Boolean Simplification using Karnaugh Map(K-Map)

Simplification of Boolean function:

- Minimize the number of terms(product terms (SOP)/ Sum terms(POS))
- Minimize the number of laterals
- Minimize the number of logic gates required to implement a Boolean function
- Minimum number of inputs to each gate

Methods for Simplifying Boolean Function:

- Law of Boolean Algebra
- Karnaugh Map (K-Map)

Karnaugh Map (K-Map)

- A pictorial form of a truth table of Boolean function is known as K-map
- Graphical representation
- 2-variable K-Map: map consisting of 4 squares because it consists of 4 minterms (i.e. 2X2 Matrix)
- 3-variable K-Map: map consisting of 8 squares because it consists of 8 minterms. (i.e. 2X4 Matrix or 4X2 Martrix)
- 4-Variable K-Map: map consisting of 16 squares because it consists of 16 minterms (i.e. 4X4 Matrix)
- 5-Variable K-map :map consisting of 32 squares because it consists of 32 minterms. (i.e. 4X8 Matrix or 8X4 Matrix)

- And so on. …

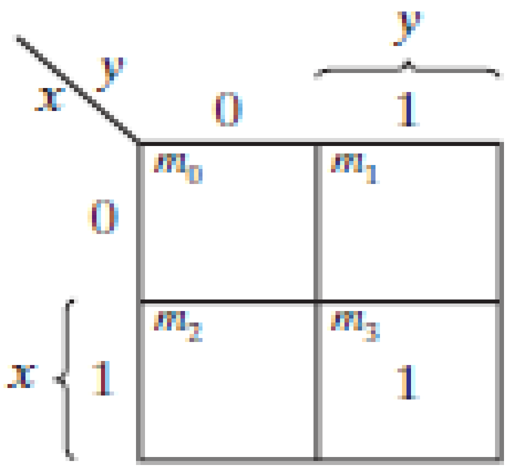
Rules for simplification using K-map:

- The minterms(incase of SOP) /Maxterm(in case of POS) arranged not in binary sequence but in a sequence similar to the Gray code.
- i.e. only one bit changes in value from one adjacent column to the next
- then push 1 in squares corresponding to minterms given in the function and push 0 to remaining square
- We form a group of adjacent squares whose values are 1 (When simplified function in SOP form) or Group of adjacent squares whose values are 0 (When simplified function in POS form)
- The grouping of adjacent squares in the form of power of two
- i.e. 2^0 or 2^1 or 2^2 or 2^3 and so on.

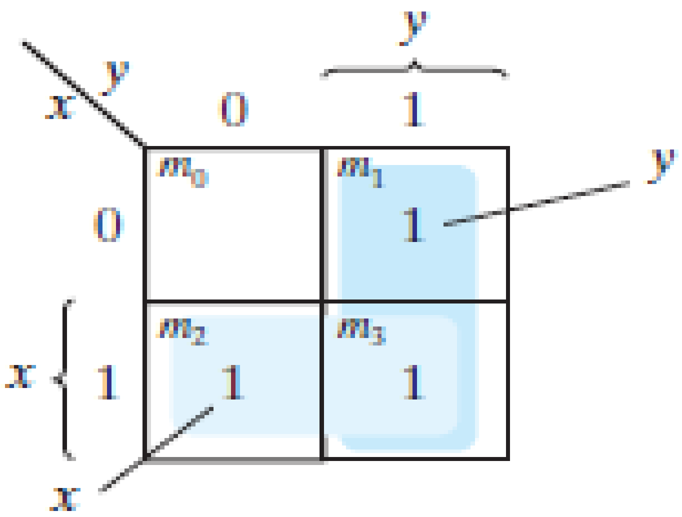
- Two squares are adjacent only when its minterm differ by only one bit position.
- When the number of adjacent squares are grouped in the form of
  - 2^0 then no laterals are reduced from term(i.e. all the laterals are present for resenting a term i. e. all n laterals present in a terms)
  - 2^1 then one laterals is reduced from a term (i.e. n-1 laterals are present in a term)
  - 2^2 then two laterals are reduced from a term (i.e. n-2 laterals are present in a term)
  - 2^3 then three laterals are reduced from a term (i.e. n-3 laterals are present in a term)
  - 2^k then k laterals are reduced from term (i.e. n-k laterals are present in a term)

Example of two variable K-map

Simplify the following Boolean Fuunction:
Example 1. F(x, y) = $\sum(3)$ = xy (in SOP)
Example 2. F(x, y) = $\sum(1, 2, 3)$ = x+y (in SOP)



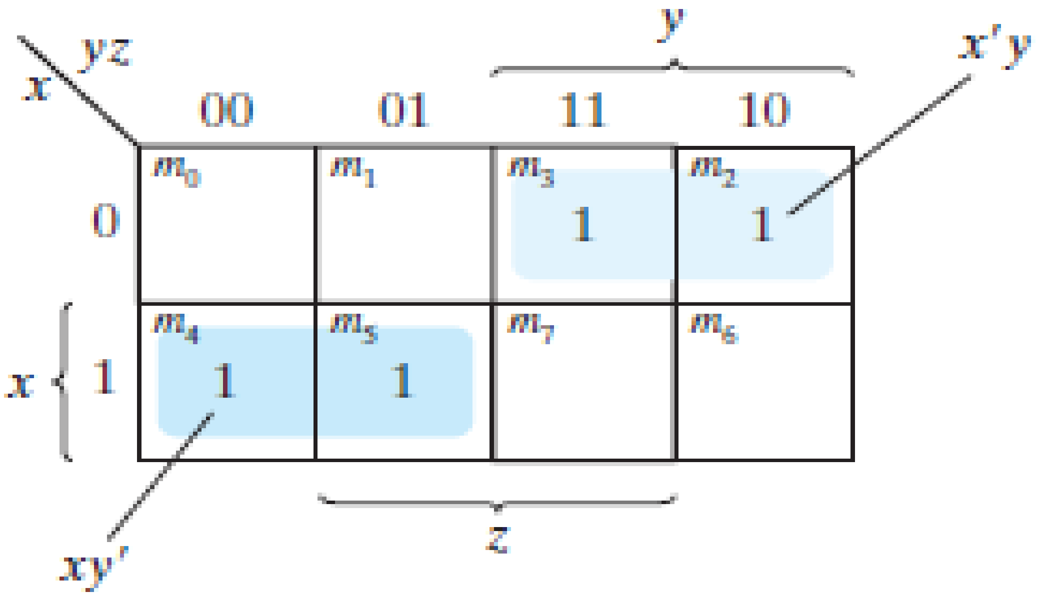
(a) xy



(b) $x + y$

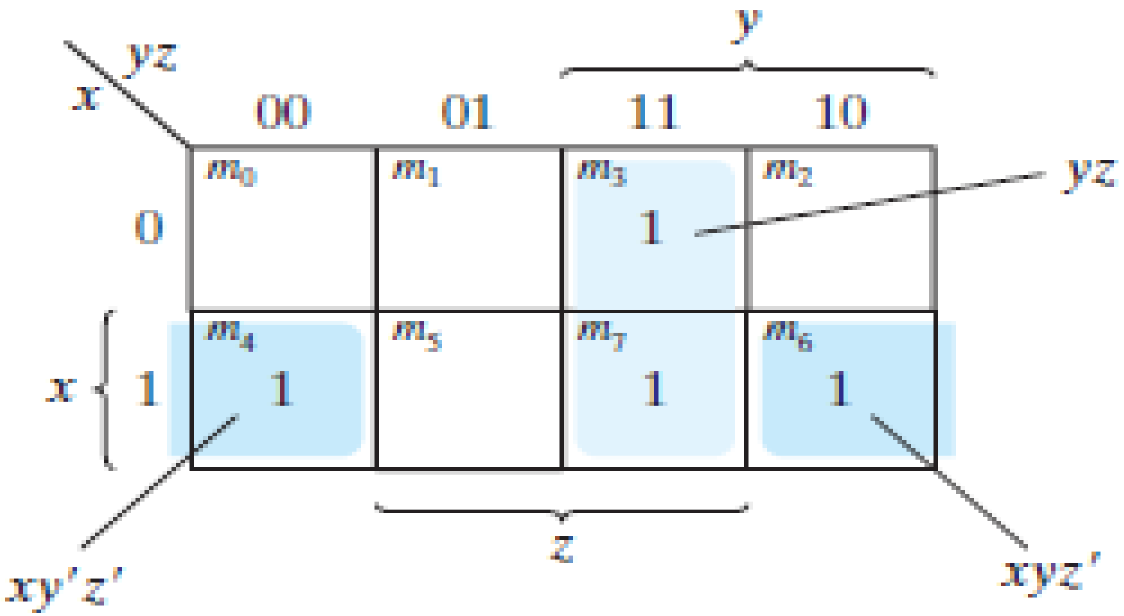
Example of two variable K-map

Simplify the following Boolean Function:
Example 2. $F(x, y, z) = \sum(2, 3, 4\ 5) = x'y + xy'$ (in SOP)



Three variable Boolean Function Simplification Example

Simplify the following Boolean Function:
Example 2. F(x, y, z) = ∑(3, 4 , 6, 7) = xz′ + yz (in SOP) (Simplified)
Ans: F(x, y, z) = xz′ + yz (in SOP) (Simplified)

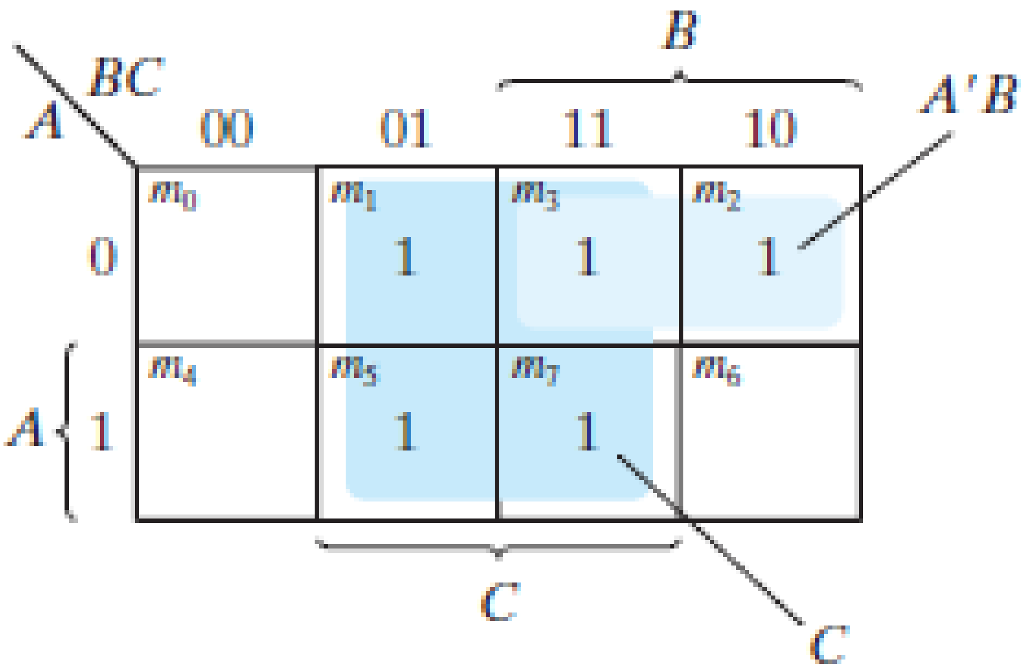


Note: xy′z′ + xyz′ = xz′

Simplify the Boolean Function:

$F(A, B, C) = A'C + A'B + AB'C + BC = \sum(1, 2, 3, 5, 7)$

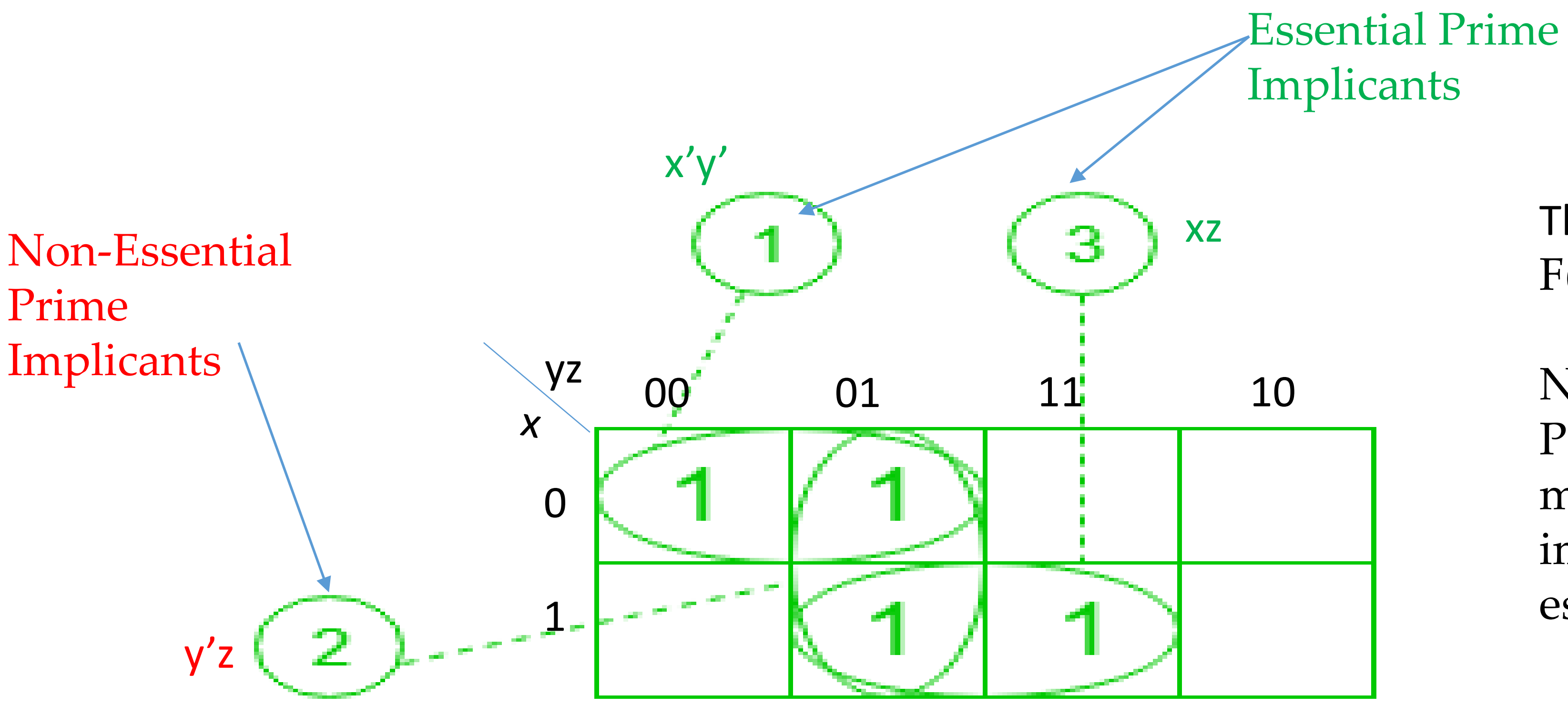
Ans: $F(A, B, C) = A'B + C$



Implicants in K-map

- **Prime implicants:** it is a product term/Sum Term obtained by combining maximum number of possible adjacent square in a map whose values are 1 (In case of Product term) or obtained by combining maximum number of possible adjacent square in a map whose values are 0 (in case of Sum term):
- There are three types of Prime Implicants
- **Essential Prime Implicants:**
  - Those prime implicants which cover at least one minterm that can't be covered by any other prime implicant.
  - It is always appear in the final solution
- **Non-Essential or Redundent Prime Implicants:**
  - The prime implicants for which each of its minterm is covered by some essential prime implicant.
  - It never appear in final solution
- **Selective prime Implecants**
  - Those prime implicants which are neither essential nor redundant.
  - It may appear in some solution or may not appear in some solution

Example of Prime Implicants

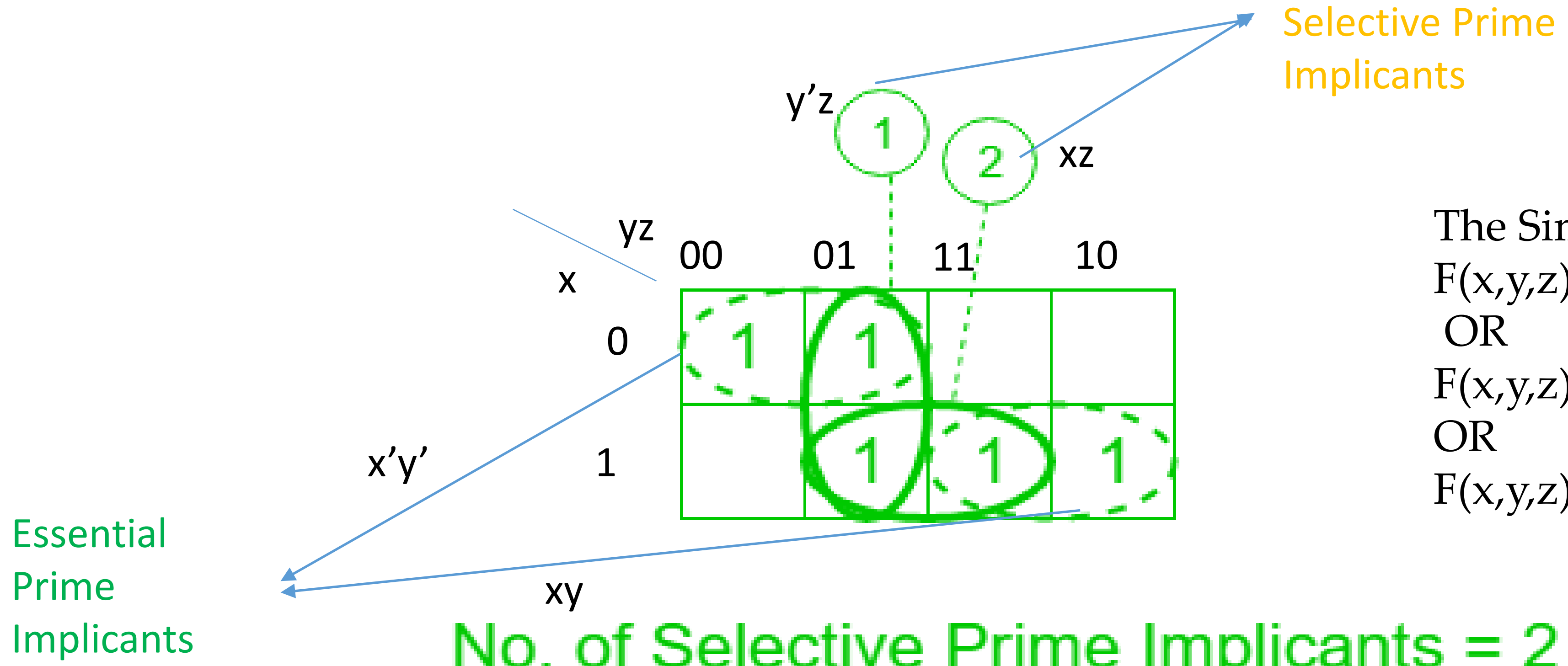


The Simplified function is written as :
F(x,y,z) = x'y' + xz

Note: We Ignore non-essential Prime Implicant because all the minterms of non-essential prime implecant are covered by essential prime implicants

No. of Prime Implicants = 3

Example of Prime Implicants….



No. of Selective Prime Implicants = 2

The Simplified function is written as:

$F(x,y,z) = x'y' + xy + y'z$

OR

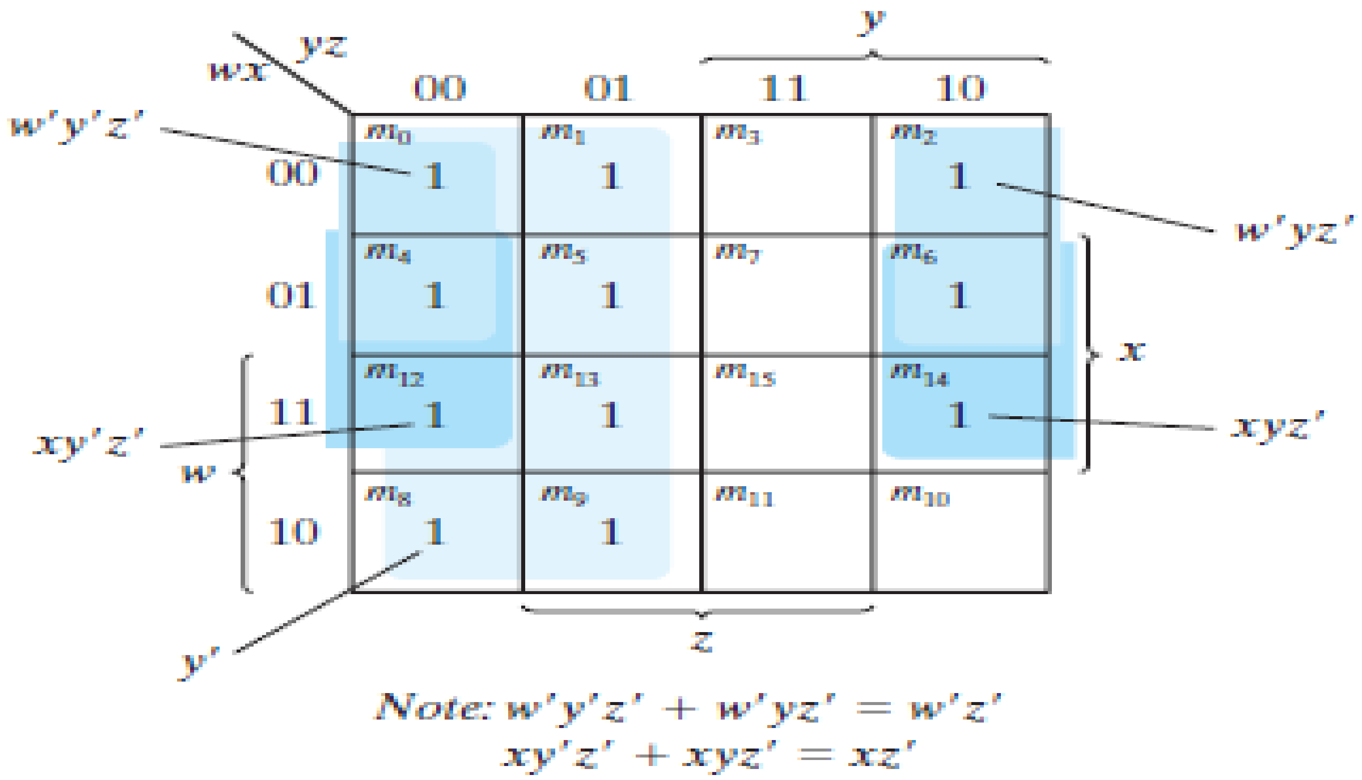
$F(x,y,z) = x'y' + xy + xz$

OR

$F(x,y,z) = x'y' + xy + xy'z$

Simplification of Four Variable Boolean Function:

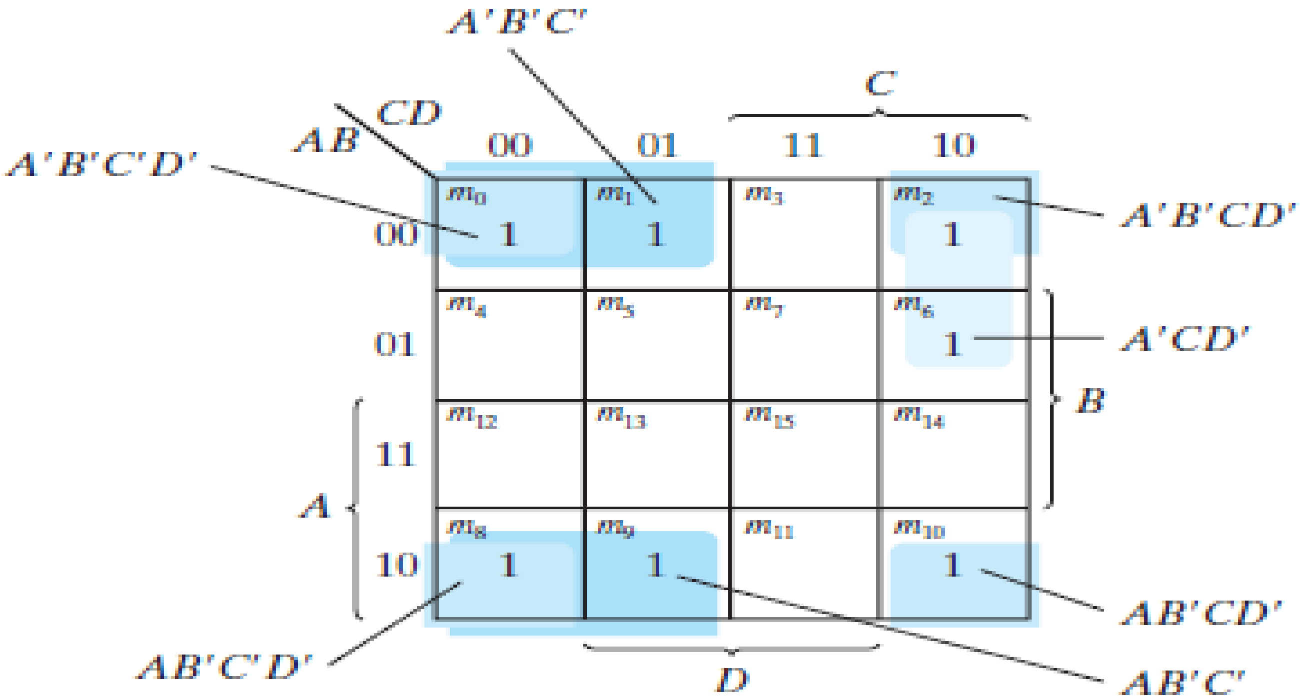
Simplify the Boolean function F(w, x, y, z)= ∑ (0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14)



Simplified Function is F(w, x, y, z)= y’ + w’z’+ xz’

Simplification of Four Variable Boolean Function……

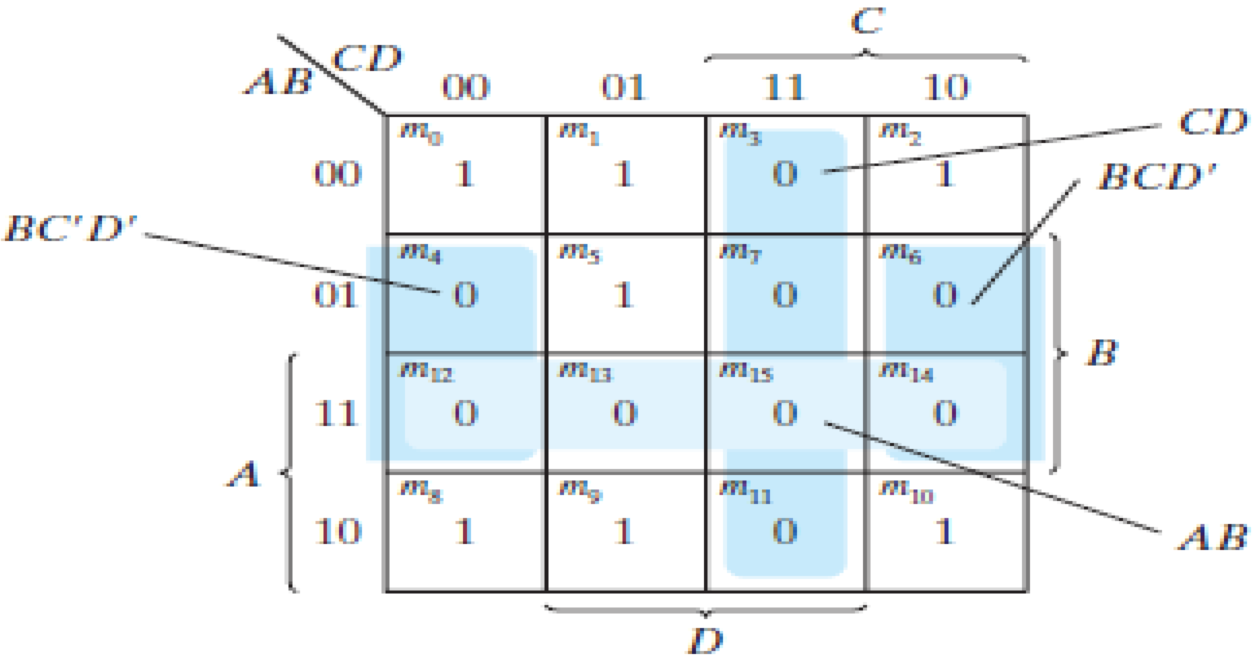
Simplify the Boolean function F(A, B, C, D)= A’B’C’ + B’CD’ + A’BCD’ + AB’C’
The Simplified function is F(A, B, C, D)= B’D’ + B’C’ + A’CD’



Note: $A'B'C'D' + A'B'CD' = A'B'D'$
$AB'C'D' + AB'CD' = AB'D'$
$A'B'D' + AB'D' = B'D'$
$A'B'C' + AB'C' = B'C'$

Simplification of POS Boolean Function

Simplify the Boolean function F(w, x, y, z)= ∑ (0, 1, 2, 5, 8, 9, 10) = ∏(3, 4, 6, 7, 11, 12, 13, 14, 15)
Simplified function in POS is F(w, x, y, z)= (C' +D')(B'+D)(A'+B')



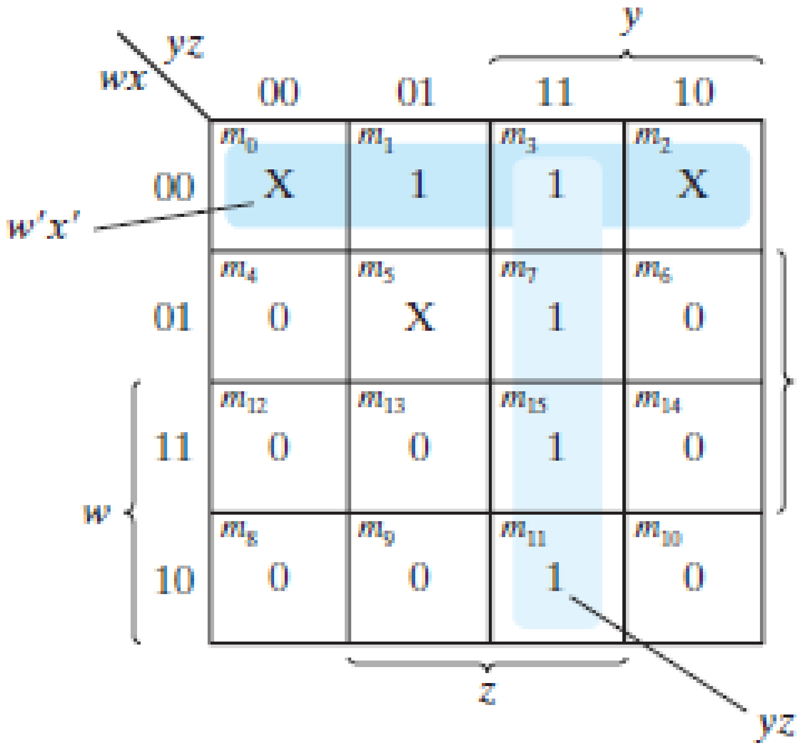
Note: $BC'D' + BCD' = BD'$

DON’T Care Condition in K-Map

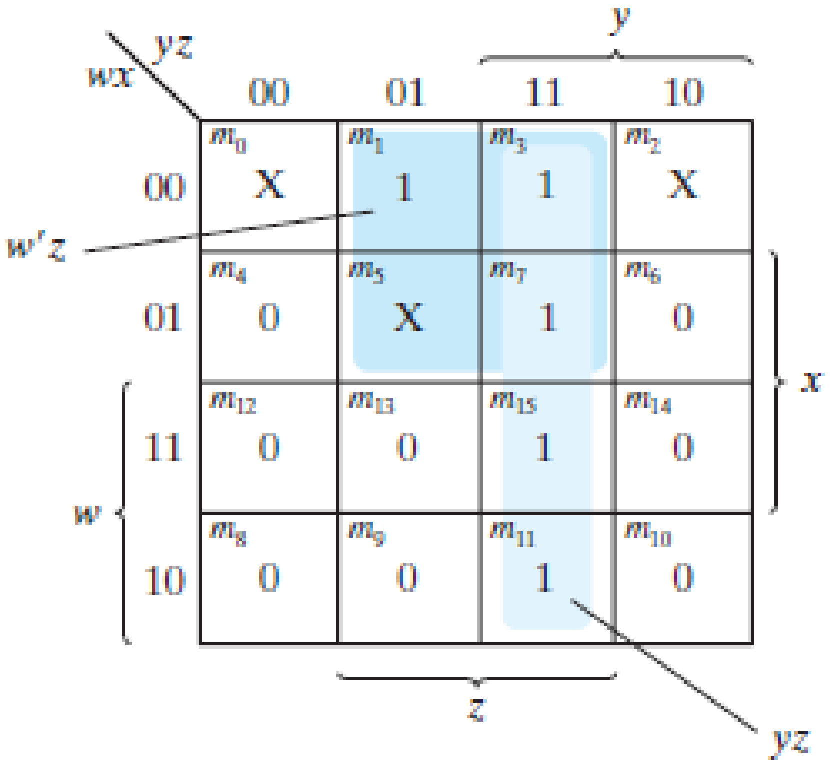
- In some application, We not know the value of some of the minterms/Maxterm
- i.e. the value of minterms/Maxterms are unspecified .
- These unspecified Minterms/Maxterm are know as DON’T Care Condition.
- We put the value ‘X’ in these Minterms/ Maxterm. The value X means it is unknown whether its value is 0 or 1
- When we form a group for simplifying Boolean function, ‘X’ is taken in the group and assumed it to be 1 in case of SOP function or it to be 0 in case of POS function as per the rule of group formation in K-map if needed otherwise ignore .

Examples of DON'T Care Condition in K-Map

Simplify the Boolean function F(w, x, y, z)=∑ (1, 3, 7, 11, 15) which has the don't care condition d(w, x, y, z)= ∑(0, 2, 5)



(a) $F = yz + w'x'$



(b) $F = yz + w'z$

- Reference Book: Digital Design by M Morris Mano, Fifth Edition, Pearson Publication

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- Thanks