## NEW ENGLAND ASSOCIATION OF MATHEMATICS LEAGUES

## PLAYOFFS – 2015

**Round 1: Arithmetic and Number Theory**

1. ____________________

2. ____________________

3. ____________________

1. For integers $x$ and $y$, $x * y = x^y + y^x$. If $2 * A = 100$, compute $A$.

2. Given $a$ and $b$ are natural numbers and $a^2 + b^3 < 100$, how many ordered pairs satisfy this inequality?

3. Each square of a 3 x 3 grid is filled with a distinct integer from 1 to 9, inclusive.

   Compute the probability that the randomly-filled grid at the right will have each of $a$, $b$, and $c$ less than 5, and each of $p$, $q$, and $r$ greater than 5.

| | | |
|---|---|---|
| $a$ | $b$ | $c$ |
| $x$ | 5 | $y$ |
| $p$ | $q$ | $r$ |

## NEW ENGLAND ASSOCIATION OF MATHEMATICS LEAGUES

## PLAYOFFS – 2015

**Round 2: Algebra 1**

**1.** ____________________

**2.** ____________________

**3.** ( __________ , __________ )

1. If $312_{(x-1)} = 211_{(x+1)}$ , find the value of $x$.

2. The square of the reciprocal of 3 less than a number is $\frac{25}{144}$.

   Compute <u>two</u> possible values of the number.

3. $A$ and $B$ denote the <u>largest</u> pair of values in the following list, where $A > B$ :

$$1^{160},\ 2^{140},\ 3^{120},\ 4^{100},\ 5^{80},\ 6^{60},\ 7^{40},\ 8^{20},\ 9^{0}$$

Compute the ordered pair $(A, B)$. Leave the answers in exponential form as shown.

## NEW ENGLAND ASSOCIATION OF MATHEMATICS LEAGUES

## PLAYOFFS – 2015

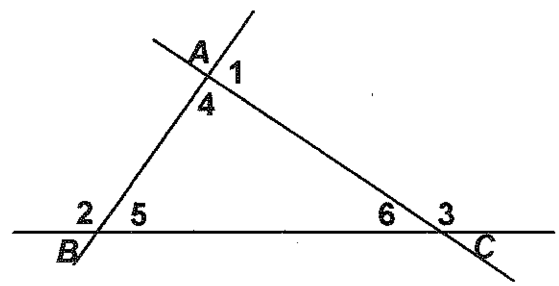
**Round 3: Geometry**

1. ______ : ______ : ______

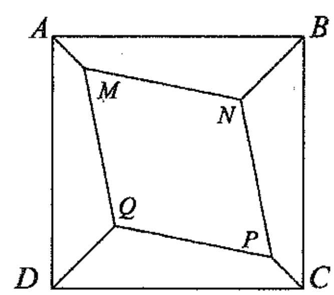
2. ____________________

3. ____________________

1. The exterior angles of $\Delta ABC$ satisfy the following condition: $m\angle 1 : m\angle 2 : m\angle 3 = 7 : 10 : 13$. Compute the ratio $m\angle 4 : m\angle 5 : m\angle 6$ .




2. $ABCD$ is a square with a side length of 16. From $A$ and $C$ segments of length 2 are drawn into the square at angles of 45 with the sides. From $B$ and $D$ segments of length 4 are drawn in a similar fashion. Compute the area of $MNPQ$.




3. Given: An isosceles triangle where the legs each have length $a$ and the base has length $c$. Let $R$ denote the radius of the circumscribed circle. Determine a simplified expression for $R^2$ in terms of $a$ and $c$

## NEW ENGLAND ASSOCIATION OF MATHEMATICS LEAGUES

**PLAYOFFS – 2015**

**Round 4: Algebra 2**

**1.** ____________________

**2.** ____________________

**3.** ____________________

1. If $S_n = i^n + i^{-n}$, where $n$ is any integer, and $i = \sqrt{-1}$, compute <u>all</u> possible values of $S_n$.

2. The roots of $\left(\dfrac{\log 0.\overline{3}}{\log 81}\right)x^2 + \left(\dfrac{\log 8}{\log 4}\right)x + \log 1000 = c$ are equal. Each log expression is understood to be a common log, i.e. $\log_{10}$. Compute $c$.

3. In the expansion of $\left(1+x^2\right)^t$ for a <u>positive</u> integer $t$, the coefficient of $x^8$ is six times the coefficient of $x^4$. If the first term in the expansion is 1, compute the ninth term in the expansion.

**NEW ENGLAND ASSOCIATION OF MATHEMATICS LEAGUES**

**PLAYOFFS – 2015**

**Round 5: Analytic Geometry**

**1.** ( ____, ____, ____, ____ )

**2.** ______________________

**3.** ( ________ , ________ )

1. Given the equation of a circle: $x^2 - 8x + y^2 + 14y + 40 = 0$. When written in simplified form, the area between a chord and an arc cut off by a central angle of 45° can be expressed as $\frac{a}{b}\left(\pi - c\sqrt{d}\right)$, where $a$, $b$, $c$, and $d$ are integers. Determine $(a, b, c, d)$

2. Given the equation $y^2 - 8y + 13 = x$, compute the number of square units in the area of the trapezoid whose vertices are on the parabola and having $x$-coordinates which are 4 more and 16 more than the $x$-coordinate of the vertex.

3. Consider an ellipse with center $(0, 0)$ and the major axis on the $y$-axis. One vertex is $(0, -2\sqrt{10})$ and one point on the ellipse is $P(-3, -4)$. A line $L_1$ through $P$ has $x$-intercept 1 and intersects a line $L_2$, which contains a focus $F(0, f)$, $f > 0$, and is perpendicular to $L_1$. The intersection of $L_1$ and $L_2$ is $Q(m, n)$. Determine the ordered pair $(m, n)$.

## NEW ENGLAND ASSOCIATION OF MATHEMATICS LEAGUES

## PLAYOFFS – 2015

**Round 6: Trig and Complex Numbers**

1. ( ________ , ________ )

2. ____________________

3. ( _____ , _____ , _____ , _____ )

1. The following can be expressed in $a+bi$ form. Compute the ordered pair $(a,b)$.

$$\left(\sqrt{2}cis\frac{7\pi}{24}\right)^4\left(\sqrt{3}cis\frac{5\pi}{18}\right)^6$$

2. Given $\cos(a-180°)=-0.8$, $\sin(b+90°)=0.6$, $0\le a<90°$, and $0\le b<90°$, compute $\cos(a+b)-\sin(a-b)$.

3. There are 4 solutions to the equation $\sin(x+17)=\cos(2x-23)$ over the interval $0\le x<360°$. If the solutions are denoted $A$, $B$, $C$ and $D$, where $A<B<C<D$, compute $(A,B,C,D)$.

## NEW ENGLAND ASSOCIATION OF MATHEMATICS LEAGUES

## PLAYOFFS – 2015

**Team Round - Place all answers on the team round answer sheet.**

1. For $x \in \left[0, \frac{\pi}{2}\right]$, determine the largest solution to

$2\sin x \cdot \cos x \cdot \cos(2x) \cdot \cos(4x) \cdot \cos(8x) \geq \frac{1}{16}$.

2. Given the parabola $y = \frac{x^2}{4p}$ with $p > 0$, let $F$ be the focal point, let $M$ lie on the parabola in the first quadrant with an $x$-coordinate of $a$, and let $O$ be the origin. If $\Delta MOF$ is isosceles with $OF = OM$, compute the ratio $\frac{a^2}{p^2}$.

3. Compute all ordered pairs $(x, y)$ which satisfy the following system of equations:

$$\begin{cases} x^2 - y^2 = 1 \\ xy - 2y + x = 2 \end{cases}$$

4. Consider two geometric sequences of positive real numbers. The first three terms of the first sequence are the same as the first three terms of the second, but in reverse order. The sum of the first six terms of one sequence is equal to 8 times the sum of the first six terms of the other sequence. If the common ratios are $r$ and $m$, where $r < m$, compute the ordered pair $(r, m)$.

5. Let $S = \{10, 11, 12, \ldots, 18, 19\}$. If seven numbers are chosen from $S$ at random, compute the probability that the sum of the remaining three numbers is divisible by 10.

6. One term of $\left(x^3 + \frac{1}{x^2}\right)^{12}$ is $Px^{11}$. Another term of the expansion is $Px^n$, $n \neq 11$. Find the ordered pair $(P, n)$.

## NEW ENGLAND ASSOCIATION OF MATHEMATICS LEAGUES

### PLAYOFFS – 2015

### *Answer Sheet*

**Round 1**

1 6

2. 31

3. $\frac{1}{315}$

**Round 2**

1. 10

2. $\frac{27}{5}$ or $\frac{3}{5}$

3. $4^{100}$, $3^{120}$

**Round 3**

1. 8 : 5 : 2

2. $272 - 96\sqrt{2}$

3. $\frac{a^4}{4a^2 - c^2}$

**Round 4**

1. 0, 2, −2

2. $\frac{21}{4}$ or 5.25

3. $165x^{16}$

**Round 5**

1. (25, 8, 2, 2)

2. 72

3. $(3, 2)$

**Round 6**

1. $\left(-54\sqrt{3}, 54\right)$

2. 0.28 or $\frac{7}{25}$

3. (32, 152, 272, 310)

**Team**

1. $\frac{41\pi}{96}$

2. $4\sqrt{5} - 8$

3. $\left(2, \pm\sqrt{3}\right)$, $\left(\pm\sqrt{2}, -1\right)$

4. $\left(\frac{1}{2}, 2\right)$

5. $\frac{1}{10}$

6. (792,1)

MASSACHUSETTS ASSOCIATION OF MATHEMATICS LEAGUES

NEW ENGLAND PLAYOFFS – 2015 - SOLUTIONS

**Round 1 Arithmetic and Number Theory**

1. We require that $2^A + A^2 = 100$ . Since the sum is even, $A$ must be even.
By brute force, $2^6 + 6^2 = 64 + 36 = 100$, yielding $A = \underline{6}$

2. If $a = 1$ to 5, $b = 1$ to 4, giving 20 ordered pairs. If $a = 6$ to 8, $b = 1$ to 3, giving 9 ordered pairs. If $a = 9$, $b = 1$ or 2, giving 2 ordered pairs. Thus, there are a total of **31** ordered pairs.

3. The number of ways of filling the grid to satisfy the stated property is $\begin{vmatrix} 4 & 3 & 2 \\ 2 & 1 & 1 \\ 4 & 3 & 2 \end{vmatrix}$, whereas a random fill could be done in $\begin{vmatrix} 9 & 8 & 7 \\ 6 & 5 & 4 \\ 3 & 2 & 1 \end{vmatrix}$ ways. Thus, the probability is

$$\frac{2(4!)^2}{9!} = \frac{\cancel{2\cdot 4}\cdot \cancel{3\cdot 2}}{9\cdot\cancel{8}\cdot 7\cdot\cancel{6}\cdot 5} = \underline{\mathbf{\frac{1}{315}}}$$

**Round 2 Algebra 1**

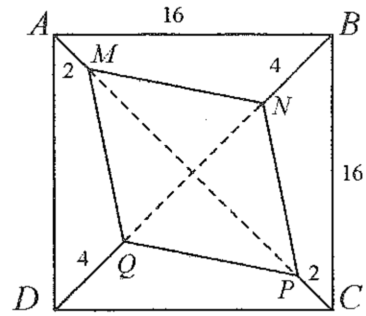
1. $312_{(x-1)} = 211_{(x+1)} \Leftrightarrow 3(x-1)^2 + (x-1) + 2 = 2(x+1)^2 + (x+1) + 1$
$\Leftrightarrow 3x^2 - 5x + 4 = 2x^2 + 5x + 4 \Leftrightarrow x^2 - 10x = x(x-10) = 0 \Rightarrow x = \underline{\mathbf{10}}$.

2. $\left(\frac{1}{a-3}\right)^2 = \frac{25}{144} \Rightarrow a = 3 \pm \frac{12}{5}$ $\boxed{a = \frac{27}{5}, \frac{3}{5}}$

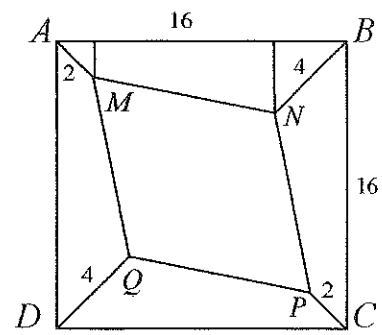
3. $1^{160}$ and $9^0$ both equal 1 and are ignored. Convert each of the in-between values to powers of 20. $2^{140} = (2^7)^{20} = 128^{20}$ , $3^{120} = (3^6)^{20} = 729^{20}$, $4^{100} = (4^5)^{20} = 1024^{20}$ ,
$5^{80} = (5^4)^{20} = 625^{20}$,
$6^{60} = (6^3)^{20} = 216^{20}$ , $7^{40} = (7^2)^{20} = 49^{20}$
The larger the base is, the larger the power. Thus, $(A,B) = \underline{(\mathbf{4^{100}, 3^{120}})}$.

## Round 3 – Geometry

1. Since the exterior angles of any triangle (one per vertex) always total 360°, we have $7n+10n+13n=30n=360 \Rightarrow n=12 \Rightarrow (m\angle 1, m\angle 2, m\angle 3)=(84,120,156)$ $\Rightarrow (m\angle 4, m\angle 5, m\angle 6)=(96,60,24)$. Thus, the required ratio is **8 : 5 : 2**




2. Method 1: Since the 4 trapezoids are congruent, $MNPQ$ is a rhombus. Since $AC = DB = 16\sqrt{2}$, then $MP = 16\sqrt{2}-4$ and $QN = 16\sqrt{2}-8$. Using the area formula $\frac{d_1 \cdot d_2}{2}$ gives $\frac{(16\sqrt{2}-4)(16\sqrt{2}-8)}{2} = \frac{512-64\sqrt{2}-128\sqrt{2}+32}{2} =$ $\boxed{272-96\sqrt{2}}$.




Method 2: Drop perpendiculars from $M$ and $N$ to $\overline{AB}$. The first is $\sqrt{2}$ units long, the second is $2\sqrt{2}$ units long. They divide $AMNB$ into two 45-45-90 right triangles with areas of $\frac{1}{2}\cdot\sqrt{2}\cdot\sqrt{2}=1$ and $\frac{1}{2}\cdot 2\sqrt{2}\cdot 2\sqrt{2}=4$, respectively, and one trapezoid whose bases have lengths of $\sqrt{2}$ and $2\sqrt{2}$ and whose height is $16-3\sqrt{2}$.

The area of the trapezoid is $\frac{1}{2}\left(16-3\sqrt{2}\right)\left(3\sqrt{2}\right)=24\sqrt{2}-9$. Thus, the area of $AMNB$ is $24\sqrt{2}-9+5=24\sqrt{2}-4$. The sum of the areas of the four congruent trapezoids is $96\sqrt{2}-16$. Then the area of $MNPQ$ is $16^2-\left(96\sqrt{2}-16\right)=\boxed{272-96\sqrt{2}}$.

3. Let $h$ denote the altitude from the vertex of the isosceles triangle to its base. Since the radius $R$ of a circle circumscribed about a triangle with sides $x$, $y$, and $z$ is $\frac{xyz}{4\cdot\text{area of }\Delta}$, we have $R=\frac{a^2c}{4\left(\frac{1}{2}hc\right)}=\frac{a^2}{2h}$. From the Pythagorean theorem,

$a^2=h^2+\left(\frac{c}{2}\right)^2 \Rightarrow h^2=\frac{4a^2-c^2}{4}$. Squaring and substituting, $R^2=\underline{\boldsymbol{\frac{a^4}{4a^2-c^2}}}$.

Alternate Solution:

The diameter $d$ of the circumscribed circle of a triangle is $\frac{\text{side}}{\sin(\text{opposite angle})}$. Consider $A$ as the vertex angle, then $\sin B=\sqrt{\frac{4a^2-c^2}{2a}}$, so $d=\frac{2a^2}{\sqrt{4a^2-c^2}}$.

## Round 4 – Algebra 2

1. $n=0\Rightarrow 1+1=\underline{2}$; $n=1\Rightarrow i+\frac{1}{i}=i+(-i)=\underline{0}$; $n=2\Rightarrow i^2+\frac{1}{i^2}=-1+(-1)=\underline{-2}$

   $n=3\Rightarrow i^3+\frac{1}{i^3}=-i+(i)=0$ … and the cycle repeats.

2. $\left(\frac{\log 0.\overline{3}}{\log 81}\right)=\log_{81}\left(3^{-1}\right)=-\log_{81}3=-\frac{1}{4},\ \frac{\log 8}{\log 4}=\log_4 8=\frac{3}{2}$

   $-\frac{1}{4}x^2+\frac{3}{2}x+3-c=0\Leftrightarrow x^2-6x-4(3-c)=0$

   Equal roots require that the discriminant be zero. $36+16(3-c)=0\Rightarrow c=\frac{84}{16}=\underline{\mathbf{\frac{21}{4}}}$

3. The expansion of $\left(1+x^2\right)^t$ is $\binom{t}{0}1+\binom{t}{1}\left(x^2\right)^1+\binom{t}{2}\left(x^2\right)^2+\binom{t}{3}\left(x^2\right)^3+\binom{t}{4}\left(x^2\right)^4+\ldots$

   Therefore, $6\cdot\binom{t}{2}=\binom{t}{4}\Rightarrow\frac{6\cancel{t(t-1)}}{2!}=\frac{\cancel{t(t-1)}(t-2)(t-3)}{4!}$

   $\Rightarrow 72=(t-2)(t-3)\Rightarrow t^2-5t-66=0\Rightarrow(t-11)(t+6)=0\Rightarrow t=11$

   Finally, the ninth term in the expansion is $\binom{11}{8}\left(x^2\right)^8=\frac{11\cdot 10\cdot 9}{1\cdot 2\cdot 3}x^{16}=\underline{\mathbf{165x^{16}}}$.

## Round 5 – Analytic Geometry

1. The equation can be rewritten as $(x-4)^2+(y+7)^2=25$, the graph of which is a circle of radius 5. The area of the triangle formed by two radii and a chord is found by $\frac{1}{2}ab\sin\theta$ which gives $\frac{25\sqrt{2}}{4}$. The area of the 45° sector is $\frac{1}{8}\cdot 25\pi$. Subtracting gives $\frac{25}{8}\left(\pi-2\sqrt{2}\right)$. The ordered quadruple is $\underline{\mathbf{(25, 8, 2, 2)}}$.

2. The given equation is that of a parabola with vertex $(-3,4)$. The required $x$-coordinates are 1 and 13. The vertices of the trapezoid are $(1,2),(1,6),(13,8),$ and $(13,0)$. The area is $\frac{1}{2}(4+8)(13-1)=\underline{\mathbf{72}}$.

3. Given ellipse $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$, with $V\left(0,-2\sqrt{10}\right)\rightarrow a^2=40$. Substituting $P(-3,-4)$ gives $\frac{9}{b^2}+\frac{2}{5}=1\rightarrow b^2=15$. Now $F(0,f)$ becomes $F(0,\sqrt{40-15})$ or $F(0,5)$. The slope of $L_1$ is $\frac{0+4}{1+3}=1$. $L_1$ has equation $x-y=1$. $L_2$ has slope $-1$ and passes through $F(0,5)$ so its equation is $x+y=5$. $\therefore$ $\underline{\boldsymbol{Q(3,2)}}$

## Round 6 – Trig and Complex Numbers

1. Applying the exponents of 4 and 6, the expression becomes $\left(4cis\frac{7\pi}{6}\right)\left(27cis\frac{5\pi}{3}\right)$.

   Multiplying gives $108cis\frac{17\pi}{6} = \underline{\mathbf{-54\sqrt{3}+54i}}$.

2. $\cos a = 0.8, \sin a = 0.6, \cos b = 0.6, \sin b = 0.8$

   $\therefore \cos(a+b) - \sin(a-b) = \left[(0.6)(0.8)-(0.6)(0.8)\right]-\left[(0.6)(0.6)-(0.8)(0.8)\right] =$

   $0-\left[0.36-(0.64)\right] = \underline{\mathbf{0.28}}$ or $\underline{\mathbf{\frac{7}{25}}}$.

3. $\sin(x+17) = \cos(2x-23) \Leftrightarrow \sin(x+17) = \sin(90-(2x-23)) = \sin(113-2x)$.

   If $\sin A = \sin B$, then $A$ and $B$ are equal or supplementary. More specifically,
   $A = B + 360k$ or $A + B = 180 + 360k$, for any integer $k$.
   Adding $360k$ generates all coterminal angles.
   $x+17 = 113-2x+360k \Rightarrow 3x = 96+360k \Rightarrow x = 32+120k \Rightarrow$ 32, 152, 272
   (for $k$ = 0,1 and 2).
   $(x+17)+(113-2x) = 180+360k \Rightarrow 130-x = 180+360k$
   $\Rightarrow x = -50-360k \Rightarrow$ 310 (for $k = -1$).
   Thus, $(A,B,C,D) = \underline{\mathbf{(32, 152, 272, 310)}}$.

## Team Round

1. Using the double angle identity $2\sin A\cos A = \sin 2A$ multiple times, we have

   $2\sin x\cos x\cos 2x\cos 4x\cos 8x \geq \frac{1}{16} \rightarrow \sin 2x\cos 2x\cos 4x\cos 8x \geq \frac{1}{16}$,

   $2\sin 2x\cos 2x\cos 4x\cos 8x \geq \frac{1}{16}\cdot 2 \rightarrow \sin 4x\cos 4x\cos 8x \geq \frac{1}{8}$,

   $2\sin 4x\cos 4x\cos 8x \geq \frac{1}{8}\cdot 2 \rightarrow \sin 8x\cos 8x \geq \frac{1}{4}$. Finally, $2\sin 8x\cos 8x \geq \frac{1}{4}\cdot 2$ gives

   $\sin 16x \geq \frac{1}{2}$. Setting $16x = \frac{\pi}{6}+2\pi k$ or $\frac{5\pi}{6}+2\pi k$ gives $x = \frac{\pi}{96}+\frac{\pi}{8}k$ or $\frac{5\pi}{96}+\frac{\pi}{8}k$. The largest possible value of $x$ will occur at the second of the two values for $k = 3$ since when $k \geq 4$ the solutions are outside the domain. When $k = 3$, $x = \underline{\mathbf{\frac{41\pi}{96}}}$.

**Team Round - continued**

2. Since the equation of the parabola is $y=\frac{x^2}{4p}$ the focal point $F=(0,p)$. The coordinates of $M=\left(a,\frac{a^2}{4p}\right)$ and since $\Delta FOM$ is isosceles with $OF=OM$, then $p^2=a^2+\frac{a^4}{16p^2}$. Thus,

$$a^4+16p^2a^2-16p^4=0 \to a^2=\frac{-16p^2\pm\sqrt{256p^4+64p^4}}{2} \to$$

$$a^2=\frac{8p^2\sqrt{5}-16p^2}{2}=p^2\left(4\sqrt{5}-8\right).$$ Thus, $\frac{a^2}{p^2}=\boxed{4\sqrt{5}-8}$.

3. $xy-2y+x=2 \Rightarrow y(x-2)=2-x \Rightarrow y=\frac{2-x}{x-2}=\frac{-1\cancel{(x-2)}}{\cancel{(x-2)}}=-1$, provided $x\neq 2$.

But remember there was no such restriction on the original equation! Substituting in the first equation, $x^2=2 \Rightarrow x=\pm\sqrt{2}$. However, if $x=2$, we have $y^2=3 \Rightarrow y=\pm\sqrt{3}$.
Thus, we have 4 ordered pairs, namely $\underline{\mathbf{\left(2,\pm\sqrt{3}\right)}}$, $\underline{\mathbf{\left(\pm\sqrt{2},-1\right)}}$.

**Team Round - continued**

4. Assume the first sequence is $A$: $a, ar, ar^2, \ldots$ and the second $B$: $b, bm, bm^2, \ldots$ . Then:

$$\left.\begin{array}{l}(1)\ a = bm^2 \\ (2)\ ar = bm\end{array}\right\} \Rightarrow \frac{1}{r} = m \quad \text{and} \quad \sum_1^7 A = 8 \cdot \sum_1^7 B \Leftrightarrow a\left(\frac{r^6-1}{r-1}\right) = 8b\left(\frac{m^6-1}{m-1}\right)$$

$$(3)\ b = ar^2$$

Substituting for $b$ and $m$, $a\left(\frac{r^6-1}{r-1}\right) = 8ar^2\left(\frac{r^{-6}-1}{r^{-1}-1}\right) = 8ar^2\left(\frac{1-r^6}{r^5-r^6}\right) = 8ar^2\left(\frac{r^6-1}{r^5(r-1)}\right)$

$\Rightarrow 1 = \frac{8}{r^3} \Rightarrow r = 2, m = \frac{1}{2}$, but we stipulated that $r < m$. Our assignment of first and second was arbitrary, so we simply reverse the roles of $r$ and $m$. $(r,m) = \underline{\boldsymbol{\left(\frac{1}{2}, 2\right)}}$.

Alternate Solution:

Sequence 1: $1, r, r^2, r^3, r^4, r^5$; sequence 2: $r^2, r, r^0, r^{-1}, r^{-2}, r^{-3}$

$S_1 = \frac{r^6-1}{r-1}$; $S_2 = \frac{\frac{r^6-1}{r^4}}{1-\frac{1}{r}} = \frac{r^6-1}{r^4} \cdot \frac{r}{r-1}$. Now $\frac{r^6-1}{r-1}$ cancels leaving $1 = \frac{8}{r^3} \rightarrow r = 2$.

5. There are ${}_{10}C_3 = \frac{10!}{3! \cdot 7!} = 120$ possible sets of triples. Given the particular numbers in $S$, the only sums divisible by 10 are 40 and 50. Here are the sets that sum to 40: $\{10,11,19\}$, $\{10,12,18\}, \{10,13,17\}, \{10,14,16\}, \{11,12,17\}, \{11,13,16\}$, $\{11,14,15\}$ and $\{12,13,15\}$. The sets that sum to 50 are: $\{13,18,19\}, \{14,17,19\}, \{15,16,19\}$ and $\{15,17,18\}$. There are a total of 12 sets, making the probability that the sum would be divisible by 10 equal to $\frac{12}{120} = \boxed{\frac{1}{10}}$.

6. The general term of the binomial expansion is $C_k^{12}\left(x^3\right)^{(12-k)} x^{-2k} = C_k^{12} x^{(36-5k)}$. So we have $36 - 5k = 11$, and $5k = 25$, so $k = 5$. Then $P = C(12, 5) = C(12, 7)$, and $n = 36 - 5(7) = 1$.
$P = \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = (11)(9)(8) = 792$. So $(P, n) =$ **<u>(792, 1)</u>**.