Université Libre de Bruxelles

Summary

Fluid mechanics and transport processes
MECA-H-300

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Appel à contribution

Synthèse OpenSource

Ce document est grandement inspiré de l'excellent cours donné par NomDuProf à l'EPB (École Polytechnique de Bruxelles), faculté de l'ULB (Université Libre de Bruxelles). Il est écrit par les auteurs susnommés avec l'aide de tous les autres étudiants et votre aide est la bienvenue ! En effet, il y a toujours moyen de l'améliorer surtout que si le cours change, la synthèse doit être changée en conséquence. On peut retrouver le code source à l'adresse suivante

`https://github.com/nenglebert/Syntheses`

Pour contribuer à cette synthèse, il vous suffira de créer un compte sur *Github.com*. De légères modifications (petites coquilles, orthographe, ...) peuvent directement être faites sur le site ! Vous avez vu une petite faute ? Si oui, la corriger de cette façon ne prendra que quelques secondes, une bonne raison de le faire !

Pour de plus longues modifications, il est intéressant de disposer des fichiers : il vous faudra pour cela installer LaTeX, mais aussi *git*. Si cela pose problème, nous sommes évidemment ouverts à des contributeurs envoyant leur changement par mail ou n'importe quel autre moyen.

Le lien donné ci-dessus contient aussi le `README` contient de plus amples informations, vous êtes invités à le lire si vous voulez faire avancer ce projet !

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Merci !

Chapitre 1

Introduction

Before beginning the summary, I want to tell you that my English level isn't perfect. Please collaborate and correct the gramatically wrong sentences.

1.1 Reminder

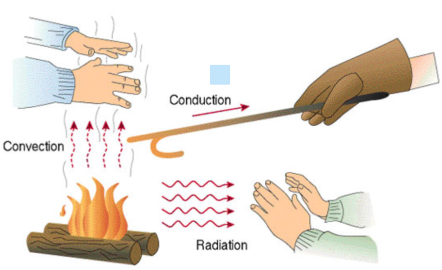
The governing equations in transport processes are the followings :

- Mass conservation :
$$\frac{\partial \rho}{\partial t} + \nabla(\rho v) = 0 \tag{1.1}$$
- Navier-Stokes :
$$\rho\left(\frac{Dv}{Dt} + v\nabla v\right) = -\nabla p + \mu\nabla^2 v \tag{1.2}$$
- Energy equation :
$$\frac{DT}{Dt} = \nabla(\alpha\nabla T) + \frac{\dot{Q}v}{\rho c} \tag{1.3}$$
- Species conservation
$$\frac{\partial \rho_A}{\partial t} + \nabla(\rho_A v_A) = r_A \tag{1.4}$$

Let's precise that there are many applications using these equations like in the aerospace and automotive industry, in safety and fire prevention or in buildings design.

1.2 Convection and diffusion

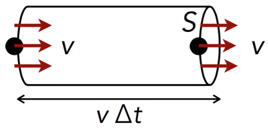
1.2.1 Definitions



Here is a picture illustrating the principles of **convection**, **conduction** and **radiation**. Imagine that you have a fire and you put your hands above. You will feel a flow of heat transmitted by convection. If someone comes with a stick, there will be conduction in the material transmitting the energy from particles to particles. Finally, if the hands are next to the fire, there is no flow but you feel the heat. The energy is transmitted by radiation.

1.2.2 Convection

Convection is a transfer always associated to **bulk (ensemble) fluid motion**. We consider a fluid with **uniform** velocity and a cylinder of section S and lenght Δt. In that time interval, the fluid in the cylinder will have crossed the section S. We are now able to express the convective flux of momentum (quantité de mouvement), energy and mass knowing that the flux of a physical quantity is given by



$$flux_A = \frac{A}{S\Delta t} \tag{1.5}$$

Mass

We know that the mass is given by density × volume and that the volume of the cylindre is $Sv\Delta t$. Using the new expression of the mass and (1.5), we can find the **flux of mass**

$$M_c = \rho v S\Delta t \qquad and \qquad J_{M_c} = \frac{M_c}{S\Delta t} = \rho v \tag{1.6}$$

Momentum

Similarly to the calcul of the mass

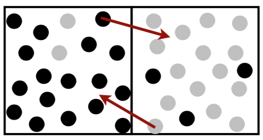
$$Q_c = mv = \rho v^2 S\Delta t \qquad and \qquad J_{Q_c} = \rho v^2 \tag{1.7}$$

Energy

The energy in the system is given by the specific heat energy of each particles [1]. If T_0 is the reference temperature,

$$E_c = \rho c_v(T - T_0)S\Delta t \qquad and \qquad J_{E_c} = \rho c_v(T - T_0) \tag{1.8}$$

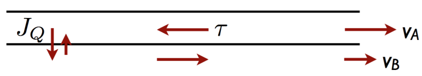
1.2.3 Diffusion



Diffusion is a transfer associated to the **particles random walk** and is due to the presence of a gradient of physical quantity (temperature for example). The picture on the left illustrates that, for an infinite time, the process will reequilibrate the gradient, difference between the two boxes.

1.3 Diffusive flux

If we consider two parallel fluid layers to bulk velocity $v_A > v_b$, the gradient of velocity will vanish [2] ($v_A = v_b$). This effect is due to the initial velocity gradient that cause the diffusion of faster particles towards the slower ones, transferring a momentum flux J_Q. In fact, the origin of the flux in the transversal direction to the flow is due to the **friction** between the two layers, parallel to the flow direction.



1. See *Chimie générale* for the expression
2. Disparaître

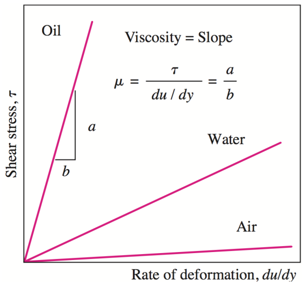
1.3.1 Shear stress

To characterize the friction between to layers, we introduce the **shear stress** that is proportional to the gradient of velocity

$$\tau = \tau\left(\frac{dv}{dy}\right) \tag{1.9}$$

According to the Newton's law, for newtonian fluids like gases and liquids, the equation becomes

$$\tau = \mu\left(\frac{dv}{dy}\right) \tag{1.10}$$



— μ is the **dynamic viscosity**, the **intrinsic resistance** of the fluid to motion $[kg/m.s]$

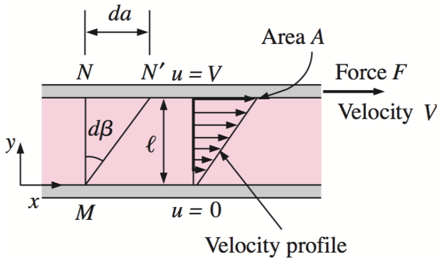
— dv/dy the gradient of velocity $[s^{-1}]$

— τ a force per unit surface $[N/m^2] = [Pa]$

If we see the shear stress to the **rate of deformation**[3] on a graph, we can see that the **viscosity** is the slope[4]. We can also see that viscosity of air is miner than water that's miner than oil.

1.3.2 Planar Couette flow

Let's consider a fluid layer between two very large plates sparated by a distance l. A constant parallel force F (drag force[5]) is applied to the upper plate and after the initial transients, it moves continuously to a constant velocity V. What are the consequences on the fluid ?



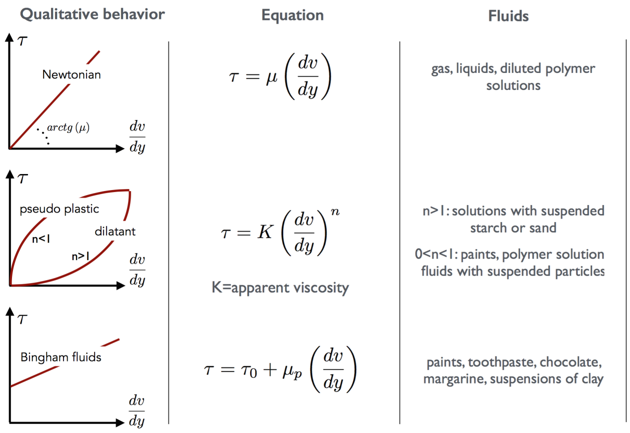
- **Empirical observations :**
  No slip conditions[6] at walls $\rightarrow u(0) = 0$ and $u(l) = V$.

- **The flow is organized in parallel layers :**
  We suppose that the flow is laminar (no turbulence), so the velocity profile is a sole function of y and is **linear** : $u(y) = V\frac{y}{l}$.

- **The drag force is imposed :**
  $\tau = \frac{F}{A} = \mu\frac{dv}{dy} = c$

3. Gradient of velocity
4. La pente
5. Force de traînée
6. Condition de non-glissement

1.3.3 Fluid rheology



Rheology is the study of the flow that establish the relation between the shear stress and the velocity gradient. Let's have a look to different fluids.
As first observation, we can see that newtonian fluids respect a linear relation for shear stress to the rate of deformation. It's not the case for others like second and third line of the table[7]. For bingham fluids, there is a critical shear stress to reach before the behavior becomes like newtonian fluids

Dynamic viscosity of liquids and gases

When we observe the table of dynamic viscosity for the water, the dynamic viscosity decreases with temperature increase for liquid water. It's not the case for the gases for wich dynamic viscosity increases with temperature.
The explanation is that, for liquids, the increase of temperature increases the gradient of velocity making the liquid more free. For gases, the increase of temperature makes that particles hit each other more frequently, decreasing the velocity.

1.4 Constitutive relations

It's an important section because it shows that transport processes have a uniform approach. Let's introduce 3 new variables : **kinematic viscosity** ν, **thermal diffusivity** α**internal energy** u **mass diffusivity** D. Let's precise that $\nu, \alpha, D = [L^2T^{-1}]$

- **Momentum diffusion flux** (Newton's law)

$$J_{Q_d} = -\mu \frac{dv}{dy} \quad \xrightarrow{\nu = \frac{\mu}{\rho}} \quad J_{Q_d} = -\nu \frac{d(\rho v)}{dy} \tag{1.11}$$

- **Energy diffusive flux** (Fourrier's law)

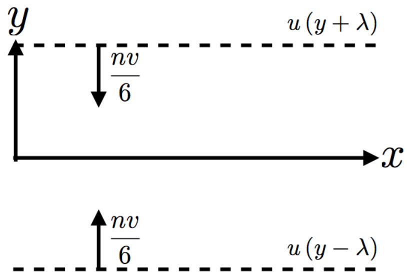
$$J_{E_d} = -k \frac{dT}{dy} \quad \xrightarrow{\alpha = \frac{k}{\rho c_v} \ and \ u = c_v (T - T_0)} \quad J_{E_d} = -\alpha \frac{d(\rho u)}{dy} \tag{1.12}$$

- **Mass diffusion flux** (Fick's law)

$$J_{M_d} = -D \frac{dc}{dy} \qquad (c = \text{concentration}) \tag{1.13}$$

7. Starch = amidon

1.5 Gas viscosity



Determination of the dynamic viscosity - Maxwell, 1860

I. *n* molecules per unit volume, each with mass *m*
II. a fraction of *n/3* moving in the *y direction*
III. *a fraction n/6* along +y et n/6 along -y
IV. *nv/6* molecules crossing a unit area in both directions, per unit time
V. *nv/6* molecules with free mean path $\lambda = 1/\pi n d^2$
VI. *nv/6 molecules with momentum m u(y+λ)* et *m u(y-λ)*

Let's consider a gas moving in the x direction with a velocity $u = u(y)$. The **kinetic theory** gives us the random velocity in y using $v = \sqrt{\frac{3k_bT}{m}}$. The development of Maxwell, above, allows us to calculate the momentum flux crossing the x axis.

$$J_{Q_d} = \left(\frac{nv}{6}\right) mu(y-\lambda) - \left(\frac{nv}{6}\right) mu(y+\lambda) \tag{1.14}$$

Here we use $u(y-\lambda)$ and $u(y+\lambda)$ because the last particles that can cross an axis on y are those who are situated in a distance of maximum λ from there.
The Tylor expansion of the flux $u(y+\lambda) = u(\lambda) + \lambda\frac{du}{dy} + \ldots$ and the constitutive equation (1.11) gives us the final expression of viscosity

$$\mu = \frac{1}{3}nm\lambda v = \frac{1}{3}\rho\lambda v = \frac{1}{\pi\sqrt{3}}\frac{\sqrt{mk_bT}}{d^2} \tag{1.15}$$