Round 3 Geometry

Angles and Triangles

GREATER BOSTON MATHEMATICS LEAGUE

MEET 2 – NOVEMBER 1998

ROUND 3 – Geometry: Angles and Triangles

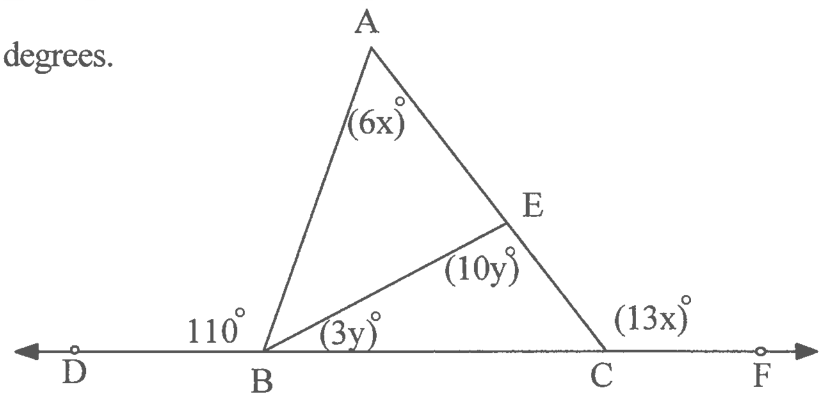
1. ____________________

2. ____________________

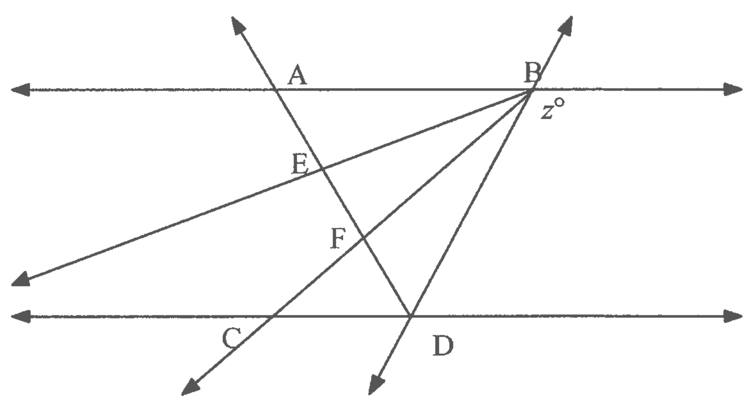
3. ____________________

DIAGRAMS ARE NOT NECESSARILY DRAWN TO SCALE
CALCULATORS ARE NOT ALLOWED ON THIS ROUND

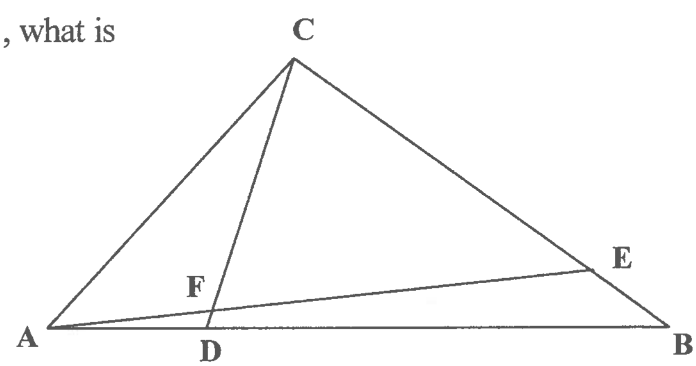
1. Find the measure of $\angle$ AEB in degrees.



2. If $\overleftrightarrow{AB}$ is parallel to $\overleftrightarrow{CD}$, $\overleftrightarrow{BE}$ and $\overleftrightarrow{BF}$ trisects $\angle$ABD, $\overleftrightarrow{DA}$ bisects $\angle$CDB, and m $\angle$BED = 77°, find z.



3. Given m $\angle$CBA : m $\angle$BAC : m $\angle$ACB = 3:4:8, $\overline{BD} \cong \overline{BC}$, and $\overline{CA} \cong \overline{CE}$, what is measure of $\angle$ AFC in degrees?



GREATER BOSTON MATHEMATICS LEAGUE

MEET 2 – NOVEMBER 1999

ROUND 3 – Geometry: Angles and Triangles

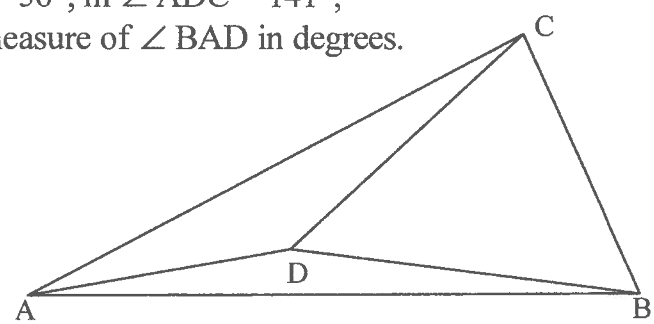
1. ____________________

2. ____________________

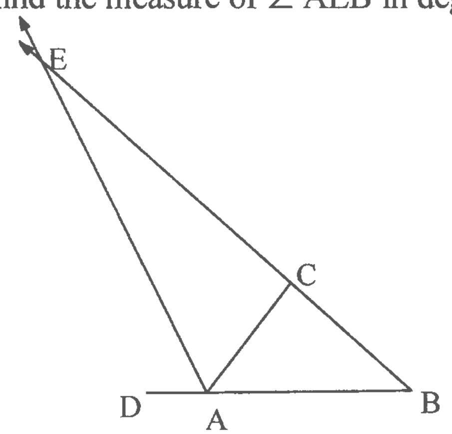
3. ____________________

DIAGRAMS ARE NOT NECESSARILY DRAWN TO SCALE
CALCULATORS ARE NOT ALLOWED ON THIS ROUND

1. Given m $\angle$ CAB = 36°, m $\angle$ BCD = 56°, m $\angle$ ADC = 141°, AC = AB, and CB = CD, find the measure of $\angle$ BAD in degrees.



2. If m $\angle$ CAD = $(2x^2)°$, m $\angle$ ABC = $(5x+2)°$, m $\angle$ ACB = $(10x+6)°$, and the bisector of $\angle$ CAD intersects $\overrightarrow{BC}$ at point E, find the measure of $\angle$ AEB in degrees.



3. The measures of consecutive angles of a convex polygon are 171°, 173°, 176°, 171°, 173°, 176°, … , 171°, 173°, 176°, the same group of three measures appearing an integral number of times. Find the number of sides for this polygon.

GREATER BOSTON MATHEMATICS LEAGUE

MEET 2 – NOVEMBER 2000

ROUND 3 – Geometry: Angles and Triangles

1. ____________________

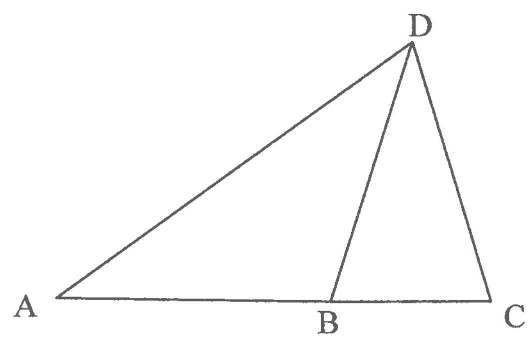
2. ____________________

3. ____________________

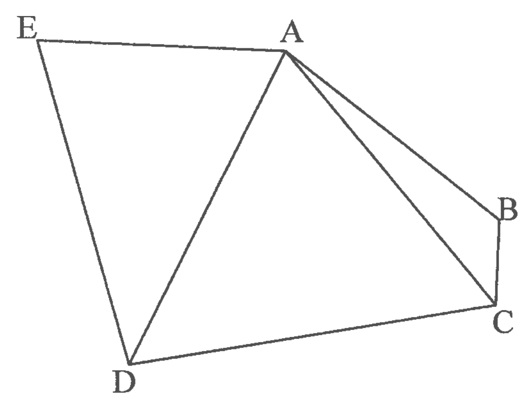
DIAGRAMS ARE NOT NECESSARILY DRAWN TO SCALE
CALCULATORS ARE NOT ALLOWED ON THIS ROUND

1. The supplement of the complement of an angle is 6° less than the supplement of the angle. Find the number of degrees in the measurement of the angle.

2. Given $\overline{ABC}$, AB = BD = CD, and $m\angle ADC = 66°$, compute the number of degrees in $m\angle C$.



3. The ratio of the measures of consecutive exterior angles of convex pentagon ABCDE at vertices A, B, C, D, and E is 2:3:4:5:6, respectively. If $\overrightarrow{AD}$ bisects $\angle CAE$ and $\angle ADE \cong \angle ACB$, compute the number of degrees in $m\angle BAC$.



GREATER BOSTON MATHEMATICS LEAGUE

MEET 2 – NOVEMBER 2001

ROUND 3 – Geometry: Angles and Triangles

1. ____________________

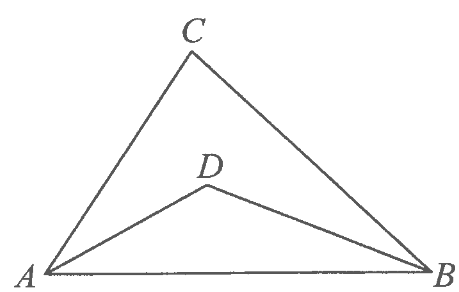
2. ____________________

3. ____________________

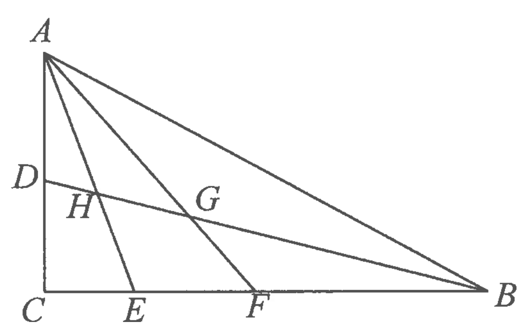
DIAGRAMS ARE NOT NECESSARILY DRAWN TO SCALE
CALCULATORS ARE NOT ALLOWED ON THIS ROUND

1. Given ΔABC with $\overline{ADC}$, $\overline{BD} \perp \overline{AC}$, and $BA:BC:BD = 2:\sqrt{2}:1$. Find the number of degrees in the measure of $\angle ABC$.

2. Given $\overline{AD}$ bisects $\angle BAC$, $\overline{BD}$ bisects $\angle ABC$, and $m\angle ACB + m\angle ADB = 210°$, find the number of degrees in $m\angle ACB$.



3. Given $\angle C$ is right, $\overline{BGHD}$, $\overline{AHE}$, $\overline{AGF}$, $\overline{ADC}$, $\overline{CEFB}$, $\overline{BD}$ bisects $\angle ABC$, $\overline{AE}$ and $\overline{AF}$ trisect $\angle BAC$. If $m\angle EHG - m\angle GFE = 10°$, find the number of degrees in $m\angle AGB$.



GREATER BOSTON MATHEMATICS LEAGUE
MEET 2 – NOVEMBER 2006

ROUND 3 – Geometry - Angles and Triangles

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2. ______________________ °

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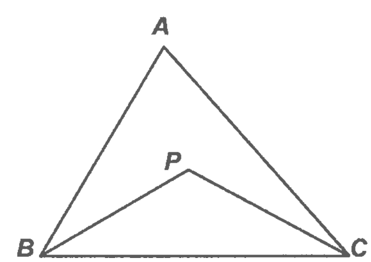
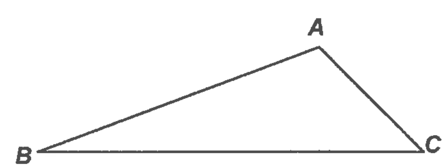
DIAGRAMS ARE NOT NECESSARILY DRAWN TO SCALE.
CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

1. The exterior angle of the base angle of an isosceles triangle is 12 more than twice the vertex angle. How many degrees are there in the vertex angle?

2. In ΔABC, $\overline{PB}$ bisects $\angle B$ and $\overline{PC}$ divides $\angle C$ so that $m\angle PCB = 2m\angle PCA$. If $PB = PC$ and $m\angle A = m\angle PCA$, find $m\angle BPC$.



3. The complement of the supplement of A is 4 less than $\frac{1}{3}$ the supplement of the complement of C. If $m\angle A = 8m\angle B$, find the measure of $\angle B$.



GREATER BOSTON MATHEMATICS LEAGUE

MEET 2 – NOVEMBER 2007

ROUND 3 – Geometry - Angles and Triangles

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2. ______________________________ °

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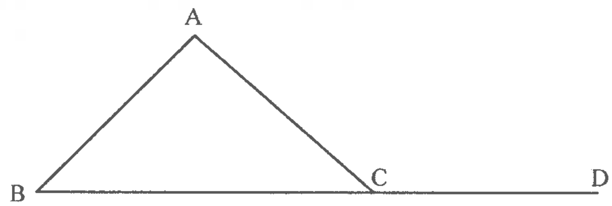
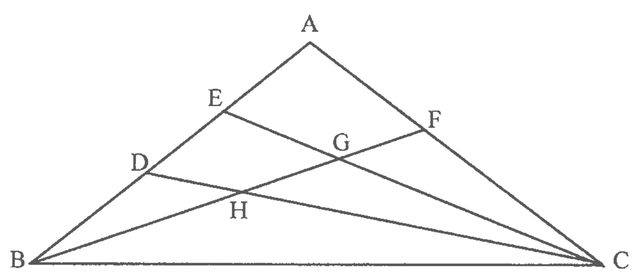
DIAGRAMS ARE NOT NECESSARILY DRAWN TO SCALE.
CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

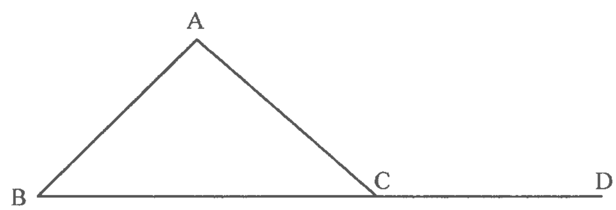
1. Given ΔABC, $m\angle A = (4x)^\circ, m\angle B = (6x-10)^\circ$ and $m\angle ACD = (9x+2)^\circ$. The supplement of the smallest angle of ΔABC is 6° less than three times the complement of an angle whose measure is t°. Find the value of t.



2. Given the diagram, $m\angle A = 68^\circ$, $m\angle ABC : m\angle ACB = 4:3$, $\angle ABC$ is bisected by $\overline{BF}$, and $\angle ACB$ is trisected by $\overline{EC}$ and $\overline{DC}$. Find $m\angle BHC - m\angle EGH$.



3. The supplement of the complement of $m\angle A$ is 8° more than the $m\angle ACD$. Find the measure $\angle B$.



GREATER BOSTON MATHEMATICS LEAGUE

MEET 2 – NOVEMBER 2008

ROUND 3 – Geometry - Angles and Triangles

1. ______________________

2. ______________________

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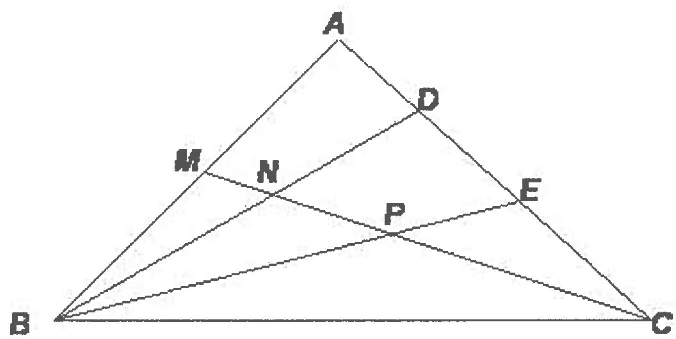
DIAGRAMS ARE NOT NECESSARILY DRAWN TO SCALE.
CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

1. In ΔABC, the ratio of the measures of the angles is $3:4:8$. In ΔDEF, $m\measuredangle D$ is the same as the measure of the smallest angle of ΔABC. If angles E and F are bisected, what is the measure of the obtuse angle formed by the bisectors?

2. In an isosceles triangle the exterior angle of one of the base angles has measure $(3x+3)^\circ$ The vertex angle has measure $(x-9)^\circ$ Find the ratio of the numerical measure of the vertex angle to the measure of a base angle.

3. In ΔABC, $\overline{MC}$ bisects $\measuredangle ACB$, $\overline{BD}$ and $\overline{BE}$ are drawn such that the ratio of the measures of $\measuredangle BMN : \measuredangle BNP : \measuredangle BPC : \measuredangle MCB = 9:10:11:3$.
Determine the numerical value of the ratio $m\measuredangle BDE : m\measuredangle ABE$.



GREATER BOSTON MATHEMATICS LEAGUE

MEET 2 – NOVEMBER 2009

ROUND 3 – Geometry - Angles and Triangles

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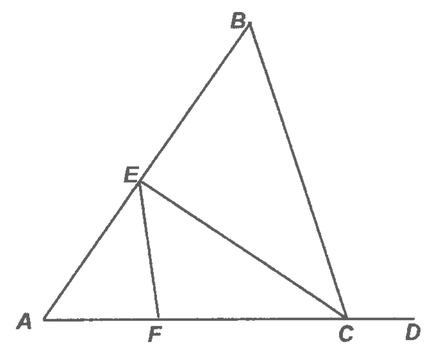
2. ______________________ °

3. ______________________

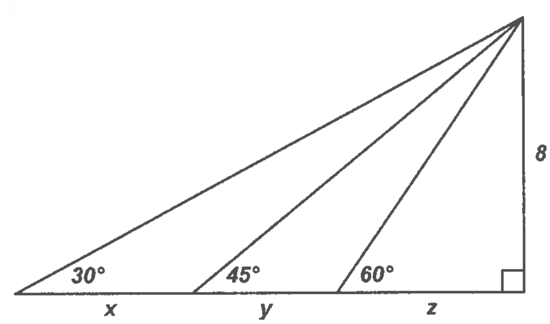
DIAGRAMS ARE NOT NECESSARILY DRAWN TO SCALE.
CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

1. The complement of the supplement of an angle G is trisected and the result is 14°. How many degrees are in the supplement of G?

2. In ΔABC, $m\angle BCD = 114°$, $\overline{CE}$ bisects $\angle ACB$, $\overline{EF}$ bisects $\angle AEC$ and $m\angle B = 28°$. How many degrees are in the complement of $\angle AEF$?



3. Find the exact numerical value of $\frac{y}{x}$ as a simplified fraction.



GREATER BOSTON MATHEMATICS LEAGUE

MEET 2 – NOVEMBER 2010

ROUND 3 – Geometry - Angles and Triangles

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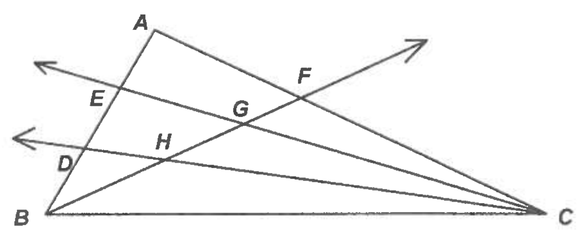
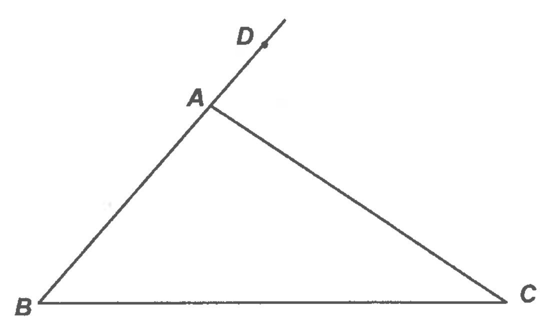
DIAGRAMS ARE NOT NECESSARILY DRAWN TO SCALE.
CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

1. The supplement of the complement of the supplement of an angle whose measure is $x°$ equals $\left(\frac{2}{3}x\right)^{\circ}$. Compute the value of x.

2. Given: $\angle ABC$ is bisected by ray $\overrightarrow{BF}$
$\angle ACB$ is trisected by rays $\overrightarrow{CD}$ and $\overrightarrow{CE}$
$m\angle BFC : m\angle BAC = 13 : 8$
$m\angle BGC = 116°$
Compute $m\angle AEC : m\angle BHC$.



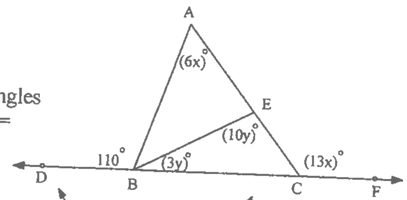
3. In ΔABC, $m\angle B = (4x)°$, $m\angle DAC = (185 - 7x)°$ and $5m\angle BAC = 6(m\angle C) - 24°$.
If the ratio of $m\angle BAC : m\angle C = P : Q$, where P and Q are relatively prime, compute the ordered triple (x, P, Q).



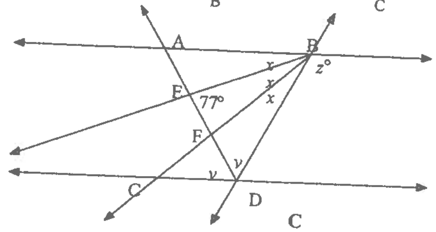
GBML 98

ROUND 3

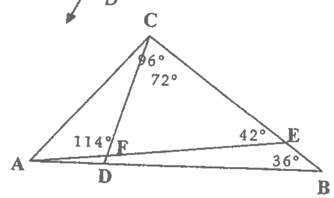
1. $13x = 3y + 10y \Rightarrow x = y$ Also, $13x + 110 + 180 - 6x = 360$ (Sum of ext. angles $= 360°$) $\Rightarrow x = 10 \Rightarrow y = 10 \Rightarrow$ m $\angle$ AEB $= 180° - 100° =$ **80°**



2. Because of the parallel lines: $3x + 2y = 180°$ and because of **Δ** BED, $2x + y = 103° \Rightarrow$ $x = 26°$ and $z = 180 - 78 =$ **102**

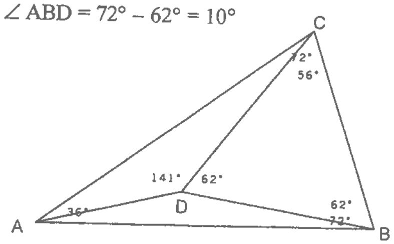


3. $3x + 4x + 8x = 180° \Rightarrow x = 12° \Rightarrow$ 36°, 48°, and 96° are the angles of the triangle. $\Rightarrow$ m $\angle$ AEC $= (180° - 96°) \div 2 = 42°$ and m $\angle$ BCD $= (180° - 36°) \div 2 = 72° \Rightarrow$ m $\angle$ AFC $= 42° + 72° =$ **114°**



GBML 99

1. m $\angle$ ACB = m $\angle$ ABC = 72°; m $\angle$ CDB = m $\angle$ CBD = 62°; m $\angle$ ADB = 360° − 141° − 62° = 157°; m $\angle$ ABD = 72° − 62° = 10° m $\angle$ BAD = 180° − 157° − 10° = 13°



2. By the exterior angle theorem: $2x^2 = 15x + 8 \Rightarrow 2x^2 - 15x - 8 = 0 \Rightarrow (2x+1)(x-8) = 0 \Rightarrow x = 8$;

Note $x = -\frac{1}{2}$ leads to a negative measure for $\angle$ ABC.

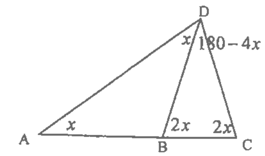
m $\angle$ CAD = 128°; m $\angle$ ABC = 42°; Therefore m $\angle$ AEB = 64° − 42° = 22°

3. The exterior angles are 9°, 7°, 4°, 9°, 7°, 4°, …, 9°, 7°, 4° where every three add to 20°; since the exterior angles must add to 360°, the number of sides = 360 ÷ 20 × 3 = 54.

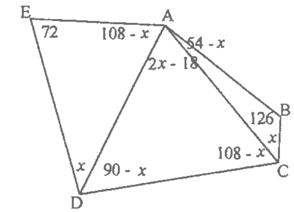
GBML 00

1. $180 - (90 - x) = (180 - x) - 6 \to 90 + x = 174 - x \to 2x = 84 \to x = 42$

2. $180 - 4x + x = 66 \to 3x = 114 \to x = 38$ $\to 2x = 76$

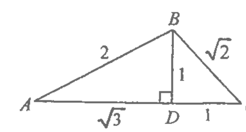


3. $2a + 3a + 4a + 5a + 6a = 360 \to 20a = 360 \to$ $a = 18 \to m\angle ABC = 180 - 3(18) = 126$;
$m\angle BCD = 180 - 4(18) = 108$;
$m\angle CDE = 180 - 5(18) = 90$
$m\angle AED = 180 - 6(18) = 72$; $m\angle ADC = 90 - x$ and
$m\angle ACD = 108 - x \to m\angle DAC = 2x - 18$;
$m\angle DAE = 108 - x \to 108 - x = 2x - 18 \to x = 42 \to$
$m\angle BAC = 54 - 42 = 12$

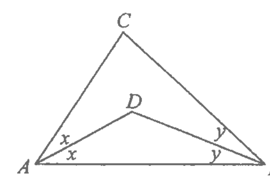


GBML 01

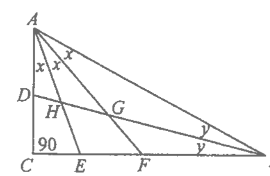
1. There is no loss of generality to let $AB = 2$, $BD = 1$ and $BC = \sqrt{2} \Rightarrow AD = \sqrt{3}$ and $DC = 1 \Rightarrow$ $m\angle ABD = 60°$ *and* $m\angle CBD = 45° \Rightarrow m\angle ABC = 105°$.



2. Let $z = x + y \Rightarrow m\angle ACB = (180 - 2z)°$ and $m\angle ADB = (180 - z)° \Rightarrow 360 - 3z = 210 \Rightarrow z = 50 \Rightarrow$ $m\angle ACB = 80°$.

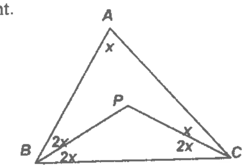


3. Let $m\angle CAE = m\angle FAE = m\angle BAF = x°$;
let $m\angle ABD = m\angle CBD = y°$; $3x + 2y = 90$;
by the exterior angle theorem, $m\angle EHG = (2x + y)°$
and $m\angle GFE = (2y + x)° \Rightarrow (2x + y) - (2y - x) = 10$
$\Rightarrow x - y = 10 \Rightarrow 2x - 2y = 20 \Rightarrow 5x = 110 \Rightarrow x = 22 \Rightarrow$
$y = 12 \Rightarrow m\angle AGB = (180 - 22 - 12)° = 146°$.



GBML 06

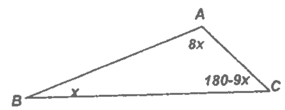
1. Let the measure of the base angle be $x°$. Then the vertex angle is $(180 - 2x)°$ and the exterior angle at the base is $(180 - x)°$. $180 - x = 12 + 2(180 - 2x) \rightarrow 180 - x = 372 - 4x$ $\rightarrow 3x = 192 \rightarrow x = 64$ Thus, the vertex angle measures $180 - 2(64) = \underline{52°}$.
2. The given information allows marking ΔABC as indicated at the right. $m\angle BPC = 180 - 4x$. Since $8x = 180 \rightarrow$ $m\angle BPC = 180 - 90 = \underline{90°}$



3. $90 - (B + C) = \frac{1}{3}(180 - (90 - C)) - 4$

$90 - (180 - 8x) = \frac{1}{3}(90 + 180 - 9x) - 4$

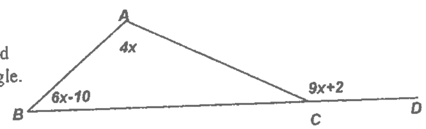
$8x - 90 = 90 - 3x - 4 \rightarrow 11x = 176 \rightarrow x = \underline{16°}$



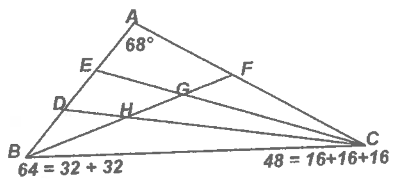
GBML 07

ROUND 3

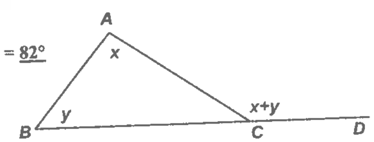
1. $(9x + 2) = 4x + (6x - 10) = 10x - 10 \rightarrow x = 12$ $\rightarrow m\angle A = 48°$, $m\angle B = 62°$, $m\angle ACB = 70°$ and $m\angle ACD = 110°$ Thus, $\angle A$ is the smallest angle. $132 = 3(90 - t) - 6 = 264 - 3t \rightarrow 3t = 132$ $\rightarrow t = \underline{44}$



2. Let $m\angle ABC = 4x$ and $m\angle ACB = 3x$ Then $7x = 180 - 68 = 112 \rightarrow x = 16$ and $m\angle BHC = 180 - 32 - 16 = 132$ and, as an exterior angle of ΔBGC, $m\angle EGH = m\angle GBC + m\angle GCB = 64°$ $\rightarrow 132° - 64° = \underline{68°}$



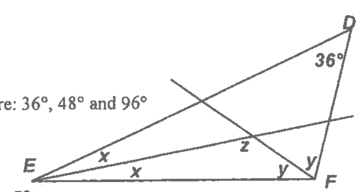
3. $180 - (90 - x) = 8 + (x + y)$ $\rightarrow 90 + x = x + y + 8$ Thus, regardless of the value of x, $y = m\angle B = \underline{82°}$



GBML 08

ROUND 3

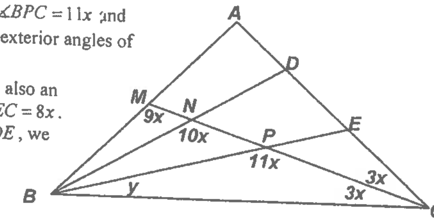
1. $3n + 4n + 8n = 180 \rightarrow n = 12 \rightarrow$ angles in ΔABC are: 36°, 48° and 96° $\rightarrow m\angle D = 36°$ Then:



$m\angle D + m\angle E + m\angle F = 36 + 2x + 2y = 180 \rightarrow x + y = 72$
But $z = 180 - (x + y) = \underline{108}$

2. Let the base angle be represented by y. $3x + 3 + y = 180 \rightarrow y = 177 - 3x$.
$2(177 - 3x)° + (x - 9)° = 180° \rightarrow x = 24°$, $y = 78° \rightarrow 24 : 78 \rightarrow \underline{4 : 13}$

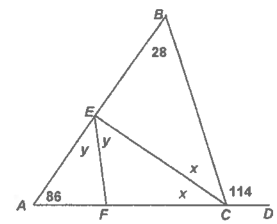
3. Let $m\measuredangle BMN = 9x$, $m\measuredangle BNP = 10x$, $m\measuredangle BPC = 11x$ and $m\measuredangle MCB = 3x$. Since $\angle BPC$ and $\angle BNP$ are exterior angles of ΔBNP and ΔBMN respectively, we have $m\measuredangle NBP = m\measuredangle MBN = x$. Since $\angle BPC$ is also an exterior angles of ΔPEC, we have $m\measuredangle PEC = 8x$. Since $\angle BEC$ is an exterior angles of ΔBDE, we have $m\measuredangle BDE = 7x$.
Thus, the required ratio is $\underline{7 : 2}$.

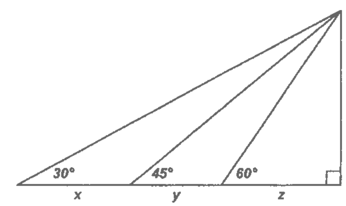


GBML 09

ROUND 3

1. $\dfrac{90 - (180 - G)}{3} = 14 \rightarrow G - 90 = 42 \rightarrow G = 132$ and the supplement is $\underline{48}$





2. $2x = 66 \rightarrow x = 33$, $114 = 28 + m\angle A \rightarrow A = 86$
$2y + 33 + 86 = 180 \rightarrow y = 30.5 \rightarrow$ complement is $\underline{59.5}$

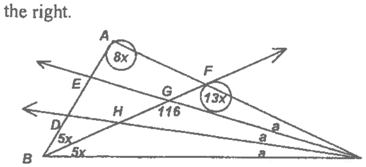
3. $45° \rightarrow y + z = 8$
$30° \rightarrow x + y + z = 8\sqrt{3} \rightarrow x = 8\sqrt{3} - 8$
$60° \rightarrow z = \dfrac{8}{\sqrt{3}} \rightarrow y = 8 - \dfrac{8}{\sqrt{3}}$
Thus, the required ratio is

$$\frac{y}{x} = \frac{8 - \frac{8}{\sqrt{3}}}{8\sqrt{3} - 8} = \frac{1 - \frac{1}{\sqrt{3}}}{\sqrt{3} - 1} = \frac{\sqrt{3} - 1}{3 - \sqrt{3}} \cdot \frac{3 + \sqrt{3}}{3 + \sqrt{3}} = \frac{3\sqrt{3} + 3 - 3 - \sqrt{3}}{9 - 3} = \frac{2\sqrt{3}}{6} = \underline{\frac{\sqrt{3}}{3}}$$

GBML 10

1. Supplement of x: $180 - x$
Complement of $(180 - x)$: $90 - (180 - x) = x - 90$
Supplement of $(x - 90)$: $180 - (x - 90) = 270 - x$
Thus, $270 - x = \frac{2}{3}x \rightarrow 810 - 3x = 2x \rightarrow 5x = 810 \rightarrow x = \underline{162}$.

2. The given information is marked in the diagram at the right.
$13x$ is an exterior angle of $\Delta ABF \rightarrow m\angle ABF = 5x$
$\overrightarrow{BF}$ bisects $\angle ABC \rightarrow m\angle FBC = 5x$ also
$\overrightarrow{CD}$ and $\overrightarrow{CE}$ trisects $\angle ACB$
$\rightarrow \angle ACE = \angle ECD = \angle DCB = a$



ΔBGC: $5x + 2a = 64$
ΔBFC: $18x + 3a = 180 \rightarrow a = 60 - 6x$
Substituting, $5x + 2(60 - 6x) = 64 \rightarrow 7x = 56 \rightarrow x = 8$, $a = 12$
$\rightarrow m\angle AEC = 180 - (64 + 12) = 104$
$\rightarrow m\angle BHC = 180 - (40 + 12) = 128$
$\rightarrow m\angle AEC : m\angle BHC = 104 : 128 = \underline{13 : 16}$

3. As the supplement of $\angle DAC$, $m\angle DAC = 180 - (185 - 7x) = 7x - 5$.
Applying the exterior angle theorem or invoking the sum of the interior angles of a triangle, $m\angle C = 185 - 11x$.
Therefore, $5(7x - 5) = 6(185 - 11x) - 24$
$\rightarrow 35x - 25 = 1110 - 66x - 24$
$\rightarrow 101x = 1111 \rightarrow x = 11$.
$(7x - 5) : (185 - 11x) = 72 : 64 = 9 : 8$
$\rightarrow (x, P, Q) = \underline{\mathbf{(11, 9, 8)}}$.