An Introduction to "Dimensionality"
Some Warping Required

Jan Armendariz-Bones

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1 Introduction

This paper is about the ideas of dimensionality, both in the relatively physical sense of Linear Algebra as well as into the non-standard way of thinking for fractals.

1.1 A Brief Summary of Linear Algebra

First we begin by reviewing some general axioms and definitions from linear algebra.

Definition 1. *(Vector Space Axioms) We say that a space X with scalars in a field $\mathbb{F}$ is said to be a vector space if the following conditions are satisfied:*

- *(Associativity) For any u, v, and w in the space X, we have that*
$$u + (v + w) = (u + v) + w.$$
- *(Commutativity) For any $u, v \in X$ we have that*
$$u + v = v + u.$$
- *(Identity Element) There exists a vector $e \in X$ such that for any $v \in X$, $ev = ve = v$.*
- *(Zero Vector) There exists a vector $0 \in X$ such that for any $v \in X$, $0 + v = v + 0 = v$.*
- *(Inverse Element) For any $v \in X$, there exists another vector – denoted as the element $-v$ – such that*
$$v + (-v) = 0.$$
- *(Associativity of Scalars)For any two scalars $a, b \in \mathbb{F}$ and $v \in X$,*
$$a(bv) = b(av).$$
- *(Identity scalar) There exists a $1 \in \mathbb{F}$ such that for any $v \in X$,*
$$1v = v.$$

Remark: If we have a subset, Y, of a vector space, X, that satisfies the vector space axioms, we call Y a *subspace* of X.

When we consider any vector space, we need some way to refer to an arbitrary point in space. In $\mathbf{R}^2$, we would say that a point in the space is described as (x, y) where x and y are real numbers, denoting how far they are from the origin with respect to the x and y axes. Now within that description, there was a fundamental choice between the "building blocks" of the vector space. Those blocks being $(1, 0)$ and $(0, 1)$ respectively. How can we determine what we can choose as those blocks?

Definition 2. *(Linear Combination) For any two vectors, u and v, we call the linear combination of the two the collection of vectors such that we have vectors of the form*

$$au + bv,$$

where a and b are scalars.

Definition 3. *(Span) We say that a collection of vectors $\{v_1, v_2, \dots, v_n\}$ span a vector space V when:*

- *Every vector in space can be represented as a linear combination of our*

Remark: We call the collection of vectors that span the vector space a *basis* for the space V.

2 Outline

- General Linear Algebra definition of dimensionality.
  - LADR (uses term degree):
    A polynomial $p \in P(\mathcal{F})$ is said to have degree m if there exists scalars $(a_i)_1^n$ – where $a_n \neq \mathbf{0}$ in the scalar field such that for any $z \in \mathcal{F}$), we have that

    $$p(z) = \sum_1^n a_i b_i.$$

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- Manifolds and atlases.
  - Abstract Algbera (Visual Algbera): The dimension of a field is the same as the vector space dimension, however the adjoining of an element "adds" a dimension to the field.
    * Consider $\mathbb{R}(i) \cong \mathbb{C}$. From the perspective of the field $\mathbb{R}$, there is two dimensions; with the bases 1 and i.
    * Hausdorff dimension: How can I separate these two things??
      · Consider the difference between a (Koch) fractal and a square. One seems to take up less area than the other? **not a trick question!** $A(C) < A(Sq)$.
      · What is the "minimum" hypervolume growth necessary for each?!?
      · Why is this fractal so wacky?!
      · Remark that there is infinite perimeter, yet finite area...