Report on the submission *On the computational complexity of detecting possibilistic locality*, by Andrew W. Simmons.

The manuscript concerns an interesting computational decision problem arising in quantum mechanics, by way of a so-called "Bell experiment". A system is considered involving various quantities, which can be measured independently and without communication by different experimenters. The experimenters can choose one of several different possible measurements to make. Experimental observations then lead to an estimate of probabilities for outcomes in terms of the combinations of measurements chosen by the experimenters. In simplified form: some combinations of outcomes may be found to be possible, while others to be impossible. The system is said to have an explanation based on a local hidden variable, if there is a family of values for the possible measurements that can be made by the experimenters that determine the outcomes independently of the combination of measurements that they choose to make. In general it turns out that this is not necessarily the case: it is possible in such a system for the choice of measurements to be made by the experimenters (simultaneously and without any communication) to influence the observed outcome (Bell's Theorem).

Given a table of possibilities: can it be explained by a local hidden-variable model? This decision problem lies in **NP** and is known to be **NP**-complete in general. Various parameters that might be restricted: the number of experimenters (the dimension), the number of possible measurements, the number of possible outcomes. Abramsky, Gottlob and Kolaitis showed that the problem is **NP**-complete in dimension 3 (and higher) when the available measurements have at least two outcomes each, as well as dimension 2 when there are at least 3 outcomes for the experimenters. When these particular parameters are lowered beyond these stated thresholds, the problem becomes tractable. The present manuscript finds a genuine intermediate set-up between these cases by dropping the uniformity in the number of outcomes in measurements. More specifically, the manuscript considers dimension 2, but the measurements available to one experimenter have just two outcomes each, while those available to the other have 3. The main contribution of the article is that detecting a local hidden-variable model for such a table of possibilities is **NP**-complete. This is achieved by way of first showing a case of **NP**-completeness for a 2-robust satisfiability problem (do all assignments on 2-variables extend to a solution?), and then subsequent reduction to the local hidden-variable model problem. There is also an extended contribution (Theorem 3) toward a further possible intractability result which is left open.

Overall I found the results to be interesting and I would support publication. I think the article could be improved with some further explanation to the reader less expert in the quantum mechanical aspects of the article.

First, I would have liked some more explanation of Hardy's example, given that it is some sort of archetypal representative of the kind of configurations that are throughout the article, and it is the first to appear in the article. I found it necessary to consult references [1] and [8], where it was more clearly explained. In the present article all that is stated is that "we can quickly see". I can now see that to someone familiar with the idea it probably is quick to see, but it is still of interest to a wider audience.

Second, I found Section IV quite challenging to read, though perhaps this is due to relative unfamiliarity with some of the concepts. Perhaps some more explanation and background there would be useful.

First column, page 1. Sometimes **NP**-complete, sometimes **NP**-Complete. Also, include the citation for the *Avis et al* article: it's currently not in the references.

Throughout: capitalize the first letter when referring to numbered items, as in Figure 2, Lemma 3, Section IV.

Last symbol before Definition 1. Should 1/8 be 3/8?

Sentence before Theorem 1. It is good to avoid using "this" too often, as it is often ambiguous as to what the "this" is referring to. In the sentence before Theorem 1, you presumably mean the statement in the rest of the sentence, but the initial interpretation is what has just been read (the previous sentence), until it becomes clear that it can't really be referring to that.

Definition 2. This is not actually the definition of r-robust satisfiability given in [1]. In [1] it is required that the assignments not violate a constraint (in your case a 3SAT clause) nor any projection of a constraint. It doesn't affect anything in your subsequent arguments, as in the case of 2-robust satisfiability for 3SAT, none of this comes into play. Provided that no clause contains only one or two variables, an assignment from two variables cannot violate any clause, nor any projection of a clause. (The projections of 3SAT relations of course offer no restriction on pairs and singletons at all.)

Page 4, item 1. Neither nor (not neither or).

Page 4, third last line of proof of Lemma 1. sitll$\mapsto$still

Page 5, first paragraph (the critical step of finishing the proof). Is it possible to slightly expand the explanation here? The rest of the proof is nicely developed.

Page 5, half way down first column. deterministic is misspelt.

Page 5, start of paragraph after the end of the proof of Theorem 3. The use of "this" in the first sentence here is particularly confusing. What is "this problem"? Presumably you mean the problem that is the domain of the reduction in the proof of the lemma, but I think that grammatically you are referring to the reduction itself when you write "this". Then further into the sentence what is "this proof". Is it the one you just referred to, or the one that is the overall point of the statement?

Page 5, bottom. When you write "one can show", do you mean that you can and are not giving the proof, or that it is shown in reference [9]? I wasn't actually familiar with the NPA hierarchy, so the comment was lost on me.

Page 6, first sentence. There is something wrong with the sentence at the moment.

Page 6. Perhaps to be more self contained, some brief introduction of the Bloch sphere concept might be useful.

Page 6, sentence finishing paragraph 1. I'm not sure there is an Equation IV in the article, but regardless of this possible typo, can the sentence be rewritten (and possibly split into two sentences)?

Page 6, a few lines further down. This is the first reference to Figure 2; is there a reason why the figure appears on page 2?

Page 6, penultimate line of penultimate paragraph: there is a space missing before "eg".

Page 7, first paragraph of column 2. What do you mean by "opposite sense". I think I can guess, but it should be stated explicitly. With regards to the computational issue that remains unresolved, there have been a number of other robust satisfiability problems recently shown to be **NP**-complete, and these might possibly be of interest to you in this sort of setting. Indeed the article [Lucy Ham, Gap theorems for robust satisfiability: Boolean CSPs and beyond. Theoretical Computer Science 676 (2017), 69–91] completely classifies the tractability of 2-robust satisfiability for all fixed template constraint problems over two element domains (subject to some amendment of the local compatibility condition omitted from your Definition 2). I think the problem you are interested in is a restriction of a fixed template CSP, rather than the full fixed template CSP itself, but there might still be other avenues of attack to be found here.