Channel Estimation for an OFDM-Based MIMO System

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Abstract

We consider the problem of channel estimation for an orthogonal frequency-division multiplexing (OFDM)-based multiple-input multiple-output (MIMO) wireless local area network system. We show that an existing channel estimation approach, which is based on a sub-carrier level orthogonal training symbol design and the finite impulse response channel model, can suffer from severe performance degradations for realistic OFDM channels. As such, we propose a new channel estimation approach, which employs a modified version of this training symbol design and a polynomial fitting/interpolation method. Our simulation results show that our new channel estimation approach significantly outperforms the existing one for the realistic OFDM channels.

Index Terms—Channel estimation, OFDM, MIMO system, Wireless LAN standards.

I. Introduction

Orthogonal frequency-division multiplexing (OFDM) has been selected as the basis for several similar new high-speed wireless local area network (WLAN) standards [1], including IEEE 802.11a, IEEE 802.11g, and HIPERLAN/2, which support a data rate up to 54 Mbps. Future WLAN standards, such as the IEEE 802.11n [2], need to support transmission data rates higher than 54 Mbps. One promising way of increasing the transmission data rate is to deploy multiple antennas at both the transmitter and receiver ends to exploit the huge channel capacity offered by such a system in a multipath-rich environment. The corresponding system is referred to as a multiple-input multiple-output (MIMO) WLAN system. Several OFDM-based MIMO WLAN systems have been proposed for the multipath-rich, time-invariant, and frequency-selective fading channels; see, e.g., [3], [4] and the references therein. Channel response estimation is important for these systems, and it is usually performed based on training.

In this paper, we consider the training-based channel response estimation for the OFDM-based MIMO system described in [4], which has $M = 2$ transmit antennas and $N = 2$ (N can be larger than 2) receive antennas (this system is referred to as the MIMO system for short in the sequel). A large number of training-based channel response estimation approaches have been proposed for systems like the one considered herein. The training symbols for a majority of these approaches can be casted into two classes: Class I—sub-carrier level orthogonal training symbols (see, e.g., [5], [6]) and Class II—symbol level orthogonal training symbols (see, e.g., [3], [4]).

For systems with Class I training symbols, one transmit antenna (denoted as Tx 1) uses one half of all the sub-carriers appropriated for training and the other (denoted as Tx 2) uses the remaining half, and the transmissions from the two transmit antennas do not interfere with each other—the training symbols from the two transmit antennas are orthogonal to each other at the sub-carrier level. The Class I training symbols were proposed based on the assumption that an OFDM channel can be represented by a finite impulse response (FIR) filter, and a maximum-likelihood (ML) estimation method is used with these symbols to estimate the FIR channel response in time domain; see [5] for such an existing channel estimation approach, which is referred to as the ML-FIR approach herein. An advantage of ML-FIR is that the accompanying Class I training symbols can be of the same length as those for a single-input single-output (SISO) system. Yet, as pointed out in [7], [8], the FIR channel model is only a poor approximation of the realistic OFDM channels. For the realistic OFDM channels, channel estimates obtained from approaches such as ML-FIR suffer from severe performance degradations.

For systems with Class II training symbols, each transmit antenna uses all the sub-carriers appropriated for training, and the transmissions from the two transmit antennas are separated using on/off transmission (see, e.g., [3]) or plus/minus transmission (see, e.g., [4])—the training symbols are orthogonal to each other at the symbol level. An advantage of using Class II training symbols is that the simple sub-carrier training method, which is the same as for the SISO case [8], can be used for channel response estimation in frequency domain (FD), which can obviate the necessity of the FIR model assumption. Yet, the length of the Class II training symbols is usually twice as long as that for a SISO system, and hence has a larger overhead.

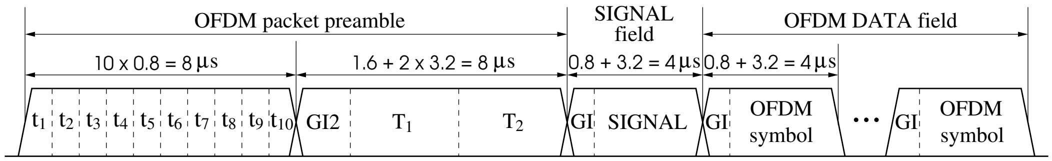
Our focus herein is to improve the overhead/channel estimation performance trade-offs for the MIMO system. We propose a new channel estimation approach, which is based on a modified version of the Class I training symbols. The new approach, which exploits the polynomial approximation [9] of the FD channel response, is referred to as the polynomial fitting/interpolation (PF/I) approach. PF/I estimates channel response in FD by first applying polynomial fitting to the sub-carriers containing the training symbols and then performing interpolation to cover the sub-carriers without training symbols. Our simulation results show that PF/I can significantly outperform ML-FIR for realistic OFDM channels. This suggests that the polynomial model is superior to the FIR model for channel response estimation for the OFDM signaling.

II. Background

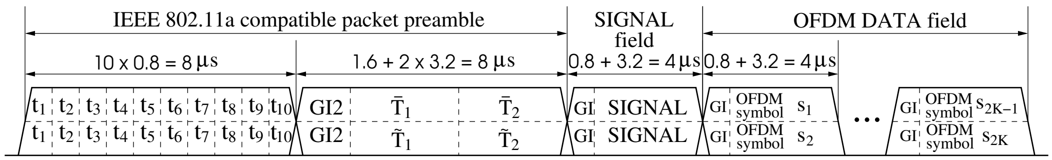
Before describing the background for the MIMO channel response estimation—the MIMO preamble and channel model—we first give an overview of the IEEE 802.11a standard, based on which the MIMO system is designed.

A. Overview of IEEE 802.11a

The OFDM packet preamble of a SISO system, as shown in Fig. 1(a), consists of 10 identical short OFDM training symbols $\mathrm{t}_i, i = 1, 2, \ldots, 10$, each of which contains $N_C = 16$ samples, and 2 identical long OFDM training symbols $\mathrm{T}_i, i = 1, 2$, each of which contains $N_S = 64$ samples. (The nominal bandwidth of the OFDM signal is 20 MHz and the in-phase/quadrature sampling interval t_S is 50 ns.) Between the short and long OFDM



(a)



(b)

Fig. 1. Packet structure for the OFDM-based WLANs: (a) an IEEE 802.11a conformable SISO system and (b) the MIMO system.

training symbols there is a long guard interval (GI2) consisting of $2N_C = 32$ data samples, which is the cyclic prefix (CP) for T_1. The long training symbols can be used to estimate the SISO channel response.

The information carrying data are encoded in the OFDM DATA field. Each OFDM data symbol employs $N_S = 64$ sub-carriers, 52 of which are used for data or pilot symbols. There are also 12 null sub-carriers with one in the center and the other 11 on the two ends of the frequency band. The OFDM data symbols are obtained via taking IFFT of the data symbols, pilot symbols, and nulls on these N_S sub-carriers. To eliminate the inter-symbol interference, each OFDM data symbol is preceded by a CP or guard interval (GI) containing the last N_C samples of the OFDM data symbol.

B. MIMO Preamble with Modified Class I Training Symbols

For the MIMO system considered herein, two packets, with corresponding preambles, are transmitted simultaneously from Tx 1 and Tx 2. To be backward compatible with the SISO preamble, the same short training symbols as in the SISO one are used for both Tx 1 and Tx 2. However, the long training symbols cannot be used directly.

The Class I training symbols, i.e., the long OFDM training symbols for the MIMO system, are obtained by first splitting orthogonally [5], [6] the original IEEE 802.11a FD training sequence, denoted by x's, to obtain two training sequences with zeros inserted at appropriate places, as shown in Table I, and then taking IFFTs of the two training sequences. Note that we have modified the Class I training symbols by adding two additional training bits, each at one end of the bandwidth, to improve the channel estimation performance. Let the corresponding time-domain (TD) training symbols for Tx 1 and Tx 2 be denoted as $\bar{\mathrm{T}}_1$ and $\tilde{\mathrm{T}}_1$, respectively, and let $\bar{\mathrm{T}}_2 = \bar{\mathrm{T}}_1$ and $\tilde{\mathrm{T}}_2 = \tilde{\mathrm{T}}_1$. Then we have the MIMO preamble design, as shown in Fig. 1(b). The MIMO preamble with the modified Class I training symbols is shorter than the one given in [4], which was designed to be IEEE 802.11a backward compatible, and hence the former has a smaller overhead than the latter. Note that the design in Fig. 1(b) is also backward compatible with IEEE 802.11a since exactly the same FD training sequence as IEEE 802.11a is transmitted on the original 52 sub-carriers and the two additional sub-carriers can be discarded automatically by the SISO system.

TABLE I

FD TRAINING SEQUENCES FOR THE MODIFIED CLASS I TRAINING SYMBOLS FOR THE MIMO SYSTEM.

$[n_S-1]_{N_S}$	-27	-26	-25	-24	...	-2	-1	0	1	2	...	24	25	26	27
Tx 1	-1	0	x	0	...	0	x	0	0	x	...	x	0	x	0
Tx 2	0	x	0	x	...	x	0	0	x	0	...	0	x	0	-1

C. Realistic OFDM Channel Model

For the SISO system, the TD channel response is sometimes modeled by an exponentially decaying FIR filter with length L_F

$$h_e(t) = \sum_{l_F=0}^{L_F-1} h_{l_F}^{(\mathrm{t})} \delta(t - l_F t_S), \tag{1}$$

where

$$h_{l_F}^{(\mathrm{t})} \sim \mathcal{N}\left(0, \left(1 - e^{-1/t_n}\right) e^{-l_F/t_n}\right) \tag{2}$$

(zero-mean Gaussian) with $\left\{h_{l_F}^{(\mathrm{t})}\right\}_{l_F=0}^{L_F-1}$ being independent of each other, $t_n = t_r/t_S$, t_r being the root-mean-square delay spread of the frequency-selective fading channel, $L_F = \lceil 10t_n \rceil + 1$, and $\lceil x \rceil$ denoting the smallest integer not less than x. While the FIR model is convenient for simulation purposes, it simulates only a very special case of the true OFDM channel, where the time delays are constrained to the multiples of the sampling instants [7], [8].

We modify (1) slightly to obtain a *generic* model for realistic OFDM channels as

$$h_m(t) = \sum_{l_F=0}^{L_F-1} h_{l_F}^{(\mathrm{t})} \delta(t - l_F t_S - t_{l_F}), \tag{3}$$

where for $l_F = 0, 1, \ldots, L_F - 1$, the multipath gain $h_{l_F}^{(\mathrm{t})}$ is as in (2) and the time delay t_{l_F} is uniformly distributed over $[0, t_S]$. We also assume that $\left\{h_{l_F}^{(\mathrm{t})}\right\}_{l_F=0}^{L_F-1}$ and $\{t_{l_F}\}_{l_F=0}^{L_F-1}$ are independent of each other. The channel model in (3) is more realistic than the one in (1) due to the flexible time delays $\{t_{l_F}\}_{l_F=0}^{L_F-1}$ introduced in (3).

Consider the base-band processing and an ideal low-pass filtering before A/D conversion at the receiver. The corresponding FD channel response (also called channel gain) of (3) on sub-carrier n_S, $n_S = 1, 2, \ldots, N_S$, is then given by

$$h_{n_S} = \sum_{l_F=0}^{L_F-1} h_{l_F}^{(\mathrm{t})} e^{-j2\pi \frac{l_F t_S + t_{l_F}}{N_S t_S} [n_S-1]_{N_S}}, \tag{4}$$

where $[n_S - 1]_{N_S}$ is equal to $n_S - 1$ for $n_S \leq N_S/2$ and $n_S - 1 - N_S$ for $n_S > N_S/2$.

For the MIMO system, each channel can be modeled by (3). For path l_F, $l_F = 0, 1, \ldots, L_F - 1$, the path gains for the MIMO channels are assumed to be independent of each other (correlations can be easily added), whereas the time delays are assumed the same for all the channels.

III. CHANNEL ESTIMATION

For the MIMO system, the task of channel estimation is to estimate the FD channel gain matrix

$$\mathbf{H} = \begin{bmatrix} h_{n_S}^{(1,1)} & h_{n_S}^{(1,2)} \\ h_{n_S}^{(2,1)} & h_{n_S}^{(2,2)} \end{bmatrix} \in \mathbb{C}^{2\times 2} \tag{5}$$

for the 52 data/pilot carrying sub-carriers, where $h_{n_S}^{(n,m)}$ denotes the channel gain from Tx m to receive antenna (Rx) n on the n_Sth sub-carrier.

To estimate the channel responses on those sub-carriers without training symbols, we must exploit the channel correlations among sub-carriers. One simple way is to resort to channel models to approximate the OFDM channels. The FIR model of [5], [6] is an example and is widely used. An alternative model, which we advocate here, is the recently introduced piecewise polynomial model [9]. It has been successfully used to improve the channel response estimation accuracy for SISO OFDM channels. It is preferred over the FIR counterpart for realistic OFDM channels since: (a) the FD channel response of an FIR filter *must be periodic* in N_S (as can be seen in (4) by dropping the t_{l_F}'s) whereas the FD channel response of a realistic OFDM channel *cannot be periodic* unless $t_{l_F} = 0$ for all l_F, which is clear from (4), and (b) the FD channel response described by a polynomial *can be flexible* (without this periodicity constraint). An additional advantage of the polynomial approximation is that it is not sensitive to the responses of the filters used at the transmitter and receiver since the effects of these filters can be easily absorbed into the polynomials.

Consider the estimation of the channels from Tx 1 and Tx 2 to Rx n, $n = 1, 2$. As in [9], we use piecewise polynomials to approximate the FD channel responses. We group the sub-carriers into 4 sets, with each set corresponding to a polynomial. This grouping depends on the transmit antenna: for Tx 1, the 4 sets are, respectively,

$$\begin{aligned} \mathcal{S}_{1,1} &= \{-27, -26, \ldots, -13\}, \\ \mathcal{S}_{1,2} &= \{-15, -14, \ldots, -1\}, \\ \mathcal{S}_{1,3} &= \{-1, 0, \ldots, 14\}, \\ \mathcal{S}_{1,4} &= \{12, 13, \ldots, 26\}; \end{aligned}$$

and for Tx 2, the 4 sets are, respectively,

$$\begin{aligned} \mathcal{S}_{2,1} &= \{-26, -25, \ldots, -12\}, \\ \mathcal{S}_{2,2} &= \{-14, -12, \ldots, 1\}, \\ \mathcal{S}_{2,3} &= \{1, 2, \ldots, 15\}, \\ \mathcal{S}_{2,4} &= \{13, 14, \ldots, 27\}. \end{aligned}$$

We remark that the above sets are grouped in such a way that each set contains 8 sub-carriers with training symbols, as can be seen from Table I.

A polynomial of order P can be used to approximate the FD channel response on each of the above sets. For example, for channel $h_{n_S}^{(n,1)}$ on $\mathcal{S}_{1,1}$ ($-27 \le [n_S - 1]_{N_S} \le -13$), we have

$$h_{n_S}^{(n,1)} = \sum_{p=0}^{P} \alpha_p^{(n,1,1)} \left([n_S - 1]_{N_S} - f^{(n,1,1)} \right)^p + e_{n_S,P}^{(n,1,1)}, \tag{6}$$

where $\alpha_p^{(n,1,1)}$, $p = 0, 1, \ldots, P$, is the pth (complex) polynomial coefficient, $e_{n_S,P}^{(n,1,1)}$ is the model error depending on P and $f^{(n,1,1)} = -20$ is used to shift the initial location of the polynomial. (For the above three-letter-superscript indexing, the first two are used to refer to the channel between Tx 1 and Rx n, and the last two the sub-carrier set.)

Now, let us consider the new PF/I method for the MIMO system by considering $h_{n_S}^{(n,1)}$ on $\mathcal{S}_{1,1}$ as an example. Let $\mathbf{v}_1 = [-7 \ \ -5 \ \ -3 \ \ -1 \ 1 \ 3 \ 5 \ 7]^T$ (with T denoting the transpose) and $\mathbf{V}_1$ be the $8 \times (P+1)$ matrix with the ith column being $\mathbf{v}_1^{\cdot(i-1)}$ (element-wise power). Let $\hat{\mathbf{h}}_C^{(n,1,1)} \in \mathbb{C}^{8\times 1}$ be the channel gain estimates on the sub-carriers with training symbols, i.e., sub-carriers $\{-27, -25, -23, -21, -19, -17, -15, -13\}$, obtained via sub-carrier training in the same way as for the SISO system. Then the polynomial coefficients obtained via polynomial fitting (PF) can be written as

$$\begin{aligned}\hat{\boldsymbol{\alpha}}^{(n,1,1)} &= [\hat{\alpha}_0^{(n,1,1)} \ \ \hat{\alpha}_1^{(n,1,1)} \ \ \ldots \ \ \hat{\alpha}_P^{(n,1,1)}]^T \\ &= (\mathbf{V}_1^T\mathbf{V}_1)^{-1}\mathbf{V}_1^T\hat{\mathbf{h}}_C^{(n,1,1)}. \end{aligned} \tag{7}$$

Let $\hat{\mathbf{h}}_F^{(n,1,1)}$ be the PF counterpart of $\hat{\mathbf{h}}_C^{(n,1,1)}$. Then $\hat{\mathbf{h}}_F^{(n,1,1)}$ can be obtained using

$$\hat{\mathbf{h}}_F^{(n,1,1)} = \mathbf{V}_1\hat{\boldsymbol{\alpha}}^{(n,1,1)}. \tag{8}$$

Let $\mathbf{v} = [-6 \ \ -4 \ \ -2 \ 0 \ 2 \ 4 \ 6]^T$ and $\mathbf{V}$ be the matrix formed from $\mathbf{v}$ in the same way as $\mathbf{V}_1$ from $\mathbf{v}_1$. Let $\hat{\mathbf{h}}_I^{(n,1,1)}$ be the channel response estimate on sub-carriers $\{-26, -24, -23, -20, -18, -16, -16\}$ via polynomial interpolation. Then, we have

$$\hat{\mathbf{h}}_I^{(n,1,1)} = \mathbf{V}\hat{\boldsymbol{\alpha}}^{(n,1,1)}. \tag{9}$$

Combining $\hat{\mathbf{h}}_F^{(n,1,1)}$ and $\hat{\mathbf{h}}_I^{(n,1,1)}$, we obtain the estimate of the channel response from Tx 1 to Rx n on $\mathcal{S}_{1,1}$.

In exactly the same way, we can estimate the channel responses from Tx 1 to Rx n on $\mathcal{S}_{1,2}$ and $\mathcal{S}_{1,4}$ as well as those from Tx 2 to Rx n on $\mathcal{S}_{2,1}$, $\mathcal{S}_{2,3}$, and $\mathcal{S}_{2,4}$. As for channels from Tx 1 to Rx n on $\mathcal{S}_{1,3}$ and from Tx 2 to Rx n on $\mathcal{S}_{2,2}$, the response can be obtained similarly. Let $\mathbf{v}_2 = [-8 \ \ -5, \ \ -3 \ \ -1 \ 1 \ 3 \ 5 \ 7]^T$ and $\mathbf{V}_2$ be the matrix formed from $\mathbf{v}_2$ in the same way as $\mathbf{V}_1$ from $\mathbf{v}_1$. Let $\hat{\boldsymbol{\alpha}}^{(n,1,3)}$, $\hat{\mathbf{h}}_C^{(n,1,3)}$, $\hat{\mathbf{h}}_F^{(n,1,3)}$, and $\hat{\mathbf{h}}_I^{(n,1,3)}$ be the counterparts of $\hat{\boldsymbol{\alpha}}^{(n,1,1)}$, $\hat{\mathbf{h}}_C^{(n,1,1)}$, $\hat{\mathbf{h}}_F^{(n,1,1)}$, and $\hat{\mathbf{h}}_I^{(n,1,1)}$. Then we have, corresponding to (7), (8), and (9), respectively,

$$\hat{\boldsymbol{\alpha}}^{(n,1,3)} = (\mathbf{V}_2^T\mathbf{V}_2)^{-1}\mathbf{V}_2^T\hat{\mathbf{h}}_C^{(n,1,3)}, \tag{10}$$

$$\hat{\mathbf{h}}_F^{(n,1,3)} = \mathbf{V}_2\hat{\boldsymbol{\alpha}}^{(n,1,3)}, \tag{11}$$

$$\hat{\mathbf{h}}_I^{(n,1,3)} = \mathbf{V}\hat{\boldsymbol{\alpha}}^{(n,1,3)}. \tag{12}$$

IV. Numerical Examples

We provide two numerical examples to demonstrate the performance of our new channel estimation approach. Due to the fact that 52 out of 64 sub-carriers are used in the OFDM-based WLAN systems, the SNR used herein is the symbol-to-noise-power ratio normalized by $52/64$. For a given SNR, we use the same total transmission power for both the MIMO and SISO systems.

Fig. 2 shows the mean-squared-error (MSE) comparison of PF/I and ML-FIR as a function of P for the MIMO system with the modified Class I training symbols over realistic OFDM channels generated according to (3) with $t_r = 50$ ns and SNR being 20 and 30 dB, respectively. (For ML-FIR, P is the length of FIR channel.) Fig. 3 shows the MSE comparison of PF/I (with $P = 5$) and ML-FIR ($P = 19$) as a function of SNR

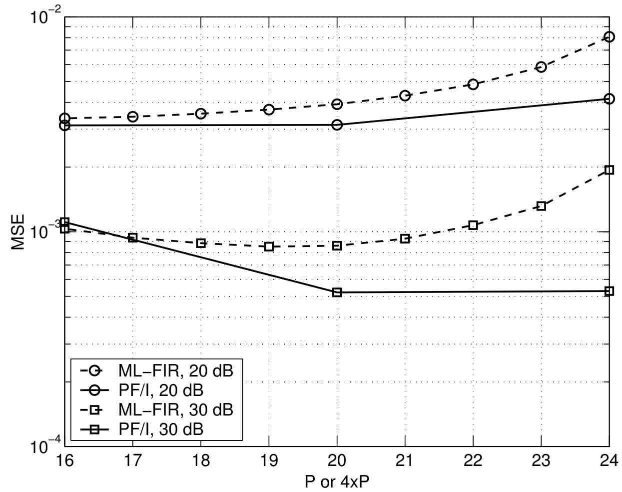


Fig. 2. MSE versus P or $4P$ comparison between PF/I and ML-FIR.

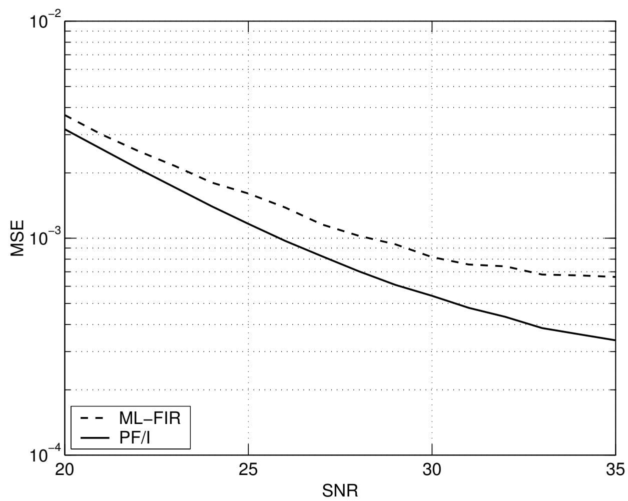


Fig. 3. MSE versus SNR comparison between PF/I and ML-FIR.

under the same simulation conditions as for Fig. 2. We can see from the figures that our new PF/I approach significantly outperforms ML-FIR for realistic OFDM channels, especially at high SNRs which are desired for high data rate MIMO systems [4]. This suggests that the polynomial model is superior to its FIR counterpart for channel estimation for the OFDM signaling.

V. Concluding Remarks

We have proposed a new channel estimation approach, a PF/I method based on a modified Class I training symbol design, for an OFDM-based MIMO WLAN system. Simulation results have shown that the new channel estimation approach can significantly outperform existing ones.

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