

# Recommender Systems

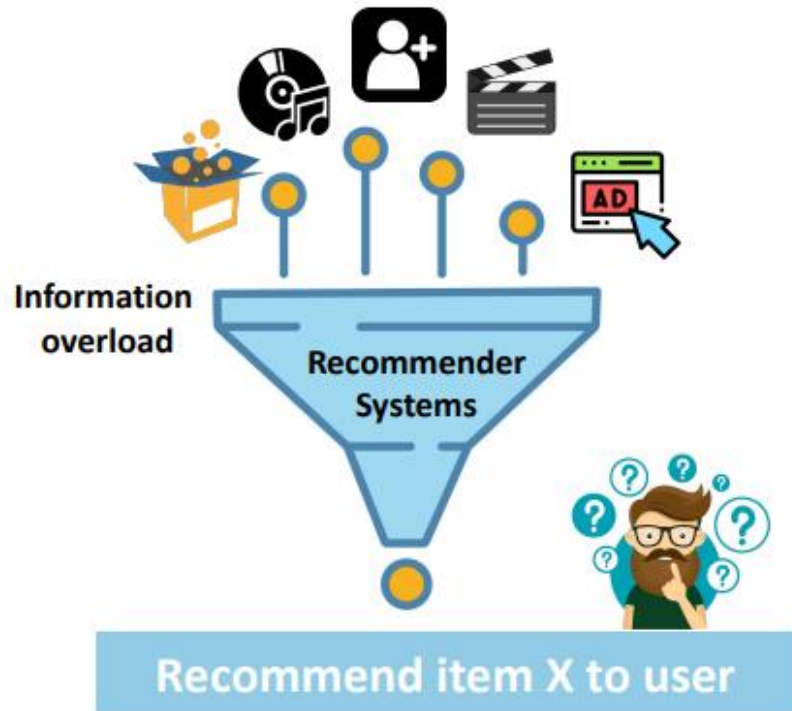
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The Age of Search has come to an end  
...long live the Age of Recommendation!

Chris Anderson "The Long Tail"



# Recommender systems



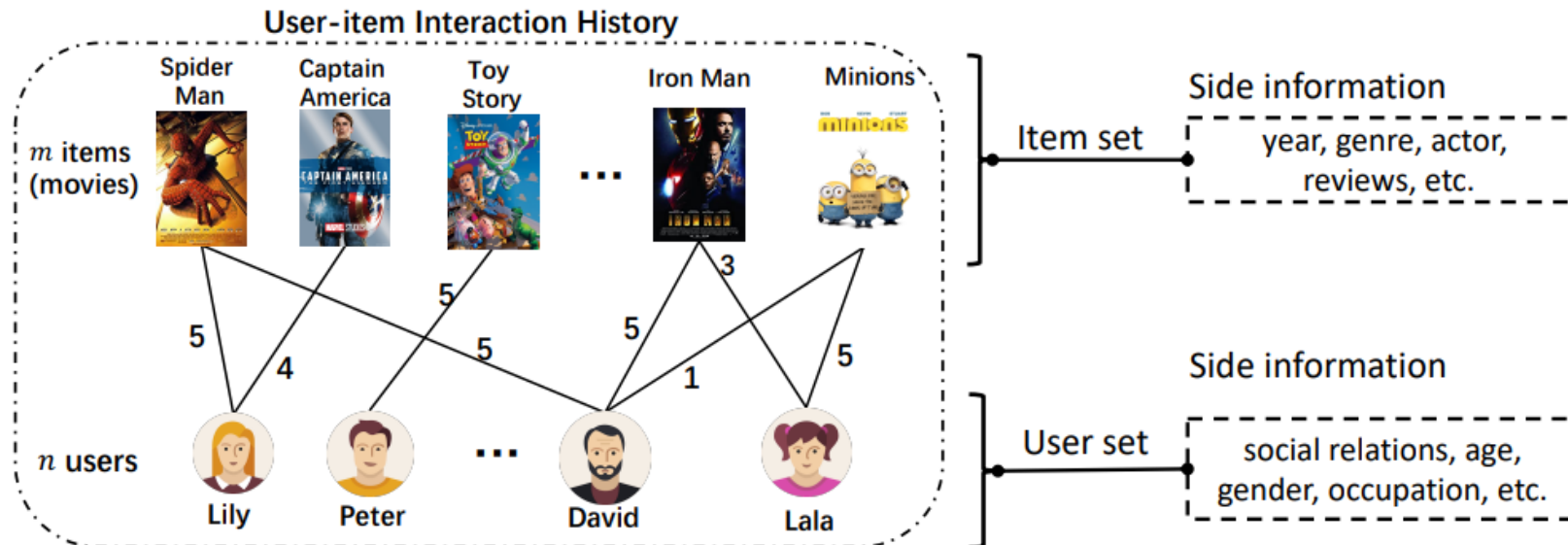
**Items** can be: Products, News, Movies, Videos, Friends, etc.

**Personalized Web-based applications that provide users with personalized recommendations about content they may be interested in**

# Problem formulation



Historical user-item interactions or additional side information (e.g., social relations, item's knowledge, etc.)

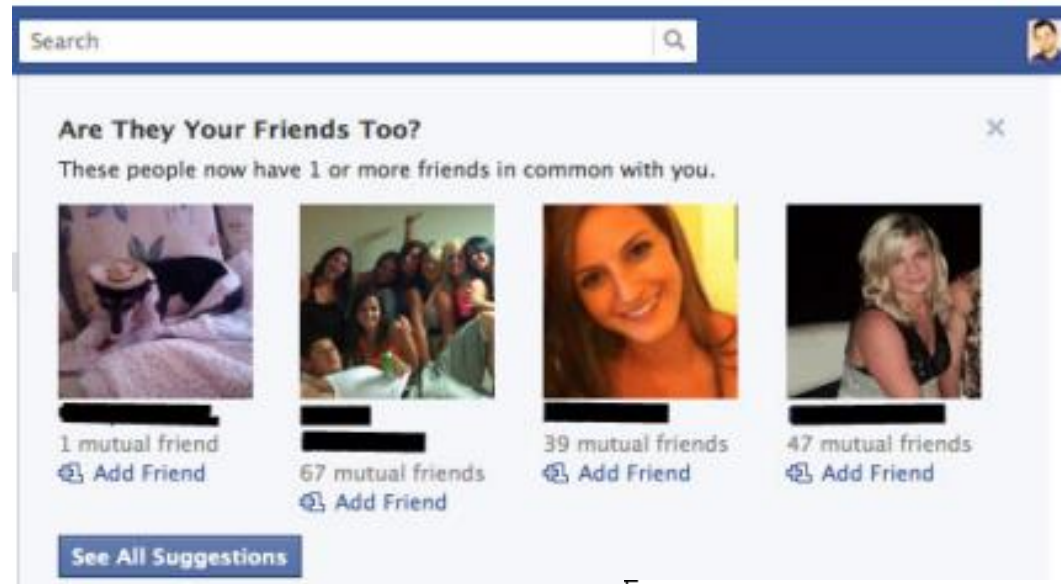


Tutorial website: <https://deeprs-tutorial.github.io>

# Problem formulation



Predict how likely a user would interact with a target Item (e.g., click, view, or purchase)



Tutorial website: <https://deeprs-tutorial.github.io>

Applications

# Application: E-commerce,

- Recommendation has been widely applied in online services



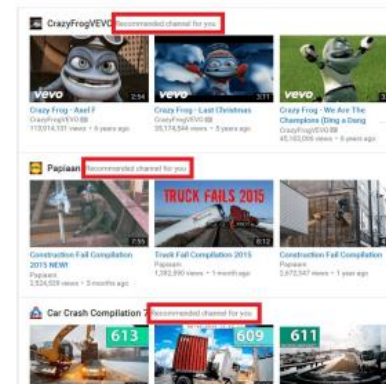
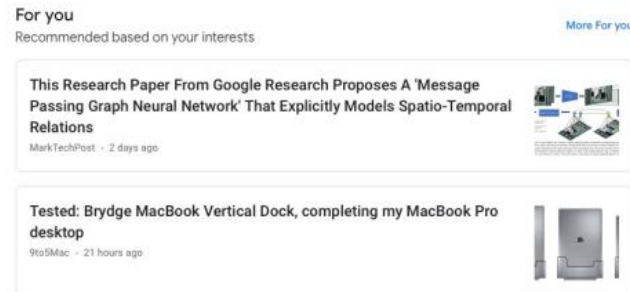


# Application: Content sharing

- Recommendation has been widely applied for content sharing



## News/Video/Image Recommendation

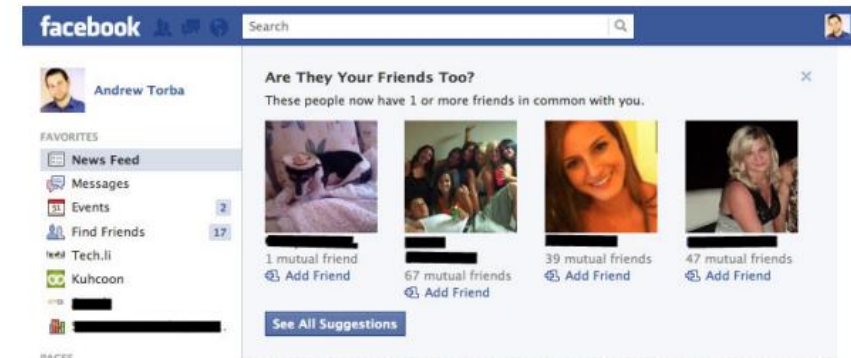


# Application: Social

- Recommendation has been widely applied in social networks



Friend Recommendation





# The value of recommendations



**75%**

of consumers tried a new  
shopping behavior during  
the pandemic<sup>1</sup>



**71%**

of consumers expect  
personalization<sup>2</sup>



**76%**

of consumers get frustrated  
when they don't find it<sup>2</sup>

Source: [McKinsey](#).

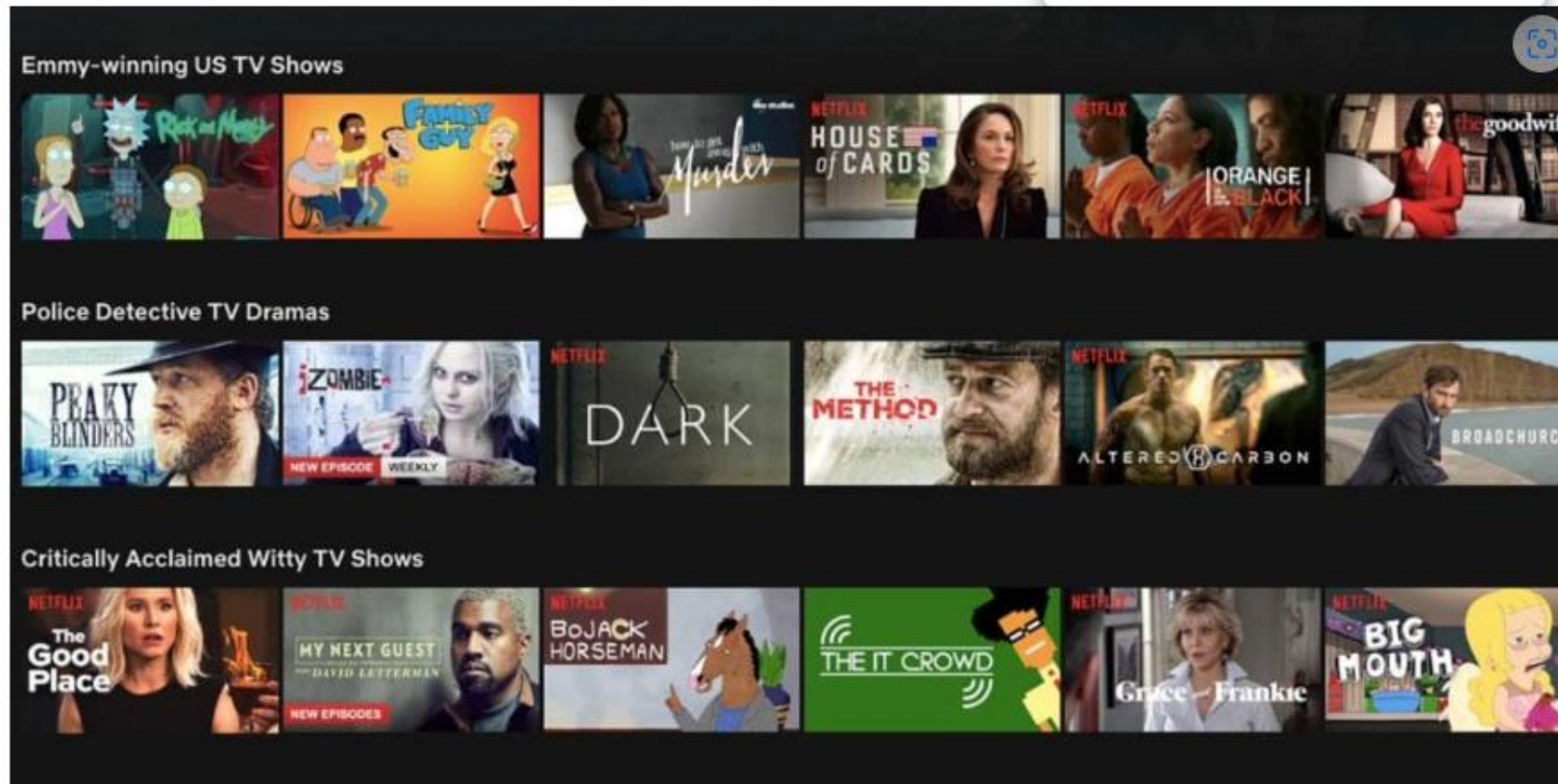
# The value of recommendations: Amazon

- [According to McKinsey](#), 35% of Amazon purchases are thanks to recommendation systems.



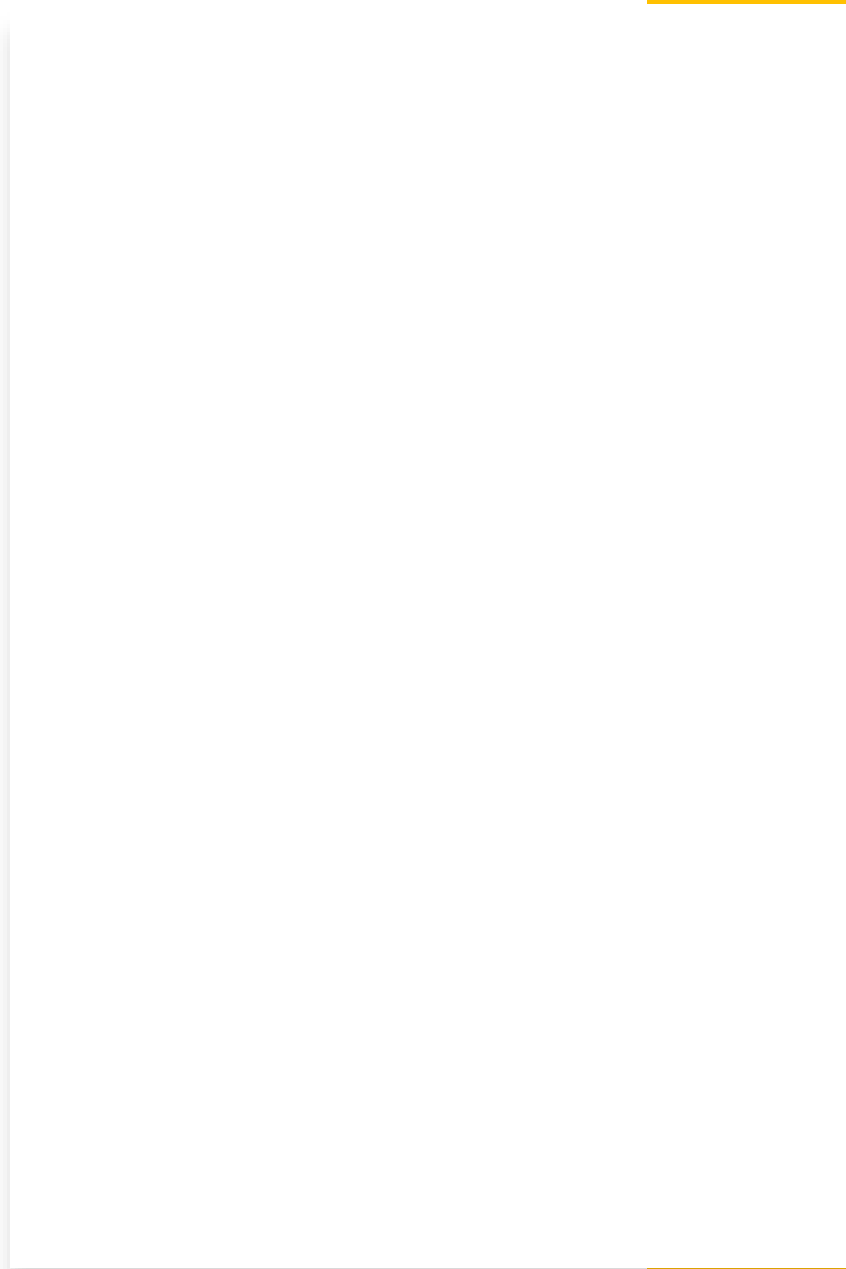
# The value of recommendations: Netflix

- McKinsey study we mentioned above highlights that 75% of Netflix viewing is driven by recommendations. In fact, Netflix is so obsessed with providing the best results for users that they held [data science competitions](#) called [Netflix Prize](#) where one with the most accurate movie recommendation algorithm wins a prize worth \$1,000,000.
- [How Netflix's Recommendations System Works](#)

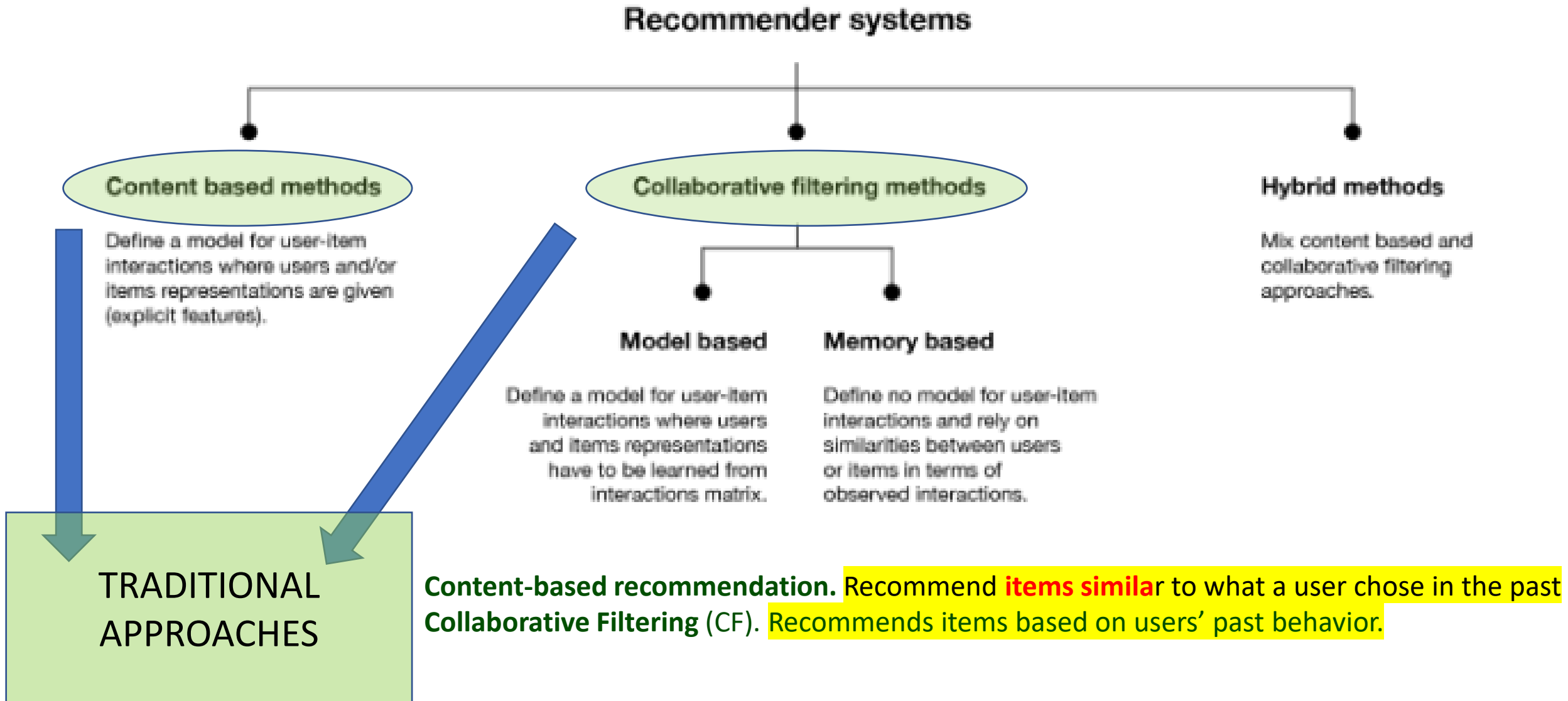




# Taxonomy

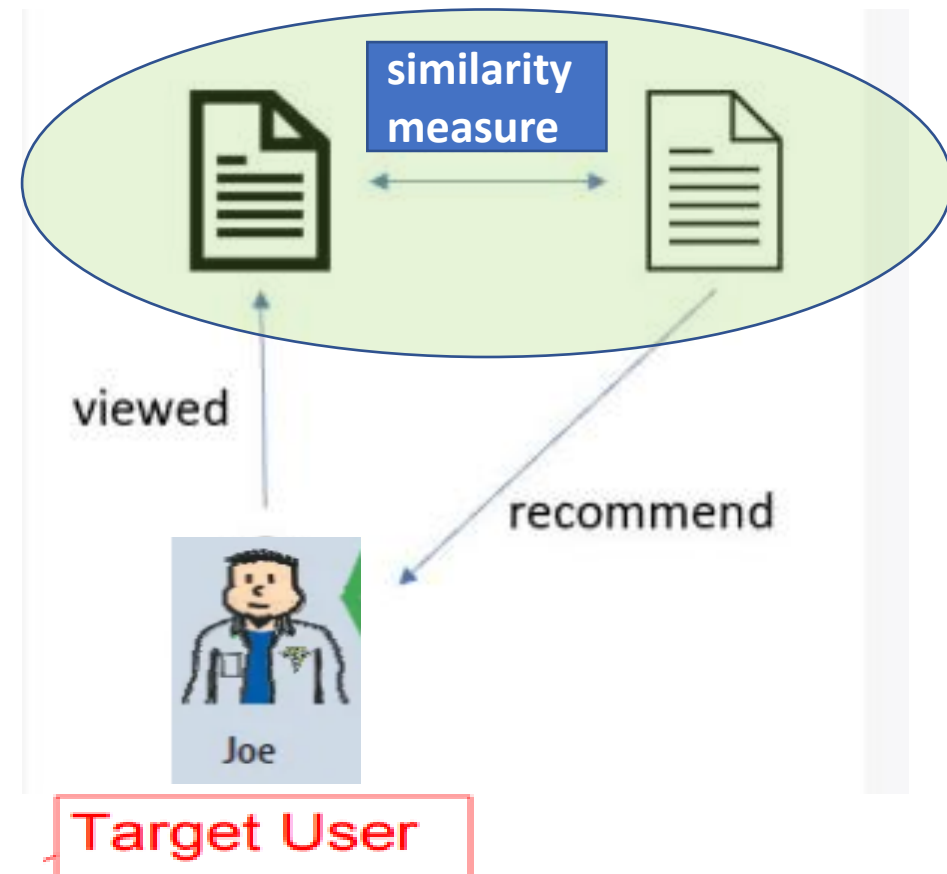


# Taxonomy



# FOCUS: Content-Based Recommendation

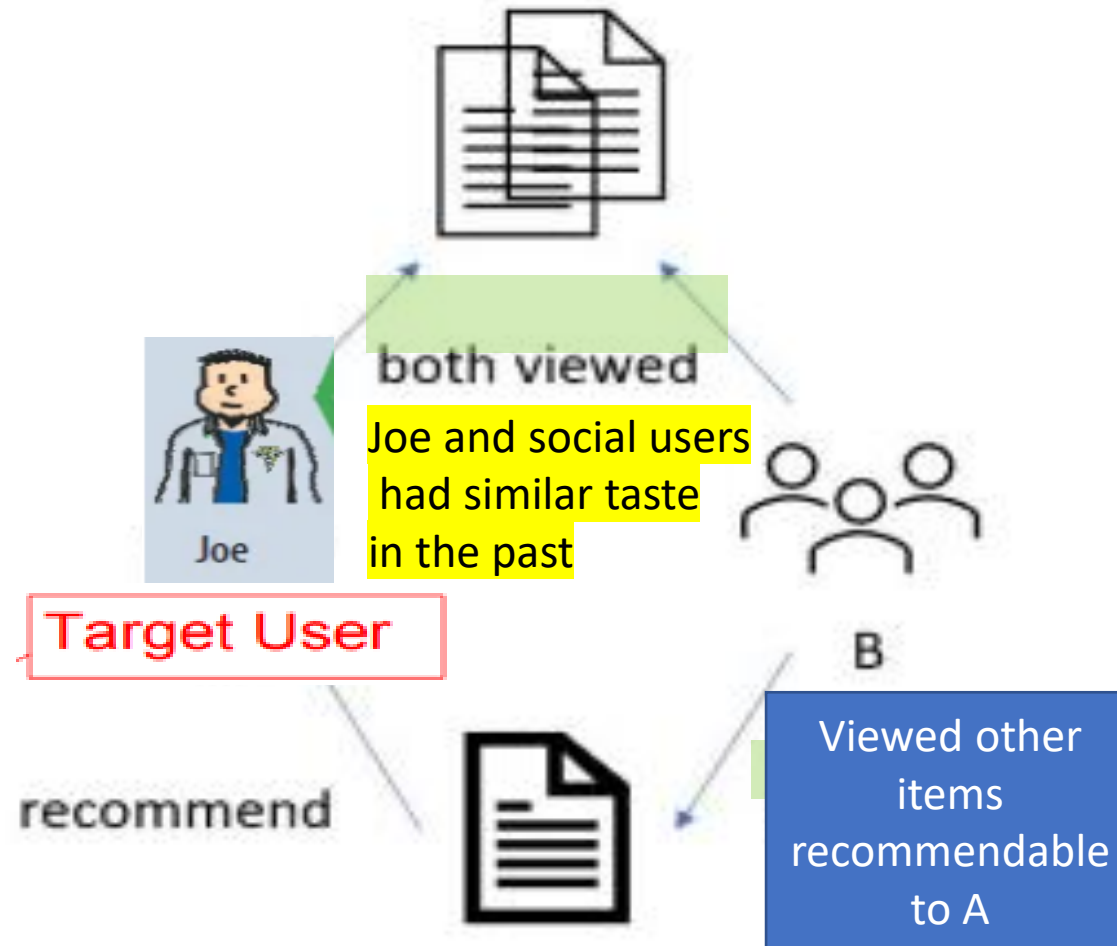
- Recommend items similar to what a user chose in the past
- Often **uses similarity measure** between items - representations (Salton)





# FOCUS: Collaborative filtering

- Past interactions between users and items produce new recommendations.
- Assumptions:
  1. Similar users share the same interest and
  2. Similar items are liked by a user.



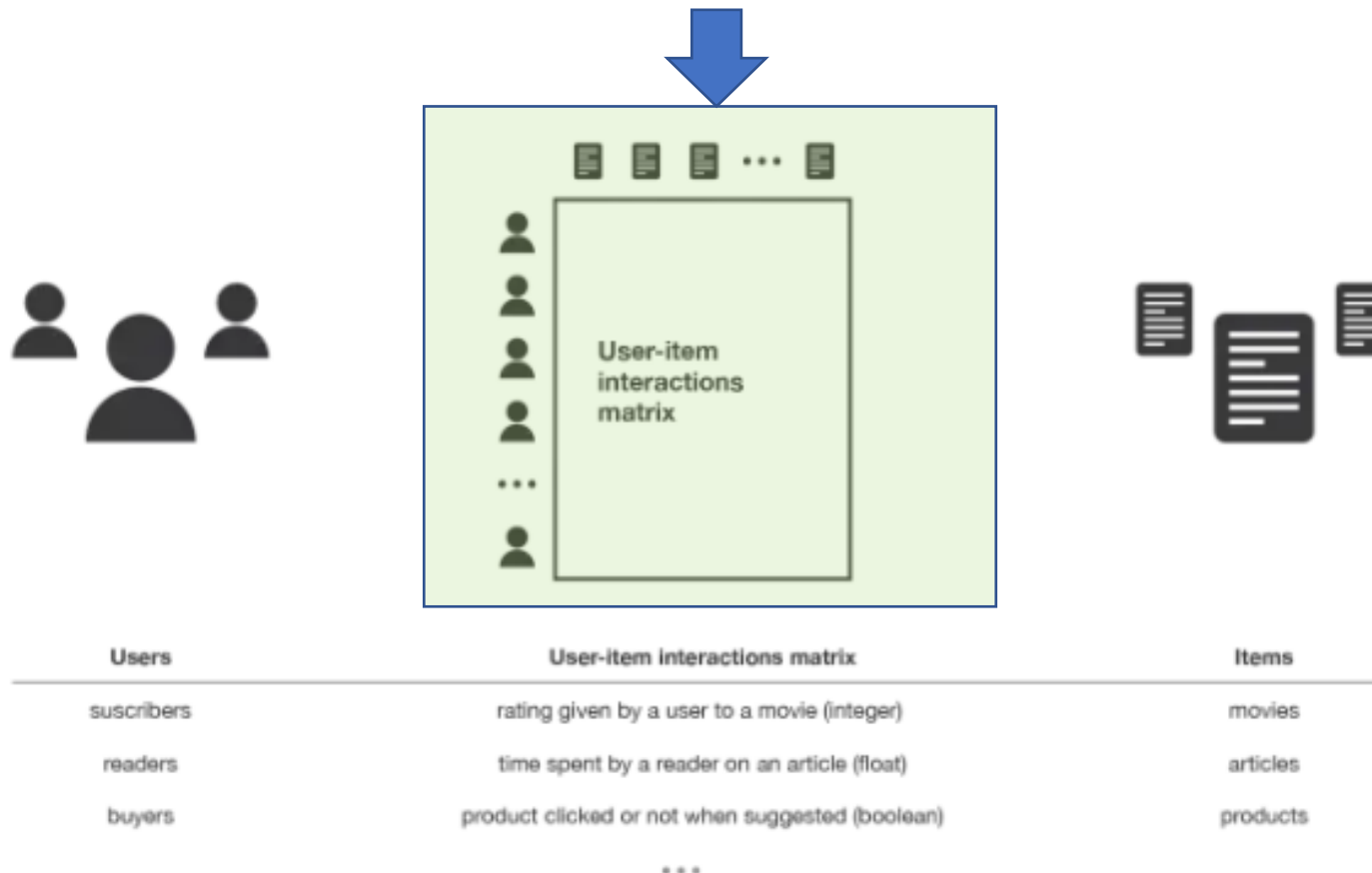
User's preferences could be predictive of Joe's preferences in the future -

# FOCUS: Collaborative filtering



Historical user-item interactions or additional side information (e.g., social relations, item's knowledge, etc.)

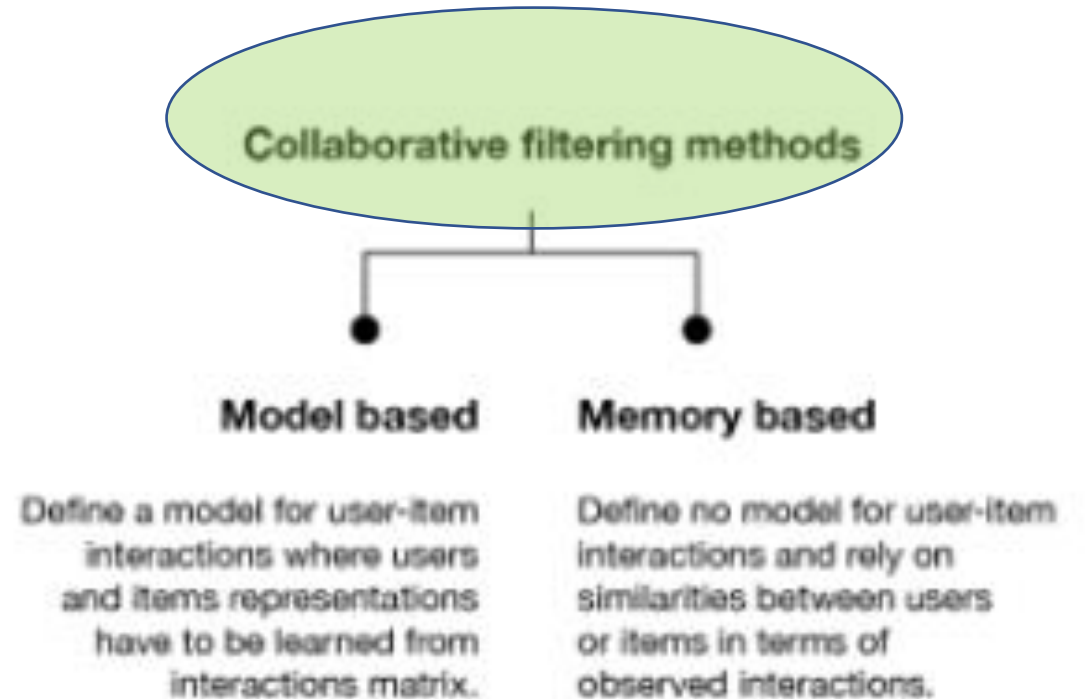
- Typical approach: **past user-item interactions** (U/I matrix) are stored in the **"user-item matrix"**.



**user-item interactions matrix** provides information to recommends items based on users' past behavior.

# Taxonomy of Collaborative Filtering

- Memory based
  - directly works with values of recorded interactions,
  - no model assumed ,
  - essentially based on nearest neighbours search (e.g., find the closest users from a user of interest and suggest the most popular items among these neighbours).
- Model based
  - assume an underlying “generative” model that explains user-item interactions and try to discover it in order to make new predictions.





# Memory based CF

## Collaborative filtering methods



### Memory based

Define no model for user-item interactions and rely on similarities between users or items in terms of observed interactions.

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# Memory Based CF

## Collaborative filtering methods



### Memory based

Define no model for user-item interactions and rely on similarities between users or items in terms of observed interactions.

- There are two categories of Memory Based CF:
  - **User-based**: measure the similarity between target users and other users
  - **Item-based**: measure the similarity between the items that target users rates/interacts with and other items

# User-based CF (“user-centred” )

CF assumption: similar users share the same interest and similar items are liked by a user

- tries to identify users with the most similar “interactions profile” (nearest neighbours)
  - in order to suggest items that are the most popular among these neighbours (and that are “new” to our user). It represents users based on their interactions with items and evaluate distances between users.



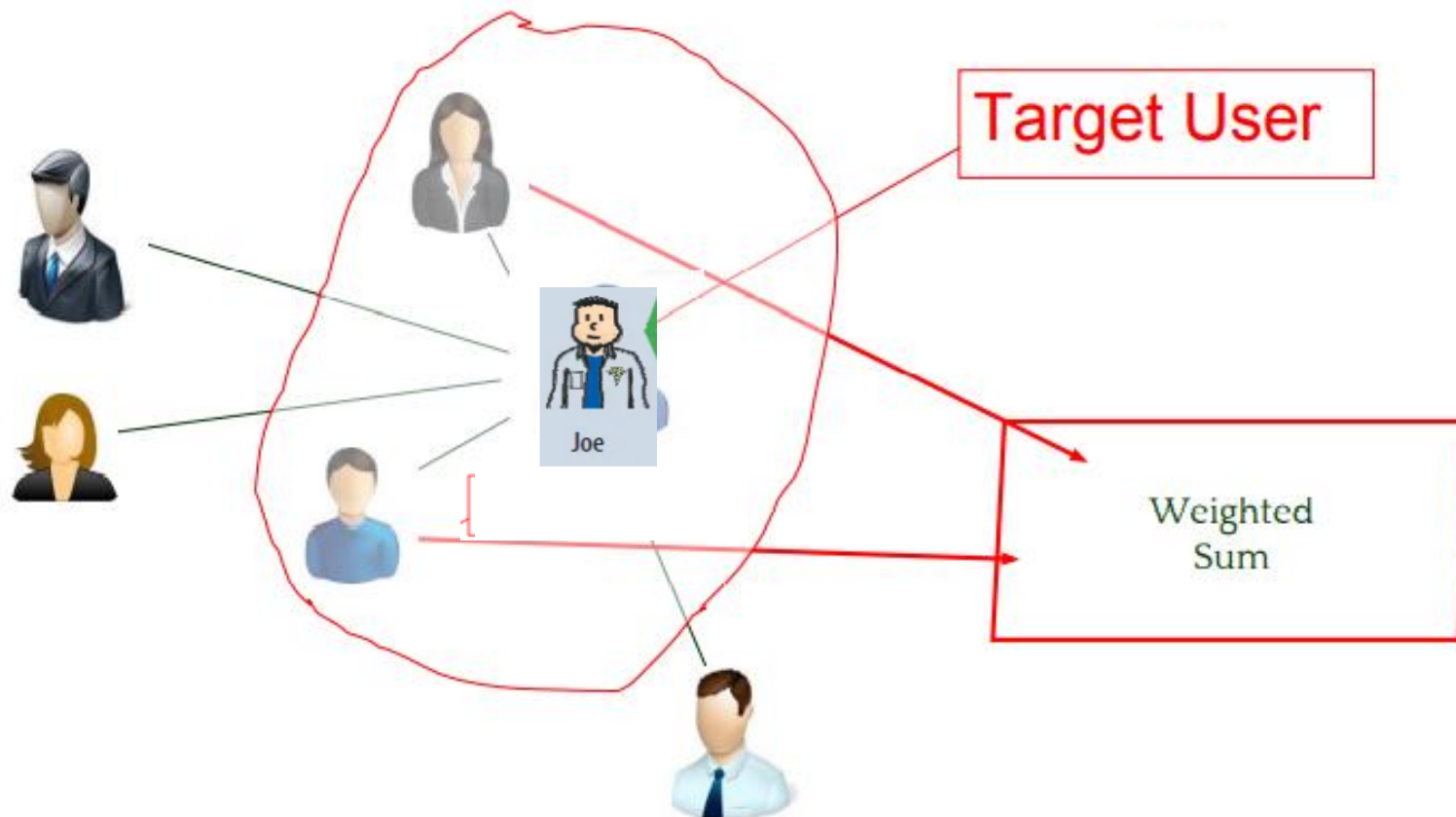
# User-based: intuition

- Consider user  $u$  **Target User**
- Find  $N$  other users similar to  $u$ , **rating-wise**
- Predict  $u'$  rating based on the other  $N$  users.

1

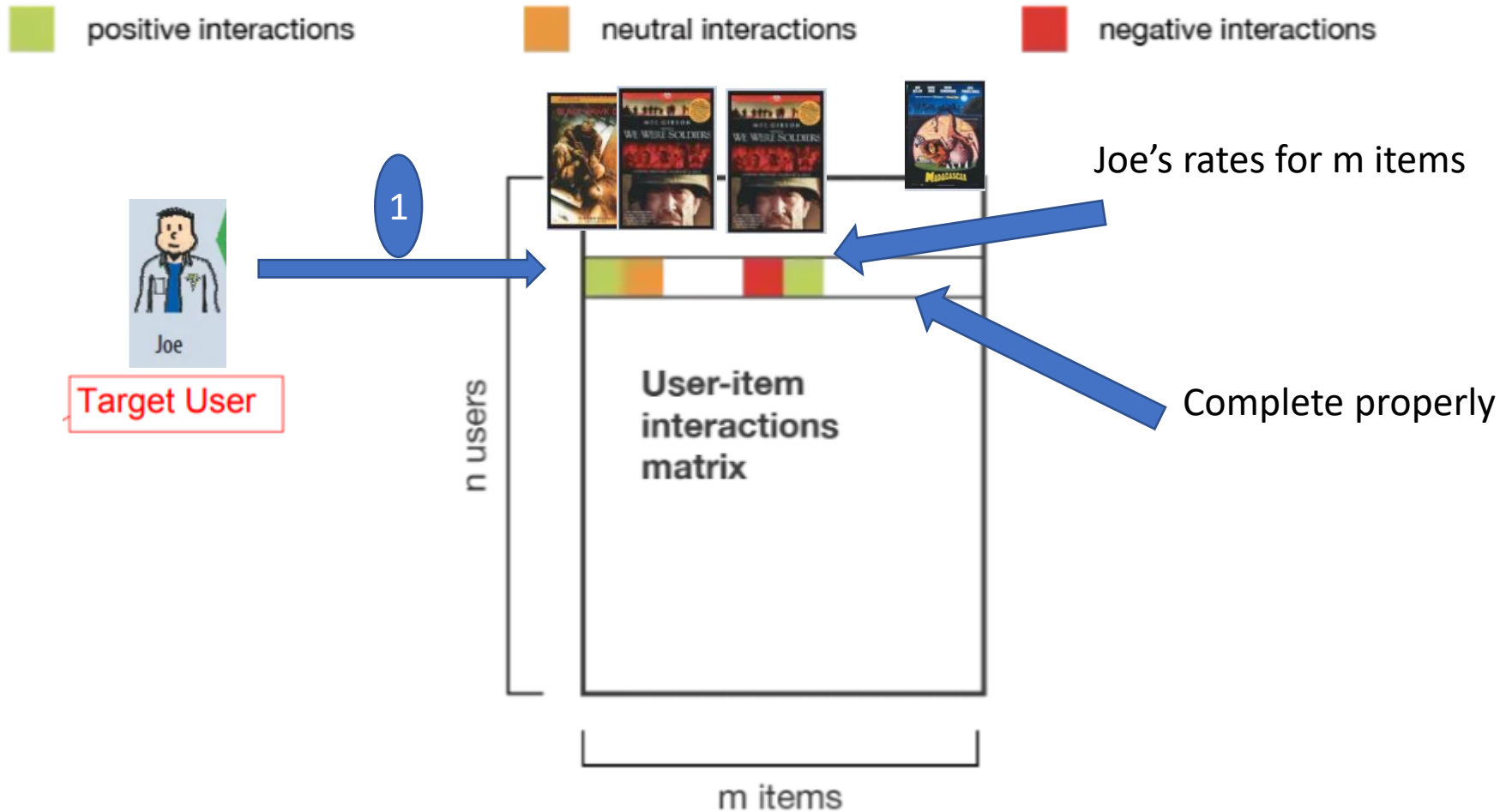
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identify users with the most similar “interactions profile”



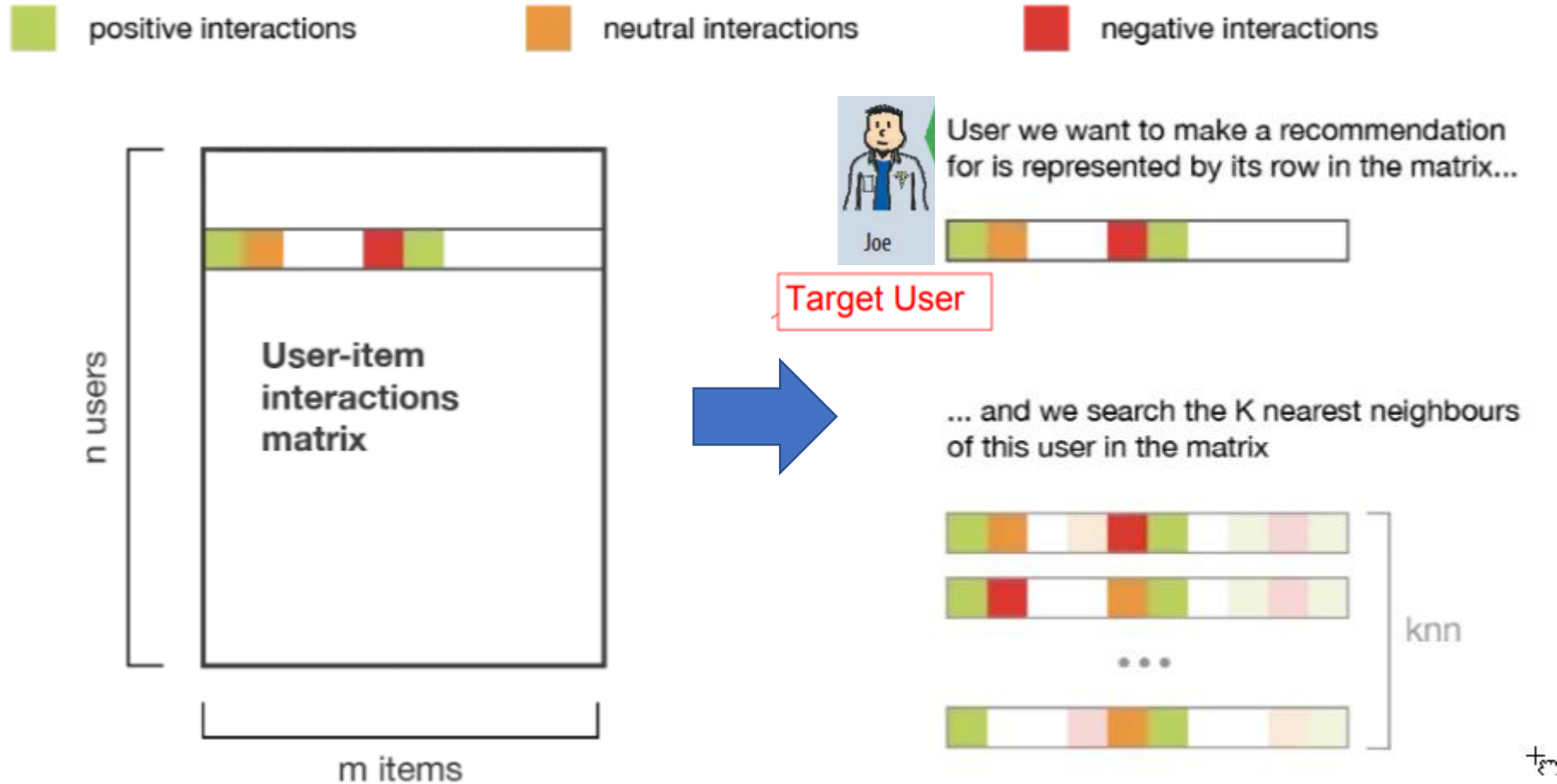
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# User-User CF: Intuition

- Consider user  $u$  **Target User**
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# User-User CF: Intuition

- Consider user  $u$  **Target User**
- Find  $N$  other users similar to  $u$ , rating-wise
- Predict  $u'$  rating based on the other  $N$  users.

2

■ positive interactions    
 ■ neutral interactions    
 ■ negative interactions



TARGET USER

User we want to make a recommendation for is represented by its row in the matrix...



... and we search the  $K$  nearest neighbours of this user in the matrix



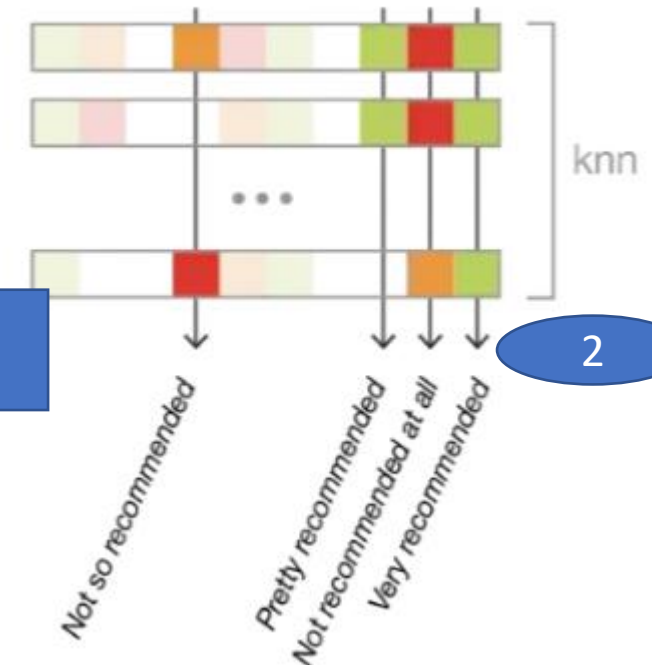
User we want to make a recommendation for is represented by its row in the matrix...



**Target User**

We can then recommend the most popular items among the  $K$  nearest neighbours

SIMILAR USERS



2

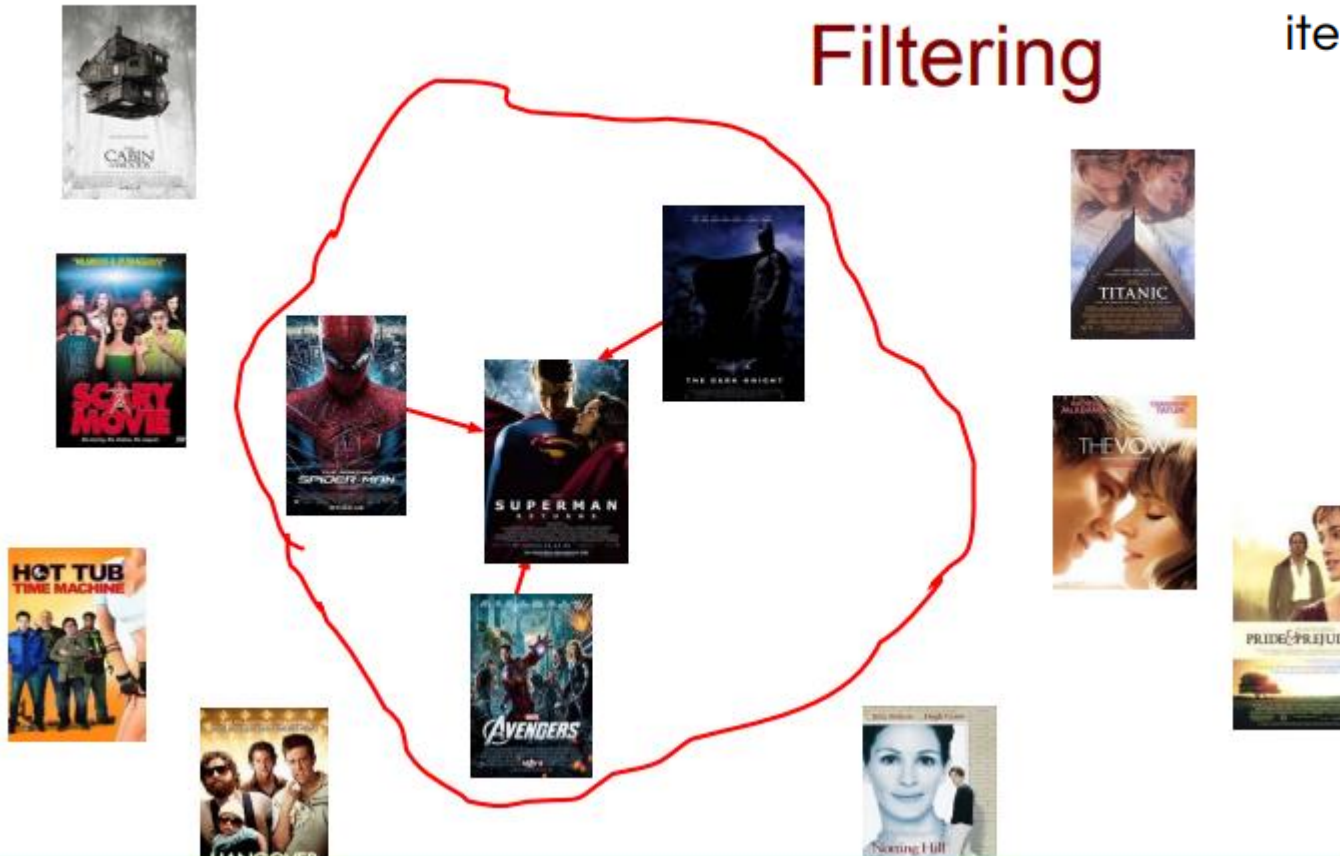
# Item- based (“item-centred” )

CF assumption: similar users share the same interest and similar items are liked by a user

- Find items similar to the ones the user already “positively” interacted with.
  - Two items are considered to be similar if most of the users that have interacted with both of them did it in a similar way.

# Item- based: intuition

## Item-Item Collaborative Filtering



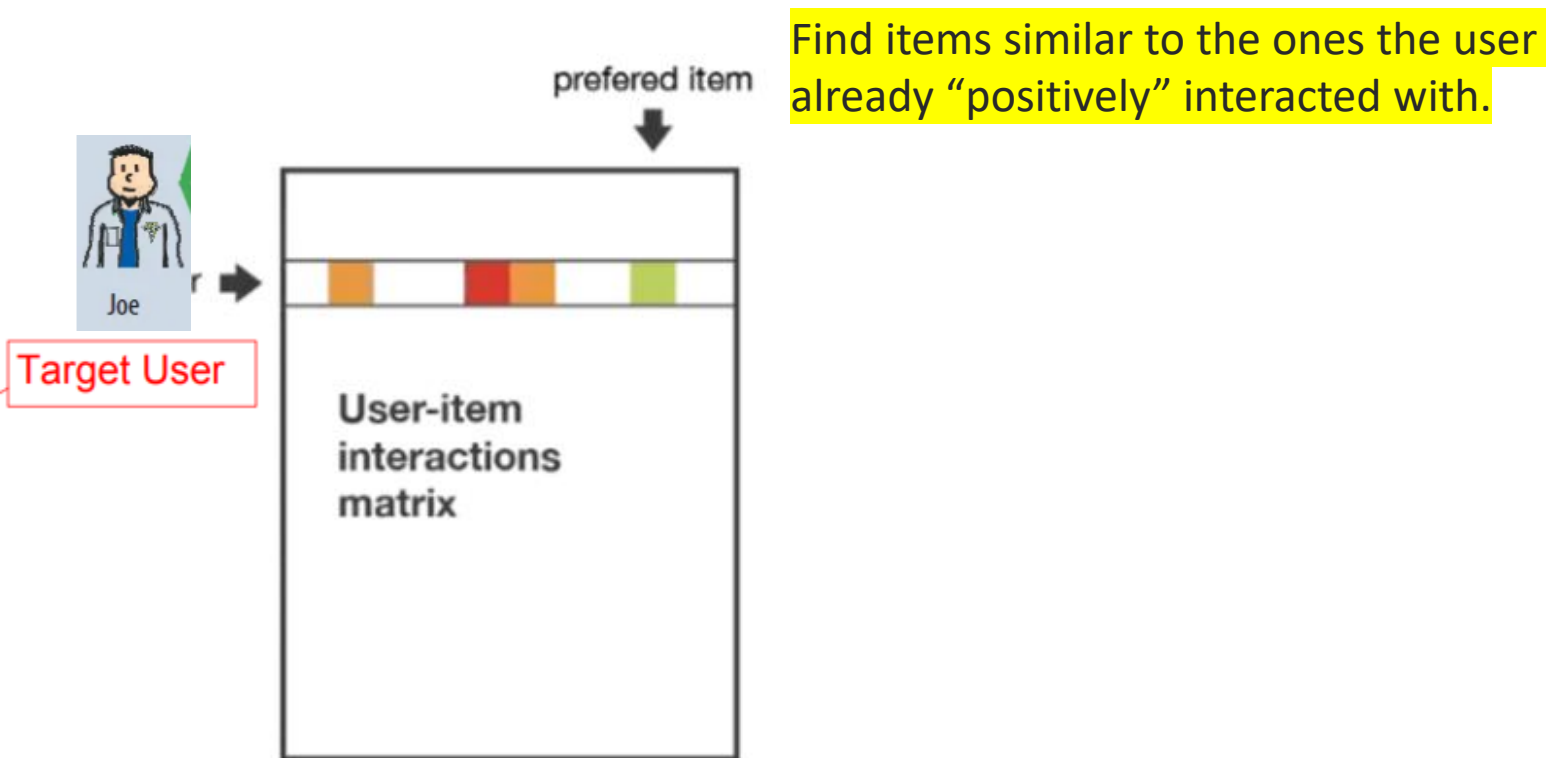
- Look into the items the **target user has rated**
- Compute how similar they are to the target item
  - Similarity **only using** past **ratings** from other users!
- Select k most similar items.
- Compute Prediction by taking weighted average on the target user's ratings on the most similar items.

Find items similar to the ones the user already “positively” interacted with.



# Item-Item CF: Intuition

■ positive interactions      ■ neutral interactions      ■ negative interactions

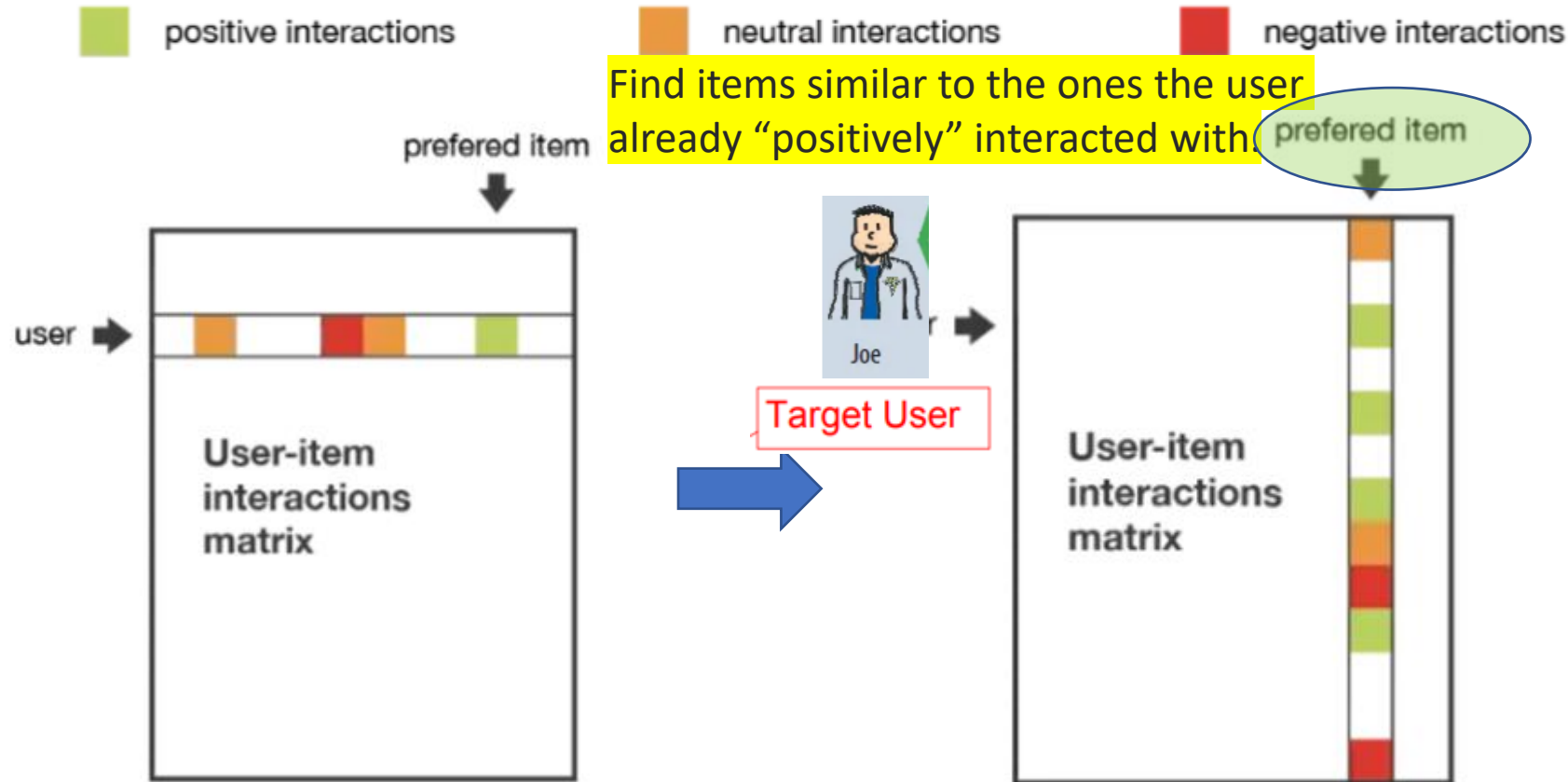


We identify the preferred item of user we want to make recommendation for.

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- Compute how similar they are to the target item
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# Item-Item CF: Intuition

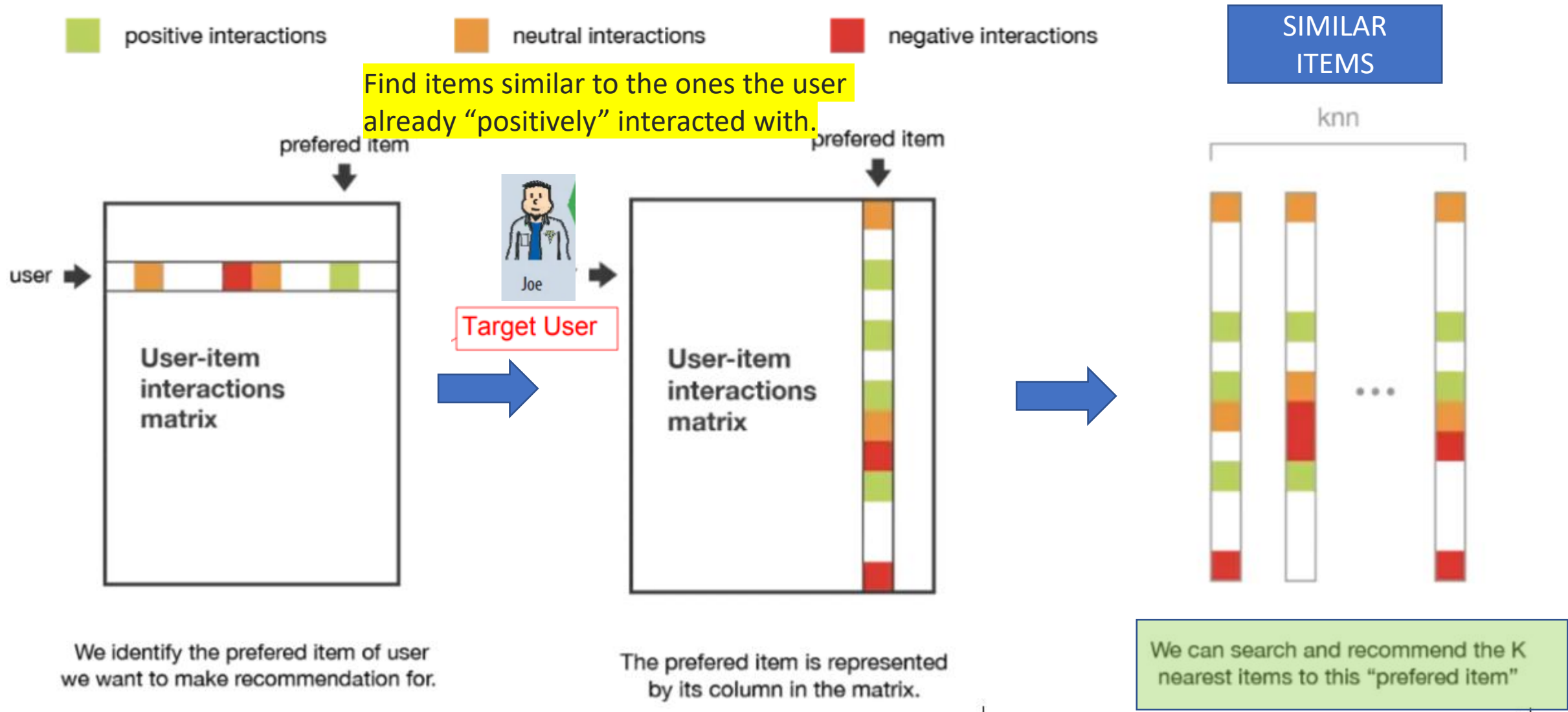
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- Select k most similar items.
- Compute Prediction by taking weighted average on the target user's ratings on the most similar items.



We identify the preferred item of user we want to make recommendation for.

The preferred item is represented by its column in the matrix.

# Item-Item CF: Intuition



# Recap

- Collaborative recommendation systems
  - “People who agreed in the past are likely to agree in the future”
- Content based
  - ▪ Recommend items similar to what the user liked in the past
- Knowledge based ▪
  - Use domain knowledge to retrieve items that match user needs
- Hybrid ▪
  - Combines above designs. Most common in practice

Can be considered  
for extended  
tasks in our lab's  
challenge !



# Model Based

Matrix Factorization

- Conventional collaborative filtering model is based on **Matrix Factorization (MF)**.

# Matrix factorization for recommendations

- Matrix factorization is an extensively used technique in collaborative filtering recommendation systems.
- Objective is to factorize a user-item matrix into two low-ranked matrices, the user-factor matrix and the item-factor matrix

## MATRIX FACTORIZATION TECHNIQUES FOR RECOMMENDER SYSTEMS

Yehuda Koren, Yahoo Research

Robert Bell and Chris Volinsky, AT&T Labs—Research

As the Netflix Prize competition has demonstrated, matrix factorization models are superior to classic nearest-neighbor techniques for producing product recommendations, allowing the incorporation of additional information such as implicit feedback, temporal effects, and confidence levels.

Such systems are particularly useful for entertainment products such as movies, music, and TV shows. Many customers will view the same movie, and each customer is likely to view numerous different movies. Customers have proven willing to indicate their level of satisfaction with particular movies, so a huge volume of data is available about which movies appeal to which customers. Companies can analyze this data to recommend movies to particular customers.

### RECOMMENDER SYSTEM STRATEGIES

Broadly speaking, recommender systems are based on one of two strategies. The *content filtering* approach creates a profile for each user or product to characterize its nature. For example, a movie profile could include attributes regarding its genre, the participating actors, its director, and so forth. User profiles might include information or answers provided in a questionnaire. The profiles allow programs to associate users with matching products. Of course,

**M**odern consumers are inundated with choices. Electronic retailers and content providers offer a huge selection of products, with unprecedented opportunities to meet a variety of special needs and tastes. Matching consumers with the most appropriate products is key to enhancing user satisfaction. Therefore, more retailers have built recommender systems, which analyze user interest in products to provide personalized recommendations.



556



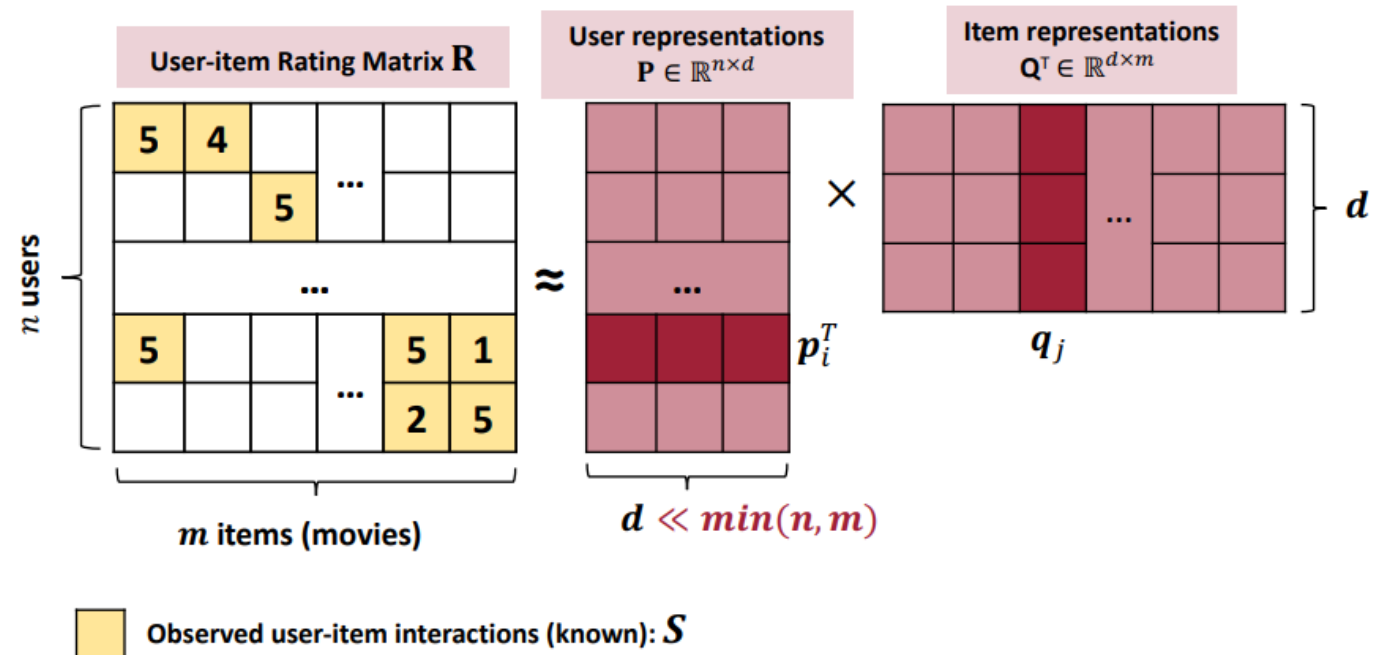
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# Matrix factorization

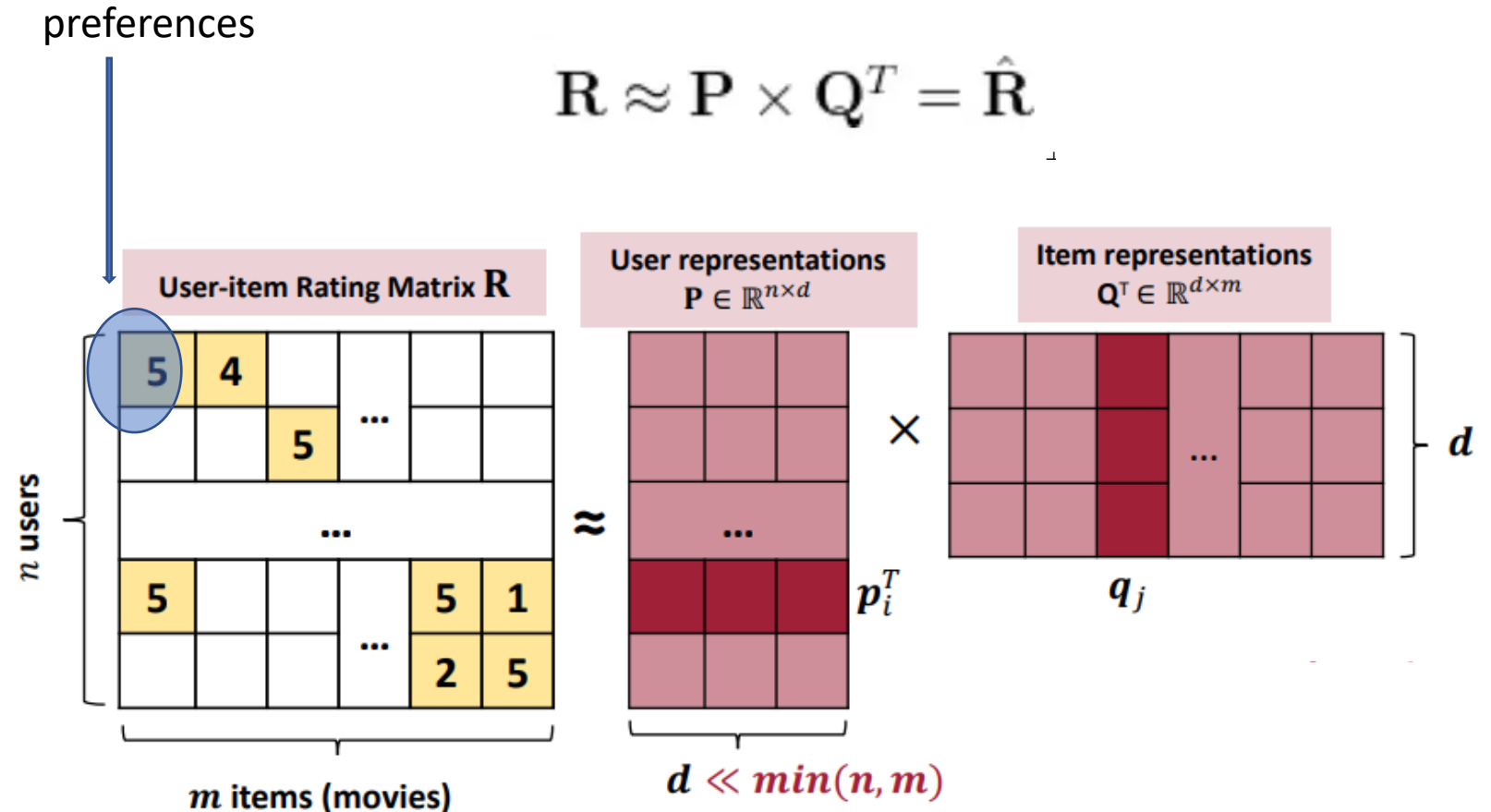
- matrix factorization can be thought of as finding 2 matrices whose product is the original matrix.



$$R \approx P \times Q^T = \hat{R}$$

# Main idea

- Assume there exists a latent space of features in which we can represent both users and items and
- such that the interaction between a user and an item can be obtained by computing the dot product of corresponding dense vectors in that space.



Generate latent features when multiplying two different kinds of entities.

# More formally

- user vector represents their preferences, item vector represents its features, dot product measures similarity
- Find  $d$ -length vectors for each user and item such that

## Task: rating prediction in Netflix

Given  $n \times m$  matrix  $\mathbf{R}$ , the goal is to learn:

**Users/Items representations:**  $\mathbf{P} \in \mathbb{R}^{n \times d}$ ,  $\mathbf{Q} \in \mathbb{R}^{m \times d}$

Objective with rating reconstruction error:

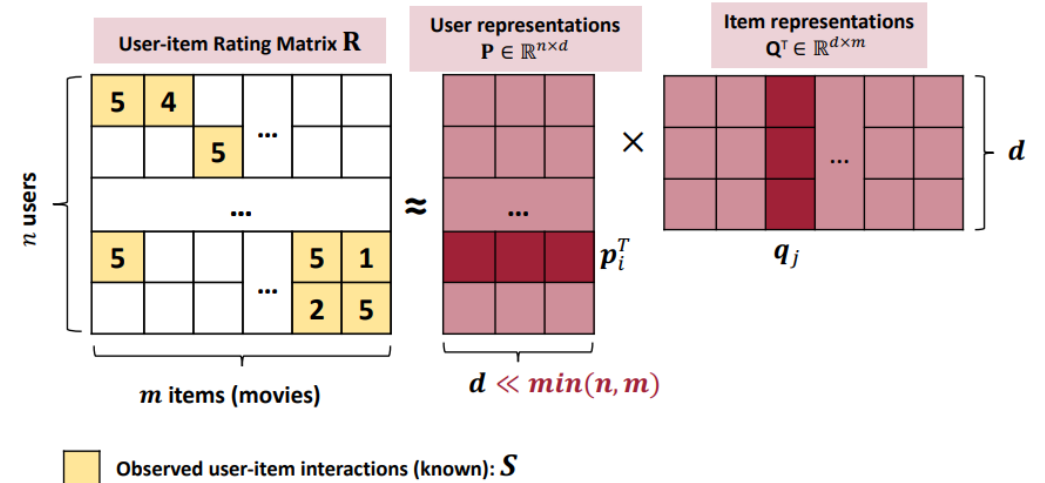
$$\min_{\mathbf{P}, \mathbf{Q}} \sum_{i,j \in S} (r_{ij} - \hat{r}_{ij})^2 = \sum_{i,j \in S} (r_{ij} - \mathbf{p}_i^T \mathbf{q}_j)^2$$

observed rating score

predicted rating score

We want the error as lower as possible by properly fitting these parameters

9



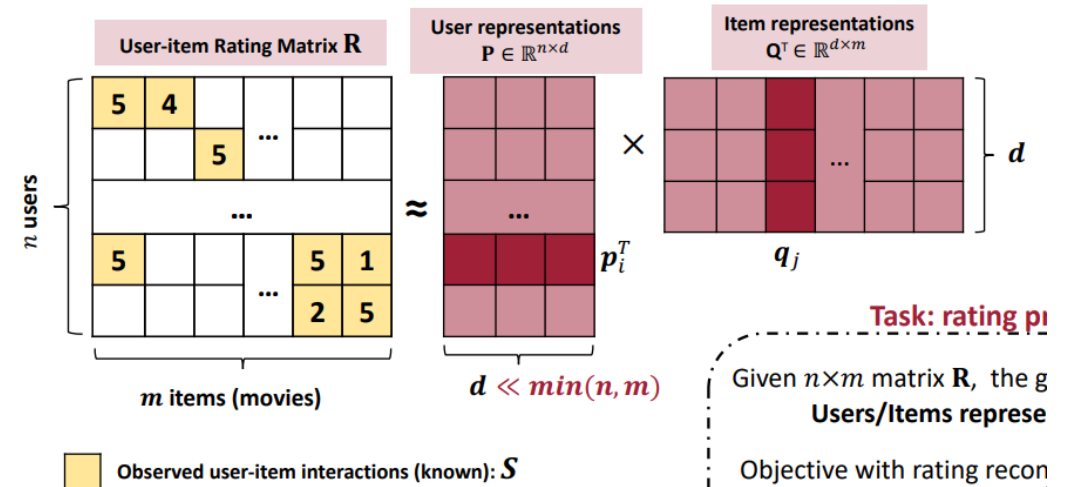
$$\mathbf{R} \approx \mathbf{P} \times \mathbf{Q}^T = \hat{\mathbf{R}}$$

1

# Matrix factorization learning: Notation

- One obvious method to find matrix  $q$  and  $p$  is the gradient descent method.

- $u$  – user,  $i$  – item
- $r_{ui}$  – rating
- $\hat{r}_{ui}$  – predicted rating
- $b, b_u, b_i$  – bias
- $q_i, p_u$  – latent factor vectors (length  $k$ )



# Assumptions and notation

- Prediction  $\hat{r}_{ij}$  for  $\langle \text{user}, \text{item} \rangle$  pair  $i, j$ :

(for a while assume centered data without bias)

$$\hat{r}_{ui} = q_i^T p_u$$

- vector multiplication
- user-item interaction via latent factors

illustration (3 factors):

- user ( $p_u$ ): (0.5, 0.8, -0.3)
- item ( $q_i$ ): (0.4, -0.1, -0.8)

- $u$  – user,  $i$  – item
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# Loss formulation

- Prediction  $\hat{r}_{ij}$  for  $\langle \text{user}, \text{item} \rangle$  pair  $i, j$ :

(for a while assume centered data without bias)

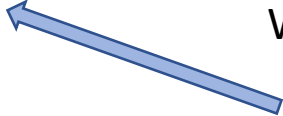
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- we want to minimize “squared errors” (related to RMSE, more details later)

$$\min_{q,p} \sum_{(u,i) \in T} (r_{ui} - q_i^T p_u)^2$$

We want the error as lower as possible by properly fitting these parameters



# Problem formulation and regularization

- regularization to avoid overfitting (standard machine learning approach)

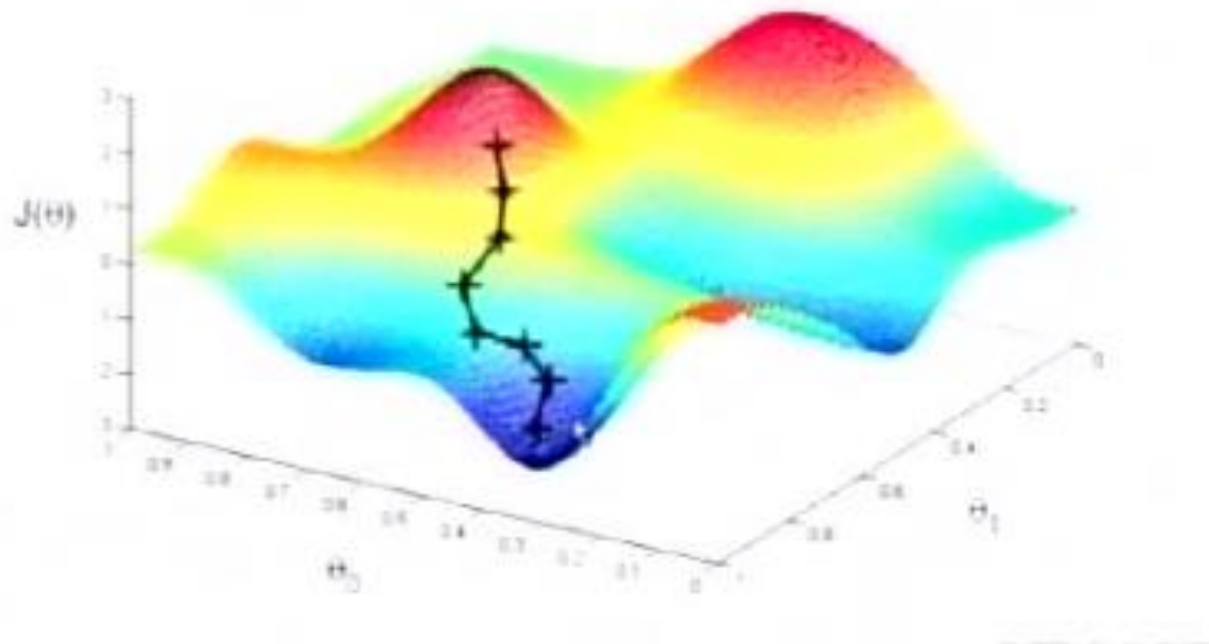
$$\min_{q,p} \sum_{(u,i) \in T} (r_{ui} - q_i^T p_u)^2 + \lambda(\|q_i\|^2 + \|p_u\|^2)$$

- we want to minimize “squared errors” (related to RMSE, more details later)

$$\min_{q,p} \sum_{(u,i) \in T} (r_{ui} - q_i^T p_u)^2$$

How to find the minimum?

# Gradient Descent



How to find the minimum?

- we want to minimize “squared errors” (related to RMSE, more details later)

$$\min_{q,p} \sum_{(u,i) \in T} (r_{ui} - q_i^T p_u)^2$$



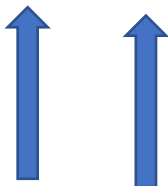
# Matrix factorization learning

- we want to minimize “squared errors” (related to RMSE, more details later)

$$\min_{q,p} \sum_{(u,i) \in T} (r_{ui} - q_i^T p_u)^2$$

+

- Loss for prediction where true rating is  $r_{ij}$ :

$$L(r_{ij}, \hat{r}_{ij}) = (r_{ij} - \hat{r}_{ij})^2 = (r_{ij} - \sum_{f=1}^F U_{if} \cdot V_{jf})^2$$


Let me use matrix notation here

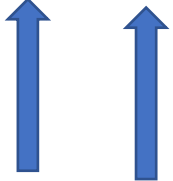
# Loss differentiation ....

- we want to minimize “squared errors” (related to RMSE, more details later)

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+

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Let me use matrix notation here

- Gradient of loss function for sample  $\langle i, j \rangle$  :

$$\frac{\partial L(r_{ij}, \hat{r}_{ij})}{\partial U_{if}} = \frac{\partial (r_{ij} - \sum_{f=1}^F U_{if} \cdot V_{jf})^2}{\partial U_{if}} = -2(r_{ij} - \sum_{f=1}^F U_{if} \cdot V_{jf}) V_{jf}$$

$$\frac{\partial L(r_{ij}, \hat{r}_{ij})}{\partial V_{jf}} = \frac{\partial (r_{ij} - \sum_{f=1}^F U_{if} \cdot V_{jf})^2}{\partial V_{jf}} = -2(r_{ij} - \sum_{f=1}^F U_{if} \cdot V_{jf}) U_{if}$$

— for  $f = 1$  to  $F$

# Loss differentiation ....

- Gradient of loss function for sample  $\langle i, j \rangle$  :

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— for  $f = 1$  to  $F$

- Let's simplify the notation:

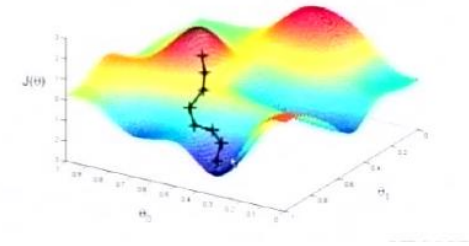
let  $e = r_{ij} - \sum_{f=1}^F U_{if} \cdot V_{jf}$  (the prediction error)

$$\frac{\partial L(r_{ij}, \hat{r}_{ij})}{\partial U_{if}} = \frac{\partial e^2}{\partial U_{if}} = -2e V_{jf}$$
$$\frac{\partial L(r_{ij}, \hat{r}_{ij})}{\partial V_{jf}} = \frac{\partial e^2}{\partial V_{jf}} = -2e U_{if}$$

— for  $f = 1$  to  $F$

# Gradient descent

Gradient Descent



1. Decide on  $F$  = dimension of factors
2. Initialize factor matrices with small random values
3. Choose one sample from training set
4. Calculate loss function for that single sample
5. Calculate gradient from loss function
6. Update  $2 \cdot F$  model parameters a single step using gradient and learning rate
7. Repeat from 3) until stopping criterion is satisfied

- Set learning rate =  $\eta$
- Then the factor matrix updates for sample  $\langle i, j \rangle$  are:

$$U_{if} = U_{if} + 2\eta e V_{jf}$$

$$V_{jf} = V_{jf} + 2\eta e U_{if}$$

– for  $f = 1$  to  $F$

# Gradient descent with the regularized step

- Must use some form of regularization (usually  $L_2$ ):

$$L(r_{ij}, \hat{r}_{ij}) = (r_{ij} - \sum_{f=1}^F U_{if} \cdot V_{jf})^2 + \lambda \sum_{f=1}^F U_{if}^2 + \lambda \sum_{f=1}^F V_{jf}^2$$

- Update rules become:

$$\begin{aligned} U_{if} &= U_{if} + 2\eta(eV_{jf} - \lambda U_{if}) \\ V_{jf} &= V_{jf} + 2\eta(eU_{if} - \lambda V_{jf}) \end{aligned}$$

– for  $f = 1$  to  $F$

# Deep recommender systems

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A deep model based RS!

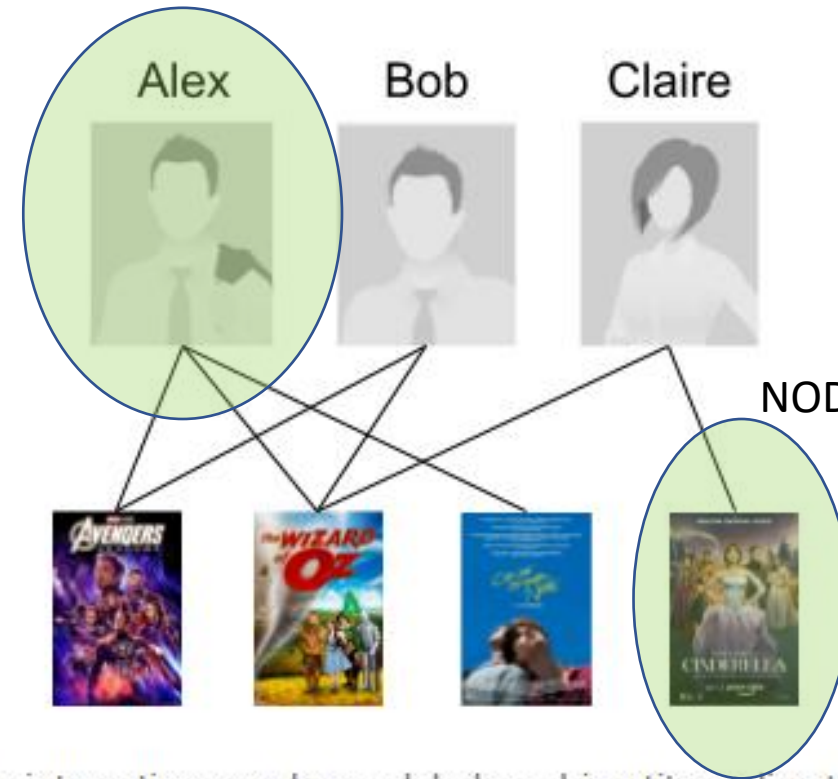




# User item relationships as bipartite graphs

- Recommender system can be naturally modelled as a bipartite graph:
- Two types of nodes: **users** and **items**.
  - Edges connect users and items Indicates user-item interaction (e.g., click, purchase, review etc.)
  - Often associated with timestamp (timing of the interaction)

NODE: USER TYPE



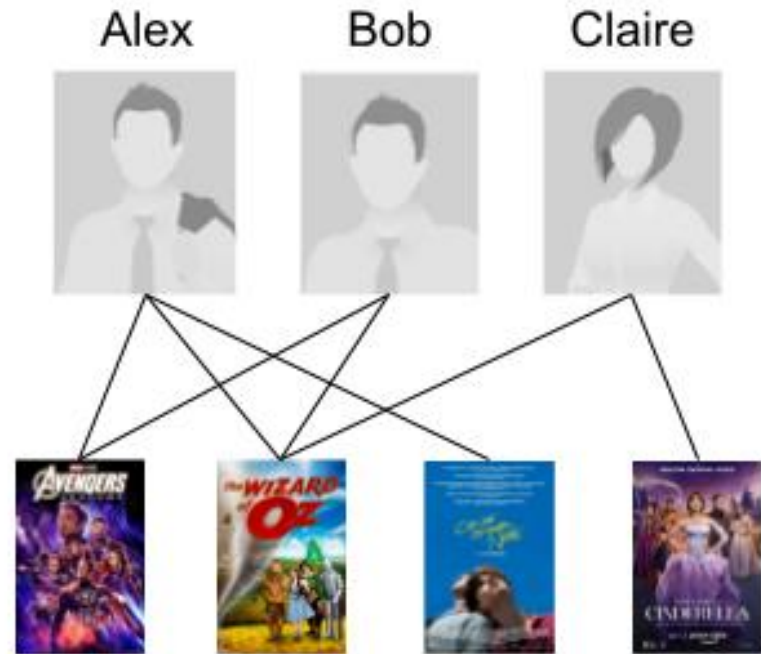
NODE: ITEM TYPE

User-Item interactions can be modeled as a bipartite undirected graph.

# Notation

- **Notation:**

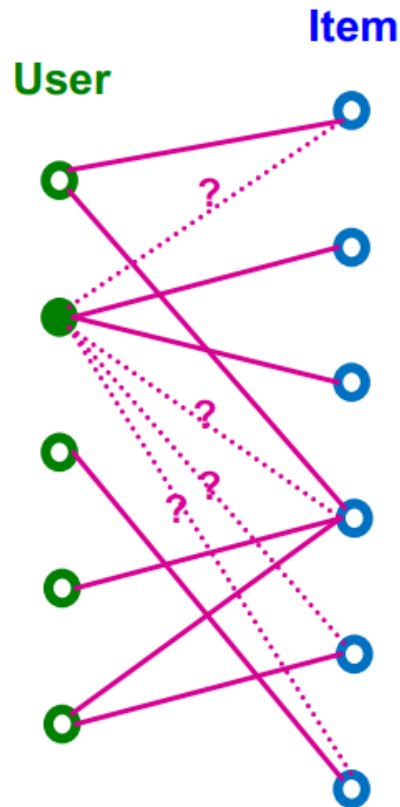
- $U$ : A set of all users
- $V$ : A set of all items
- $E$ : A set of observed user-item interactions
  - $E = \{(u, v) \mid u \in U, v \in V, u \text{ interacted with } v\}$



User-Item interactions can be modeled as a bipartite undirected graph.



# Problem formulation



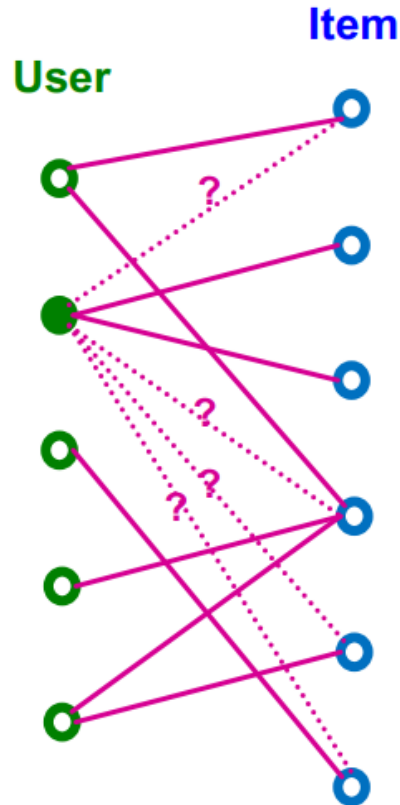
## ■ Given

- Past user-item interactions

## ■ Task

- Predict new items each user will interact in the future.
- Can be cast as **link prediction** problem.
  - Predict new user-item interaction edges given the past edges.

# Problem formulation

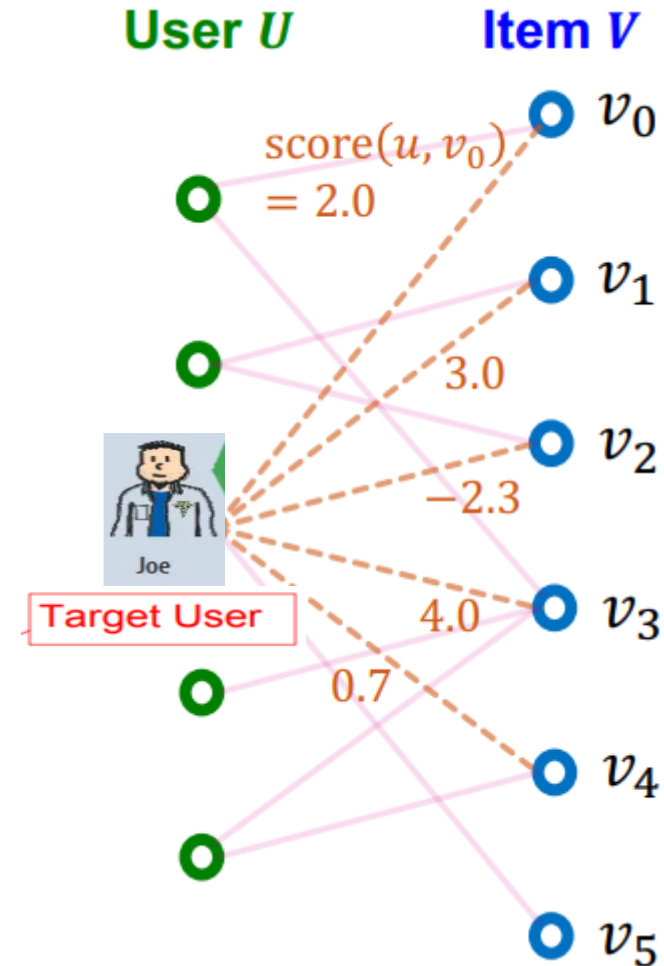


- For each user, we recommend  $K$  items.
  - In order for recommendation to be effective,  $K$  needs to be much smaller than the total number of items (up to billions)
  - $K$  is typically in the order of 10—100.
- The goal is to include as many positive items as possible in the top- $K$  recommended items.
  - Positive items = Items that the user will interact with in the future.

## TARGET

For each user, we recommend  $K$  items.

- To get the top- $K$  items, we need a score function for user-item interaction:
  - For  $u \in U, v \in V$ , we need to get a real-valued scalar  $\text{score}(u, v)$ .
  - $K$  items with the largest scores for a given user  $u$  (excluding already-interacted items) are then recommended.

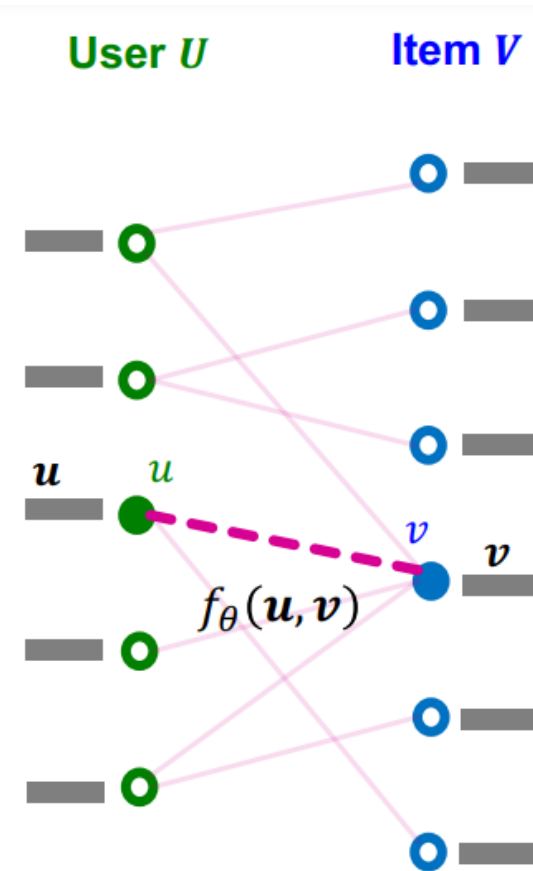


For  $K = 2$ , recommended items for user  $u$  would be  $\{v_1, v_3\}$ .

# EMBEDDING based k top scores

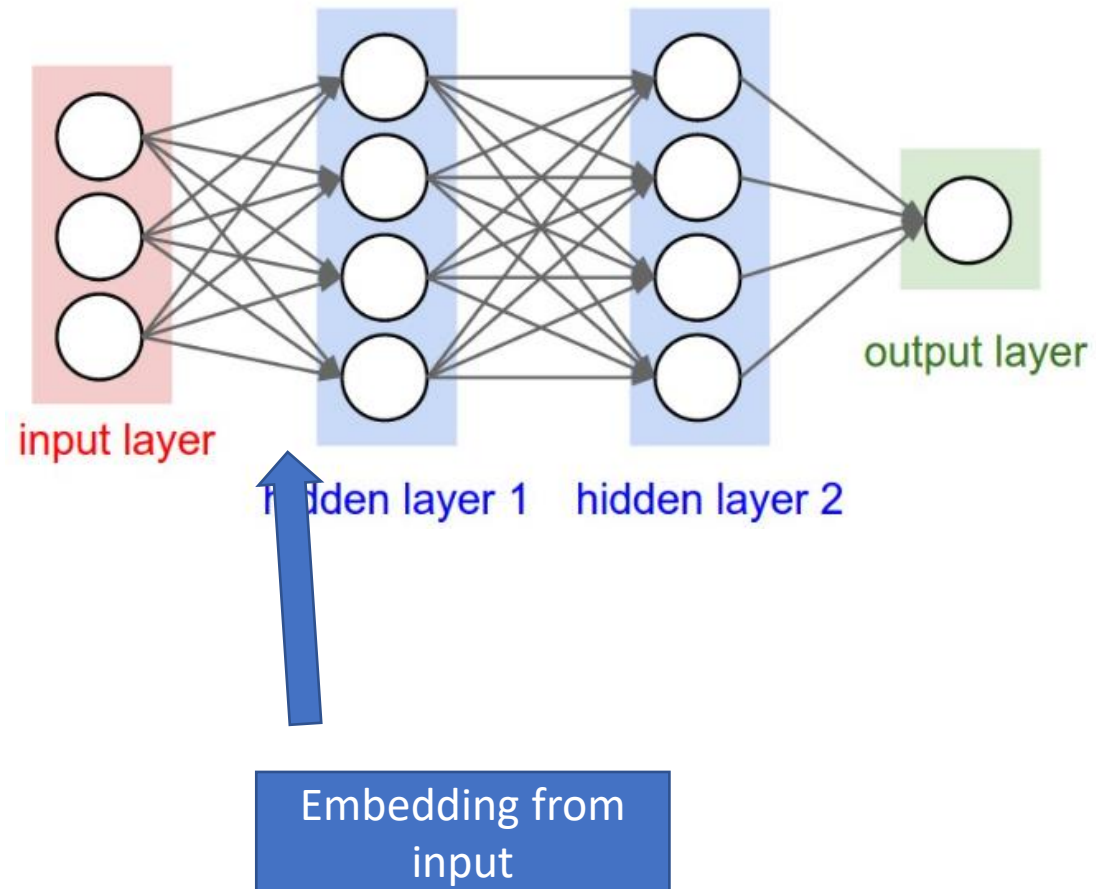
- We consider **embedding-based models** for scoring user-item interactions.

- For each user  $u \in U$ , let  $\mathbf{u} \in \mathbb{R}^D$  be its  $D$ -dimensional embedding.
- For each item  $v \in V$ , let  $\mathbf{v} \in \mathbb{R}^D$  be its  $D$ -dimensional embedding.
- Let  $f_\theta(\cdot, \cdot): \mathbb{R}^D \times \mathbb{R}^D \rightarrow \mathbb{R}$  be a parametrized function.
- Then,  $\text{score}(u, v) \equiv f_\theta(\mathbf{u}, \mathbf{v})$

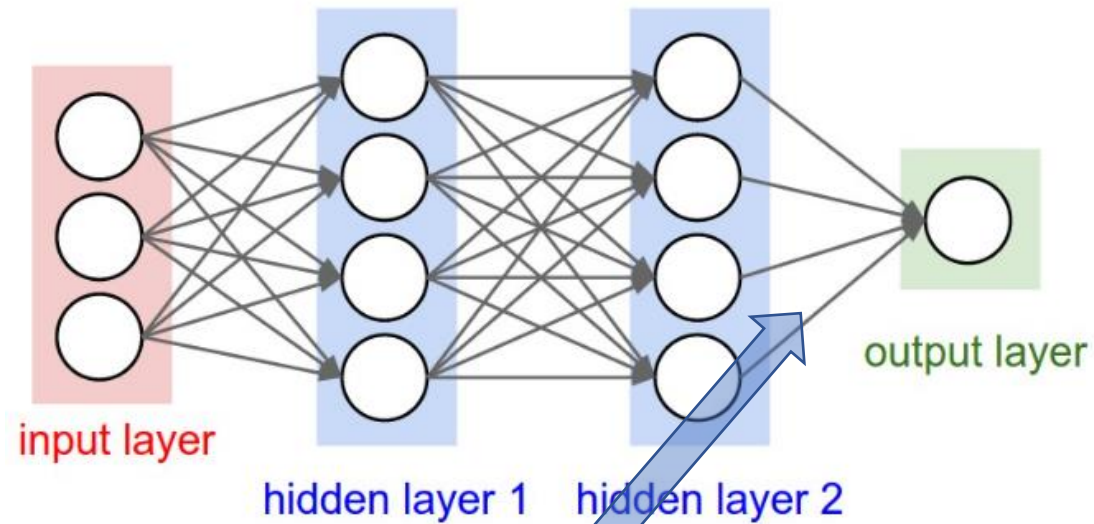


Can be a NN

# Remarks: Embedding ?



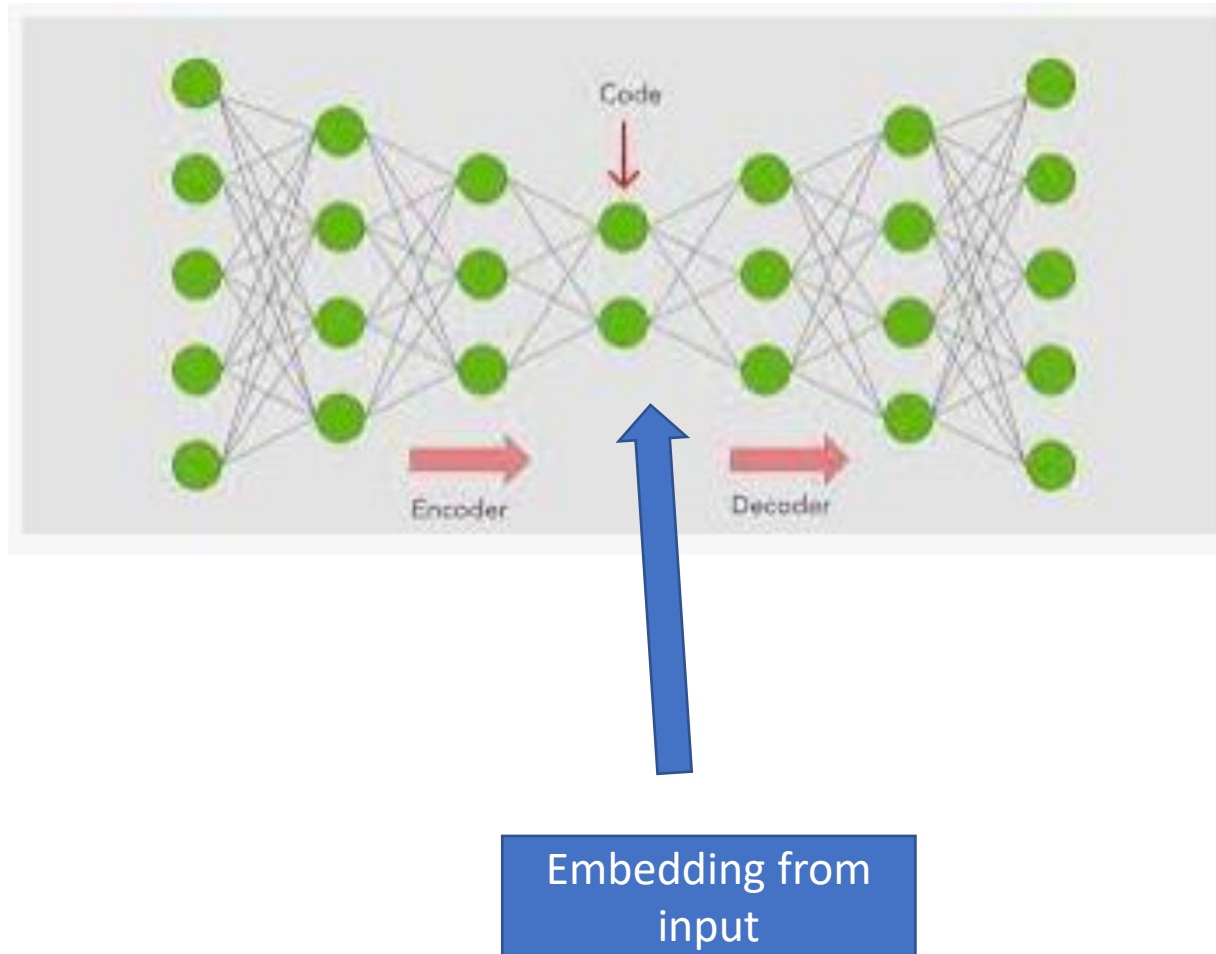
# Remarks: Embedding ?



Another latent  
representation  
from hidden layer  
n2

$$h2 = \sigma(h1)$$

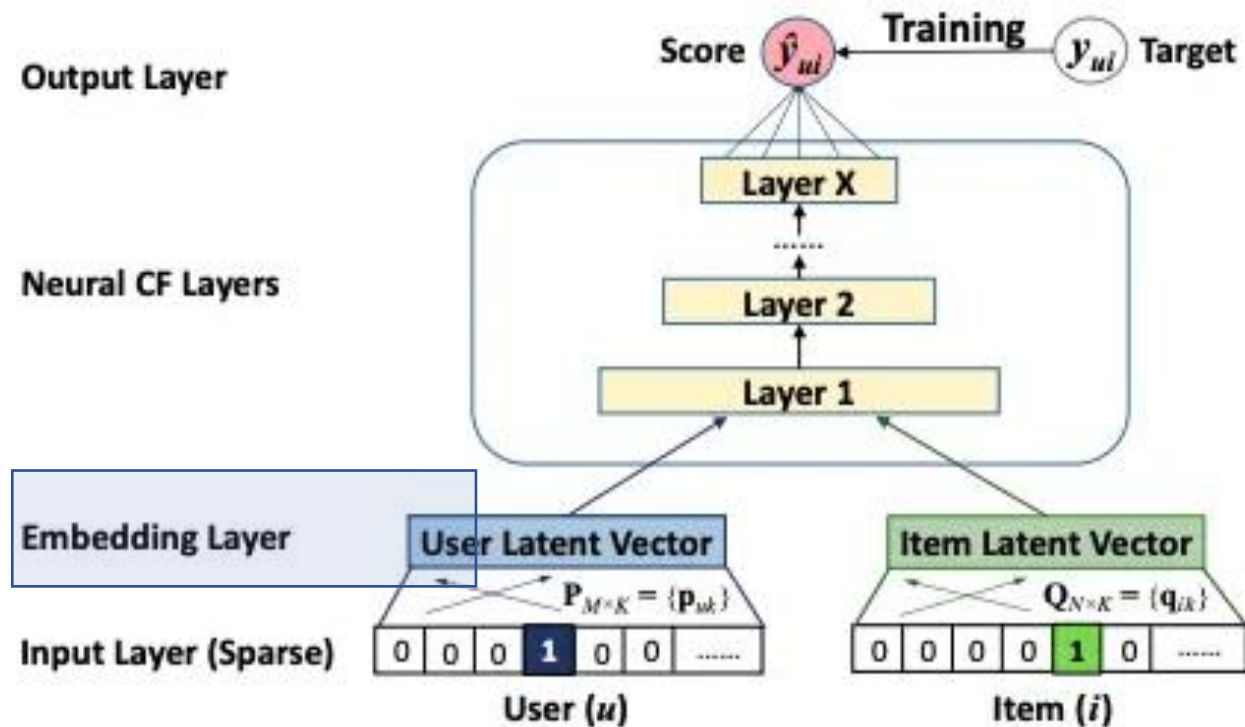
# Remarks: Embedding with autoencoder



# A naive deep mechanism

- We consider **embedding-based models** for scoring user-item interactions.

dense or  
latent vectors  
for the  
sparse inputs

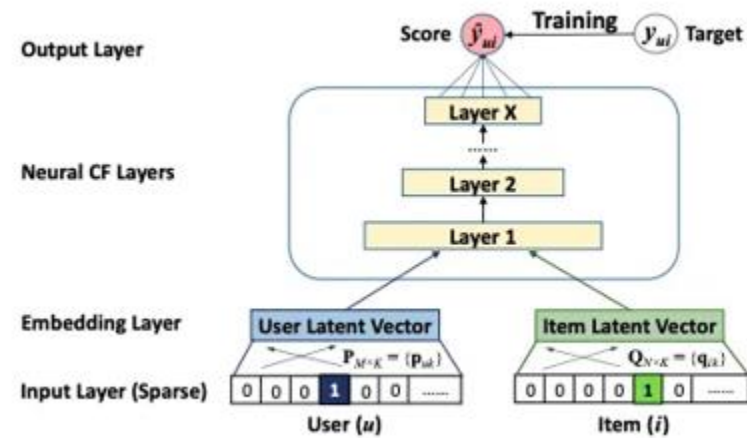


Basically both items and users  
are one-hot encoded.

Fig.2 from "Neural Collaborative Filtering" by X He, L Liao, H Zhang, L Nie, X Hu, TS Chua — Proceedings of the 26th international conference on world wide web, 2017



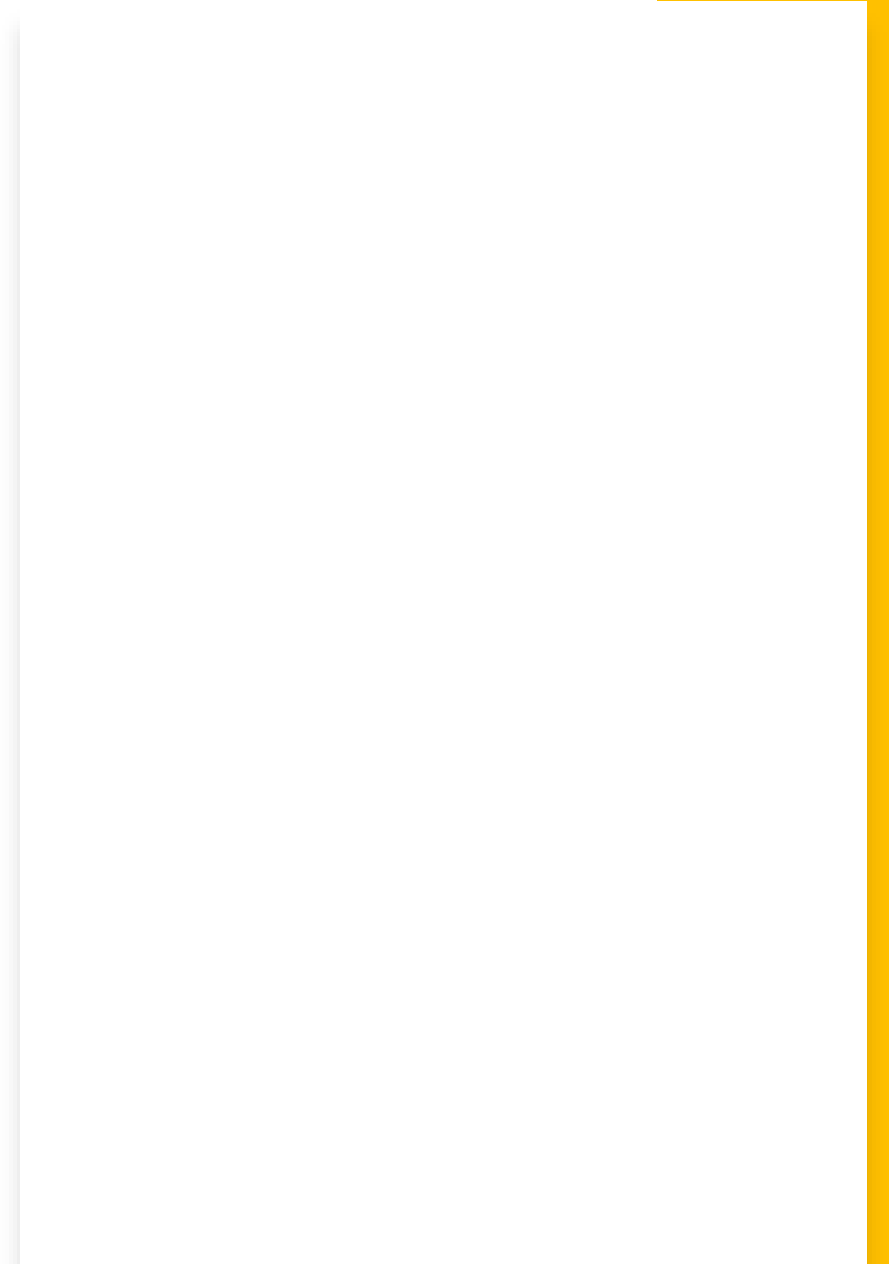
# Remark: A deep mechanism



- The model itself does *not explicitly* capture graph structure
  - The graph structure is *only implicitly* captured in the training objective.



Some related issues with  
RS



# Problem: Serendipity

- You watch a cat video ->
  - get recommended more cat videos ->
    - watch more cat videos ->
      - recommended more cat videos...
- Interesting at first but becomes repetitive: never explores new interests
- Potential solutions: -
  - Look at how similar users shifted interests in the past –
- Domain knowledge that “if user needs X, they are more likely to need Y in the future”

# Problem: Cold Start

- User doesn't have an informative history of interactions on the platform –
- New user or the interaction is sparse by nature (buying cars) –
- Potential solution: - Query the user for more information - E.g. import contacts when signing up for social media, ask what type of car (size, # seats, model, etc.) the user needs

# Problem: Feature Extraction

- So far, we had to manually pick out the features that we think are relevant –
- Manual labelling becomes infeasible when –
  - There are simply too many items –
  - Items cannot be described by a few discrete labels. for example, images and videos –
- Potential solution: - Neural methods that can learn useful features from data

# More problems

- Scalability - how to create efficient recommendation systems that work with millions or billions of users and items, when pairwise metrics are infeasible
- Large Datasets - how to process large amounts of data, which is possibly unstructured Sparse Ratings - how to create confident recommendations to users about items even if little information was provided about those items Lack of user profiles –

# References

- Koren, Yehuda, Robert Bell, and Chris Volinsky. *Matrix factorization techniques for recommender systems*. Computer 42.8 (2009): 30-37.
- Koren, Yehuda, and Robert Bell. *Advances in collaborative filtering*. Recommender Systems Handbook. Springer US, 2011. 145-186.