A Long Ass List of Problems for 33X

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The idea for this page came from a YouTube video called "10,000" problems in Analysis, which can be found here!

Of course, the goal is to compile a long list of problems ranging from very difficult to near trivial, all to get a better grasp of what it means to do *real* advanced calculus problems.

Question 1

Source: Mathematics Discord (discord message)
By using the Cauchy Criterion for convergence, show that the sequence defined by

$$\{x_n\}_1^\infty = \frac{1}{1^2} + \frac{1}{2^2} + \cdots + \frac{1}{n^2}.$$

converges.

Question 2

Source: Spring 1981 UC Berkley Mahtematics PhD Prelims, Question 16.
Let $f(x)$ be defined as a real-valued function for all $x \geq 1$, such that $f(1) = 1$ and

$$f'(x) = \frac{1}{x^2 + (f(x))^2}.$$

Prove that

$$\lim_{x \to \infty} f(x)$$

exists and the limit is *less than* $1 + \frac{\pi}{4}$.