Solution to Book # 9.72

My apologies everyone- I realize I had the wrong slides open and that may have caused undue confusion when working through this problem.

The slide in the lecture was referring to the theorem that a sum of compound Poisson's is itself compound Poisson. Rather I meant to show Slide 9 from Jan 28- the theorem underpinning the sparse vector algorithm. That is the following:

If $S \sim$ compound Poisson($\lambda, \Pr(X = x_i) = \pi_i$), $i = 1, \ldots, m$ then

$$S = x_1 N_1 + \ldots + x_m N_m,$$

where the N_i's

- represent the number of claims of amount x_i
- are mutually independent
- are Poisson($\lambda_i = \lambda \pi_i$)

Now you don't *need* this to solve it like I said. You can go about it in the top down way I mentioned in class. I'll present both solutions though. As a side note. I know the solutions manual is out there, but this is generally why I don't like using it for anything but a numerical check of the answer. It can give the wrong impression there is only one answer.

Top Down Solution:

Consider that:

$$E[N] = 500(0.01) + 500(0.02) = 15$$

Which means if S is modelled as a compound Poisson:

$$E[S] = E[N]E[X] = 15((x)p_1 + (2x)p_2)$$

Where x and $2x$ are the severities of class 1 and class 2 that occur with frequency p_1 and p_2 respectively. So how do we find p_i?

Note that:

$$p_1 = P(Class1|Claim) = \frac{P(Claim|Class1)}{P(Claim)} P(Class1)$$

If:

- $P(Claim|Class1) = 0.01$
- $P(Class1) = 500/1000 = 0.5$
- $P(Claim) = P(Claim|Class1)P(Class1) + P(Claim|Class2)P(Class2) = (0.01)(0.5) + (0.02)(0.5) = 0.015$

Then:

$$p_1 = \frac{0.01}{0.015} 0.5 = 1/3$$

And this is intuitive as it's the relative frequency we see in the table. Finally:

$$Var(S) = 15E[X^2] = 15((1/3)x^2 + (2/3)(2x)^2)$$

And we can solve for x given the variance.

Bottom Up Solution: We have that:

$$S = (500\mathbf{1}_1)x + (500\mathbf{1}_2)2x$$

where $\mathbf{1}_1$ is the indicator of a claim in class one i.e. $E[\mathbf{1}_1] = 0.01$.

We know from the theorem for the sparse vector algorithm that if S is compound Poisson we can match $(500\mathbf{1}_1)$ with $N_1 \sim Poi(p_1\lambda)$ and $(500\mathbf{1}_2)$ with $N_2 \sim Poi(p_2\lambda)$. If $E[500\mathbf{1}_1] = 5$ and $E[500\mathbf{1}_2] = 10$ then:

$$\begin{aligned} p_1\lambda &= 5 \\ p_2\lambda &= 10 \end{aligned}$$

Solving gives $p_1 = 1/3$, $p_2 = 2/3$ and $\lambda = 15$ as desired.