Assigment # 1

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Q1) $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3}$

for a function F_x, $f^{(n)}$ is used to approximate the Lagrange form of the remainder

$$R_n = \left| \frac{f^{(n+1)}(c)\,(x - x_0)^{n+1}}{(n+1)!} \right|$$ Where c is when Maclaurin series is calculated

only if its $(n+1)^{th}$ derivative is continuous

$f(x) = \ln(1+x)$

$= f^{(n+1)}(x) = \frac{-6}{(1+x)^n}$

$f^{(n+1)}(c) = \frac{-6}{(1+c)^4}$

Remainder is:

$R_4 = \left| \frac{-6}{(1+c)^4} \times \frac{x^4}{4!} \right|$

$= \frac{x^4}{4(1+c)^4} < 5\times10^{-4}$

Max remainder value when $c=0$

$\frac{x^4}{4} < 5\times10^{-4}$

$= x < \sqrt[4]{20\times10^{-4}}$

$x < 0.211$

$\therefore$ the value of x is less than 0.211

(Q2) a) Lagrange interpolating polynomial

$$P(x) = L_{1,0}(x)y_0 + L_{1,1}(x)y_1$$

$$= \frac{(x-x_1)}{(x_0-x_1)} y_0 + \frac{(x-x_0)}{(x_1-x_0)} y_1 \rightarrow (1)$$

b) Newton's interpolating polynomial

$$f(x) = y_0 + f[x_0, x_1](x-x_0)$$

where $f[x_0, x_1] = \frac{y_1 - y_0}{x_1 - x_0}$

$$f(x) = y_0 + \frac{y_1 - y_0}{x_1 - x_0}(x - x_0) \rightarrow (2)$$

c) Linear Spline

$$S_0(x) = y_0 + \frac{y_1 - y_0}{x_1 - x_0}(x - x_0) \rightarrow (3)$$

d) Bezier Curve:

$(x_0, y_0) \rightarrow P_1$ $\quad (x_1, y_1) \rightarrow P_2$

$$B(t) = (1-t)P_1 + tP_2$$

$$= (1-t)\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} + t\begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$$

$$B(t) = \begin{pmatrix} x_0 + t(x_1 - x_0) \\ y_0 + t(y_1 - y_0) \end{pmatrix}$$

$$x = x_0 + t(x_1 - x_0)$$

$$y = y_0 + t(y_1 - y_0)$$

$$t = \frac{x - x_0}{x_1 - x_0}$$

$$y = y_0 + \left(\frac{x - x_0}{x_1 - x_0}\right)(y_1 - y_0) \rightarrow (4)$$

Since (4) = (2) and (3)

$$(4): y = \frac{y_0(x_1 - x_0) + (x - x_0)(y_1 - y_0)}{(x_1 - x_0)}$$

$$= \frac{x_1 y_0 - x_0 y_0 + x y_1 - x y_0 - x_0 y_1 + x_0 y_0}{(x_1 - x_0)}$$

$$y = \frac{(x - x_0)y_1 - (x - x_1)y_0}{x_1 - x_0}$$

which is equivalent to

$$y = \frac{(x - x_1)}{(x_0 - x_1)} y_0 + \frac{(x - x_0)}{(x_1 - y_0)} y_1$$

equal to ①

∴ Bazier curve is equivalent to other forms

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Q3) The Bezier curve will be given by:

(a) $P(t) = (1-t)^4 P_0 + 4(1-t)^3 t P_1 + 6(1-t)^2 t^2 P_2 + 4(1-t)t^3 P_3 + t^4 P_4$

t ranges from 0 - 1.

$$P(t) = (1-t)^4 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + 4(1-t)^3 t \begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix} + 6(1-t)^2 t^2 \begin{bmatrix} 3 \\ 1 \\ 6 \end{bmatrix} + 4(1-t)t^3 \begin{bmatrix} 4 \\ 2 \\ 6 \end{bmatrix} + t^4 \begin{bmatrix} 5 \\ 4 \\ 5 \end{bmatrix} \Rightarrow (i)$$

(b) Plug $t = 0.35$ into (i)

$$P(0.35) = (1-0.35)^4 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + 4(1-0.35)^3 (0.35) \begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix} + 6(1-0.35)^2 (0.35)^2 \begin{bmatrix} 3 \\ 1 \\ 6 \end{bmatrix} + 4(1-0.35)(0.35)^3 \begin{bmatrix} 4 \\ 2 \\ 6 \end{bmatrix} + (0.35)^4 \begin{bmatrix} 5 \\ 4 \\ 5 \end{bmatrix}$$

$$= (0.18) \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + 0.38 \begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix} + 0.31 \begin{bmatrix} 3 \\ 1 \\ 6 \end{bmatrix} + 0.11 \begin{bmatrix} 4 \\ 2 \\ 6 \end{bmatrix} + 0.02 \begin{bmatrix} 5 \\ 4 \\ 5 \end{bmatrix}$$

$$P(0.35) = \begin{bmatrix} 0.18 + 0.79 + 0.93 + 0.44 + 0.1 \\ 0.36 + 0.38 + 0.31 + 0.22 + 0.08 \\ 0.54 + 1.9 + 1.86 + 0.66 + 0.1 \end{bmatrix}$$

$$P(0.35) = \begin{bmatrix} 2.44 \\ 1.35 \\ 5.06 \end{bmatrix}$$

(c) Take derivative of (i)

$$P'(t) = 4(1-t)^3 P_0 + 4P_1[3(1-t)^2(-1)t + (1-t)^3] + 6P_2[2(1-t)(-1)t^2 + 2t(1-t)^2] + 4P_3[(-1)t^3 + 3t^2(1-t)] + 4t^3 P_4$$

Plug the values for $t = 0.5$,

$$P'(0.5) = -4(1-0.5)^2\begin{bmatrix}1\\2\\3\end{bmatrix} + 4(1-0.5)^3\begin{bmatrix}2\\1\\5\end{bmatrix} - 12\cancel{\{} \cancel{+12\,t\,0.5}$$

$$\cancel{+12\,t\,0} - 12(1-0.5)^2(0.5)\begin{bmatrix}2\\1\\5\end{bmatrix} - 12(1-0.5)(0.5)^2\begin{bmatrix}3\\1\\6\end{bmatrix}$$

$$+12(1-0.5)^2(0.5)\begin{bmatrix}3\\1\\6\end{bmatrix} + 12(1-0.5)(0.5)^2\begin{bmatrix}4\\2\\6\end{bmatrix}$$

$$-4(0.5)^3\begin{bmatrix}4\\2\\6\end{bmatrix} + 4(0.5)^3\begin{bmatrix}5\\4\\5\end{bmatrix}$$

$$P'(0.5) = \begin{bmatrix}-0.5+1-3-4.5+6-2+2.5\\-1+0.5-1.5-1.5+1.5+3-1+2\\-1.5+2.5-7.5-9+9+9-3+2.5\end{bmatrix}$$

$$P'(0.5) = \begin{bmatrix}4\\3\\2\end{bmatrix}$$

(d) Acceleration at $t = 0.75$

$$P''(t) = 12P_0(1-t)^2(-1) + 12P_1(1-t)^3(-1)$$

$$-12P_1[2(1-t)(-1)t + (1-t^2)]$$

$$-12P_2[(-1)t^2 + 2t(1-t)] + 12P_3[(-1)t^2 + 2t(1-t)]$$

$$-12t^2P_3 + 12t^2P_4$$

$$= 12(1-t)^2[P_0 - 2P_1 + P_2] - 24(1-t)t[P_1 - 2P_2 + P_3]$$

$$+12t^2[P_2 - 2P_3 + P_4]$$

Plug values

$$P''(0.75) = 12(1-0.75)^2\left[\begin{pmatrix}1\\2\\3\end{pmatrix} - \begin{pmatrix}4\\2\\10\end{pmatrix} + \begin{pmatrix}3\\1\\6\end{pmatrix}\right]$$

$$-24(1-0.75)(0.75)\left[\begin{pmatrix}2\\1\\5\end{pmatrix} - \begin{pmatrix}6\\2\\12\end{pmatrix} + \begin{pmatrix}4\\2\\6\end{pmatrix}\right]$$

$$+12(0.75)^2\left[\begin{pmatrix}3\\1\\6\end{pmatrix} - \begin{pmatrix}8\\4\\12\end{pmatrix} + \begin{pmatrix}5\\4\\5\end{pmatrix}\right]$$

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$$P''(0.75) = 0.75 \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} - 4.5 \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} + 6.75 \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$$

$$P''(0.75) = \begin{bmatrix} 0 - 0 + 0 \\ 0.75 - 4.5 + 6.75 \\ -0.75 + 4.5 - 6.75 \end{bmatrix}$$

$$P''(0.75) = \begin{bmatrix} 0 \\ 3 \\ -3 \end{bmatrix}$$

Q4) For Cubic Spline:

$$S_i(x) = a_i + b_i(x-x_i) + c_i(x-x_i)^2 + d_i(x-x_i)^3$$

Checking Left End Conditions:

$S_i(x_i) = f_i, \quad 0 \le i \le n-1$

$a_i + b_i(x_i - x_i) + c_i(x_i - x_i)^2 + d_i(x_i - x_i)^3 = f_i$

$a_i = f_i \rightarrow (1)$

Right End conditions:

$S_i(x_{i+1}) = f_{i+1}, \quad 0 \le i \le n-1$

$a_i + b_i(x_{i+1} - x_i) + c_i(x_{i+1} - x_i)^2 + d_i(x_{i+1} - x_i)^3 = f_{i+1}$

$a_i + b_i h_i + c_i h_i^2 + d_i h_i^3 = f_{i+1} \quad (h_i = x_{i+1} - x_i)$

$a_i + b_i h_i + c_i h_i^2 + d_i h_i^3 = a_{i+1} \rightarrow (2)$

Continuity of $S'(x)$:

$S_i'(x_{i+1}) = S_{i+1}'(x_{i+1}); \quad 0 \le i \le n-2$

$b_i + 2c_i(x_{i+1} - x_i) + 3d_i(x_{i+1} - x_i)^2$

$= b_{i+1} + 2c_{i+1}(x_{i+1} - x_{i+1})^2 + 3d_{i+1}(x_{i+1} - x_{i+1})^2$

Continuity of $S''(x)$:

$S_i''(x_{i+1}) = S_{i+1}''(x_{i+1}), \quad 0 \le i \le n-2$

$2c_i + 6d_i(x_{i+1} - x_i) = 2c_{i+1} + 6d_{i+1}(x_{i+1} - x_{i+1})$

$c_i + 3d_i h_i = c_{i+1} \rightarrow (4)$

$d_i = \dfrac{c_{i+1} - c_i}{3h_i} \rightarrow (5)$

Putting (5) in (2):

$$a_i + b_i h_i + c_i h_i^2 + \frac{c_{i+1} - c_i}{3h_i} h_i^3 = a_{i+1}$$

$$a_i + b_i h + \frac{h_i^2}{3}(2c_i - c_{i+1}) = a_{i+1} \rightarrow (6)$$

Putting (5) in (3):

$$b_i + 2c_i h_i + \frac{3(c_{i+1} - c_i) h_i^2}{3h_i} = b_{i+1}$$

$$b_i + h_i(c_i + c_{i+1}) = b_{i+1} \rightarrow (7)$$

from (6),

$$b_i h_i = a_{i+1} - a_i - \frac{h_i^2}{3}(2c_i + c_{i+1})$$

$$b_i = \frac{a_{i+1} - a_i}{h_i} - \frac{h_i}{3}(2c_i + c_{i+1}) \rightarrow (8)$$

Putting $i = i-1$ in (7) & (8): $b_{i-1} + h_{i-1}(c_{i-1} + c_i) = b_i \rightarrow (9)$

$$b_{i-1} = \frac{a_i - a_{i-1}}{h_{i-1}} - \frac{h_{i-1}}{3}(2c_{i-1} + c_i) \rightarrow (10)$$

Put (8) and (10) in (9)

$$\frac{a_i - a_{i-1}}{h_{i-1}} = \frac{h_{i-1}}{3}(2c_{i-1} - c_i) + h_{i-1}(c_{i-1} + c_i)$$

$$= \frac{a_{i+1} - a_i}{h_i} - \frac{h_i}{3}(2c_i + 2c_{i+1})$$

$$\frac{a_{i+1} - a_i}{h_{i-1}} - \frac{a_i - a_{i-1}}{h_{i-1}} = \frac{h_i}{3}(2c_i + c_{i+1}) - \frac{h_{i-1}}{3}(2c_{i-1} + c_i)$$

$$+ h_{i-1}(c_{i-1} + c_i)$$

$$3\left[\frac{F_{i+1} - F_i}{h_i} - \frac{F_i - F_{i-1}}{h_{i-1}}\right] = 2h_i c_i + 2h_{i-1} c_i + h_i c_{i+1} + h_{i-1} c_{i-1}$$

$$3[F[x_i, x_{i+1}] - F[x_{i-1}, x_i]] = h_{i-1} c_{i-1} + 2(h_{i-1} + h_i) c_i + h_i c_{i+1}$$

$$h_{i-1} c_{i-1} + 2(h_{i-1} + h_i) + h_i c_{i+1} = 3(F[x_i, x_{i+1}] - F[x_{i-1}, x_i])$$

For natural Cubic Spline,

$$S''(x_0) = S''(x_n) = 0$$

$$\Rightarrow c_0 = 0 \rightarrow (12) \Rightarrow c_n = 0 \rightarrow (13)$$

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Using (11), (12), (13), we get this matrix:

$$
\left[\begin{array}{ccccc|c}
c_0 & c_1 & c_2 & \cdots & c_n & \\
1 & 0 & 0 & \cdots & 0 & 0 \\
h_0 & 2(h_0+h_1) & h_1 & & & 3[F[x_1,x_2]-F[x_0,x_1]] \\
0 & h_1 & 2(h_1+h_2) & h_2 & & 3[F[x_2,x_3]-F[x_1,x_2]] \\
\vdots & \vdots & \vdots & \vdots & 0 & \vdots \\
0 & h_{n-2} & 2(h_{n-2}+h_{n-1}) & h_{n-1} & & 3(F[x_{n-1},x_n]-F[x_{n-2},x_{n-3}]) \\
0 & \cdots\ 0 & 0 & & 1 & 0
\end{array}\right]
$$

The above matrix is tridiagonal matrix:
Only the main diagonal, sub and super diagonal entries are not 0

(b)

x	1.0	1.2	1.4	1.6	1.8	2.0	2.2
y	2.1783	3.3201	4.0552	4.9530	6.0496	7.3891	9.0250

we can see,

$h=2,\ n=6$

$S_i(x)=a_i+b_i(x-x_i)+c_i(x-x_i)^2+d_i(x-x_i)^3 \rightarrow (4)\ ,\ x_i\le x\le x_{i+1}$

Using iterative matrix:

$$
\left[\begin{array}{ccccccc|c}
c_0 & c_1 & c_2 & c_3 & c_4 & c_5 & c_6 & \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
h_0 & 2(h_0+h_1) & 0 & 0 & 0 & 0 & 0 & 3(F[x_1,x_2]-F[x_0,x_1]) \\
0 & h_1 & 2(h_1+h_2) & h_2 & 0 & 0 & 0 & 3(F[x_2,x_3]-F[x_1,x_2]) \\
0 & 0 & h_2 & 2(h_2+h_3) & 0 & 0 & 0 & 3(F[x_3,x_4]-F[x_2,x_3]) \\
0 & 0 & 0 & h_3 & 2(h_3+h_4) & h_4 & 0 & 3(F[x_4,x_5]-F[x_3,x_4]) \\
0 & 0 & 0 & 0 & h_4 & 2(h_4+h_0) & h_5 & 3(F[x_5,x_0]-F[x_4,x_5]) \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0
\end{array}\right]
$$

$$= \left[\begin{array}{ccccccc|c} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.2 & 0.8 & 0.2 & 0 & 0 & 0 & 0 & -6.1005 \\ 0 & 0.2 & 0.8 & 0.2 & 0 & 0 & 0 & 2.4405 \\ 0 & 0 & 0.2 & 0.8 & 0.2 & 0 & 0 & 2.982 \\ 0 & 0 & 0 & 0.2 & 0.8 & 0.2 & 0 & 3.6435 \\ 0 & 0 & 0 & 0 & 0.2 & 0.8 & 0.2 & 4.446 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

Using (a)

$S_0(x) = -14.712792x^3 + 44.136875x^2 - 37.839383x + 10.5931$

$S_1(x) = 22.723956x^3 - 90.633825x^2 + 123.885217x - 54.09674$

$S_2(x) = -5.0085472x^3 + 25.84875x^2 - 39.181883x - \cancel{5.97956}$ 22.00124

$S_3(x) = 1.822708x^3 - 6.947125x^2 + 13.282117x - 5.97956$

$S_4(x) = 3.230208x^3 - 14.547625x^2 + 26.963017x - 14.1881$

$S_5(x) = -8.056042x^3 + 53.169875x^2 - 108.471893x + 76.1019$

Bazier Curve

$P_0 \begin{pmatrix} 1.0 \\ 2.1783 \end{pmatrix}$, $P_1 = \begin{pmatrix} 1.2 \\ 3.3201 \end{pmatrix}$, $P_2 = \begin{pmatrix} 1.4 \\ 4.0552 \end{pmatrix}$, $P_3 = \begin{pmatrix} 1.6 \\ 4.9530 \end{pmatrix}$

$P_5 \begin{pmatrix} 1.8 \\ 6.0496 \end{pmatrix}$, $P_6 = \begin{pmatrix} 2.0 \\ 7.3891 \end{pmatrix}$, $P_7 = \begin{pmatrix} 2.2 \\ 9.0250 \end{pmatrix}$

Bazier Curve is given as:

$$B_{p_0, p_1 \ldots p_i}(t) = \sum_{i=0}^{6} \binom{6}{i} t^i (1-t)^{6-i} P_i$$

$$B_{p_0,p_1 \ldots p_6}(t) = \binom{6}{0} t^0 (1-t)^6 \begin{pmatrix} 1 \\ 2.1783 \end{pmatrix} + \binom{6}{1} + (1-t)^5 \begin{pmatrix} 1.2 \\ 3.32 \end{pmatrix}$$
$$+ \binom{6}{2} t^2 (1-t)^4 \begin{pmatrix} 1.4 \\ 4.0552 \end{pmatrix} + \binom{6}{3} t^3 (1-t)^3 \begin{pmatrix} 1.6 \\ 4.9530 \end{pmatrix}$$
$$+ \binom{6}{4} t^4 (1-t)^2 \begin{pmatrix} 1.8 \\ 6.0496 \end{pmatrix} + \binom{6}{5} t^5 (1-t) \begin{pmatrix} 2.0 \\ 7.3891 \end{pmatrix} + \binom{6}{6} t^6 (1-t)^0 \begin{pmatrix} 2.2 \\ 9.025 \end{pmatrix}$$

$$B_{p_1, p_2 \ldots p_6}(t) = \begin{pmatrix} 1.2t + 1 \\ -0.5399t^6 + 3.2478t^5 - 7.4995t^4 + 11.388t^3 - 6.1005t^2 + 6.8506t + 2.1783 \end{pmatrix}$$

Solving the tridiagonal system:

$c_0 = 0$

$c_1 = \cancel{5} -8.827375$

$c_2 = 4.807$

$c_3 = 1.801875$

$c_4 = 2.8955$

$c_5 = 4.833625$

$c_6 = 0$

Using (5)

$d_0 = -14.71229167$

$d_1 = 22.72395833$

$d_2 = -5.008541667$

$d_3 = 1.822708333$

$d_4 = 3.230208333$

$d_5 = -8.056041667$

Using (1)

$a_0 = 2.1783$

$a_1 = 3.3201$

$a_2 = \cancel{4.056}\ 4.0552$

$a_3 = 4.9530$

$a_4 = 6.0496$

$a_5 = 7.3891$

Using (8)

$b_0 = 6.297491667$

$b_1 = 4.532016667$

$b_2 = 3.727941667$

$b_3 = 5.0497166067$

$b_4 = 5.989191667$

Using (9)

$b_5 = 753501.6667$

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(Q5)

x_i	0.4	0.6	0.8
$f(x_i)$	0.0256	0.1296	0.4096

Order = 2 due to quadratic interpolation

Lagrange interpolation

$$l_0(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} \qquad l_1(x) = \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}$$

$$l_0'(x) = \frac{2x - x - x_2}{(x_0-x_1)(x_0-x_2)} \qquad l_1'(x) = \frac{2x - x_0 - x_2}{(x_1-x_0)(x_1-x_2)}$$

$$l_2(x) = \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}$$

For $f(x) = l_0(x) f_0 + l_1(x) f_1 + l_2(x) f_2$

$$f'(x) = l'_0(0) \cancel{f(0)} f_0 + l'_1(x) f_1 + l'_2(x) f_2 \quad \text{-(i)}$$

$$l_2'(x) = \frac{\cancel{2}\, x - x_0 - x_1}{(x_2-x_0)(x_2-x_1)}$$

for

$$f_2''(x) = 2\left[\frac{f_0}{(x_0-x_1)(x_0-x_2)} + \frac{f_1}{(x_1-x_0)(x_1-x_2)} + \frac{f_2}{(x_2-x_0)(x_2-x_1)}\right] \quad \text{-(ii)}$$

$f'_2(x) = 0.8$, putting this value in (i)

$$f'_2(0.8) = \frac{(2(0.8) - 0.6 - 0.8)}{(0.4-0.6)(0.4-0.8)}^{0.0256} + \frac{(2(0.8) - 0.4 - 0.8)}{(0.6-0.4)(0.6-0.8)}^{0.1296}$$

$= 0.064 - 1.296 + 3.072$

$f_2'(0.8) = 1.84$

for $f_2''(0.8) = 2\left[\frac{0.0256}{\cancel{(0.4)(0.6)}(0.4-0.6)(0.4-0.8)} + \frac{0.1296}{(0.6-0.4)(0.6-0.8)} + \frac{0.4096}{(0.8-0.4)(0.8-0.6)}\right]$

$= 2[0.32 + (-3.24) + 5.12]$

$2(2.2)$

$f_2''(0.8) = 4.4$

Since $f(x) = x^4$

$f'(x) = 4x^3$

$f''(x) = 12x^2$

$f'''(x) = 24x$

$\max_{0.4 \le x \le 0.6} |24x| = |24(0.8)| = 19.2$

$|E'_2(0.8)| \le \frac{h^2}{3} M_3 = \frac{0.04}{3}(19.2) = 0.256$

and

$|E''_2(0.8) \le h M_3 = (0.2)(19.2) = 3.84$

Q6)

T	f(T)	f'(T)	f''(T)
22	4181	$\frac{4179-4181}{42-22}$ 0.1	$\frac{0.7-(-0.1)}{52-22} = 0.26$
42	4179	$\frac{4186-4179}{52-42}$ 0.7	$\frac{0.43-0.7}{82-42} = \frac{-27}{4000}$
52	4186	$\frac{4199-4186}{82-52}$ 0.43	$\frac{1-0.43}{100-52} = \frac{19}{1600}$
82	4199	$\frac{4217-4199}{100-82}$ 1	
100	4217		

$f(T) = b_0$

for $T_1 = 82$

$f(T_1) = 4199$

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for $T_2 = 40$
$f(T_2) = 4181$
for $T_3 = 90$
$f(T_3) = 4199$

$f'(T)$

$f'(T) = b_0 + b_1(T - T_0)$
for $T_1 = 82$
$f'(T_1) = 4199$ (as in table)
for $T_2 = 40$
Since $22 \le T_2 \le 42$
so let $T_0 = 22,\ T_1 = 42$
$f'(T_2) = 4181 + (-0.1)(40 - 22)$
$= 4179.2$
for $T_3 = 90$
Since $82 \le T_3 < 100$
let $T_0 = 82, T_1 = 100$
$f'(T_3) = 4199 + 1(90 - 82)$
$= 4207$

$f''(T) = b_0 + b_1(T - T_0) + b_2(T - T_0)(T - T_1)$
for $T_1 = 82°$
$f''(82) = 4199$
for $T_2 = 40°$
Since $22 \le 40 \le 42$
let $T_0 = 22,\ T_1 = 42$
$f''(40) = 4181 + (-0.1)(40 - 22) + 0.26(40 - 22)(40 - 42)$
$4181 + (-1.8) + (-9.36)$
$= 4169.84$
$T_3 = 90$
let $T_0 = 82, T_1 = 100$
$f''(90) = 4199 + 1(90 - 82) + \frac{19}{1600}(90 - 82)(90 - 100)$
$= 4199 + 8 + (-0.95)$
$= 4206.05$ //

Scanned with CamScanner