An aggregate loss is modeled by a Cmpd Poi with \lambda=1.5 and:

severity $f(x) = \dfrac{\alpha\theta^{\alpha}}{(x+\theta)^{\alpha+1}}$ $x > 0$

$\alpha = 4$; $\theta = 2$

Use NP2 to find $P(S > 9) = ?$

In order to use NP2 I need;

$E[S] = \lambda\mu$ $\quad \gamma_1(S) = \dfrac{\lambda\mu_3'}{\sqrt{\lambda}(\mu_2')^{3/2}}$

$Var(S) = \lambda\mu_2'$

Need $\mu_k' = E[X^k]$; for Pareto;

Simplify & test

$\mu = \dfrac{\theta}{\alpha-1} = 2/3$, $\mu_2' = \dfrac{2\theta^2}{(\alpha-1)(\alpha-2)} = 4/3$

$\mu_3' = \dfrac{6\theta^3}{(\alpha-1)(\alpha-2)(\alpha-3)} = 8$

We can show;

$$E[S] = 1.5 \times 2/3 = 1$$

$$Var(S) = 1.5 \times 4/3 = 2$$

$$\gamma_1(S) = \frac{8}{\sqrt{1.5}\,(4/3)^{3/2}} = 3\sqrt{2} \approx 4.2$$

"NP1" $P(S > 4) \approx 1 - P_r\left(\frac{S - E[S]}{Var(S)} \le 4\sqrt{2}\right)$

$$\left(\frac{4-1}{\sqrt{2}}\right)$$

NP2 $P(S > 4) \approx 1 - \Phi(\underline{s})$

$$s = \sqrt{\frac{9}{\gamma_1^2} + \frac{6 \cdot 2\sqrt{2}}{\gamma_1} + 1} - \frac{3}{\gamma_1}$$

$$\approx \sqrt{5\tfrac{1}{2}} - \frac{\sqrt{2}}{2} \approx 1.6$$

$$\Rightarrow P(S > 4) \approx 1 - \Phi(1.6) \approx 0.05$$