# Round 2 Algebra 1

## Simultaneous Linear equations, Word Problems, Matrices

**GREATER BOSTON MATHEMATICS LEAGUE**

**MEET 2 – NOVEMBER 1998**

**ROUND 2** – Simultaneous Linear Equations, Word Problems, Matrices

1. ( ______ , ______ )

2. ______________

3. ______________

**CALCULATORS ARE NOT ALLOWED ON THIS ROUND**

1. Find the ordered pair, $(x, y)$, solution to the following system of equations:

$$\begin{cases} \dfrac{x}{2} - \dfrac{y}{3} = 4 \\ \dfrac{x - 4y}{14} = 2 \end{cases}$$

2. Given the matrix equation, $\begin{pmatrix} 6 & -1 \\ 5 & 4 \end{pmatrix} \cdot \begin{pmatrix} x & y \\ -8 & 9 \end{pmatrix} = \begin{pmatrix} 5y + 16 & 2x - 3 \\ a & b \end{pmatrix}$,

find the sum $a + b + x + y$.

3. If sixty coins consisting of nickels, dimes, and quarters are worth exactly five dollars and fifteen cents, what is the most number of quarters you can have ?

# GREATER BOSTON MATHEMATICS LEAGUE

## MEET 2 – NOVEMBER 1999

**ROUND 2** – Simultaneous Linear Equations, Word Problems, Matrices

1. ____________________

2. __(________, ________)__

3. ____________________

**CALCULATORS ARE NOT ALLOWED ON THIS ROUND**

1. Given the following system of equations in $x$ and $y$, find $x + y$ in terms of $a$:

$$\begin{cases} 3x + 4y = -a \\ 9x - 8y = 7a \end{cases}$$

2. Find the ordered pair, $(x, y)$, solution to the following matrix equation:

$$\begin{pmatrix} x & y \\ 2 & 4 \end{pmatrix}\begin{pmatrix} 1 & y-4 \\ 2 & x \end{pmatrix} = \begin{pmatrix} -1 & -50 \\ 10 & 6 \end{pmatrix}$$

3. The ratio of Kaitlin's age now to her father's age six years ago is 1:2. In twelve years the ratio of their ages will be 5:9. Find Kaitlin's age in years now.

# GREATER BOSTON MATHEMATICS LEAGUE

## MEET 2 – NOVEMBER 2000

**ROUND 2** – Simultaneous Linear Equations, Word Problems, Matrices

1. ( , )

2. ____________________

3. ____________________

**CALCULATORS ARE NOT ALLOWED ON THIS ROUND**

1. Find the ordered pair $(x, y)$ which is a solution to the following matrix equation.
$$\begin{pmatrix} x & 2 \\ y & -1 \end{pmatrix}\begin{pmatrix} 4 \\ 7 \end{pmatrix}=\begin{pmatrix} y-10 \\ 4-x \end{pmatrix}$$

2. Find the 2-digit whole number such that three times its ten's digit is one less than seven times its unit's digit and when the number formed by reversing its digits is subtracted from the number, the result is 45.

3. One amount of money is invested at 5% and another amount is invested at 8%. The total yearly interest from both investments is \$620. If the interest rates were reversed on the two amounts, the annual interest would be increased by \$60. What is the total number of dollars invested?

**GREATER BOSTON MATHEMATICS LEAGUE**

**MEET 2 – NOVEMBER 2001**

**ROUND 2** – Simultaneous Linear Equations, Word Problems, Matrices

1. $ ________________

2. (____, ____)

3. ________________

**CALCULATORS ARE NOT ALLOWED ON THIS ROUND**

1. In *Mathland's* monetary system, 3 *alphas* and 5 *betas* are worth \$4.34 while 5 *alphas* and 3 *betas* are worth \$4.30. Find the total value of 1 *alpha* and 1 *beta* in dollars and cents.

2. Given the following matrix multiplication, find the ordered pair $(A, B)$.

$$\begin{pmatrix} 5 & A \\ 3 & 2B \end{pmatrix}\begin{pmatrix} B & 3A \\ 2 & -1 \end{pmatrix}=\begin{pmatrix} -19 & C \\ D & -12 \end{pmatrix}$$

3. Al and his children Bill and Carol have ages that total 78 years. In 10 years Al will be twice Bill's age then and 6 years ago Al was five times Carol's age then. How many years old is Al now?

# GREATER BOSTON MATHEMATICS LEAGUE
# MEET 2 – NOVEMBER 2006

## ROUND 2 – Simultaneous Linear equations, Word Problems, Matrices

1. $________________

2. ( ________ , ________ )

3. ( ________ , ________ )

**CALCULATORS ARE NOT ALLOWED ON THIS ROUND.**

1. I have 120 coins consisting of dimes and quarters exclusively. Two times the number of dimes is 25 more than three times the number of quarters. How much money (in dollars and cents) are these 120 coins worth?

2. Find the ordered pair $(x, y)$ in terms of $A$ satisfying the following matrix equation:

$$\begin{pmatrix} 4 & -9 \\ 6 & -12 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 8A \\ 11A \end{pmatrix}$$

3. Find the ordered pair $(x, y)$ in terms of $A$ and $B$ that satisfies the following system of equations:

$$\frac{3x}{4A} - \frac{2y}{3B} = 1$$
$$\frac{2x}{3A} - \frac{5y}{9B} = 1$$

# GREATER BOSTON MATHEMATICS LEAGUE

## MEET 2 – NOVEMBER 2007

**ROUND 2** – Simultaneous Linear equations, Word Problems, Matrices

1. ______________________

2. ______________________

3. ______________________

**CALCULATORS ARE NOT ALLOWED ON THIS ROUND.**

1. Find $\left\{(x, y) \mid \frac{x}{2}+\frac{8y}{3}=a \text{ and } \frac{3x}{2}-2y=\frac{a}{2}\right\}$ in terms of $a$.

2. If $\begin{pmatrix} x & 5 \\ 3x & -7 \end{pmatrix}\begin{pmatrix} 4 \\ y \end{pmatrix}=\begin{pmatrix} 6 \\ -4 \end{pmatrix}$, determine the ordered pair $(x, y)$

3. $A$, $B$, and $C$ are $2\times 2$ matrices.
   $A=\begin{pmatrix} 4 & 3 \\ 2 & 1 \end{pmatrix}$ and $B=\begin{pmatrix} -2 & 0 \\ 6 & -4 \end{pmatrix}$ and $A^{-1}\cdot B+\begin{pmatrix} -8 & 7 \\ 11 & -10 \end{pmatrix}=C$, find $C^{-1}$.

# GREATER BOSTON MATHEMATICS LEAGUE

## MEET 2 – NOVEMBER 2008

**ROUND 2** – Simultaneous Linear equations, Word Problems, Matrices

1. ____________________

2. ____________________

3. ____________________

**CALCULATORS ARE NOT ALLOWED ON THIS ROUND.**

1. Find all ordered pairs $(x, y)$ which make the following statement true.

$$\begin{bmatrix} 2 & y \\ x & 5 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 3x \\ 2y \end{bmatrix}$$

2. In a 3-digit base 10 number, the tens digit is two more than the hundreds digit and the sum of all three digits is 13. If the units digit and the hundreds digit are reversed, the result is 99 less than the original number. Find the original number.

3. What is the least amount of money that would satisfy the following conditions?

   The ratio of the number of dimes to quarters is 5 : 6 <u>and</u> the ratio of the number of nickels to dimes is 5 : 3.

# GREATER BOSTON MATHEMATICS LEAGUE

## MEET 2 – NOVEMBER 2009

**ROUND 2** – Simultaneous Linear equations, Word Problems, Matrices

1. ( __________ , __________ )

2. ( ________ , ________ , ________ )

3. ______________________________

## CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

1. Solve this system. Express your answer as an ordered pair $(x, y)$, where $x$ and $y$ are each in terms of $a$.

$$4x+3y=-\frac{1}{a}$$
$$\frac{x}{2}-\frac{y}{3}=\frac{2}{a}$$

2. If 3 apples plus 2 bananas weigh 14 ounces, 9 cantaloupes minus 15 apples weigh 4 ounces, and 8 bananas minus 3 cantaloupes weigh 15 ounces, express the weight in ounces of these items as an ordered triple in the order apples, bananas, cantaloupes.

3. Given: $A=\begin{pmatrix} 8 & 6 \\ 4 & 4 \end{pmatrix}$, $B=\begin{pmatrix} 9 & 6 \\ 5 & 3 \end{pmatrix}$ and $\left(A^{-1}\right)\left(B^{-1}\right)=\frac{1}{k}\begin{pmatrix} 21 & D \\ E & F \end{pmatrix}$
Compute the value of $K+D+E+F$.

# GREATER BOSTON MATHEMATICS LEAGUE

## MEET 2 – NOVEMBER 2010

**ROUND 2** – Simultaneous Linear equations, Word Problems, Matrices

1. P : D = ________ : ________

2. ________________________________

3. ________________________________

**CALCULATORS ARE NOT ALLOWED ON THIS ROUND.**

1. Al had 50 coins consisting of pennies and dimes. He imagined his pennies became dimes and his dimes became nickels. With these changes the amount of money he would have would be 2 cents more than what he actually had. Compute the ratio of pennies to dimes that he had originally.

2. Find all possible ordered pairs $(x, y)$ given the following system of equations, where $a$ and $b$ are fixed nonzero constants:

$$a^2x + b^2y = b^2$$
$$2a^2x - 7b^2y = b^2$$

3. Al's age 4 years ago was one more than 3 times Sue's age at that time. Measured from today, Sue's age in 40 years will equal three times Al's age 17 years ago. In how many years (from today) will Al's age be twice Sue's age? Assume Al and Sue have the same birthday.

**MASSACHUSETTS MATHEMATICS LEAGUE**
**MARCH 2004**
**ROUND 1: SIM. EQUATIONS & DET.**

**ANSWERS**

**A)**_______________

**B)**_______________

**C)**_______________

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**A)** A rectangular cafeteria has a perimeter of 800 feet, and an area of 30, 000 square feet. Find the number of feet in the length and in the width of the cafeteria.

**B)** Lia is offered two different salary options to sell health club memberships. Plan A is: \$500 weekly in addition to 6% of her sales; while Plan B is: \$700 weekly in addition to 1% of her sales. Lia realized that she could get the same salary for a week under both plans. What is that salary?

C) Solve for x: $\begin{vmatrix} 1 & -3 & x \\ x & 4 & -1 \\ -2 & 4 & 1 \end{vmatrix} = 40$

**MASSACHUSETTS MATHEMATICS LEAGUE**
**MARCH 2005**
**ROUND 1: ALGEBRA 2 SIMULTANEOUS EQUATIONS & DETERMINANTS**

**ANSWERS**

**A)**_______________

**B)**_______________

**C)**_______________

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A) If $(a, b, c)$ is the solution to the following system, evaluate $abc$

$$\begin{cases} a + 2b + 3c = 3 \\ 3b + 2c = 7 \\ 3b + 4c = 17 \end{cases}$$

B) For real numbers $s$ and $t$, $s + t = 17$ while $\sqrt{s}\sqrt{t} = 7$. In simplified radical form $s = a \pm b\sqrt{c}$ with $b>0$. Find the value of $c + b + a$.

C) For what value(s) of the constant c will the following system have no real solutions for $(x, y, z)$?

$$\begin{cases} x + 2y + 3z = 5 \\ 2x + cz = y + 1 \\ cx + 6z = 8 + 2y \end{cases}$$

**MASSACHUSETTS MATHEMATICS LEAGUE**
**MARCH 2006**
**ROUND 1 ALGEBRA 2: SIMULTANEOUS EQUATIONS & DETERMINANTS**

**ANSWERS**

**A)**_______________

**B)**_______________

**C)**( ______ , ______ )

A) Find $x$ if $\begin{vmatrix} 1 & 1 & 1 \\ x & 2 & x \\ 0 & x & 1 \end{vmatrix} = 5$

B) Find all ordered pairs $(x, y)$ that satisfy this system:

$$\frac{-1}{1-x} = \frac{1}{2y+1}$$
$$(x-1)^2 + (2y+1)^2 = 50$$

C) If $A$ is the sum of the $x$-coordinates of the ordered pairs $(x, y)$ satisfying:

$$(1+x\sqrt{2})^2(1-x\sqrt{2})^2 = y^2$$
$$3x = y - 1$$

and $N$ is the <u>number</u> of ordered pairs satisfying the system, find $(A, N)$.

**MASSACHUSETTS MATHEMATICS LEAGUE**
**CONTEST 6 – MARCH 2007**
**ROUND 1 ALG 2: SIMULTANEOUS EQUATIONS AND DETERMINANTS**

**ANSWERS**

A) $a =$ ____________ $b =$ ____________

B) ______________________________

C) ______________________________

A) Determine the values of $a$ and $b$ such that the solution $(x, y)$ of the system $\begin{cases} ax + by = 5 \\ ax - by = 15 \end{cases}$ is (5, 15).

B) Determine the numerical value of $\begin{vmatrix} 3 & a \\ b & 4 \end{vmatrix}$ if the system of equations represented by

$\begin{bmatrix} 3 & a \\ b & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ -6 \end{bmatrix}$ intersect at the point $P(5, 1)$.

C) The identity $\dfrac{A}{x} + \dfrac{B}{2x-1} + \dfrac{C}{x+3} \equiv \dfrac{-21}{2x^3 + 5x^2 - 3x}$ is true for exactly one ordered triple $(A, B, C)$.
Determine the value of $A + B + C$.

**MASSACHUSETTS MATHEMATICS LEAGUE**
**CONTEST 6 – MARCH 2008**
**ROUND 1 ALG 2: SIMULTANEOUS EQUATIONS AND DETERMINANTS**

**ANSWERS**

A) ______________________________

B) ______________________________

C) (pizza, sub, spag) = ( \$___ , \$___ , \$___ )

A) The system $\begin{cases} y = 2x+1 \\ Ax+By=-9 \end{cases}$ represents a pair of perpendicular lines that intersect at $\left(\frac{1}{2}, 2\right)$.
Determine the ordered pair $(A, B)$.

B) Find all real valuesof $x$ for which $\begin{vmatrix} 1 & x & -3 \\ 2 & 0 & -1 \\ -3 & 2x-3 & x \end{vmatrix} = -7$

C) After the math meet, the bus stopped at the Pythagoras House of Pizza.
The mathletes noticed the following:
3 pizzas and 2 meatball subs cost \$4 more than 5 spaghetti with meat sauce.
Twice the cost of "one meatball sub and one spaghetti with meat sauce" would be \$2 less than 3 pizzas.
The combined cost of one meatball sub and one spaghetti with meat sauce is \$3 more than the cost of one pizza. Find the cost of each item.

GBML 98

**ROUND 2**

1. $\begin{cases} \frac{x}{2} - \frac{y}{3} = 4 \\ \frac{x-4y}{14} = 2 \end{cases} \Rightarrow \begin{cases} 3x - 2y = 24 \\ x - 4y = 28 \end{cases} \Rightarrow \begin{cases} -6x + 4y = -48 \\ x - 4y = 28 \end{cases} \Rightarrow -5x = -20 \Rightarrow x = 4$ and $y = -6$
$\Rightarrow$ **(4, -6)** is the answer.

2. $\begin{pmatrix} 6 & -1 \\ 5 & 4 \end{pmatrix} \cdot \begin{pmatrix} x & y \\ -8 & 9 \end{pmatrix} = \begin{pmatrix} 5y+16 & 2x-3 \\ a & b \end{pmatrix} \Rightarrow 6x + 8 = 5y + 16$ and $6y - 9 = 2x - 3 \Rightarrow$
$6x - 5y = 8$ and $-2x + 6y = 6 \Rightarrow x = 3$ and $y = 2 \Rightarrow a = -17$ and $b = 46 \Rightarrow$
$a + b + x + y = \mathbf{34}$

3. Let $n$ = number of nickels, $d$ = number of dimes, and $q$ = number of quarters:
$n + d + q = 60$ and $5n + 10d + 25q = 515 \Rightarrow n + 2d + 5q = 103 \Rightarrow d + 4q = 43$
To find the maximum number of quarters let $d = 3 \Rightarrow q = \mathbf{10}$

GBML 99

1. $\begin{cases} 3x + 4y = -a \\ 9x - 8y = 7a \end{cases} \Rightarrow 6x + 8y = -2a \Rightarrow 15x = 5a \Rightarrow x = \frac{a}{3} \Rightarrow y = \frac{-a}{2} \Rightarrow x + y = -\frac{a}{6}$

2. $\begin{pmatrix} x & y \\ 2 & 4 \end{pmatrix} \begin{pmatrix} 1 & y-4 \\ 2 & x \end{pmatrix} = \begin{pmatrix} -1 & -50 \\ 10 & 6 \end{pmatrix} \Rightarrow x + 2y = -1$ and $2y - 8 + 4x = 6 \Rightarrow 4x + 2y = 14$
$\Rightarrow 3x = 15$ and $x = 5 \Rightarrow y = -3$. Therefore $(x, y) = (5, -3)$

3. Let $x$ = Kaitlin's age now; $y$ = her father's age now; $\frac{x}{y-6} = \frac{1}{2}$ and $\frac{x+12}{y+12} = \frac{5}{9} \Rightarrow$
$2x = y - 6$ and $9x + 108 = 5y + 60 \Rightarrow y = 2x + 6$ and $5y - 9x = 48 \Rightarrow$
$10x + 30 - 9x = 48 \Rightarrow x = 18$

GBML 00

1. $\begin{pmatrix} x & 2 \\ y & -1 \end{pmatrix} \begin{pmatrix} 4 \\ 7 \end{pmatrix} = \begin{pmatrix} y-10 \\ 4-x \end{pmatrix} \rightarrow \begin{cases} 4x + 14 = y - 10 \\ 4y - 7 = 4 - x \end{cases} \rightarrow \begin{cases} 4x + 24 = y \\ 4y + x = 11 \end{cases} \rightarrow$
$16x + 96 + x = 11 \rightarrow 17x = -85 \rightarrow x = -5,\ y = 4 \rightarrow$ ordered pair solution is $(-5, 4)$.

2 Let $t$ = ten's digit, $u$ = unit's digit; $\rightarrow \begin{cases} 3t = 7u - 1 \\ (10t + u) - (10u + t) = 45 \end{cases} \rightarrow \begin{cases} -7u + 3t = -1 \\ 9t - 9u = 45 \end{cases} \rightarrow$
$\begin{cases} -7u + 3t = -1 \\ 3u - 3t = -15 \end{cases} \rightarrow -4u = -16 \rightarrow u = 4 \rightarrow t = 9 \rightarrow$ number is 94

3. $\begin{cases} .05x + .08y = 620 \\ .08x + .05y = 680 \end{cases} \rightarrow \begin{cases} .40x + .64y = 4960 \\ -.40x - .25y = -3400 \end{cases} \rightarrow .39y = 1560 \rightarrow y = 4000 \rightarrow$
$.05x + 320 = 620 \rightarrow .05x = 300 \rightarrow x = 6000 \rightarrow x + y = 10000$

GBML 01

1. $3\alpha + 5\beta = 4.34$ and $5\alpha + 3\beta = 4.30 \Rightarrow 8\alpha + 8\beta = 8.64 \Rightarrow \alpha + \beta = \$1.08$

2 $\begin{cases} 2A + 5B = -19 \\ 9A - 2B = -12 \end{cases} \Rightarrow \begin{cases} 4A + 10B = -38 \\ 45A - 10B = -60 \end{cases} \Rightarrow 49A = -98 \Rightarrow$
$A = -2 \Rightarrow -4 + 5B = -19 \Rightarrow B = -3 \Rightarrow (A, B) = (-2, -3)$.

3. Let $a$ = Al's current age; $b$ = Bill's current age; $c$ = Carol's current age $\Rightarrow$
$\begin{cases} a + b + c = 78 \\ a + 10 = 2(b + 10) \\ a - 6 = 5(c - 6) \end{cases} \Rightarrow \begin{cases} a + b + c = 78 \\ a = 2b + 10 \\ a = 5c - 24 \end{cases} \Rightarrow \begin{cases} a + b + c = 78 \\ b = \frac{a-10}{2} \\ c = \frac{a+24}{5} \end{cases} \Rightarrow a + \frac{a-10}{2} + \frac{a+24}{5} = 78 \Rightarrow$
$10a + 5a - 50 + 2a + 48 = 780 \Rightarrow 17a = 782 \Rightarrow a = 46$.

~~GBML 01~~

GBML 06

**ROUND 2**

1. Let $D$ and $Q$ denote the number of dimes and quarters respectively.
Then $D + Q = 120$ and $2D = 3Q + 25 \rightarrow 2(120 - Q) = 3Q + 25$ or $5Q = 240 - 25 = 215$
Thus, $Q = 43$ and $D = 77$ and the value of the 120 coins is \$10.75 + \$7.70 = **\$18.45**

2. The given matrix equation is equivalent to the system $\begin{cases} 4x - 9y = 8A \\ 6x - 12y = 11A \end{cases}$

Solving simultaneously, $\begin{matrix} 12x - 27y = 24A \\ 12x - 24y = 22A \end{matrix} \rightarrow -3y = 2A \rightarrow y = -2A/3$ and substituting back into the first equation $4x + 6A = 8A \rightarrow x = A/2$. Thus, the required ordered pair is $\underline{\left(\frac{A}{2}, -\frac{2A}{3}\right)}$.

3. Clearing out fractions by multiplying the 1st equation by $12AB$ ($\rightarrow 9Bx - 8Ay = 12AB$) and the second 2nd by $9AB$ ($\rightarrow 6Bx - 5Ay = 9B$) and then causing the $x$-term (or $y$-term) to drop out using linear combination is the standard approach, but tedious in this case.
Multiply the 1st equation by (-5/3) $\rightarrow -\frac{5}{4}\left(\frac{x}{A}\right) + \frac{10}{9}\left(\frac{y}{B}\right) = -\frac{5}{3}$ and multiplying the 2nd equation by 2 $\rightarrow \frac{4}{3}\left(\frac{x}{A}\right) - \frac{10}{9}\left(\frac{y}{B}\right) = 2$. Adding, $\left(\frac{4}{3} - \frac{5}{4}\right)\left(\frac{x}{A}\right) = 2 - \frac{5}{3} \rightarrow \frac{1}{12}\left(\frac{X}{A}\right) = \frac{1}{3}$
$\rightarrow x = 4A$ Substituting in the 1st equation, $3 - \frac{2y}{3B} = 1 \rightarrow y = 3B \rightarrow$ $\underline{\mathbf{(3A, 4B)}}$

GBML 07

1. $3\left(\frac{x}{2} + \frac{8y}{3} = a\right) - \left(\frac{3x}{2} - 2y = \frac{a}{2}\right) \rightarrow 10y = \frac{5a}{2} \rightarrow y = \frac{a}{4}$
Substituting in the 2nd equation, $\frac{3x}{2} - \frac{a}{2} = \frac{a}{2} \rightarrow x = \frac{2a}{3}$ Thus $(x, y) = \underline{\left(\mathbf{\frac{a}{4}, \frac{2a}{3}}\right)}$

2. $\begin{pmatrix} x & 5 \\ 3x & -7 \end{pmatrix}\begin{pmatrix} 4 \\ y \end{pmatrix} = \begin{pmatrix} 6 \\ -4 \end{pmatrix} \rightarrow \begin{cases} 4x + 5y = 6 \\ 12x - 7y = -4 \end{cases} \rightarrow \begin{cases} -12x - 15y = -18 \\ 12x - 7y = -4 \end{cases} \rightarrow y = 1 \rightarrow \underline{\left(\mathbf{\frac{1}{4}, 1}\right)}$

3. For any invertible 2 x 2 matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, $A^{-1} = \frac{1}{\det(A)}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$.

Thus, $A = \begin{pmatrix} 4 & 3 \\ 2 & 1 \end{pmatrix} \rightarrow A^{-1} = -\frac{1}{2}\begin{pmatrix} 1 & -3 \\ -2 & 4 \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} & \frac{3}{2} \\ 1 & -2 \end{pmatrix}$

If $B = \begin{pmatrix} -2 & 0 \\ 6 & -4 \end{pmatrix}$, then $C = A^{-1} \cdot B + \begin{pmatrix} -8 & 7 \\ 11 & -10 \end{pmatrix}$

$= \begin{pmatrix} -\frac{1}{2} & \frac{3}{2} \\ 1 & -2 \end{pmatrix} \cdot \begin{pmatrix} -2 & 0 \\ 6 & -4 \end{pmatrix} + \begin{pmatrix} -8 & 7 \\ 11 & -10 \end{pmatrix} = \begin{pmatrix} 1+9-8 & 0-6+7 \\ -2-12+11 & 0+8-10 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ -3 & -2 \end{pmatrix}$

and $C^{-1} = \frac{1}{-1}\begin{pmatrix} -2 & -1 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ -3 & -2 \end{pmatrix}$. Note: $C$ is its own inverse!

GBML 08

**ROUND 2**

1. $8 + y = 3x,\ 4x + 5 = 2y \rightarrow 4x + 5 = 2(3x - 8) \rightarrow x = \frac{21}{2},\ y = \frac{47}{2} \rightarrow \left(\mathbf{\frac{21}{2}, \frac{47}{2}}\right)$

2. $\begin{cases} t = h + 2 \\ h + t + u = 13 \\ 100u + 10t + u = 100h + 10t + u - 99 \end{cases} \rightarrow \begin{cases} 2h + u = 11 \\ h - u = 1 \end{cases} \rightarrow 3h = 12 \rightarrow h = 4, u = 3, t = 6$
$\rightarrow$ $\underline{\mathbf{463}}$

3. $\begin{cases} D : Q = 5 : 6 \\ N : D = 5 : 3 \end{cases} \leftrightarrow \begin{cases} Q : D = 6 : 5 = 18 : 15 \\ N : D = 5 : 3 = 25 : 15 \end{cases} \rightarrow N : Q : D = 25 : 18 : 15$
Thus, the least amount of money is 25 nickels, 18 quarters and 15 dimes, i.e. \$1.25 in nickels, \$1.50 in dimes, and \$4.50 in quarters $\rightarrow$ $\underline{\mathbf{\$7.25}}$

GBML 09

**ROUND 2**

1. $\begin{cases} 4x + 3y = -\frac{1}{a} \\ \frac{x}{2} - \frac{y}{3} = \frac{2}{a} \end{cases} \rightarrow \begin{cases} 4x + 3y = -\frac{1}{a} \\ 3x - 2y = \frac{12}{a} \end{cases} \rightarrow \begin{cases} 8x + 6y = -\frac{2}{a} \\ 9x - 6y = \frac{36}{a} \end{cases} \rightarrow 17x = \frac{34}{a} \rightarrow x = \frac{2}{a}$
Substituting in the first equation, $3y = -\frac{8}{a} - \frac{1}{a} = -\frac{9}{a} \rightarrow y = -\frac{3}{a}$ Thus, $(x, y) = \underline{\left(\mathbf{\frac{2}{a}, -\frac{3}{a}}\right)}$

2. $\begin{cases} (1)\ 3A + 2B = 14 \\ (2)\ 9C - 15A = 4 \\ (3)\ 8B - 3C = 15 \end{cases} \rightarrow \begin{cases} 15A + 10B = 70 \\ 9C - 15A = 4 \end{cases} \rightarrow 10B + 9C = 74$
Combining with (3) $\rightarrow$ $(B, C) = \left(\frac{7}{2}, \frac{13}{3}\right)$
Substituting for $B$ is (1) $\rightarrow A = \frac{7}{3} \rightarrow (A, B, C) = \underline{\left(\mathbf{\frac{7}{3}, \frac{7}{2}, \frac{13}{3}}\right)}$

3. $A = \begin{pmatrix} 8 & 6 \\ 4 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 9 & 6 \\ 5 & 3 \end{pmatrix} \rightarrow \det(A) = 8(4) - 6(4) = 8,\ \det(B) = 9(3) - 6(5) = -3$
Since the inverse of an invertible 2 x 2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $\frac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$,
$A^{-1} = \frac{1}{8}\begin{bmatrix} 4 & -6 \\ -4 & 8 \end{bmatrix}$ and $B^{-1} = -\frac{1}{3}\begin{bmatrix} 3 & -6 \\ -5 & 9 \end{bmatrix}$
$A^{-1} \cdot B^{-1} = \frac{1}{8} \cdot \begin{bmatrix} 4 & -6 \\ -4 & 8 \end{bmatrix} \cdot -\frac{1}{3}\begin{bmatrix} 3 & -6 \\ -5 & 9 \end{bmatrix} = -\frac{1}{24}\begin{bmatrix} 4 & -6 \\ -4 & 8 \end{bmatrix}\begin{bmatrix} 3 & -6 \\ -5 & 9 \end{bmatrix}$
The following shows the computations for the product of the two matrices.
The entry $A_{ij}$ is the sum of the products of the corresponding entries in row $i$ and column $j$ . . .

(continued)

so aligning the matrices $\begin{matrix} & [B] \\ [A] & [AB] \end{matrix}$ makes it easier to keep track of which row and which column are being used.

$$\begin{bmatrix} \boxed{3} & -6 \\ \boxed{-5} & 9 \end{bmatrix}$$
$$\downarrow$$
$$\begin{bmatrix} 4 & -6 \\ -4 & 8 \end{bmatrix}\begin{bmatrix} 3 & -6 \\ -5 & 9 \end{bmatrix} = \begin{bmatrix} \boxed{4} & \boxed{-6} \rightarrow \\ -4 & 8 \end{bmatrix}\begin{bmatrix} (4)(3)+(-6)(-5) & (4)(-6)+(-6)(9) \\ (-4)(3)+(8)(-5) & (-4)(-6)+(8)(9) \end{bmatrix} = \begin{bmatrix} 42 & -78 \\ -52 & 96 \end{bmatrix}$$

Thus, $A^{-1}\cdot B^{-1} = -\frac{1}{24}\cdot\begin{bmatrix} 42 & -78 \\ -52 & 96 \end{bmatrix} = -\frac{1}{12}\cdot\begin{bmatrix} 21 & -39 \\ -26 & 48 \end{bmatrix}$

and $K+D+E+F = -12-39-26+48 = \underline{-29}$

GBML 10

**ROUND 2**

1. Let $(P, D) = (x, 50-x)$. Then:
   Imagined – Acual $= \big(10x+5(50-x)\big)-\big(x+10(50-x)\big) = 2$
   $5x + 250 - 500 + 9x = 2 \rightarrow 14x = 252 \rightarrow x = 18 \rightarrow (18, 32) \rightarrow \underline{\mathbf{9:16}}$.

2. $\left.\begin{matrix} a^2x+b^2y=b^2 \\ 2a^2x-7b^2y=b^2 \end{matrix}\right\} \rightarrow 9b^2y = b^2 \rightarrow y = \frac{1}{9}$

   Substituting for $y$, $a^2x+\frac{1}{9}b^2 = b^2 \rightarrow 9a^2x+b^2 = 9b^2 \rightarrow 9a^2x = 8b^2 \rightarrow x = \frac{8b^2}{a^2}$

   $\rightarrow (x, y) = \underline{\left(\frac{\mathbf{8b^2}}{\mathbf{9a^2}}, \frac{\mathbf{1}}{\mathbf{9}}\right)}$

3. Let $(A, S)$ denote Al's and Sue's ages now.
   Four years ago, $A - 4 = 1 + 3(S-4) \rightarrow A = 3S - 7$
   Also, $S - 40 = 3(A - 17) \rightarrow S = 3A - 91$
   $\rightarrow A = 3(3A-91) - 7 = 9A - 280 \rightarrow 8A = 280 \rightarrow A = 35, S = 14$
   In $x$ years, we require that $35 + x = 2(14 + x) \rightarrow x = \underline{7}$.

# MASSACHUSETTS MATHEMATICS LEAGUE
## MARCH 2004
## ROUND 1: SIM. EQUATIONS & DET.

**ANSWERS**

A) $\ell = 300,\ w = 100$

B) 740

C) 2, -19/4

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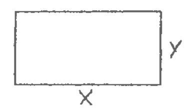
**A)** A rectangular cafeteria has a perimeter of 800 feet, and an area of 30, 000 square feet. Find the number of feet in the length and in the width of the cafeteria.




$x + y = 400, \quad xy = 30{,}000$

$x(400-x) = 30{,}000, \quad x^2 - 400x + 30{,}000 = 0$

$(x-100)(x-300) = 0$

$x = 300, \quad y = 100$

**B)** Lia is offered two different salary options to sell health club memberships. Plan A is: \$500 weekly in addition to 6% of her sales; while Plan B is: \$700 weekly in addition to 1% of her sales. Lia realized that she could get the same salary for a week under both plans. What is that salary?

$y = .06x + 500, \quad y = .01x + 700$

$.06x + 500 = .01x + 700, \quad .05x = 200, \quad x = 4000$

$y = 740$

**C)** Solve for x: $\begin{vmatrix} 1 & -3 & x \\ x & 4 & -1 \\ -2 & 4 & 1 \end{vmatrix} = 40$

$$\begin{matrix} 1 & -3 & x & 1 & -3 \\ x & 4 & -1 & x & 4 \\ -2 & 4 & 1 & -2 & 4 \end{matrix}$$

$4 - 6 + 4x^2 + 3x + 4 + 8x = 40$

$4x^2 + 11x + 2 = 40, \quad 4x^2 + 11x - 38 = 0$

$(4x + 19)(x - 2) = 0$

$x = 2$ or $-19/4$

MML 05

**Round One:**

A. c = 5 so b = – 1 so a = –10 so abc = 50.

B. st = 49 = s(17-s) Use quadratic formula to get $s = \dfrac{17 \pm \sqrt{93}}{2} = 8.5 \pm 0.5\sqrt{93}$

C. No solution if determinant of coef. matrix = 0 Solving $\begin{vmatrix} 1 & 2 & 3 \\ 2 & -1 & c \\ c & -2 & 6 \end{vmatrix} = 0$ we have $-6 + 2c^2 - 12 + 2c + 3c - 24 = 0$ so c = -6 or 3.5

MML 06 **Round One:**

A. $(2 + 0 + x^2) - (0 + x^2 + x) = 5$ means $2 - x = 5$, so $x = -3$.

B. First equation simplifies to $x - 1 = 2y + 1$; sub for $2y + 1$ in second to get $2(x-1)^2 = 50$, so $x - 1 = \pm 5$. If $x = 6, y = 2$; if $x = -4, y = -3$.

C. First eqtn: $y^2 = [(1 + x\sqrt{2})(1 - x\sqrt{2})]^2$ so $y = \pm(1 - 2x^2)$ so $3x + 1 = 1 - 2x^2$ meaning $x = 0$ or $x = -3/2$; or $3x + 1 = 2x^2 - 1$ meaning $x = 2$ or $x = -1/2$
The sum of the four numbers is 0.

MML 07 **Round 1**

A) Adding and substituting $x = 5$, $2ax = 20 \rightarrow$ **$\underline{a = 2}$**
Subtracting and substituting $y = 15$, $2by = -10 \rightarrow$ **$\underline{b = -1/3}$**

B) The matrix equation is equivalent to: $\begin{cases} 3x + ay = 7 \\ bx + 4y = -6 \end{cases}$. Substituting, $\begin{cases} 15 + a = 7 \\ 5b + 4 = -6 \end{cases}$
$\rightarrow (a, b) = (-8, -2)$
Evaluating the determinant, $12 - (-8)(-2) =$ **$\underline{-4}$**

C) Think of -21 as $0x^2 + 0x - 21$

As a single fraction, the left hand side is $\dfrac{A(2x^2 + 5x - 3) + B(x^2 + 3x) + C(2x^2 - x)}{2x^3 + 5x^2 - 3x}$

Re-arranging terms, $\dfrac{(2A + B + 2C)x^2 + (5A + 3B - C)x - 3A}{2x^3 + 5x^2 - 3x}$

Thus, $\begin{cases} 2A + B + 2C = 0 \\ 5A + 3B - C = 0 \\ -3A = -21 \end{cases} \rightarrow A = 7$ and $\begin{cases} B + 2C = -14 \\ 3B - C = -35 \end{cases} \rightarrow (B, C) = (-12, -1)$

and the required sum is 7 + (-12) + (-1) = **$\underline{-6}$**.

MML 08

A) The slope of the first line is 2 → slope of the second line $= -\dfrac{A}{B} = -\dfrac{1}{2} \rightarrow B = 2A$

Thus, $A\left(\frac{1}{2}\right) + 2A(2) = -9 \rightarrow A = -2 \rightarrow (A, B) =$ **$\underline{(-2, -4)}$**

B) Determining the determinant using the weaving technique,

$$\begin{matrix} 1 & x & -3 & 1 & x \\ 2 & 0 & -1 & 2 & 0 \\ -3 & 2x-3 & x & -3 & 2x-3 \end{matrix} \rightarrow [1(0)x + x(-1)(-3) + -3(2)(2x-3)] - [-3(0)(-3) + (2x-3)(-1)(1) + x(2)(x)]$$

$\rightarrow 3x - 12x + 18 - 2x^2 + 2x - 3 = -7 \rightarrow 2x^2 + 7x - 22 = (2x + 11)(x - 2) = 0 \rightarrow$ **$\underline{-11/2, +2}$**

C) Let $(x, y, z)$ denote the cost of (pizza, sub, spaghetti)
Then (a) $3x + 2y = 4 + 5z \rightarrow$ $3x + 2y - 5z = 4$,
(b) $2(y + z) = 3x - 2 \rightarrow$ $-3x + 2y + 2z = -2$
(c) $x = y + z - 3 \rightarrow$ $x - y - z = -3$
(a) – (b) $\rightarrow 4y - 3z = 2 \rightarrow 4y - 3z = 2$
3(c) + (b) $\rightarrow -y - z = -11 \rightarrow -4y - 4z = -44 \rightarrow -7z = -42 \rightarrow z = 6, y = 5$ and $x = 8 \rightarrow$ **<u>($8, $5, $6)</u>**