The Lottery Ticket Hypothesis

Finding Sparse, Trainable Neural Networks

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March 5, 2020

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Presentation outline

1. The Lottery Ticket Hypothesis (LTH)
2. Towards a Better Understanding of the LTH
3. On the Generalization Properties of Lottery-Winners

An overview of popular Deep Learning Models

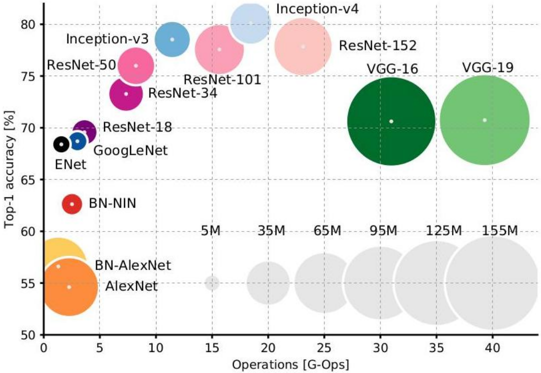

Figure: Image taken from Canziani A. et al. An Analysis of Deep Neural Network Models for Practical Applications.

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- Reduce computational and memory requirements
- Fit large models on e.g. smartphones
- Faster and more efficient inference
- (In some cases) lead to better performance

The Lottery Ticket Hypothesis (LTH)

- An idea which is **not new** ...

598 Le Cun, Denker and Solla

Optimal Brain Damage

Yann Le Cun, John S. Denker and Sara A. Solla
AT&T Bell Laboratories, Holmdel, N. J. 07733

ABSTRACT

We have used information-theoretic ideas to derive a class of practical and nearly optimal schemes for adapting the size of a neural network. By removing unimportant weights from a network, several improvements can be expected: better generalization, fewer training examples required, and improved speed of learning and/or classification. The basic idea is to use second-derivative information to make a tradeoff between network complexity and training set error. Experiments confirm the usefulness of the methods on a real-world application.

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- "It is better to retain the weights from the initial training phase than it is to re-initialize the pruned model."

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The Lottery Ticket Hypothesis: *A randomly-initialized dense neural network contains a subnetwork that is initialized such that -when trained in isolation- it can match the test accuracy of the original network after training for at most the same number of iterations.*

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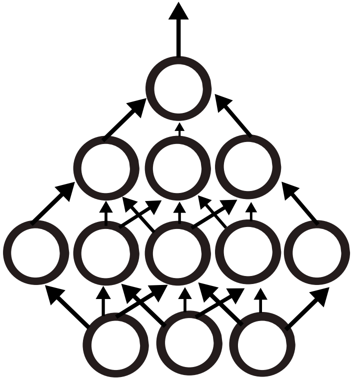

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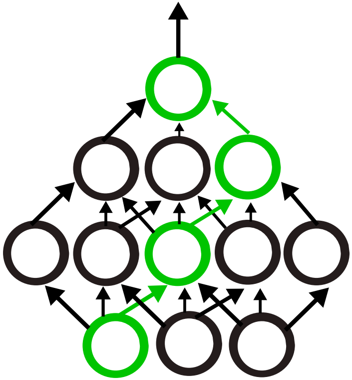

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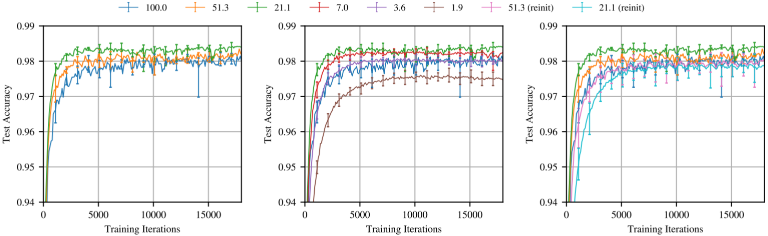
Winning-Tickets in Fully-Connected Networks

- Consider a MLP which gets trained on the MNIST dataset
- Weights get pruned based on their **magnitude**
- Winning tickets get compared to sparse models which get randomly reinitialized $\theta' \sim \mathcal{D}_\theta$ and are therefore not the winners of the lottery anymore

The Lottery Ticket Hypothesis (LTH)

- An overall performance of sparse models winners of the LTH



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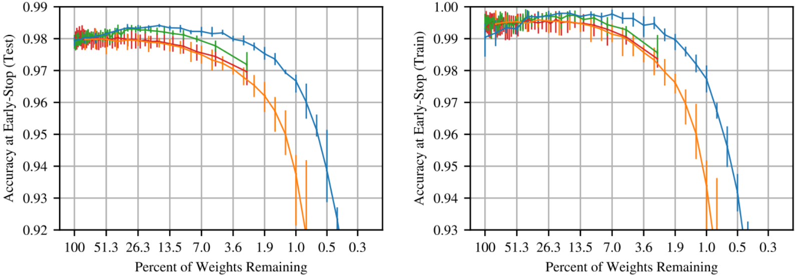
- A comparison with randomly initialized sparse models



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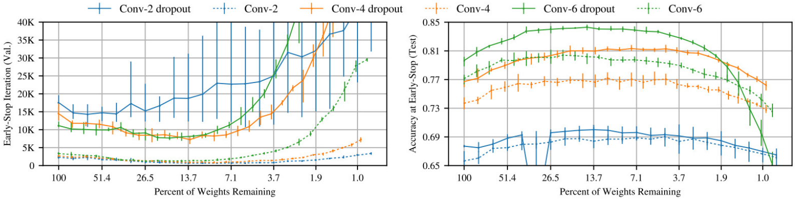
Winning-Tickets in Convolutional Networks

- Consider a VGG-like architecture which gets trained on the CIFAR-10 dataset
- Comparison to the **Dropout** regularization technique
- Extension to more popular architectures such a ResNet-50

The Lottery Ticket Hypothesis (LTH)

- Convolutional Networks and Dropout regularization



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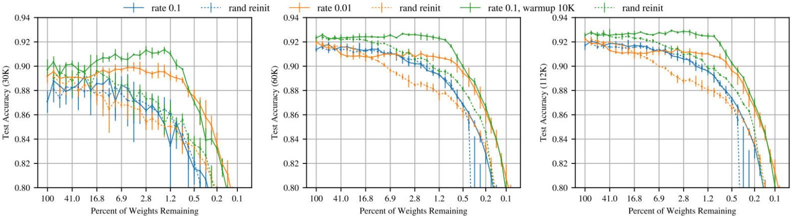

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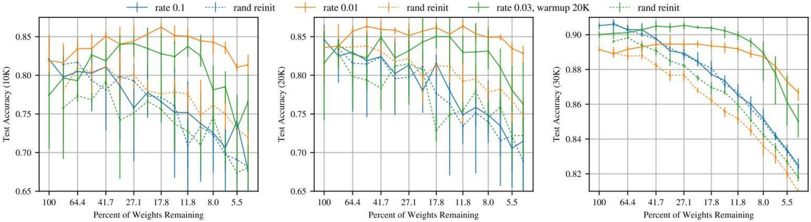

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- Additional extensions include different optimization algorithms, regularizers, etc ...
- Show that pruned models can be trained **from scratch**
- Albeit this seems to be harder to achieve when CNNs are considered

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    2. Winning tickets are a compromise between large overparametrized models and too small ones
    3. If understood this could lead to better weights initialization strategies (we're far from this)

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    3. The paper only considers 2 classification problems on 2 relatively simple datasets
    4. Presents this new deep-learning phenomenon but does not give any insights on why this happens

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- New formalization of the LTH phenomenon
- Researchers started to focus whether winning-initializations could be transferred among domains

Towards a Better Understanding of the LTH

Let's take a look again at the way winning tickets should be found...

1. Randomly initialize a network $f(x; \theta_0)$ where $\theta_0 \sim \mathcal{D}_\theta$
2. Train the network for j iterations
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Towards a Better Understanding of the LTH

To be be a successful lottery-ticket hunter ...

- In order to find winning-tickets in CNNs **late-resetting** needs to be used.
- Train the original θ parameters for k iterations, without going through any pruning
- Then start the usual procedure for finding winning-tickets
- k is an hyperparameter that needs to be tuned, therefore not leading to a fully trained model!

Towards a Better Understanding of the LTH

If we take a look at the new LTH scheme ...

1. Randomly initialize a network $f(x;\theta_0)$ where $\theta_0 \sim \mathcal{D}_\theta$
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Towards a Better Understanding of the LTH

- Once this **rewinding trick** was introduced finding the winners of the lottery became much easier
- The presence of lottery-winners has been found in a large set of architectures trained on even larger datasets
- Therefore characterizing the phenomenon better (when do these weights appear?)
- Not providing an answer to **why** these weights appear at the early stages of training

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Why should we care?

Once answered, all these questions could give insights about **what** makes some weights so special to be the winners of the initialization lottery!

On the Generalization Properties of the LTH

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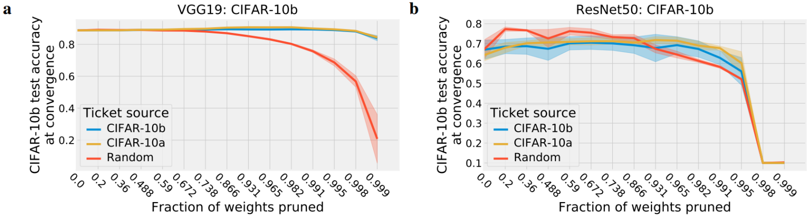
- Do winning tickets generalize within the **same** data distribution?



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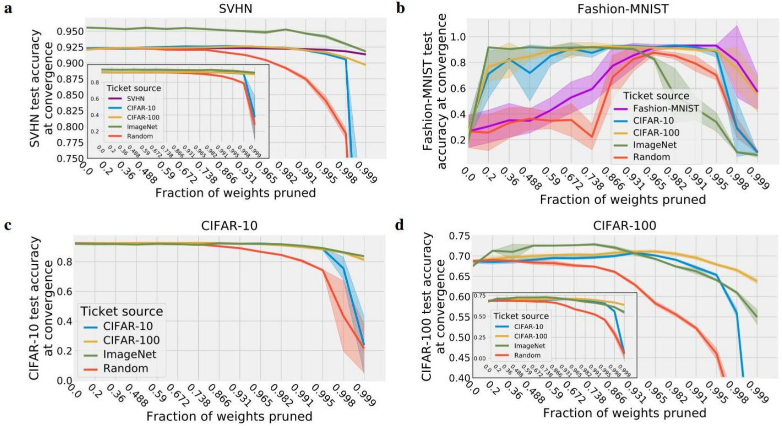
- Do winning tickets generalize between **several** photorealistic data distributions?



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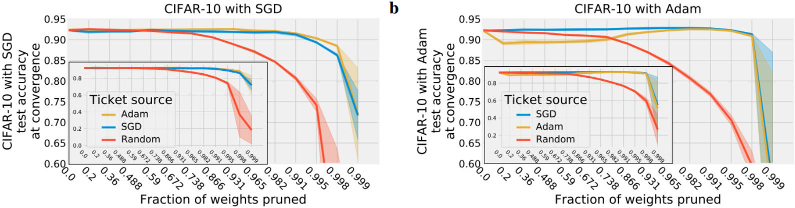
- Do winning tickets generalize across **optimizers**?



On the Generalization Properties of the LTH

- It does seem that winning-initializations are very **general** and that they are independent from a considered experimental setup
- **However** although different datasets have been used, all images come from the same source domain (natural images)
- Do lottery winners generalize to very different distributions?
- Can we really find one ticket to rule them all?! and therefore understand the LT phenomenon even more?

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- Do winning tickets contain inductive biases which are domain independent?

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- Can this help us characterize the LTH even better?

On the Generalization Properties of the LTH

An additional benefit that so far has not been considered ...

Abstract

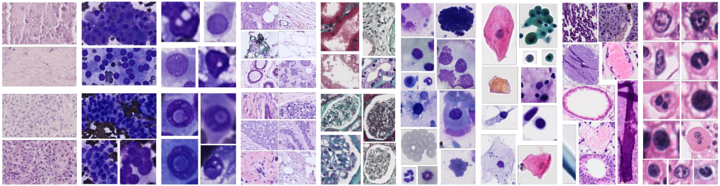
Neural network pruning techniques can reduce the parameter counts of trained networks by over 90%, decreasing storage requirements and improving computational performance of inference without compromising accuracy. However, contemporary experience is that the sparse architectures produced by pruning are difficult to train from the start, which would similarly improve training performance.

We find that a standard pruning technique naturally uncovers subnetworks whose initializations made them capable of training effectively. Based on these results, we articulate the *lottery ticket hypothesis*: dense, randomly-initialized, feed-forward networks contain subnetworks (*winning tickets*) that—when trained in isolation—reach test accuracy comparable to the original network in a similar number of iterations. The winning tickets we find have won the initialization lottery: their connections have initial weights that make training particularly effective.

We present an algorithm to identify winning tickets and a series of experiments that support the lottery ticket hypothesis and the importance of these fortuitous initializations. We consistently find winning tickets that are less than 10-20% of the size of several fully-connected and convolutional feed-forward architectures for MNIST and CIFAR10. Above this size, the winning tickets that we find learn faster than the original network and reach higher test accuracy.

On the Generalization Properties of the LTH

- In training conditions where data is scarce and therefore it results hard to train large networks identifying the lottery winners can be particularly important
- Can we avoid finding new lottery winners for each dataset and simply reuse already pruned models?



(a) Necrosis (b) ProliferativePattern (c) CellInclusion (d) Breast (e) Glomeruli (f) MouseLba (g) HumanLba (h) Lung (i) BoneMarrow

Figure: Image taken from Mormont R. et al. Comparison of deep transfer learning strategies for digital pathology.

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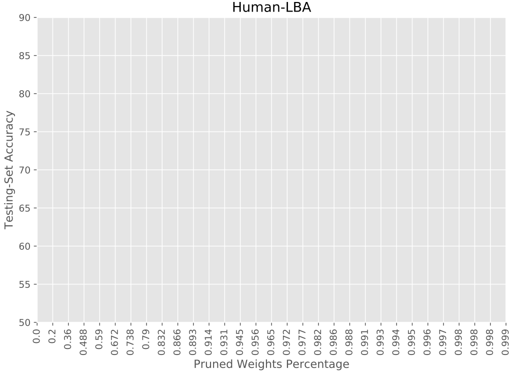

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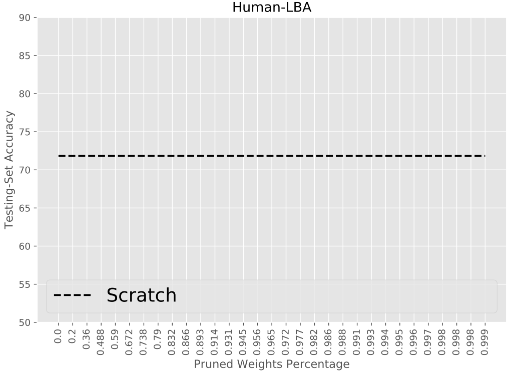

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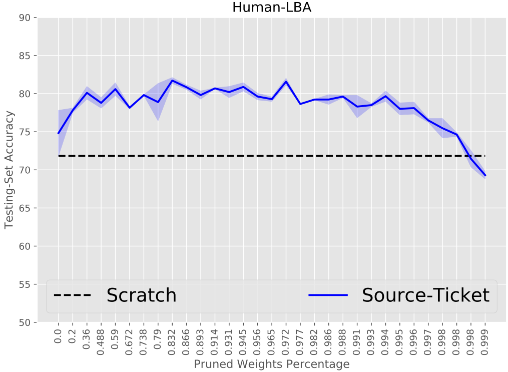

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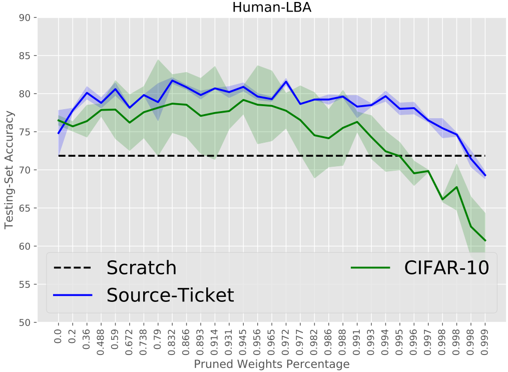

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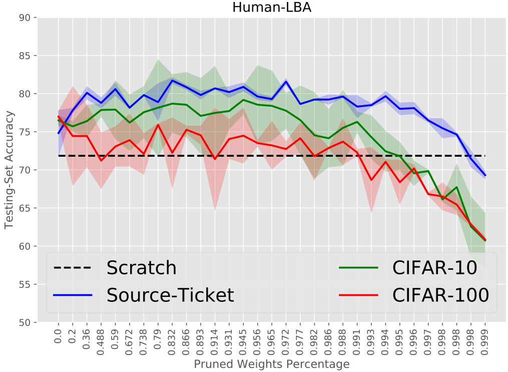

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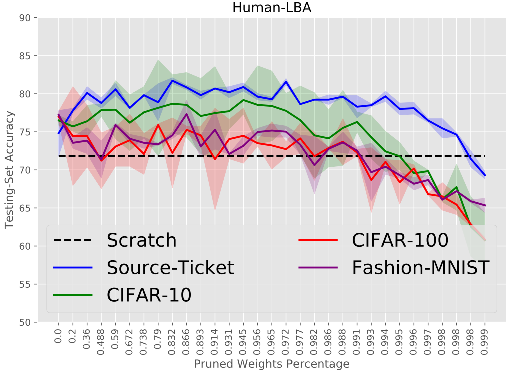

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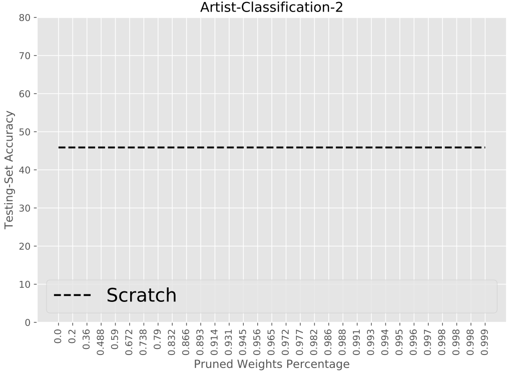

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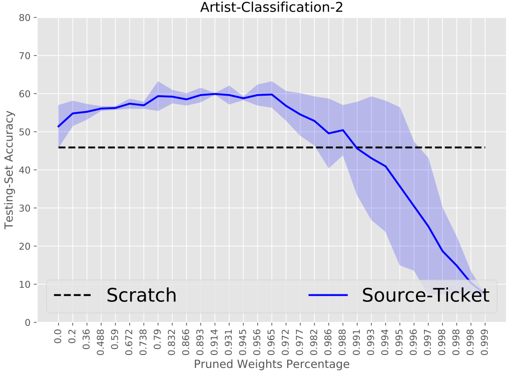

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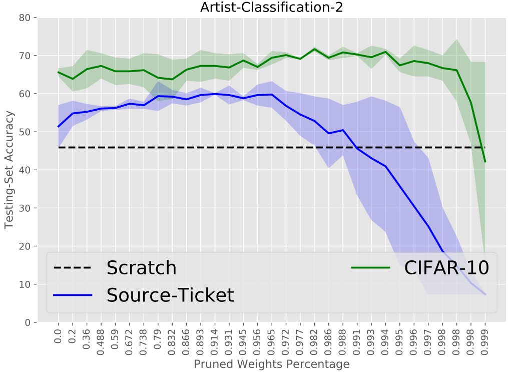

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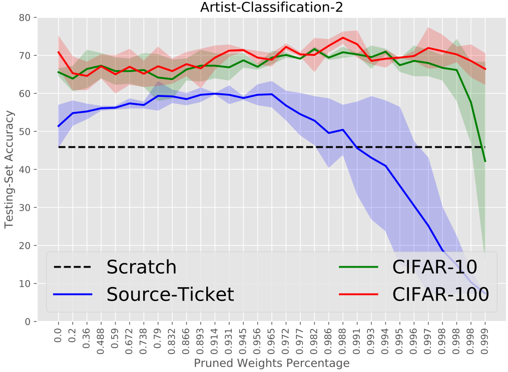

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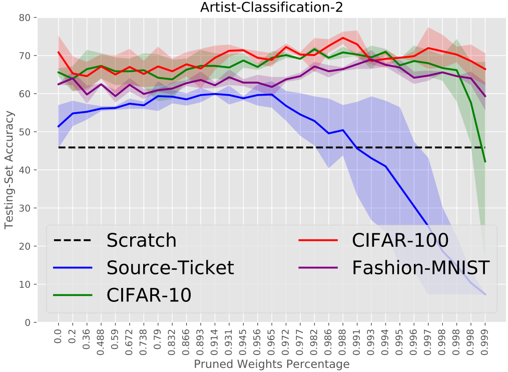

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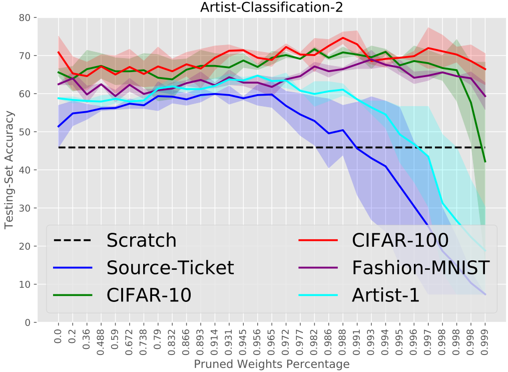

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Last but not least: what makes some weights so special?!

We still have no idea!

References and Credits

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