Cache Efficient Parallel Partition Algorithms
An In-Place Exclusive Read/Write Memory Algorithm

Our Research Question

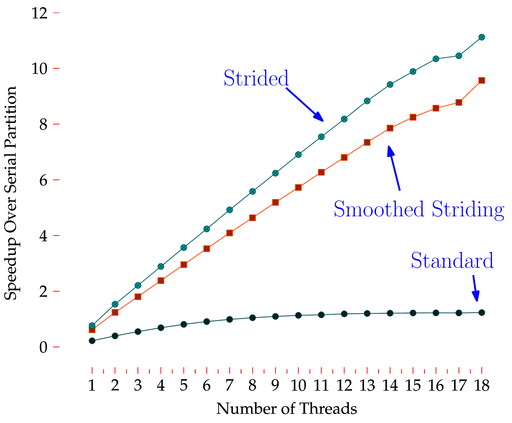
Can we create an algorithm with *theoretical guarantees* that is *fast in practice*?

Result

We created the *Smoothed Striding Algorithm*.
Key Features:

- linear work and polylogarithmic span
  (like the Standard Algorithm)
- fast in practice
  (like the Strided Algorithm)
- theoretically optimal cache behavior
  (unlike any past algorithm)

Smoothed Striding Algorithm's Performance



Strided Versus Smoothed-Striding Algorithm

Strided Algorithm
[Francis and Pannan, 92; Frias and Petit, 08]

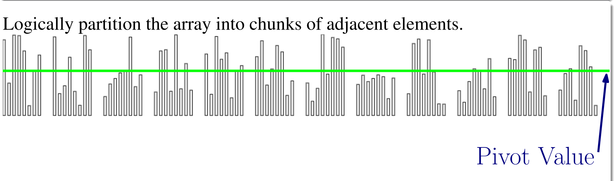
- Good cache behavior in practice
- Worst case span is $T_\infty \approx n$
- On random inputs span is $T_\infty = \tilde{O}(n^{2/3})$

Smoothed-Striding Algorithm

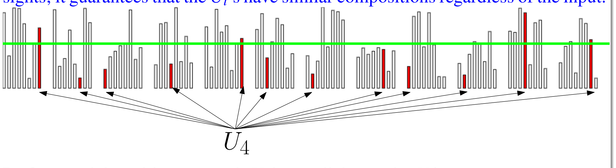
- Provably optimal cache behavior
- Span is $T_\infty = O(\log n \log\log n)$ with high probability in n
- Uses randomization *inside* the algorithm

Smoothed Striding Algorithm

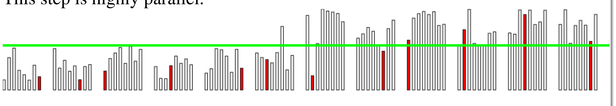
Logically partition the array into chunks of adjacent elements.



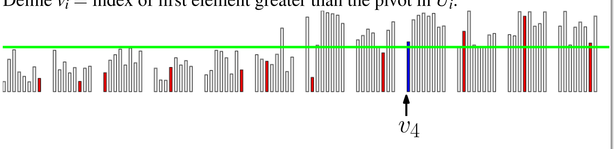
Form groups U_i that contain a random element from each chunk.
This randomization step was one of our key insights; it guarantees that the U_i's have similar compositions regardless of the input.



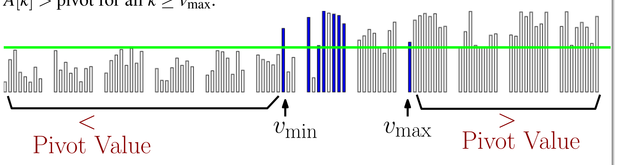
Perform serial partitions on each U_i in parallel over the U_i's.
This step is highly parallel.



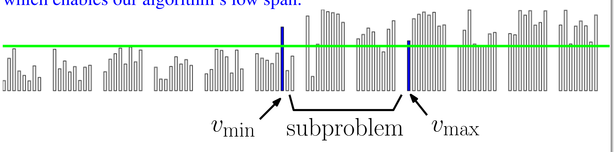
Define v_i = index of first element greater than the pivot in U_i.



Identify leftmost and rightmost v_i. Note that $A[k] \leq$ pivot for all $k < v_{\min}$, and $A[k] >$ pivot for all $k \geq v_{\max}$.



Recursively partition the subarray.
This step was previously impossible; adding randomization enables this step, which enables our algorithm's low span.

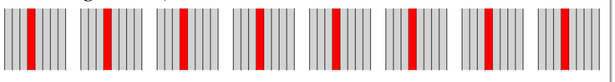


A Key Challenge

How do we store the U_i's if they are all random?

Storing which elements make up each U_i takes too much space!

Strided Algorithm P_i.



Smoothed-Striding Algorithm U_i.

How to Store the Groups

Key Insight: While each U_i does need to contain a random element from each chunk, the U_i's don't need to be *independent*.

We store U_1, and all other groups are determined by a "circular shift" of U_1 (wraparound within each chunk).