Round 2
Inequalities and Absolute Value

GREATER BOSTON MATHEMATICS LEAGUE

MEET 4 – JANUARY 1999

ROUND 2 – Inequalities and Absolute Value

1. ____________________

2. ____________________

3. ____________________

CALCULATORS ARE NOT ALLOWED ON THIS ROUND

1. Solve the following equation for x: $6x^2 = 19|x| - 10$

2. **How many integers** satisfy the following system of inequalities?

   $|3x + 8| < 23$ and $|4x - 2| \geq 10$

3. Solve the following inequality for x: $\dfrac{3}{2x - 8} \geq \dfrac{x + 2}{x^2 - 4x}$

GREATER BOSTON MATHEMATICS LEAGUE

MEET 4 – JANUARY 2000

ROUND 2 – Inequalities and Absolute Value

1. ______________________

2. ______________________

3. ______________________

CALCULATORS ARE NOT ALLOWED ON THIS ROUND

1. Find **how many integers** solve the following system of inequalities:

$$\{x \mid |x| < 13 \text{ and } 1 - 3x > 8\}$$

2. Solve the following inequality for x:

$$\left\{x \mid \frac{2}{3x} \le \frac{1}{x-1}\right\}$$

3. Solve the following equation for x:

$$\left\{x \mid \left|\sqrt{4x^2 - 12x + 9} - 18\right| = 7\right\}$$

GREATER BOSTON MATHEMATICS LEAGUE

MEET 4 – JANUARY 2001

ROUND 2 – Inequalities and Absolute Value

1. ____________________

2. ____________________

3. ____________________

CALCULATORS ARE NOT ALLOWED ON THIS ROUND

1. Find all values of x, $x \in \Re$, satisfying the equation, $|2x-3| = 12 - |6-4x|$

2. Find all values of x, $x \in \Re$, satisfying the equation, $3x^2 + 8x - 4 = |3x+4|$

3. Find all values of x, $x \in \Re$, satisfying the inequality,
$\left\{ x \,\middle|\, \dfrac{x-4}{x^2-3x} \le 1 \right\}$

GREATER BOSTON MATHEMATICS LEAGUE

MEET 4 – JANUARY 2002

ROUND 2 – Inequalities and Absolute Value

1. ____________________

2. ____________________

3. ____________________

CALCULATORS ARE NOT ALLOWED ON THIS ROUND

1. Find all values of x, $x \in \Re$, satisfying the inequality, $\left\{ x \,\middle|\, \frac{x}{|x|-2} > 0 \right\}$

2. Find all values of x, $x \in \Re$, satisfying the equation, $\left\{ x \,\middle|\, |2x-3| = |x+6| + 3 \right\}$

3. Find all values of x, $x \in \Re$, satisfying the inequality,
$\left\{ x \,\middle|\, \frac{4}{x+6} - \frac{4}{2-x} \geq \frac{x^2}{12-4x-x^2} \right\}$

GREATER BOSTON MATHEMATICS LEAGUE
MEET 4 – JANUARY 2006

ROUND 2 – Inequalities and Absolute Value

1. ____________________

2. ____________________

3. ____________________

CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

1. Determine the largest integer which satisfies both $|3x - 5| < 6$ and $2|x - 4| > 3$.

2. The solution set of $\dfrac{x^2 + 2x + 3}{12 - kx} \geq 0$ is $\{x|\ x < \dfrac{5}{6}\}$.

   If $k = \dfrac{a}{b}$, a reduced rational number, find $a + b$.

3. Solve over the reals. $\dfrac{x}{3} - \dfrac{3}{x-2} \geq \dfrac{5}{4}$

GREATER BOSTON MATHEMATICS LEAGUE
MEET 4 – JANUARY 2007

ROUND 2 – Inequalities and Absolute Value

1. { ________________ }

2. ____________________

3. ____________________

CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

1. List all integral values of x that satisfy both of the following inequalities: $|x-3| \leq 5$ and $|x-2| > 3$.

2. Find all real values of x that satisfy the following inequality: $\dfrac{7}{x^2-4} \geq \dfrac{9}{x+2}$.

3. Find all real values of x that satisfy the following inequality: $|3x^2 - 10x + 4| \leq 4$

GREATER BOSTON MATHEMATICS LEAGUE

MEET 4 – JANUARY 2008

ROUND 2 – Inequalities and Absolute Value

If you would like to receive email announcements regarding upcoming competitions, please print your email on the reverse side of this paper when you have finished answering the problems.

1. ______________________

2. ______________________

3. ______________________

CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

1. Find all values of $x \in \Re$ which satisfy $\frac{2}{3} - \frac{x^2 - 1}{4} \geq \frac{1}{2}x + \frac{1}{6}$

2. Find all values of $x \in \Re$ which satisfy $|3x + 2| + 2 \geq |x^2|$

3. Find all values of $x \in \Re$ which satisfy $|2x + 1| \geq |x| + 3$

GREATER BOSTON MATHEMATICS LEAGUE

MEET 4 – JANUARY 2009

ROUND 2 – Inequalities and Absolute Value

1. ______________________

2. ______________________

3. ______________________

CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

1. Find the <u>sum</u> of all possible solutions. $|x-3|^2 = 13|x-3| - 36$

2. Find the sum of all integer values of x which satisfy the following inequality:

$$\frac{2x-1}{4} - \frac{5-4x}{3} \geq \frac{3x^2-68}{12}$$

3. Solve for x over the reals. $\left|\frac{3x}{2x-7}\right| < 3$

GREATER BOSTON MATHEMATICS LEAGUE

MEET 4 – JANUARY 2010

ROUND 2 – Inequalities and Absolute Value

1. $k =$ __________

2. ____________________

3. $\{ x \mid$ ____________________ $\}$

CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

1. The set $\left\{ x \mid \frac{3}{4}x - \frac{2x+3}{3} + \frac{6-x}{6} \le \frac{5}{12} \right\}$ is equivalent to $\{x \mid x \ge k\}$. Find the value of *k*.

2. Find all values of *x* which satisfy the following statement: $|2x-1| = |4-x| + 3$

3. Find $\left\{ x : \left| \frac{x^2-2}{x+4} \right| < x+1 \right\}$.

GREATER BOSTON MATHEMATICS LEAGUE
MEET 4 – JANUARY 2011

ROUND 2 – Inequalities and Absolute Value

1. ______________________________

2. { x : ________________________ }

3. { x : ________________________ }

CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

1. Compute the <u>sum</u> of all possible values of x which make the following statement true.

$$\left| \, |x-7| \; - \; 6 \; \right| = 4$$

2. Solve for x: $\left\{ x : |3x-1| \le \dfrac{3}{4} \text{ and } |2x-3| > \dfrac{5}{2},\ x \in \text{Reals} \right\}$

3. Find $\left\{ x : |3x-2| + 2x \le |5-x| \right\}$

MASSACHUSETTS MATHEMATICS LEAGUE
OCTOBER 2003
ROUND 5: INEQUALITIES & ABSOLUTE VALUES

ANSWERS

`A)_____________

B)_____________

C)_____________

A) Solve for x: $x^3 < 5x^2 + 24x$.

B) Solve for x: $|4 - 2x| = x^2 - 3x + 2$

C) Solve for x: $\frac{1}{x^2} - \frac{5}{x} < 24$

MASSACHUSETTS MATHEMATICS LEAGUE
OCTOBER 2005
ROUND 5 INEQUALITIES & ABSOLUTE VALUE

ANSWERS

A)______________

B)______________

C)______________

A) **Find all real x for which** $4x^2 + 20x + 25 \leq 0$

B) **Find the sum of all solutions to**

$$|x(x-13)| = 30$$

C) **Find all real x for which**

$$13x^3 > 50x^2 - 44x - 8$$

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2006
ROUND 5 INEQUALITIES & ABSOLUTE VALUE

ANSWERS

A) ______________________

B) ______________________

C) ______________________

A) It cost a publishing company \$250 to setup for the printing of a brochure. After that, the cost of printing, materials, etc. is 19¢ per brochure. If the publisher sells the brochure for 40¢ per copy, how many copies minimum must be sold before any profit is realized?

B) How many integer solutions are there that satisfy both $|2x - 1| \le 23$ and $|x + 1| > 4$?

C) Find the domain of the real-valued function defined by $f(x) = \sqrt{\dfrac{x+4}{12-4x-x^2}}$.

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2007
ROUND 5 INEQUALITIES & ABSOLUTE VALUE

ANSWERS

A) ______________________________

B) ______________________________

C) ______________________________

A) Find the area of the region defined by $\begin{cases} y \le |x| \\ x \le 2 \\ x \ge -1 \\ y \ge 0 \end{cases}$

B) Solve for x over the reals. $$\frac{|x-3|(x-4)}{(x+5)^3} \ge 0$$

C) Determine the set of values of x (over the reals) for which the following inequality is satisfied:

$$\frac{1}{x} \le \frac{1}{x-1} - \frac{1}{2}$$

ROUND 2

'99 GBML

1. $6x^2 = 19|x| - 10 \Rightarrow 6|x|^2 - 19|x| + 10 = 0 \Rightarrow (3|x|-2)(2|x|-5) = 0 \Rightarrow$ the solutions for x are $\pm\frac{2}{3}$ and $\pm\frac{5}{2}$

2. $|3x+8| < 23$ and $|4x-2| \ge 10 \Rightarrow -23 < 3x+8 < 23$ and $4x-2 \ge 10$ or $4x-2 \le -10$
$\Rightarrow -10\frac{1}{3} < x < 5$ and $x \ge 3$ or $x \le -2 \Rightarrow x = -10, -9, \ldots -2, 3, 4$ which are **11** possibilities.

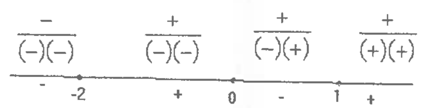
3. $\frac{3}{2x-8} \ge \frac{x+2}{x^2-4x} \Rightarrow \frac{3}{2(x-4)} - \frac{x+2}{x(x-4)} \ge 0 \Rightarrow \frac{3x-2(x+2)}{2x(x-4)} \ge 0 \Rightarrow \frac{1}{2x} \ge 0$ and $x \ne 4$
$\Rightarrow$ **$x > 0$ and $x \ne 4$**

'00 GBML

Round 2

1. $-13 < x < 13 \Rightarrow x = -12, -11, \ldots, 11, 12;\ 1-3x > 8 \Rightarrow -3x > 7 \Rightarrow x < -\frac{7}{3} \Rightarrow x = -3, -4, \ldots$
The intersection of these two sets of integers = – 12, – 11, …– 3, which has 10 solutions.

2. $\frac{2}{3x} - \frac{1}{x-1} \le 0 \Rightarrow \frac{2x-2-3x}{3x(x-1)} \le 0 \Rightarrow \frac{-x-2}{3x(x-1)} \le 0 \Rightarrow \frac{x+2}{3x(x-1)} \ge 0 \Rightarrow$ key numbers are:
-2 (included) 0 and 1 (excluded); on the number line:



$\Rightarrow$ solution is $\{x \mid -2 \le x < 0 \text{ or } x > 1\}$

3. $\left|\sqrt{4x^2-12x+9} - 18\right| = 7 \Rightarrow \left|\sqrt{(2x-3)^2} - 18\right| = 7 \Rightarrow \left||2x-3| - 18\right| = 7 \Rightarrow$
$|2x-3| - 18 = \pm 7 \Rightarrow |2x-3| = 11$ *or* $25 \Rightarrow 2x-3 = \pm 11$ *or* $\pm 25 \Rightarrow$
$x = \frac{3\pm 11}{2}$ *or* $\frac{3\pm 25}{2} \Rightarrow x = -11, -4, 7, 14$

'01 GBML

Round 2

1. $|2x-3| = 12 - |6-4x| \to |2x-3| = 12 - 2|3-2x| \to |2x-3| = 12 - 2|2x-3| \to 3|2x-3| = 12$
$\to |2x-3| = 4 \to 2x-3 = \pm 4 \to x = -\frac{1}{2}, \frac{7}{2}$

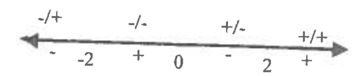
2. $3x^2+8x-4 = |3x+4| \to$ case (i): $x \ge -\frac{4}{3}: 3x^2+8x-4 = 3x+4 \to 3x^2+5x-8 = 0 \to$
$(3x+8)(x-1) = 0 \to x = 1$; case(ii): $x < -\frac{4}{3}: 3x^2+8x-4 = -3x-4 \to 3x^2+11x = 0 \to$
$x(3x+11) = 0 \to x = -\frac{11}{3}$; the 2 solutions are $-\frac{11}{3}, 1$

3. $\frac{x-4}{x^2-3x} \le 1 \to \frac{x-4-x^2+3x}{x^2-3x} \le 0 \to \frac{-x^2+4x-4}{x(x-3)} \le 0 \to \frac{x^2-4x+4}{x(x-3)} \ge 0 \to \frac{(x-2)^2}{x(x-3)} \ge 0$
$\to$ key values for x are 0 and 3 (excluded) and 2 (included); considering the value of the rational expression for each section of the number line reaches the following conclusion:
$x < 0$ or $x > 3$ or $x = 2$

'02 GBML

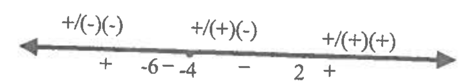
ROUND 2 – Inequalities and Absolute Value

1. To solve $\frac{x}{|x|-2} > 0$, identify the key values for x, which are –2, 0, 2 (values that make the numerator and denominator 0). Now section off the number line (see below):
Therefore the solution is $-2 < x < 0$ or $x > 2$.



2. To solve $|2x-3| = |x+6| + 3$, consider the key values for x (values that make each absolute value expression = 0) which are –6 and 1.5. Now consider three cases:
(i) $x \ge 1.5$: $2x-3 = x+6+3 \Rightarrow x = 12$, which satisfies the restriction on x.
(ii) $-6 \le x \le 1.5$: $3-2x = x+6+3 \Rightarrow x = -2$, which satisfies the restriction on x.
(iii) $x \le -6$: $3-2x = -x-6+3 \Rightarrow x = 6$, which does not satisfy the restriction on x.
Therefore the solutions for x are –2, 12 only.

3. $\frac{4}{x+6} - \frac{4}{2-x} \ge \frac{x^2}{12-4x-x^2} \Rightarrow \frac{4}{x+6} + \frac{4}{x-2} \ge \frac{-x^2}{x^2+4x-12} \Rightarrow$
$\frac{x^2}{x^2+4x-12} + \frac{4}{x+6} + \frac{4}{x-2} \ge 0 \Rightarrow \frac{x^2}{(x+6)(x-2)} + \frac{4(x-2)}{(x+6)(x-2)} + \frac{4(x+6)}{(x-2)(x+6)} \ge 0$
$\Rightarrow \frac{x^2+4x-8+4x+24}{(x+6)(x-2)} \ge 0 \Rightarrow \frac{x^2+8x+16}{(x+6)(x-2)} \ge 0 \Rightarrow \frac{(x+4)^2}{(x+6)(x-2)} \ge 0.$
Key values: –6, 2(excluded) and –4 (included). Now, section off the number line
$\Rightarrow x < -6$ or $x > 2$ or $x = -4$.



'06
GBML

ROUND 2 – Inequalities and Absolute Value

1. 1^{st} condition: $-6<3x-5<+6 \rightarrow -1<3x<11 \rightarrow -1/3<x<11/3$ (a segment w/open endpoints)
   2^{nd} condition: $2(x-4)<-3$ or $2(x-4)>+3 \rightarrow x<5/2$ or $x>11/2$ (2 rays w/open endpoints)
   Taking the overlap, we get $-\frac{1}{3}<x<\frac{5}{2}$. The largest integer in this interval is <u>2</u>.
2. The numerator may be rewritten as $(x+1)^2+2$ and since this expression is always positive, the denominator determines the sign of the fraction. Ignoring the numerator, $12-kx>0$ must produce $x<5/6$.
   Thus, $12>kx \rightarrow 12/k>x$ (if k is positive) or $12/k<x$ (if k is negative)
   The later condition does not match the known solution set.
   $12/k = 5/6 \rightarrow k = 72/5 \rightarrow a+b = \underline{77}$
3. Manipulating the inequality: $\frac{x^2-2x-9}{3(x-2)}-\frac{5}{4}\ge 0 \Rightarrow \frac{4x^2-8x-36-15x+30}{12(x-2)}$
   $\Rightarrow \frac{(4x+1)(x-6)}{12(x-2)}\ge 0$ The critical points are: -1/4, 2 and 6. All factors become negative for x values smaller than their critical values and positive for larger values.
   A zero in the numerator is allowed, BUT a zero is rejected in the denominator!
   Sign of Quotient: – + – +
   -1/4 2 6
   Thus, we have: $\underline{-\frac{1}{4}\le x<2 \text{ or } x\ge 6}$

'07
GBML

ROUND 2 – Inequalities and Absolute Value

1. $|x-3|\le 5 \rightarrow -2\le x\le 8$ and $|x-2|>3 \rightarrow x<-1$ or $x>5$
   The overlap is <u>**-2, 6,7, 8**</u>
2. $\frac{7}{x^2-4}\ge\frac{9}{x+2}\rightarrow \frac{7-9(x-2)}{x^2-4}\ge 0\rightarrow \frac{-9x+25}{(x-2)(x+2)}\ge 0$
   The 3 critical values divide the number line into 4 intervals. Testing each interval determines that the quotient is non-negative for <u>**$x<-2$**</u> or <u>**$2<x\le 25/9$**</u>
3. $|3x^2-10x+4|\le 4 \rightarrow -4\le 3x^2-10x+4\le 4 \rightarrow 3x^2-10x+4\ge -4$ and $3x^2-10x+4\le 4$

   $3x^2-10x+8=(3x-4)(x-2)\ge 0 \rightarrow x\le 4/3$ or $x\ge 2$
   $3x^2-10x = x(3x-10)\le 0 \rightarrow 0\le x\le 10/3$
   Taking the overlap, we have <u>**$0\le x\le 4/3$ or $2\le x\le 10/3$**</u>

'08
GBML

ROUND 2

1. $\frac{2}{3}-\frac{x^2-1}{4}\ge\frac{1}{2}x+\frac{1}{6}\rightarrow 8-3x^2+3\ge 6x+2\rightarrow 3x^2+6x-9\le 0\rightarrow x^2+2x-3\le 0$
   $\rightarrow (x+3)(x-1)\le 0 \rightarrow \underline{-3\le x\le 1}$
2. Domain: $x<-2/3$ (-2/3) $x\ge -2/3$

$x<-2/3$		$x\ge -2/3$
$(-3x-2)+2\ge x^2$		$(3x+2)+2\ge x^2$
$x^2+3x\le 0$		$3x+4\ge x^2$
$x(x+3)\le 0$		$x^2-3x-4\le 0$
$-3\le x\le 0$		$(x+1)(x-4)\le 0 \rightarrow -1\le x\le 4$
over domain: $-3\le x<-2/3$	or	$-2/3\le x\le 4$

Combining, <u>**$-3\le x\le 4$**</u>

3. (-1/2) (0)

$-2x-1>-x+3$	$2x+1\ge -x+3$	$2x+1\ge x+3$
$-x\ge 4$	$3x\ge 2$	$x\ge 2$
$x\le -4$	$x\ge 2/3$	
with domain – ok	outside domain – rejected	with domain – ok

Thus, the solution is <u>**$x\le -4$ or $x\ge 2$**</u>.

GBML
'09

ROUND 2

1.
$|x-3|^2-13|x-3|+36=(|x-3|-9)(|x-3|-4)=0$
$|x-3|=9 \rightarrow x-3=\pm 9\rightarrow x=12,-6$
$|x-3|=4 \rightarrow x-3=\pm 4\rightarrow x=7,-1$
Thus, the required sum is <u>12</u>.

2.
Clearing denominators, $3(2x-1)-4(5-4x)\ge 3x^2-68$
$6x-3-20+16x\ge 3x^2-68$
$22x-23\ge 3x^2-68$
$0\ge 3x^2-22x-45=(3x+5)(x-9)\rightarrow -\frac{5}{3}\le x\le 9$
Thus, integer solutions are: -1, 0, 1, 2, …, 9 and the sum is <u>44</u>.

3.
$\rightarrow -3<\frac{3x}{2x-7}<3 \rightarrow$ (1) $\frac{3x}{2x-7}>-3$ <u>and</u> (2) $\frac{3x}{2x-7}<3$

(1) $\frac{3x}{2x-7}+3>0\rightarrow\frac{9x-21}{2x-7}>0\rightarrow\frac{3x-7}{2x-7}>0\rightarrow x<\frac{7}{3}$ or $x>\frac{7}{2}$

(2) $\frac{3x}{2x-7}<3\rightarrow\frac{-3x+21}{2x-7}<0\rightarrow\frac{x-7}{2x-7}>0\rightarrow x<\frac{7}{2}$ or $x>7$

7/3 7/2 7

Taking the overlap (<u>and</u>), $\underline{x<\frac{7}{3} \text{ or } x>7}$.

GBML
'10

ROUND 2

1. $\left\{x\middle|\frac{3}{4}x-\frac{2x+3}{3}+\frac{6-x}{6}\le\frac{5}{12}\right\}\rightarrow 9x-8x-12+12-2x\le 5\rightarrow -x\le 5\rightarrow x\ge -5\rightarrow k=\underline{-5}$
2. 1/2 4

$-2x+1=4-x+3$	$2x-1=4-x+3$	$2x-1=-4+x+3$
$x=\underline{-6}$	$3x=8$	$x=0$ (extraneous)
	$x=\underline{\frac{8}{3}}$	

ROUND 2 - continued

3. $\left\{x:\left|\frac{x^2-2}{x+4}\right|<x+1\right\} \rightarrow -(x+1)<\frac{x^2-2}{x+4}<x+1$, <u>provided $x>-1$</u> (insuring that $\left|\frac{x^2-2}{x+4}\right|\geq 0$)

$\rightarrow -(x+1)<\frac{x^2-2}{x+4}$ <u>and</u> $\frac{x^2-2}{x+4}<x+1$

First condition: $\frac{x^2-2}{x+4}+(x+1)=\frac{2x^2+5x+2}{x+4}=\frac{(2x+1)(x+2)}{x+4}>0$

Critical points at –4, –2, and –1/2. All factors are negative for $x<-4$ and the sign of the expression alternates as each critical point is passed.

The quotient is positive for $\left\{x\,|\,-4<x<-2 \text{ or } x>-\frac{1}{2}\right\}$.

Second condition: $\frac{x^2-2}{x+4}-(x+1)<0 \rightarrow \frac{x^2-2-\left(x^2+5x+4\right)}{x+4} \rightarrow \frac{-5x-6}{x+4}<0 \rightarrow \frac{5x+6}{x+4}>0$

Critical points at –4 and –6/5. Both factors are negative for $x<-4$ and the sign of the expression alternates as each critical point is passed.

The quotient is positive for $\left\{x\,|\,x<-4 \text{ or } x>-\frac{6}{5}\right\}$.

Taking the intersection of the two conditions and the pre-condition $(x>-1)$, we have $\underline{x>-\frac{1}{2}}$.

ROUND 2

1. $\big||x-7|-6\big|=4 \rightarrow |x-7|=6\pm 4=10,2 \rightarrow x-7=\pm 10, \pm 2$
$\rightarrow x=7\pm 10, 7\pm 2$. Determining the four numbers and adding them is not necessary. Clearly, the expressions 7 ± 10 and 7 ± 2 each denote two numbers equidistant from 7. Thus, the sum is $4(7)=\underline{\mathbf{28}}$.

2. $|3x-1|\leq\frac{3}{4}\rightarrow$

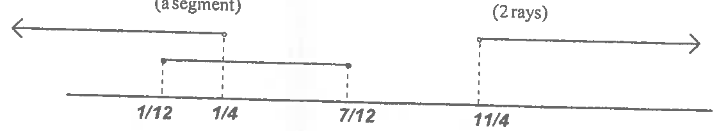
$$-\frac{3}{4}\leq 3x-1\leq\frac{3}{4}$$
$$\frac{1}{4}\leq 3x\leq\frac{7}{4}$$
$$\frac{1}{12}\leq x\leq\frac{7}{12}$$

(a segment)

and $|2x-3|>\frac{5}{2}\rightarrow$

$$2x-3<-\frac{5}{2} \text{ or } 2x-3>\frac{5}{2}$$
$$2x<\frac{1}{2} \text{ or } 2x>\frac{11}{2}$$
$$x<\frac{1}{4} \text{ or } x>\frac{11}{4}$$

(2 rays)



3. $|3x-2|+2x\leq|5-x|$ has critical points at: $x=2/3$ and 5.
In each case, we look at an equivalent equation <u>valid only within the specified domain.</u>
Case 1: $x<2/3$
$\rightarrow -3x+2+2x\leq 5-x \rightarrow 2\leq 5$ (which is always true) – these values accepted.

Case 2: $2/3\leq x\leq 5$

$\rightarrow 3x-2+2x\leq 5-x \rightarrow 6x\leq 7 \rightarrow x\leq\frac{7}{6} \rightarrow \frac{2}{3}\leq x\leq\frac{7}{6}$

Case 3: $x>5$

$\rightarrow 3x-2+2x\leq -5+x \rightarrow 4x<-3 \rightarrow x\leq-\frac{3}{4} \rightarrow\varnothing$ (i.e. no solution, outside domain!)

Combining the solutions, we have $\underline{\left\{x\,|\,x\leq\frac{7}{6}\right\}}$.

GBML
'11

MASSACHUSETTS MATHEMATICS LEAGUE
OCTOBER 2003
ROUND 5: INEQUALITIES & ABSOLUTE VALUES
ANSWERS

MML
'03

A) $x<-3, 0<x<3$

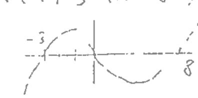
B) ~~2or~~ 3

C) $x>1/8, x<-1/3$

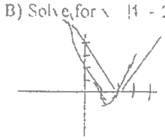
$x^3-5x^2-24x<0$

$x(x+3)(x-8)<0$

ans $x<-3,\ 0<x<8$



B) Solve for x: $|4-2x|=x^2-3x+2$



$4x-2x=x^2-3x+2$

$x^2-x-2=0$

$(x+1)(x-2)=0$

$x=-1, x=2$

no intersection for $x>2$

if $x>2$

$2x-4=x^2-3x+2$

$0=x^2-5x+6$

$x=$ ~~6 or -1~~ 2 or 3

$\therefore x=$ ~~2~~ 3

C) Solve for x: $\frac{1}{x^2}-\frac{5}{x}<24$

$\frac{1}{x^2}-\frac{5}{x}-24<0$

$-24x^2-5x+1<0$

$24x^2+5x-1>0$

$(8x-1)(3x+1)>0$

ANS $x>\frac{1}{8},\ x<-\frac{1}{3}$

MML '05

Round Five:

A. $(2x+5)^2 \le 0$ only if $(2x+5)^2 = 0$ so $2x+5=0$, $x = -2.5$

B. $x^2 - 13x = 30$ so $(x-10)(x-3)=0$ or $x^2 - 13x = -30$ so $(x-15)(x+2)=0$ sum is $10 + 3 + 15 - 2$

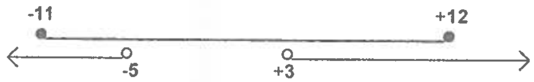
C. By synthetic division testing or calculator table x–2 is a factor of $13x^3 - 50x^2 + 44x + 8 = (x-2)(x-2)(13x+2)$ If $x \ne 2$ first two are positive product so $13x + 2 > 0$ if $x > -2/13$

MML '06

Round 5

A) $40x \ge 19x + 25000 \rightarrow 21x \ge 25000 \rightarrow x \ge 1190.47+ \rightarrow x_{min} = \underline{\mathbf{1191}}$

B) $|2x-1| \le 23 \rightarrow -23 < 2x - 1 < +23 \rightarrow -11 \le x \le +12$
$|x+1| > 4 \rightarrow x + 1 < -4$ <u>or</u> $x + 1 > +4 \rightarrow x < -5$ or $x > +3$



Thus, the overlap contains integers from -11 to -6 inclusive as well as integers from 4 to 12 inclusive, a total of 6 + 9 = <u>15</u> integers.

C) The expression under the square root, i.e. the radicand, must be non-negative.

$\dfrac{x+4}{12-4x-x^2} = \dfrac{x+4}{(6+x)(2-x)}$ The critical values are -4, -6 and +2.

Two factors $(x+4)$ and $(6+x)$ are negative for values of x less than the critical value and positive for values of x greater than the critical value.
For $(2-x)$ the situation is reversed.
The following diagram summarizes this situation:

```
(x + 4) --------------------------------0++++++++++++++++++++++++++++++
(6 + x) -----------------0++++++++++++++++++++++++++++++++++++++++++++++
(2 - x) ++++++++++++++++++++++++++++++++++++++++++++++0------------
Section   1              2              3              4
_______________-6________-4______________________2________
      2 neg          1 neg          0 neg              1 n
```

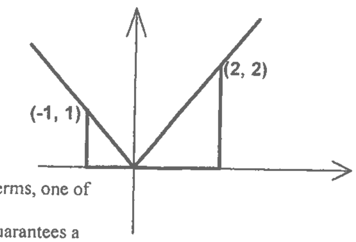
Thus, in section 1 (<u>$x < -6$</u>) and section 3 (<u>$-4 \le x < 2$</u>), the quotient is non-negative.
Note: Only -4 is included, since the other critical values would cause division by zero.

MML '07

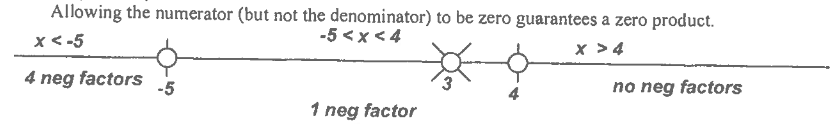
Round 5

A) The region consists of 2 right triangles

Area $= \frac{1}{2}\cdot 1\cdot 1 + \frac{1}{2}\cdot 2\cdot 2 = \underline{2.5}$



B) The quotient on the left-hand side is comprised of 5 terms, one of which is never negative.
Having an <u>even</u> number of factors that are negative guarantees a positive product.
Allowing the numerator (but not the denominator) to be zero guarantees a zero product.

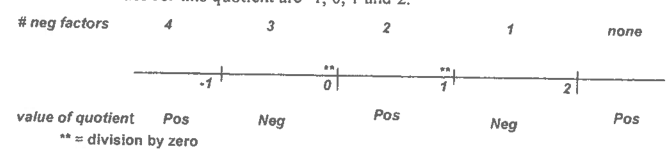


$\rightarrow \underline{x < -5 \text{ or } x \ge 4 \text{ or } x = 3}$

C) $\frac{1}{x} \le \frac{1}{x-1} - \frac{1}{2} \rightarrow \frac{1}{x-1} - \frac{1}{x} - \frac{1}{2} \ge 0 \rightarrow \frac{2x - 2(x-1) - x(x-1)}{2x(x-1)} \ge 0 \rightarrow \frac{2 + x - x^2}{2x(x-1)} \ge 0$

$\rightarrow \frac{(2-x)(1+x)}{2x(x-1)} \ge 0 \rightarrow \frac{(x-2)(x+1)}{2x(x-1)} \le 0$

The critical values for this quotient are -1, 0, 1 and 2.



Thus, the solution intervals are: $\underline{-1 \le x < 0 \text{ or } 1 < x \le 2}$.