Team Round

GREATER BOSTON MATHEMATICS LEAGUE

MEET 1 – OCTOBER 1998

TEAM ROUND

3 pts. 1. ________________

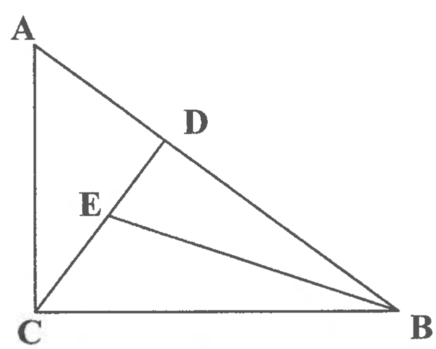
3 pts. 2. ________________

4 pts. 3. ________________

SAT APPROVED CALCULATORS ALLOWED ON THIS ROUND

1. Given that the division, $2x5y_6 \div 15_6$, in base 6 produces a whole number result, find all possible ordered pairs (x,y).

2. Given AC = 3, BC = 4, $\angle$ ACB is right, $\angle$ ADC is right, and $\overline{BE}$ bisects $\angle$ ABC, find the distance from E to $\overline{BC}$.



3. Find the last two digits in $2^{1998} + 3^{1998}$.

GREATER BOSTON MATHEMATICS LEAGUE

MEET 1 – SEPTEMBER 1999

TEAM ROUND

5 pts. 1. ____________________

5 pts. 2. ____________________

SAT APPROVED CALCULATORS ALLOWED ON THIS ROUND
except for the **TI-89 Calculator**, which is not allowed on the Team Round

1. How many 3-digit whole numbers have exactly 10 positive integral factors?

2. Given θ is an acute angle of a right triangle whose sides have whole number lengths and $\csc\theta = x + 0.05$, where x is a whole number, compute $\sec\theta$. Put the result(s) in the form $\frac{a}{b}$ where a and b are relatively prime whole numbers.

GREATER BOSTON MATHEMATICS LEAGUE

MEET 1 – SEPTEMBER 2000

TEAM ROUND

3 pts. 1. ____________________

3 pts. 2. ____________________

4 pts. 3. ____________________

SAT APPROVED CALCULATORS ARE ALLOWED IN THE TEAM ROUND, EXCEPT FOR CALCULATORS WITH SYMBOLIC MANIPULATION PROGRAMS, (FOR EXAMPLE THE TI-89) WHICH ARE NOT ALLOWED IN THIS ROUND.

1. The greatest common factor of three distinct natural numbers 21, x, and y is 21, and their least common multiple is 462. How many ordered pairs (x, y), $x < y$, satisfy these conditions?

2. Given the polynomial in x and y, $4x^4 + ky^4$, is factorable over the rationals, write the factored form for this polynomial for the minimum value for k, where k is an odd integer greater than 1.

3. Right triangle ABC has legs $\overline{AC}$ and $\overline{BC}$. If AC = 105 and the lengths of $\overline{AB}$ and $\overline{BC}$ are whole numbers, find the smallest possible value for $\sin(\angle A)$ as a rational number in reduced form.

GREATER BOSTON MATHEMATICS LEAGUE

MEET 1 – OCTOBER 2001

TEAM ROUND **(12 MINUTES LONG)**

3 pts. 1. ________________

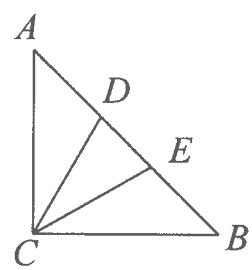
3 pts. 2. $(\_\_\_\_,\_\_\_\_)$

4 pts. 3. ________________

SAT APPROVED CALCULATORS ARE ALLOWED IN THE TEAM ROUND, EXCEPT FOR CALCULATORS WITH SYMBOLIC MANIPULATION PROGRAMS, (FOR EXAMPLE THE TI-89) WHICH ARE NOT ALLOWED IN THIS ROUND.

1. Given n is a positive integer such that $2^n + 1$ is relatively prime with 15. Consider the set of all such n. Find the sum of the first 1000 elements in this set.

2. Given ΔABC is right isosceles with $\angle ACB$ right, $\overline{CD}$ and $\overline{CE}$ trisect $\angle ACB$, and $\dfrac{DE}{BC} = \sqrt{r} - \sqrt{s}$, find the ordered pair (r,s).



3. Find the area bounded by $9x^2 - 9xy + 9x + 2y^2 = 18$ and the positive coordinate axes.

GREATER BOSTON MATHEMATICS LEAGUE

MEET 1 – OCTOBER 2006

TEAM ROUND

3 pts.	1.	______________________
3 pts.	2.	______________________
4 pts.	3.	______________________

SAT APPROVED CALCULATORS ARE ALLOWED ON THIS ROUND.

1. Farmer MacDonald brought 21 chickens and 15 turkeys to market. He sold some of each of them to the local butcher for a total of \$84. If he received \$2.25 for each chicken and \$6 for each turkey, determine the maximum number of animals that were <u>not</u> sold?

2. Dick's stamp collection consists of 3 books. One-eighth of his stamps are in the first book, <u>several</u> sevenths are in the second book and there are 2006 stamps in the third book. Find all possible solutions for the number of stamps in Dick's entire collection?

3. Two natural numbers are simultaneously chosen at random from a box containing the natural numbers 2, 3, 4, …, 29, 30. In how many ways can the two numbers be selected from the box so that their product has exactly four factors? The order in which the two numbers are selected is irrelevant.

GREATER BOSTON MATHEMATICS LEAGUE

MEET 1 – OCTOBER 2007

TEAM ROUND

3 pts. 1. ______________________________

3 pts. 2. ______________________________

4 pts. 3. ______________________________

SAT APPROVED CALCULATORS ARE ALLOWED ON THIS ROUND.

1. The sum of four positive numbers is 100. If you increase the first by three, decrease the second by three, multiply the third by three, and divide the fourth by three, the results in each case are the same. Find the largest of the original numbers.

2. $2x^2 - 2Ax + A^2 - B^2 = 0$ has integer roots $r_1 > 0$ and $r_2 < 0$.
If $A : B = 1 : 5$, find the ordered pair (A, B) such that $A + B$ is a minimum and both r_1 and r_2 are two-digit integers.

3. The numbers ABC_8 and ABA_9 are three-digit numbers and BB_{15} is a two-digit number.
Find all possible ordered triples (A, B, C) which satisfy the following equation:
$ABC_8 = ABA_9 - BB_{15}$
Note: The left-most digit of each numeral must be non-zero.

GREATER BOSTON MATHEMATICS LEAGUE

MEET 1 – OTOBER 2008

TEAM ROUND

3 pts.	1.	______________________
3 pts.	2.	______________________
4 pts.	3.	______________________

SAT APPROVED CALCULATORS ARE ALLOWED ON THIS ROUND.

1. Find all values of $x \in \Re$, for which the following statement is true:

$$9^x \cdot 2^{2x} + 8^{\frac{1}{3}} \cdot 9^{\frac{1}{2}} = 21 \cdot 3^{x-1} \cdot 4^{\frac{x}{2}}$$

2. Find the four smallest natural numbers N greater than 40 for which the product of the proper factors equals N.

   Note: All proper factors of N are less than N.

3. Find all ordered pairs (x, y) which satisfy the following system of equations.

   $2x+3y=30-\sqrt{2x+3y}$ and $\sqrt{2x-3y}=12-2x+3y$

GREATER BOSTON MATHEMATICS LEAGUE

MEET 1 – OCTOBER 2009

TEAM ROUND

3 pts. 1. ____________________

3 pts. 2. ____________________

4 pts. 3. ____________________

CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

1. Find all values of $x \in \mathbb{R}$ that satisfy the following inequality:

$$\frac{x^4 - 2x^3 - 7x^2 + 20x - 12}{x^3 + 5x} \le 0$$

2. How many three digit numbers that contain only the digits 0, 1, 2, 4, 5, 6, 7, or 9 without repetition are divisible by 11 and 4, but not by 3.

3. w, y, and z are integers. The following system $\left\{\begin{matrix} 6w + 7y - 9z = 0 \\ 7w + 10y - 13z = 0 \end{matrix}\right\}$ has many ordered triples satisfying it. Find the ordered triple (w, y, z) for which both y and z are the smallest possible positive integers.

GREATER BOSTON MATHEMATICS LEAGUE

MEET 1 – OCTOBER 2010

TEAM ROUND

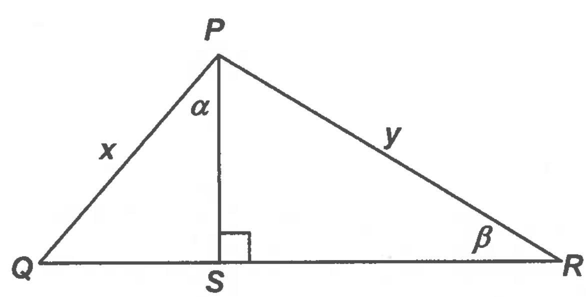
3 pts.	1.	______________________
3 pts.	2.	$x =$ ____________________
4 pts.	3.	__________ : __________

CALCULATORS ARE <u>NOT</u> ALLOWED ON THIS ROUND.

1. How many positive 4-digit multiples of 4 contain <u>none</u> of the digits 6, 7, 8, 9 and 0?

2. Find the value of x in terms of y that satisfies $\dfrac{2x^2-3y^2+2x-3y-xy}{2x^2-12y^2+5xy}=2$.

3. Given the diagram at the right and
$$\begin{cases} \tan\alpha+\cot\beta=0.975 \\ \cot\beta-\tan\alpha=0.525 \end{cases}$$
Find the ratio $\dfrac{x}{y}$ as two relatively prime positive integers.

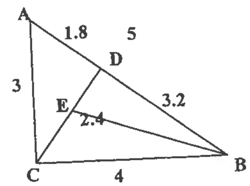


GBML 1998

TEAM ROUND

1. $2x5y_6 + 15_6 = (2 \cdot 216 + 36x + 5 \cdot 6 + y) \div (1 \cdot 6 + 5) = (462 + 36x + y) \div 11$
If this is a whole number and since 462 is divisible by 11, then $36x + y$ is divisible by 11 $\Rightarrow 3x + y$ is divisible by 11; x and y are integers from 0 to 5; If $x = 0 \Rightarrow y = 0$. If $x = 1$, $y = 8$, impossible. If $x = 2$, $y = 5$. If $x = 3$, $y = 2$. If $x = 4$, $y = 10$, impossible. If $x = 5$, $y = 7$, impossible. $\Rightarrow$ **(0,0) (2,5) and (3,2)** are the solutions.

2. The distance from E to $\overline{BC}$ = DE
The ratio of DE: EC = 4:5 $\Rightarrow$
$DE = \frac{4}{9}(2.4) = \frac{4}{9} \cdot \frac{12}{5} = \frac{16}{15} \approx 1.0667$



3. Using a calculator to generate powers of 2 and 3, you observe that 2^{22} ends in 04 and 3^{22} ends in 09 $\Rightarrow$ The last 2 digits of the powers of 2 and 3 repeat every 20 times; 1998 = 18 mod 20 ; 2^{18} ends in 44 and 3^{18} ends in 89; 44 + 89 = 133 $\Rightarrow$ **33** is the answer.

GBML 1999

TEAM ROUND

1. Any whole number with 10 factors is of the form p^9 or p^4q where p and q are primes
$2^9 = 512$, $3^9 > 1000$ and does not qualify; $2^4 = 16$, q = 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61; $3^4 = 81$, $q = 2$, 5,7, 11; $5^4 = 625$ and so does not qualify.
Therefore there are **20** possibilities

2. .05 = 1/20 $\Rightarrow$ one leg has length 20 $\Rightarrow c^2 - a^2 = 20^2 \Rightarrow (c+a)(c-a) = 20^2$; Now consider all possible sets of simultaneous equations where $c + a$ and $c - a$ are even factor of 20^2
$c + a = 200$, $c - a = 2 \Rightarrow c = 101$ and $a = 99$, which is the only one where c ends in a 1 $\Rightarrow$
$\sec\theta = \mathbf{\frac{101}{99}}$ [Note c is of the form $20x + 1 \Rightarrow c$ has a units digit equal to 1.]

GBML 2000

TEAM ROUND

1. $462 \div 21 = 22$; $x = 21a$, $y = 21b$, $\text{LCM}(a,b) = 22$;
$a \neq 1: (a,b) = (2,11), (2,22), (11,22)$; therefore the number of ordered pairs is 3.

2. In order for $4x^4 + ky^4$ to be factorable, when you complete its square the term being added and subtracted must itself be a perfect square, which is $2(2x^2)(\sqrt{k}\,y^2) \Rightarrow \sqrt{k}$ is a perfect square. Since k is odd and greater than 1, then the smallest value for k is 81.
$4x^4 + 81y^4 = 4x^4 + 36x^2y^2 + 81y^4 - 36x^2y^2 = (2x^2 + 9y^2)^2 - (6xy)^2 =$
$(2x^2 + 6xy + 9y^2)(2x^2 - 6xy + 9y^2)$

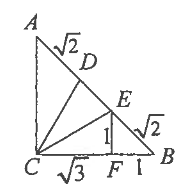
3. Let BC = a and AB = $c \rightarrow c^2 - a^2 = 105^2 = 3^2 \cdot 5^2 \cdot 7^2$; In order for $\sin(\angle A)$ to be as small as possible, $c + a$ and $c - a$ must be factors of 105^2 and their difference must be non-zero, yet as small as possible; The largest factor of 105^2 less than 105 = 75;
$75 \times 147 = 105^2$; $\begin{cases} c + a = 147 \\ c - a = 75 \end{cases} \rightarrow c = 111,\ a = 36;\ \sin(\measuredangle A) = \frac{a}{c} = \frac{36}{111} = \frac{12}{37}$

GBML 2001

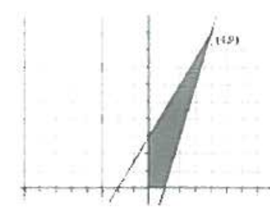
TEAM ROUND

1. $2^n + 1 \equiv (-1)^n + 1 \bmod 3 \Rightarrow$ all odd values for n are 0 mod3;
$2^{2n} + 1 = 4^n + 1 \equiv (-1)^n + 1 \bmod 5 \Rightarrow$ all even values for n that non-multiples of 4 are divisible by 5; the set is $\{4,8,12,16,\ldots\}$; the sum of the first 1000 elements in this set is $500 \times 4004 = 2002000$

2. Draw $\overline{EF} \perp \overline{BC}$; since a ratio is required, there is no loss of generality to let $EF = 1 \Rightarrow BE = AD = \sqrt{2}$ and $CF = \sqrt{3}$;
therefore $BC = \sqrt{3} + 1 \Rightarrow AB = \sqrt{2}(\sqrt{3} + 1) = \sqrt{6} + \sqrt{2} \Rightarrow$
$DE = \sqrt{6} - \sqrt{2} \Rightarrow \frac{DE}{BC} = \frac{\sqrt{6} - \sqrt{2}}{\sqrt{3} + 1} = \frac{(\sqrt{6} - \sqrt{2})(\sqrt{3} - 1)}{(\sqrt{3} + 1)(\sqrt{3} - 1)} =$
$\frac{4\sqrt{2} - 2\sqrt{6}}{2} = 2\sqrt{2} - \sqrt{6} = \sqrt{8} - \sqrt{6} \Rightarrow (r,s) = (8,6)$.



3. $9x^2 - 9xy + 9x + 2y^2 = 18 \Rightarrow 9x^2 - 9xy + 9x + 2y^2 - 18 = 0$
$\Rightarrow (3x - 2y + 6)(3x - y - 3) = 0 \Rightarrow 3x - 2y + 6 = 0$
and $3x - y - 3 = 0$; these two lines intersect at $(4,9)$;
the first line has intercepts $(0,3)$ and $(-2,0)$;
the second line has intercept $(1,0)$;
the shaded area $= \frac{1}{2} \cdot 3 \cdot 9 - \frac{1}{2} \cdot 2 \cdot 3 = \frac{21}{2}$



GBML 2006

TEAM ROUND

1.
Let C and T respectively denote the number of chickens and turkeys sold.
$(9/4)C + 6T = 84 \rightarrow 9C + 24T = 336 \rightarrow 3C + 8T = 112 \rightarrow C = (112 - 8T)/3$
Separating the numerator into parts that must be divisible by 3 and parts that are not necessarily divisible by 3, $C = (111 - 9T + 1 + T)/3 \rightarrow C = 37 - 3T + \frac{1+T}{3}$

If $T = 2, 5, 8, 11$ or 14, the last term is an integer.
The corresponding values of C are: 32, 24, 16, 8 and 0.
Thus, the possible transactions were $(C, T) = (8, 11)$ or $(16, 8)$, leaving either **17** (or 12) unsold.

2.
Let x denote the number of stamps in Dick's collection.
Let y denote the number of sevenths in book 2. ($y \geq 2$)

Then $\frac{x}{8} + \frac{xy}{7} + 2006 = x \rightarrow 7x + 8xy + 56(2006) = 56x \rightarrow 2^4 \cdot 7 \cdot 17 \cdot 59 = 49x - 8xy$

$\rightarrow x = \frac{2^4 \cdot 7 \cdot 17 \cdot 59}{49 - 8y}$ Clearly, the numerator is only divisible by the primes 2, 7, 17 and 59.

$(y, 49 - 8y) = (2, 33), (3, 25), (4, 17), (5, 9), (6, 1)$
$y = 4 \rightarrow x =$ **6,608** and $y = 6 \rightarrow x =$ **112,336**

3.
Since 4 can be factored only as $2 \cdot 2$ or $4 \cdot 1$ (implying exponents used in the prime factorization of the products we seek must be either $1 \cdot 1$ or $3 \cdot 0$), the integers with exactly 4 positive factors must either be the product of two distinct primes or a perfect cube of a prime.
The 10 primes in the box are: 2, 3, 5, 7, 11, 13, 17, 19, 23, and 29
Thus, there are ${}_{10}C_2 = 45$ possible pair products, plus the perfect cubes 8, 27 and 125 $\rightarrow$ **48**

GBML 2007

TEAM ROUND

1. $A + B + C + D = 100$ and $A + 3 = B - 3 = 3C = \frac{D}{3}$

$A + (A+6) + \frac{A+3}{3} + 3(A+3) = 100$

$\rightarrow A = 15\frac{3}{4}$ and $(B, C, D) = \left(21\frac{3}{4}, 6\frac{1}{4}, 56\frac{1}{4}\right)$

Thus, the largest number is $\mathbf{56\frac{1}{4}}$

2. The roots of $2x^2 - 2Ax + A^2 - B^2 = 0$ are $\frac{2A \pm \sqrt{4A^2 - 8(A^2 - B^2)}}{4} = \frac{A \pm \sqrt{2B^2 - A^2}}{2}$

Let $A = k$ and $B = 5k$. Then the roots simplify to $\frac{k \pm \sqrt{49k^2}}{2} = \frac{k \pm 7k}{2} = 4k, -3k$

Since both roots are two-digit integers, $k \geq 4 \rightarrow (A, B) =$ **(4, 20)**

3. Since A, B and C are digits in base 8, it follows that $0 \leq A, B, C \leq 7$.
However, since A is a leftmost digit, $A \neq 0$.
$ABC_8 = ABA_9 - BB_{15} \rightarrow 64A + 8B + C = 81A + 9B + A - 15B - B$
$\rightarrow C = 18A - 15B = 3(6A - 5B) \rightarrow$
$C = 0$ (and $6A = 5B$) $\rightarrow (A, B) = (0, 0)$ – rejected or $(5, 6)$
$C = 3$ (and $6A - 5B = 1$) $\rightarrow (A, B) = (1, 1)$ or $(6, 7)$
$C = 6$ (and $6A - 5B = 2$) $\rightarrow (A, B) = (2, 2)$ only
Thus, there are 4 ordered triples: **(5, 6, 0)**, **(1, 1, 3)**, **(6, 7, 3)** and **(2, 2, 6)**

Check:

(5, 6, 0)	$560_8 \stackrel{?}{=} 565_9 - 66_{15} \rightarrow$ (base 10) $368 = 464 - 96$ √
(1, 1, 3)	$113_8 \stackrel{?}{=} 111_9 - 11_{15} \rightarrow$ (base 10) $75 = 91 - 16$ √
(6, 7, 3)	$673_8 \stackrel{?}{=} 676_9 - 77_{15} \rightarrow$ (base 10) $443 = 555 - 112$ √
(2, 2, 6)	$226_8 \stackrel{?}{=} 222_9 - 22_{15} \rightarrow$ (base 10) $150 = 182 - 32$ √

GBML 2008

TEAM ROUND

1. $3^{2x} \cdot 2^{2x} + 2 \cdot 3 = 21 \cdot \frac{1}{3} \cdot 3^x \cdot 2^x \rightarrow 6^{2x} - 7 \cdot 6^x + 6 = 0 \rightarrow (6^x - 6)(6^x - 1) = 0 \rightarrow$

$6^x = 0$ or $6^x = 1$ (Both check)

2. The required numbers must be represented by the product of two prime numbers.
They are:

$2 \cdot 23 = 46$
$3 \cdot 17 = 51$
$3 \cdot 19 = 57$
$5 \cdot 11 = 55$

3. $2x + 3y + \sqrt{2x + 3y} = 30 \rightarrow$
Let $A = 2x + 3y \rightarrow$
$A + \sqrt{A} - 30 = 0 \rightarrow$
$\left(A^{\frac{1}{2}} + 6\right)\left(A^{\frac{1}{2}} - 5\right) = 0 \rightarrow \cancel{A^{\frac{1}{2}} + 6 = 0}\quad A^{\frac{1}{2}} - 5 = 0 \rightarrow$
$\sqrt{2x + 3y} = 5 \rightarrow 2x + 3y = 25$

Likewise
$2x - 3y + \sqrt{2x - 3y} = 12 \rightarrow$
Let $B = 2x - 3y \rightarrow$
$B + \sqrt{B} - 12 = 0 \rightarrow$
$\left(B^{\frac{1}{2}} + 4\right)\left(B^{\frac{1}{2}} - 3\right) = 0 \rightarrow$
$\sqrt{2x - 3y} = 3 \rightarrow 2x - 3y = 9$

Two equations and two unknowns… $x = \frac{17}{2}, y = \frac{8}{3}$

GBML 2005

TEAM ROUND – page 1

1. By synthetic substitution the numerator factors into the product of binomials.

	1	−2	−7	20	−12	
2	1	0	−7	6	0	$(x^3 - 7x + 6)$
1	1	1	−6	0		$(x^2 + x - 6) = (x+3)(x-2)$

$\frac{x^4 - 2x^3 - 7x^2 + 20x - 12}{x^3 + 5x} = \frac{(x-2)^2(x-1)(x+3)}{x(x^2+5)}$

Since $x^2 + 5 > 0$ for all real x, it may be ignored. Likewise, $(x - 2)^2$ may be ignored, **except when $x = 2$**. Thus, we examine $\frac{(x-1)(x+3)}{x} \leq 0$ which has critical points at −3, 0 and 1.

As we move from left to right on the number line, each time we pass a critical point one less term in this quotient is negative. On the left all three terms are negative. Thus, the solution contains all reals for which $\underline{x \leq -3}$ and $\underline{0 < x \leq 1}$, but don't forget to include $\underline{x = 2}$ also.

2. Let $\underline{A}\ \underline{B}\ \underline{C}$ denote the 3-digit number. Then:
$\div 11 \rightarrow C + A - B = 11m$, i.e. must be a multiple of 11
$\div 4 \rightarrow 10B + C = 4n$
But $A + B + C \neq 3k$
Starting with the second condition, we can set up a table for the rightmost two digits

BC	$C-B$	A	ABC	Verdict
04	4	7	704	ok
12	1	–	- - -	
16	5	6	616	rejected (dupl. digit)
20	-2	2	220	rejected (dupl. digit)
24	2	9	924 rej	rejected (div. by 3)
40	-4	4	440	rejected (dupl. digit)
52	-3	3	352	ok
56	1	–	- - -	
60	-6	6	660	rejected (dupl. digit)
64	-2	2	264	rejected (div. by 3)
72	-5	5	572	ok
76	-1	1	176	ok
92	-7	7	792	rejected (div. by 3)
96	-3	3	396	rejected (div. by 3)

3. $\begin{cases} 6w+7y-9z=0 \\ 7w+10y-13z=0 \end{cases}$ Eliminating w, $\begin{cases} 60w+70y-90z=0 \\ -49w-70y+91z=0 \end{cases} \rightarrow 11w+z=0 \rightarrow z=-11w$
Substituting in the first equation, $6w + 7y + 99w = 0 \rightarrow 7y = -105w \rightarrow y = -15w$
$w = -1$ insures that both y and z are positive and as small as possible.
$\rightarrow$ **(–1, 15, 11)**

ARML 2010

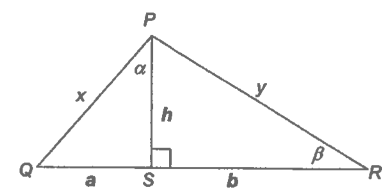
TEAM ROUND

1. The available digits are 1, 2, 3, 4 and 5. In each of the leftmost two positions there are 5 choices, since divisibility by 4 is entirely dependent of the rightmost two positions.
This two-digit number must be divisible by 4.
The rightmost two digits must be 12, 24, 32, 44 or 52, i.e. 5 choices.
Thus, there are 5^3 = **125** possible 4-digit numbers.

Alternate solution: The smallest 4-digit integers divisible by 4 will be of the form 11_ _.
The first candidate is 1112. Adding 4 and eliminating those integers which use digits greater than 4, we get 1124, 1132, 1144, 1152. The leftmost two digits could be 12, 13, 14, 15, 21, 22, …, 55. Again, we get 5(25) = **125**.

2. Since $2x^2-12y^2+5xy=(2x-3y)(x+4y)\neq 0$, $x\neq \frac{3y}{2}, -4y$
Cross multiplying, $2x^2-3y^2+2x-3y-xy=4x^2-24y^2+10xy$
$\rightarrow 2x^2-11xy-21y^2-2x+3y=0 \rightarrow (2x-3y)(x+7y)-(2x-3y)=0$
$\rightarrow (2x-3y)(x+7y-1)=0 \rightarrow x = \underline{1-7y}$ only

3. $\begin{cases} \tan\alpha+\cot\beta=0.975 \\ \cot\beta-\tan\alpha=0.525 \end{cases} \rightarrow \begin{matrix} \frac{a}{h}+\frac{b}{h}=0.975 \\ \frac{b}{h}-\frac{a}{h}=0.525 \end{matrix}$



Adding, $\frac{b}{h}=\frac{1.500}{2}=\frac{3}{4}$
Subtracting, $\frac{a}{h}=\frac{0.450}{2}=0.225=\frac{225}{1000}=\frac{9}{40}$.

If $b = 30$ and $h = 40$, then $a = 9$.

Pythagorean triples (9, 40 ___) and (30, 40, ___) $\rightarrow x = 41$ and $y = 50$ and $\frac{x}{y} = \underline{\mathbf{\frac{41}{50}}}$.