GREATER BOSTON MATHEMATICS LEAGUE

MEET 5 – MARCH 1999

ROUND 4 – Algebra 2

1. $b=$ ________________

2. ________________

3. ________________

CALCULATORS ARE NOT ALLOWED ON THIS ROUND

1. Given the quadratic equation in x, $x^2 - ax + b = 0$, such that the difference of its roots is 1, find b in terms of a.

2. The solution for x for the equation, $2^{2x-3} = 3^{2-x}$, can be put in the form $\log_b a$ where a and b are positive integers. Under these conditions, find the smallest possible value for $a+b$.

3. Solve the following equation for x: $\sqrt[3]{8x+16} + \sqrt[3]{x^2+4x+4} = 15$

GREATER BOSTON MATHEMATICS LEAGUE

MEET 5 – MARCH 2000

ROUND 4 – Algebra 2

1. ____________________

2. ____________________

3. ____________________

CALCULATORS ARE NOT ALLOWED ON THIS ROUND

1. Given a rectangle whose length is 8 cm longer than its width and the ratio of its area to its perimeter equals 3 cm^2: 2 cm , find the number of square centimeters in the area of this rectangle.

2. Solve the following equation for x. Put the result in simplest radical form.

   $\log_3 2 + \log_9 7 = \log_{27} x$

3. Given the function, f, such that $f(x) = kx^2 + 6x + 4k$, find all real k such that the minimum value of f is positive.

GREATER BOSTON MATHEMATICS LEAGUE

MEET 5 – MARCH 2001

ROUND 4 – Algebra 2

1. ____________________

2. (,) ____________

3. ____________________

CALCULATORS ARE NOT ALLOWED ON THIS ROUND

1. Find all values for x satisfying the following equation: $\log_{2/3}(x+3) = -2 + \log_{2/3}(x-2)$

2. Find the ordered pair (x, y), where x and y are rational, satisfying the following equation: $12^{x+y} = 6 \cdot 18^{x-2y}$

3. Find all values for x satisfying the following inequality: $\dfrac{x}{|x-2|} > 2$

GREATER BOSTON MATHEMATICS LEAGUE

MEET 5 – MARCH 2002

ROUND 4 – Algebra 2
Problems submitted by Maimonides.

1. ____________________

2. ____________________

3. ____________________

CALCULATORS ARE NOT ALLOWED ON THIS ROUND

1. Solve the following equation for x: $\log_{25} x = 18\log_{x^4} 5$.

2. Four positive numbers form a geometric sequence. The sum of these four numbers divided by the sum of first two numbers is 37. If the first number is a, find the fourth number in terms of a.

3. If k is added to each of the numbers 4, 124, and 316, the results are the squares of consecutive terms of an arithmetic sequence. Solve for k.

GREATER BOSTON MATHEMATICS LEAGUE
MEET 5 – MARCH 2006

ROUND 4 – Algebra 2: Open

1. ____________________

2. ____________________

3. ____________________

CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

1. Evaluate: $\dfrac{4\ln e^3 + 8\ln\dfrac{1}{\sqrt[3]{e}}}{\log_{16} 8 - \log_9 243}$

   (Note: $\ln = \log_e$)

2. Find all values of x, $x \in \Re$, which make the following statement true.

$$\sqrt{2x-1} = 4\sqrt[4]{2x-1} - 3$$

3. The roots of the equation $3x^2 - 7x - 1 = 0$ are r_1 and r_2. Find the equation whose roots are $\dfrac{1}{2r_1}$ and $\dfrac{1}{2r_2}$. Write the equation in the form $Ax^2 + Bx + C = 0$, where A, B and C are integers and the GCF(A, B, C) =1.

GREATER BOSTON MATHEMATICS LEAGUE
MEET 5– March 2007

ROUND 4 – Algebra 2: Open

1. ______________________

2. ______________________

3. ______________________

CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

1. Solve the following equation for x. Put the result in simplest radical form.

$$\frac{1}{\log_4 9} + \log_9 11 = \log_3 x$$

2. The positive odd integer are arranged in a pattern indicated in the diagram below. What number will be found in the 20th row, 16th column?

⇦columns⇨

rows⇳	1	2	3	4	5
1	1				
2	3	5			
3	7	9	11		
4	13	15	17	19	
5	21	23	25	27	29

3. Given the equation $4x^2 + kx + 2 = 0$. Find <u>all</u> values of k for which 3 times the positive difference of the roots equals the sum of the roots.

GREATER BOSTON MATHEMATICS LEAGUE

MEET 5 – MARCH 2008

ROUND 4 – Algebra 2: Open

1. ______________________

2. ______________________

3. ______________________

CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

1. Compute the exact value in simplified form. $\left(\left(2\cdot\sqrt[3]{343}+7^0\right)^{-1}\cdot\sqrt{\frac{1}{81}+\frac{1}{144}}\right)^{-\frac{1}{3}}$

2.

3. Find all values of x which satisfy $x^2+2\le|x^2-3x-4|$.

GREATER BOSTON MATHEMATICS LEAGUE

MEET 5 – FEBRUARY 2009

ROUND 4 – Algebra 2: Open

1. ______________________

2. ______________________

3. (_____ , _____ , _____ , _____)

CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

1. If $\log 5 = m$ and $\log 7 = p$, find $\log \sqrt[4]{\dfrac{392}{25}}$ in terms of m and p.
   Note: The above expressions are common logs, i.e. base 10.

2. Factor completely over the integers: $4^{x+1} - 2^{x+5} - 36$

3. The equation $x^2 - 2ax + 6b = 0$ has <u>positive</u> integral roots p and q, where $p > q$. Twelve times the sum of the roots equals five times the product of the roots. Determine the quadruple (a, b, p, q) for which this is true.

GREATER BOSTON MATHEMATICS LEAGUE

MEET 5 – FEBRUARY 2010

ROUND 4 – Algebra 2: Open

1. ______________________________

2. ______________________________

3. ______________________________

CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

1. Determine the integer k for which the sum of the following natural numbers is 627.

$$4k-10,\ 4k-9,\ 4k-8,\ \ldots\ 4k+9,\ 4k+10,\ 4k+11$$

2. Find all values of x, $x \in$ {real numbers}, such that

$$\log_4 \sqrt{2} + \log_9 \sqrt[3]{3} = \log_{64} \sqrt{2} + \left(\log_{A^2} 16\right)\left(\log_x A\right)$$

3. The three roots of the following equation form an arithmetic sequence. Find all rational values of k for which this is true.

$$4x^3 - 48x^2 + k^2x - 5k - x = 0$$

GREATER BOSTON MATHEMATICS LEAGUE

MEET 5 – MARCH 2011

ROUND 4 – Algebra 2: Open

1. $x =$ ____________________

2. (________ , ________)

3. ____________________

CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

1. Given: $L_1 : \{(x, y) \mid 3x + 5y = 28\}$ and $L_2 : \{(x, y) \mid 4x - 7y = 10\}$
Find the value of the x-intercept of the line through P ($P \in L_1 \cap L_2$) whose x-intercept is three times its y-intercept.

2. The equation $2x^3 + 6x^2 + Kx + J = 0$ has one root twice the other, while the third root is 3. Compute the ordered pair (K, J).

3. Solve for x over the reals. $\dfrac{x-1}{3x} - \dfrac{x}{x-1} + \dfrac{1}{x^2 - x} < 0$

'99 '11

ROUND 4

1. Since the difference of the roots is 1, call the roots r and $r+1$; $2r+1=a$ and $r(r+1)=b$
$\Rightarrow r=\frac{a-1}{2}$ and $b=\left(\frac{a-1}{2}\right)\left(\frac{a-1}{2}+1\right)=\left(\frac{a-1}{2}\right)\left(\frac{a+1}{2}\right)=\mathbf{\frac{a^2-1}{4}}$ or $\mathbf{\frac{1}{4}a^2-\frac{1}{4}}$

2. $2^{2x-3}=3^{2-x}\Rightarrow(2x-3)\log 2=(2-x)\log 3\Rightarrow 2x\log 2-3\log 2=2\log 3-x\log 3\Rightarrow$
$x(2\log 2+\log 3)=3\log 2+2\log 3\Rightarrow x\log 12=\log 72\Rightarrow x=\log_{12}72\Rightarrow a+b=\mathbf{84}$

3. $\sqrt[3]{8x+16}+\sqrt[3]{x^2+4x+4}=15\Rightarrow\sqrt[3]{8(x+2)}+\sqrt[3]{(x+2)^2}=15\Rightarrow$
$(x+2)^{2/3}+2(x+2)^{1/3}-15=0\Rightarrow\left((x+2)^{1/3}+5\right)\left((x+2)^{1/3}-3\right)=0\Rightarrow$
$(x+2)^{1/3}=-5$ or $3\Rightarrow x+2=-125$ or $27\Rightarrow x=\mathbf{-127}$ or $\mathbf{25}$

'00

ROUND 4

1. Call the width of the rectangle $x\Rightarrow$ length is $x+8\Rightarrow$ area is $x(x+8)$ and the perimeter is $4x+16$; $\frac{x^2+8x}{4x+16}=\frac{3}{2}\Rightarrow x^2+8x=6x+24\Rightarrow x^2+2x-24=0\Rightarrow$
$(x+6)(x-4)=0\Rightarrow x=4\Rightarrow area=4\cdot 12=48$

2. $\log_3 2+\log_9 7=\log_{27}x\Rightarrow\frac{\log 2}{\log 3}+\frac{\log 7}{\log 9}=\frac{\log x}{\log 27}\Rightarrow\frac{\log 2}{\log 3}+\frac{\log 7}{2\log 3}=\frac{\log x}{3\log 3}\Rightarrow$
$\left(\frac{\log 2}{\log 3}+\frac{\log 7}{2\log 3}\right)6\log 3=\left(\frac{\log x}{3\log 3}\right)6\log 3\Rightarrow 6\log 2+3\log 7=2\log x\Rightarrow$
$\log(2^6\cdot 7^3)=\log x^2\Rightarrow x=\sqrt{2^6\cdot 7^3}=2^3\cdot 7\sqrt{7}=56\sqrt{7}$ [Note x must be greater than 0.]

3. The minimum value for the quadratic function occurs at its vertex. $\Rightarrow x=-\frac{b}{2a}$
$\Rightarrow x=-\frac{3}{k}\Rightarrow f\left(-\frac{3}{k}\right)=k\left(-\frac{3}{k}\right)^2+6\left(-\frac{3}{k}\right)+4k=\frac{9}{k}-\frac{18}{k}+4k=4k-\frac{9}{k}$;
$4k-\frac{9}{k}>0\Rightarrow\frac{4k^2-9}{k}>0\Rightarrow\frac{(2k-3)(2k+3)}{k}>0$; key numbers for this inequality are $-\frac{3}{2}, 0, \frac{3}{2}$; Since the parabola opens up, $k>0$, The values for k that makes the fraction greater than 0 in relation to these conditions are: $k>\frac{3}{2}$

'01

ROUND 4

1. $\log_{2/3}(x+3)=-2+\log_{2/3}(x-2)\rightarrow 2=\log_{2/3}(x-2)-\log_{2/3}(x+3)\rightarrow$
$2=\log_{2/3}\left(\frac{x-2}{x+3}\right)\rightarrow\frac{x-2}{x+3}=\frac{4}{9}\rightarrow 9x-18=4x+12\rightarrow 5x=30\rightarrow x=6.$

2. $12^{x+y}=6\cdot 18^{x-2y}\rightarrow(2^2\cdot 3)^{x+y}=(2\cdot 3)(2\cdot 3^2)^{x-2y}\rightarrow 2^{2x+2y}\cdot 3^{x+y}=2^1\cdot 3^1\cdot 2^{x-2y}\cdot 3^{2x-4y}$
$\rightarrow\begin{cases}2x+2y=1+x-2y\\x+y=1+2x-4y\end{cases}\rightarrow\begin{cases}x+4y=1\\-x+5y=1\end{cases}\rightarrow 9y=2\rightarrow y=\frac{2}{9}\rightarrow x=\frac{1}{9}\rightarrow(x,y)=\left(\frac{1}{9},\frac{2}{9}\right).$

3. $\frac{x}{|x-2|}>2\rightarrow x\neq 2$ and since $|x-2|>0\rightarrow x>2|x-2|$;
if $x>2$: $x>2(x-2)\rightarrow x>2x-4\rightarrow -x>-4\rightarrow x<4$;
if $x<2$: $x>2(2-x)\rightarrow x>4-2x\rightarrow 3x>4\rightarrow x>\frac{4}{3}$; therefore the solution to the inequality is $\frac{4}{3}<x<4$ and $x\neq 2$ or equivalently $\frac{4}{3}<x<2$ or $2<x<4$

'02

ROUND 4 – Algebra 2

1. $\log_{25}x=18\log_{x^4}5\Rightarrow\frac{\log x}{\log 25}=\frac{18\log 5}{\log x^4}\Rightarrow\frac{\log x}{2\log 5}=\frac{18\log 5}{4\log x}\Rightarrow(\log x)^2=9(\log 5)^2\Rightarrow$
$\log x=\pm 3\log 5\Rightarrow\log x=\log 5^{\pm 3}\Rightarrow x=5^{\pm 3}=125,\ \frac{1}{125}.$

2. Let the four terms be $a, ar, ar^2, ar^3\Rightarrow\frac{a+ar+ar^2+ar^3}{a+ar}=37\Rightarrow\frac{1+r+r^2+r^3}{1+r}=37\Rightarrow$
$\frac{(1+r)(1+r^2)}{1+r}=37\Rightarrow 1+r^2=37\Rightarrow r=6\Rightarrow$ fourth term is $216a$.

3. $(i)\,4+k=(a-d)^2$, $(ii)\,124+k=a^2$, and $(iii)\,316+k=(a+d)^2\Rightarrow$
$(ii)-(i):120=2ad-d^2$ and $(iii)-(ii):192=2ad+d^2$. Subtracting these equations:
$2d^2=72\Rightarrow d=\pm 6\Rightarrow\pm 12a+36=192\Rightarrow a=\pm 13\Rightarrow 124+k=169\Rightarrow k=45.$

'06

ROUND 4 – Algebra 2: Open

1. $= \dfrac{4(3)+8(-\frac{1}{3})}{\frac{3}{4}-\frac{5}{2}} \cdot \dfrac{12}{12} = \dfrac{144-32}{9-30} = \dfrac{112}{-21} = \underline{-\dfrac{16}{3}}$

2. Let $y = \sqrt[4]{2x-1}$. Then the original equation becomes $y^2 = 4y - 3 \rightarrow y^2 - 4y + 3 = 0$
$\rightarrow (y-3)(y-1) = 0 \rightarrow y = 3$ or $1 \rightarrow 2x - 1 = 81$ or $1 \rightarrow x = \underline{\mathbf{1, 41}}$

3. Normalized equation: $x^2 - \frac{7}{3}x - \frac{1}{3} = 0 \rightarrow r_1 + r_2 = \frac{7}{3},\ r_1r_2 = -\frac{1}{3}$

sum of new roots: $\dfrac{1}{2r_1} + \dfrac{1}{2r_2} = \dfrac{r_1+r_2}{2r_1r_2} = \dfrac{7/3}{-2/3} = -\dfrac{7}{2}$

product of new roots: $\dfrac{1}{2r_1} \cdot \dfrac{1}{2r_2} = \dfrac{1}{4r_1r_2} = \dfrac{1}{-4/3} = -\dfrac{3}{4}$

New equation: $x^2 + \frac{7}{2}x - \frac{3}{4} = 0 \rightarrow \underline{\mathbf{4x^2 + 14x - 3 = 0}}$

'07

ROUND 4 – Algebra 2: Open

1. $\log_9 4 + \log_9 11 = \log_9 44 = \log_3 x = \log_9(x^2) \rightarrow x^2 = 44 \rightarrow x = \underline{\mathbf{+2\sqrt{11}}}$ \
(The negative root is rejected since the argument of the log function must be positive.)

2. The gaps between the values in the first column (1, 3, 7, 13, …) are increasing by 2.
This tells us that the rule generating these values is quadratic, namely $An^2 + Bn + C$
$(1, 1), (2, 3)$ and $(3, 7) \rightarrow A + B + C = 1,\ 4A + 2B + C = 3$ and $9A + 3B + C = 7$
Subtracting, $3A + B = 2,\ 5A + B = 4$ and subtracting again $\rightarrow (A,B,C) = (1, -1, 1) \rightarrow n^2 - n + 1$
Thus, the first entry in row 20 is $20^2 - 20 + 1 = 381$
To find the entry in the 16th column, add 30. $\rightarrow$ **411**

3. The sum of the roots is $-k/4$. Since the roots of a quadratic in general are $\dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$,
the positive difference between the roots is $\dfrac{\sqrt{b^2-4ac}}{a}$ (provided $a > 0$).
Thus, $3\left(\dfrac{\sqrt{k^2-32}}{4}\right) = -\dfrac{k}{4} \rightarrow 9(k^2-32) = k^2 \rightarrow 8k^2 = 9(32) \rightarrow k = \pm 6$. However, for $k = +6$,
$4x^2 + 6x - 2 = 2(2x+1)(x+1) = 0$ has a root sum of $-3/2$ and a positive root difference of $+1/2$. Thus, $k = +6$ is extraneous and the only value of k is $\underline{-6}$.

'08

ROUND 4

1. $\left(\left(2\cdot\sqrt[3]{343}+7^0\right)^{-1}\cdot\sqrt{\dfrac{1}{81}+\dfrac{1}{144}}\right)^{-\frac{1}{3}} = \left(\dfrac{1}{2\cdot 7+1}\cdot\sqrt{\dfrac{81+144}{81\cdot 144}}\right)^{-\frac{1}{3}}$
$= \left(\dfrac{1}{15}\cdot\dfrac{15}{9\cdot 12}\right)^{-\frac{1}{3}} = (4\cdot 27)^{+\frac{1}{3}} = \underline{\mathbf{3\sqrt[3]{4}}}$

2.

3. Since $|x^2 - 3x - 4| = |(x+1)(x-4)| = \begin{cases} x^2-3x-4 & \text{if } x \le -1 \text{ or } x \ge 4 \\ -x^2+3x+4 & \text{if } -1 < x < 4 \end{cases}$, the original equation is equivalent to:
For $x \le -1$ or $x \ge 4$, $x^2 + 2 \le x^2 - 3x - 4 \rightarrow 3x < -6 \rightarrow \underline{x \le -2}$ (acceptable)
For $-1 < x < 4$, $x^2 + 2 \le -x^2 + 3x + 4 \rightarrow 2x^2 - 3x - 2 \le 0 \rightarrow (2x+1)(x-2) \le 0$
$\rightarrow \underline{\mathbf{-1/2 \le x \le 2}}$ (acceptable)

'09

ROUND 4

1. $\log\sqrt[4]{\dfrac{392}{25}} = \log\left(\left(\dfrac{8\cdot 49}{25}\right)^{\frac{1}{4}}\right) = \dfrac{1}{4}(\log 8 + \log 49 - \log 25) = \dfrac{1}{4}(3(1-m)+2p-2m)$
$= \underline{\mathbf{\dfrac{3+2p-5m}{4}}}$

2. $4^{x+1} - 2^{x+5} - 36 = \left(2^{x+1}\right)^2 - 2^4 2^{x+1} - 36 = \left(2^{x+1}-18\right)\left(2^{x+1}+2\right) = 2\left(2^x-9\right)2\left(2^x+1\right)$
$= \underline{\mathbf{4\left(2^x-9\right)\left(2^x+1\right)}}$

3. The sum of the roots is $2a$ and the product of the roots is $6b$.
Thus, $12(2a) = 5(6b)$ $4a = 5b \rightarrow \begin{cases} p+q = 2a \\ pq = 6b \end{cases}$
Dividing, $\dfrac{p+q}{pq} = \dfrac{a}{3b} = \dfrac{4a}{12b} = \dfrac{5b}{12b} = \dfrac{5}{12} \rightarrow 12(p+q) = 5pq \rightarrow 12q = 5pq - 12p$
$\rightarrow p = \dfrac{12q}{5q-12}$ Since p and q are both positive integers, we start with $q = 3$.
$q = 3 \rightarrow p = \dfrac{36}{15-12} = 12$ $q = 4 \rightarrow p = \dfrac{48}{20-12} = 6$ $q = 6 \rightarrow p = \dfrac{72}{30-12} = 4$
Since $p > q$, the search is over and $(p, q) = (6, 4) \rightarrow (a, b) = (5, 4) \rightarrow$ **(5, 4, 6, 4)**

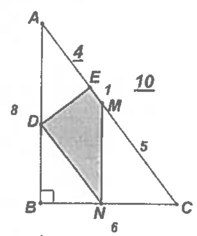
\10

ROUND 4

1. This progression is arithmetic with common difference of 1 and contains 22 terms Excluding $4k + 11$, the sum of the first and last, the second and next to last, etc. always produces sums of $8k$. Thus the sum is $10(8k) + 4k + (4k + 11) = 88k + 11 = 627 \rightarrow k = \underline{7}$.

ROUND 4 - continued

2. First note that the last term is equivalent to $\log_x 4$.



$$\left(\log_{A^2} 16\right)\left(\log_x A\right) = \frac{4\log 2}{\log\left(A^2\right)} \cdot \frac{\log A}{\log x} = \frac{2\log 2}{\log x} = \frac{\log 4}{\log x} = \log_x 4$$

$4^A = 2^{\frac{1}{2}}$, $9^B = 3^{\frac{1}{3}}$ and $64^C = 2^{\frac{1}{2}} \rightarrow \frac{1}{4} + \frac{1}{6} = \frac{1}{12} + \log_x 4 \rightarrow \frac{1}{3} = \log_x 4 \rightarrow x = 4^3 = \underline{64}$

3. Let $(r_1, r_2, r_3) = (a-d, a, a+d)$, where a and d denote the first term and common difference of the arithmetic progression respectively. Then the sum of the roots (call it S) is simply $3a$. Rewriting the equation, we have

$$4x^3 - 48x^2 + k^2x - 5k - x = 0 \rightarrow 4x^3 + (-48)x^2 + \left(k^2 - 1\right)x - 5k = 0$$

Using the root-coefficient relationships, $S = \frac{+48}{4} = 12 = 3a \rightarrow a = 4$ and the roots of the equation are $4 - d$, 4 and $4 + d$. Since 4 is a root we have

$$\underbrace{4\left(4^3\right) - 48\left(4^2\right)}_{16\cdot 16 \;-\; 48\cdot 16} + \left(k^2 - 1\right)4 - 5k = 0 \rightarrow 4k^2 - 5k - 32\cdot 16 - 4 = 0 \rightarrow 4k^2 - 5k - 516 = 0$$

$\rightarrow (k - 12)(4k + 43) = 0 \rightarrow k = \underline{-\frac{43}{4}, 12}$

\11

ROUND 4

1. $\begin{aligned}(4)3x + 5y &= 28\\(-3)4x - 7y &= 10\end{aligned} \rightarrow 41y = 112 - 30 = 82 \rightarrow P(x, y) = P(6, 2)$

Suppose the x-intercept (a) is located at $(a, 0)$ and the y-intercept (b) is located at $(0, b)$.

Thus, $a = 3b$ and the slope $m = \frac{b-0}{0-a} = \frac{b}{-a} = \frac{b}{-3b} = -\frac{1}{3}$.

The equation of the required line is $(y - 2) = -\frac{1}{3}(x - 6) \rightarrow 3y - 6 = -x + 6 \rightarrow x + 3y = 12$

Substituting, $a + 3(0) = 12 \rightarrow a = \underline{12}$.

2. Suppose the roots are a_1, a_2 and a_3. The sum of the roots is $\left(-\frac{6}{2}\right) = -3$.

$\rightarrow a_1 + a_2 + a_3 = a_1 + 2a_1 + 3 = 3a_1 + 3 = -3 \rightarrow a_1 = -2$, $a_2 = -4$ and $a_3 = 3$.

$(x+2)(x+4)(x-3) = \left(x^2 + 6x + 8\right)(x-3) = x^3 + 3x^2 - 10x - 24 = 0$

Multiply through be 2 to get a lead coefficient of 2, $2x^3 + 6x^2 - 20x - 48 = 0$.

Thus, $(K, J) = (-20, -48)$.

Alternate Method:

The original equation must be satisfied by $x = -2 \rightarrow -16 - 2K + 24 + J = 0 \rightarrow -2K + J = -8$ (1)

The original equation must be satisfied by $x = 3 \rightarrow 54 + 3K + 54 + J = 0 \rightarrow 3K + J = -108$ (2)

Subtracting (2) from (1), $-5K = 100 \rightarrow K = -20$

Substituting is (2), $-60 + J = -108 \rightarrow J = -48$

Thus, $(J, K) = \underline{(-20, -48)}$.

3. $\frac{x-1}{3x} - \frac{x}{x-1} + \frac{1}{x^2 - x} < 0 \rightarrow \frac{(x-1)^2 - x(3x) + 3}{3x(x-1)} < 0 \rightarrow \frac{-2x^2 - 2x + 4}{3x(x-1)} < 0 \rightarrow \frac{-2(x+2)(x-1)}{3x(x-1)} < 0$

Canceling the common factor and dividing through by -2, $\frac{x+2}{x} > 0$, provided $x \neq 1$.

Thus, $\underline{x < -2 \text{ or } x > 0 \text{ and } x \neq 1}$.