Round 5
Conic Sections

GREATER BOSTON MATHEMATICS LEAGUE

MEET 4 – JANUARY 1999

ROUND 5 – Conics

1. ____________________

2. ____________________

3. ____________________

CALCULATORS ARE NOT ALLOWED ON THIS ROUND

1. Find in simplest radical form the distance from the center of $x^2 + y^2 + 6x - 16y - 3 = 0$ to the vertex of $y^2 + 4y - 5x + 14 = 0$

2. Given the ellipse $4x^2 + 9y^2 = 36$, find the focus of the parabola whose vertex is at the lower y-intercept of the ellipse and which passes through the x-intercepts of the ellipse.

3. Given the conic with foci (7, -3) and (-1, -3) such that the difference of the distances from any point on this conic to the foci is 4, find the distance between its x intercepts in simplest radical form.

GREATER BOSTON MATHEMATICS LEAGUE

MEET 4 – JANUARY 2000

ROUND 5 – Conics

1. (,)

2. ____________

3. ____________

CALCULATORS ARE NOT ALLOWED ON THIS ROUND

1. Find the coordinates of the focus of the parabola, $\{(x, y) \mid y^2 - 16y + 6x + 79 = 0\}$

2. Given the circle, $\{(x, y) \mid x^2 + y^2 - 12x + 12y + 36 = 0\}$, a tangent line is drawn from the point $P\ (15, -6)$ to this circle. Find the distance from P to the point of tangency.

3. Given the ellipse, $\left\{(x, y) \mid \frac{(x+2)^2}{16} + \frac{(y-4)^2}{25} = 1\right\}$, a hyperbola's foci are the ellipse's vertices and the hyperbola's vertices are the ellipse's foci. Find the y-coordinates of the points on the hyperbola whose x-coordinate is 6.

GREATER BOSTON MATHEMATICS LEAGUE

MEET 4 – JANUARY 2001

ROUND 5 – Conics

1. ______________________

2. (____,____) (____,____)

3. (_____,_____) _______

CALCULATORS ARE NOT ALLOWED ON THIS ROUND

1. Given the conic, $\left\{(x,y)\middle|2x^2-y^2=10\right\}$, find the equations of both asymptotes in slope-intercept form, which is $y=mx+b$.

2. Find the coordinates of the two points of intersection of line $\ell,\left\{(x,y)\middle|x-2y+6=0\right\}$, with the parabola whose vertex is the origin and whose focus is the point $P\ (2,0)$.

3. Given conic C, $\left\{(x,y)\middle|3x^2+y^2=1\right\}$, a circle is drawn having the same center as conic C and containing its foci. Find in simplest form the coordinates of the point in the first quadrant where the circle intersects conic C.

GREATER BOSTON MATHEMATICS LEAGUE

MEET 4 – JANUARY 2002

ROUND 5 – Conics

1. $(\_\_\_\_\_, \_\_\_\_\_)$

2. ____________________

3. ____________________

CALCULATORS ARE NOT ALLOWED ON THIS ROUND

1. Given the conic C, $\left\{(x,y)\middle| y^2+4y+12x-32=0\right\}$, find the coordinates of its focus.

2. A circle contains the point $P(2,9)$ and is tangent to both axes. Find all possible values for the radius of this circle.

3. The conic C, $\left\{(x,y)\middle| 12x^2-4y^2-72x-24y+24=0\right\}$, has a focus F in quadrant III with coordinates (a,b). If point $P(a,d)$ lies on conic C, find all possible values for d.

GREATER BOSTON MATHEMATICS LEAGUE
MEET 4 – JANUARY 2006

ROUND 5 – Conics

1. ____________________

2. ____________________

3. ____________________

CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

1. Find the equation of the parabola whose focus is $F(7, 3)$ and whose directrix is $x = 1$.

2. Given: $x^2 + 4y^2 = 36$ and $25x^2 - 4y^2 = 100$

   Determine the number of units in the circumference of the circle whose diameter has one endpoint at the smaller x-intercept of the hyperbola and the other endpoint at the larger y-intercept of the ellipse.

3. Given the hyperbola $\frac{(x-3)^2}{9} - \frac{(y+2)^2}{4} = 1$

   Point $P(8, y)$ is on the hyperbola. How far is P from the closest asymptote?

GREATER BOSTON MATHEMATICS LEAGUE
MEET 4 – JANUARY 2007

ROUND 5 – Conics

1. ________________ **units**

2. ________________ **units^2**

3. ____________________

CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

1. What is the shortest distance from point $Q(7, 13)$ to the conic

$$C_1 : \left\{(x, y) \mid (x+1)^2 + (y+2)^2 = 16\right\} ?$$

2. Find the area of the quadrilateral whose vertices are the foci and the endpoints of the minor axis of the conic $C_2 : \left\{(x,\ y) \middle| 9x^2 - 44 + 25y^2 - 54x + 100y = 0\right\}$

3. The conic $C_3 : \left\{(x,\ y) \middle| x^2 + y^2 - 8x - 6y - 20 = 0\right\}$ intersects the x-axis at points A and B. Find the equation of the parabola whose vertex is the center of conic C_3 and contains the points A and B.

GREATER BOSTON MATHEMATICS LEAGUE

MEET 4 – JANUARY 2008

ROUND 5 – Conics

If you would like to receive email announcements regarding upcoming competitions, please print your email on the reverse side of this paper when you have finished answering the problems.

1. ______________________

2. ______________________

3. ______________________

CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

1. An ellipse is in the third quadrant and tangent to both axes. Its major axis is parallel to the x-axis and its length is twice that of its minor axis. The sum of the lengths of the axes is 24. Determine the equation of the ellipse.

2. Find the equation of the set of points equidistant from the line $x = -4$ and the point $P(4, 3)$.

3. Find the distance from the center of the first conic C_1 to the focus of the second conic C_2.

$$C_1: 9x^2 + 24y + 4y^2 - 36x + 36 = 0 \quad \text{and} \quad C_2: x^2 - 8y = 39 - 6x$$

GREATER BOSTON MATHEMATICS LEAGUE

MEET 4 – JANUARY 2009

ROUND 5 – Conics

1. ______________________

2. ______________________

3. (__________ , __________)

CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

1. Find the length of the line segment whose endpoints are:
   the focus of the conic C_1: $\{(x, y) \mid y^2 - 12x - 15 = 6y\}$ and
   the center of the conic C_2: $\{(x, y) \mid 2x^2 - 8x + 16y + 2y^2 + 1 = 0\}$

2. Find the area of the convex quadrilateral whose vertices are the endpoints of the minor axis and the foci of the ellipse: $\{(x, y) \mid x^2 + 32y + 4y^2 - 6x + 57 = 0\}$

3. The parabola $y = Ax^2 + Bx + 4$ passes through (1, 28) and $\left(\frac{1}{2}, 14\right)$.
   Determine the coordinates of the vertex of this parabola.

GREATER BOSTON MATHEMATICS LEAGUE

MEET 4 – JANUARY 2010

ROUND 5 – Conics

1. ______________________________

2. ______________________________

3. $J($_____, _____$)$ $K($_____, _____$)$

CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

1. Find the equation of the circle whose diameter is the minor axis of the conic $C: 9x^2 + 4y^2 - 36x + 24y + 36 = 0$

2. Given the conic $C: 4x^2 - y^2 + 24x + 4y + 28 = 0$
   Find the equation of the asymptote of the conic C with positive slope.
   Express the equation in $y = mx + b$ form.

3. Ann was looking at the graph of a parabola and noted the following information about it:

   - the coordinates of the vertex V are $(6, 0)$.
   - a line parallel to the x-axis intersects the parabola at point $P(10.5, 9)$ and point Q.
   - another line parallel to the x-axis intersects the parabola at points $J(a, b)$ and $K(c, d)$, where $a < c$.

   The chord $\overline{PQ}$ is 5 units longer than the chord $\overline{JK}$.
   Find the (x, y) coordinates of the points J and K.

GREATER BOSTON MATHEMATICS LEAGUE
MEET 4 – JANUARY 2011

ROUND 5 – Conics

1. (_____ , _____), (_____ , _____)

2. ______________________________

3. (__________ , __________)

CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

1. Determine the coordinates of the endpoints of the vertical chord through the center of the circle $x^2+y^2-6x+8y=0$.

2. The endpoints of the major axis of an ellipse are the same as the foci of
$C_1\!:\left\{(x,y)\,|\,\dfrac{x^2}{64}-\dfrac{y^2}{36}=1\right\}$.
The foci of the ellipse are the same as the vertices of C_1.
Compute the length of a focal chord of the ellipse.

3. A chord $\overline{RS}$ passes through the focus of the parabola P_1: $\{(x,y)\,|\,y^2-8x=4y+12\}$ and has one endpoint at $R(6, 10)$. Find the coordinates of the other endpoint S.

ROUND 5

'99 GBML

1. $x^2+y^2+6x-16y-3=0 \Rightarrow x^2+6x+9+y^2-16y+64=76 \Rightarrow (x+3)^2+(y-8)^2=76$
$\Rightarrow$ center = (-3, 8)
$y^2+4y-5x+14=0 \Rightarrow y^2+4y+4=5x-14+4 \Rightarrow (y+2)^2=5(x-2) \Rightarrow$
vertex = (2, -2). Distance between these points = $\sqrt{5^2+(-10)^2} = \mathbf{5\sqrt{5}}$

2. The vertex of the parabola is (0, -2) and the x-intercepts are (3, 0) and (-3, 0) $\Rightarrow$ Equation of the parabola is $x^2=4p(y+2) \Rightarrow$ Since $(3,0)$ is a point on the parabola, then $9=8p$ and $p=\frac{9}{8} \Rightarrow focus = \left(0,-2+\frac{9}{8}\right)=\left(0,-\frac{7}{8}\right)$

3. foci (7, -3) and (-1, -3) and the diff. of dist. = 4 $\Rightarrow$ hyperbola has center (3, -3) and $2a=4 \Rightarrow a=2;\ 2c=8 \Rightarrow c=4 \Rightarrow b^2=c^2-a^2=12 \Rightarrow$ equation of the hyperbola is $\frac{(x-3)^2}{4}-\frac{(y+3)^2}{12}=1$; $y=0$: $\frac{(x-3)^2}{4}-\frac{9}{12}=1 \Rightarrow (x-3)^2=7 \Rightarrow x=3\pm\sqrt{7} \Rightarrow$
distance between the x intercepts = $\mathbf{2\sqrt{7}}$

'00 GBML

Round 5

1. $y^2-16y+6x+79=0 \Rightarrow y^2-16y+64=-6x-79+64 \Rightarrow (y-8)^2=-6(x+2\tfrac{1}{2}) \Rightarrow$
vertex = $(-2\tfrac{1}{2},8)$ and since the parabola opens left, the focus is p units left of vertex.
$4p=6 \Rightarrow p=1\tfrac{1}{2} \Rightarrow$ focus $=(-2\tfrac{1}{2}-1\tfrac{1}{2},8)=(-4,8)$

2. $x^2+y^2-12x+12y+36=0 \Rightarrow x^2-12x+36+y^2+12y+36=36 \Rightarrow$
$(x-6)^2+(y+6)^2=36 \Rightarrow$ center $=(6,-6)$ and radius $=6$; $(15,-6)$ is 9 units from the center $\Rightarrow$ distance from $(15,-6)$ to the point of tangency $=\sqrt{9^2-6^2}=3\sqrt{3^2-2^2}=3\sqrt{5}$

3. The hyperbola has the same center $(-2,4)$ as the ellipse; the vertices of the ellipse are 5 units up and down from the center $\Rightarrow c=5$ for the hyperbola; for the ellipse, $c^2=a^2-b^2 \Rightarrow c^2=25-16=9 \Rightarrow c=3 \Rightarrow a=3$ for the hyperbola; for the hyperbola, $c^2=a^2+b^2 \Rightarrow 25=9+b^2 \Rightarrow b^2=16$; from these facts the equation of the hyperbola is $\frac{(y-4)^2}{9}-\frac{(x+2)^2}{16}=1$; let $x=6$: $\frac{(y-4)^2}{9}-\frac{(6+2)^2}{16}=1 \Rightarrow \frac{(y-4)^2}{9}-4=1 \Rightarrow$
$(y-4)^2=45 \Rightarrow y-4=\pm3\sqrt{5} \Rightarrow y=4\pm3\sqrt{5}$

Round 5

'01 GBML

1. $2x^2-y^2=10 \rightarrow \frac{x^2}{5}-\frac{y^2}{10}=1 \rightarrow$ asymptotes are $\frac{x^2}{5}=\frac{y^2}{10} \rightarrow y^2=2x^2 \rightarrow y=\pm\sqrt{2}x$

2. The equation of the parabola is $y^2=8x$ and since $x=2y-6 \rightarrow y^2=8(2y-6) \rightarrow$
$y^2-16y+48=0 \rightarrow (y-4)(y-12)=0 \rightarrow y=4,12 \rightarrow x=2,18$ respectively $\rightarrow$
points of intersection are $(2,4),(18,12)$

3. $3x^2+y^2=1 \rightarrow \frac{x^2}{1/3}+\frac{y^2}{1}=1 \rightarrow c^2=1-\frac{1}{3}=\frac{2}{3} \rightarrow$ equation of the circle is
$x^2+y^2=\frac{2}{3} \rightarrow$ subtracting this from the original equation, $2x^2=\frac{1}{3} \rightarrow x^2=\frac{1}{6} \rightarrow y^2=\frac{1}{2}$
since the point of intersection is in quadrant I $\rightarrow$ point $=\left(\frac{\sqrt{6}}{6},\frac{\sqrt{2}}{2}\right)$

ROUND 5 – Conics

'02 GBML

1. $y^2+4y+12x-32=0 \Rightarrow y^2+4y+4=-12x+36 \Rightarrow (y+2)^2=-12(x-3) \Rightarrow$ vertex = $(3,-2)$. This parabola "opens" to the left and since $4p=12 \Rightarrow p=3 \Rightarrow$ focus is 3 units to the left of the vertex $\Rightarrow$ focus = $(3-3,-2)=(0,-2)$.

2. Since the circle is tangent to both axes, its center must lie on the line $y=x$.
Let $O(h,h)$ be its center $\Rightarrow (2-h)^2+(9-h)^2=h^2 \Rightarrow h^2-4h+4-18h+81=0$
$\Rightarrow h^2-22h+85=0 \Rightarrow (h-5)(h-17)=0 \Rightarrow h=5,\ 17$.

3. $12x^2-4y^2-72x-24y+24=0 \Rightarrow 12(x^2-6x+9)-4(y^2+6y+9)=-24+108-36 \Rightarrow$
$12(x-3)^2-4(y+3)^2=48 \Rightarrow \frac{(x-3)^2}{4}-\frac{(y+3)^2}{12}=1 \Rightarrow$ center of hyperbola = $(3,-3)$; foci are on the same horizontal line as the center and $c^2=4+12 \Rightarrow c=4 \Rightarrow$ foci are $(3\pm4,-3) \Rightarrow$ focus in quadrant III is $(-1,-3)$; when $x=-1 \Rightarrow \frac{(-1-3)^2}{4}-\frac{(y+3)^2}{12}=1$
$\Rightarrow 4-\frac{(y+3)^2}{12}=1 \Rightarrow \frac{(y+3)^2}{12}=3 \Rightarrow (y+3)^2=36 \Rightarrow y=-3\pm6=-9,\ 3$.

'06
GBML

ROUND 5 – Conics

1. Since the vertex must be the midpoint of the horizontal segment from the focus $F(7, 3)$ to $\mathcal{D}$, the directrix, the vertex is at (4, 3). The parabola opens away from the directrix, i.e. to the right. The standard equation is $(y-k)^2 = 4p(x-h)$, where (h, k) is the vertex and p denotes the directed distance from the vertex to the focus ($p = 3$).
Thus, the equation must be $\underline{\mathbf{(y-3)^2 = 12(x-4)}}$ or $\underline{\mathbf{y^2 - 12x - 6y + 57 = 0}}$

An alternative solution (using the definition of a parabola as a set of points equidistant from a fixed point and a fixed line):
You are encouraged to draw a sketch.
Let $P(x, y)$ be a point on the parabola. $PF = \sqrt{(x-7)^2+(y-3)^2}$ and $P\mathcal{D} = x-1$
Squaring both sides, $(x-7)^2 + (y-3)^2 = (x-1)^2 \rightarrow -14x + 49 + y^2 - 6y + 9 = -2x + 1$
$\rightarrow y^2 - 12x - 6y + 57 = 0$.

2. Letting $y = 0$ in the 2nd eqtn. $\rightarrow x = \pm 2 \rightarrow$ endpoint at (-2, 0)
Letting $x = 0$ in the 1st eqtn. $\rightarrow y = \pm 3 \rightarrow$ endpoint at (0, 3)
Diameter $= \sqrt{13} \rightarrow C = \underline{\pi\sqrt{13}}$

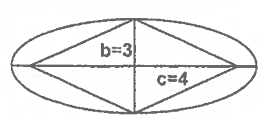
3. Substituting $x = 8$ into the equation of the hyperbola, we get $(y+2)^2/4 = 16/9$
$\rightarrow y = -2 \pm 8/3 = 2/3$ or $-14/3$
It's a horizontal hyperbola with $a = 3$, $b = 2$ and center at (3, -2).
The asymptotes intersect at (3, -2) and have slopes of $\pm 2/3$.
The equations of the asymptotes are $(y + 2) = \pm(2/3)(x-3)$
$\rightarrow 2x - 3y - 12 = 0$ and $2x + 3y = 0$
Now use the point to line distance formula between (8, 2/3) and $2x$-$3y$-12=0
$$\frac{|8(2)+(2/3)(-3)+(-12)|}{\sqrt{2^2+(-3)^2}} = \frac{16-2-12}{\sqrt{13}} = \frac{2}{\sqrt{13}} = \frac{2\sqrt{13}}{13}$$

'07
GBML

ROUND 5 – Conics

1. C_1 is a circle. Thus, the distance from P to C_1 is the distance from P to the center of the circle minus the radius of the circle. The center is at (-1, -2) and the radius is 4.
The required distance is $\sqrt{(7+1)^2+(13+2)^2}\ - 4 = \sqrt{289}\ - 4 = 17 - 4 = \underline{\mathbf{13}}$

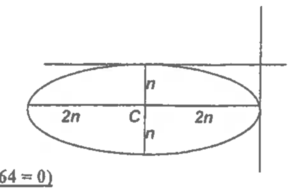
2. $9x^2 - 54x + 25y^2 + 100y = 44 \rightarrow 9(x^2 - 6x + 9) + 25(y^2 + 4y + 4) = 44 + 81 + 100 = 225$
$\rightarrow \frac{(x-3)^2}{25} + \frac{(y+2)^2}{9} = 1 \rightarrow$ center $C(3, -2)$, $a = 5$, $b = 3$ and $c = 4$ (horizontal ellipse)
and area $= 2\left(\frac{1}{2}\cdot 8\cdot 3\right) = \underline{\mathbf{24}}$



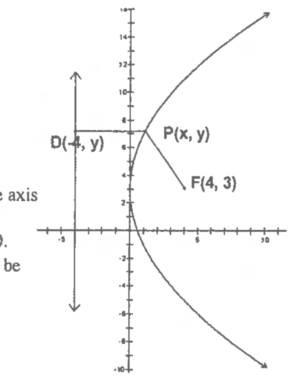
3. $(x^2 - 8x + 16) + (y^2 - 6y + 9) = 20 + 16 + 9 = 45 \rightarrow (x-4)^2 + (y-3)^2 = 45$
The vertex is at (4, 3) and the x-intercepts are: $(x-4)^2 + (0-3)^2 = 45 \rightarrow 4 \pm \sqrt{36} \rightarrow$ $A(10, 0)$ and $B(-2, 0)$ Since points A and B are equidistant from the vertex, the axis of symmetry must be the vertical line $x = 4$ and the equation must be of the form $y = p(x-h)^2 + k$, where (h, k) denotes the vertex. Since A is on the parabola, its coordinates must satisfy the equation and we have $0 = p(10-4)^2 + 3 \rightarrow p = -1/12$
Thus, the required equation is $\underline{\mathbf{y = -\frac{1}{12}(x-4)^2 + 3}}$ (or equivalent)

GBML
'08

ROUND 5



1. Let $4n$ and $2n$ denote the lengths of the major and minor axes respectively. $6n = 24 \rightarrow n = 4$
$\rightarrow$ the center $C(-8, -4)$, $a = 8$, and $b = 4$
$\rightarrow$ equation: $\underline{\mathbf{\frac{(x+8)^2}{64} + \frac{(y+4)^2}{16} = 1}}$ or $\underline{\mathbf{(x^2 + 4y^2 + 16x + 32y + 64 = 0)}}$



2. $PF = PD \rightarrow \sqrt{(x-4)^2+(y-3)^2} = x+4$
Squaring both sides and expanding the x-terms,
$x^2 - 8x + 16 + (y-3)^2 = x^2 + 8x + 16$
Canceling, $16x = (y-3)^2 \rightarrow \underline{x = \frac{1}{16}(y-3)^2}$
or $\underline{\mathbf{y^2 - 16x - 6y + 9 = 0}}$
Alternative: Knowing the description defines a parabola whose axis of symmetry is horizontal and which opens to the right, the equation must be of the form $(y-k)^2 = 4p(x-h)$, where $p > 0$.
The focus $F(4,3)$ and the directrix is $x = -4$ forces the vertex to be $V(0, 3)$ and $p = +4$. Thus, we arrive at the same equation.

Round 5 - continued

3. Ellipse: $9x^2 + 24y + 4y^2 - 36x + 36 = 0 \rightarrow 9(x^2 - 4x + 4) + 4(y^2 + 6y + 9) = -36 + 9\cdot 4 + 4\cdot 9$
$\rightarrow 9(x-2)^2 + 4(y+3)^2 = 36 \rightarrow \frac{(x-2)^2}{4} + \frac{(y+3)^2}{9} = 1 \rightarrow$ center $C(2, -3)$
Parabola: $x^2 - 8y = 39 - 6x \rightarrow x^2 + 6x + 9 = 8y + 39 + 9 \rightarrow (x+3)^2 = 8(y+6)$
$\rightarrow V(-3, -6)$, $p = +2$ (opening up) $\rightarrow F(-3, -4) \rightarrow CF = \sqrt{(2+3)^2+(-3+4)^2} = \underline{\sqrt{26}}$

GBML
'09

ROUND 5

1.
C_1: $y^2 - 6y + 9 = 12x + 15 + 9 \rightarrow (y-3)^2 = 12(x+2) \rightarrow 4p = 12 \rightarrow p = 3$
Thus, C_1 is a parabola with vertex $V(-2, 3)$ and focus $F(1, 3)$

$$\begin{array}{llll} & 2x^2 - 8x & + 2y^2 + 16y & = -1 \\ C_2: & x^2 - 4x & + y^2 + 8y & = -\frac{1}{2} \end{array}$$
$$\left(x^2 - 4x + 4\right) + \left(y^2 + 8y + 16\right) = (x-2)^2 + (y+4)^2 = -\frac{1}{2} + 20$$
Thus, C_2 is a circle with center at (2, –4).

$FC = \sqrt{(1-2)^2+(3+4)^2} = \sqrt{50} = \underline{5\sqrt{2}}$

2.
$x^2 - 6x + 9 + 4\left(y^2 + 8y + 16\right) = -57 + 9 + 64 \rightarrow (x-3)^2 + 4(y+4) = 16$
$\rightarrow \frac{(x-3)^2}{16} + \frac{(y+4)^2}{4} = 1 \rightarrow (a, b) = (4, 2)$ and $a^2 - b^2 = c^2 \rightarrow c = 2\sqrt{3}$
Since the major and minor axes are perpendicular, the area of the quadrilateral may be computed as half the product of the diagonals. The minor axis has length $2b$ and the distance between the foci is $2c$. Thus, the area is simply $bc = \underline{8\sqrt{3}}$

GBML '10

3.

Substituting, $\begin{array}{l} 28 = A+B+4 \\ 14 = \frac{1}{4}A + \frac{1}{2}B + 4 \end{array} \rightarrow \begin{array}{l} 28 = A+B+4 \\ 56 = A+2B+16 \end{array} \rightarrow \begin{array}{l} 24 = A+B \\ 40 = A+2B \end{array} \rightarrow (A, B) = (8, 16)$

Thus, $y = 8x^2+16x+4$ or $y-4 = 8(x^2+2x+1)-8 \rightarrow (y-4) = 8(x+1)^2$ and the vertex of the parabola is $\underline{(-1, 4)}$.

ROUND 5

1. $9x^2+4y^2-36x+24y+36=0 \rightarrow 9(x^2-4x+4)+4(y^2+6y+9) = -36+36+36=36$

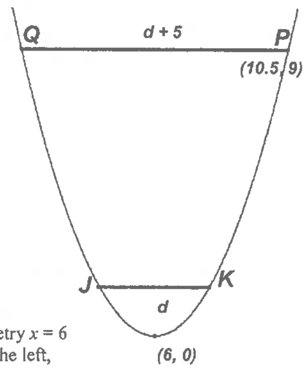
$\rightarrow \frac{(x-2)^2}{4}+\frac{(y+3)^2}{9}=1 \rightarrow$ Ellipse w/center @ (2, –3), $a=3$, $b=r=2$

$\rightarrow \underline{(x-2)^2+(y-3)^2=4}$ or $\underline{x^2+y^2-4x-6y+9=0}$

2. $4x^2-y^2+24x+4y+28=0 \rightarrow 4(x^2+6x+9)-(y^2-4y+4) = -28+36-4=4$

$\rightarrow \frac{(x+3)^2}{1}-\frac{(y-2)^2}{4}=1 \rightarrow$ Hyperbola w/center @ (–3, 2), $a=1$, $b=2$

$\rightarrow$ asymptotes: $(y-2) = \pm\frac{2}{1}(x+3) \rightarrow \underline{y=2x+8}$



3. The equation of the parabola must be of the form

$$(x-h)^2 = 4p(y-k)$$

Vertex at (6, 0) $\rightarrow (x-6)^2 = 4py$

$P(10.5, 9) \rightarrow \left(\frac{9}{2}\right)^2 = 4p(9) \rightarrow 4p = \frac{9}{4}$

Therefore, the equation of the parabola is $(x-6)^2 = \frac{9}{4}y$.

$y=9 \rightarrow (x-6)^2 = \frac{81}{4} \rightarrow x = 6 \pm \frac{9}{2} \rightarrow PQ = 9 \rightarrow JK = 4$

(or note that P and Q are symmetric w.r.t. the axis of symmetry $x = 6$ and since P is 4.5 units to the right, Q must be 4.5 units to the left, i.e. at (1.5, 9) and, consequently $PQ = 9$ and $JL = 4$)

Suppose the coordinate of K and J are (x_1, y'), (x_2, y') respectively.

Then $JK = x_2 - x_1 = 4$.

$(x-6)^2 = \frac{9}{4}y \rightarrow x = 6 \pm \frac{3}{2}\sqrt{y} \rightarrow JK = 3\sqrt{y} = 4 \rightarrow y = \frac{16}{9}$

$\rightarrow x = 6 \pm \frac{3}{2}\cdot\sqrt{\frac{16}{9}} = 6 \pm \frac{3}{2}\cdot\frac{4}{3} = 6 \pm 2 = 8, 4 \rightarrow J\ \underline{\left(4, \frac{16}{9}\right)}\quad K\underline{\left(8, \frac{16}{9}\right)}$

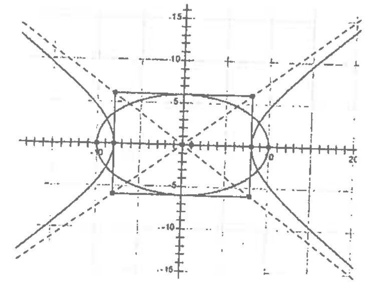
GBML '11

ROUND 5

1. $x^2+y^2-6x+8y=0 \rightarrow (x^2-6x+\underline{9})+(y^2+8y+\underline{16}) = 9+16=25$

$\rightarrow (x-3)^2+(y+4)^2=5^2 \rightarrow$ Center: (3, –4) Radius: 5

$\rightarrow$ endpoints of vertical diameter: $\underline{(3, 1), (3, -9)}$



2. Hyperbola: (horizontal), $(a, b) = (8, 6) \rightarrow c = 10$

Ellipse: (horizontal), $(a, c) = (10, 8) \rightarrow b = 6$

Focal chord: $\frac{2b^2}{a} = \frac{72}{10} = \underline{\frac{36}{5}}$ (or $\underline{7.2}$)

3. $y^2-8x=4y+12 \rightarrow (y-2)^2 = 8(x+2)$

$\rightarrow$ horizontal parabola w/vertex(–2, 2), $a=2 \rightarrow$ focus (0, 2)

$\overline{RS}: (y-10) = \frac{4}{3}(x-6) \rightarrow 3y-30 = 4x-24 \rightarrow x = \frac{3(y-2)}{4}$

Substituting, $(y-2)^2 = 6(y-2)+16 = 6y+4$

$\rightarrow y^2-10y = y(y-10) = 0$

$\rightarrow y = 0, 10 \rightarrow S\underline{\left(-\frac{3}{2}, 0\right)}$