

This algorithm first encodes the original (uncompressed) data in complex manifolds, and then it uses a wirtinger derivative to calculate the derivatives used in the training process. (2).

$$\frac{\partial f}{\partial z} = \frac{1}{2}\left(\frac{\partial f}{\partial Z_{Real}} - i\cdot\frac{\partial f}{\partial z_{imaginary}}\right)$$
$$\frac{\partial f}{\partial Z} = \frac{1}{2}\left(\frac{\partial f}{\partial Z_{Real}} + i\cdot\frac{\partial f}{\partial z_{imaginary}}\right), where\, Z\, is\, the\, conjugate\, of\, z\, \mathbf{in}\, this\, case. (2).$$

Differentiability:

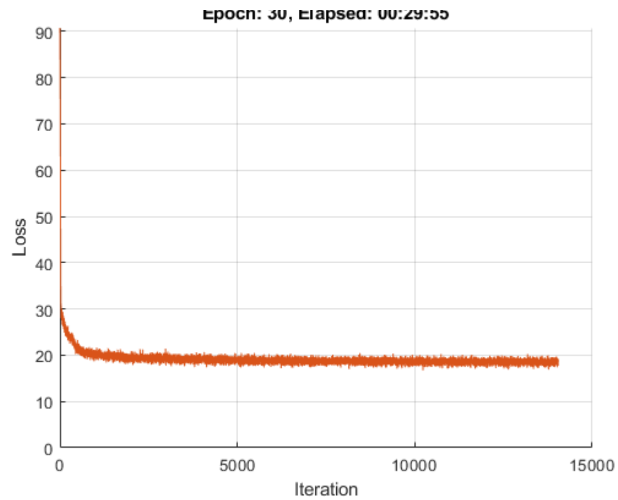
Note that to be differentiable at all, the conditions for this are different to real variables. In the case of 1 complex variable it would have to satisfy the cauchy-riemann equations shown in (5),
and it has to be continuous in the neighbourhood of interest.

The data was first changed to complex manifolds by using the complex command in matlab across all numerical data components, to convert this to complex double format.

After that I just ran the algorithm with everything else exactly the same as it was, including the data sets and layers used to make the autoencoder work.

Results.

The original is shown here (from (1)).



Below is the results obtained by using complex manifolds to do it, note that the number of iterations needed for comparable accuracy is much lower than it is with real valued data (modelled by using real manifolds in the above case, which very often are not differentiable everywhere).

Why does this work?

The reason why this needs fewer iterations to get the same accuracy or better, is because unlike real variables complex ones are not semi continuous over manifolds, that have more than 4 dimensions.

i.e. real variables in this context are bounded above or below over a measure (which is used to calculate the probability distributions, and the probabilites from it), but unlike complex variables they are not typically bounded both above and below. Hence they are not continuous everywhere.

Number of epochs used to do this was 30, although from the graph this does converge with fewer epochs than this.

Software used: MATLAB 2022A with the Deep Learning toolkit installed.

References.

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By Thomas.N.Toseland.