Round 5 Trigonometry

Angular and Linear Velocity; Right Triangle Trigonometry

GREATER BOSTON MATHEMATICS LEAGUE

MEET 1 – OCTOBER 1998

ROUND 5 – Trigonometry: Angular and Linear Velocity; Right Triangle

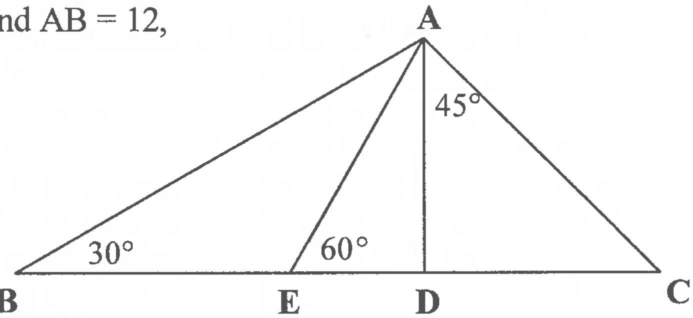
1. ____________________

2. ____________________

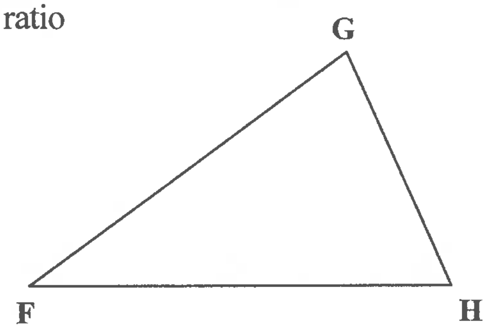
3. ____________________

CALCULATORS ARE NOT ALLOWED ON THIS ROUND
DIAGRAMS ARE NOT NECESSARILY DRAWN TO SCALE

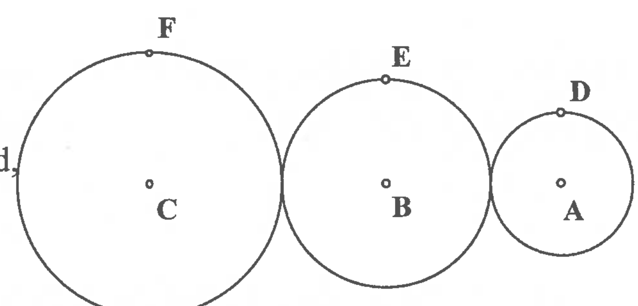
1. Given the diagram with $\overline{AD} \perp \overline{BC}$ and AB = 12, find AC – BE.



2. Given $\tan \angle F = \frac{3}{4}$ and $\tan \angle H = \frac{24}{7}$, find the ratio of GF to FH.



3. Given externally tangent circles centered at A, B, and C with radii of 8cm, 12cm, and 15cm respectively, circle A is rotating about A at $\frac{\pi}{5}$ radians per second, which rotates circle B about B, which in turn rotates circle C about C. Find the fewest number of seconds so that points D, E, and F will be located again at exactly the same positions as now.



GREATER BOSTON MATHEMATICS LEAGUE

MEET 1 – SEPTEMBER 1999

ROUND 5 – Trigonometry: Angular and Linear Velocity; Right Triangle

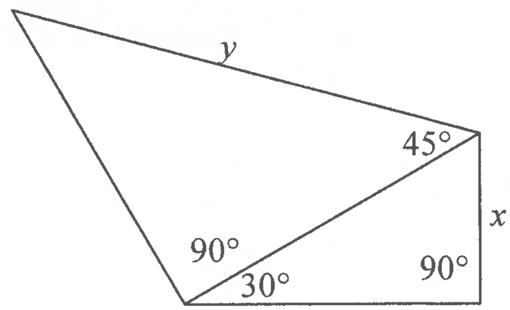
1. ____________________

2. ____________________

3. ____________________

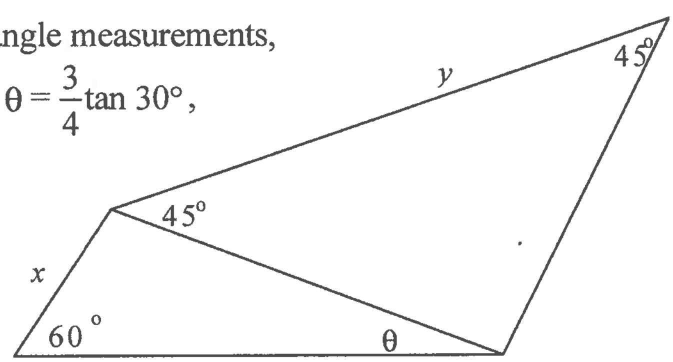
CALCULATORS ARE NOT ALLOWED ON THIS ROUND
DIAGRAMS ARE NOT NECESSARILY DRAWN TO SCALE

1. Given the diagram on the right with indicated lengths x , y and angle measurements. Compute the ratio of x to y.



2. A car is travelling at 25 meters per second and has a wheel radius of 375 millimeters. How many minutes does it take a point at the bottom of the wheel to turn through 800 revolutions?

3. Given the diagram with indicated angle measurements, indicated lengths x and y, and $\tan\theta = \frac{3}{4}\tan 30°$, compute the ratio of y to x.



GREATER BOSTON MATHEMATICS LEAGUE

MEET 1 – SEPTEMBER 2000

ROUND 5 – Trigonometry: Angular and Linear Velocity; Right Triangle

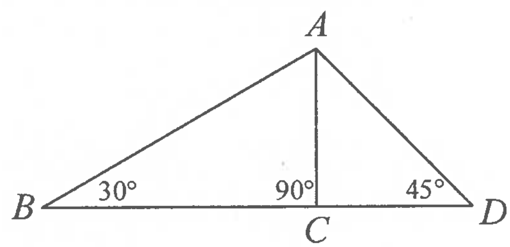
1. ____________________

2. ____________________

3. ____________________

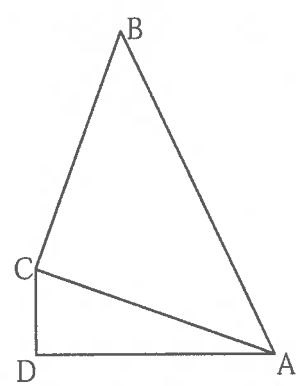
CALCULATORS ARE NOT ALLOWED ON THIS ROUND.
DIAGRAMS ARE NOT NECESSARILY DRAWN TO SCALE.

1. Given the diagram on the right, if $\overline{BCD}$ and $BC = 6$, find the length of $\overline{AD}$.



2. The front wheel of an old fashioned bicycle has a radius of 6 inches while the back wheel has a radius of $1\frac{3}{4}$ feet. If the front wheel, while traveling, is rotating at 315 revolutions per minute, the back wheel makes how many revolutions in 1 second?

3. Given $m\angle CAB = m\angle ABC = 45°$, $\tan(\angle CAD) = \frac{\sqrt{2}}{4}$, $m\angle D = 90°$, and $AB = 12$, find the perimeter of quadrilateral ABCD.



GREATER BOSTON MATHEMATICS LEAGUE

MEET 1 – OCTOBER 2001

ROUND 5 – Trigonometry: Angular and Linear Velocity; Right Triangle

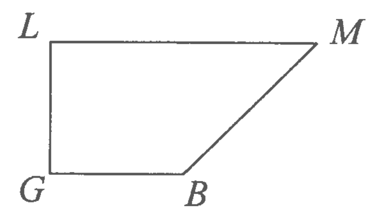
1. ________________

2. ________________

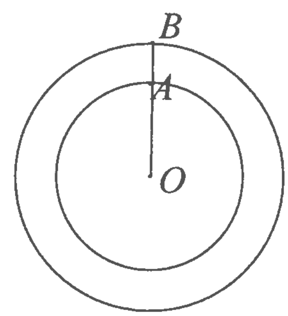
3. ________________

CALCULATORS ARE NOT ALLOWED ON THIS ROUND.
DIAGRAMS ARE NOT NECESSARILY DRAWN TO SCALE.

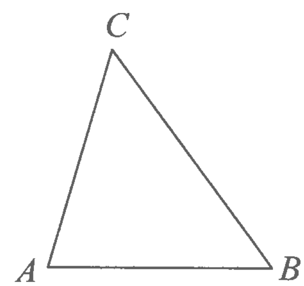
1. Given $GL = GB = 1$, $m\measuredangle G = 90°$, $m\measuredangle B = 135°$, and $\overline{GB} \parallel \overline{ML}$, find the perimeter of quadrilateral $GBML$.



2. Given concentric circles centered at point O with points A and B collinear with O, $AO = 6\text{cm}$ and $AB = 2\text{cm}$. A particle at A is rotating clockwise around the inner circle at $32\pi\ \text{cm/sec}$ and a particle at B is rotating clockwise around the outer circle at $30\pi\ \text{cm/sec}$. What is total number of revolutions traveled by both particles the first time that they are back to this original position?



3. Given $\sin A = .96$, $\sin B = .8$, and the perimeter of $\Delta ABC = 4$, find the length of $\overline{AB}$.



GREATER BOSTON MATHEMATICS LEAGUE

MEET 1 – OCTOBER 2006

ROUND 5 – Trig: Angular and Linear Velocity, Right Triangles

1. ______________________________

2. ______________________________

3. ______________________________

CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

1. In an isosceles triangle, the legs are each 23 inches long and the base is 14 inches long. Determine the exact value of the cosine of the complement of a base angle.

2. A point on a circle of radius 6 cm is moving around the circle at $\pi/12$ radians per second. Determine the linear velocity of the midpoint of a radius of this circle (in cm/sec).

3. A ship sailing due east sights a buoy 30° north of east. Continuing due east, four miles later, it sights the same buoy 60° north of east. If the easterly course is maintained, how close will the ship come to the buoy? If necessary, express your answer in simplified radical form. (For these small distances the curvature of the earth may be ignored.)

GREATER BOSTON MATHEMATICS LEAGUE

MEET 1 – OCTOBER 2007

ROUND 5 – Trig: Angular and Linear Velocity, Right Triangles

1. ____________________

2. ____________________

3. ____________________

CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

1. If θ is acute and $\cot\theta = \dfrac{7\sqrt{2}}{12}$, compute $\sin\left(90^{\circ} - \theta\right)$.

2. A tool rotates at 10,000 RPM (revolutions per minute). If the rotation speed is increased by p%, the tool rotates through 72° in 0.001 second. Compute p.

3. Given: In ΔABC, $m\angle A = 45°$, $AC = 14$ and $BC = 10$
   If $\angle B$ is as large as possible, compute $\sin C$.

GREATER BOSTON MATHEMATICS LEAGUE

MEET 1 – OCTOBER 2008

ROUND 5 – Trig: Angular and Linear Velocity, Right Triangles

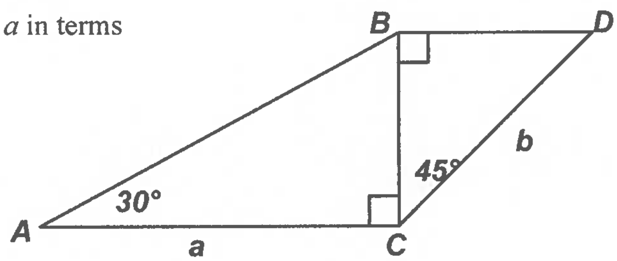
1. ______________________________

2. ______________________________

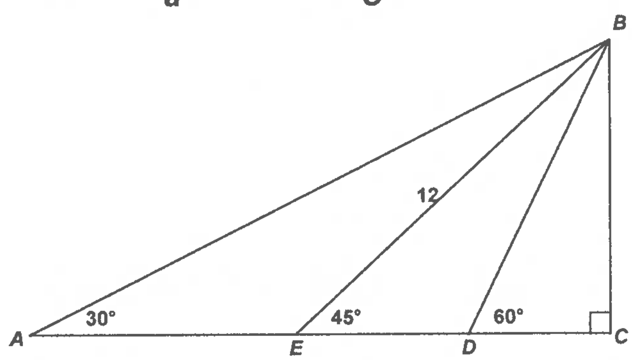
3. ______________________________ft/sec

CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

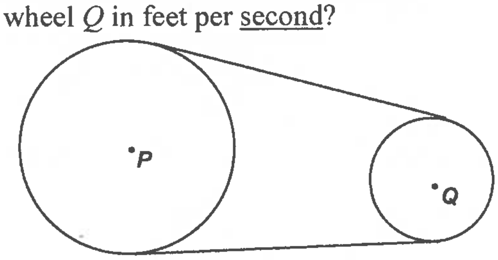
1. Given the diagram at the right, find a in terms of b in simplified form.



2. If $BE = 12$, find $AE - ED$.



3. A wheel P is making 10 revolutions per minute, while a second wheel Q connected to the first wheel by a belt making 900 revolutions per hour. The radius of wheel P is 2 feet, what is the linear velocity of a point on the rim of wheel Q in feet per second?



GREATER BOSTON MATHEMATICS LEAGUE

MEET 1 – OCTOBER 2009

ROUND 5 – Trig: Angular and Linear Velocity, Right Triangles

1. ______________________

2. ______________________

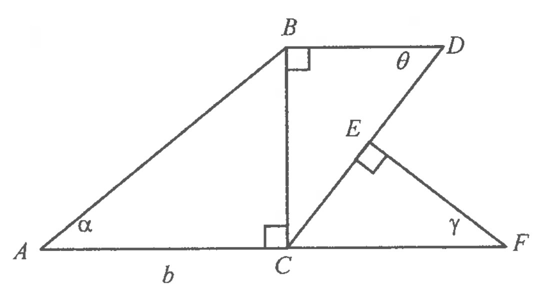
3. ______________________

CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

1. In a right triangle, $\cos\theta = \frac{3}{7}$. Find $\cot\theta + \csc\theta$ in simplest form.

2. The linear velocity of a point on the rim of a wheel is π ft./sec. The radius of the wheel is 18 inches. Find the number of revolutions made by the wheel in 40 minutes.

3. Given: $DE = EC$. Express CF in the simplified form $\frac{p}{q}$, where q is a constant and p is a product in terms of $b, \alpha, \theta,$ and γ.



GREATER BOSTON MATHEMATICS LEAGUE

MEET 1 – OCTOBER 2010

ROUND 5 – Trig: Angular and Linear Velocity, Right Triangles

1. ____________________ minutes

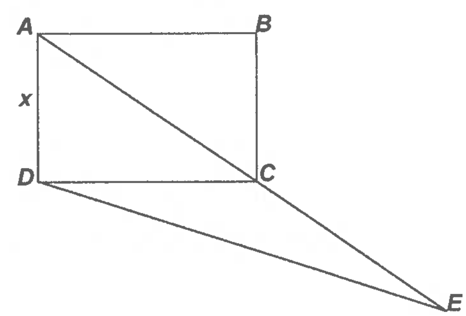
2. ____________________

3. ____________________

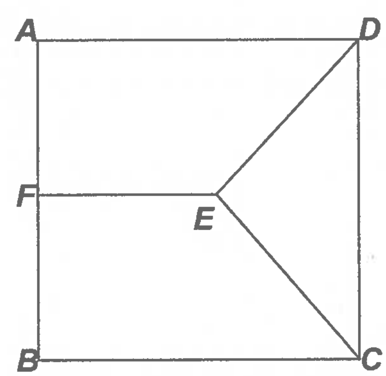
CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

1. A car travels at 32 meters per second and has a wheel radius of 36 centimeters. Compute the time (in minutes) it take a point on the rim of the wheel to turn through 4000 revolutions.

2. $ABCD$ is a rectangle, $DC = 2AD$, $AC = CE$ and $AD = x$. Compute DE in terms of x.



3. $ABCD$ is a square. $AB = 6$, $\overline{EF} \perp \overline{AB}$, $EC = ED = FE$. Compute $\sin(\angle EDC)$.



MASSACHUSETTS MATHEMATICS LEAGUE
DECEMBER 2004
ROUND 1 TRIG: RT ANGLE, LAWS SINES & COSINES

ANSWERS

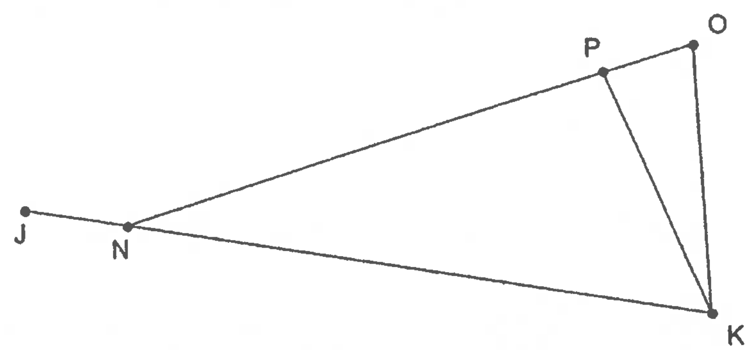
A)______________

B)______________

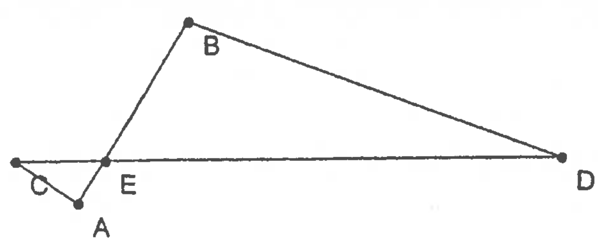
C)______________

A) If $\sec(x)=2.2$ and $\tan(x) < \cos(x) - 2$, find the exact value of $\csc(x)$ in simplified radical form

B) If ΔNOK is isosceles with $NO = NK = 18$, $OP = 2$, and $\cos(\angle JNO) = -0.75$ find PK in simplified radical form.



C) Given $DB = 91$, $AC = 7$, $EC = 8$, $\angle D = 30°$, and $\overline{AC} \perp \overline{AB}$ find the exact length of $\overline{AB}$.



MASSACHUSETTS MATHEMATICS LEAGUE
DECEMBER 2005
ROUND 1 TRIG: RT ANGLE, LAWS SINES & COSINES

ANSWERS

A)_______________

B)_______________

C)_______________

A) In triangle ABC with hypotenuse $\overline{AB}$, $\sin(\angle A)=0.28$. Find $\tan(\angle B)$, expressing your answer as a fraction $\frac{a}{b}$ with a, b relatively prime.

B) In acute ΔDEF $\sin(\angle D) = \sin(\angle F) + \frac{1}{3}$ while $EF = ED + \frac{2}{3}$

If $\sin(\angle F) = \frac{5}{9}$, find the exact value of EF in simplified form

C) In acute ΔABC $\cot(\angle A) = 0.75$ while $\tan(\angle B) = 2.40$ If the perimeter of ΔABC is 420, find the triangle's area.

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 3 – DECEMBER 2006
ROUND 1 TRIG: RIGHT ANGLE PROBLEMS, LAWS OF SINE AND COSINE

ANSWERS

A) ______________________________

B) ______________________________

C) (________ , ________)

A) An equilateral triangle has sides of length 6.
Points A, B, C, D, E and F are trisection points of the sides.
What is the exact length of a segment that

- connects two of these points not on the same side of the triangle and
- is not parallel to any sides of the triangle?

Express your answer as an exact value in simplified form.

B) In ΔABC, $m\angle B = 150°$, $a = BC = 10$ and $b = AC = 15$.
Determine the exact value of $\sin(B + C)$.

C) The perimeter of a regular n-sided polygon is p. A simplified expression for the apothem of the polygon in terms of p and n may be written in the form $\dfrac{p\cot(\frac{X}{n})}{Yn}$, where $\dfrac{X}{n}$ is the degree-measure of an angle whose vertex is at the center of the regular polygon.
Determine the ordered pair (X, Y).

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 3 – DECEMBER 2007
ROUND 1 TRIG: RIGHT ANGLE PROBLEMS, LAWS OF SINES AND COSINES

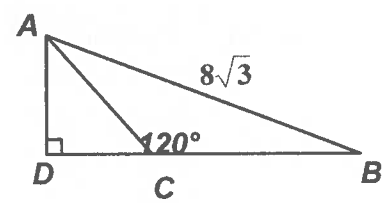
ANSWERS

A) ______________________

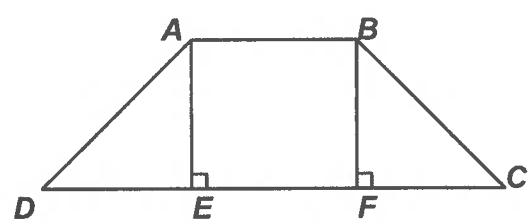
B) ____________ : ____________

C) ______________________

A) Given: ΔABC is isosceles, $m\angle ACB = 120°$ $AB = 8\sqrt{3}$
Compute AD.



B) The area of an isosceles trapezoid is 840 square units.
Altitudes $\overline{AE}$ and $\overline{BF}$ divide the longer base into three segments of equal length. If the length of an altitude of the trapezoid is 1 unit less than the length of the longer base, what is the ratio of the perimeter of the trapezoid to the altitude of the trapezoid?

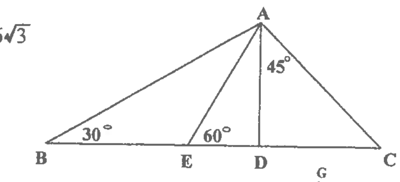


C) In ΔABC, $AB = 4$, $AC = 6$, $m\angle C = 30°$.
Determine all possible values for the exact length of $\overline{BC}$ in simplified radical form.

GBML 1998

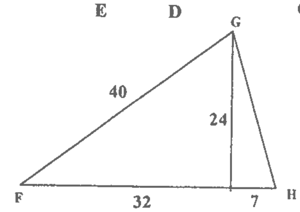
ROUND 5

1. $AB = 12 \Rightarrow AD = 6 \Rightarrow AC = 6\sqrt{2}$, $BD = 6\sqrt{3}$ and $DE = 2\sqrt{3} \Rightarrow BE = 4\sqrt{3} \Rightarrow$
$AC - BE = 6\sqrt{2} - 4\sqrt{3}$



2. $\frac{3}{4} = \frac{24}{32}$ Draw a perpendicular from G to FH

ratio of GF to FH = 40:39

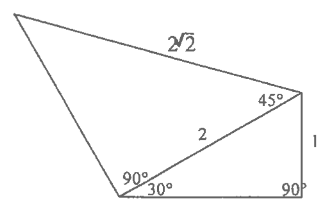


3. For points D, E, and F to be located in the exact same location a second time, each circle needs to rotate some whole number of revolutions. Call the circumferences of circles A, B, and C **C_A, C_B,** and **C_C; C_A** $= 16\pi$ cm; **C_B** $= 24\pi$ cm; **C_C** $= 30\pi$ cm; The least common multiple of these three circumferences is 240π cm, which is 15 revolutions of circle A. One revolution of circle A takes 10 secs. $\Rightarrow$ 15 revolutions take 150 sec.

GBML 1999

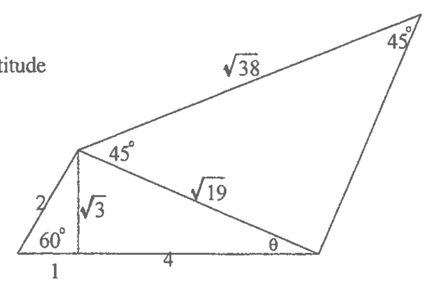
ROUND 5

1. Let $x = 1 \Rightarrow y = 2\sqrt{2} \Rightarrow x{:}y =$ **$\sqrt{2}$:4**



2. Circumference = 0.75π meters; 0.75π meters × 800 = 600π meters; 600π meters ÷ 1500 meters per minute = **0.4π** minutes

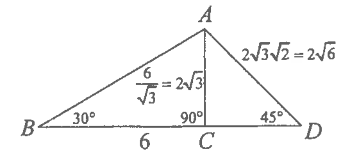
3. Let $x = 2$ for convenience. Draw the altitude as indicated on the diagram.
$\tan\theta = \frac{3}{4}\frac{\sqrt{3}}{3} = \frac{\sqrt{3}}{4} \Rightarrow$ leg of the
45-45-90° $\Delta = \sqrt{19} \Rightarrow y = \sqrt{38} \Rightarrow$
ratio of y to $x =$ **$\sqrt{38}$:2**



GBML 2000

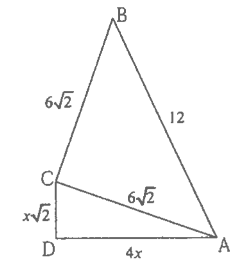
ROUND 5

1. $AC = \frac{6}{\sqrt{3}} = 2\sqrt{3} \rightarrow AD = 2\sqrt{3}\sqrt{2} = 2\sqrt{6}$



2. For the front wheel, $C = 12\pi$ in.; 315 rpm $= \frac{315 \cdot 12\pi \text{ in.}}{60\text{sec}} = 63\pi$ in./sec.
For the back wheel, $C = 3.5\pi$ ft. $= 42\pi$ in. $\frac{63\pi}{42\pi} = 1.5$ rev/sec

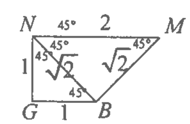
3. $(4x)^2 + (x\sqrt{2})^2 = (6\sqrt{2})^2 \rightarrow 18x^2 = 72 \rightarrow x = 2 \rightarrow$
perimeter of ABCD $= 12 + 6\sqrt{2} + 2\sqrt{2} + 8 = 20 + 8\sqrt{2}$.



GBML 2001

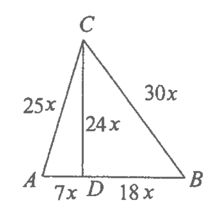
ROUND 5

1. Draw $\overline{BN}$. $BN = \sqrt{2} \Rightarrow BM = \sqrt{2} \Rightarrow MN = 2 \Rightarrow$
Perimeter of $GBML = 4 + \sqrt{2}$



2. The angular velocity of the particle at $A = \frac{32\pi}{12\pi}$ rev/sec $= \frac{8}{3}$ rev/sec.
The angular velocity of the particle at $B = \frac{30\pi}{16\pi}$ rev/sec $= \frac{15}{8}$ rev/sec.
After 24 sec. particle at A traveled 64 revolutions and the particle at B traveled 45 revolutions for a total of 109 revolutions.

3. Draw altitude $\overline{CD}$. $\sin A = \frac{24}{25}$, $\sin B = \frac{4}{5} = \frac{24}{30}$; let $CD = 24x \Rightarrow$
$AC = 25x$, $AD = 7x$, $BC = 30x$, and $DB = 18x \Rightarrow$ perimeter =
$80x = 4 \Rightarrow x = 0.05$; $AB = 25x = 1.25$.



GBML 2006

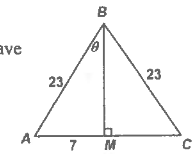
ROUND 5

1.

By the Pythagorean theorem $BM = \sqrt{23^2 - 7^2} = \sqrt{480} = 4\sqrt{30}$

Since θ is the complement of a base angle, applying SOHCAHTOA, we have

$\cos(\theta) = \underline{\dfrac{4\sqrt{30}}{23}}$



2.

The point rotates 15° each second, completing one revolution in 24 seconds.

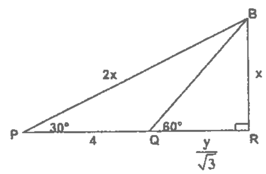
The circumference of the circle = $2\pi(6)$ = (d)24 → $d = \pi/2$ cm

Thus, the point on the circumference travels $\pi/2$ cm /sec, i.e. its linear velocity is $\pi/2$ cm/sec.

The midpoint of a radius travels only half as far in the same time period, i.e. its linear velocity is $\underline{\pi/4}$ cm/sec.

3.

Consider the diagram at the right. We must find BR.



$4x^2 = (4 + \frac{x}{\sqrt{3}})^2 + x^2 \rightarrow 3x^2 = 16 + \frac{8x}{\sqrt{3}} + \frac{x^2}{3}$

$\rightarrow 8x^2 - 8\sqrt{3}x - 48 = 0 \rightarrow x^2 - \sqrt{3}x - 6 = 0$

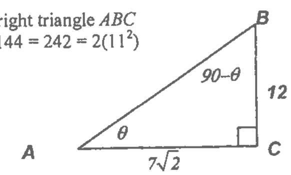
Using the quadratic formula $x = \dfrac{\sqrt{3} \pm \sqrt{3+24}}{2} = \dfrac{\sqrt{3} \pm 3\sqrt{3}}{2} = \underline{2\sqrt{3}}$

GBML 2007

ROUND 5

1. Since θ is acute, it may be represented as an angle in right triangle ABC as illustrated in the diagram to the right. $AB^2 = 98 + 144 = 242 = 2(11^2)$

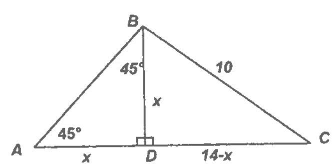
$\rightarrow AB = 11\sqrt{2} \rightarrow \sin(90 - \theta) = \dfrac{7\sqrt{2}}{11\sqrt{2}} = \underline{\dfrac{7}{11}}$



2. 10000 RPM = $\dfrac{10000 \text{ rev}}{60 \text{ sec}} = \dfrac{10000(360)°}{60 \text{ sec}} \rightarrow \dfrac{10000\left(1+\frac{p}{100}\right)360}{60(1000)} = 72 \rightarrow 60\left(1+\frac{p}{100}\right) = 72$

$\rightarrow 1+\frac{p}{100} = \frac{6}{5} \rightarrow p = \underline{20}$

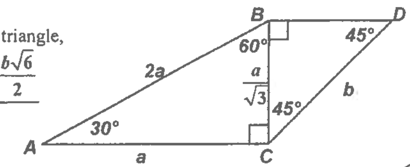
3. Using the Pythagorean Theorem in ΔBCD,
$x^2 + (14 - x)^2 = 100$
$\rightarrow x^2 + 196 - 28x + x^2 = 100$
$\rightarrow 2x^2 - 28x + 96 = 0$
$\rightarrow x^2 - 14x + 48 = (x - 6)(x - 8) = 0$
$\rightarrow x = 6$ or 8
Thus, BCD is a scaled 3-4-5 triangle and, clearly, $x = 6$ produces the largest $\angle B$ and
$\sin C = \underline{\dfrac{3}{5}}$ or $\underline{0.6}$



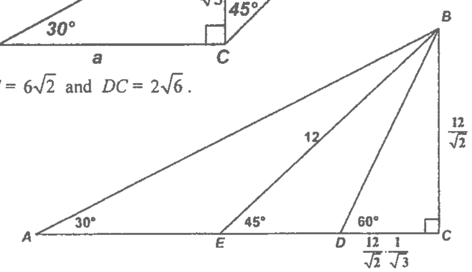
GBML 2008

ROUND 5

1. Since ΔBCD is an isosceles right triangle,
$b = \dfrac{a}{\sqrt{3}} \cdot \sqrt{2}$. Thus, $a = \dfrac{\sqrt{3}b}{\sqrt{2}} = \underline{\dfrac{b\sqrt{6}}{2}}$



2. From the diagram we see that $BC = EC = 6\sqrt{2}$ and $DC = 2\sqrt{6}$.
Thus, $DE = 6\sqrt{2} - 2\sqrt{6}$
$AC = BC \cdot \sqrt{3} = 6\sqrt{2} \cdot \sqrt{3} = 6\sqrt{6}$
But $AE = AC - EC = 6\sqrt{6} - 6\sqrt{2}$
Finally, $AE - ED =$
$\left(6\sqrt{6} - 6\sqrt{2}\right) - \left(6\sqrt{2} - 2\sqrt{6}\right) =$
$\underline{8\sqrt{6} - 12\sqrt{2}}$



3. Wheel Q turns at 900 rev/hour ⇔ 60 rev/minute ⇔ $\frac{1}{4}$ rev/second

Since the angular velocity of the two wheels is in a 10 : 15 = 2 : 3 ratio, the radii of the wheels must also be in a 2 : 3 ratio, implying that the radius of wheel Q is

(2/3)2 = 4/3 feet = 16 inches. Thus, a linear velocity is $\frac{1}{4}$ rev/sec is equivalent to

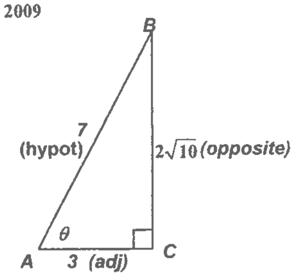
$\frac{1}{4} \cdot 2 \cdot \pi \cdot 16 = \underline{8\pi}$ feet/sec

GBML 2009

Detailed Solutions for GBML Meet 1 – OCTOBER 2009

ROUND 5

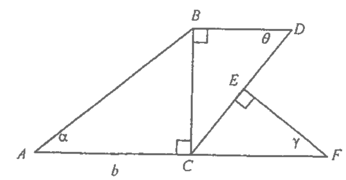
1. $\cot\theta + \csc\theta = \dfrac{3}{2\sqrt{10}} + \dfrac{7}{2\sqrt{10}} = \dfrac{10}{2\sqrt{10}}$
$= \dfrac{5}{\sqrt{10}} = \underline{\dfrac{\sqrt{10}}{2}}$



2. The circumference of the circle is 36π inches = 3π feet
Thus, a point with a linear velocity of π ft/sec makes $\frac{1}{3}$ rev/sec or 20 rev/min or
20(40) = $\underline{800}$ revolutions in 40 minutes.

3. In ΔABC, $\frac{BC}{b} = \tan\alpha$.

In ΔBCD, $\frac{BC}{CD} = \frac{BC}{2CE} = \sin\theta$.



In ΔCEF, $\frac{CE}{CF} = \sin\gamma$.

Substituting for BC in the 2nd equation, $\frac{b\tan\alpha}{2CE} = \sin\theta$ or $CE = \frac{b\tan\alpha}{2\sin\theta}$

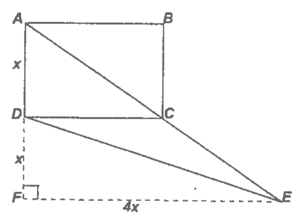
From the 3rd equation, $CF = \frac{CE}{\sin\gamma}$ and we have $CF = \frac{b\tan\alpha}{2\sin\theta\sin\gamma} = \underline{\frac{b\cdot\tan\alpha\cdot\csc\theta\cdot\csc\gamma}{2}}$

GBML 2010

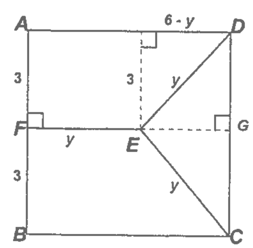
ROUND 5

1. The circumference of the wheel is $\frac{72\pi}{100}$ meters.
   In 4000 revolutions, the point on the rim of the wheel travels down the road $\left(\frac{72\pi}{100}\right)4000 = 40(72\pi)$ meters. To travel this distance at $32\frac{m}{\sec}$ would take $\frac{40(72\pi)}{32} = 5(18)\pi = 90\pi \sec = \underline{\frac{3\pi}{2}}$ minutes.

2. Locate F as indicated in the diagram.
   Clearly, $\Delta ADC \sim \Delta AFE$ and $\frac{AD}{AF} = \frac{1}{2} \rightarrow FE = 4x$
   $\rightarrow DE = \underline{x\sqrt{17}}$



3. Drop a perpendicular from E to $\overline{AD}$. Then:
   $3^2 + (6-y)^2 = y^2 \rightarrow 27 + 36 - 12y = 0 \rightarrow y = \frac{15}{4}$
   $\rightarrow EG = 6 - \frac{15}{4} = \frac{9}{4} \rightarrow \sin(\angle EDC) = \frac{9/4}{15/4} = \underline{\frac{3}{5}}$
   Note ΔDEG is similar to the 3 – 4 – 5 right triangle.



MML 12/04

Round One:

A. Since tan(x)<0 we are in fourth quadrant with right triangle having sides of 11,5, and $4\sqrt{6}$ so csc(x) = $\frac{-11\sqrt{6}}{24}$.

B. Law of Cosines: $PK^2 = 16^2 + 18^2 - 2(16)(18)(0.75) = 148$ so PK=$2\sqrt{37}$

C. Pythagorus gives AE=$\sqrt{15}$ Law of Sines gives BE= sin ∠ D (BD) / sin ∠ BED.
   Since sin ∠ CEA = 7/8, BE = 0.5 (91)(8/7) = 52

MML 12/05

Round One:

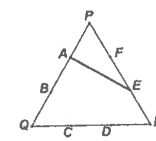
A. BC/AB = 0.28 =7/25 so one rt Δ has AC = 24 by Pythagoras. Tan(∠B)=AC/BC

B. Law of Sines: $\frac{\sin(\angle D)}{EF} = \frac{\sin(\angle F)}{ED}$ so $\frac{5/9 + 1/3}{x + 2/3} = \frac{5/9}{x}$ thus
   8/9 x = 5/9 x +10/27 so ED = x = 10/9 and EF = 16/9

C. Draw altitude CD. From ∠A AD = 3x, CD = 4x. From ∠B BD = 5y, CD=12y so 12y=9x and AD = 9y. Pythagoras gives BC = 13y and AC = 15y while AB = 14y. Perimeter gives y =10 thus area is 0.5(14y)(12y) = 8400

MML 12/06

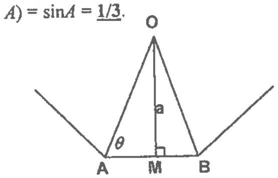
Round 1



A) Using the law of cosine, $AE^2 = 2^2 + 4^2 - 2(2)(4)\cos 60°$
   $= 20 - 16(1/2) = 12 \rightarrow PQ = \underline{2\sqrt{3}}$.

B) Using the law of sine, $\frac{\sin A}{10} = \frac{\sin 150°}{15} \rightarrow \sin A = \frac{2}{3}\cdot\frac{1}{2} = \frac{1}{3}$
   The given information (2 sides and the non-included angle) is the ambiguous case, but since $\angle B$ is obtuse, there is exactly one triangle satisfying the given conditions.
   Since $A + B + C = 180°$, $B + C = 180 - A$ and $\sin(B + C) = \sin(180 - A) = \sin A = \underline{1/3}$.

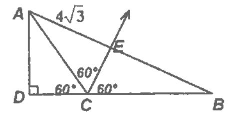
C) $m\angle BOA = (360/n)° \rightarrow \theta = 90 - 180/n$ and $AM = ½(p/n) = p/(2n)$
   $\tan(\theta) = (OM)/(AM) = (2na)/p \rightarrow a = p\tan(\theta)/(2n)$
   Replacing the angle θ by its complement and the trig function by its cofunction, $\rightarrow \frac{p\cot(\frac{180}{n})}{2n} \rightarrow (X, Y) = \underline{(180, 2)}$.



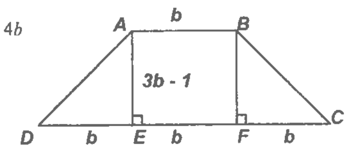
MMC 12/07

Round 1

A) ΔABC is a 30-60-90 triangle. Draw altitude $\overline{CE}$ from C to $\overline{AB}$.
$\overline{AB}$ must be the base in isosceles triangle ABC.
Therefore, E must also be a midpoint of $\overline{AB}$ and ΔACE must also be a 30-60-90 triangle congruent to $\Delta ACD \rightarrow AD = \underline{4\sqrt{3}}$
($ADCE$ is a kite)



B) $A = 840 = \frac{h}{2}(b_1 + b2) = (3b-1)(4b)/2 \rightarrow 1680 = 12b^2 - 4b$
$\rightarrow 3b^2 - b - 420 = (3b+35)(b-12) = 0 \rightarrow b = 12$
ΔADE, ΔBCF are 12 – 35 – 37 right triangles
$\rightarrow Per = 74 + 48 = 122$, $AE = 35 \rightarrow$ **122 : 35**

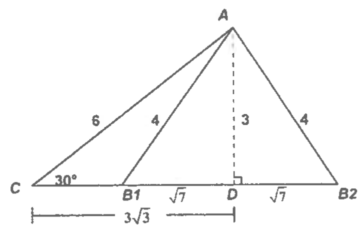


C) This is the ambiguous case, where we have information about two sides and the non-included angle. In general, there could be 0, 1 or 2 possible solutions. In this problem there are two solutions.

Using the law of sine, $\frac{AB}{\sin C} = \frac{AC}{\sin B} \rightarrow \frac{4}{.5} = \frac{6}{\sin B}$

$\rightarrow \sin B = 3/4 \rightarrow \cos B = \pm\frac{\sqrt{7}}{4}$

Dropping an altitude from A creates a 30-60-90 triangle $ACD \rightarrow \overline{AD}$, the side opposite 30°, must have length 3 and $\overline{CD}$, the side opposite 60°, must have length $3\sqrt{3}$.



Clearly, referring to the diagram, the negative cosine value is associated with an obtuse angle ($\angle B$ in ΔACB_1) and the positive cosine value is associated with the acute angle ($\angle B$ in ΔACB_2)
Thus, $BC = \underline{\sqrt{7} + 3\sqrt{3} \text{ or } 3\sqrt{3} - \sqrt{7}}$.

Alternate solution: Using only 30 – 60 – 90 right triangles (2 diagrams are possible)