Round 1 – Algebra 1

GREATER BOSTON MATHEMATICS LEAGUE

MEET 3 – DECEMBER 1998

ROUND 1 – Algebra 1: Fractions and Word Problems

1. ____________________

2. ____________________

3. ____________________

CALCULATORS ARE NOT ALLOWED ON THIS ROUND

1. Simplify $\dfrac{2x+7}{x^2+2x+4} - \dfrac{x^2+3x+26}{8-x^3}$

2. The rate of flow of water into a pool via an inlet pipe is twice the rate out via a drain. When the pool is one quarter full, the drain is opened for eight minutes and then closed. The inlet pipe is now opened and fills the pool in forty minutes. How many minutes would it take the inlet pipe to fill the pool if it was empty to begin with and the drain is kept closed?

3. Arthur and Betsy are painting a house. One day Arthur painted for six hours, and Betsy painted for two hours and one-third of the house was painted. The next day Arthur painted for three hours and Betsy painted for six hours, and the house was completely painted. Assuming each one works at his or her own constant rate throughout, how long in hours would it take Arthur by himself to paint the entire house?

GREATER BOSTON MATHEMATICS LEAGUE

MEET 3 – DECEMBER 1999

ROUND 1 – Algebra 1: Fractions and Word Problems

1. ______________________

2. ______________________

3. ______________________

CALCULATORS ARE NOT ALLOWED ON THIS ROUND

1. Solve the following equation for x: $\dfrac{18}{x^2-9}+\dfrac{2}{3-x}=\dfrac{x}{x+3}$

2. Bob can type a long dictated manuscript in 18 hours while Alice can type it in 15 hours. Alice and Bob work for 6 hours typing different parts of the manuscript and then stop. Charles types the rest of the manuscript in 8 hours. How many hours would it have taken Charles and Bob, working together, to type the entire manuscript?

3. The rational expression, $\dfrac{x^2-2x-15}{3x^2-5x+k}$, can be reduced for what value(s) of k.

GREATER BOSTON MATHEMATICS LEAGUE

MEET 3 – DECEMBER 2000

ROUND 1 – Algebra 1: Fractions and Word Problems

1. ____________________

2. ____________________

3. ____________________

CALCULATORS ARE NOT ALLOWED ON THIS ROUND

1. Alex takes twice as long as Mary to change a tire. Together they can change a tire in 12 minutes. How many minutes would it take Alex to change a tire working alone?

2. A motor boat travels a certain distance, *d*, upstream against a 4 kilometers per hour current and then returns downstream to its starting point. Traveling that same total distance, 2*d*, would have taken 50% more time on a lake without any current. What is the exact number of kilometers per hour for the speed of the motorboat without any current?

3. Find all values for x satisfying the following equation: $\dfrac{x^4 - x^2 - 2x - 1}{x^3 - 1} = \dfrac{5}{2}$

GREATER BOSTON MATHEMATICS LEAGUE

MEET 3 – DECEMBER 2001

ROUND 1 – Algebra 1: Fractions and Word Problems

1. ____________________

2. ____________________

3. ____________________

CALCULATORS ARE NOT ALLOWED ON THIS ROUND

1. The ratio of girls to boys at a math meet is 2:3. If two more girls and eight more boys compete, then the ratio of girls to boys would be 5:8. Find the total number of students at the meet originally.

2. Solve the following equation for x:

$$\frac{x-1-\dfrac{1}{x-1}}{x-3+\dfrac{2}{x}}=1$$

3. Jill, a master carpenter, and two trainees, Jack and Jim, are building a room to a house. If each one worked alone, Jack would take six hours longer to build the room than Jill and Jim takes a third longer than Jack. Jack and Jill without Jim worked for six hours and then stopped. Jim by himself finished building the room in twenty-two hours. How many hours would it have taken Jill to build the room from start to finish without any help?

GREATER BOSTON MATHEMATICS LEAGUE
MEET 3 – DECEMBER 2005

ROUND 1 – Algebra 1: Fractions and Word Problems

1. ______________________

2. ______________________

3. ________________ **years**

CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

1. Simplify completely. Assume $x \neq 0$.

$$x(1 - \frac{1}{x^2}) \div (\frac{1}{x} - 1)$$

2. I'm thinking of a date <u>D</u> in December.
Consider the five consecutive dates: P, Q, D, R and S. The sum $Q + D + R$ is a multiple of 5. Either P or S is prime, but not both. Determine all possible values for the date D.

3. Sue's age 5 years ago is equal to $\frac{1}{4}$ of her dad's age 6 years from now. 18 years from today the ratio of their ages will be 12 : 7. How old is Sue today?

GREATER BOSTON MATHEMATICS LEAGUE

MEET 3 – DECEMBER 2006

ROUND 1 – Algebra 1: Fractions and Word Problems

1. ______________________

2. ______________________

3. __________ : __________

CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

1. Four consecutive integers are such that the third is 6/5 of the first. What is the fourth integer in the sequence?

2. The ratio of red to blue marbles in a bowl is 3 : 2. If six more red marbles were added and three blue ones taken away, the ratio of blue to red would be 3 : 8. How many marbles were originally in the bowl?

3. Jody has \$4.30 more in quarters than in dimes. If the dimes were quarters and the quarters were dimes, she would increase the total amount of money in her possession by \$1.20. What was the original ratio of the number of dimes to the number of quarters?

GREATER BOSTON MATHEMATICS LEAGUE

MEET 3 – DECEMBER 2007

ROUND 1 – Algebra 1: Fractions and Word Problems

1. ____________ : ____________

2. ______________________________

3. plane: _________ train: __________

CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

1. Find the ratio of $a:b$ if $\dfrac{\dfrac{2a+b}{a+b}-\dfrac{1}{2}}{1-\dfrac{a}{a+b}}=3$

2. A cyclist sprints back and forth between points A and B twice.
His average rates for the 4 sprints are 40 mph, 30 mph, 20 mph and x mph.
Find the exact value of x, if his overall average speed is 25 mph.

3. The distance from Boston to Montreal is 720 miles. A plane makes the trip in 4 less hours than a train. If the first $\frac{1}{3}$ of the trip was traveled by train, and the remaining distance by plane, the trip would take four hours. Find the rate of the plane and train in miles per hour.

GREATER BOSTON MATHEMATICS LEAGUE

MEET 3 – DECEMBER 2008

ROUND 1 – Algebra 1: Fractions and Word Problems

1. ______________________

2. (_____ , _____)

3. ______________________

CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

1. If $\dfrac{3x+2a}{3a-x}=9$, find the value of $\dfrac{x+2a}{x-a}$.

2. Some M&Ms were divided among 3 children so that the first child had two more M&Ms than $\frac{1}{3}$ of all the M&Ms. The second child had one more than half of the remaining M&Ms, and the third child had 14 M&Ms. If A and B denote the number of M&Ms received by the first and second child respectively, determine the ordered pair (A, B).

3. If I divide a two digit number by 2 more than the sum of the digits, I get a quotient of 3 and a remainder of 4. If I reverse the order of the digits and divide by 3 times the sum of the digits, my quotient is 2 and the remainder is 16. Find the original two digit number.

GREATER BOSTON MATHEMATICS LEAGUE

MEET 3 – DECEMBER 2009

ROUND 1 – Algebra 1: Fractions and Word Problems

1. $(J, K) = (\_\_\_\_\_\_, \_\_\_\_\_\_)$

2. ____________________ gallons

3. ________________________

1. Find the ordered pair (J, K) such that J and K are the smallest positive integers which satisfy the following statement:

$$\frac{4}{7} \text{ of } \frac{5K}{3J} \text{ is equivalent to } \frac{35}{8} \text{ of } \frac{2J}{3K}$$

2. A radiator is $\frac{2}{3}$ full of an 80% antifreeze solution. $3\frac{1}{2}$ gallons are drained off and then the radiator is filled with a 50% antifreeze solution. The radiator now has a 63% antifreeze solution. Compute the capacity of the radiator in gallons.

3. Mary had 35 coins consisting of nickels and dimes. However, Mary thought all the dimes were quarters. She actually spent \$1.75 and now what she actually has left is \$0.30 less than $\frac{1}{3}$ of what she thinks she has left. How much money did Mary really have originally?

GREATER BOSTON MATHEMATICS LEAGUE

MEET 3 – DECEMBER 2010

ROUND 1 – Algebra 1: Fractions and Word Problems

1. ______________________

2. ______________________

3. ______________________

CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

1. Simplify completely: $\dfrac{\frac{3}{4}-\frac{x-1}{x}}{\frac{x}{16}-\frac{1}{x}}$

2. Orville had two mixtures of antifreeze with water. Mixture A was 40% antifreeze, while mixture B was 60% antifreeze. Orville wanted a combined mixture where the number of quarts of mixture A to the number of quarts of mixture B would be 3 : 2. When combining the mixtures, Wilbur mistakenly reversed the ratio. The percent of antifreeze in Wilbur's combined mixture exceeded the percent of antifreeze in the combined mixture Orville intended by k%. Compute k.

3. A and B working together complete a job when A works for 2/3 of a week and B works for $1\frac{1}{3}$ weeks. They can also complete the job when A works 5/12 of a week and B works for $1\frac{1}{2}$ weeks. Compute the ordered pair (a, b), where a and b denote the number of weeks respectively it would take A and B to complete the job, each working alone.

MASSACHUSETTS MATHEMATICS LEAGUE
OCTOBER 2003
ROUND 4: FRACTIONS & MIXED NUMBERS
NON-CALCULATOR

ANSWERS

A)_______________

B)_______________

C)_______________

A) If $\frac{1}{a(b+1)} + \frac{1}{b(a+1)} = \frac{1}{(a+1)(b+1)}$, what is the value of $\frac{1}{a} + \frac{1}{b}$?

B) The numerator of a fraction is two less than the denominator. When both the numerator and the denominator are increased by five, the result is 4/3 of the original fraction. ~~What~~ Find ~~was the original fraction~~? all possible original fractions.

C) If $\frac{x-3y}{x+2y} = 4\frac{2}{3}$, what is the value of $\frac{3x}{4y}$ expressed as a fraction?

MASSACHUSETTS MATHEMATICS LEAGUE
OCTOBER 2005
ROUND 4 ALG 1: FRACTIONS & MIXED NUMBERS

******* NO CALCULATORS ON THIS ROUND ******

ANSWERS

A)______________

B)______________

C)______________

A) A ream (500 sheets) of letter size paper (eight and a half by eleven inches) is $2\frac{1}{8}$ inches thick. If the volume of a single sheet as a simplified fraction is $\frac{a}{b}$ cubic inches, find $a+b$.

B) Egyptians wrote fractions as sums of unit fractions ($\frac{1}{n}$). If we write $\frac{5}{18}$ as $\frac{1}{a}+\frac{1}{b}$ and $\frac{4}{9}$ as $\frac{1}{b}+\frac{1}{a}+\frac{1}{b}$, find the value of $a+b$.

C) My hose will fill my pool in 14 hours; the pool's drain empties the pool in 8 hours. At 6 a.m. my pool was empty so I closed the drain and turned on the hose. When the pool was half full, the drain accidentally opened! At what time did my pool become empty again? (Specify a.m. or p.m.)

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2006
ROUND 4 ALG 1: FRACTIONS & MIXED NUMBERS

******* NO CALCULATORS ON THIS ROUND *******

ANSWERS

A) ______________________

B) ______________________

C) (_______ , _______)

A) Determine the positive difference between A and B.

$$A = 2 - \frac{1}{3 + \frac{1}{4}} \qquad B = \frac{\frac{1}{3} + \frac{1}{4}}{\frac{2}{3} - \frac{1}{4}}$$

B) Solve for x: $\left(\frac{1+x}{1-x}\right) = 4 - 3\left(\frac{1-x}{1+x}\right)$

C) The fraction $\frac{17}{25}$ can be expressed as the sum of three unit fractions, the first of which is $\frac{1}{2}$, that is $\frac{17}{25} = \frac{1}{2} + \frac{1}{A} + \frac{1}{B}$. Find (A, B), where $0 < A < B$.

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2007
ROUND 4 ALG 1: FRACTIONS & MIXED NUMBERS

ANSWERS

A) ______________________

B) ______________________

C) ______________________

***** NO CALCULATORS ON THIS ROUND *****

A) Compute: $$\frac{2\frac{7}{16} - 3\frac{3}{4}}{5\frac{5}{8} + 7\frac{1}{2}}$$

B) Solve for x. $$\frac{x-3}{x+1} - \frac{2x-8}{x^2-1} = \frac{3}{x-1}$$

C) If the integer $n \geq 1$ and $\frac{(n+1)!(2n-2)!}{(n-1)!(2n)!} = \frac{2}{7}$, compute n.

Recall: $n! = n(n-1)(n-2) \ldots \cdot 2 \cdot 1$ and $0! = 1$

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2010
ROUND 4 ALG 1: FRACTIONS & MIXED NUMBERS

ANSWERS

A) ______________________

B) (__________ , __________)

C) ______________________ mph

***** NO CALCULATORS ON THIS ROUND ****

A) Express the average of $\frac{1}{4}$, $1\frac{1}{5}$ and 1.5 as a simplified ratio of integers.

B) Let $a \oplus b = \dfrac{a+2b}{2a-b}$. Compute the ordered pair (a, b) for which $a \oplus b = 2$ and $a - b = 2$.

C) During rush hour a Microsoft employee averages only 40 mph on the ride from home to office. After a long day at the office, he returns home late at night over the same route. What average speed (in mph) on his return trip insures that his overall average speed is 55 mph, assuming he does not stop and is not stopped for speeding?

MASSACHUSETTS MATHEMATICS LEAGUE
FEBRUARY 2004
ROUND 4: WORD PROBLEMS

ANSWERS

A)_______________

B)_______________

C)_______________

A) What is the larger of the two numbers whose sum is ten, if the sum of their reciprocals is 8/15.

B) An elevator went from the bottom of a tower to the top at a speed of 4 meters/second. It remained at the top for ninety seconds, and then returned to the bottom at a speed of 5 m/sec. If the total trip took 4 5 minutes, how high is the tower?

CONSECUTIVE
C) The sum of the squares of three ~~positive~~ odd integers is 967 more than the sum of the squares of the two even integers between them. Calculate the sum of the five consecutive integers.

MASSACHUSETTS MATHEMATICS LEAGUE
FEBRUARY 2005
ROUND 4 ALGEBRA ONE: WORD PROBLEMS

ANSWERS

A)_________gallons

B)_______________

C)__________hour

A) How many gallons of pure water must be added to a gallon of alcohol 75% pure to make a mixture only 15% pure?

B) I am half as old as my mother was when my brother was twelve years younger than I am now. My brother was born when my mother was 26. If the sum of my brother's and my own current ages is 36, how old was my mother when I was born?

C) I paddled my canoe upstream for 6 hours. I then rested for an hour as my canoe drifted downstream, then paddled back downstream to my starting point in just 2 hours. How much earlier would I have gotten back if I had headed back immediately rather than resting for that hour? Express your answer as a reduced fraction of an hour.

MASSACHUSETTS MATHEMATICS LEAGUE
FEBRUARY 2006
ROUND 4 ALGEBRA ONE: WORD PROBLEMS

ANSWERS

A)_____________

B)__________**mins**

C)______________

A) I have a mixture of quarters and Sacagawea dollar coins the same size. I gave \$16.50 worth of these coins to a teller, but he mistakenly counted 3 of the quarters as dollars. If he credited me X **cents** for these coins and the minimum number of coins I could have given the teller is K, find the numeric value of $X + K$.

B) I jog at 12 feet per second and my little sister jogs at a constant slower rate. If we run in opposite directions on a quarter mile track, we pass each other every minute. If we run in the same direction, how many minutes will it take me to lap her? (Recall: 1 mile = 5280 feet!)

C) A chemist adds 20 liters of an alcohol and water solution that is 30% alcohol to 10 liters of an original solution of alcohol and water. He finds the percentage of alcohol in the resulting mixture is 6 percentage points higher than in the original solution. What was the percentage of alcohol of the original solution?

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 – FEBRUARY 2008
ROUND 4 ALG 1: WORD PROBLEMS

ANSWERS

A) _______________________________

B) _____________________________ %

C) _____________________________ oz.

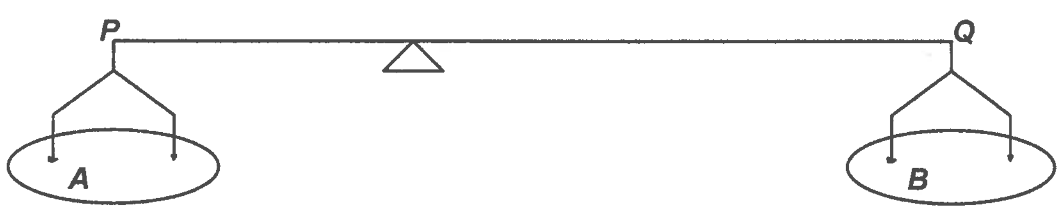
A) 35 students registered for enrichment courses in a summer program at Brewster Academy. Here is registration information for the science courses.

Course	# students registered
Biology	19
Chemistry	17
Physics	11
Chemistry and Physics	5
Biology and Physics	7
Biology and Chemistry	12
All 3 courses	2

How many of the 35 students did not register for any of these science courses?

B) Three men work at equal rates work 8 hours per day. If they complete 80% of a job in four 8-hour days, what percent of the job would be completed by one of these men in 6 hours?

C) A supply of cylinders of equal but unknown mass are available and used as follows:
8 cylinders in pan A balance a single 9 oz. cube in pan B.
A single 25 oz cube in pan A balances 2 cylinders in pan B.
How much (in ounces) does each cylinder weigh?
Note that the fulcrum is not at the midpoint of $\overline{PQ}$.



GBML '98

ROUND 1

1. $$\frac{2x+7}{x^2+2x+4}-\frac{x^2+3x+26}{8-x^3}=\frac{2x+7}{x^2+2x+4}+\frac{x^2+3x+26}{x^3-8}=$$
$$\frac{(x-2)(2x+7)}{(x-2)(x^2+2x+4)}+\frac{x^2+3x+26}{(x-2)(x^2+2x+4)}=\frac{2x^2+3x-14+x^2+3x+26}{(x-2)(x^2+2x+4)}=$$
$$\frac{3x^2+6x+12}{(x-2)(x^2+2x+4)}=\frac{3(x^2+2x+4)}{(x-2)(x^2+2x+4)}=\frac{3}{x-2}$$

2. The time to fill or empty the tank is inversely proportional to the rate of flow $\Rightarrow$
If x = time to fill the pool, then $2x$ = time to empty the pool $\Rightarrow$
Equation: $\frac{40}{x}-\frac{8}{2x}=\frac{3}{4}\Rightarrow\frac{40}{x}-\frac{4}{x}=\frac{3}{4}\Rightarrow\frac{36}{x}=\frac{3}{4}\Rightarrow$ x = **48 min.**

3. Let a = Arthur's rate, b = Betsy's rate;
$\frac{6}{a}+\frac{2}{b}=\frac{1}{3};\ \frac{3}{a}+\frac{6}{b}=\frac{2}{3}\Rightarrow\frac{1}{a}+\frac{2}{b}=\frac{2}{9}\Rightarrow$
$\frac{5}{a}=\frac{1}{9}\Rightarrow\Rightarrow$ a = **45** hours.

GBML '99

ROUND 1

1. $\frac{18}{x^2-9}+\frac{2}{3-x}=\frac{x}{x+3}\Rightarrow\frac{18}{x^2-9}-\frac{2}{x-3}=\frac{x}{x+3}\Rightarrow 18-2(x+3)=x(x-3)\Rightarrow$
$18-2x-6=x^2-3x\Rightarrow x^2-x-12=0\Rightarrow(x-4)(x+3)=0\Rightarrow x=4$ since -3 is an extraneous solution to the original equation.

2. $\frac{6}{15}+\frac{6}{18}=\frac{11}{15}\Rightarrow$ Charles has $\frac{4}{15}$ of the manuscript left to type. $\Rightarrow\frac{4}{15}=\frac{8}{x}\Rightarrow x=30$ hrs
$\frac{x}{18}+\frac{x}{30}=1\Rightarrow x=\frac{45}{4}$ *or* 11.25 *hrs.*

3. $\frac{x^2-2x-15}{3x^2-5x+k}=\frac{(x-5)(x+3)}{3x^2-5x+k}$; this is reducible only if either 5 or -3 are zeros of the bottom polynomial $\Rightarrow 3(5)^2-5(5)+k=0$ *or* $3(-3)^2-5(-3)+k=0\Rightarrow k=-42$ *or* -50

ROUND 1 GBML '00

1. Let t = number of minutes for Mary $\rightarrow 2t$ = minutes for Alex $\rightarrow\frac{12}{t}+\frac{12}{2t}=1\rightarrow 2t=36$

2. Let d = distance one way, r = rate of the boat in still water $\rightarrow\frac{d}{r+4}+\frac{d}{r-4}=\frac{3}{2}\cdot\frac{2d}{r}\rightarrow$
$\frac{1}{r+4}+\frac{1}{r-4}=\frac{3}{r}\rightarrow r(r-4)+r(r+4)=3(r^2-16)\rightarrow$
$r^2+4r+r^2-4r=3r^2-48\rightarrow r^2=48\rightarrow r=4\sqrt{3}$

3. $\frac{x^4-x^2-2x-1}{x^3-1}=\frac{5}{2}\rightarrow\frac{x^4-(x+1)^2}{(x-1)(x^2+x+1)}=\frac{5}{2}\rightarrow\frac{(x^2-x-1)(x^2+x+1)}{(x-1)(x^2+x+1)}=\frac{5}{2}\rightarrow$
$\frac{x^2-x-1}{x-1}=\frac{5}{2}\rightarrow 2x^2-2x-2=5x-5\rightarrow 2x^2-7x+3=0\rightarrow$
$(2x-1)(x-3)=0\rightarrow x=\frac{1}{2},3$

ROUND 1 – Algebra 1: Fractions and Word Problems

GBML 101

1. Let $2x$ = number of girls and $3x$ = number of girls $\Rightarrow 5x$ = total number of students $\Rightarrow$
$\frac{2x+2}{3x+8}=\frac{5}{8}\Rightarrow 16x+16=15x+40\Rightarrow x=24\Rightarrow 5x=120.$

2. $$\frac{x-1-\frac{1}{x-1}}{x-3+\frac{2}{x}}=1\Rightarrow\frac{\frac{(x-1)^2}{x-1}-\frac{1}{x-1}}{\frac{x^2-3x+2}{x}}=1\Rightarrow\frac{\frac{x^2-2x}{x-1}}{\frac{(x-2)(x-1)}{x}}=1\Rightarrow$$
$$\frac{x\cancel{(x-2)}}{x-1}\cdot\frac{x}{(x-1)\cancel{(x-2)}}=1\Rightarrow\frac{x^2}{(x-1)^2}=1\Rightarrow\frac{x}{x-1}=\pm1\Rightarrow x=x-1\ or\ x=1-x\Rightarrow x=\frac{1}{2}.$$

3. Let x = hours for Jill to do the job $\Rightarrow x+6$ = hours for Jack and $\frac{4}{3}(x+6)$ = hours for Jim
$\Rightarrow\frac{6}{x}+\frac{6}{x+6}+\frac{22}{\frac{4}{3}(x+6)}=1\Rightarrow\frac{6}{x}+\frac{6}{x+6}+\frac{33}{2(x+6)}=1\Rightarrow\frac{6}{x}+\frac{45}{2(x+6)}=1$
$\Rightarrow 12(x+6)+45x=2x(x+6)\Rightarrow 12x+72+45x=2x^2+12x\Rightarrow 0=2x^2-45x-72\Rightarrow$
$(2x+3)(x-24)=0\Rightarrow x=24$ hours.

GBML '05

Algebra 1: Fractions and Word Problems

1. $x(1-\frac{1}{x^2})\div(\frac{1}{x}-1)=x(\frac{x^2-1}{x^2})(\frac{x}{1-x})=\cancel{x}(\frac{(x-1)(x+1)}{\cancel{x}(1-x)})=\underline{-(x+1)}$

   Let y denote the original expression. Then, excluding $x = 0$, the equation becomes $y = -x - 1$. The graph is a straight line with a slope of -1, but the point at $(0, -1)$, i.e. the y-intercept is missing. We say the graph has a hole or discontinuity at $(0, -1)$.

2. Represent P, Q, D, R and S as n-2, n-1, n, n+1 and n+2 respectively. Then $(n-1) + n + (n+1) = 3n = 5k \rightarrow n = 5k/3$. Since these two quantities are equal and we know n is an integer, $5k/3$ must also represent an integer. This implies k must take on values of 3, 6, 9, 12, 15 which produce possible dates of 5, 10, 15, 20 and 25. n must be odd otherwise P and S will both be even and therefore not prime. Thus, we need only test the three odd candidates. 3-5-7 and 13-13-17 are each rejected since P and S are both prime $n = 25 \rightarrow P = 23$ (prime) and $S = 27$ (composite) Thus, $D =$ **25**.

3. Let S denote Sue's age now and D denote Dad's age now. Then

   #1: $(S-5)=(D+6)/4 \rightarrow D=4S-26$

   #2: $\frac{D+18}{S+18}=\frac{12}{7}$

   Substituting, $\frac{(4S-26)+18}{S+18}=\frac{4S-8}{S+18}=\frac{12}{7}$ (since Dad must be older than Sue)

   $28S-56=12S+216 \rightarrow 16S=272 \rightarrow S=\underline{\mathbf{17}}$

GBML '06

ROUND 1

1. Let $x, x+1, x+2$ and $x+3$ denote the 4 consecutive integers. Then $(x+2) = 6x/5 \rightarrow x = 10 \rightarrow t_4 = \underline{\mathbf{13}}$

2. $R : B = 3 : 2$ and $(B-3) : (R+6) = 3 : 8 \rightarrow R = \frac{3}{2}B$ and $8B - 24 = 3R + 18$

   $\rightarrow 8B-24=\frac{9}{2}B+18 \rightarrow 16B-48=9B+36 \rightarrow 7B=84 \rightarrow B=12, R=18$

   $\rightarrow$ total: **30**

3. $25Q = 10D + 430$ and $(10Q + 25D) - (25Q + 10D) = 120$
   $5Q = 2D + 86$ and $15D - 15Q = 120 \rightarrow 15D - 3(2D + 86) = 120 \rightarrow 9D = 378 \rightarrow D = 42$
   Substituting, $5Q = 84 + 86 = 170 \rightarrow Q = 34$
   Thus, $D : Q = 42 : 34 =$ **21 : 17**.

GBML '07

ROUND 1

1. $\dfrac{\frac{2(2a+b)-(a+b)}{2(a+b)}}{\frac{a+b-a}{a+b}}=\frac{3a+b}{2b}=3 \rightarrow 3a+b=6b \rightarrow 3a=5b \rightarrow \frac{a}{b}=\underline{\frac{5}{3}}$

2. Let d = denote the distance between points A and B. Then:

   $\dfrac{4d}{\frac{d}{40}+\frac{d}{30}+\frac{d}{20}+\frac{d}{x}}=\dfrac{4}{\frac{1}{40}+\frac{1}{30}+\frac{1}{20}+\frac{1}{x}}\cdot\frac{120x}{120x}=\frac{480x}{3x+4x+6x+120}=\frac{480x}{13x+120}=25$

   $\rightarrow 325x+3000=480x \rightarrow 155x=3000 \rightarrow x=\underline{\mathbf{\frac{600}{31}}}$

3. $\dfrac{240}{\left(\frac{720}{x+4}\right)}+\dfrac{480}{\left(\frac{720}{x}\right)}=4 \rightarrow \frac{x+4}{3}+\frac{2}{3}x=4 \rightarrow x+4+2x=12 \rightarrow x=\frac{8}{3}, x+4=\frac{20}{3}$

   $\dfrac{720}{\frac{8}{3}}=\underline{\mathbf{270}}$ mph by plane, $\dfrac{720}{\frac{20}{3}}=\underline{\mathbf{108}}$ mph by train

GBML '08

ROUND 1

1. Cross multiplying $\frac{3x+2a}{3a-x}=\frac{9}{1}$, we have $3x+2a=27a-9x \rightarrow 12x=25a \rightarrow x=\frac{25a}{12}$

   Substituting in $\frac{x+2a}{x-a}$, $\dfrac{\frac{25a}{12}+2a}{\frac{25a}{12}-a}\cdot\frac{12}{12}=\frac{25a+24a}{25a-12a}=\frac{49a}{13a}$. Thus, the required expression has a numerical value of $\underline{\mathbf{\frac{49}{13}}}$. [The original equation $\frac{3x+2a}{3a-x}=3$ was meaningless if $a = 0$, since substituting 0 for a produces $\frac{3x+0}{0-x}=3$ or $-3 = 3$ which is a false statement.]

2. If T denote the total number of M&Ms, then the first child has $A=\left(\frac{T}{3}+2\right)$ M&Ms and the second child has $B=\frac{1}{2}\left(\frac{2}{3}T-2\right)+1$ or simply $B=\left(\frac{T}{3}\right)$ M&Ms.

   Thus, $\left(\frac{T}{3}+2\right)+\left(\frac{T}{3}\right)+14=T \rightarrow \frac{T}{3}=16 \rightarrow T=48 \rightarrow (A, B) = \underline{\mathbf{(18, 16)}}$

3. $\frac{10t+u}{t+u+2}=3R4 \rightarrow 10t+u=3(t+u+2)+4$ or $7t-2u=10$

   $\frac{10u+t}{3(t+u)}=2R16 \rightarrow 10u+t=2(3(t+u))+16 \rightarrow 4u-5t=16$

   Adding, $\begin{cases}4u-5t=16\\-4u+14t=20\end{cases} \rightarrow 9t=36 \rightarrow (t, u) = (4, 9)$ and the original number is **49**.

GBML '09

ROUND 1

1. $\frac{4}{7}\cdot\frac{5K}{3J}=\frac{2J}{3K}\cdot\frac{35}{8} \rightarrow \frac{20K}{21J}=\frac{35J}{12K} \rightarrow 20K\cdot 12K = 35J\cdot 21J \rightarrow 16K^2 = 49J^2$

   $\rightarrow \frac{J^2}{K^2}=\frac{16}{49}$. $J, K > 0 \rightarrow (J, K) = \underline{(4, 7)}$

2. $\frac{2}{3}(.80)-\frac{7}{2}(.80)+\left(\frac{x}{3}+\frac{7}{2}\right)(.50)=x(.63)$

   $\frac{2}{3}(80)+\frac{7}{2}(80)+\frac{2x+7}{6}(50)=6(63x)$

   $4x(80)-21(80)+(2x+21)50=63x$

   $320x-1680+100x+1050=378x$

   $420x-378x=630$

   $42x=630$

   $x=\frac{630}{42}=\underline{15}$

3. Actual $\left|\begin{array}{l} x \text{ dimes} \\ 35-x \text{ nickels}\end{array}\right.$ Perceived $\left|\begin{array}{l} x \text{ quarters} \\ 35-x \text{ nickels}\end{array}\right.$

   Value (¢) of amount: $10x+5(35-x)=5x+175$ $\quad 25x+5(35-x)=20x+175$

   Now $5x+175-175=\frac{1}{3}(20x+175-175)-30 \rightarrow 5x=\frac{20x}{3}-30 \rightarrow \frac{5x}{3}=30 \rightarrow x=18$

   18 dimes + 17 nickels $\rightarrow$ **$2.65**

   Check: $265-175 \stackrel{?}{=} \frac{1}{3}(20\cdot 18)-30 \leftrightarrow 90=90$

GBML '10

ROUND 1

1. $\left(\frac{\frac{3}{4}-\frac{x-1}{x}}{\frac{x}{16}-\frac{1}{x}}\right)\cdot\frac{16x}{16x}=\frac{12x-16x+16}{x^2-16}=\frac{-4x+16}{x^2-16}=\frac{-4(x-4)}{(x+4)(x-4)}=\underline{\frac{-4}{x+4}}$

2. Let $3x$ and $2x$ denote the available number of quarts of solutions A and B respectively. Both mixture are between 40 and 60%.

   Orville's mixture (Y%): $.4(3x)+.6(2x)=\frac{Y}{100}(5x) \rightarrow 40(3x)+60(2x)=Y(5x) \rightarrow 5Y=240 \rightarrow Y=48$

   Wilbur's mixture (Z%): $.4(2x)+.6(3x)=\frac{Z}{100}(5x) \rightarrow 40(2x)+60(3x)=Z(5x) \rightarrow 5Z=260 \rightarrow Z=52$

   Thus, the increase is 4% $\rightarrow k=\underline{4}$.

3. Let $x=\frac{1}{a}$ and $y=\frac{1}{b}$ represent their individual rates. $\begin{cases}\frac{1}{a}\cdot\frac{2}{3}+\frac{1}{b}\cdot\frac{4}{3}=1\\ \frac{1}{a}\cdot\frac{5}{12}+\frac{1}{b}\cdot\frac{3}{2}=1\end{cases} \rightarrow$

   $\begin{cases}2x+4y=3 \ (***)\\ 5x+18y=12\end{cases} \rightarrow \begin{cases}18x+36y=27\\ 10x+36y=24\end{cases}$ Subtracting, $8x=3 \rightarrow x=\frac{3}{8}$. Substituting in

   (***), $\frac{3}{4}+4y=3 \rightarrow 4y=3-\frac{3}{4}=\frac{9}{4} \rightarrow y=\frac{9}{16}$. Therefore, $(a, b)=\left(\frac{8}{3},\frac{16}{9}\right)$

MML 10/03

MASSACHUSETTS MATHEMATICS LEAGUE
OCTOBER 2003
ROUND 4: FRACTIONS & MIXED NUMBERS
NON-CALCULATOR

ANSWERS

A) -1

B) $-10/-8 \rightarrow 3/5$

C) $-111/44$

A) If $\frac{1}{a(b+1)}+\frac{1}{b(a+1)}=\frac{1}{(a+1)(b+1)}$, what is the value of $\frac{1}{a}+\frac{1}{b}$?

$b(a+1)+a(b+1)=ab$ $\quad$ So $\frac{1}{a}+\frac{1}{b}=\frac{b+a}{ba}=-1$

$ab+b+ab+a=ab$

$b+a=-ba$

$\frac{b+a}{ba}=-1$

B) The numerator of a fraction is two less than the denominator. When both the numerator and the denominator are increased by five, the result is 4/3 of the original fraction. What was the original fraction?

$\frac{x-2}{x}$ = orig fract $\quad \frac{x+3}{x+5}=\frac{4(x-2)}{3x} \quad x^2+3x-40=0$

$(x+8)(x-5)=0$

$3x(x+3)=(4x-8)(x+5) \quad x=-8,\ x=5$

$3x^2+9x=4x^2+12x-40$ $\quad$ ANS $\frac{-10}{-8}$, $\frac{3}{5}$

C) If $\frac{x-3y}{x+3y}=4\frac{2}{3}$, what is the value of $\frac{3x}{4y}$ expressed as a fraction?

$\frac{x-3y}{x+3y} \quad \frac{14}{3} \quad 3x-9y=14x+42y$

$-11x=37y$

$\frac{x}{y}=-\frac{37}{11}, \quad \frac{3x}{4y}=-\frac{111}{44}$

MML 10/05

Round Four:

A. $(8\frac{1}{2} \times 11 \times 2\frac{1}{8}) / 500 = (17 \times 11 \times 17) / (2 \times 6 \times 500) = \frac{3179}{8000}$ which is irreducible.

B. 1/b = 4/9 – 5/18 =1/6 so b=6 , a =9.

C. ½ = n/8 – n/14 yields n = 28/3 = 9 and 1/3 hours after 1 p.m.

MML 10/06

Round 4

A) $A = 2 - 4/13 = \frac{22}{13}$ and $B = \frac{4+3}{8-3} = \frac{7}{5}$ Computing the difference, $\frac{22(5)-7(13)}{13(5)} = \boxed{\frac{19}{65}}$

B) Let $A = \left(\frac{1+x}{1-x}\right)$ Then the equation becomes $A = 4 - 3(1/A) \rightarrow A^2 - 4A + 3 = (A-3)(A-1) = 0$

$\rightarrow A = 1$ or 3. Substituting for A, $1 + x = 1 - x \rightarrow x = \underline{0}$ or $1 + x = 3(1 - x) \rightarrow 4x = 2 \rightarrow x = \underline{½}$

C) Method 1:

$\frac{1}{A} + \frac{1}{B} = \frac{17}{25} - \frac{1}{2} = \frac{9}{50} \rightarrow 50A + 50B = 9AB \rightarrow B = \frac{50A}{9A-50}$

B is an integer if and only if $9B$ is an integer. $9B = \frac{450A}{9A-50} = 50 + \frac{2500}{9A-50}$

Thus, $9A$- 50 must be a positive factor of 2500. The smallest possible value of A for which this is true is $A = 6$ and $B = 75 \rightarrow$ **(6, 75)**. In fact, the only ordered pairs are (6, 75) and (75, 6).

Method 2:

$\frac{17}{25} - \frac{1}{2} = \frac{9}{50} = 0.18$

The next largest unit fraction which is less than or equal to 0.18 is $\frac{1}{6} \approx 0.83 \rightarrow A = 6$

$\frac{9}{50} - \frac{1}{6} = \frac{4}{300} = \frac{1}{75} \rightarrow B = 75$ Therefore, $(A, B) =$ **(6, 75)**

MML 10/07

Round 4

A) $\frac{2\frac{7}{16} - 3\frac{3}{4}}{5\frac{5}{8} + 7\frac{1}{2}} = \frac{\left(\frac{39}{16} - \frac{15}{4}\right)16}{\left(\frac{45}{8} + \frac{15}{2}\right)16} = \frac{39-60}{90+120} = \frac{-21}{210} = -\frac{1}{10}$ or $-\underline{0.1}$

B) Multiplying by the LCD = $(x + 1)(x - 1)$, $(x-3)(x-1) - 2x + 8 - 3x - 3 = 0$
$\rightarrow x^2 - 4x + 3 - 5x + 5 = 0 \rightarrow x^2 - 9x + 8 = (x-1)(x-8) = 0 \rightarrow x = \underline{8}$ ($x = 1$ is extraneous)

C) $\frac{(n+1)!(2n-2)!}{(n-1)!(2n)!} = \frac{(n+1)n\cancel{(n-1)!}\cancel{(2n-2)!}}{\cancel{(n-1)!}(2n)(2n-1)\cancel{(2n-2)!}} = \frac{(n+1)n}{(2n)(2n-1)} = \frac{2}{7} \rightarrow 7n^2 + 7n = 8n^2 - 4n$
$\rightarrow n^2 - 11n = n(n-11) = 0 \rightarrow n = \underline{11}$

Here's the original question C (nixed by the proofreaders):
For extremely large positive values of n, the following fraction approaches a fixed value L. Compute L.

$$\frac{(n+1)!(2n-2)!}{(n-1)!(2n)!}$$

You might want to try your hand at solving this question before peeking at the solution below.

$$\frac{(n+1)!(2n-2)!}{(n-1)!(2n)!} = \frac{(n+1)n\cancel{(n-1)!}\cancel{(2n-2)!}}{\cancel{(n-1)!}(2n)(2n-1)\cancel{(2n-2)!}} = \frac{n^2+n}{4n^2-2n}$$

Dividing numerator and denominator by n^2, we have $\frac{1+\frac{1}{n}}{4+\frac{2}{n}}$

As n takes on extremely large positive values (i.e. approaches infinity), the fractional terms in the numerator and denominator each approach zero and the overall fraction approaches **1/4** or **0.25**

MML 10/10

Round 4 Fraction & Mixed numbers

A) $\frac{59}{60}$ B) (8, 6) C) 88 mph

MML 2/04

A) What is the larger of the two numbers whose sum is ten, if the sum of their reciprocals is 8/15.

$x, 10-x$ $\frac{1}{x} + \frac{1}{10-x} = \frac{8}{15}$

$15(10-x) + 15x = 8(10x - x^2)$

$150 = 80x - 8x^2$, $8x^2 - 80x + 150 = 0$

$4x^2 - 40x + 75 = 0$ $(2x-5)(2x-15) = 0$ $x = \frac{5}{2}, \frac{15}{2}$ ANS 15/2

B) An elevator went from the bottom of a tower to the top at a speed of 4 meters/second. It remained at the top for ninety.seconds, and then returned to the bottom at a speed of 5 m/sec. If the total trip took 4.5 minutes, how high is the tower?

x = ht of elevator $\frac{x}{4} + 90 + \frac{x}{5} = 270$, $\frac{x}{4} + \frac{x}{5} = 180$

$5x + 4x = 180 \cdot 20$

$\frac{9x}{9} = \frac{180 \cdot 20}{9}$, $x = 20 \cdot 20 = 400$

C) The sum of the squares of three positive odd integers is 967 more than the sum of the squares of the two even integers between them. Calculate the sum of the five consecutive integers. $x, x+2, x+4$

$x^2 + (x+2)^2 + (x+4)^2 = 967 + (x+1)^2 + (x+3)^2$

$x^2 + 4x + 4 + 8x + 16 = 967 + 2x + 1 + 6x + 9$

$x^2 + 12x + 20 = 8x + 977$, $x^2 + 4x - 957 = 0$

$(x+33)(x-29) = 0$, $x = 29$, $x+1 = 30$, $x+2 = 31$, $x+3 = 32$, $x+4 = 33$

OR $x = -33 \rightarrow -155$ ANS ± 155

MML 2/04

Round Four:

A. The ratio of alcohol is $0.15 = \frac{0.75}{1+n}$ solving yields n = 4.

B. I am x now. "Then" my mother was 2x and my brother x-12 so 2x = x – 12 + 26 so x=14 any my brother is 22. My mother was 8 yrs older when I was born or 34.

C. Let r= paddling speed in still water, s = speed of current. 6(r-s) = s + 2(r+s) so s=(4/9)r and upstream rate is (5/9)r, downstream (13/9)r. Immediate return would have taken $\frac{Dist}{time} = \frac{6(r-s)}{r+s} = \frac{6(5/9)r}{(13/9)r} = \frac{30}{13}$ instead of the three hrs it took (1 resting)

MML 2/06

Round Four:

A. The absolute minimum number of coins would be 16 dollars and 2 quarters, but since there must be at least 3 quarters, we have 15 dollars and 6 quarters → $K = 21$.
The teller's mistake credited my account 3(75) = 225 extra cents → $X = 1875$.

B. In one minute 12(60)+x(60)=1320 so sister jogs at 10 ft/sec. In same direction I gain 2 ft/sec or 120 ft/minute. 1320/120 = 11.

C. Original mix was $n/100$. $\frac{n+6}{100} = \frac{(n/100)10 + .30(20)}{30}$ Solving, $n = 21$.

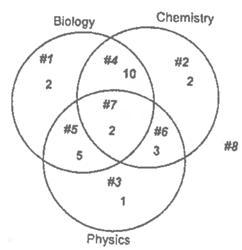
MML 2/08

Round 4

A) Using a Venn Diagram to separate the overlaps,

Biology	#1,4,5,7
Chemistry	#2,4,6,7
Physics	#3,5,6,7
Chemistry and Physics	#6,7
Biology and Physics	#5,7
Biology and Chemistry	#4,7
All 3 courses	#7

None = 35 – (2+10+2+5+2+3+1) = **<u>10</u>**



B) Let x denote the percent of the job completed. Then $\frac{3\cdot 4}{80} = \frac{1\cdot\frac{3}{4}}{x} \rightarrow 12x = 60 \rightarrow x = \underline{5}$

C) Let z denote the mass of each cylinder and (x, y) the distances of pans A and B from the balance point respectively.
Equating the clockwise and counterclockwise torques keeps the system in equilibrium.
Thus, $8zx = 9y \rightarrow y = \frac{8zx}{9}$ and $25x = 2zy \rightarrow y = \frac{25x}{2z}$
Equating and canceling the x's in the numerator (since $x \neq 0$)
$\rightarrow \frac{8z}{9} = \frac{25}{2z} \rightarrow z^2 = \frac{225}{16} \rightarrow z = \underline{\frac{15}{4} \text{ (or } 3\frac{3}{4}\text{, 3.75)}}$