Round 3
Similar Polygons, Circles and Areas Related to Circles

GREATER BOSTON MATHEMATICS LEAGUE

MEET 4 – JANUARY 1999

ROUND 3 – Similar Polygons, Circles and Areas Related to Circles

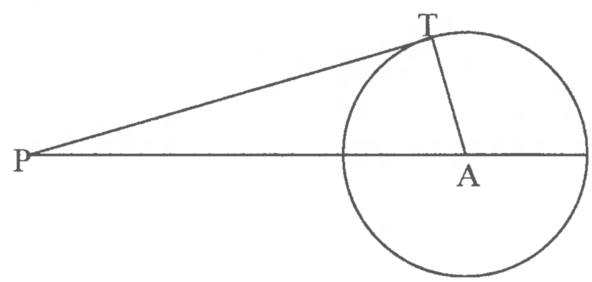
1. ____________________

2. ____________________

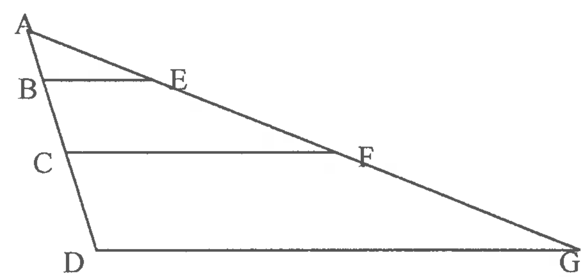
3. ____________________

DIAGRAMS ARE NOT NECESSARILY DRAWN TO SCALE
CALCULATORS ARE NOT ALLOWED ON THIS ROUND

1. From point P, 9 inches from the closest point on a circle centered at A with diameter of length 7 inches, one tangent is drawn to the circle with T, the point of tangency. Find the number of square inches in the area of Δ PAT.



2. Given $\overline{BE} \parallel \overline{CF} \parallel \overline{DG}$, AB:BC:CD = 2:3:4, and the area of BEFC = 126, find the area of CFGD.



3. Given Δ RST with $\overline{RS} \perp \overline{ST}$, RS = 6, and ST = 8, find the area of the region interior to the circumscribed circle and exterior to the inscribed circle of Δ RST.

GREATER BOSTON MATHEMATICS LEAGUE

MEET 4 – JANUARY 2000

ROUND 3 – Similar Polygons, Circles and Areas Related to Circles

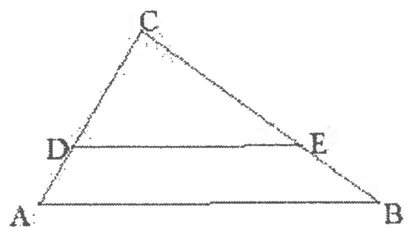
1. ____________________

2. ____________________

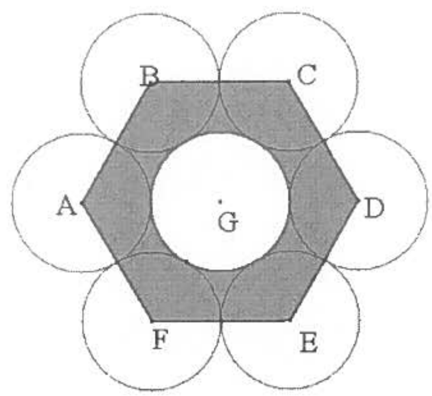
3. ____________________

DIAGRAMS ARE NOT NECESSARILY DRAWN TO SCALE
CALCULATORS ARE NOT ALLOWED ON THIS ROUND

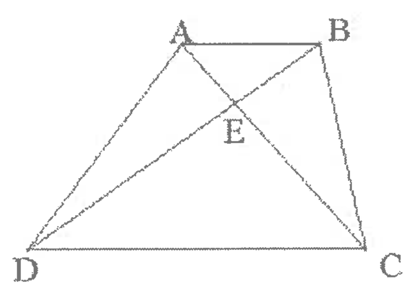
1. Given $\overline{DE}$ parallel to $\overline{AB}$, CD:DA = 2:1, and the area of trapezoid ABED = 45 cm^2, find the number of square centimeters in the area of Δ CDE.



2. Given congruent circles of radius 2 cm. tangent externally in pairs, whose centers form the regular hexagon, ABCDEF, and circle G tangent to all six circles, find the shaded area. (See the figure.)



3. Given $\overline{AB}$ parallel to $\overline{CD}$, and AE:EC = 2:5, find the ratio of the area of Δ ABE to the area of trapezoid ABCD.



GREATER BOSTON MATHEMATICS LEAGUE

MEET 4 – JANUARY 2001

ROUND 3 – Similar Polygons, Circles and Areas Related to Circles

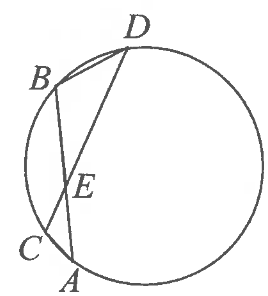
1. ____________________

2. ____________________

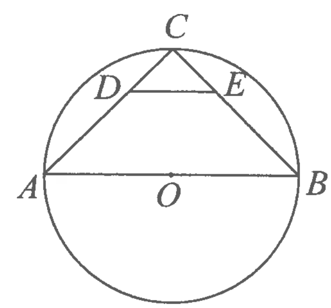
3. ____________________

DIAGRAMS ARE NOT NECESSARILY DRAWN TO SCALE
CALCULATORS ARE NOT ALLOWED ON THIS ROUND

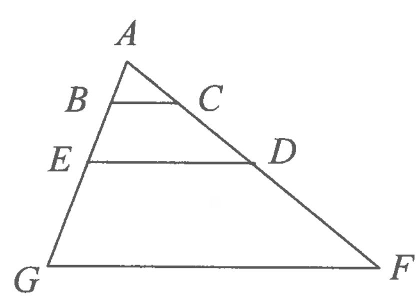
1. The circle on the right has chords $\overline{AB}$ and $\overline{CD}$ intersecting at point E. If $CE = 4$, $ED = 12$, and $BE = 8$, find the ratio of the area of ΔACE to the area of ΔBDE.



2. Given circle, center O, diameter $\overline{AB}$, isosceles $\Delta\ ACB$, $\overline{DE} \parallel \overline{AB}$, $\overline{ADC}$, $\overline{BEC}$, $AD{:}DC = 2{:}1$ and the circumference of circle O is 24π cm, find the number of square centimeters in the area of quadrilateral $ABED$.



3. In the diagram on the right, $\overline{BC} \parallel \overline{ED} \parallel \overline{GF}$, $BC : GF = 1 : 5$, and $AC : CD = 2 : 3$, find the ratio of the area of trapezoid $BCDE$ to the area of ΔAGF.



GREATER BOSTON MATHEMATICS LEAGUE

MEET 4 – JANUARY 2002

ROUND 3 – Similar Polygons, Circles and Areas Related to Circles

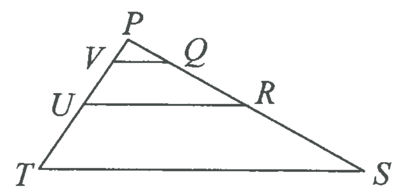
1. ____________________

2. ____________________

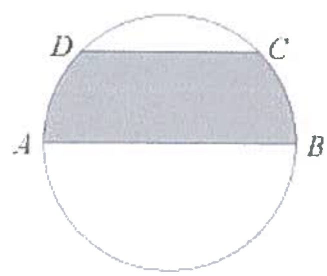
3. ____________________

DIAGRAMS ARE NOT NECESSARILY DRAWN TO SCALE
CALCULATORS ARE NOT ALLOWED ON THIS ROUND

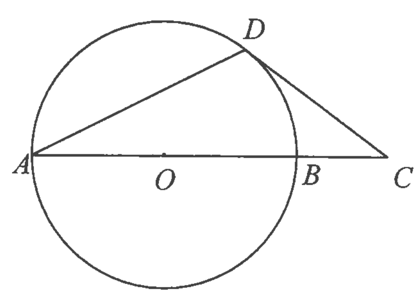
1. Given $\overline{PQRS}$, $\overline{PVUT}$, $\overline{QV} \parallel \overline{RU} \parallel \overline{ST}$, and $PQ:QR:RS = 1:2:3$, find the ratio of the area of ΔPQV to the area of trapezoid *RSTU.*



2. Given $\overline{AB}$ is a diameter of the circle to the right, $AB = 20$, and $m\widehat{AD} = m\widehat{BC} = 45°$, find the area of the shaded region.



3. Given circle centered at O, $\overline{AOBC}$, $\overline{CD}$ tangent to the circle at point D, $BC = 6$, and $CD = 12$, find the area of ΔACD.



GREATER BOSTON MATHEMATICS LEAGUE
MEET 4 – JANUARY 2006

ROUND 3 – Similar Polygons, Circles and Areas Related to Circles

1. ______________ **units^2**

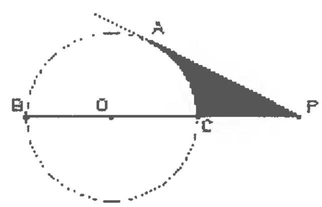
2. ______________ **units^2**

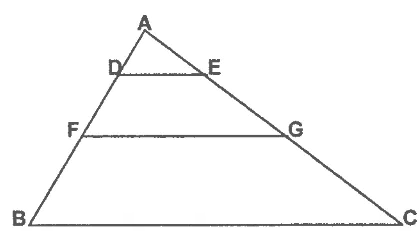
3. _____ **:** _____ **:** _____

CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

1. A 30° central angle extends through two concentric circles of radii 7 units and 11 units. Determine the exact number of square units between the two concentric circles and in the interior of the central angle.

2. $\overline{PA}$ is tangent to circle O whose circumference is 8π units. $OP = 2(OB)$. Find, in simplified form, the exact area of the shaded region.





3. Given ΔABC, where
   $\overline{DE} \parallel \overline{FG} \parallel \overline{BC}$
   $AD = 4$, $DF = 6$ and $FB = 8$.

   Find the ratio of the area of ΔADE to the area of trapezoid $DFGE$ to the area of trapezoid $FBCG$. Express your answer as a simplified ratio of integers.

GREATER BOSTON MATHEMATICS LEAGUE
MEET 4 – JANUARY 2007

ROUND 3 – Similar Polygons, Circles and Areas Related to Circles

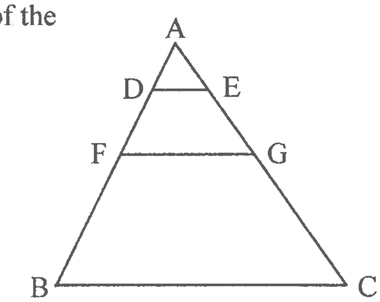
1. ________ : ________

2. ____________________

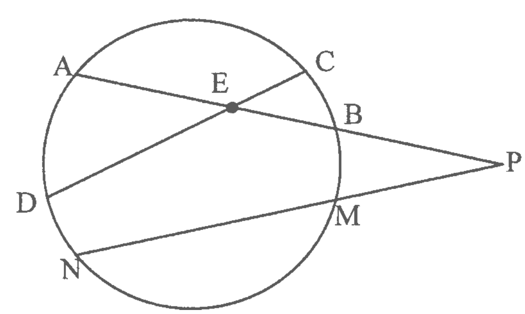
3. ________ : ________

CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

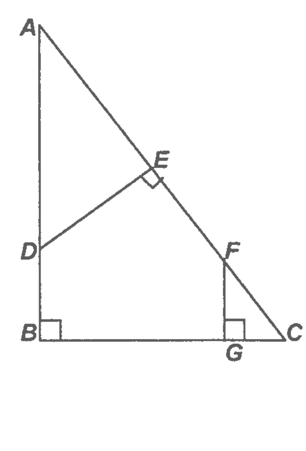
1. In ΔABC, $\overline{DE} \parallel \overline{FG} \parallel \overline{BC}$, $AD:DF:FB = 2:5:8$. Find the ratio of the area of quadrilateral $DEGF$ to quadrilateral $FGCB$.



2. In a circle, chords $\overline{AB}$ and $\overline{DC}$ intersect at point E. Secants $PBEA$ and PMN are shown. $EC = 4$, $DC = 13$, $AB = 15$, $AE:EB = 4:1$, $PB = 6$, and $NM = AE - 1$. Find PM.



3. In right ΔABC, $\overline{DE} \perp \overline{AC}$, $\overline{FG} \parallel \overline{AB}$, $AD = 8$, $DB = 4$, $GC = 3$, $AF:AC = 2:3$. Find the ratio of $EF:DE$.



GREATER BOSTON MATHEMATICS LEAGUE

MEET 4 – JANUARY 2008

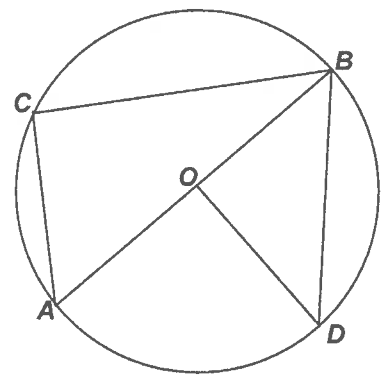
ROUND 3 – Similar Polygons, Circles and Area

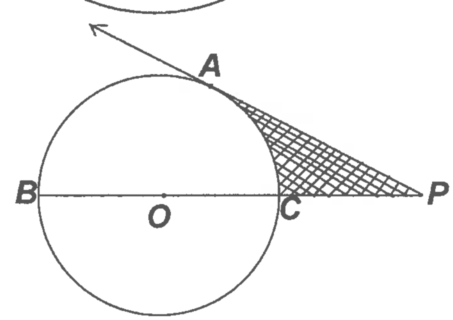
If you would like to receive email announcements regarding upcoming competitions, please print your email on the reverse side of this paper when you have finished answering the problems.	1. ____________ 2. ____________ 3. ____________

DIAGRAMS ARE NOT NECESSARILY DRAWN TO SCALE.
CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

1. In circle O, $\overset{\frown}{AD} \cong \overset{\frown}{DB}$ and $2m\overset{\frown}{AC} = m\overset{\frown}{CB}$. Compute $\dfrac{BC}{BD}$ in simplified radical form.

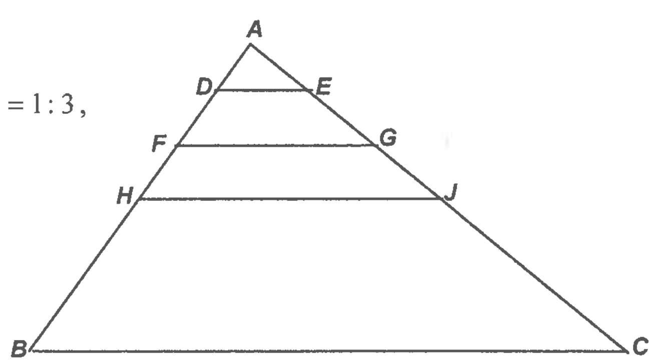
   Note: Each arc above refers to a minor arc.





2. $PC = OC$, $\overrightarrow{PA}$ is tangent to circle O and $\overline{PB}$ is a secant. Find the ratio of the area of the shaded region bounded by the tangent $\overrightarrow{PA}$, the secant $\overleftrightarrow{BP}$, and the circle O to the area of ΔAOC.

3. Given: ΔABC with $\overline{DE} \parallel \overline{FG} \parallel \overline{HJ} \parallel \overline{BC}$
   If $AD : DF = 2:3$, $AD : AH = 1:4$ and $AF : AB = 1:3$, find the ratio of the area of trapezoid $FGJH$ to the area of trapezoid $HJCB$.



GREATER BOSTON MATHEMATICS LEAGUE

MEET 4 – JANUARY 2009

ROUND 3 – Similar Polygons, Circles and Area

1. ______________________

2. __________ : __________

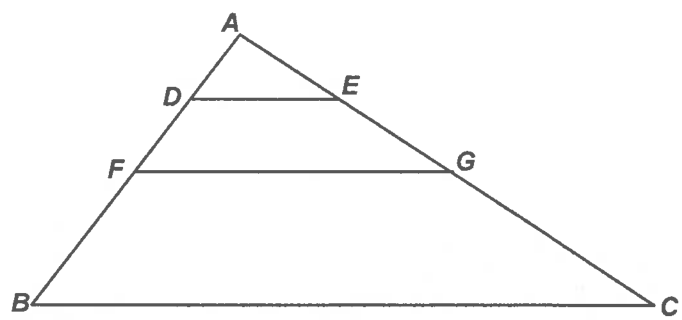
3. ______________________

DIAGRAMS ARE NOT NECESSARILY DRAWN TO SCALE.
CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

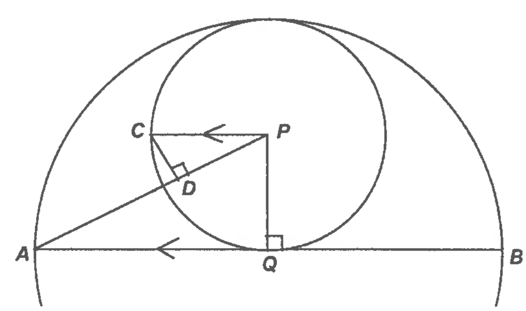
1. Find the longest distance from the point $P(3, 13)$ to the circle

$$C:\{x^2 - 10y + y^2 + 24 + 6x = 0\}$$

2. In the triangle ABC, $\overline{DE} \parallel \overline{FG} \parallel \overline{BC}$.
$AD = DF$ and $AF = FB$. The area of trapezoid $DEGF$ is 9 square units. What is the ratio of the area of triangle ADE to the area of trapezoid $FGCB$?



3. Circle P is inscribed in a semicircle of circle Q as indicated. If $AB = 8$, find the length of $\overline{CD}$.



GREATER BOSTON MATHEMATICS LEAGUE

MEET 4 – JANUARY 2010

ROUND 3 – Similar Polygons, Circles and Area

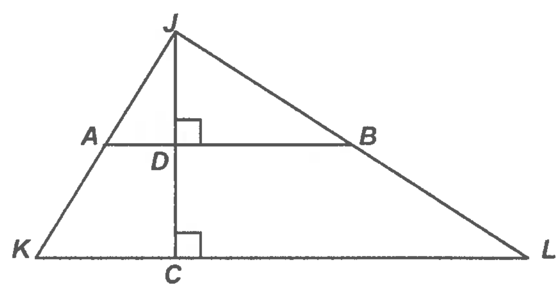
1. $AB =$ __________ $JD =$ __________

2. __________________________ inches^2

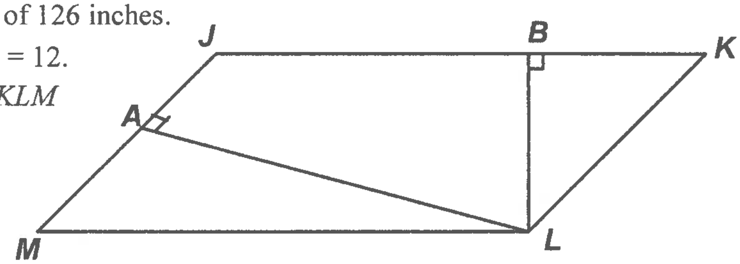
3. ______________________________

DIAGRAMS ARE NOT NECESSARILY DRAWN TO SCALE.
CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

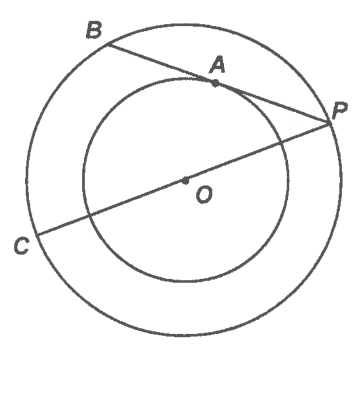
1. In ΔJKL, $\overline{JC} \perp \overline{KL}$, $\overline{AB} \parallel \overline{KL}$, $JC = 8$ and $KL = 24$. Compute the lengths of $\overline{AB}$ and $\overline{JD}$ so that the area of ΔJAB equals the area of quadrilateral $ABLK$.



2. Parallelogram $JKLM$ has a perimeter of 126 inches. $\overline{LA} \perp \overline{MJ}, \overline{LB} \perp \overline{JK}$, $LA = 15$ and $LB = 12$. Compute the area of parallelogram $JKLM$ in square inches.



3. Two concentric circles with center of O have radii R and r, with $R = 3r$. Find a simplified expression for the area of ΔPBC in terms of r, if $\overline{POC}$ is a diameter of the larger circle and chord $\overline{PB}$ is tangent to the smaller circle at A.



GREATER BOSTON MATHEMATICS LEAGUE
MEET 4 – JANUARY 2011

ROUND 3 – Similar Polygons, Circles and Area

1. ______________________ cm

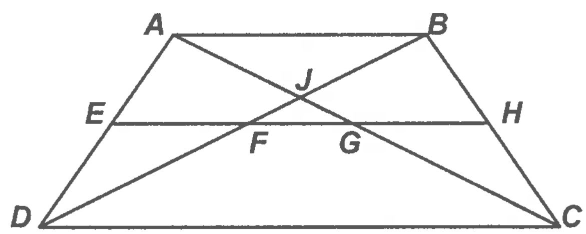
2. ________ : ________ : ________

3. ______________________ sq. units

DIAGRAMS ARE NOT NECESSARILY DRAWN TO SCALE.
CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

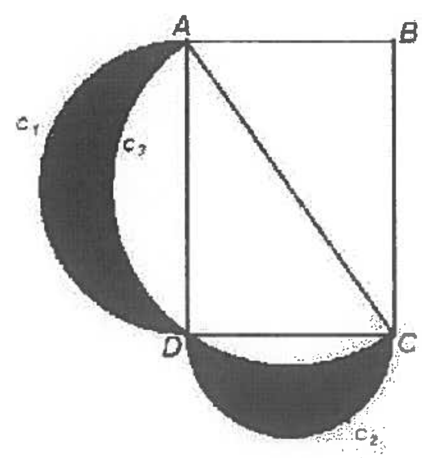
1. A chord in circle O has length 48 cm and is 8 cm closer to the center than a chord of length 40 cm. How much further from the center will a 30 cm chord be than the 48 cm chord?

2. Given: Isosceles trapezoid $ABCD$
with median $\overline{EFGH}$
$AB : DC = 2 : 5$ and $FG = 7.5$



Express area(ΔDHC) : area(ΔFJG) : area(ΔAJB) as a simplified ratio of integers.

3. The area of rectangle $ABCD$ is 40 units^2. The perimeter of rectangle $ABCD$ is 26 units.
C_1 is a semicircle drawn on $\overline{AD}$.
C_2 is a semicircle drawn on $\overline{CD}$.
C_3 is a semicircle drawn on $\overline{AC}$.
Compute the area of the shaded region.



MASSACHUSETTS MATHEMATICS LEAGUE
JANUARY 2004
ROUND 5: SIMILAR POLYGONS

ANSWERS

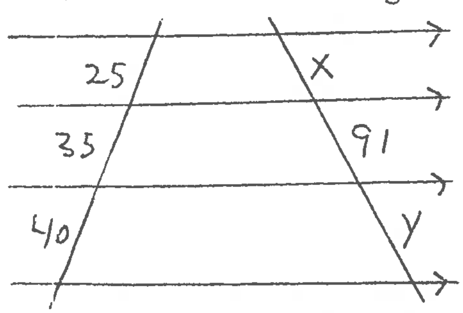
A)______________

B)______________

C)______________

A) There are two solid cubes made of the same material where the edge of one cube is three times the edge of the other. If the smaller cube weighs 2.3 grams, calculate to the nearest tenth, the weight of the larger cube.

B) In the figure shown, lines k, l, m, and n are parallel, with transversal segment lengths given. Calculate the sum of the lengths of segments x and y.



C) In regular hexagon ABCDEF, G is on $\overline{FC}$ so that $\angle CBG = 45^0$. Calculate in simple radical form, the ratio of $\overline{GC}$ *to* $\overline{CB}$.

MASSACHUSETTS MATHEMATICS LEAGUE
FEBRUARY 2004
ROUND 5: GEOMETRY CIRCLES
NON-CALCULATOR

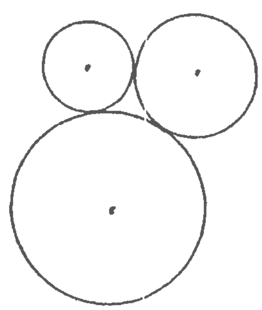
ANSWERS

A)_______________

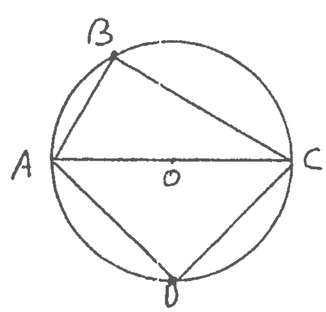
B)_______________

C)_______________

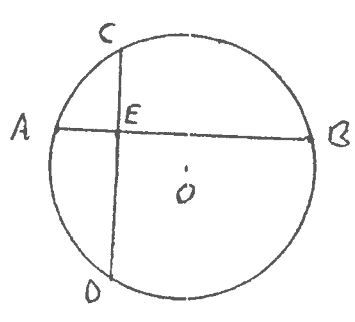
A) Three circles of areas π, 4π, *and* 9π are drawn tangent to each other. Calculate the area of the triangle formed by connecting the centers of the three circles.



B) In the figure, $\overline{AC}$ is a diameter of circle O, $\widehat{AB} = \frac{1}{2}\widehat{BC}$, D is the midpoint of $\widehat{AC}$. Find the value of BC/AD in simplified radical form.



C) In circle O, $\overline{CD} \perp \overline{AB}$, CE = 5, CD = 14, and the ratio of AE to AB is 1 to 6. The area of circle O is $k\pi$. What is the value of k?



MASSACHUSETTS MATHEMATICS LEAGUE
JANUARY 2005
ROUND 5 GEOMETRY: SIMILAR POLYGONS

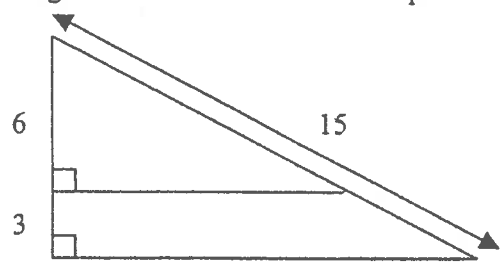
ANSWERS

A)________ sq units

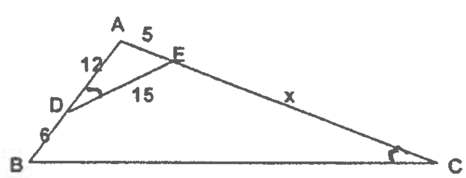
B)______________

C)______________

A) In the figure what is the area of the trapezoid?



B) If m∠ACE = m∠ADE and EC = x, express the exact value of x as a decimal.



C) The ratio of the perimeters of 2 regular hexagons is 4:3 . If the smaller diagonal of the smaller hexagon has length $4\sqrt{3}$ find the sum of the areas of the two hexagons in the simplified radical form $\dfrac{A\sqrt{B}}{C}$.

MASSACHUSETTS MATHEMATICS LEAGUE
FEBRUARY 2005
ROUND 5 GEOMETRY: CIRCLES

******* NO CALCULATORS ON THIS ROUND ******

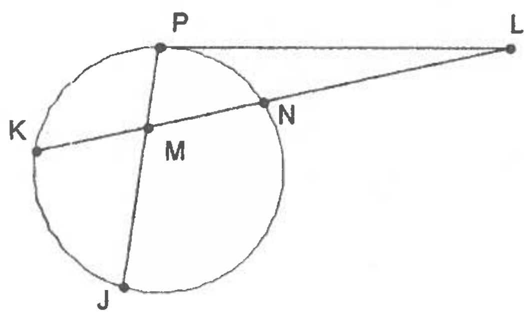
ANSWERS

A)______________

B)______________

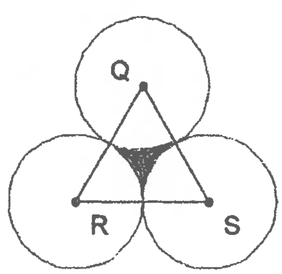
C)______________

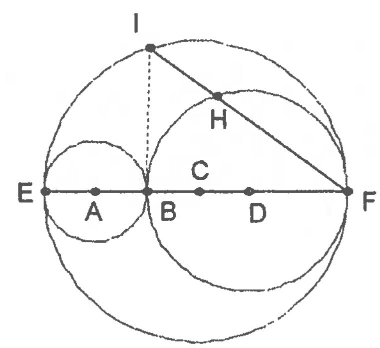
A) Given MJ=9, LN=13, M the midpoint of $\overline{KN}$, and PM=4 find the exact length of the tangent $\overline{PL}$



B) Circles Q, R, and S (as shown on the left below) are externally tangent and each has a radius of 3. Find the exact area of the total shaded region.

C) On the right below circles centered at A, C, and D are mutually tangent at E, B, and F. The largest circle has radius 12 and the smallest has radius 4. If $\overline{IB}$ is a tangent to the smaller circles find HF in simplified radical form.





MASSACHUSETTS MATHEMATICS LEAGUE
JANUARY 2006
ROUND 5 GEOMETRY: SIMILAR POLYGONS

ANSWERS

A)________ sq units

B)______________

C)______________

A) A 3-4-5 triangle is enlarged to make a similar triangle with hypotenuse 50 units long. What is the area of the enlarged triangle?

B) A right Δ has integer sides and one side has length 5. A second Δ with a perimeter of 1 is similar to the first Δ. Find the maximum possible difference between the areas of the two triangles. Express the answer as a simplified fraction $\frac{a}{b}$.

C) $ABCDEF$ is a regular hexagon of side 10 cm. M is the midpoint of $\overline{AB}$ and N the midpoint of $\overline{CD}$. X is the intersection of $\overline{ME}$ and $\overline{NF}$. Find the exact length MX in simplified radical form.

MASSACHUSETTS MATHEMATICS LEAGUE
FEBRUARY 2006
ROUND 5 GEOMETRY: CIRCLES

***** NO CALCULATORS ON THIS ROUND ****

ANSWERS

A)_______________

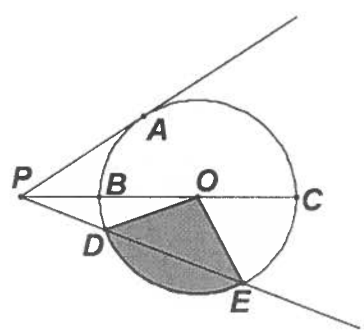
B)_______________

C)_______________

A) A chord of length 96 cm is 20 cm from the center of the circle.
How far is the midpoint of the chord from the furthest point on the circle?

B) Two chords $\overline{AB}$ and $\overline{CD}$ intersect at E. If $AE = 5x - 3$, $CE = 3x - 1$, $BA = 6x - 2$, and $DC = 5x - 1$, find all possible lengths for AE.

C) In the dagram (not to scale) $\overline{PA}$ is tangent to the circle with center O.
$PO = 7\sqrt{7}$, $PD = DE$ and AP = $7\sqrt{6}$. Find the exact area of sector ODE.



MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 4 – JANUARY 2007
ROUND 5 GEOMETRY: SIMILARITY OF POLYGONS

ANSWERS

A) ____________________________ $units^2$

B) ____________________________ feet

C) ______________ : ______________

A) A segment connects the midpoints of the legs of a right triangle with sides of length 3, 4 and 5, dividing the right triangle into a triangle and a trapezoid. What is the area of the trapezoid?

B) A building has a light mounted 15 feet above the ground. A person 6 feet tall is standing 10 feet from the base of the building.
Exactly how long is the person's shadow?
(Assume the person and the building are perpendicular to level ground.)

C) Given: $\Delta ABC \sim \Delta CAD$, $AB = 12$ and $CD = 27$. Determine the simplified ratio of the area of the circle inscribed in ΔABC to area of the circle inscribed in ΔCAD.

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 4 – JANUARY 2008
ROUND 5 GEOMETRY: SIMILARITY OF POLYGONS

ANSWERS

A) (__________ , __________)

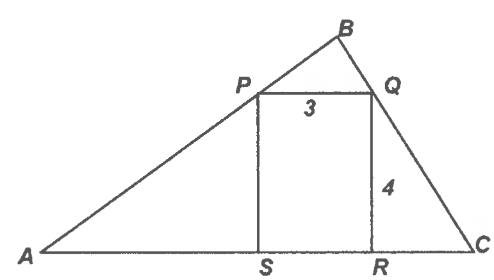
B) ______________________________

C) ______________________________

A) ΔABC is a right triangle with legs $AB = 3$ and $BC = 4$. $\Delta DEF \sim \Delta ABC$ and $DF = 6$. Determine the ordered pair (DE, EF).

B) A line parallel to the short sides of a 12 x 25 rectangle subdivides the rectangle into two similar noncongruent rectangles. Determine the area of the larger of these two rectangles.

C) If $PQRS$ is a 3 x 4 rectangle as illustrated, $\overline{AB} \perp \overline{BC}$ and $RC = 3$, compute the perimeter of ΔABC.



MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 – FEBRUARY 2008
ROUND 5 GEOMETRY: CIRCLES

ANSWERS

A) ______________________ ft/min

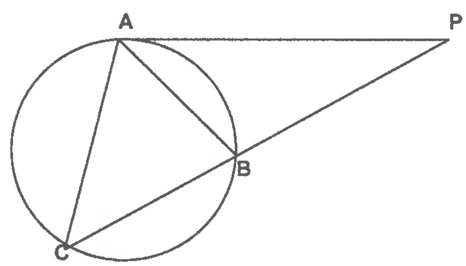
B) ______________________

C) ______________________

***** NO CALCULATORS ON THIS ROUND *****

A) A wheel of radius 6 inches rotates at 2 revolutions per second. In terms of π, how fast does a point on the circumference turn in feet per minute?

B) $\overline{PA}$ is tangent to circle O at A, $PA = 5x - 3$, $PB = 3x - 1$, $BC = 7x - 11$ and $AC = 2x + 3$ Compute the perimeter of ΔAPC.



C) In circle O, perpendicular chords $\overline{AB}$ and $\overline{CD}$ intersect at point P. $AP = 12$, $PB = 28$ and $CP = 14$. Compute $AD - PO$.

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 4 - JANUARY 2011
ROUND 5 GEOMETRY: SIMILARITY OF POLYGONS

ANSWERS

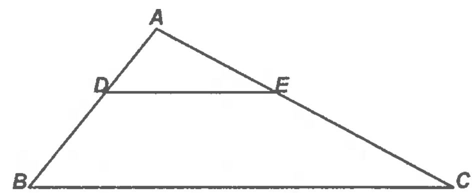
A) ______________ : ______________

B) ______________ : ______________

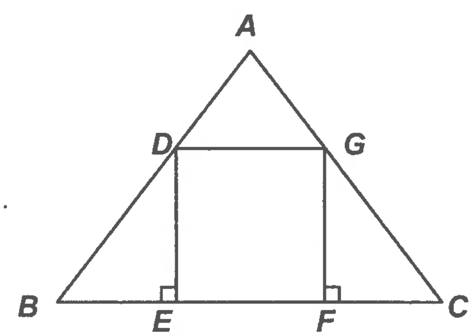
C) ______________________________

***** NO CALCULATORS ON THIS ROUND *****

A) Given: $\overline{DE} \parallel \overline{BC}, DE = 10, BC = 25$
Compute the ratio of the area of ΔADE to the area of trapezoid $DECB$.



B) ΔABC is equilateral, $\overline{DG} \parallel \overline{BC}$.
The area $BDGC$ is $\frac{15}{16}$th the area of ΔABC.
Compute the ratio of the area of ΔABC to the area of ΔBED.

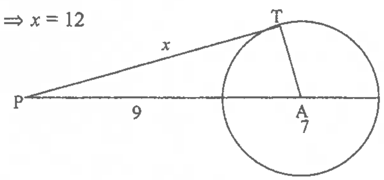


C) Given: Four regular hexagons A, B, C and D
A has an area of $\frac{9\sqrt{3}}{4}$ square units.
A longer diagonal in B has length $4\sqrt{6}$.
Sides of regular hexagon C have the same length as a shorter diagonal of A.
Sides of regular hexagon D have the same length as a shorter diagonal of B.
Compute the <u>sum</u> of the areas of hexagons C and D.

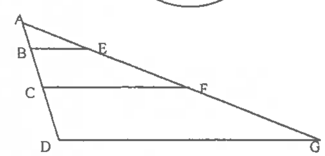
'99 GBML

ROUND 3

1. To find the tangent segment: $x^2 = 9 \cdot 16 \Rightarrow x = 12$
$\Rightarrow$ area of Δ PAT $= \left(\frac{1}{2}\right)\left(\frac{7}{2}\right)(12) = \mathbf{21}\ in^2$



2. AB:BC:CD = 2:3:4 $\Rightarrow$ AB:AC:AD = 2:5:9 $\Rightarrow$
Δ ABE: ΔACF: ΔADG = 4:25:81 $\Rightarrow$
Δ ABE = $4x$, Δ ACF = $25x$, and Δ ADG = $81x \Rightarrow$
BEFC = 126 = $21x \Rightarrow x = 6$ and CFGD = $56x$ = **336**



3. The radius of the circumscribed circle = half the hypotenuse = 5. To find the radius of the inscribed circle, use the formula $\frac{1}{2}Pr = A \Rightarrow 12r = 24$, and $r = 2 \Rightarrow$ area of the region = $25\pi - 4\pi = \mathbf{21\pi}$

'00 GBML

Round 3

1. Since CD:DA = 2:1 $\Rightarrow$ CD:CA= 2:3 $\Rightarrow$ area of Δ CDE: area of Δ CAB = 4:9; let area of Δ CDE = $4x$, then area of Δ CAB = $9x \Rightarrow$ area of ABED = $5x = 45 \Rightarrow x = 9$ and area of Δ CDE = 36 cm.^{2}

2. The length of long diagonal of the regular hexagon = twice the length of its side $\Rightarrow$ AD = 8 cm. $\Rightarrow$ radius of circle G = 2 cm. Shaded area = area of ABCDEF – area of circle G = $6\frac{4^2\sqrt{3}}{4} - \pi \cdot 2^2 = 24\sqrt{3} - 4\pi$ cm.^{2}

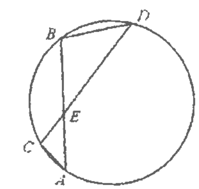
3. Since AE:EC = 2:5 $\Rightarrow$ area of Δ AEB: area of Δ CED = 4:25 and area of Δ AEB: area of Δ BEC = 2:5; BE:ED = 2:5 $\Rightarrow$ area of Δ AEB: area of Δ AED = 2:5; let area of Δ AEB = $4x \Rightarrow$ area of Δ DEC = $25x$, area of Δ AED = $10x$, and area of Δ BEC = $10x$; area of trapezoid ABCD = $4x+25x+10x+10x = 49x \Rightarrow$ area of Δ AEB: trapezoid ABCD = 4:49

'01 GBML

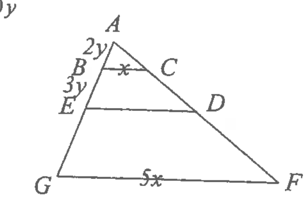
Round 3

1. $CE \cdot ED = AE \cdot ED \rightarrow AE = 6$; $\Delta AEC \sim \Delta DEB \rightarrow$ ratio of their areas = $\left(\frac{AE}{DE}\right)^2 = \left(\frac{6}{12}\right)^2 = 1:4$

2. $\Delta\, ABC$ is right isosceles; $C = 24\pi \rightarrow d = AB = 24 \rightarrow$
$AC = BC = 12\sqrt{2} \rightarrow CD = CE = 4\sqrt{2}$;
area of ABED = area of Δ ABC – area of Δ DEC =
$\frac{1}{2}\left(12\sqrt{2}\right)^2 - \frac{1}{2}\left(4\sqrt{2}\right)^2 = 144 - 16 = 128\ \text{cm}^2$



3. $\Delta ABC \sim \Delta AGF \rightarrow \frac{AB}{AG} = \frac{BC}{GF} \rightarrow \frac{2y}{AG} = \frac{1}{5} \rightarrow AG = 10y$
$\rightarrow$ height of trapezoid $BCDE$: height of ΔAGF = 3:10; call the respective heights $3h$ and $10h$.
$\Delta AED \sim \Delta AGF \rightarrow \frac{AE}{AG} = \frac{ED}{GF} \rightarrow \frac{5y}{10y} = \frac{ED}{5x} \rightarrow$
$ED = 2.5x$. Area of trapezoid $BCDE$ =
$\frac{1}{2}3h(x + 2.5x) = 5.25hx$; area of $\Delta AGF = \frac{1}{2}(5x)(10h) = 25xh$;
ratio of the areas = $\frac{5.25hx}{25hx} = \frac{21}{100}$

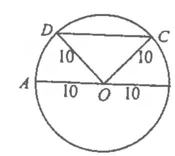


'02 GBML

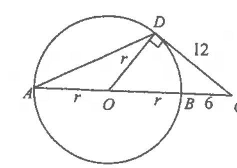
ROUND 3 – Similar Polygons, Circles and Areas Related to Circles

1. Since $PQ:QR:RS = 1:2:3 \Rightarrow PQ:PR:PS = 1:3:6 \Rightarrow$ area of ΔPQV: area of ΔPRU: area of ΔPST = 1:9:36 $\Rightarrow$ area of ΔPQV: area of trapezoid $RSTU$ = $1:36-9 = 1:27$.

2. Let O be the center of the circle. Draw radii $\overline{OC}$ and $\overline{OD}$. The shaded area consists of right ΔCOD and two 45° sectors. Therefore the area = $\frac{1}{2}(10)(10) + \frac{90}{360}(10)^2\pi = 50 + 25\pi$.



3. Draw radius $\overline{OD}$. Call its length r. $CD^2 = CA \cdot CB \Rightarrow$
$12^2 = 6(2r+6) \Rightarrow 2r + 6 = 24 \Rightarrow r = 9$. To find the area of ΔACD, you need the length of the altitude from D. Since ΔCOD is right $\Rightarrow$ if h = altitude, then $\frac{15h}{2} = \frac{12 \cdot 9}{2} \Rightarrow h = \frac{36}{5}$.
Therefore the area of $\Delta ACD = \frac{1}{2}(24)(\frac{36}{5}) = \frac{432}{5}$.



'06 GBML

ROUND 3 – Similar Polygons, Circles and Areas Related to Circles

1. The ring (annulus) between the concentric circles has area $\pi(11^2 - 7^2) = 72\pi$
   The 30° central angle slices out 1/12 of each circle and we have $\frac{72\pi}{12} = \underline{6\pi}$.

2. Let the radius of the circle be denoted r. $C = 8\pi \rightarrow r = 4$. A radius drawn to the point of contact of a tangent line forms a right angle. $OP = 2(OB) \rightarrow CP = r = 4$
   By the Pythagorean theorem, $AP = 4\sqrt{3}$ and ΔPAO is a 30-60-90 right triangle, since the sides are in a $1 : 2 : \sqrt{3}$ ratio. Thus, the area of the shaded region is
   Area(ΔPAO) – Area(60° sector) $= \frac{1}{2}(4)\sqrt{3}(4) - \frac{1}{6}\pi(4)^2 = \underline{8\sqrt{3} - \frac{8}{3}\pi}$ or $\underline{\frac{24\sqrt{3}-8\pi}{3}}$

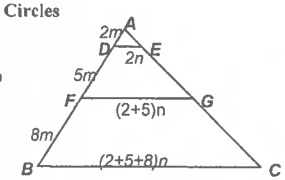
3. $\Delta ADE \sim \Delta AFG \sim \Delta ABC$ with corresponding sides in a ratio of 4 : 10 : 18 or 2 : 5 : 9 → their areas are in a ratio of 4 : 25 : 81.
   Subtracting the areas of the pairs of overlapping triangles, the required ratio is
   4 : 25-4 : 81-25 = **4 : 21 : 56**

'07 GBML

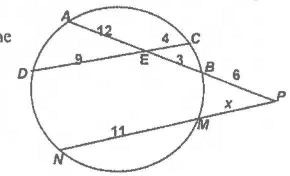
ROUND 3 – Similar Polygons, Circles and Areas Related to Circles

1. $AD : DF : FB = 2 : 5 : 8 \rightarrow DE : FG : BC = 2 : 7 : 15$
   Since the quadrilaterals are trapezoids, the required ratio

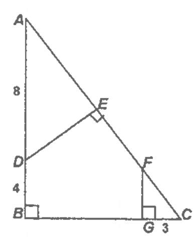
$$\frac{\text{Area}(DEGF)}{\text{Area}(FGCB)} = \frac{\frac{1}{2}(5)(2+7)}{\frac{1}{2}(8)(7+15)} = \underline{\mathbf{45 : 176}}$$



2. Applying the given information produces the diagram to the right. Let $PM = x$. Then, since $PB(PA) = PM(PN)$, we have $6(6 + 15) = x(x + 11) = 0$
   $\rightarrow x^2 + 11x - 126 = (x + 18)(x - 7) = 0 \rightarrow x = \underline{7}$



3. $AF : AC = 2 : 3 \rightarrow AF : FC = 2 : 1$
   $\overline{FG} \parallel \overline{AB} \rightarrow BG : GC = 2 : 1$
   Therefore, $BG = 6$ and $AC = 15$, $AF = 10$, $FC = 5$ and $FG = 4$
   $\Delta ADE \sim \Delta ACB \rightarrow \frac{AD}{AC} = \frac{DE}{CB} \rightarrow \frac{8}{15} = \frac{DE}{9}$
   $\rightarrow DE = 24/5$ and $EF = 18/5$
   Thus, the required ratio $EF : DE = \underline{3 : 4}$.



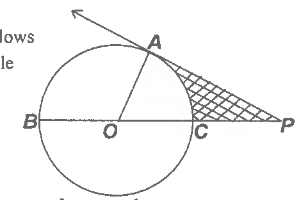
GBML '08

ROUND 3

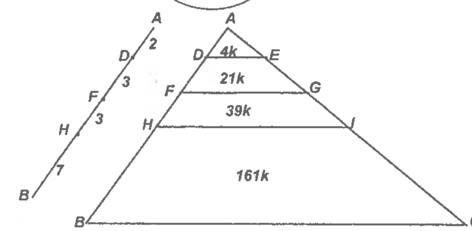
1. Let $OA = OB = r$. Since $\overline{AB}$ is a diameter, $m\widehat{AC} = 60°$ and $m\widehat{CB} = 120°$
   $\rightarrow \Delta BAC$ is a 30-60 right triangle and $BC = r\sqrt{3}$
   Since ΔBOD is an isosceles right triangle, $BD = r\sqrt{2}$.
   Thus, $\frac{BC}{BD} = \frac{r\sqrt{3}}{r\sqrt{2}} = \underline{\frac{\sqrt{6}}{2}}$

2. Area (shaded region) = Area(triangle) – Area(sector)
   Since $\angle OAP$ is a right angle, $OA = r$ and $OP = 2r$, it follows that $AP = r\sqrt{3}$ and ΔOAP is a 30 – 60 – 90 right triangle

$$\rightarrow \frac{\frac{1}{2} \cdot r\sqrt{3} \cdot r - \frac{1}{6}\pi r^2}{\frac{1}{6}\pi r^2} = \frac{\frac{\sqrt{3}}{2} - \frac{\pi}{6}}{\frac{\pi}{6}} = \underline{\frac{3\sqrt{3} - \pi}{\pi}}$$



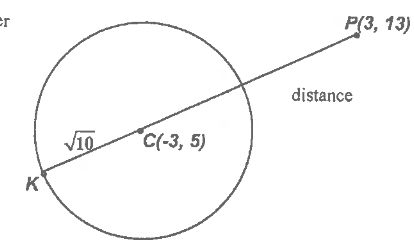
3. $AD : DF = 2 : 3$
   $AD : DH = 1 : 4 = 2 : 8 \rightarrow FH = 3$
   $AF = AB = 1 : 3 = 5 : 15 \rightarrow HB = 7$
   $DE : FG = 2 : 5$ and $\Delta ADE \sim \Delta AFG$
   → area(ΔADE) : area(ΔAFG) = 4 : 25
   If area(ΔADE) = $4k$, then
   Area($DEGF$) = $25k - 4k = 21k$
   Similarly, area($FGIH$) = $64k - 25k = 39k$
   and the area($HICB$) = $225k - 64k = 161k$
   Thus, the required ratio is **39 : 161**



GBML '09

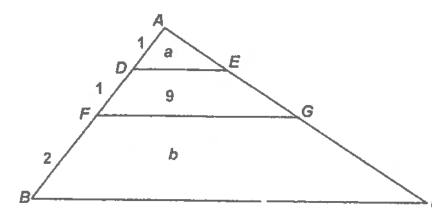
ROUND 3

1.
Completing the square, $(x^2+6x+9)+(y^2-10y+25) = -24+9+25 = 10$
The circle $(x+3)^2+(y-5)^2 = (\sqrt{10})^2$ has its center at (–3, 5) and a radius of $\sqrt{10}$.
$PC = \sqrt{(13-5)^2+(3-{}^-3)^2} = 10$ and the longest from P to a point on the circle is $\underline{10+\sqrt{10}}$, as indicated in the diagram.



2.
$\frac{1^2}{2^2} = \frac{a}{a+9} \rightarrow a + 9 = 4a \rightarrow a = 3$

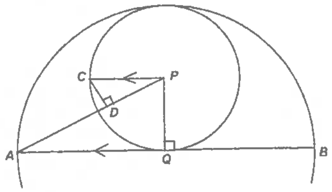
$\frac{1^2}{4^2} = \frac{a}{a+9+b} \rightarrow \frac{1}{16} = \frac{3}{12+b} \rightarrow b = 36$
Thus, the required ratio $a : b = 3 : 36 = \underline{\mathbf{1 : 12}}$



3.
$AB = 8 \rightarrow AQ = 4$, $PQ = 2$ and $AP = 2\sqrt{5}$

$\Delta CDP \sim \Delta PQA \rightarrow \dfrac{CD}{PQ} = \dfrac{CP}{PA} \rightarrow \dfrac{CD}{2} = \dfrac{2}{2\sqrt{5}}$

$\rightarrow CD = \underline{\dfrac{2\sqrt{5}}{5}}$



GBML '10

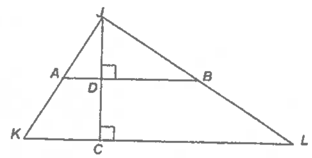
ROUND 3

1. $\Delta JAB \sim \Delta JKL \rightarrow \dfrac{JD}{AB} = \dfrac{JC}{KL} = \dfrac{8}{24} = \dfrac{1}{3}$

   Area(ΔJAB) $\sim \dfrac{1}{2} \cdot x \cdot 3x$ = area$(ABLK) = \dfrac{1}{2}(3x+24)(8-x)$

   $\rightarrow x^2 = (8+x)(8-x) \rightarrow x^2 = 32 \rightarrow x = 4\sqrt{2}$

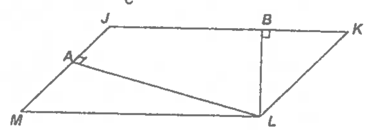
   $\rightarrow JD = \underline{4\sqrt{2}}$, $AB = \underline{12\sqrt{2}}$



2. Let $(JK, JM) = (x, y)$ Then: $\begin{cases} 2x+2y = 126 \\ 12x = 15y \end{cases}$

   $\rightarrow \begin{cases} y = 63 - x \\ 4x = 5y \end{cases} \rightarrow 4x = 5(63-x) \rightarrow 9x = 5(63)$

   $\rightarrow x = 35$ Therefore, Area = 12(35) = **420**

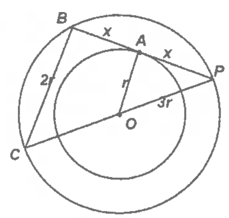


ROUND 3 – continued

3. Area$(\Delta PBC) = \dfrac{1}{2}(2r)(2x) = 2rx$

   In ΔPAO, $x^2 + r^2 = 9r^2 \rightarrow x = 2\sqrt{2}r$.

   Thus, Area = $2r \cdot 2\sqrt{2}r = \underline{4\sqrt{2}r^2}$.



GBML '11

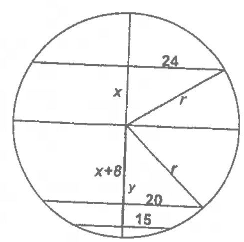
ROUND 3

1. $24^2 + x^2 = 20^2 + (x+8)^2 \rightarrow 576 = 400 + 16x + 64$

   $\rightarrow 16x = 112 \rightarrow x = 7 \rightarrow r = 25$

   $y^2 + 15^2 = 25^2 \rightarrow y^2 = 400 \rightarrow y = 20$

   $\rightarrow$ **13** cm further away



2. Let the bases $(AB, CD) = (2x, 5x)$ and the altitude from A to $\overline{DC}$ be denoted h. Let $EF = GH = a$.
   Since $\Delta DEF \sim \Delta DAB$ and $DE : DA = 1 : 2$, we have $a = \dfrac{1}{2}(2x) = x$.

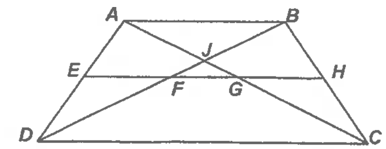
   As a median $EH = \dfrac{AB+CD}{2} \rightarrow 2a + 7.5 = \dfrac{7x}{2}$

   $\rightarrow 4a + 15 = 7x \rightarrow 3a = 15 \rightarrow a = x = 5$.

   Since $\Delta FGJ \sim \Delta BAJ$ and $AB : GF = 10 : 7.5 = 4 : 3$, the altitude from J in ΔFGJ is $\dfrac{3}{7} \cdot \dfrac{h}{2} = \dfrac{3h}{14}$ and in ΔBAJ it is $\dfrac{4}{7} \cdot \dfrac{h}{2} = \dfrac{2h}{7}$

   Now the required ratio may be written: $\dfrac{1}{2} \cdot 5x \cdot \dfrac{h}{2} \dfrac{9}{16} : \dfrac{1}{2} \cdot \dfrac{4}{7} \cdot \dfrac{h}{2} \cdot 2x : \dfrac{1}{2} \cdot 2x \cdot \dfrac{4}{7} \cdot \dfrac{h}{2}$

   $\rightarrow 5 : \dfrac{9}{16} \cdot \dfrac{4}{7} \cdot 2 : 2 \cdot \dfrac{4}{7} \rightarrow 5 : \dfrac{9}{14} : \dfrac{8}{7} \rightarrow$ **70 : 9 : 16**.



3.
Let $(AD, CD, AC) = (2a, 2b, 2c)$. Then:
The radii of c_1, c_2 and c_3 are a, b and c respectively.

The area of the shaded region is given by $\dfrac{\pi a^2}{2} + \dfrac{\pi b^2}{2} - \dfrac{\pi c^2}{2} + \dfrac{1}{2} \cdot 2a \cdot 2b$ or $\dfrac{\pi}{2}(a^2 + b^2 - c^2) + 2ab$

But, by the Pythagorean Theorem, $(2a)^2 + (2b)^2 = (2c)^2 \rightarrow a^2 + b^2 = c^2 \rightarrow a^2 + b^2 - c^2 = 0$

Therefore, the area of the shaded region is simply 2ab.

Since the area$(ABCD) = 4ab = 40$, we have $2ab = \underline{20}$

Note: This **avoids** the need to solve for a and b individually.

Area(ABCD) = $4ab \rightarrow ab = 10$ and Per$(ABCD) = 4(a+b) = 26$

$2\left(a + \dfrac{10}{a}\right) = 13 \rightarrow 2a^2 - 13a + 20 = (2a-3)(a-5) = 0 \rightarrow (a, b) = (5, 1.5)$ (or vice versa)

MML '04 JANUARY

MASSACHUSETTS MATHEMATICS LEAGUE
JANUARY 2004
ROUND 5: SIMILAR POLYGONS

ANSWERS

A) 62.1

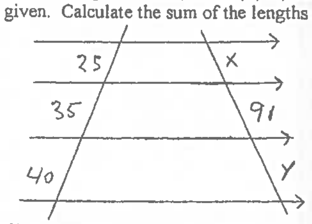
B) $x = 65,\ y = 104$

C) $(\sqrt{3}-1) : 1$

A) There are two solid cubes made of the same material where the edge of one cube is three times the edge of the other. If the smaller cube weighs 2.3 grams, calculate to the nearest tenth, the weight of the larger cube.

$\frac{w}{2.3} = \frac{27}{1}$ $\quad w = 2.3(27) = 62.1$

B) In the figure shown, lines k, l, m, and n are parallel, with transversal segment lengths given. Calculate the sum of the lengths of segments x and y.

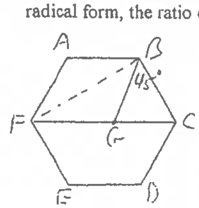


$\frac{91}{35} = \frac{13}{5}$

$x = \frac{25}{1} \cdot \frac{13}{5} = 65$

$y = \frac{40}{1} \cdot \frac{13}{5} = 104$

C) In regular hexagon ABCDEF, G is on $\overline{FC}$ so that $\angle CBG = 45°$. Calculate in simple radical form, the ratio of $\overline{GC}$ *to* $\overline{CB}$.



$\angle FBC = 90°$, so $\angle FBG = 45°$, and BG is an $\angle$ bisector. Call $BC = 1$, Then $BF = \sqrt{3}$, and $FC = 2$. $FG = GC\sqrt{3}$ so $GC\sqrt{3} + GC = 2$, so $GC = \frac{2}{\sqrt{3}+1} = \sqrt{3} - 1$

MML '04 FEB

MASSACHUSETTS MATHEMATICS LEAGUE
FEBRUARY 2004
ROUND 5: GEOMETRY CIRCLES
NON-CALCULATOR

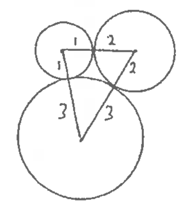
ANSWERS

A) 6

B) $\sqrt{6}/2$

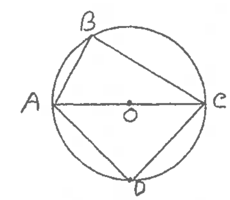
C) 85

A) Three circles of areas π, 4π, *and* 9π are drawn tangent to each other. Calculate the area of the triangle formed by connecting the centers of the three circles.



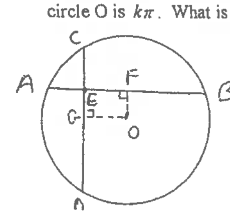
3, 4, 5 $\triangle$, area $= \frac{1}{2} \cdot 3 \cdot 4 = 6$

B) In the figure, $\overline{AC}$ is a diameter of circle O, $\widehat{AB} = \frac{1}{2}\widehat{BC}$, D is the midpoint of $\widehat{AC}$. Find the ratio of BC to AD in simplified radical form.



Let $OA = OC = 1$, Then $AB = 1$, $BC = \sqrt{3}$, $AD = \sqrt{2}$. $BC/AD = \sqrt{3}/\sqrt{2} = \sqrt{6}/2$

C) In circle O, $\overline{CD} \perp \overline{AB}$, CE = 5, CD = 14, and the ratio of AE to AB is 1 to 6. The area of circle O is $k\pi$. What is the value of k?



$GE = 2$, $AE = x$, $EB = 5x$, $5x^2 = 45$, $x = 3$, $EB = 15$, $EF = 6$, $FB = 9$, $OF = GE = 2$, $OB^2 = 2^2 + 9^2 = 85 = k$

MML '05 JAN

Round Five:

A. The larger triangle is 9-12-15 so its area is 54. The smaller triangle is scaled by 2/3 so it's area is 4/9 the larger triangle; thus the trapezoid is 5/9 of 54 or **30**

B. By AA, ΔADE ~ ΔACB. Note carefully the order of the vertices. Thus, 5/(12 + 6) = 12/(x + 5) → x = 38.2

C. If the smaller hexagon has area A the larger has area 16/9 A. so sum is 25/9A. In the smaller hexagon the altitude of one of the 6 equilateral triangles is $2\sqrt{3}$ so the triangle's area is $4\sqrt{3}$ and the hexagon's area = $24\sqrt{3}$ so sum is $\frac{200\sqrt{3}}{3}$

FEB.

..ound Five:

A. Power of pt M = 9(4)=36=MK(MN) so MK=MN=6. Power of pt L= LK(LN) = 25(13) = LP^2.

B. Equil. triangle of side 6 has area $9\sqrt{3}$ plus three 300° sectors = 3(5/6)9π

C. IB is alt to hyp of rt triangle EIF so IF is geom.. mean of FB & FE = $8\sqrt{6}$. HF:IF = DF:CF = 2:3 so HF = (2/3) $8\sqrt{6}$.

MML '06 JAN

Round Five:

A. The larger triangle is scaled by 10 so its area is scaled by 100. The smaller triangle has area 0.5(3)(4) = 6.

B. Δ#1 is 3-4-5 or 5-12-13. Max difference comes from 5-12-13 whose area and perimeter are both 30. Δ#2 is 5/20, 12/30, 13/30 and area is 1/30.

C. MN = 15 (midline) $\Delta MNX \sim \Delta EFX$ ratio 3:2 so MX is 3/5 of ME. AE twice altitude of equil Δ w/side 10 = $10\sqrt{3}$ and ME is hypotenuse of $\Delta AME = \sqrt{300+25} = 5\sqrt{13}$, so $MX = 3\sqrt{13}$

FEB.

Round Five:

A. Rt. triangle with radius as hypotenuse has legs of 20 and ½ (96), so hypotenuse is 52 [4x(5 - 12 - 13) triangle]. 20 + 52 = 72

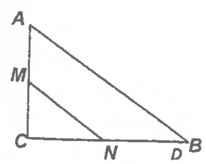
B. $(AE)(BE)=(DE)(CE)$, so $(5x-3)(x+1) = (3x-1)(2x)$. Solve quadratic to get x = 1 or 3. Both give all positive lengths so AE = 2 or 12.

C. Rt ΔPOA gives OA = 7. If $DE = x$, $x(2x) = (7\sqrt{6})^2 = 294$, so $x = DE = 7\sqrt{3}$
Thus, m∠DOE = 120° and sector is 1/3 of the circle.

MML '07 JAN

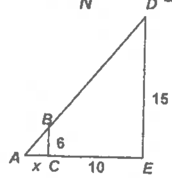
Round 5

A) The area of the 3-4-5 triangle is 6 units². The line connecting the midpoints of the legs is parallel to the hypotenuse and cuts off a triangle similar to the original 3-4-5. Since the ratio of their corresponding sides is 1 : 2, their areas are in a ratio of 1 : 4. Since the area of ΔMNC is ¼ the area of ΔABC, the area of the trapezoid is ¾ the area of ΔABC = **4.5**.



B) Since $\Delta ABC \sim \Delta ADE$, $\frac{x}{x+10} = \frac{6}{15} = \frac{2}{5} \rightarrow 5x = 2x + 20$

$\rightarrow 3x = 20 \rightarrow x = \boxed{6\frac{2}{3}}$



C) $\Delta ABC \sim \Delta CAD \rightarrow \frac{AB}{CA} = \frac{AC}{CD} \rightarrow AC^2 = AB(CD) = 12(27) = 18^2 \rightarrow AC = 18$.

Since the ratio of the radii of the inscribed circles is the same as the ratio of the corresponding sides, $\frac{r_1}{r_2} = \frac{AB}{AC} = \frac{12}{18} = \frac{2}{3} \rightarrow \frac{A_1}{A_2} = \boxed{\frac{4}{9}}$

MML '07 FEB.

Round 5

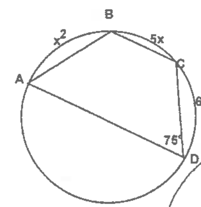
A) Appealing to the diagram, as inscribed angles,

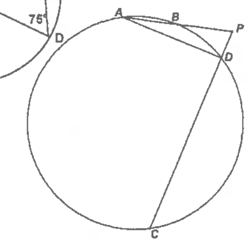
m∠$D = \frac{1}{2}(x^2 + 5x) = 75$

$\rightarrow x^2 + 5x - 150 = (x + 15)(x - 10) = 0$

$\rightarrow x = 10$

$\rightarrow$ m∠$A = \frac{1}{2}(5x + 6x) = \frac{1}{2}(110) = \underline{55°}$

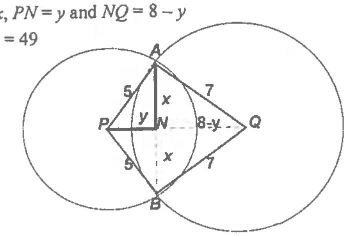




B) Let $PD = x$. Then $PB(PA) = PD(PC) \rightarrow 4(7) = x(x + 12)$
$\rightarrow x^2 + 12x - 28 = (x + 14)(x - 2) = 0 \rightarrow x = 2$ Applying the Pythagorean theorem, $DA^2 = 49 - 4 = 45 \rightarrow DA = 3\sqrt{5}$

C) $APBQ$ is a kite and $\overline{PQ}$ perpendicularly bisects $\overline{AB}$.
Let N be the point of intersection of $\overline{PQ}$ and $\overline{AB}$. Let $AN = x$, $PN = y$ and $NQ = 8 - y$
Then: $x^2 + y^2 = 25$ and $(8 - y)^2 + x^2 = 49 \leftrightarrow x^2 + y^2 - 16y + 64 = 49$
Substituting, $25 - 16y + 64 = 49 \rightarrow y = \frac{5}{2} \rightarrow x^2 = \frac{75}{4}$

$\rightarrow x = \frac{5\sqrt{3}}{2} \rightarrow AB = \boxed{5\sqrt{3}}$

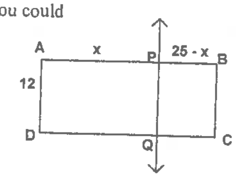


MML '08 JAN

Round 5

A) $AC = 5 \rightarrow$ the scale factor is 6/5, the legs of ΔDEF are slightly longer than the legs in ΔABC.
Specifically, $\frac{6}{5}(3,4) = \left(\frac{18}{5}, \frac{24}{5}\right)$

B) If you don't want to experiment with various subdivisions of 25, you could approach the problem algebraically. Suppose the side of length 25 is divided into lengths of x and $(25 - x)$.
Then the ratio of corresponding sides (short to long) is:
$\frac{12}{x} = \frac{25-x}{12} \rightarrow x^2 - 25x + 144 = (x - 9)(x - 16) = 0$
$\rightarrow x$ = 9 or 16 (Since x must be greater than 12, 9 is rejected.)
x = 16 → area = 16(12) = **192**.



C) QRC is a 3-4-5 right triangle.

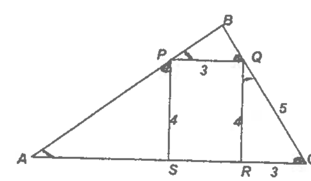
$\Delta PBQ \sim \Delta QRC$ and the scale factor is $\frac{3}{5}$

$\rightarrow BQ = \frac{3}{5}(3) = \frac{9}{5}$ and $BP = \frac{3}{5}(4) = \frac{12}{5}$

$\Delta ASP \sim \Delta QRC$ and the scale factor is $\frac{4}{3}$

$\rightarrow AS = \frac{4}{3}(4) = \frac{16}{5}$ and $AP = \frac{4}{3}(5) = \frac{20}{3}$

Thus, the perimeter of ΔABC is $8 + \frac{21}{5} + \frac{36}{3} + 3 = 23 + 4.2 = \underline{27.2}$



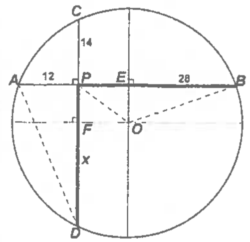
MML '08 FEB

Round 5

A) A point on the circumference moves 24π inches per second.

Converting, $24\pi \frac{\cancel{\text{in}}}{\cancel{\text{sec}}} \cdot \frac{1\text{ ft}}{12\ \cancel{\text{in}}} \cdot \frac{60\ \cancel{\text{sec}}}{1\text{ min}} = \underline{\mathbf{120\pi}}$ ft/min

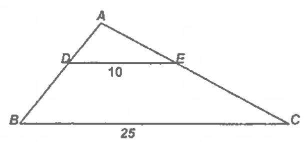
B) $(5x-3)^2 = (3x-1)(10x-12) \rightarrow 25x^2 - 30x + 9 = 30x^2 - 46x + 12$
$\rightarrow 5x^2 - 16x + 3 = (5x-1)(x-3) = 0 \rightarrow x = 3$ [1/5 is extraneous]
$PA = 12$, $PB = 8$, $BC = 10$, $AC = 9$
Thus, the perimeter of $\Delta APC = (12 + 18 + 9) = \underline{\mathbf{39}}$.



C) Let $x = PD$. Applying the product-chord theorem, $14x = 12(28)$
$\rightarrow x = 24$. Since E and F are midpoints, $AE = 20 \rightarrow PE = 8$ and
$CF = 19 \rightarrow PF = OE = 5$.
Thus, in right ΔPEO, $PO^2 = 8^2 + 5^2 \rightarrow PO = \sqrt{89}$ and
in right ΔAPD, $AD^2 = 12^2 + 24^2 \rightarrow AD = 12\sqrt{5}$
$\rightarrow AD - PO = \underline{\mathbf{12\sqrt{5} - \sqrt{89}}}$

MML '11 JAN

Round 5



A) $\Delta ADE \sim \Delta ABC$ and the ratio of corresponding sides is 2 : 5.
Thus the ratio of the areas is 4 : 25.
Therefore, the ratio of the required areas is 4 : (25 – 4) = $\underline{\mathbf{4 : 21}}$.

B) Let s denote the length of the side of equilateral triangle ABC

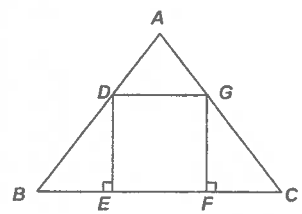
$|ABC| = \frac{s^2\sqrt{3}}{4}$, where $|ABC|$ denotes the area of ΔABC

Area(Trap $BDGC$) = 15/16 area (ΔABC) $\rightarrow AD : AB = 1 : 4$

$\rightarrow BD = \frac{3}{4}s$, $BE = \frac{3}{8}s$ and $DE = \frac{3}{8}s\sqrt{3}$

Thus, $a = |BDE| = \frac{1}{2}\left(\frac{3}{8}s\right)\left(\frac{3}{8}s\sqrt{3}\right) = \frac{9}{128}\sqrt{3}s^2$

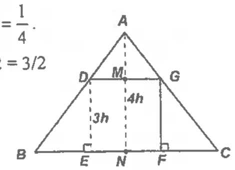
Taking the required ratio, we have $\frac{1/4}{9/128} = \underline{\mathbf{32 : 9}}$.



B) Alternate solution #1

Drop a perpendicular from A to $\overline{BC}$, intersecting $\overline{DG}$ and $\overline{BC}$ at M and N respectively.
$\frac{\text{area}(BDGC)}{\text{area}(ABCD)} = \frac{15}{16} \rightarrow \frac{area(\Delta ADG)}{area(\Delta ABC)} = \frac{1}{16}$. $\Delta ADG \sim \Delta ABC \rightarrow \frac{AD}{AB} = \frac{AM}{AN} = \frac{1}{4}$.
Let $AD = 1$ and $AM = h$. Then: $DG = EF = 1$, $AB = BC = 4$, $BE = (4-1)/2 = 3/2$
and $DE = MN = 3h$.

$$\frac{area(\Delta ABC)}{area(\Delta BED)} = \frac{\frac{1}{2}BC \cdot AN}{\frac{1}{2}BE \cdot DE} = \frac{4 \cdot 4h}{\frac{3}{2} \cdot 3h} = \frac{16}{9/2} = \underline{\mathbf{32 : 9}}.$$



Alternate solution #2 (Norm Swanson): Let $BE = FC = 6$ and $EF = 4$. $\rightarrow \frac{\frac{1}{2}4h \cdot 16}{\frac{1}{2}3h \cdot 6} = \underline{\frac{32}{9}}$

In a regular hexagon (with side s), the lengths of the diagonals are either $2s$ or $\sqrt{3}s$.

Area(A) = $6\left(\frac{(s_A)^2\sqrt{3}}{4}\right) = \frac{9\sqrt{3}}{4} \rightarrow s_A = \frac{\sqrt{6}}{2}$; long diag$_B = 4\sqrt{6} \rightarrow s_B = 2\sqrt{6}$

Thus, $s_C = \frac{\sqrt{6}}{2} \cdot \sqrt{3} = \frac{3}{2}\sqrt{2}$ and $s_D = 2\sqrt{6} \cdot \sqrt{3} = 6\sqrt{2}$

The sum of the areas is $\frac{3}{2}\sqrt{3}\left(\frac{9}{4} \cdot 2 + 36 \cdot 2\right) = \frac{3}{2}\sqrt{3}\left(\frac{153}{2}\right) = \underline{\mathbf{\frac{459}{4}\sqrt{3}}}$.