Round 4 Algebra 2/Precalculus

Logs, Exponents, Radicals and equations involving them

GREATER BOSTON MATHEMATICS LEAGUE

MEET 3 – DECEMBER 1998

ROUND 4 – Algebra 2– Logs, Exponents, Radicals and equations involving them

1. ____________________

2. ____________________

3. ____________________

CALCULATORS ARE NOT ALLOWED ON THIS ROUND

1. Solve the following equation for x: $\log_6(2x-1)+\log_6(3x-11)=2$

2. If $\log_b 24 = x$ and $\log_b \sqrt[3]{\frac{2}{9}} = y$, find $\log_b 2$ in terms of x and y.

3. Any solution to the equation, $\dfrac{\log_3 x}{\log_x 3} = \log_9 \sqrt[3]{x^4} + \log_8 16^2$, can be put in the form 3^a. Find all possible values for a.

GREATER BOSTON MATHEMATICS LEAGUE

MEET 3 – DECEMBER 1999

ROUND 4 – Algebra 2– Logs, Exponents, Radicals and equations involving them

1. ____________________

2. ____________________

3. ____________________

CALCULATORS ARE NOT ALLOWED ON THIS ROUND

1. If $\left(3+\sqrt{2}\right)^2-\sqrt[4]{4}+\dfrac{2}{\sqrt{2}-1}=x+y\sqrt{2}$, where x and y are rational, find the sum, $x+y$.

2. Solve for x: $\log_7\left(x^3-27\right)=-\log_7\left(x-3\right)=2$

3. If $\log_{a^2} b+\log_{a^3} 2b=\log_a \sqrt[6]{x}$, solve for x in terms of b.

GREATER BOSTON MATHEMATICS LEAGUE

MEET 3 – DECEMBER 2000

ROUND 4 – Algebra 2– Logs, Exponents, Radicals and equations involving them

1. ____________________

2. ____________________

3. ____________________

CALCULATORS ARE NOT ALLOWED ON THIS ROUND

1. Find all positive values for x satisfying the following equation: $\left(\sqrt[10]{x}\right)\left(\sqrt[5]{x}\right)=\left(\sqrt{7}\right)\left(\sqrt[20]{x}\right)$

2. Find all values for x satisfying the following equation: $\log_8 \sqrt{2} = \log_x \sqrt{3} - \log_x \sqrt[3]{9}$

3. Find all values for x satisfying the following equation: $\log_9 x = \log_{25} 125 - \log_x 3$

GREATER BOSTON MATHEMATICS LEAGUE

MEET 3 – DECEMBER 2001

ROUND 4 – Algebra 2– Logs, Exponents, Radicals and equations involving them

1. ____________________

2. ____________________

3. ____________________

CALCULATORS ARE NOT ALLOWED ON THIS ROUND

1. Solve for x if $\dfrac{x}{\sqrt{x}\cdot\sqrt[6]{x}}=\sqrt[4]{5x}$.

2. Solve the following equation for x: $\log_6 x+\log_6(x-1)=\log_6(x-2)+1$.

3. Given $\dfrac{\log_3 4-\log_{27}4}{\log_{\sqrt[3]{5}}2+\log_{125}\sqrt{2}-\log_{25}32}=\log_b a$ where a and b are integers, find the least possible value for $a+b$.

GREATER BOSTON MATHEMATICS LEAGUE
MEET 3 – DECEMBER 2005

ROUND 4 – Algebra 2: Logs, Exponents, Radicals and Equations involving them

1. ______________________

2. ______________________

3. ______________________

CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

1. $\dfrac{\sqrt[6]{6} \cdot \sqrt[4]{3}}{\sqrt{2}}$ may be expressed as the simplified radical expression $\dfrac{1}{2}\sqrt[N]{3^x \cdot 2^y}$. What is the numerical value of the product $x \cdot y \cdot N$?

2. Find all real values of x which satisfy the statement:

$$\log_4(x+1) - \log_8 16 + \log_4(x-1) = \log_{27} 9$$

3. Given: $\log_4 9 = a$ and $\log_4 36 = b$

Determine, in simplest form, $\log_8 \dfrac{216}{81}$ strictly in terms of a or strictly in terms of b.

GREATER BOSTON MATHEMATICS LEAGUE

MEET 3 – DECEMBER 2006

ROUND 4 – Algebra 2: Logs, Exponents Radicals and equations involving them

1. ______________________

2. ______________________

3. ______________________

CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

1. Solve for x: $\log_{25}\sqrt{5}+\log_{36}6=\log_x 27$

2. Find all x, x ε R, that satisfy the following statement:

$$(3x^{2/3})^2+(1)^{-2/3}=10\sqrt[3]{x^2}$$

3. Find $\log_A B$ if $\dfrac{\left(9^{x+2}\right)\left(125^{x-1}\right)\left(\dfrac{1}{5}\right)^{-x}}{\left(25^{2x-3}\right)\left(27^{x-1}\right)\left(\dfrac{1}{3}\right)^{x+2}}=3^A\cdot 5^B$

GREATER BOSTON MATHEMATICS LEAGUE

MEET 3 – DECEMBER 2007

ROUND 4 – Algebra 2: Logs, Exponents Radicals and equations involving them

1. ______________________________

2. ______________________________

3. ______________________________

CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

1. If x and y are positive integers and $\dfrac{\sqrt{x}}{\sqrt[3]{y}} = \dfrac{4}{5}$, determine the smallest possible 4-digit value of $x + y$.

2. Determine all possible values of x (over the reals) for which

$$\log_2 x^3 + (\log_2 x^2)^2 = \log_x x, \text{ where } x > 0$$

3. Find all values of x, over the reals, which make the following statement true

$$\log_8\left(\frac{x+3}{x-1}\right)^3 + \log_4\left(\frac{x^2-1}{x}\right)^2 = 3$$

GREATER BOSTON MATHEMATICS LEAGUE

MEET 3 – DECEMBER 2008

ROUND 4 – Algebra 2: Logs, Exponents Radicals and equations involving them

1. ______________________

2. A = ______________________

3. ______________________

CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

1. Find x such that $\sqrt{\left(\left(\frac{16}{81}\right)^{-\frac{1}{3}}\right)^{\frac{9}{2}}} = \sqrt[4]{x^3}$.

2. If $\log_8 18 = A$ and $\log_2 9 = B$, find A in terms of B.

3. Given: $\log_6 \frac{A}{72} + \frac{\log 2}{\log 6} + \left(\log_{36} A^2\right)^2 = 0$
   Find all possible real values of A.

GREATER BOSTON MATHEMATICS LEAGUE

MEET 3 – DECEMBER 2009

ROUND 4 – Algebra 2: Logs, Exponents Radicals and equations involving them

1. ______________________________

2. ______________________________

3. ______________________________

1. For what real values of x is the following statement true:

$$\log_9 x^2 + \left(\log_9 x\right)^{-1} = 3$$

2. Compute all real values of x for which

$$\sqrt{x-4} - 2^{-3\log_8 2} - \log_2 \frac{1}{32} = \sqrt{x+16} + \log_4 32 \,.$$

3. Compute all real values of x for which

$$\sqrt{\frac{9}{4}+4} + \left(\frac{25}{16}\right)^{\frac{3}{2}}\left(\frac{1}{8}\right)^{-\frac{4}{3}} = \sqrt{4x^4+4x^4+x^4}$$

GREATER BOSTON MATHEMATICS LEAGUE

MEET 3 – DECEMBER 2010

ROUND 4 – Algebra 2: Logs, Exponents Radicals and equations involving them

1. ______________________

2. ______________________

3. ______________________

CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

1. Given: $\log_2\left(\log_3\left(\log_4 x^{\frac{3}{2}}\right)\right)=1$

   If $x = a^b$,where a and b are positive integers and a has the smallest possible value, compute the sum $a + b$.

2. Solve for x over the reals.

   $$\log_4\sqrt{2}+\log_9\sqrt[3]{3}=\log_{64}\sqrt{2}+\log_x 4$$

3. Find <u>all</u> possible values of x for which $\log_3 x+\log_9 18+\log_x 9=\log_{\frac{1}{2}}\frac{1}{16}+\log_9 2$.

MASSACHUSETTS MATHEMATICS LEAGUE
DECEMBER 2003
ROUND 4: LOGS & EXPONENTIALS
NON-CALCULATOR

ANSWERS

A)_______________

B)_______________

C)_______________

A) Solve for x: $\log_9 x = \log_{16} 320 - \log_{16} 5$

B) Solve for x: $\left(\frac{1}{4}\right)^{x-x^2} = 8^{2-x}$

C) Solve for x: $\log_2(-2x-1) - \log_{\sqrt{2}} 2 + \log_2(-x+3) = 0$

MASSACHUSETTS MATHEMATICS LEAGUE
DECEMBER 2004
ROUND 4 ALG 2: LOGS & EXPONENTIAL FUNCTIONS

******* NO CALCULATORS ON THIS ROUND ******

ANSWERS

A)______________

B)______________

C)______________

A) Find all real solutions for: $\log_2 x + \log_2(x-6) = 4$

B) Solve for x: $8^{\frac{2\log_4 3}{3}} - e^{\ln 5} = x^2 - 7^{(\log_7 3 + \log_7 x)}$

C) The graph of the exponential function $f(x) = a\,b^x$ passes through the points $(1, \frac{1}{2})$ and $(3, \frac{3}{8})$. Find the exact value of a+b

MASSACHUSETTS MATHEMATICS LEAGUE
DECEMBER 2005
ROUND 4 ALG 2: LOGS & EXPONENTIAL FUNCTIONS

******* NO CALCULATORS ON THIS ROUND ******

ANSWERS

A)______________

B)______________

C) ______________

A) Evaluate: $\log_5 625 + \log 0.001 - \log_9 3 + \ln(e)$

B) Solve for x: $\log_5 x + \log_5(x - 20) = 3$

C) Solve for x given x>1 and $x^{\log_2(1/x)} = \dfrac{4}{x^3}$

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 3 - DECEMBER 2006
ROUND 4 ALG 2: LOG & EXPONENTIAL FUNCTIONS

ANSWERS

A) ______________________________

B) ______________________________

C) ______________________________

***** NO CALCULATORS ON THIS ROUND ****

A) If A and B are the roots of $3x^2 - 22x + 27 = 0$, then what is the exact value of $\log_3 A + \log_3 B$?

B) If $a = \log_8 45$ and $b = \log_2 7.5$ and $\log_3 2 = \dfrac{1}{c}$, then find a simplified expression for b in terms of a and c.

C) Determine the domain of the real-valued function f: $f(x) = \log_3\left(\dfrac{(x^2 + 3x - 4)}{(2x - 3)^2}\right)$.

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 3 - DECEMBER 2007
ROUND 4 ALG 2: LOG & EXPONENTIAL FUNCTIONS

ANSWERS

A) ______________________________

B) $A =$ __________ $B =$ ____________

C) ______________________________

***** NO CALCULATORS ON THIS ROUND *****

A) Determine all possible values of x for which $(\log_3 x)^2 - \frac{7}{3}\log_5 125 - 6\log_3 x = 0$

B) Given: $a = \log_{36}(8)$
In terms of a, find a simplified expression for $\log_{216}(48)$.

C) Compute all possible values of x for which $3^{3\log_3 x + 1} - 2^{2\log_2 x} = 2x^4$

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 3 - DECEMBER 2009
ROUND 4 ALG 2: LOG & EXPONENTIAL FUNCTIONS

******* NO CALCULATORS IN THIS ROUND *******

ANSWERS

A) ______________________

B) ______________________

C) $x =$ ______________________

A) Compute $\log_9\left(1+\frac{\sqrt{6}}{3}\right)+\log_9\left(1-\frac{\sqrt{6}}{3}\right)$

B) Compute all real values of x for which $\log_4 x^3 - 2\log_{16} x + \log_{64} x^6 = 3$
Any radicals in your answer must have the smallest possible index.

C) Solve for x over the reals. $\dfrac{2^x - 2^{-x}}{2} = -1.875$

GBML '98

ROUND 4

1. $\log_6(2x-1)+\log_6(3x-11)=2 \Rightarrow (2x-1)(3x-11)=36 \Rightarrow$
$6x^2-25x-25=0 \Rightarrow (6x+5)(x-5)=0 \Rightarrow x=5$ only since $x=-\frac{5}{6}$ does not check $\Rightarrow$
Solution = **5**

2. $\log_b 24 = x$ and $\log_b \sqrt[3]{\dfrac{2}{9}} = y \Rightarrow 3\log_b 2+\log_b 3 = x$ and $\log_b 2 - 2\log_b 3 = 3y \Rightarrow$
$6\log_b 2 + 2\log_b 3 = 2x$ and $\log_b 2 - 2\log_b 3 = 3y \;\Rightarrow\; 7\log_b 2 = 2x+3y \Rightarrow \log_b 2 = \mathbf{\dfrac{2x+3y}{7}}$

3. $\dfrac{\log_3 x}{\log_x 3} = \log_9 \sqrt[3]{x^4} + \log_8 16^2 \Rightarrow \dfrac{\dfrac{\log x}{\log 3}}{\dfrac{\log 3}{\log x}} = \dfrac{\frac{4}{3}\log x}{2\log 3} + \dfrac{\log\left(2^4\right)^2}{3\log 2} \Rightarrow \left(\log_3 x\right)^2 - \frac{2}{3}\log_3 x - \frac{8}{3} = 0$
Let $a=\log_3 x \Rightarrow 3a^2-2a-8=0 \Rightarrow (3a+4)(a-2)=0 \Rightarrow a=-\frac{4}{3}$ **or 2**

GBML '99

ROUND 4

1. $\left(3+\sqrt{2}\right)^2-\sqrt[4]{4}+\dfrac{2}{\sqrt{2}-1}=11+6\sqrt{2}-\sqrt{2}+2\left(\sqrt{2}+1\right)=11+6\sqrt{2}-\sqrt{2}+2\sqrt{2}+2=$
$13+7\sqrt{2} \Rightarrow x+y=20$

2. $\log_7\left(x^3-27\right)-\log_7(x-3)=2 \Rightarrow \log_7\left(\dfrac{x^3-27}{x-3}\right)=2 \Rightarrow x^2+3x+9=49 \Rightarrow$
$x^2+3x-40=0 \Rightarrow (x+8)(x-5)=0 \Rightarrow x=5$ Note x = -8 is extraneous to the original equation.

3. $\log_{a^2} b+\log_{a^3} 2b=\log_a \sqrt[6]{x} \Rightarrow \dfrac{\log b}{\log a^2}+\dfrac{\log 2b}{\log a^3}=\dfrac{\frac{1}{6}\log x}{\log a} \Rightarrow \dfrac{\log b}{2\log a}+\dfrac{\log 2b}{3\log a}=\dfrac{\frac{1}{6}\log x}{\log a} \Rightarrow$
$3\log b+2\log 2b=\log x \Rightarrow \log\left(b^3(2b)^2\right)=\log x \Rightarrow x=4b^5$

GBML '00

ROUND 4

1. $\left(\sqrt[10]{x}\right)\left(\sqrt[5]{x}\right)=\left(\sqrt{7}\right)\left(\sqrt[20]{x}\right) \rightarrow \dfrac{x^{\frac{1}{5}}x^{\frac{1}{10}}}{x^{\frac{1}{20}}}=7^{\frac{1}{2}} \rightarrow x^{\frac{1}{4}}=7^{\frac{1}{2}} \rightarrow x=7^2 \rightarrow x=49$

2. $\log_8\sqrt{2}=\log_x\sqrt{3}-\log_x\sqrt[3]{9} \rightarrow \dfrac{\log\sqrt{2}}{\log 8}=\dfrac{1}{2}\log_x 3-\dfrac{2}{3}\log_x 3 \rightarrow$
$\dfrac{\frac{1}{2}\log 2}{3\log 2}=-\dfrac{1}{6}\log_x 3 \rightarrow \dfrac{1}{6}=-\dfrac{1}{6}\log_x 3 \rightarrow \log_x 3=-1 \rightarrow x=\dfrac{1}{3}$

3. $\log_9 x=\log_{25}125-\log_x 3 \rightarrow \dfrac{1}{2}\log_3 x=\dfrac{3}{2}-\dfrac{1}{\log_3 x}$;
Let $y=\log_3 x \rightarrow \dfrac{1}{2}y=\dfrac{3}{2}-\dfrac{1}{y} \rightarrow y^2=3y-2 \rightarrow y^2-3y+2=0 \rightarrow$
$(y-1)(y-2)=0 \rightarrow \log_3 x=1,2 \rightarrow x=3,9$

GBML '01

ROUND 4 – Algebra 2– Logs, Exponents, Radicals and equations involving them

1. $\dfrac{x}{\sqrt{x}\cdot\sqrt[6]{x}}=\sqrt[4]{5x} \Rightarrow \dfrac{x}{x^{1/2}x^{1/6}}=5^{1/4}x^{1/4} \Rightarrow x^{1-1/2-1/6-1/4}=5^{1/4} \Rightarrow x^{1/12}=5^{1/4} \Rightarrow x=\left(5^{1/4}\right)^{12}=5^3=125$.

2. $\log_6 x+\log_6(x-1)=\log_6(x-2)+1 \Rightarrow \log_6 x+\log_6(x-1)=\log_6(x-2)+\log_6 6 \Rightarrow$
$\log_6 x(x-1)=\log_6 6(x-2) \Rightarrow x^2-x=6x-12 \Rightarrow x^2-7x+12=0 \Rightarrow$
$(x-3)(x-4)=0 \Rightarrow x=3,4$. (Both solutions satisfy the original equation.)

3. $\dfrac{\log_3 4-\log_{27}4}{\log_{\sqrt[3]{5}}2+\log_{125}\sqrt{2}-\log_{25}32}=\dfrac{\dfrac{2\log 2}{\log 3}-\dfrac{2\log 2}{3\log 3}}{\dfrac{\log 2}{\frac{1}{3}\log 5}+\dfrac{\frac{1}{2}\log 2}{3\log 5}-\dfrac{5\log 2}{2\log 5}}=\dfrac{\left(2-\dfrac{2}{3}\right)\dfrac{\log 2}{\log 3}}{\dfrac{3\log 2}{\log 5}+\dfrac{\log 2}{6\log 5}-\dfrac{5\log 2}{2\log 5}}=$
$\dfrac{\dfrac{4\log 2}{3\log 3}}{\left(3+\dfrac{1}{6}-\dfrac{5}{2}\right)\dfrac{\log 2}{\log 5}}=\dfrac{\dfrac{4\log 2}{3\log 3}}{\dfrac{2\log 2}{3\log 5}}=\dfrac{2\log 5}{\log 3}=\dfrac{\log 25}{\log 3}=\log_3 25 \Rightarrow a+b=28$.

GBML '05

ROUND 4 Algebra 2 – Logs, Exponents, Radicals and Equations involving them

1. $\dfrac{\sqrt[6]{6}\cdot\sqrt[4]{3}}{\sqrt{2}}=\dfrac{6^{\frac{1}{6}}\cdot 3^{\frac{1}{4}}}{2^{\frac{1}{2}}}=\dfrac{6^{\frac{2}{12}}\cdot 3^{\frac{3}{12}}}{2^{\frac{6}{12}}}=\left(\dfrac{6^2\cdot 3^3}{2^6}\right)^{\frac{1}{12}}=\left(\dfrac{2^2\cdot 3^5}{2^6}\right)^{\frac{1}{12}}=\sqrt[12]{\dfrac{3^5}{2^4}}$
$=\sqrt[12]{\dfrac{3^5}{2^4}\cdot\dfrac{2^8}{2^8}}=\dfrac{1}{2}\sqrt[12]{2^8\cdot 3^5} \rightarrow x\cdot y\cdot N=5(8)(12)=\underline{\mathbf{480}}$

2. $\log_4(x+1)-\log_8 16+\log_4(x-1)=\log_{27}9$
$\Leftrightarrow \log_4(x^2-1)-\dfrac{4}{3}=\dfrac{2}{3} \Leftrightarrow x^2-1=4^2=16 \rightarrow x^2=17 \rightarrow x=\pm\sqrt{17}$
BUT the negative root is extraneous since substitution in the original equation produces the log of a negative which is not allowed.

Thus, $x=\underline{+\sqrt{17}}$ <u>only</u>

Remember: OUTPUT from the log function can be negative, but INPUT to the log function can never be negative.

3. $\log_4 9 = a \to \log_2 3 = a \to$
$\log_4 36 = b \to \log_2 6 = b \to 1 + \log_2 3 = b$ or $1 + a = b$

$\log_8 \frac{216}{81} = 3\log_8 6 - 4\log_8 3 = 3\log_8 2 - \log_8 3 = 3 \cdot \frac{1}{3}\log_2 2 - \frac{1}{3}\log_2 3 = 1 - \frac{a}{3} = \underline{\frac{3-a}{3}}$

Substituting b-1 for a, clearly $\underline{\frac{4-b}{3}}$ is equivalent.

GBML '06

ROUND 4

1. $\log_{25}\sqrt{5} + \log_{36} 6 = \log_x 27 \to ¼ + ½ = ¾ = \log_x 27 \to x^{3/4} = 27 \to x = 27^{4/3} \to x = 3^4 = \underline{\mathbf{81}}$

2. Let $Y = x^{2/3}$ Then $(3x^{2/3})^2 + (1)^{-2/3} = 10\sqrt[3]{x^2} \to 9Y^2 - 10Y + 1 = (Y-1)(9Y-1) = 0$
$\to Y = 1$ or $1/9 \to x^{2/3} = 1$ or $3^{-2} \to x = (1)^{3/2}$ or $(3^{-2})^{3/2} \to x = \underline{\mathbf{1, 1/27}}$

3. $\dfrac{(9^{x+2})(125^{x-1})\left(\frac{1}{5}\right)^{-x}}{(25^{2x-3})(27^{x-1})\left(\frac{1}{3}\right)^{x+2}} = \dfrac{3^{2x+4}5^{3x-3}5^x}{5^{4x-6}3^{3x-3}3^{-x-2}} = \dfrac{3^{2x+4}5^{4x-3}}{5^{4x-6}3^{2x-5}} = 3^9 5^3 \to A = 9, B = 3 \to \log_9(3) = \underline{\mathbf{1/2}}$

GBML '07

ROUND 4

1. Let $x = 16n^2$ and $y = 125n^3$. Then:
$x = 1 \to x + y = 16 + 125 = 141$
$x = 2 \to x + y = 64 + 1000 = \underline{\mathbf{1064}}$ (the smallest 4-digit sum)

2. Let $N = \log_2 x$. Then $3N + 4N^2 = 1 \to 4N^2 + 3N - 1 = (4N-1)(N+1) = 0 \to N = \frac{1}{4}, -1$
$\to x = \underline{\frac{1}{2} \text{ or } \sqrt[4]{2}}$

3. $\log_2\left(\frac{x+3}{x-1}\right) + \log_2\left(\frac{x^2-1}{x}\right) = 3 \to \log_2\left(\frac{x+3}{x-1}\right)\left(\frac{x^2-1}{x}\right) = 3 \to \left(\frac{x+3}{x-1}\right)\left(\frac{x^2-1}{x}\right) = 8$
Provided $x \neq 1$ and $x \neq 0$, this equation simplifies to $(x+3)(x+1) = 8x \to x^2 + 4x + 3 = 8x$
$\to x^2 - 4x + 3 = (x-3)(x-1) = 0 \to x = \underline{3}$ only

GBML '08

ROUND 4

1. $\sqrt{\left(\left(\frac{16}{81}\right)^{-\frac{1}{3}}\right)^{\frac{9}{2}}} = \sqrt[4]{x^3} \to \left(\left(\left(\frac{16}{81}\right)^{-\frac{1}{3}}\right)^{\frac{9}{2}}\right)^{\frac{1}{2}} = x^{\frac{3}{4}} \to \left(\frac{16}{81}\right)^{-\frac{3}{4}} = x^{\frac{3}{4}}$

Raising both sides to the $\frac{4}{3}$ power, we have $x = \left(\frac{16}{81}\right)^{-1} = \underline{\mathbf{\frac{81}{16}}}$

2. $\log_8 18 = A \to 8^A = 18 \to 2^{3A} = 9 \cdot 2$ and $\log_2 9 = B \to 2^B = 9$
Substituting, $2^{3A} = 2^B \cdot 2 = 2^{B+1}$. Thus, $3A = B + 1$ and $A = \underline{\mathbf{\frac{B+1}{3}}}$

3. $\log_6 \frac{A}{72} + \frac{\log 2}{\log 6} + (\log_{36} A^2)^2 = 0 \to \log_6 \frac{A}{2 \cdot 6^2} + \log_6 2 + (\log_6 A)^2 = 0$
$\to (\log_6 A - \log_6 2 - 2) + \log_6 2 + (\log_6 A)^2 = 0$
Let $N = \log_6 A$. Then: $N^2 + N - 2 = (N-1)(N+2) = 0 \to N = \log_6 A = 1, -2$
$\to A = 6^1, 6^{-2} = \underline{\mathbf{6, \frac{1}{36}}}$

GBML '09

ROUND 4

1. $\log_9 x^2 + (\log_9 x)^{-1} = 3 \to 2\log_9 x + \frac{1}{\log_9 x} = 3 \to 2(\log_9 x)^2 - 3\log_9 x + 1 = 0$

Let $A = \log_9 x$. Then: $2A^2 - 3A + 1 = (2A-1)(A+3) = 0 \to A = \frac{1}{2}, 3 \to x = \underline{\mathbf{3, 9}}$

2. $\sqrt{x-4} - 2^{-3\log_8 2} - \log_2 \frac{1}{32} = \sqrt{x+16} + \log_4 32$

$\sqrt{x-4} - \frac{1}{2} + 5 = \sqrt{x+16} + \frac{5}{2} \to \sqrt{x-4} + 2 = \sqrt{x+16} \to x - 4 + 4\sqrt{x-4} + 4 = x + 16$
$\to 4\sqrt{x-4} = 16 \to \sqrt{x-4} = 4 \to x - 4 = 16 \to x = \underline{\mathbf{20}}$

3. $\sqrt{\frac{9}{4} + 4} + \left(\frac{25}{16}\right)^{\frac{3}{2}}\left(\frac{1}{8}\right)^{-\frac{4}{3}} = \sqrt{4x^4 + 4x^4 + x^4} \to \sqrt{\frac{25}{4}} + \left(\frac{125}{64}\right)(16) = \sqrt{9x^4}$
$\to \frac{5}{2} + \frac{125}{4} = 3x^2 \to 10 + 125 = 135 = 12x^2 \to x^2 = \frac{135}{12} = \frac{45}{4}$
Taking the square root of both sides, $|x| = \frac{3\sqrt{5}}{2} \to x = \underline{\mathbf{\pm\frac{3\sqrt{5}}{2}}}$

GBML '10

ROUND 4

1. $\log_2\left(\log_3\left(\log_4 x^{\frac{3}{2}}\right)\right) = 1 \to \log_3\left(\log_4 x^{\frac{3}{2}}\right) = 2^1 = 2 \to \log_4 x^{\frac{3}{2}} = 3^2 = 9$
$\to x^{\frac{3}{2}} = 4^9 \to x = (4^9)^{\frac{2}{3}} = 4^6 = 2^{12} \to a + b = \underline{\mathbf{14}}$

2. $4^A = 2^{\frac{1}{2}}$, $9^B = 3^{\frac{1}{3}}$ and $64^C = 2^{\frac{1}{2}} \to \frac{1}{4} + \frac{1}{6} = \frac{1}{12} + \log_x 4 \to \frac{1}{3} = \log_x 4 \to x = 4^3 = \underline{\mathbf{64}}$

3. $\log_3 x + \log_9 18 + \log_x 9 = \log_{\frac{1}{2}} \frac{1}{16} + \log_9 2$

$\to \log_3 x + \log_x 9 + \log_9 18 - \log_9 2 = \log_{\frac{1}{2}} \frac{1}{16} \to \log_3 x + \log_x 9 + \log_9\left(\frac{18}{2}\right) = 4 \to \log_3 x + \log_x 9 = 3$
If $A = \log_3 x$, $\log_3 x + \log_x 9 = 3 \to A + \frac{2}{A} = 3 \to A^2 - 3A + 2 = (A-2)(A-1) = 0$
$\to A = 1, 2 \to x = 3^1, 3^2 = \underline{\mathbf{3, 9}}$.

MML 12/03

MASSACHUSETTS MATHEMATICS LEAGUE
DECEMBER 2003
ROUND 4: LOGS & EXPONENTIALS
NON-CALCULATOR

ANSWERS

A) 27

B) 3/2, -2

C) -1

A) Solve for x: $\log_9 x = \log_{16} 320 - \log_{16} 5$

$\log_9 x = \log_{16} \frac{320}{5} = \log_{16} 64 = \frac{3}{2}$

$x = 9^{3/2} = 27$

B) Solve for x: $\left(\frac{1}{4}\right)^{x-x^2} = 8^{2-x}$

$2^{-2(x-x^2)} = 2^{3(2-x)}$

$-2x + 2x^2 = 6 - 3x$

$2x^2 + x - 6 = 0$

$(2x-3)(x+2) = 0$

$x = 3/2, -2$

C) Solve for x: $\log_2(-2x-1) - \log_{\sqrt{2}} 2 + \log_2(-x+3) = 0$

$\log_2(-2x-1)(-x+3) = \log_{\sqrt{2}} 2 = 2$

$2x^2 - 6x + x - 3 = 2^2 = 4$

$2x^2 - 5x - 7 = 0 \quad (2x-7)(x+1) = 0$

$x = \frac{7}{2}, -1$, only -1 checks.

MML 12/04

Round Four:

A. $x(x-6) = 16$ so $x = 8$ or $x = -2$ Since -2 has no log the solution is $x = 8$.

B. Use log properties simplify to $3 - 5 = x^2 - 3x$ to $0 = (x-1)(x-2)$

C. Divide $3/8 = a b^3$ by $½ = a b$ to get $¾ = b^2$ so $b = \sqrt{3}/2$, $a = \sqrt{3}/3$

MML 12/05

Round Four:

A. $4 + (-3) - ½ + 1 = 3/2$.

B. $\log_5(x^2 - 20x) = \log_5(125)$ so $x^2 - 20x - 125 = 0$ so $(x-25)(x+5) = 0$, exclude -5 as not in domain of $\log_5$ function.

C $\log_x\left(\frac{4}{x^3}\right) = \log_2\left(\frac{1}{x}\right)$ so $\log_x(4) - 3 = -\log_2(x)$ and if $a = \log_x 2$ we solve

$2a - 3 = \frac{-1}{a}$ to get $a = ½$ or 1 so $x = 4$ or 2.

MML 12/06

Round 4

A) Note that A and B, the roots of the quadratic equation, are each positive numbers $\left(\frac{22 \pm \sqrt{22^2 - 12(27)}}{6}\right)$ and we can let $x = \log_3 A + \log_3 B = \log_3(AB)$

But AB, the product of the roots of the quadratic, is given by the constant term divided by the lead coefficient → $27/3 = 9$

Thus, $x = \log_3(9) = \underline{2}$.

B) $8^a = 2^{3a} = 45$ and $2^b = 7.5$ or $2^{b+1} = 15$. Dividing, $2^{3a-(b+1)} = 3$

$\rightarrow 3a - b - 1 = \log_2 3 = \frac{1}{\log_3 2} = c \rightarrow b = \underline{3a - 1 - c}$

C) $(2x-3)^2 > 0$ for all x except 3/2. The critical points in the numerator of the argument $(x-1)(x+4)$ are 1 and -4. The product is positive for $x < -4$ or $x > 1$ and negative in between. Since the log of zero or negative values is undefined, the domain is restricted to $x < -4$ or $x > 1$ (excluding $x = 3/2$).

MML 12/07

Round 4

A) $(\log_3 x)^2 - 6\log_3 x - 7 = (\log_3 x + 1)(\log_3 x - 7) = 0 \rightarrow \log_3 x = -1, +7 \rightarrow x = \underline{\frac{1}{3}, 2187}$

B) $a = \log_{36}(8) \rightarrow 2a = \log_6(8)$

If $N = \log_{216}(48)$, then $3N = \log_{\sqrt[3]{216}}(48) = \log_6(48) = \log_6(6 \cdot 8) = 1 + \log_6(8) = 1 + 2a$

Thus, $N = \underline{\frac{2a+1}{3}}$

C) $3^{3\log_3 x + 1} - 2^{2\log_2 x} = 2x^4 \rightarrow 3x^3 - x^2 = 2x^4 \rightarrow x^2(2x^2 - 3x + 1) = x^2(x-1)(2x-1) = 0$

$\rightarrow x = 0$ (extraneous), $\underline{1, \frac{1}{2}}$

MML 12/09

Round 4 Alg 2: Log and Exponential Functions

A) $-\frac{1}{2}$ B) $2\sqrt{2}$ C) $x = -2$