In this article the author considers the question of deciding possibilistic locality/nonlocality from empirical possibility tables. Previous results have shown certain scenarios (scenarios dictate the shape of the possibility table) to allow for efficient algorithms and identified others in which the problem becomes NP-Hard. The author identifies the remaining scenario which does not fall into the domain of applicability of previous results, and completes the characterisation of the complexity of deciding possibilistic locality by showing (2,3)-Possloc to be NP-Hard. This is the first main contribution.

The second contribution is that the author considers the issue of deciding complexity for classes of empirical possibility tables which are realisable by quantum theory (previous results hold for non-signalling tables, of which quantum realisable tables are a subclass). This is a very difficult topic on which relatively little is known. While the author does not resolve the issue, he does make notable progress by showing how to quantumly embed a large class of 3-SAT instances into (2,3)-Possloc. Moreover the reasoning and discussion here are interesting in themselves, and I believe they will be of interest and value to researchers working on this and other topics relating to quantum realisability.

Overall, if I were to judge only on the basis of the scientific content, I would be very glad to recommend this article for publication. However, I refrain from making this recommendation yet for the following reason. There are issues with the presentation, which I strongly believe needs to be significantly reworked before the article is in a suitable form to be published. I provide some general comments for the author below, together with many specific recommendations. I do think that these issues can be remedied in an updated version of the manuscript, and if so I do think that this article would make for a very nice publication.

**\*General Comment\***

Speaking as someone reasonably well acquainted with the topics considered in the article, I already found that I had to work hard to keep up. Potentially this article could be of interest to quite a wide audience of researchers working in a number of fields. However, in the current form I believe that, aside a handful of specialists on these precise issues, the interested reader is not presented with enough of the basics to easily engage with the content.

As a general recommendation, I think the article would greatly benefit from a Background section immediately following the introduction in which the basic concepts are clearly introduced. For instance this could contain some explanation of how possibility tables could arise, what they are supposed to correspond to in the physical context, various devices for reasoning on possibility tables which are used throughout the article, such as the notion of possibilistic no signalling, deterministic grids, etc. It would also be a good place to introduce, some basics of the quantum aspects of the paper (which ultimately are not many, and putting this here will reassure non-quantum readers), and introducing in some detail the very interesting mode of reasoning based on the equatorial plane intersected with the Bloch sphere from figures 2 and 3.

A second general recommendation would be to stress the generality of the problem from the outset, mentioning that locality is ultimately a feature of possibility tables, which in the case of quantumly realisable tables relates to Bell, Hardy, etc. It would also be good to reassure the non-quantum reader that the article could be read and understood without specialist knowledge of quantum theory, highlighting the sections where no quantum theory is involved and explaining that those aspects that are used in the remaining sections are introduced in the background section.

**\*Specific Comments and Suggestions\***

Abstract:

\* “quantitatively” is not a good word here. Indeed “qualitatively” would appear to be more suitable to me. If we think of quantifying nonlocality one would immediately tend to imagine Bell inequality violations, but these are not directly relevant to logical nonlocality.

\* “table of possibilities” is too vague even for an abstract. You could mention that these may arise from empirical data for instance.

Introduction:

\* A suggestion would be to mention how such tables may arise in other, non-physical contexts. For instance I believe that Abramsky has considered how in the context of relational databases locality corresponds to the existence of universal relations, etc. This point is only a suggestion, however.

~~\* When Bell experiments are introduced, I think it would be important for the non-quantum reader to be reassured that the salient features are independent of understanding how the correlations arise.~~

\* Two notions are mentioned without any explanation or indication of what they are: “correlation polytopes” and “compatibility graphs”. These do not necessarily have to be explained in great detail here, but they do need to be better related to what has been said in the previous paragraph for instance. If no further comment is made, the risk is that the non-specialist reader will switch off already.

\* Typo: seperate->separate

Section II:

\* “a table of probabilities for a Bell experiement” is too vague. This could be remedied by explaining in a background section how probability/possibility tables arise.

\* On the topic of how possibility tables arise from possibility tables, there are some non-trivial questions here, some of which are addressed for instance by Abramsky’s Relational Hidden Variables paper, or Mansfield and Soares Barbosa’s Extendability paper.

\* The wrong probability is given above definition 1. Unless the labelling is changed, this should surely be 3/8

\* I think it needs to be made even more explicitly clear at some point in this section that the complexity problem being considered relates to bipartite scenarios.

\* e.g. “with j and k being the maximum number…” is a difficult sentence to parse, and could perhaps be reworked.

\* Tables should be numbered and referred to at the appropriate points in the text (throughout the article, and not only in this section); likewise for figures, which don’t seem to be referred to everywhere that could be relevant in the main text

\* Figure 2 is a very interesting device, but it’s not explained sufficiently, nor how it relates to possibility tables or paradoxes or locality. This badly needs explanation especially for non specialists who will be unable to guess at its significance.

\* The proof of theorem 1 would appear to me to be rather different to that which can be found in Mansfield and Fritz. Perhaps this is worth mentioning explicitly somewhere too.

\* “Constructing the possibility table for this steering scenario…” I would have thought that a given scenario could give rise to many different possibility tables. Perhaps we have different notions of scenario in mind, but as there’s no definition of what a steering scenario is I can’t be sure. This is typically the kind of term which needs a clear definition somewhere.

\* It needs to be explained somewhere why the Table at the top of p3 is quantum realisable. This is the kind of thing that quantum specialists will be able to deduce, with some consideration, but where others will be lost.

\* The notion of tables which are “quantumly accessible” are mentioned for the first time on this page. I think this could have been introduced earlier (e.g. in a background section).

\* The fact that “the third measurement row acts to convert…” will be clear to those who are used to dealing with such tables, but this needs more explanation for other readers. This requires the kind of reasoning which could for instance be briefly introduced in a background section.

\* “completion of the highlighted 1”. The term completion has not been defined and will not be clear to those who have not recently read and retained the terminology of e.g. the Mansfield and Fritz paper.

\* The fact that the family of tables has a maximum probability of paradox of 1/2 could do with further explanation or proof.

\* The word “violation” is used here. I think it needs to be explained why probability of paradox relates to violation of a Bell inequality which will not be immediately clear for most readers

\* “the paradoxical probability for Hardy’s own…” should perhaps more accurately be the maximum quantum paradoxical probability?

Section III:

\* Something needs to be cleared up around Definition 2. It’s not explained what variables correspond to in the probability tables, and perhaps an example would help. This again could be contained in a background section or could be easily relatable to material having appeared earlier in a background section.

\* I find Definition 3 to be confusingly stated. Instead of using brackets, would it not be clearer to first define 0-validity and after words say that 1-valid is defined similarly.

\* It was not clear to me why only clauses of the kinds found in equations (1) and (2) are looked at.

\* I think that the proof of Lemma 1 would benefit from presenting the reader first with an outline before going into the details. I found myself trying to follow many details without a global idea of where it was supposed to be leading me

\* It was also confusing to me to read “Now, to test whether or not the instance is 2-robust” when from the statement of the Lemma I thought that this was being assumed from the outset

\* The final paragraph being placed at the start of the proof would have helped

\* Typo: “sitll”

\* When in the proof of Lemma 2 I see “we add a measurement to the party…” I can’t help but feel that measurement (or what it corresponds to in a possibility table) and party should have been defined clearly before now. Those not familiar with Bell experiments, etc, and tables of the kind that are under consideration, are likely to have difficulty in following at this point again otherwise.

\* The term “deterministic grid” is used near the beginning of p5. This is very specialised terminology and has not been defined elsewhere in the article.

\* The deduction at the end of the proof of Lemma 2 relies on no-signalling, but this has not been explained anywhere, and the non-specialist reader would surely have trouble in reproducing the reasoning here

\* The final paragraph of this section could actually be stated as a result (a proposition perhaps) in its own right

\* “any 1” in this paragraph should also read “some 1”

Section IV:

\* The first sentence here would be fine if it was simply recalling something from earlier in the article, but it’s too late and too brief to be introducing such a point at this stage. The notion of generalised no-signalling distribution should be made clear in a background section perhaps.

\* In the first paragraph of the proof of Theorem 3, I failed to understand why PVMs can be taken without loss of generality. Does it have something to do with the idea that convex mixtures can only contain less zeroes maybe? In any case this certainly needs more explanation, as I’m not convinced of it as it stands

\* The NPA hierarchy is mentioned at the end of page 9. I think that at least a brief sketch of what the NPA hierarchy is or is useful for would be useful here. This may interest many readers, but as it is risks alienating readers who are not already familiar with it.

\* Near the beginning of p6 there is a typo: “we will now present a constructions will”

\* Reference is made to equation IV. There’s something dodgy with the reference, which is crucial here, so the argument became somewhat unclear to me at that point.

\* The pointer to a generalised Hardy paradox “given above” is not clear enough. Also, I wonder if it’s not a bit confusing to refer to generalised Hardy paradox especially given that there are many alternative generalisations, and that (if I located the correct one in this instance) the paradox in question is not really in the original Hardy form. Perhaps there is some better terminology here, though I only mention this as a thought rather than a firm recommendation for a change.

\* Aside from introducing the diagrams earlier, I would like to be told what I’m supposed to interpret as positive and negative literals in figure 3

\* The final bullet point on p6 left me wondering why the statement holds. Again, perhaps this needs more explanation and is again the kind of reasoning that could be introduced in a background section

As additional comments, I quite liked the discussion of why it is not clear that relaxing the assumptions, on the left hand column of p6, would not clearly lead to being able to embed more instances. This is very insightful. I also found the conclusion section to be very good.