

Round 2 Algebra 1

Problem Solving (Word Problems)

GREATER BOSTON MATHEMATICS LEAGUE

MEET 1 – OCTOBER 1998

ROUND 2 – Algebra 1 – Word Problems

1. _____
2. _____
3. _____

CALCULATORS ARE NOT ALLOWED ON THIS ROUND

1. The ratio of three natural numbers is 3:5:12. Three times the smallest number increased by twice the largest number is twenty-one more than six times the remaining number. Find the sum of these three natural numbers.
2. Manuel's father's age is now two years more than four times Manuel's age. In six years Manuel will be half of what his father's age was when Manuel was born. How old is Manuel now?
3. Three trains leave from the same location thirty minutes apart going along the exact same straight track. The first train to leave is travelling at a speed eight kilometers per hour slower than the second train. The third train's speed is two kilometers per hour faster than the first train. Three hours after the first train has left, all trains are still travelling along the track. If the distance then between the second and third trains is three times the distance between the first and second trains, what is the speed of the first train in kilometers per hour?

GREATER BOSTON MATHEMATICS LEAGUE

MEET 1 – SEPTEMBER 1999

ROUND 2 – Algebra 1 – Word Problems

1. _____
2. _____
3. _____

CALCULATORS ARE NOT ALLOWED ON THIS ROUND

1. A motorist and a bicyclist set out in opposite directions from the same location, the motorist leaving at 8:00 AM, the bicyclist at 8:40 AM. The motorist is travelling twice as fast as the bicyclist and at 12:20 PM on that same day they are 296 kilometers apart. Compute the speed of the motorist in kilometers per hour.
2. A 16 ounce can of nuts contain 10 ounces of peanuts costing \$.08 per ounce and the rest cashews costing \$.40 per ounce. If a certain amount of peanuts are replaced with cashews, and the price of the nuts in the can increases 25%, compute the number of ounces of peanuts that are now in the can.
3. Three positive numbers add to 30 with one twice another. If twice the sum of the two smallest is nine more than the largest, compute the smallest of the three numbers.

GREATER BOSTON MATHEMATICS LEAGUE

MEET 1 – SEPTEMBER 2000

ROUND 2 – Algebra 1 – Word Problems

1. _____

2. _____

3. _____ %

CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

1. A car and a bicycle set out in the same direction, from the same location on the same morning, the bicyclist leaving at 9:00 AM, the motorist at 10:24 AM. The car is averaging 12 MPH faster than the bicycle and at 12:30 PM on that same day the car has overtaken the bicycle. How many miles has each of the vehicles traveled when they meet?
2. An inheritance was divided among three heirs in the ratio of 4:3:2. If the recipient with the largest share gave \$1000 to each of the other heirs, then her amount would be \$24,000 less than twice the sum of the other two. What is the total dollar value in the inheritance?
3. Brand A sells cans of mixed nuts advertising 28% cashews, 16% walnuts, and the rest peanuts. Brand B sells its cans of mixed nuts advertising 25% more cashews and 12.5% more walnuts than brand A, and the rest peanuts. If the ratio of the costs per pound of cashews to walnuts to peanuts is 4:2:1 and a can of brand B holds the same weight in nuts as a can of brand A, what percent higher in price should a can of brand B's mixed nuts cost than a can of brand A's mixed nuts?

GREATER BOSTON MATHEMATICS LEAGUE

MEET 1 – OCTOBER 2001

ROUND 2 – Algebra 1 – Word Problems

1. _____

2. _____

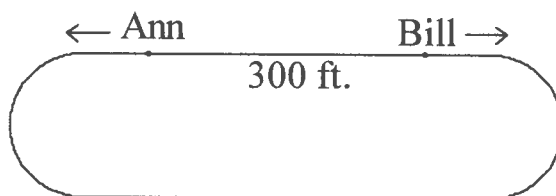
3. _____

CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

1. Kaitlin has three times as much money as her father. If she gives him \$250, she would then have \$20 less than twice his new amount. How many dollars did Kaitlin have originally?

2. A 20 liter acid solution is formed by mixing a certain amount of a 15% acid solution with an amount one-fifth more than that of a 25% acid solution and the rest a 30% acid solution. If this mixture is 25.8% acid, how many liters of the 15% acid are used in this solution?

3. Ann and Bill stand 300 feet apart on a track and run away from each other, in opposite directions. (See the diagram below.) They pass each other in 30 seconds. Ann completes one lap 60 seconds after they pass and Bill completes one lap 90 seconds after they pass. If Ann and Bill had a race one lap around this track, Ann would beat Bill by f feet. Solve for f .



GREATER BOSTON MATHEMATICS LEAGUE

MEET 1 – OCTOBER 2006

ROUND 2 – Algebra 1 – Word Problems

1. _____ mph

2. _____

3. \$ _____

CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

1. Al runs three miles at the rate of 5mph. He returns over the same route by ambulance. The entire trip took 41 minutes. What was the average speed of the ambulance in mph?
2. On an 18 foot seesaw, the fulcrum (balance point) is located at the 10' mark. Alice is seated on the 17' mark and Beth is seated on the 1' mark. If Alice weighs 156 lbs, Beth weighs 108 lbs and Mary weighs 40 lbs, at what mark should Mary be seated so that the seesaw will balance?
3. The ratio of nickels to dimes to quarters is 1:2:3. If the ratio of nickels to dimes to quarters for the same number of coins were 3:1:2, the value of the coins would be reduced by \$2. Originally, in dollars and cents, by how much did the value of the quarters exceed the value of the nickels?

GREATER BOSTON MATHEMATICS LEAGUE

MEET 1 – OCTOBER 2007

ROUND 2 – Algebra 1 – Word Problems

1. _____
2. _____
3. _____

CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

1. If the perimeter of a rectangle is 46 and its length is 1 less than three times its width, compute the length of a diagonal.

2. The ratio of the mother's age to the daughter's age is $7 : 2$, while the ratio of the father's age to the son's age is $10 : 3$. If each age is a natural number and the sum of all their ages is 101, determine the sum of the ages of the parents.

3. Find all two-digit natural numbers which equal six times the sum of its digits increased by 8.

GREATER BOSTON MATHEMATICS LEAGUE

MEET 1 – OCTOBER 2008

ROUND 2 – Algebra 1 – Word Problems

1. _____
2. _____
3. _____

CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

1. In a fraction, the numerator and denominator are relatively prime one-digit numbers and the denominator itself is not prime. The numerator is one more than one third of the denominator. What number added to this fraction produces a result of $\frac{4}{3}$?
2. Two unequal 2 - digit numbers have the same pair of digits. Three times the smaller reduced by $\frac{1}{5}$ of their positive difference equals the larger. What are the numbers?
3. A chemist has 5 gallons of a 40% sulfuric acid solution. He wanted to add pure sulfuric acid to make a 60% sulfuric acid solution but mistakenly added a gallon of water. Now, how many gallons of pure sulfuric acid must be added to get a 60% solution?

GREATER BOSTON MATHEMATICS LEAGUE

MEET 1 – OCTOBER 2009

ROUND 2 – Algebra 1 – Word Problems

1. _____

2. _____ feet

3. (_____ , _____)

CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

1. Find all 2-digit natural numbers with the property that each one is equal to 3 more than 5 times the sum of its digits.

2. An 18' see-saw has its fulcrum so that the lengths of its two sections are in the ratio 2:1. Larry weighs 60 lbs, Moe weighs 40 lbs and Curly weighs 48 lbs. If Larry sits on the end of the short side, while Moe sits on the end of the long side, how far from Larry should Curly sit so that the see-saw balances.

3. The sum of the ages of Al and Jen is 35 years and 0 months. They have the same birthday (Month/Day), but Al is older than Jen. Al's age in 15 years will be 5 times Jen's age 5 years ago. In how many years and months will the ratio of Al's age to Jen's age be 10 : 7? Express your answer as an ordered pair of the form (years, months).

GREATER BOSTON MATHEMATICS LEAGUE

MEET 1 – OCTOBER 2010

ROUND 2 – Algebra 1 – Word Problems

1. _____ by _____ cents
2. _____
3. _____ years

CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

1. Bill had 40 coins consisting of nickels, dimes and quarters. He had three times as many dimes as nickels. The value of all the coins was \$6.10. A magician transforms his original nickels into dimes, his original dimes into quarters, and his original quarters into nickels. Did the value of the coins become greater ($>$) or less ($<$), and by how much?
2. Dick had 60 quarts of an alcohol and water mixture which was labeled 25% alcohol. He wanted to increase the % of alcohol to 40% by adding k quarts of a mixture that was labeled 70% alcohol. Instead, he erroneously added k quarts of pure alcohol and the alcohol content increased to $(40 + x)\%$ due to his error. Compute x , the percent by which he missed his target.
3. The sum of the ages of Holly's parents is 10 times Holly's age now. The sum of her parent's ages when Holly was born is the square of her age now. Her mother is 2 years older than her father now. How old is Holly's mother now?

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MASSACHUSETTS MATHEMATICS LEAGUE

FEBRUARY 2004

ROUND 4: WORD PROBLEMS

ANSWERS

A) _____

B) _____

C) _____

A) What is the larger of the two numbers whose sum is ten, if the sum of their reciprocals is $\frac{8}{15}$.

B) An elevator went from the bottom of a tower to the top at a speed of 4 meters/second. It remained at the top for ninety seconds, and then returned to the bottom at a speed of 5 m/sec. If the total trip took 45 minutes, how high is the tower?

C) The sum of the squares of three ^{consecutive} ~~positive~~ odd integers is 967 more than the sum of the squares of the two even integers between them. Calculate the sum of the five consecutive integers.

**MASSACHUSETTS MATHEMATICS LEAGUE
FEBRUARY 2005
ROUND 4 ALGEBRA ONE: WORD PROBLEMS**

ANSWERS

A) _____gallons

B) _____

C) _____hour

- A) How many gallons of pure water must be added to a gallon of alcohol 75% pure to make a mixture only 15% pure?
- B) I am half as old as my mother was when my brother was twelve years younger than I am now. My brother was born when my mother was 26. If the sum of my brother's and my own current ages is 36, how old was my mother when I was born?
- C) I paddled my canoe upstream for 6 hours. I then rested for an hour as my canoe drifted downstream, then paddled back downstream to my starting point in just 2 hours. How much earlier would I have gotten back if I had headed back immediately rather than resting for that hour? Express your answer as a reduced fraction of an hour.

MASSACHUSETTS MATHEMATICS LEAGUE
FEBRUARY 2006
ROUND 4 ALGEBRA ONE: WORD PROBLEMS
ANSWERS

A) _____

B) _____ mins

C) _____

- A) I have a mixture of quarters and Sacagawea dollar coins the same size. I gave \$16.50 worth of these coins to a teller, but he mistakenly counted 3 of the quarters as dollars. If he credited me X cents for these coins and the minimum number of coins I could have given the teller is K , find the numeric value of $X + K$.
- B) I jog at 12 feet per second and my little sister jogs at a constant slower rate. If we run in opposite directions on a quarter mile track, we pass each other every minute. If we run in the same direction, how many minutes will it take me to lap her? (Recall: 1 mile = 5280 feet!)
- C) A chemist adds 20 liters of an alcohol and water solution that is 30% alcohol to 10 liters of an original solution of alcohol and water. He finds the percentage of alcohol in the resulting mixture is 6 percentage points higher than in the original solution.
What was the percentage of alcohol of the original solution?

**MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 – FEBRUARY 2007
ROUND 4 ALG 1: WORD PROBLEMS**

ANSWERS

A) _____

B) _____

C) _____

- A) Express Scripts uses three types of bottles to ship pills to their customers. Bottle A holds 12 fewer pills than bottle B . Bottle C holds twice as many pills as bottle A . If 2 A bottles, 4 B bottles and 3 C bottles hold 240 pills. How many pills does an A bottle hold?
- B) The first book in a trilogy by Anne McCaffrey contains 67 more pages of text than the second, while the third contains 24 fewer pages of text than the second. The total number of pages of text in the trilogy is 751 pages. If all the chapters in the first book contain exactly P pages of text, where $P > 1$, what is the maximum number of chapters possible in the first book?
- C) Of the 585 girls at Northeast Statistics High School last year, 170 played fall sports, 165 played winter sports and 150 played spring sports. Twenty two girls played sports in all three seasons, while 80 played only in the fall, 84 played only in the winter, and 71 played only in the spring. How many girls did not play sports in any of the three seasons?

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 – FEBRUARY 2008
ROUND 4 ALG 1: WORD PROBLEMS

ANSWERS

- A) _____
- B) _____ %
- C) _____ oz.

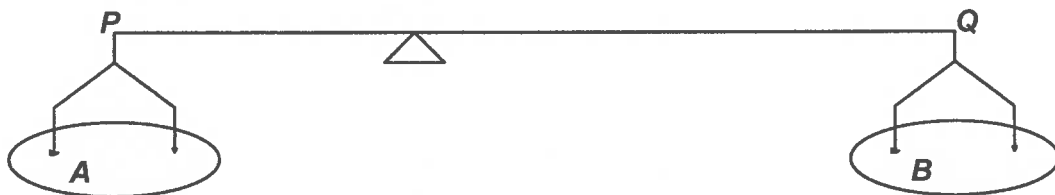
- A) 35 students registered for enrichment courses in a summer program at Brewster Academy. Here is registration information for the science courses.

<u>Course</u>	<u># students registered</u>
Biology	19
Chemistry	17
Physics	11
Chemistry and Physics	5
Biology and Physics	7
Biology and Chemistry	12
All 3 courses	2

How many of the 35 students did not register for any of these science courses?

- B) Three men work at equal rates work 8 hours per day. If they complete 80% of a job in four 8-hour days, what percent of the job would be completed by one of these men in 6 hours?

- C) A supply of cylinders of equal but unknown mass are available and used as follows:
8 cylinders in pan *A* balance a single 9 oz. cube in pan *B*.
A single 25 oz cube in pan *A* balances 2 cylinders in pan *B*.
How much (in ounces) does each cylinder weigh?
Note that the fulcrum is not at the midpoint of \overline{PQ} .



GBML 1998

ROUND 2

1. Call the three numbers $3x$, $5x$, and $12x$. Equation: $9x + 24x = 30x + 21 \Rightarrow x = 7$;
Sum of the numbers $= 20x = 140$

2. Manuel's age $= x$; Manuel's father's age $= 4x + 2$
Manuel's age in 6 years $= x + 6$; Manuel's father's age when Manuel was born $= 3x + 2$
Equation: $2(x + 6) = 3x + 2 \Rightarrow x = 10$

	rate	time	distance
1st train	$x - 8$	3	$3x - 24$
2nd train	x	2.5	$2.5x$
3rd train	$x - 6$	2	$2x - 12$

Equation:
 $2.5x - 2x + 12 = 3(3x - 24 - 2.5x)$
 $\Rightarrow x = 84 \Rightarrow x - 8 = 76$

ROUND 2

1. Let $x =$ speed of the bicyclist and $2x =$ speed of the motorist. Time traveled by motorist $= 13/3$ hours and time travelled by the bicyclist $= 11/3$ hours. Equation is:
 $\frac{13}{3}2x + \frac{11}{3}x = 296 \Rightarrow \frac{37}{3}x = 296 \Rightarrow x = 24$ and $2x = 48$ kph.

2. $10 \times 0.8 + 6 \times 40 = 3.20$; $3.20 \times 1.25 = 4.00$ (new cost of the nuts)
equation: $.08(10 - x) + .40(6 + x) = 4 \Rightarrow .32x = .80 \Rightarrow x = 2.5 \Rightarrow 7.5$ oz of peanuts now

3. The three numbers are x , $2x$, and $30 - 3x$

Case I: $2x$ the largest: equation is $2(x + 30 - 3x) = 2x + 9 \Rightarrow 60 - 4x = 2x + 9 \Rightarrow x = \frac{17}{2}$
and $30 - 3x = \frac{17}{2} = \frac{9}{2}$

Case II: $30 - 3x$ the largest: equation is $2(x + 2x) = 30 - 3x + 9 \Rightarrow 9x = 39 \Rightarrow x = \frac{13}{3}$

GBML 2000

ROUND 2

1. Let $x =$ speed of the bicycle $\rightarrow x + 12 =$ speed of the car:
 $3.5x = 2.1(x + 12) \rightarrow 1.4x = 2.1 \cdot 12 \rightarrow x = 18 \rightarrow$ distance traveled $= 63$ miles.

2. Let the three amounts be $4x$, $3x$, and $2x$:
 $4x - 2000 = 2(5x + 2000) - 24000 \rightarrow 4x - 2000 = 10x - 20000$
 $\rightarrow 6x = 18000 \rightarrow 9x = 27000$

3. Let $x =$ weight of the nuts in brand A's can $\rightarrow x =$ weight of the nuts in brand B's can.
Let $c =$ cost per pound of peanuts $\rightarrow 2c =$ cost per pound of walnuts and $4c =$ cost per pound of cashews.

cost of nuts in brand A's can: $(.28x)(4c) + (.16x)(2c) + (.56x)(c) = 2xc$
cost of nuts in brand B's can: $(.35x)(4c) + (.18x)(2c) + (.47x)(c) = 2.23xc$
percent increase $= \frac{2.23 - 2}{2} = 0.115 = 11.5\%$

GBML 2001

ROUND 2

1. Let $x =$ Katlin's father's amount:
 $3x - 250 = 2(x + 250) - 20 \Rightarrow x = 730 \Rightarrow 3x = 2190$

2. Let $x =$ amount of the 15% acid solution:
 $.15x + .25\left(\frac{6}{5}x\right) + .30\left(20 - \frac{11}{5}x\right) = 258(20) \Rightarrow .15x + .30x + 6 - .66x = 5.16 \Rightarrow$
 $.21x = .84 \Rightarrow x = 4$

3. Let $x =$ number of feet in one lap $\Rightarrow \frac{x}{90} =$ Ann's speed and $\frac{x}{120} =$ Bill's speed:
 $300 + 30\left(\frac{x}{90}\right) + 30\left(\frac{x}{120}\right) = x \Rightarrow 300 + \frac{7}{12}x = x \Rightarrow 300 = \frac{5}{12}x \Rightarrow x = 720$.
Since Ann beats Bill by 30 seconds, the distance $= \frac{30}{120}(720) = 180$ feet.

GBML 2006

ROUND 2

- Let r = the rate of the ambulance (in mph).
Time = Distance/Rate $\rightarrow 3/5 + 3/r = 41/60$ (hours) $\rightarrow 36r + 180 = 41r \rightarrow 5r = 180 \rightarrow r = \underline{36}$

	B108lbs	M40lbs	A166lbs
1	10	17	

To be in balance (or equilibrium), the clockwise torque (or tendency to turn) must equal the counterclockwise torque. Thus, Beth creates $9(108) = 972$ foot-pounds of CCW torque and Alice creates $7(156) = 1092$ foot-pounds of CW torque. Mary must be seated to the left of the fulcrum, say at a distance of x feet and $40x = 1192 - 972 = 120 \rightarrow x = 3$. Therefore, Mary must be seated at the 7' marker.

- Originally, $N : D : Q = x : 2x : 3x$ (with a value in ¢ of $5x + 20x + 75x = 100x$)
The changed ratio, with the same number of coins, implies $N : D : Q = 3x : x : 2x$ (with a new value in ¢ of $15x + 10x + 50x = 75x$) Thus, $100x = 75x + 200 \rightarrow x = 8$ and the original 8 quarters were worth $40¢$ and the original 24 quarters were worth $\$6$ producing a difference of $\underline{\$5.60}$

GBML 2007

ROUND 2

- Per = $2(L + W) = 46$ and $L = 3W - 1 \rightarrow L + W = 23 \rightarrow 3W - 1 + W = 23 \rightarrow W = 6$ and $L = 17$
Let D denote the diagonal. Then $D^2 = 6^2 + 17^2 = 325 = 25(13) \rightarrow D = \underline{5\sqrt{13}}$

- To maintain the given ratios, we use scale factors of x and y :

$$\begin{cases} M : 7x \\ D : 2x \rightarrow 9x + 13y = 101 \rightarrow x = \frac{101 - 13y}{9} = \frac{(99 - 9y) + (2 - 4y)}{9} = 11 - y + \frac{2 - 4y}{9} \\ F : 10y \\ S : 3y \end{cases}$$

- Let x and y denote the tens and units digits of the two digit number. Thus, $0 \leq x, y \leq 9$.
 $10x + y = 6(x + y) + 8 \rightarrow 4x - 5y = 8 \rightarrow x = \frac{5y + 8}{4} = \frac{4y + 8}{4} + \frac{y}{4} = y + 2 + \frac{y}{4}$
Thus, y must be a multiple of 4. $y = 0 \rightarrow x = 2 \rightarrow \underline{20}$ (Check: $20 = 6 \cdot 2 + 8$)
 $y = 4 \rightarrow x = 7 \rightarrow \underline{74}$ (Check: $74 = 6 \cdot 11 + 8$)
 $y = 8 \rightarrow x = 12$ rejected

GBML 2008

ROUND 2

- $\frac{4}{9} + x = \frac{4}{3} \rightarrow x = \frac{8}{9}$
- $3(10x + y) - \frac{1}{5}(9y - 9x) = 10y + x \rightarrow x = \frac{2y}{7} \rightarrow (x, y) = (2, 7) \rightarrow$ #s are: $\underline{22}$ and $\underline{72}$
- $4(5) + x = 0.60(6 + x) \rightarrow 2 + x = 3.6 + 6x \rightarrow 4x = 1.6 \rightarrow x = \underline{4}$

GBML 2009

ROUND 2

- Let $N = 10T + U$. $10T + U = 3 + 5(T + U) \rightarrow T = \frac{4U + 3}{5} \rightarrow U = 3, 8 \rightarrow N = \underline{33, 78}$

- The short and long sides of the see-saw are 6 and 12 feet respectively.
The torque (tendency to turn) is the product of weight and the distance from the fulcrum (pivot point). To be in equilibrium (that is, to balance), the clockwise torque must equal the counterclockwise torque. Let x = the distance (in feet) between Larry and Curly.



$$\text{Then: } 60(6) + 48(6 - x) = 40(12) \rightarrow 360 + 288 - 48x = 480 \rightarrow x = 168/48 = 21/6 = \underline{3.5} \text{ feet}$$

- Let (x, y) denote the ages (in years) of (Al, Jen). Then:
 $x + y = 35 \rightarrow (35 - y) + 15 = 5y - 25 \rightarrow 6y = 75 \rightarrow (x, y) = (22.5, 12.5)$
Now in T years, $\frac{22.5 + T}{12.5 + T} = \frac{10}{7} \rightarrow 7(45 + 2T) = 10(25 + 2T)$
 $\rightarrow 6T = 65 \rightarrow T = 10 \frac{5}{6} \rightarrow \underline{10.10}$

GBML 2010

ROUND 2

- $D \ 3x \rightarrow 5x + 10(3x) + 25(40 - 4x) = 610 \rightarrow 1000 - 65x = 610 \rightarrow 65x = 390 \rightarrow x = 6$
 $\underline{Q} \ 40 - 4x$
After magician's trick, we have 6D, 18Q and 16N, worth $60 + 450 + 80 = \$5.90$
which is less by 20 cents.

- Let n denote the number of quarts of 70% mixture which should have been added.
 $0.25(60) + 0.70n = 0.40(60 + n) \rightarrow 150 + 7n = 240 + 4n \rightarrow n = 30$

$$\text{Actual alcohol/water mixture: } \frac{15 + 30}{90} = \frac{1}{2} = 50\% \rightarrow x = \underline{10}$$

- Let h, m and d denote Holly, mom and dad's current ages respectively. Then

$$\begin{cases} (1) \ h + d = 10x \\ (2) \ (h - x) + (d - x) = x^2 \text{ Substituting } (d + 2) \text{ in } (1), \ d + 1 = 5x. \\ (3) \ h = d + 2 \end{cases}$$
Combining like terms in (2), $(h + d) - 2x = x^2 \rightarrow 10x - 2x = x^2 \rightarrow x^2 - 8x = x(x - 8) = 0$
 $x \neq 0 \rightarrow x = 8 \rightarrow d = 39 \rightarrow h = \underline{41}$.

MMML 2/04

A) What is the larger of the two numbers whose sum is ten, if the sum of their reciprocals is $8/15$.

$$x, 10-x \quad \frac{1}{x} + \frac{1}{10-x} = \frac{8}{15}$$

$$15(10-x) + 15x = 8(10x - x^2)$$

$$150 = 80x - 8x^2, \quad 8x^2 - 80x + 150 = 0$$

$$4x^2 - 40x + 75 = 0 \quad (2x-5)(2x-15) = 0 \quad x = \frac{5}{2}, \frac{15}{2}$$

B) An elevator went from the bottom of a tower to the top at a speed of 4 meters/second. It remained at the top for ninety seconds, and then returned to the bottom at a speed of 5 m/sec. If the total trip took 4.5 minutes, how high is the tower?

$$x = 60 \quad \frac{x}{4} + 90 + \frac{x}{5} = 270, \quad \frac{x}{4} + \frac{x}{5} = 180$$

$$5x + 4x = 180 \cdot 20$$

$$\frac{9x}{9} = \frac{180 \cdot 20}{9}, \quad x = 20 \cdot 20 = 400$$

C) The sum of the squares of three positive odd integers is 967 more than the sum of the squares of the two even integers between them. Calculate the sum of the five consecutive integers.

$$x, x+2, x+4$$

$$x^2 + (x+2)^2 + (x+4)^2 = 967 + (x+1)^2 + (x+3)^2$$

$$x^2 + 4x + 4 + 8x + 16 = 967 + 2x + 1 + 6x + 9$$

$$x^2 + 12x + 20 = 8x + 977, \quad x^2 + 4x - 957 = 0$$

$$(x+33)(x-29) = 0, \quad x = 29, \quad x+1 = 30, \quad x+2 = 31, \quad x+3 = 32, \quad x+4 = 33$$

$$OE \quad x = -33 \rightarrow -155$$

$$145 + 155$$

MMML 2/05

Round Four:

A. The ratio of alcohol is $0.15 = \frac{0.75}{1+n}$ solving yields $n = 4$.

B. I am x now. "Then" my mother was $2x$ and my brother $x-12$ so $2x = x - 12 + 26$ so $x = 14$ any my brother is 22. My mother was 8 yrs older when I was born or 34.

C. Let $t =$ paddling speed in still water, $s =$ speed of current, $6(t-s) = s + 24(t+s)$ so $s = (4/9)t$ and upstream rate is $(5/9)t$, downstream $(13/9)t$. Immediate return would have taken $\frac{Dist}{time} = \frac{6(t-s)}{t+s} = \frac{6(5/9)t}{(13/9)t} = \frac{30}{13}$ instead of the three hrs it took (1 resting).

MMML 2/06

Round Four:

A. The absolute minimum number of coins would be 16 dollars and 2 quarters, but since there must be at least 3 quarters, we have 15 dollars and 6 quarters $\rightarrow K = 21$.

B. The teller's mistake credited my account $3(75) = 225$ extra cents $\rightarrow X = 1875$. In one minute $12(60) + x(60) = 1320$ so sister jogs at 10 ft/sec. In same direction I gain 2 ft/sec or 120 ft/minute. $1320/120 = 11$.

C. Original mix was $n/100$. $\frac{n+6}{100} = \frac{(n/100)10 + 30(20)}{30}$ Solving, $n = 21$.

MMML 2/07

Round 4

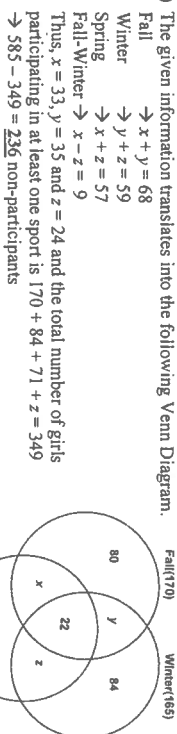
A) Let B hold x pills. Then A holds $x - 12$ and C holds $2(x - 12)$

$$24 + 4B + 3C = 240 \rightarrow 2x - 24 + 4x + 6x - 72 = 240 \rightarrow 12x = 336 \rightarrow x = 28 \rightarrow A = 16$$

B) Let $x =$ # pages of text in book 2. Then $1^{\text{st}} = x + 67$ and $3^{\text{rd}} = x - 24$

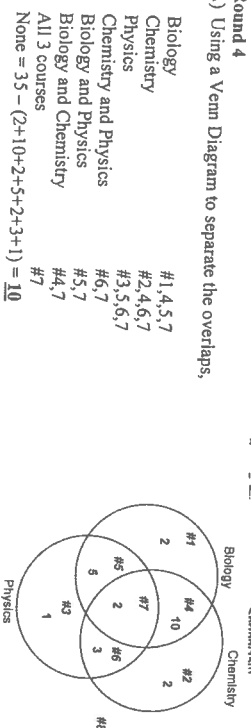
$3x + 43 = 751 \rightarrow x = 236 \rightarrow 1^{\text{st}}$ book has 303 pages. $303 = 3(101)$ Since both these factors are prime, the first book either has 3 chapters of 101 pages each or 101 chapters of 3 pages each. Thus, the maximum number of chapters is 101.

C) The given information translates into the following Venn Diagram.



Round 4

A) Using a Venn Diagram to separate the overlaps,



B) Let x denote the percent of the job completed. Then $\frac{3-4}{80} = \frac{1-\frac{3}{4}}{x} \rightarrow 12x = 60 \rightarrow x = 5$

C) Let z denote the mass of each cylinder and (x, y) the distances of pans A and B from the balance point respectively.

Equating the clockwise and counterclockwise torques keeps the system in equilibrium.

$$\text{Thus, } 8xz = 9y \rightarrow y = \frac{8xz}{9} \text{ and } 25x = 2zy \rightarrow y = \frac{25x}{2z}$$

Equating and canceling the x 's in the numerator (since $x \neq 0$)

$$\rightarrow \frac{8z}{9} = \frac{25}{2z} \rightarrow z^2 = \frac{225}{16} \rightarrow z = \frac{15}{4} \text{ (or } 3\frac{3}{4}, 3.75)$$