

Be Cautious when Using the FIR Channel Model with the OFDM-Based Communication Systems

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Abstract—Orthogonal frequency-division multiplexing (OFDM) can be used to support high data rate transmissions over time-dispersive fading channels. Many OFDM-based communication systems use packet-based transmission, where channel estimation is needed for the detection of the information-carrying symbols. The finite impulse response (FIR) channel model is simple and effective for some simulations of the OFDM-based communication systems over the time-dispersive channels; yet, it is only an approximate channel model which cannot be used in the case of accurate channel estimation. Unfortunately, many researchers have overlooked this issue and have been devising channel estimation algorithms squarely based on the FIR channel model. While the channel estimation results from these algorithms can be impressive for the FIR channels, the algorithms can hardly be applied in real-world applications. This paper explains in detail the reason why the FIR channel model is only an approximate one for the OFDM-based communication systems, trying to discourage the inappropriate usage of this model, which can lead to fruitless efforts. This paper also presents a realistic channel model for the OFDM-based communication systems, which can be used to access the channel parameter estimation algorithms realistically.

Index Terms – OFDM, channel model, channel estimation

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I. INTRODUCTION

Orthogonal frequency-division multiplexing (OFDM) technique is an effective means of supporting high data rate transmissions over time-dispersive (also called frequency-selective or multipath) fading channels. OFDM has been selected as the basis for several high-speed wireless local area networks (WLANs) standards [1], such as HIPERLAN/2, IEEE 802.11a [2], IEEE 802.11g, and IEEE 802.11n. It has also been selected as the basis for several mobile broadband wireless communication (MBWC) systems, such as WiBro (Wireless broadband) and Mobil WiMax (IEEE 802.16-2005).

Many OFDM-based communication systems, as exemplified by those conforming to the IEEE 802.11a standard, use packet-based transmission. Each packet, as shown in Figure 1, consists of an OFDM packet preamble, a SIGNAL field, and an OFDM DATA field. The preamble is designed to facilitate the estimation of channel parameters such as the carrier frequency offset (CFO), symbol timing, as well as channel response. These parameters are needed for the data symbol detection in the OFDM DATA field.

For the simulation of the time-dispersive fading channels for the OFDM-based communication systems, the FIR (finite impulse response) filter channel model [3] can be used. While this model is simple and effective for the detection/decoding performance evaluation, it cannot be used as the foundation based squarely on which (i.e., to exploit the specific characteristics) to devise signal processing algorithms for channel parameter estimation. This issue was first alluded in [4] but largely overlooked by many researchers in the wireless communication community. This neglect is evidenced by the fact that hundreds of publications in the topic of channel estimation for the OFDM-based communication systems base the algorithms squarely on the FIR model. While the results seem impressive, the algorithms can hardly be used in real applications. Even after this issue was made more clearly in a few recent publications [5], [6], [7], many researchers still extensively use the FIR model for channel estimation, as demonstrated by very recent publications.

This paper explains in detail the reason why the FIR channel model is only an approximate one for the OFDM-based communication systems, trying to discourage the inappropriate usage of this model, which can lead to fruitless efforts. This paper also presents a realistic channel model for the OFDM-based communication systems, which can be used to access the channel parameter estimation algorithms realistically.

Note that, as will be discussed later in this paper, the FIR channel model can be a very appropriate model for single-carrier communication schemes, such as CDMA.

The remainder of this paper is organized as follows. Section II overviews the generation of OFDM data symbols. Section III presents equivalent discrete channel responses for the realistic channels for the OFDM-based communication systems and discuss in detail the limitation of the FIR channel model. Section IV provides a realistic channel model which can be used for assessment of channel estimation algorithms for the OFDM-based systems. Finally, we conclude in Section V.

II. OFDM DATA MODEL

Before discussing the channel model, let us consider the generation of the OFDM data symbols, as exemplified by those specified by the IEEE 802.11a standard. Let

$$\mathbf{x}_k = \begin{bmatrix} x_{k,1} \\ x_{k,2} \\ \vdots \\ x_{k,N_S} \end{bmatrix}, \quad k = 1, 2, \dots, K, \quad (1)$$

be a vector of $N_S = 64$ symbols, where x_{k,n_S} , $n_S = 1, 2, \dots, N_S$, is the symbol modulating the n_S th subcarrier and is equal to 0 for the null subcarriers (there are 12 null subcarriers, with one being at the center of the spectrum for dc bias removal and the other 11 being at the two sides of the spectrum for bandwidth protection), 1 or -1 for the pilot tones (there are 4 pilot tones), or a member in a constellation \mathcal{C} for information-carrying subcarriers (there are 48 information-carrying subcarriers). Here K is the number of OFDM data symbols in a packet and \mathcal{C} is a finite constellation, such as BPSK, QPSK, 16-QAM, or 64-QAM. Let $\mathbf{W}_{N_S} \in \mathbb{C}^{N_S \times N_S}$ be the FFT matrix. Then the k th OFDM data symbol \mathbf{s}_k corresponding to \mathbf{x}_k is obtained by taking IFFT of \mathbf{x}_k . That is,

$$\mathbf{s}_k = \mathbf{W}_{N_S}^H \mathbf{x}_k / N_S, \quad (2)$$

where $(\cdot)^H$ denotes the conjugate transpose. To facilitate eliminating inter-symbol interference (ISI) caused by the time-dispersive channel at the receiver, \mathbf{s}_k is preceded by a cyclic prefix (CP) or guard interval (GI) $\mathbf{s}_{k,C}$ formed using the last $N_C = 16$ elements of \mathbf{s}_k .

By stacking the packet preamble and the OFDM symbols (together with their corresponding CPs) in the SIGNAL and OFDM DATA field, we obtain the entire packet $\mathbf{s} \in \mathbb{C}^{(5+K)(N_S+N_C) \times 1}$. (Here 5 is due to the fact that the length of the packet preamble is the same as that of 4 OFDM data symbols and that the length of the SIGNAL field is the same as 1 OFDM data symbol.) Passing \mathbf{s} through a pair of D/A converters at a rate of

$f_S = 20$ MHz (for both the real and imaginary parts of s) and the corresponding lower-pass filters, we obtain the (complex) baseband analog waveform $s(t)$ which is shown in Figure 2.

III. EQUIVALENT DISCRETE CHANNEL RESPONSES

Let

$$h(t) = \sum_{p=0}^{P_M} \alpha_p \delta(t - t_p) \quad (3)$$

denote the (baseband) time-domain *analog* channel impulse response of a time-invariant multipath (aka, time-dispersive) fading channel (called the *realistic channel* for short), where P_M is the number of multipaths, and α_p and $t_p = \tau_p t_S$ ($\tau_0 \leq \tau_1 \leq \dots \leq \tau_{P_M}$, $t_S = 1/f_S = 50$ ns) are the complex gain and time delay of the p th path, respectively. Then the (baseband) input to the receiver is

$$z(t) = h(t) * s(t), \quad (4)$$

as shown in Figure 2. To detect the data bits contained in each OFDM data symbol, we need to sample $z(t)$ (with sampling interval t_S) and process the sampled data in blocks of length N_S . (Note that the samples are taken at the multiples of t_S .)

Let

$$\mathbf{h}^{(t)} = [h_0^{(t)} \ h_1^{(t)} \ \dots \ h_{N_S-1}^{(t)}]^T \quad (5)$$

be the corresponding time-domain *discrete* channel response, referred to as the *equivalent* discrete channel response of $h(t)$, for the sampled data blocks. Note that by *equivalence* we mean that the discrete frequency-domain response of (5) is the same as the frequency-domain response of (3) on the (discrete) interested subcarriers; i.e., if

$$\mathbf{h} = \mathbf{W}_{N_S} \mathbf{h}^{(t)} \triangleq [h_1 \ h_2 \ \dots \ h_{N_S}]^T \quad (6)$$

is the discrete frequency-domain channel response for the sampled signals, then for an interested subcarrier n_S , we have:

$$h_{n_S} = \sum_p \alpha_p e^{-j\tau_p t_S \omega} \Big|_{\omega = \frac{2\pi[n_S-1]N_S}{N_S t_S}}, \quad (7)$$

where

$$[n_S - 1]_{N_S} = \begin{cases} n_S - 1, & n_S \leq N_S/2, \\ n_S - 1 - N_S, & n_S > N_S/2. \end{cases} \quad (8)$$

Note that Equation (7) implies that the sampling starts from $t = 0$; to avoid ISI, we need to use correct symbol timing. Note also that different symbol timing leads to different phase expression in (7).

We have two types of equivalent discrete channel responses depending on the choices of the interested subcarriers, which are used to transmit data. If we are interested in all of the $N_S = 64$ subcarriers, then we have the Type A equivalent discrete channel response; the l th, $l = 0, 1, \dots, N_S - 1$, element of $\mathbf{h}^{(t)}$ for the Type A equivalent discrete channel response can be written as

$$h_l^{(t)} = \sum_p \alpha_p e^{j \frac{\pi(\tau_p - l)}{N_S}} \frac{\sin(\pi(\tau_p - l))}{\sin(\pi(\tau_p - l)/N_S)}. \quad (9)$$

On the other hand, if we are interested in only the $N_{SC} = 52$ non-zero subcarriers with the 12 null-subcarriers being set to zero, then we have the Type B equivalent discrete channel response; the l th, $l = 0, 1, \dots, N_S - 1$, element of $\mathbf{h}^{(t)}$ for the Type B equivalent discrete channel response can be written as

$$h_l^{(t)} = \sum_p \alpha_p \left(\frac{\sin(\pi(\tau_p - l)(N_{SC} + 1)/N_S)}{\sin(\pi(\tau_p - l)/N_S)} - 1 \right). \quad (10)$$

See Appendix A.1 for the derivations of (9) and (10).

Considering the above Type A and Type B equivalent discrete channel responses, we have the following remarks pertaining to the FIR channel model for the OFDM-based communication systems, which is assumed to have a time-domain length less than N_C so that the ISI problem can be eliminated.

Remark 1: The time-domain lengths of both type of equivalent discrete channel responses usually are N_S , as shown in Figure 3. This is true even when we have only one path where t_0 is not a multiple of t_S . The reason is that, as shown in Figure 4, the transition of frequency-domain channel response from one period to another (around frequency indexes -32 and +32 in the figure) is not smooth due to fact that τ_p , ($p = 0, 1$ in this special case), is not an integer. As such, the FIR channel model is only an *approximation* of the equivalent discrete channel responses and hence an approximation of a realistic channel for the OFDM-based communication systems. Note that for an FIR channel mode, the frequency-domain channel response has a smooth transition from one period to another. Note also that the FIR channel model can be very accurate for single-carrier communication schemes, such as CDMA; this is detailed in Appendix A.2.

Remark 2: By restricting the delays in (3) to a few multiples of t_S , we have

$$h_r(t) = \sum_{l_F=0}^{L_F-1} \alpha_{l_F} \delta(t - l_F t_S). \quad (11)$$

In this case, the Type A equivalent discrete channel response of $h_r(t)$ will have a length of L_F (standing for length of FIR filter) that is smaller than N_S , and $h_r(t)$ can be *exactly* expressed by an FIR channel model. Note that this chance appears with a possibility zero in real applications.

Remark 3: Due to the above remarks, we stress that: (a) the channel parameter estimation methods for the OFDM-based communication systems cannot be designed and assessed using the FIR channel model since channel parameter estimation methods work well for the FIR channel may have degraded performance for the realistic channels [6] and (b) the design of the packet preambles cannot be based on the time-domain limited-duration property of the FIR channel model.

The following notes can be helpful in explaining a few issues about the equivalent discrete channel responses given above:

Note 1: The length of the equivalent discrete channel responses, N_S , is much longer than N_C , the length of CP. However, this will not cause the ISI problem as long as $\tau_{P_M} - \tau_0 \leq N_C - 1$ and we have the correct symbol timing. The reason is that the (sampled) receiver output over the time-dispersive channel is *not* obtained from the convolution of the equivalent discrete channel response with \mathbf{s} ; rather, it is obtained from sampling $z(t)$, the convolution of $h(t)$ and $s(t)$, as shown in Figure 2, with the portion contaminated by ISI discarded by the correct symbol timing [6].

Note 2: In the case of perfect sampling clock synchronization, i.e., the sampling clocks at the transmitter and receiver are exactly the same, an equivalent discrete channel response, be it Type A or Type B, is the same for all of the OFDM data symbols. In the case of imperfect sampling clock synchronization, which is the case for real applications, there will be sampling-shift that will cause phase rotation for the equivalent discrete channel response. This problem was addressed in detail in [6].

Note 3: If CFO simulation is needed, it should be introduced at received signal, not the transmitted signal, as detailed in Appendix A.3.

IV. A REALISTIC CHANNEL MODEL

For some simulation of the OFDM-based communication systems, the equivalent discrete channel response $\mathbf{h}^{(t)}$ can be *approximated* by an exponentially decaying FIR channel response with length L_F , denoted as

$$h_e(t) = \sum_{l_F=0}^{L_F-1} h_{l_F}^{(t)} \delta(t - l_F t_S), \quad (12)$$

where

$$h_{l_F}^{(t)} \sim \mathcal{N} \left(0, \left(1 - e^{-1/t_n} \right) e^{-l_F/t_n} \right), \quad (13)$$

with $\left\{ h_{l_F}^{(t)} \right\}_{l_F=0}^{L_F-1}$ being independent of each other, $t_n = t_r/t_S$, t_r being the root-mean-square (rms) delay spread of the frequency-selective fading channel, $L_F = \lceil 10t_n \rceil + 1$, and $\lceil x \rceil$ denoting the smallest integer not less than x . This channel model is sometimes referred to as the exponential channel model [3].

The exponential channel model is efficient for some numerical simulations, such as the detection performance simulation; however, as we have indicated earlier, this FIR channel model cannot be used to assess the channel parameter estimation methods.

The realistic channel defined by (3) is fair for the assessment of the channel parameter estimation methods, yet the direct simulation of (3) is difficult due to a potentially large P_M and the distributions of α_p . Instead, we modify (12) slightly to better characterize the realistic channels as follows:

$$h_m(t) = \sum_{l_F=0}^{L_F-1} h_{l_F}^{(t)} \delta(t - l_F t_S - t_{l_F}), \quad (14)$$

where $h_{l_F}^{(t)}$ is as in (13) and t_{l_F} , $l_F = 0, 1, \dots, L_F - 1$, is uniformly distributed over $[0, t_S]$. We also assume that $\left\{ h_{l_F}^{(t)} \right\}_{l_F=0}^{L_F-1}$ and $\{t_{l_F}\}_{l_F=0}^{L_F-1}$ are independent of each other. This new channel model, referred to as the modified exponential channel model, is more realistic than the one in (12) due to the time delays $\{t_{l_F}\}_{l_F=0}^{L_F-1}$ introduced in (14).

We have the following comments about the exponential channel model of (12) and the modified exponential channel model of (14).

Comment 1: The channel parameter estimation methods should be assessed by using the modified exponential channel model (14) first. If they work well for this model, then a large amount of simulations, especially those with detection, can be done using the exponential channel model (12) due to the points given in the following comment.

Comment 2: The exponential channel model is more efficient than the modified exponential channel model for detection performance simulations. For the exponential channel model, the sampled received data can be easily obtained by convolving the channel $\mathbf{h}_{L_F}^{(t)}$ and the transmitted signal \mathbf{s} . For the modified exponential channel model, on the other hand, the computation is much more complex, as sketched below: (a) take the FFT of the transmitted signal \mathbf{s} , (b) obtain the frequency-domain response of $h(t)$ similarly to (7), and (c) take the IFFT of the product of the above two.

Comment 3: The channel model of (14) can be easily extended to the multiple-input multiple-output (MIMO) case by using the same t_{l_F} but different $h_{l_F}^{(t)}$ for each of the MIMO channels.

V. CONCLUSIONS

As detailed in the paper, the FIR channel model cannot accurately characterize the channels for the OFDM-based systems. Therefore caution is needed when using this model for the OFDM-based systems. It is especially not appropriate to design channel estimation algorithms for the OFDM-based system squarely based on this model; when this model is not appropriate, the modified exponential channel model can be used.

Appendixes

A.1. Derivations of Equations (9) and (10)

First, let us recall that the frequency-domain channel response corresponding to the sampled signals is expressed as:

$$h_{n_S} = \sum_p \alpha_p e^{-j \frac{2\pi \tau_p [n_S - 1]_{N_S}}{N_S}}, \quad n_S = 1, 2, \dots, N_S, \quad (15)$$

where

$$[n_S - 1]_{N_S} = \begin{cases} n_S - 1, & n_S \leq N_S/2, \\ n_S - 1 - N_S, & n_S > N_S/2. \end{cases} \quad (16)$$

Second, let us derive Equation (9) with the above expression. The l th, $l = 0, 1, \dots, N_S - 1$, element of the Type A equivalent discrete channel response of $h(t)$ can be written as, due to the inverse DFT of h_{n_S} ,

$$\begin{aligned} h_l^{(t)} &= \sum_{n_S=1}^{N_S} \left(\sum_p \alpha_p e^{-j \frac{2\pi \tau_p [n_S - 1]_{N_S}}{N_S}} \right) e^{j \frac{2\pi l (n_S - 1)}{N_S}} \\ &= \sum_{n=-N_S/2}^{N_S/2-1} \left(\sum_p \alpha_p e^{-j \frac{2\pi \tau_p n}{N_S}} \right) e^{j \frac{2\pi l n}{N_S}} \\ &= \sum_p \alpha_p \sum_{n=-N_S/2}^{N_S/2-1} e^{-j \frac{2\pi (\tau_p - l) n}{N_S}} \\ &= \sum_p \alpha_p e^{j\pi(\tau_p - l)} \sum_{n=0}^{N_S-1} e^{-j \frac{2\pi (\tau_p - l) n}{N_S}} \\ &= \sum_p \alpha_p e^{j\pi(\tau_p - l)} \frac{1 - e^{-j2\pi(\tau_p - l)}}{1 - e^{-j \frac{2\pi (\tau_p - l)}{N_S}}} \\ &= \sum_p \alpha_p e^{j \frac{\pi(\tau_p - l)}{N_S}} \frac{\sin(\pi(\tau_p - l))}{\sin(\pi(\tau_p - l)/N_S)}. \end{aligned} \quad (17)$$

$$\quad (18)$$

Note that due to the failure of adequately addressing the $[\cdot]_{N_S}$ problem in (17), i.e., mistakenly using $n_S - 1$ instead of $[n_S - 1]_{N_S}$, which put the discontinued points of the periodic frequency-domain response at $\dots, -64, 0, 64, \dots$ rather than $\dots, -32, 32, \dots$ (cf. Figure 4), [4] gives an inaccurate result as:

$$h_l^{(t)} = \sum_p \alpha_p e^{-j\pi(l+(N_S-1)\tau_p)/N_S} \frac{\sin(\pi\tau_p)}{\sin(\pi(\tau_p - l)/N_S)}. \quad (19)$$

Third, let us derive Equation (10). Using the same approach as before, the l th element of the Type B equivalent discrete channel response of $h(t)$ can be written as:

$$\begin{aligned} h_l^{(t)} &= \sum_p \alpha_p \left(\sum_{n=-N_{SC}/2}^{N_{SC}/2} e^{-j\frac{2\pi(\tau_p-l)n}{N_S}} - 1 \right) \\ &= \sum_p \alpha_p \left(e^{j\frac{\pi(\tau_p-l)N_{SC}}{N_S}} \sum_{n=0}^{N_{SC}} e^{-j\frac{2\pi(\tau_p-l)n}{N_S}} - 1 \right) \\ &= \sum_p \alpha_p \left(e^{j\frac{\pi(\tau_p-l)N_{SC}}{N_S}} \frac{1 - e^{-j\frac{2\pi(\tau_p-l)(N_{SC}+1)}{N_S}}}{1 - e^{-j\frac{2\pi(\tau_p-l)}{N_S}}} - 1 \right) \\ &= \sum_p \alpha_p \left(\frac{\sin(\pi(\tau_p-l)(N_{SC}+1)/N_S)}{\sin(\pi(\tau_p-l)/N_S)} - 1 \right). \end{aligned} \quad (20)$$

A.2. The FIR channel model for single-carrier communication systems

Even though we have shown that the FIR channel model is only an approximate one for the OFDM-based communication systems, this model can be an accurate one for the single-carrier (SC) communication systems, such as the CDMA systems. This can be understood from the following two aspects.

First, the OFDM-based communication systems often aim at achieving higher transmission data rate by using larger constellations, such as 64-QAM, and thereby requiring higher signal-to-ratio (SNR) compared to the SC systems. While a certain amount of approximation is tolerable for the SC systems, it may cause serious problem for the OFDM-based systems.

Second, which is more important, the SC systems often use the limited-duration information-carrying pulses. For this kind of pulses, an integrate-and-dump (IND) filter (can be a matched filter) will have no output if there is no overlapping between the integration time and the pulse duration; see, for example, [8] and the references therein. For instance, if we use the rectangular pulse with duration t_S and use the IND filter which delivers its output every t_S second, then the non-zero outputs can be at most $\lceil \tau_{P_M} - \tau_0 + 1 \rceil$ long for the channel given in (3), which is a perfect FIR channel response. (Here $\lceil x \rceil$ stands for the smallest integer that is not smaller than x .)

The fact that the FIR channel model can be an accurate model for the SC communication systems is perhaps the main reason that some researchers use this model without scrutinizing in the OFDM-based systems.

A.3. A note on the CFO effect simulation

The CFO is the difference between the oscillators at the transmitter and receiver ends. It seems that this difference can be added at either the transmitted signal, $s(t)$, or the received signal, $z(t)$. (For notational convenience, We use the analog signals to make our points.) This can be true for the case of estimated channel parameters. Yet, for the case of perfect channel knowledge, special attention should be paid for the simulation of CFO. If the CFO is added on the received signal, everything is fine. If, on the other hand, the CFO is added on the transmitted signal, a compensation should be made to the channel response. This can be seen from the following:

$$\begin{aligned}
 & h(t) * \left(s(t) e^{j2\pi f_s \epsilon t} \right) \\
 &= \int s(t - \tau) e^{j2\pi f_s \epsilon (t - \tau)} h(\tau) d\tau \\
 &= e^{j2\pi f_s \epsilon t} \int s(t - \tau) h(\tau) e^{-j2\pi f_s \epsilon \tau} d\tau \\
 &= e^{j2\pi f_s \epsilon t} \left[z(t) * \left(h(t) e^{-j2\pi f_s \epsilon t} \right) \right], \tag{21}
 \end{aligned}$$

where $h(t) e^{-j2\pi f_s \epsilon t}$ is the actual channel response for this case.

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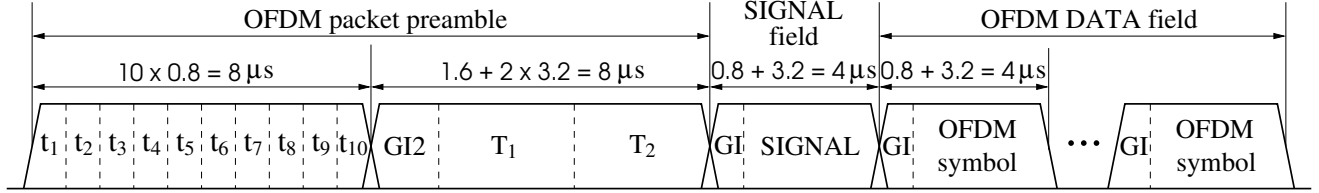


Fig. 1. Illustration of a packet defined by the IEEE 802.11a standard.

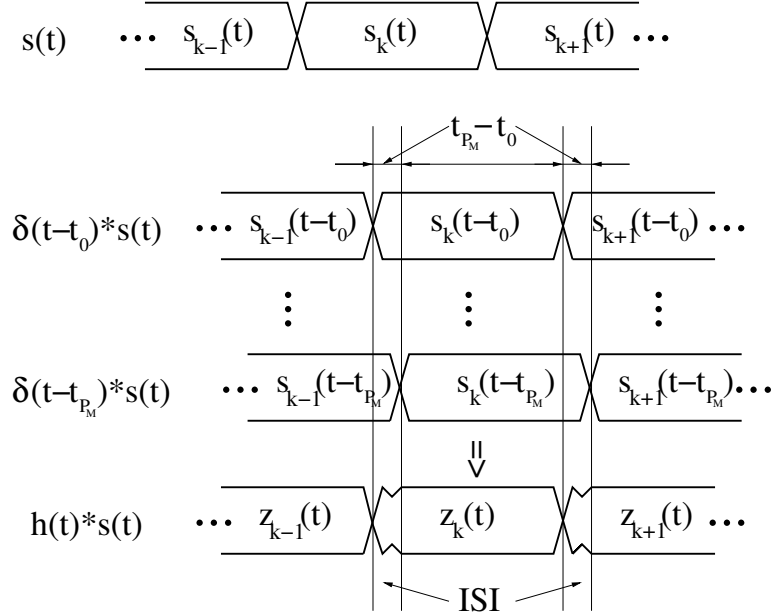
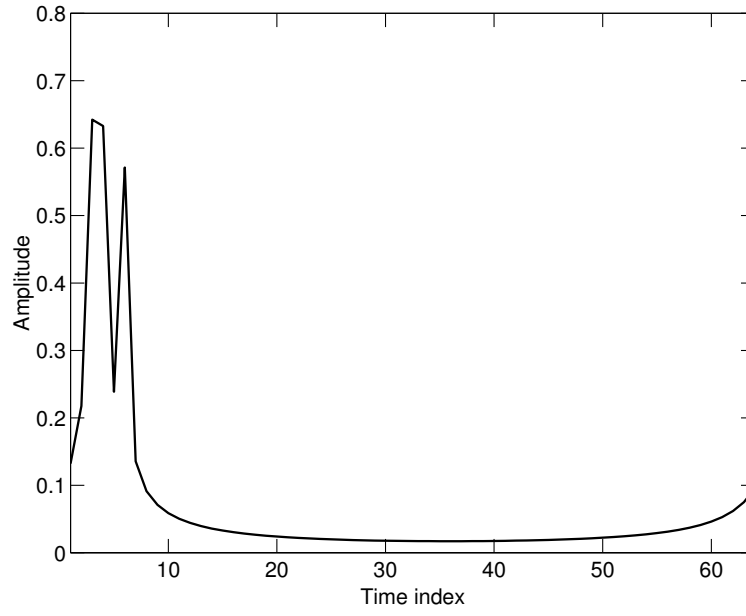
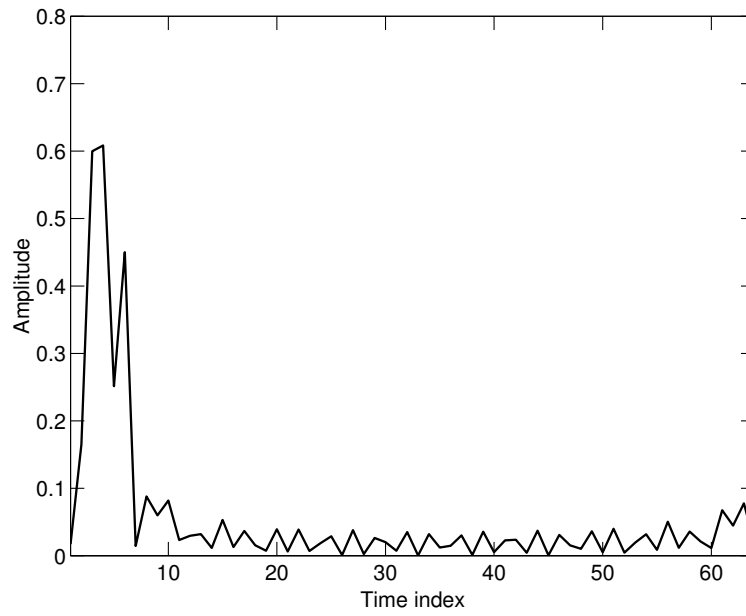


Fig. 2. Illustrations of the analog waveforms for OFDM data symbols and the output of a multipath channel.



(a)



(b)

Fig. 3. Illustration of the amplitude of two types of equivalent discrete channel response for a multipath channel described by $h(t) = \delta(t - 2.5t_s) - j0.5\delta(t - 4.8t_s)$: (a) Type A; (b) Type B.

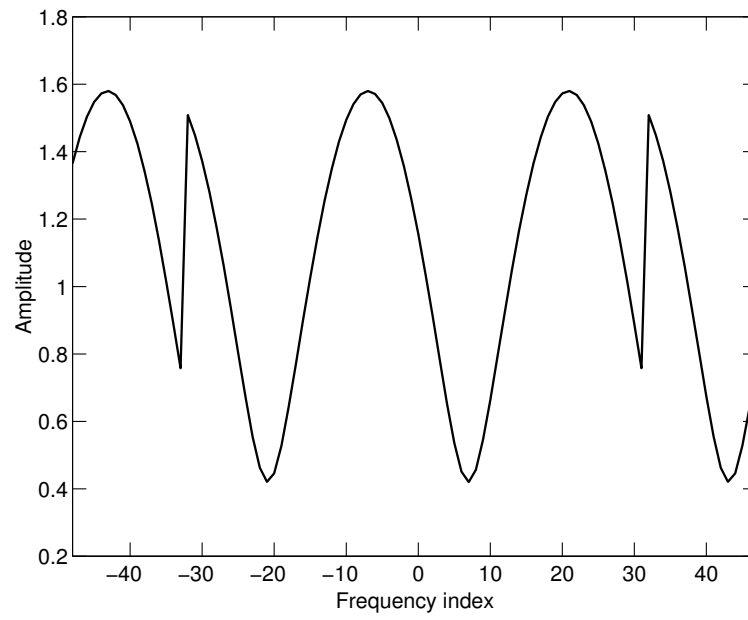


Fig. 4. Illustration of the discontinuities in the amplitude of the frequency-domain channel response for a multipath channel described by $h(t) = \delta(t - 2.5t_S) - j0.5\delta(t - 4.8t_S)$.