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SUMMARY

Aerodynamics
MECA-Y-402

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Appel à contribution

Synthèse Open Source

Ce document est grandement inspiré de l'excellent cours donné par Herman Deconinck à l'EPB (École Polytechnique de Bruxelles), faculté de l'ULB (Université Libre de Bruxelles). Il est écrit par les auteurs susnommés avec l'aide de tous les autres étudiants et votre aide est la bienvenue ! En effet, il y a toujours moyen de l'améliorer surtout que si le cours change, la synthèse doit être changée en conséquence. On peut retrouver le code source à l'adresse suivante

`https://github.com/nenglebert/Syntheses`

Pour contribuer à cette synthèse, il vous suffira de créer un compte sur *Github.com*. De légères modifications (petites coquilles, orthographe, ...) peuvent directement être faites sur le site ! Vous avez vu une petite faute ? Si oui, la corriger de cette façon ne prendra que quelques secondes, une bonne raison de le faire !

Pour de plus longues modifications, il est intéressant de disposer des fichiers : il vous faudra pour cela installer LaTeX, mais aussi *git*. Si cela pose problème, nous sommes évidemment ouverts à des contributeurs envoyant leur changement par mail ou n'importe quel autre moyen.

Le lien donné ci-dessus contient aussi un `README` contenant de plus amples informations, vous êtes invités à le lire si vous voulez faire avancer ce projet !

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Merci !

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1.2.1 Mass conservation

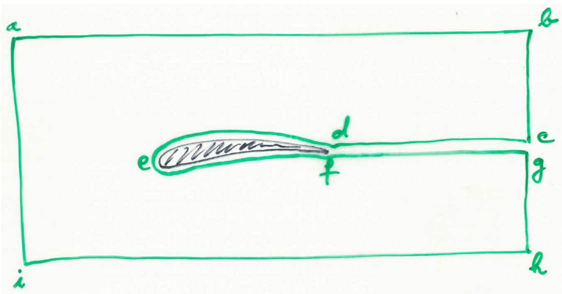

Figure 1.1

Consider the closed control volume S* (abhi) around the airfoil. It is a 2D view, but imagine that we have a 3D configuration with Z axis perpendicular to the sheet. Be aware that the normal is always perpendicular to the contour and is external! The fundamental integral form of the mass conservation equation is:

$$\frac{d}{dt}\int_V \rho\, dV + \oint_S \rho\vec{v}\, d\vec{S} = 0. \tag{1.1}$$

By applying Gauss theorem $\oint_S \vec{a}.\vec{n}\, dS = \int_V \nabla\vec{a}\, dV$, and regrouping the term in a unique integral, we obtain:

$$\int_V \left[\frac{d\rho}{dt} + \nabla.(\rho\vec{u})\right] dV = 0. \tag{1.2}$$

Considering this to be true for all volumes, the integral disappear and gives the

Continuity equation

$$\frac{d\rho}{dt} + \nabla.(\rho\vec{u}) = 0 \tag{1.3}$$

Another form can be found by introducing the material derivative $\dot{\rho} = \frac{d\rho}{dt} + (\vec{v}.\nabla)\rho$, and if we are in a steady state, the time derivative goes away.

1.2.2 Momentum equation

The general form of the momentum equation is:

$$\rho\dot{\vec{v}} = \frac{\partial\rho\vec{v}}{\partial t} + \rho(\vec{v}\nabla)\vec{v} = -\nabla p + \nabla\bar{\bar{\tau}}. \tag{1.4}$$

By considering a steady state, the time derivative goes away. If we consider the x component of the velocity, we can expend the derivative to the whole left term as:

$$\rho(\vec{v}\nabla)v_x = \nabla(\rho\vec{v}v_x) - \cancel{v_x\nabla(\rho\vec{v})} \tag{1.5}$$

where the last term is null related to (1.3) in steady state. Integrating both sides around the volume contained in the closed surface S (abcdefghi on figure) in (1.5), and applying Gauss theorem, we obtain:

$$\oint_S \vec{v}(\rho\vec{v}\vec{n})\, dS = -\oint_S p\, d\vec{S} + \oint_S \bar{\bar{\tau}}\, d\vec{S}. \tag{1.6}$$

Let's now apply this equation to the new closed contour $S* = S - \text{airfoil} - cd - fg$ (previous abhi in fact). (1.6) becomes:

$$\begin{aligned} &\oint_{S^*} \vec{v}(\rho\vec{v}\, d\vec{S}) + \cancel{\oint_{\text{airfoil}} \vec{v}(\rho\vec{v}\, d\vec{S})} + \cancel{\oint_{cd+fg} \vec{v}(\rho\vec{v}\, d\vec{S})} \\ &= -\oint_{S^*} p\, d\vec{S} - \oint_{\text{airfoil}} p\, d\vec{S} - \cancel{\oint_{cd+fg} p\, d\vec{S}} + \oint_{S^*} \bar{\bar{\tau}}\, d\vec{S} + \oint_{\text{airfoil}} \bar{\bar{\tau}}\, d\vec{S} + \cancel{\oint_{cd+fg} \bar{\bar{\tau}}\, d\vec{S}} \end{aligned} \tag{1.7}$$

where the $cd + fg$ components cancels each other if we consider that they are infinitely close to each other, as they are opposite. The airfoil integral in the left hand side is null because the wing can not be penetrated by the flow. If we manipulate the equation to refind the (1.6) shape by regrouping airfoil terms in an additional $\vec{R}$ term. Taking account the orientation of normals, the signs will be chosen in the way $\vec{R}$ is a

Force applied on the wing

$$\vec{R} = \oint_{\text{airfoil}} p\, d\vec{S} - \oint_{\text{airfoil}} \bar{\bar{\tau}}\, d\vec{S} \tag{1.8}$$

so that (1.7) becomes, after considering S* to be a contour in the **far field** so that viscous effects vanish:

$$\oint_{S^*} \vec{v}(\rho\vec{v}\, d\vec{S}) = -\oint_{S^*} p\, d\vec{S} + \cancel{\oint_{S^*} \bar{\bar{\tau}}\, d\vec{S}} - \vec{R}. \tag{1.9}$$

We still have to measure the pressure.

Uniform p along S*

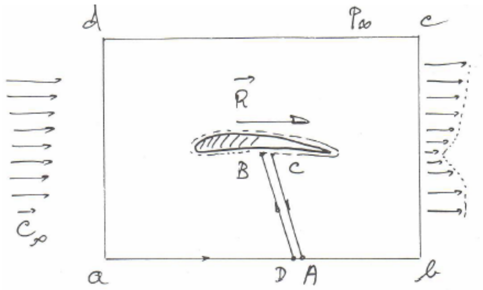

Figure 1.2

By considering this, we can compute the force only by knowing the far field parameters. Indeed, uniform pressure implies null surface integral, so that (1.9) becomes:

$$\vec{R} = -\oint_{S^*} \vec{v}(\rho\vec{v}\, d\vec{S}). \tag{1.10}$$

Let's now consider that the velocity is horizontal so that $\vec{R} = R.\vec{1}_x$, at the inlet we have $\vec{v}$ and $\vec{n}$ are opposed while at the outlet they are in the same direction:

$$\vec{R} = \int_a^d \vec{v}\, d\dot{m} - \int_b^e \vec{v}\, d\dot{m} > 0 \tag{1.11}$$

showing that there is only **drag** force.

1.3 The aerodynamic lift

Non uniform p along S*. In order to apply Bernouilli equation $p + \frac{1}{2}\rho v^2 = cst$, let's add the constants p_∞ and v_∞ to (1.9), as $\oint p_\infty\, d\vec{S} = p_\infty \oint d\vec{s} = 0$:

$$\vec{R} = -\oint_{S^*} (p - p_\infty)\, d\vec{S} - \oint_{S^*} (\vec{v} - \vec{v}_\infty)\, d\dot{m} \tag{1.12}$$

Let's express $\vec{v} = \vec{v}_\infty + \vec{\delta_c}$ with $\vec{\delta_c}$ a perturbation. Introducing this in Bernouilli equation:

$$\begin{gathered} p_\infty + \frac{1}{2}\rho \cancel{\vec{v}_\infty^2} = p + \frac{1}{2}\rho(\vec{v}_\infty + \vec{\delta_c})^2 = p + \frac{1}{2}\rho(\cancel{\vec{v}_\infty^2} + 2\vec{v}_\infty\vec{\delta_c} + \cancel{\vec{\delta_c}^2}) \\ \Rightarrow p - p_\infty = -\rho\vec{v}_\infty\vec{\delta_c}. \end{gathered} \tag{1.13}$$

If we replace this result in (1.12), we find:

$$\begin{aligned} \vec{R} &= \oint_{S^*} \rho(\vec{v}_\infty\vec{\delta_c})\, d\vec{S} - \oint_{S^*} \rho\vec{\delta_c}[(\vec{v}_\infty + \cancel{\vec{\delta_c}})\, d\vec{S}] \\ &= \oint_{S^*} \rho[(\vec{v}_\infty\vec{\delta_c})\, d\vec{S} - \vec{\delta_c}[(\vec{v}_\infty . d\vec{S})] \end{aligned} \tag{1.14}$$

by using a matrix property $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a}\vec{b})\vec{c} - (\vec{a}\vec{c})\vec{b}$:

$$= \rho\vec{v}_\infty \times \oint_{S^*} \vec{\delta_c} \times d\vec{S} = \rho\vec{v}_\infty \times \left[\oint_{S^*} \vec{v} \times d\vec{S} - \cancel{\oint_{S^*} \vec{v}_\infty \times d\vec{S}}\right] \tag{1.15}$$

and by applying Stokes theorem $\oint_S \vec{a} \times d\vec{S} = \int_V \nabla \times \vec{a}\, dV$:

$$= \rho\vec{v}_\infty \times \int (\nabla \times \vec{v})\, dV = \rho\vec{v}_\infty \times \int \vec{w}\, dV \tag{1.16}$$

where $\vec{w}$ is the **vorticity vector** of direction $\vec{1}_z$:

$$\vec{w} = \begin{vmatrix} \vec{1}_x & \vec{1}_y & \vec{1}_z \\ \partial_x & \partial_y & 0 \\ v_x & v_y & 0 \end{vmatrix} = [\partial_x v_y - \partial_y v_x]\vec{1}_z \tag{1.17}$$

This shows that the lift force is always perpendicular to the flow!

1.4 The Kutta-Joukowski formula

We will now introduce the circulation $\Gamma = -\oint \vec{v}\, d\vec{l} > 0$ around a body. The convention is to take the anticlockwise direction for $d\vec{l}$ and so for Γ to be > 0 we must have $\vec{v}$ in the clockwise direction. There is a link between the lift force and the circulation. Let's introducte **Stokes theorem**:

$$\oint \vec{a} d\vec{l} = \int_S (\nabla \times \vec{a})\, d\vec{S} \qquad \Rightarrow -\Gamma = \int_S \vec{w} d\vec{S}. \tag{1.18}$$

We remember that:

$$
\begin{aligned}
&\vec{R} = \rho\vec{v}_\infty \times \int \vec{w}\, dV = \rho\vec{v}_\infty \times \int l\vec{w}\, dS \qquad \Leftrightarrow \frac{\vec{R}}{l} = \rho\vec{v}_\infty \times \int \vec{w}\, dS \\
&\frac{\vec{R}}{l} = \rho\vec{v}_\infty \times \int \vec{w}\,(d\vec{S}.\vec{1}_z) = \rho\vec{v}_\infty \times (-\Gamma)\vec{1}_z = \rho v_\infty \Gamma\, \vec{1}_y
\end{aligned}
\tag{1.19}
$$

to finally obtain a very good approximation of the lift:

Kutta formula for lift 2D airfoil

$$|R| = \rho v_\infty \Gamma \tag{1.20}$$

Application to airfoils

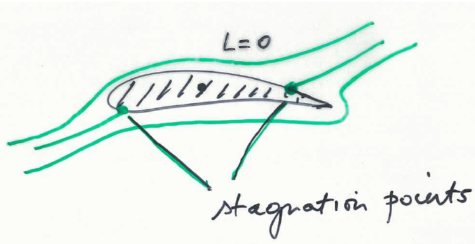

Figure 1.3

In inviscid case, if we have no vorticity that can be created, we have the kelvin theorem, in this case the circulation = 0 and so there is no lift. If we take an arbitrary contour around the airfoil we will have no circultion. In inviscid case we can never get a lift → D'alembert paradox. At the trailing edge, if the flow wants to continue on the other corner from below, the velocity must be infinity. But this is not the case in reality.

After some processes we can obtain the stagnation point on the trailing edge which satisfies the Kutta condition. So in this case if we take a contour that contains the airfoil we will have a non 0 circulation but for all contour that does not contain the airfoil it is 0. But why to put the stagnation point at the trailing edge ? This is purely physics. Gamma is free but only one corresponds to the Kutta condition.

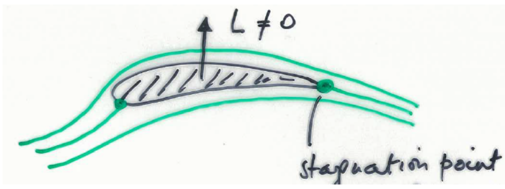

Figure 1.4

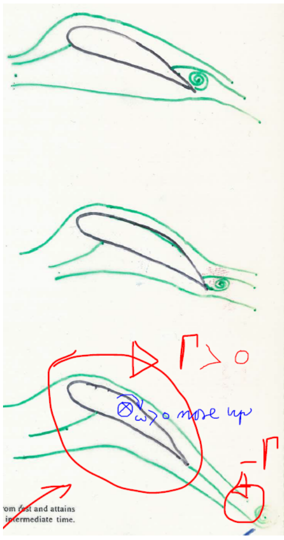

Figure 1.5

What happens is that initially we have the first kind of flow, then the formation of the starting vortex due to viscous effects (separation) which is compensate by a **bound vortex** around the airfoil that makes $\Gamma \neq 0$. Then the vortex goes away to infinity. Indeed if we take $R = \rho v_\infty \Gamma$, $\Gamma \neq 0$, so we have lift.