MATH 4281 Risk Theory–Ruin and Credibility

Module 2: Ruin Theory (cont.)

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Reinsurance Review

Reinsurance definitions review

- Proportional
  - quota share: the proportion is the same for all risks
  - surplus: the proportion can vary from risk to risk
- Non-Proportional
  - (individual) excess of loss: on each individual loss (X_i)
  - stop loss: on the aggregate loss (S)
- Cheap (reinsurance premium is the expected value), or non cheap (reinsurance premium is loaded)

Reinsurance definitions review

- In proportional reinsurance the retained proportion α defines who pays what:
  - the insurer pays $Y = \alpha X$
  - the reinsurer pays $Z = (1 - \alpha)X$
- In (individual) excess of loss reinsurance for each individual loss X, the reinsurer pays the excess over a retention (excess point) d
  - the insurer pays $Y = \min(X, d)$
  - the reinsurer pays $Z = (X - d)_+$
- These are the examples we will study today.

Applying Ruin Theory to Reinsurance

Why would Ruin theory help?

- Ruin theory gives us a model to study *options* about reinsurance
- Note that even if $\psi(u)$ can't be calculated, we can still play with the adjustment coefficient R and have qualitative results about $\psi(u)$.
- We can adjust the adjustment coefficient (hence its name..) to meet a goal, such as
  - maximize $R \Leftrightarrow$ minimize $\psi(u)$
  - find the cheapest reinsurance such that $\psi(u)$ is inferior to some level

Assumptions

- Let $0 \leq h(x) \leq x$ be the amount paid by the reinsurer for a claim with amount x i.e:
    - $h(X) = (1 - \alpha)X$ for proportional reinsurance.
    - $h(X) = (X - d)_+$ for excess of loss reinsurance.
- Reinsurance is non cheap and that the loading on reinsurance premiums is $\xi > \theta > 0$. So the reinsurance premium say c_h is:

$$c_h = (1 + \xi)\lambda E[h(X)]$$

Assumptions

- With reinsurance, the Cramér-Lundberg process becomes

$$U(t) = u + (c - c_h)t - \sum_{i=1}^{N(t)} (X_i - h(X_i)).$$

- With reinsurance, the adjustment coefficient, R_h, is then the non-trivial solution to

$$\lambda \left[m_{X-h(X)}(r) - 1 \right] = (c - c_h)r.$$

Equivalently,

$$\lambda + (c - c_h)\, r = \lambda \int_0^\infty e^{r[x-h(x)]} p(x)\, dx.$$

Example 1: Proportional Reinsurance

Suppose claims form a compound Poisson process, with $\lambda = 1$ and $p(x) = 1$ for $0 < x < 1$. Further premiums are received continuously at the rate of $c = 1$. Find the adjustment coefficient if proportional reinsurance is purchased with $\alpha = 0.5$ and with reinsurance loading equal to $\xi = 100\%$.

Solution

Solution

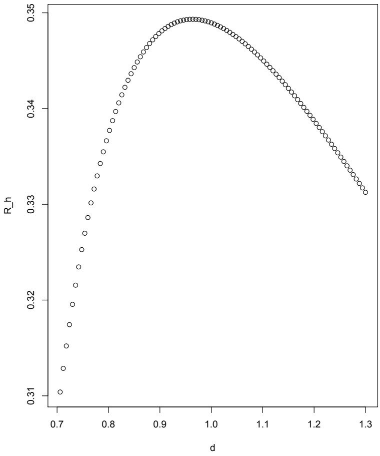
Example 2: Excess of Loss Reinsurance

Consider the Cramér-Lundberg process with X ~exp(1), $\theta = 0.25$ and $\xi = 0.4$. There is proportional reinsurance with retention α, i.e. $X - h(X) = \alpha X$. What is the retention α that will maximize R_h?

Solution

Solution

Solution



Use software to maximize R_h, we have

$$d^* = 0.9632226$$

and

$$R_h^* = 0.3493290,$$

which is much higher (better) than the best we could achieve with proportional reinsurance.

A Theorem

If

- We are in a Cramér-Lundberg setting
- We are considering two reinsurance treaties, one of which is excess of loss
- Both treaties have same expected payments and same premium loadings

then

- The adjustment coefficient with the excess of loss treaty will always be at least as good (high) as with any other type of reinsurance treaty