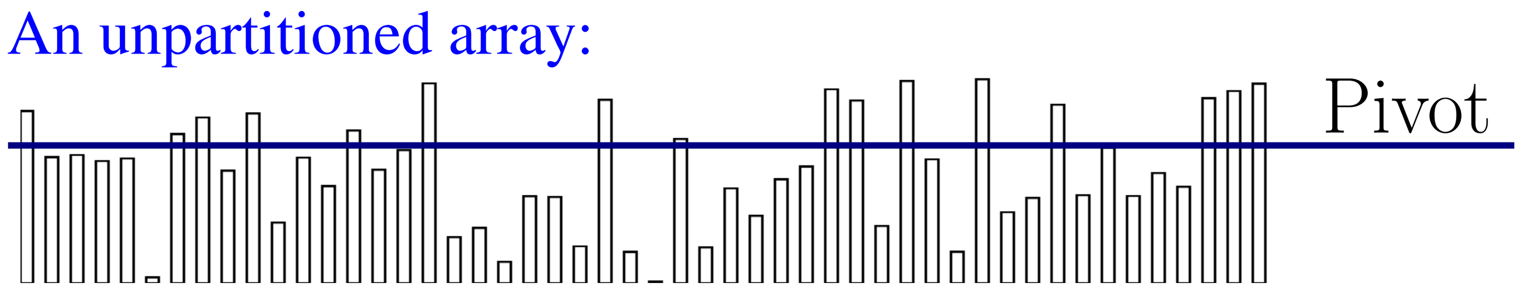
Cache Efficient Parallel Partition Algorithms
An In-Place Exclusive Read/Write Memory Algorithm

What is The Partition Problem?

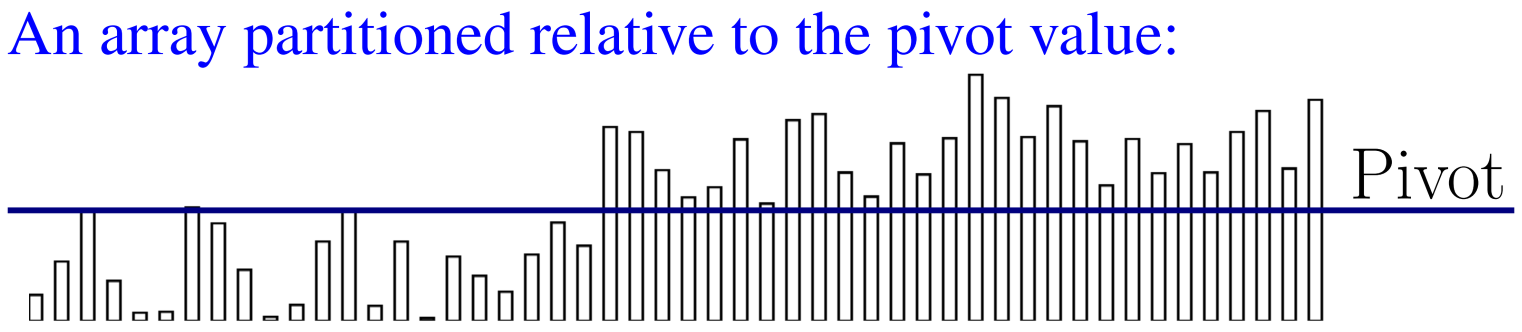
Explanation: The *Partition Problem* is to reorder the elements in a list so that elements in the same group occur in the same part of the list.

Example: A common way of grouping elements is based on whether they exceed or fall short of a certain "pivot" value.

An unpartitioned array:



An array partitioned relative to the pivot value:

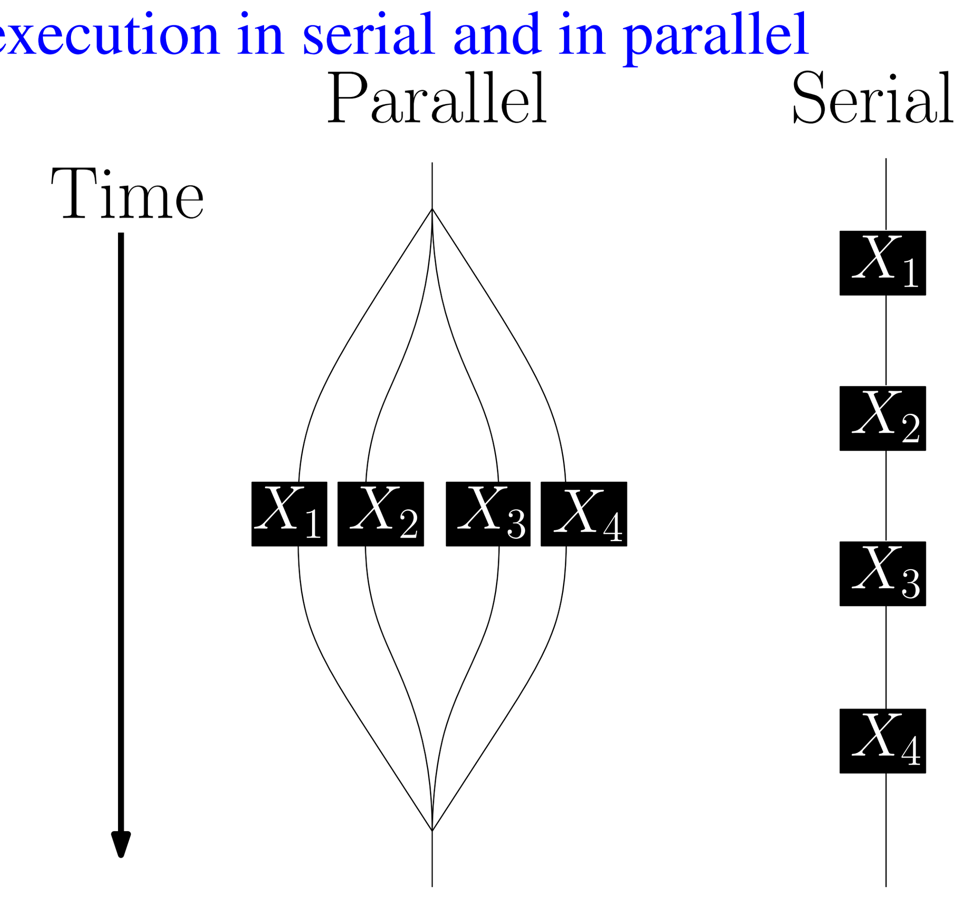


What is A Parallel Algorithm?

Explanation: Whereas a typical (i.e. serial) algorithm runs on a single processor, a *parallel algorithm* runs on $p \geq 1$ processors.

Example: Many tasks have parts that can be performed concurrently; such tasks can be performed faster with parallel computing.

Program execution in serial and in parallel



What is Cache Efficiency?

Explanation: *Cache* is a small part of memory that can be accessed much faster than ordinary RAM. When data is already loaded into Cache a program can rapidly access it; this is called a *cache hit*. When data needed by a program isn't in cache it must be loaded into cache; this is called a *cache miss*, and takes time.

Remark: An algorithm with very few cache misses is *Cache Efficient*; cache efficiency leads to faster performance in practice.

Factors in Cache-Efficiency:

- Perform low number of passes over the data
- Don't use extra memory, i.e. are *In-Place*
- Deal with elements that are close in memory together

Previous Work on the Partition Problem

The "Standard Algorithm" is theoretically optimal with span $O(\log n)$, but slow in practice due to poor cache behavior.
The fastest algorithms in practice lack theoretical guarantees

- Lock-based and atomic-variable based algorithms
  [Michael Axtmann, Sascha Witt, Daniel Ferizovic, and Peter Sanders, 2017; Philip Heidelberger, Alan Norton, and John T. Robinson, 1990; Philippas Tsigas and Yi Zhang, 2003]
  Not Exclusive Read/Write Memory
- The Strided Algorithm
  [Francis and Pannan, 92; Frias and Petit, 08]
  No locks or atomic-variables, but no bound on span

Our Research Question

Can we create an algorithm with *theoretical guarantees* that is *fast in practice*?

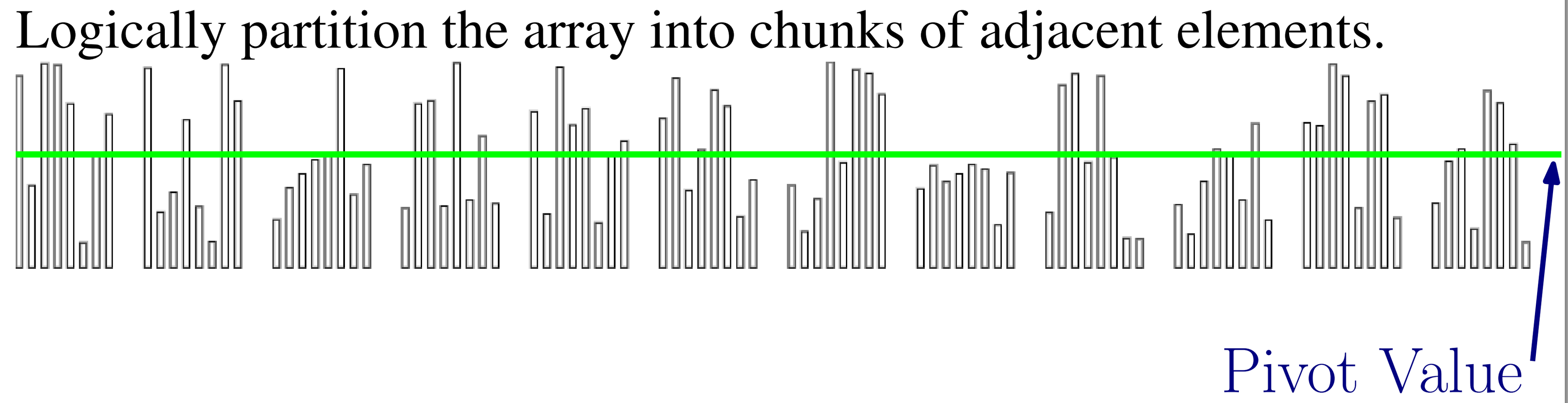
Result

We created the *Smoothed Striding Algorithm*.
Key Features:

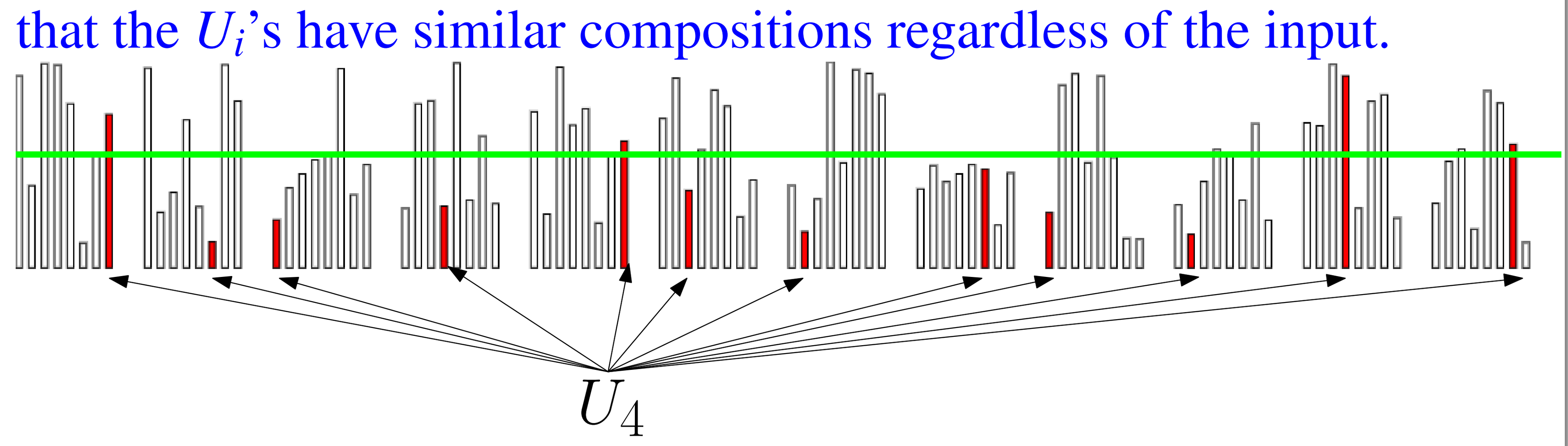
- linear work and polylogarithmic span
  (like the Standard Algorithm)
- fast in practice
  (like the Strided Algorithm)
- theoretically optimal cache behavior
  (unlike any past algorithm)

Smoothed Striding Algorithm

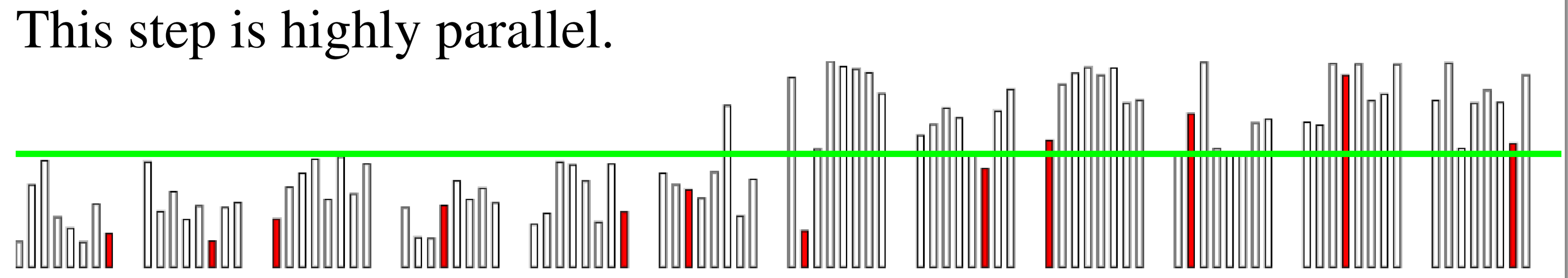
Logically partition the array into chunks of adjacent elements.



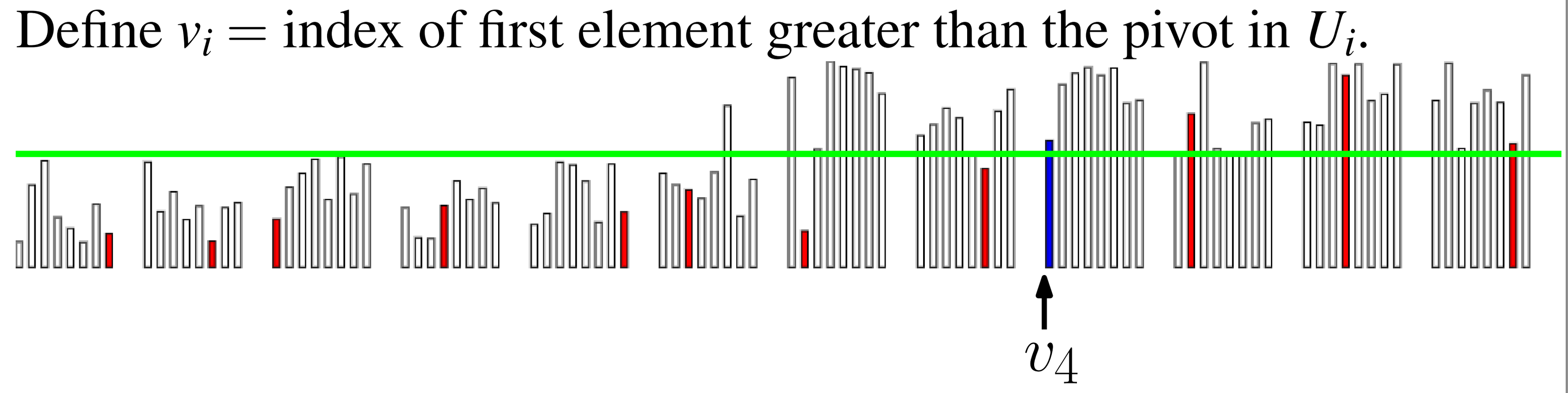
Form groups U_i that contain a random element from each chunk.
This randomization step was one of our key insights; it guarantees that the U_i's have similar compositions regardless of the input.



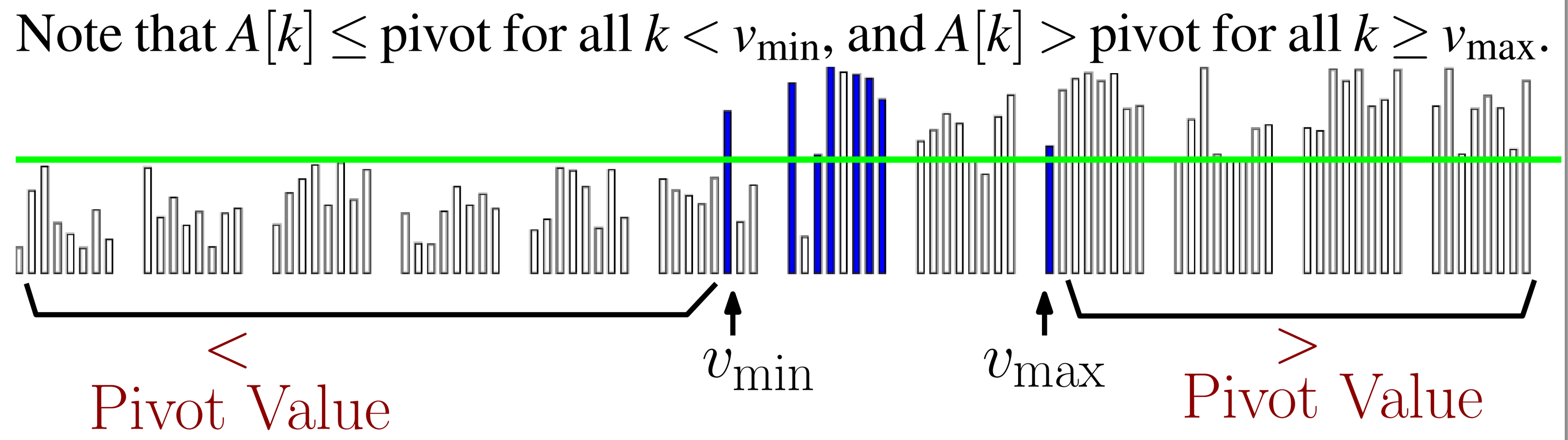
Perform serial partitions on each U_i in parallel over the U_i's.
This step is highly parallel.



Define v_i = index of first element greater than the pivot in U_i.



Identify leftmost and rightmost v_i.
Note that $A[k] \leq$ pivot for all $k < v_{\text{min}}$, and $A[k] >$ pivot for all $k \geq v_{\text{max}}$.



Recursively partition the subarray.
This step was previously impossible; adding randomization enables this step, which enables our algorithm's low span.