NEW ENGLAND ASSOCIATION OF MATHEMATICS LEAGUES

PLAYOFFS – 2005

Round 1 Arithmetic and Number Theory

1. ____(___, ___)____

2. ________________

3. ________________

1. Mary needs to get a B average in math. Her test scores so far are 72, 81, 77, and 92. What is the range of scores she can get on the final test of the term so her average will be a B for the term? The maximum score on any one test is 100. (A grade of B is given for an average between 80 and 89 inclusive.) Express your answer as the ordered pair (min, max) where min is the minimum score she can get and max is the maximum score.

2. Two fractions are such that their numerators and denominators are the four one digit prime numbers. Determine the largest possible sum for these fractions.

3. Determine the smallest integer value of n, $n > 1$, such that the set of consecutive counting numbers, $S = \{1, 2, 3, 4, \ldots, n\}$ contains exactly 8.75 times as many multiples of 2 as of 17.

NEW ENGLAND ASSOCIATION OF MATHEMATICS LEAGUES

PLAYOFFS – 2005

Round 2 Algebra 1

1. ______________________

2. ______________________

3. ______________________

1. If $25^{3x-2} = \left(\frac{1}{125}\right)^{4-x}$, determine the value of x.

2. In Tom's pocket, there are four times as many quarters as nickels but there are not enough nickels to make change for a quarter. 15 more than the square of the number of nickels is equal to 8 times the number of nickels. If there are no other coins in Tom's pocket, what is the total value of these coins in cents.

3. If $a \oplus b = ab + b$, then find all a such that the following equation has two distinct real solutions in x: $x \oplus (a \oplus x) = -\frac{1}{8}$.

NEW ENGLAND ASSOCIATION OF MATHEMATICS LEAGUES

PLAYOFFS – 2005

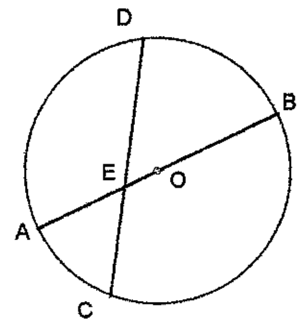
Round 3 – Geometry

1. ____________________

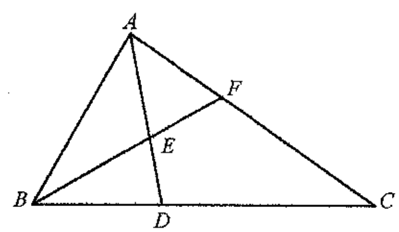
2. ____________________

3. ____________________

1. In circle O, diameter $\overline{AB}$ has length 12 units. Chord $\overline{CD}$ intersects $\overline{AB}$ at E. AE:EB = 3:5 and CE = 4. Find the number of units in the length of $\overline{ED}$.



2. In ΔABC, $\overline{BF}$ bisects $\angle ABC$ and $\frac{BE}{EF}=\frac{BC}{BA}$. If AB = 2 and BC = 3, express the ratio $\frac{AE}{ED}$ in the form $\frac{a}{b}$ where a and b are relatively prime.



3. In a 5-12-13 triangle, a segment is drawn parallel to the hypotenuse and 1/3 the way from the hypotenuse to the opposite vertex. Another segment is drawn parallel to the first and 1/3 the way from the previous segment to the opposite vertex. Determine the number of square units in the trapezoid whose bases are the two drawn segments.

NEW ENGLAND ASSOCIATION OF MATHEMATICS LEAGUES

PLAYOFFS – 2005

Round 4 – Algebra 2

1. ____________________

2. ____________________

3. ____________________

1. A is the sum of all terms of an infinite geometric sequence whose fourth term is 4 and whose seventh term is $\frac{1}{2}$. B is the sum of all terms of an infinite geometric sequence whose fifth term is $\frac{1}{3}$ and whose common ratio is $\frac{1}{3}$. Determine the value of $A + B$.

2. If $f(x) = 2x + 1$, determine all values of x such that $f\left(\frac{1}{f^{-1}(x)}\right) = f^{-1}\left(\frac{1}{f(x)}\right)$. Note: f^{-1} is the inverse function of f.

3. If $[x]$ stands for the greatest integer less than or equal to x, solve $x[x] = 2005$.

NEW ENGLAND ASSOCIATION OF MATHEMATICS LEAGUES

PLAYOFFS – 2005

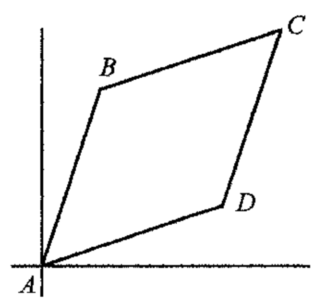
Round 5 – Analytic Geometry

1. ____________

2. (, ,)

3. ____________

1. Given $A(0,0)$, $ABCD$ is a rhombus of side 5 where the slope of $\overline{AB}$ is 2 and the slope of $\overline{AD}$ is $\frac{1}{2}$. Find the coordinates of C.



2. The equation of a hyperbola is $xy - 4y + 6x - 28 = 0$. A circle is such that its diameter is the segment connecting the vertices of the hyperbola. If the equation of the circle is written as $(x-h)^2 + (y-k)^2 = r^2$, where r is its radius, determine the ordered triple (h,k,r).

3. Let k be a positive number. If the area of the region bounded by the x-axis and the graph of $y = |x| + |x-1| - k$ is 16, determine the value of k.

NEW ENGLAND ASSOCIATION OF MATHEMATICS LEAGUES

PLAYOFFS – 2005

Round 6 – Trig and Complex Numbers

1. ______________________

2. ______________________

3. ______________________

1. Simplify $\frac{(8-i)^2}{1-2i}$ where $i=\sqrt{-1}$. Express the answer in the form $a+bi$

2. If $\tan(a+b)=\frac{7}{3}$, $\tan b=\frac{2}{3}$, and a and b are acute, what is the value of $\tan(b-a)$?

3. For $0 \le x \le 2\pi$, if $\frac{\cos 2x+1}{\cos 2x-1}=-\sec^2 x$, determine the value of $\cos^2 x$.

MASSACHUSETTS ASSOCIATION OF MATHEMATICS LEAGUES

NEW ENGLAND PLAYOFFS – 2005

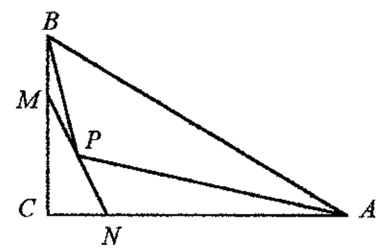
Team Round

1. ____________________ **4.** ____________________

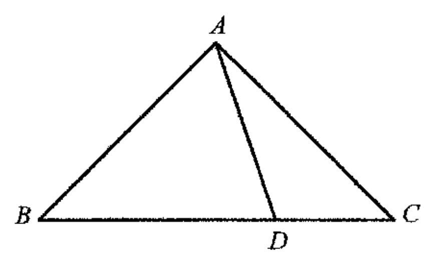
2. ____________________ **5.** ____________________

3. ____________________ **6.** ____________________

1. In ΔABC, $AC = 8$, $BC = 6$, and $AB = 10$. If $m\angle MNC = m\angle ABC$ and P is the midpoint of $\overline{MN}$, determine the length of $\overline{MN}$ so that the area of ΔBPA is half the area of ΔABC.



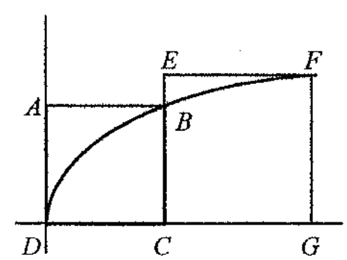
2. In isosceles ΔABC, $m\angle BAC = 108°$. $\overline{AD}$ trisects $\angle BAC$ and $BD > DC$. Find $\dfrac{BD}{DC}$.



3. A particle is moving on the curve defined by the parametric system $x = 1 - 2\cos^2\theta$ and $y = \cos\theta$. The particle is closest to the origin when $\theta = \cos^{-1} a$ for $0 \le \theta \le \dfrac{\pi}{2}$. Determine the value of a.

4. The first term of an infinite geometric series is 2 and the sum of the series lies less than 1/10 from 2. Find the values that the common ratio r can take on assuming that $r \neq 0$.

5. $ABCD$ and $EFGC$ are squares and the curve $y = k\sqrt{x}$ passes through the origin D and points B and F. Determine $\dfrac{FG}{BC}$.



6. Each side of a regular dodecagon $A_1A_2A_3....A_{12}$ is 2 units long. Determine the area of the pentagon $A_1A_2A_3A_4A_5$.

NEW ENGLAND ASSOCIATION OF MATHEMATICS LEAGUES

PLAYOFFS – 2005

Answer Sheet

Round 1

1. (78, 100)
2. $\frac{31}{6}$
3. 70

Round 2

1. $-\frac{8}{3}$
2. 315
3. $a < -1 \text{ or } a > -\frac{1}{2}$

Round 3

1. $\frac{135}{16}$
2. $\frac{16}{9}$
3. $\frac{200}{27}$

Round 4

1. $\frac{209}{2}$
2. -1
3. $-\frac{401}{9}$

Round 5

1. $\left(3\sqrt{5}, 3\sqrt{5}\right)$
2. $\left(4, -6, 2\sqrt{2}\right)$
3. $\sqrt{33}$

Round 6

1. $19 + 22i$
2. $\frac{1}{99}$
3. $\frac{-1+\sqrt{5}}{2}$

Team

1. 4.8
2. $\frac{1+\sqrt{5}}{2}$
3. $\frac{\sqrt{6}}{4}$
4. $-\frac{1}{19} < r < \frac{1}{21}$
5. $\frac{1+\sqrt{5}}{2}$
6. $5+2\sqrt{3}$

MASSACHUSETTS ASSOCIATION OF MATHEMATICS LEAGUES

NEW ENGLAND PLAYOFFS – 2005 - SOLUTIONS

Round 1 Arithmetic and Number Theory

1. So far she has 322 points. She needs a total between 400 and 445 inclusive. However, the most on one test is 100.

2. $\frac{7}{2}+\frac{5}{3}=\frac{31}{6}$

3. For $n = 17$ to 33, the ratios increase from 8:1 to 16:1 by 1's at every other number. From $n = 34$ to $n = 50$, the ratios increase from 17:2 to 25:2 by halves at every other number. From $n = 51$ to 67, the ratios increase from 25:3 to 33:3 by thirds at every other number. Thus, our first solution must occur in the 4th interval of 17's. At $n = 68$ the ratio is 34:4 and at $n = 70$, the ratio is 35:4 $= 8\frac{3}{4}$, making $n = \boxed{70}$.

Round 2 Algebra 1

1. $5^{2(3x-2)} = 5^{-3(4-x)} \rightarrow 6x-4=-12+3x$

2. Let n = number of nickels. $n^2+15=8n$ has solutions 3 and 5. 5 doesn't satisfy the condition. So there are 3 nickels and 12 quarters for \$3.15.

3. $x \oplus (a \oplus x) = x \oplus (ax+x) = ax^2+x^2+ax+x=-\frac{1}{8} \rightarrow 8(a+1)x^2+8(a+1)x+1=0$.

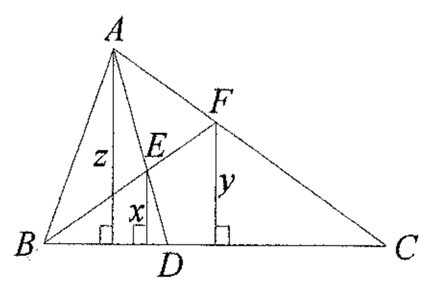
   By the discriminant: $(8(a+1))^2 - 4\cdot 8(a+1) > 0 \rightarrow (a+1)(2a+1)>0$.

Round 3 – Geometry

1. Denote the segments on the diameter as $3x$ and $5x$, getting $3x+5x=12 \rightarrow x=1.5$. Let ED = y.
   $\frac{9}{2}\cdot\frac{15}{2}=4y \rightarrow y=\frac{135}{16}$

2. Consider the area of the trapezoid to be the difference of the areas of two right triangles.
   $\frac{1}{2}\cdot 8\cdot\frac{10}{3}-\frac{1}{2}\cdot\frac{16}{3}\cdot\frac{20}{9}$

3. Since $\overline{BF}$ bisects $\angle ABC$ and $AB=2$, $BC=3$, then $\frac{AB}{BC}=\frac{AF}{FC}=\frac{EF}{BE}=\frac{2}{3}$. Drop perpendiculars with length x, y, and z from points E, F, and A respectively.



By similar triangles, $\frac{x}{y}=\frac{3}{5}$ and $\frac{y}{z}=\frac{3}{5} \Rightarrow \frac{x}{z}=\frac{9}{25} \Rightarrow$

$\frac{ED}{DA}=\frac{9}{25} \Rightarrow \frac{AE}{ED}=\frac{16}{9}$.

Round 4 – Algebra 2

1. $\frac{1}{2}=4r^3 \to r=\frac{1}{2}$. Now determine the first terms. They are 27 and 16. $\frac{27}{1-\frac{1}{3}}+\frac{16}{1-\frac{1}{2}}=$

$\frac{81}{2}+64=\frac{209}{2}$

2. From $f(x)=2x+1$ we obtain $f^{-1}(x)=\frac{x-1}{2}$. Thus, $f\left(\frac{1}{f^{-1}(x)}\right)=f\left(\frac{2}{x-1}\right)=$

$\frac{4}{x-1}+1=\frac{x+3}{x-1}$. Also, $f^{-1}\left(\frac{1}{f(x)}\right)=f^{-1}\left(\frac{1}{2x+1}\right)=\frac{\frac{1}{2x+1}-1}{2}=\frac{-x}{2x+1}$. Thus,
$\frac{x+3}{x-1}=\frac{-x}{2x+1} \to x^2+2x+1=0 \to x=\boxed{-1}$.

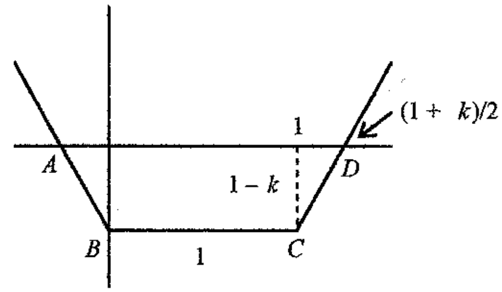
3. Note that $44^2=1936$ and $45^2=2025$. If $44 \le x < 45$, then
$x[x]=2005 \to x=\frac{2005}{44}=45\frac{25}{44}$ and that is greater than 45. If $-45 \le x < -44$, then $[x]=-45$ and we have $x[x]=2005 \to x=\frac{2005}{-45}=-44\frac{25}{45}$ and that works. Ans: $-\frac{2005}{45}=$
$\boxed{-\frac{401}{9}=-44\frac{5}{9}}$.

Round 5 – Analytic Geometry

1. Given $B(a,2a)$ and $AB=5$, then $a^2+(2a)^2=5^2 \to B\left(\sqrt{5},2\sqrt{5}\right)$. Similarly, we have $D\left(2\sqrt{5},\sqrt{5}\right)$. Thus, we have $C\left(3\sqrt{5},3\sqrt{5}\right)$.
2. The equation of the hyperbola is also $(x-4)(y+6)=4$. It is a rectangular hyperbola centered at (4, -6) with vertices at (6, -4) and (2, -8). The distance between these is $4\sqrt{2}$, so the radius of the circle is $2\sqrt{2}$.

3. $-16 = 1(1-k) + 2 \cdot \frac{1}{2}(1-k)\left(\frac{1+k}{2} - 1\right) \rightarrow$

$k^2 = 33 \rightarrow k = \boxed{\sqrt{33}}$.



Round 6 – Trig and Complex Numbers

1. $\frac{63-16i}{1-2i} \cdot \frac{1+2i}{1+2i} = \frac{95+110i}{5} = 19 + 22i$

2. $\frac{\tan a + \frac{2}{3}}{1 - \frac{2}{3}\tan a} = \frac{7}{3} \rightarrow \frac{3\tan a + 2}{3 - 2\tan a} = \frac{7}{3} \rightarrow \tan a = \frac{15}{23}$. $\therefore \tan(b-a) = \frac{\frac{2}{3} - \frac{15}{23}}{1 + \frac{2}{3} \cdot \frac{15}{23}} = \frac{1}{99}$

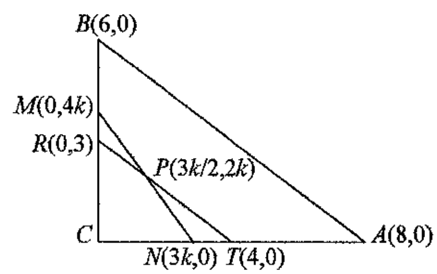
3. $\frac{\cos(2x)+1}{\cos(2x)-1} = \frac{(2\cos^2 x - 1) + 1}{(1 - 2\sin^2 x) - 1} = -\frac{\cos^2 x}{\sin^2 x}$. Setting $-\frac{\cos^2 x}{\sin^2 x} = -\sec^2 x$ yields

$\cos^4 x + \cos^2 x - 1 = 0 \rightarrow \cos^2 x = \boxed{\frac{-1+\sqrt{5}}{2}}$.

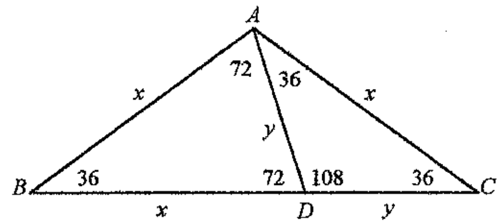
Team Round

1. Place the triangle with C at the origin. P must lie on the midline of ΔBCA and given $\Delta BCA \sim \Delta NCM$, let $M = (0,4k)$ and $N = (3k,0)$. $P\left(\frac{3k}{2}, 2k\right)$ must satisfy the equation for the midline $\overline{RT}$ which is $y = -\frac{3}{4}x + 3$ giving $2k = -\frac{3}{4}\left(\frac{3k}{2}\right) + 3 \rightarrow$

$k = \frac{24}{25}$. Since $MN = 5k$, $MN = \boxed{\frac{24}{5} = 4.8}$.



2. From ΔABC, $\dfrac{x+y}{\sin 108} = \dfrac{x}{\sin 36}$ and from ΔBDC, $\dfrac{x}{\sin 108} = \dfrac{y}{\sin 36}$. Thus, $\dfrac{x+y}{x} = \dfrac{x}{y}$. This gives $x^2 - xy + y^2 = 0 \rightarrow$ $\left(\dfrac{x}{y}\right)^2 - \dfrac{x}{y} - 1 = 0 \rightarrow \dfrac{x}{y} = \dfrac{BD}{DC} = \boxed{\dfrac{1+\sqrt{5}}{2}}$.

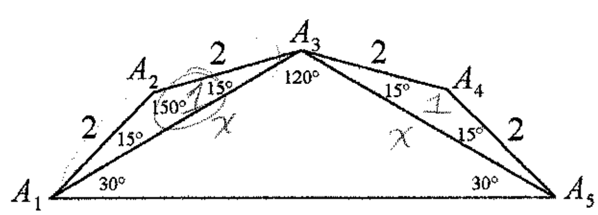


3. Let the point $P(x,y)$ lie on the curve. The distance from P to the origin equals $\sqrt{x^2+y^2}$ = $\sqrt{4\cos^4\theta - 3\cos^2\theta + 1}$. The radicand is quadratic in $\cos^2\theta$ reaching its minimum at $\cos^2\theta = -\dfrac{-3}{2\cdot 4} = \dfrac{3}{8}$. The minimum occurs at $\cos\theta = \sqrt{\dfrac{3}{8}}$. Thus, $a = \sqrt{\dfrac{3}{8}} = \boxed{\dfrac{\sqrt{6}}{4}}$.

4. $\left|\dfrac{2}{1-r} - 2\right| < \dfrac{1}{10} \rightarrow -\dfrac{1}{10} < \dfrac{2r}{1-r} < \dfrac{1}{10} \rightarrow \dfrac{r-1}{10} < 2r$ and $2r < \dfrac{1-r}{10}$. $1-r>0$ for convergence. Solving gives $\boxed{-\dfrac{1}{19} < r < \dfrac{1}{21}}$. Note: $r \neq 0$ need not be stated.

5. Let $BC = a$ and $FG = b$. Then, from point $B(a,a)$ we obtain $a = k\sqrt{a} \rightarrow k = \sqrt{a}$. From point $F(a+b,b)$ we obtain $b = k\sqrt{a+b} = \sqrt{a(a+b)}$. Thus, $b^2 = a^2 + ab \rightarrow$ $\left(\dfrac{b}{a}\right)^2 - \dfrac{b}{a} - 1 = 0 \rightarrow \dfrac{b}{a} = \boxed{\dfrac{1+\sqrt{5}}{2}}$.

6. Each interior angle is 150° and so the area of each of $\Delta A_1A_2A_3$ and $\Delta A_3A_4A_5$ is $\dfrac{1}{2}\cdot 2\cdot 2\cdot \sin 150° = 1$. Let $A_1A_3 = x$.



Then $x^2 = 2^2 + 2^2 - 2\cdot 2\cdot 2\cos 150° = 8 + 4\sqrt{3}$. The area of $\Delta A_1A_3A_5 = \dfrac{1}{2}x^2 \sin 120° = 3 + 2\sqrt{3}$. Thus, the total area is $\boxed{5+2\sqrt{3}}$.

MASSACHUSETTS ASSOCIATION OF MATHEMATICS LEAGUES

NEW ENGLAND PLAYOFFS – 2004

Round 1 Arithmetic and Number Theory

1. ______________________

2. ______________________

3. ______________________

1. If $N = 9{,}998^2 + 4(9{,}998)$, determine the number of digits in N.

2. The number $1 - 0.66_9$ is what number in base 3.

3. Given n is a positive integer, $n \leq 2004$, n equals the sum of 3 consecutive positive integers, and n equals the sum of 4 consecutive positive integers. How many different values are there for n?

NEW ENGLAND ASSOCIATION OF MATHEMATICS LEAGUES

NEW ENGLAND PLAYOFFS – 2004

Round 2 Algebra 1

1. ____________________

2. ____________________

3. ____________________

1. There are 24 students in a classroom. Six move from the left side of the classroom to the right side and now the right side has as many students as the left side used to have. How many did the left side have originally?

2. $\dfrac{x^{-3/2}\cdot\sqrt[5]{y}}{x^{-7/3}\cdot\sqrt[10]{y}}$ can be written as $\sqrt[n]{x^a y^b}$ where a and b are integers. What is the minimum possible sum of a, b, and n?

3. Given $2^{\pi}x+\left(2^{\pi}+5\right)y=3^{\sqrt{2}}x+\left(3^{\sqrt{2}}+5\right)y$, determine the value of $\dfrac{x}{y}$.

NEW ENGLAND ASSOCIATION OF MATHEMATICS LEAGUES

PLAYOFFS – 2004

Answer Sheet

Round 1

1. 8
2. $.021_3$ (or .021 with base 3 implied)
3. 166

Round 2

1. 15
2. 58
3. -1

Round 3

1. 276
2. $\sqrt{35}$
3. 36 (36°)

Round 4

1. 33
2. $-\sqrt{2} < x < \sqrt{2}$
3. $x < 0$

Round 5

1. $\frac{7}{5}$
2. (4, 32)
3. $-\sqrt{3}$

Round 6

1. $-\frac{1}{2}$
2. $f(y) = \dfrac{2y}{1-y^2}$
3. 167

Team

1. $9\left(\frac{\pi}{2} - \sqrt{2}\right)$ or equivalent
2. $\dfrac{492}{25}$
3. $n\left(\dfrac{3-n^2}{2}\right)$ $\left(\text{or } \dfrac{3n-n^3}{2} \text{ or } \dfrac{3}{2}n - \dfrac{1}{2}n^3\right)$
4. 8014
5. (-2, 3)
6. $\left(\dfrac{7}{2}, -\dfrac{25}{7}\right)$

NEW ENGLAND ASSOCIATION OF MATHEMATICS LEAGUES

PLAYOFFS – 2004 – Solutions

Round 1

1. $N = \left(9{,}998^2 + 4 \cdot 9{,}998 + 4\right) - 4 = (9{,}998 + 2)^2 - 4 = 10{,}000^2 - 4 =$
$\left(10^4\right)^2 - 4 = 10^8 - 4$. Since 10^8 has 9 digits, N has 8 digits.

2. $1 - 0.66_9 = 1 - \left(\frac{6}{9} + \frac{6}{81}\right) = 1 - \left(\frac{2}{3} + \frac{2}{27}\right) = 1 - 0.202_3 = 0.021_3$

3. $n = 3x + 3 = 4y + 6$ for positive integers x and y. Therefore $4y = 3(x-1)$. Since $4y + 6 \le 2004 \Rightarrow y \le 499.5$, then the possible values for y are 3, 6, 9, ... 498, which are 166 possibilities.

Round 2

1. Let x = the original number of students on the left side and $24 - x$ be the original number on the right side. If 6 students move from left to right then the right side has $30 - x$. Thus, $x = 30 - x$ so $x = 15$.

2. $\dfrac{x^{-3/2} \cdot \sqrt[5]{y}}{x^{-7/3} \cdot \sqrt[10]{y}} = x^{5/6} \cdot y^{1/10} = \sqrt[30]{x^{25} y^3}$

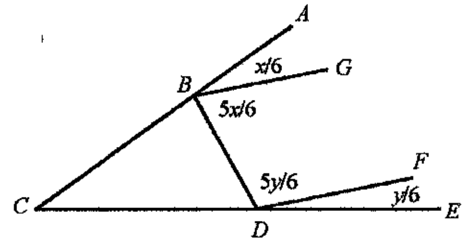
3. Let $2^{\pi} = a$ and $3^{\sqrt{2}} = b$. Then $ax + (a+5)y = bx + (b+5)y$ gives
$(a-b)x = [(b+5) - (a+5)]y = (b-a)y$. Then $\frac{x}{y} = -1$.

Round 3

1. Draw $\overline{BE} \perp \overline{DC}$, then BE = 12 EC = 12, DC = 29

2. Let $AB = 2x$ and $AD = 2y$. Using ΔAEF, $x^2 + y^2 = \left(2\sqrt{3}\right)^2 = 12$. Using ΔFBC,
$x^2 + (2y)^2 = \left(\sqrt{13}\right)^2 \to x^2 + 4y^2 = 13$. Subtracting the first from the second gives
$3y^2 = 1 \to y = \frac{1}{\sqrt{3}}$. Then $x^2 + \frac{1}{3} = 12 \to x = \sqrt{\frac{35}{3}}$. Hence, $\frac{x}{y} = \frac{AB}{AD} = \sqrt{35}$.

3. Let $m\angle ABD = x \rightarrow m\angle GBD = \frac{5x}{6}$. Let $m\angle BDE = y \rightarrow m\angle BDF = \frac{5y}{6}$. Since $\overline{BG} \parallel \overline{DF}$, then $\frac{5x}{6}+\frac{5y}{6}=180$ giving $x+y=216$. Since $m\angle CBD = 180-x$ and $m\angle BDC = 180-y$, then $m\angle C = 180-(180-x+180-y) = x+y-180 = 216-180 = 36$.

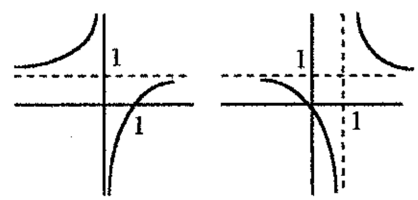


Round 4

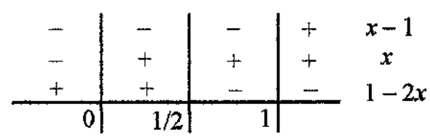
1. $M:N = 3:8 \rightarrow N = \frac{8}{3}M$. $M^2 - 3N = M \rightarrow M^2 - 8M = M$

2. For $\sqrt{1-[x^2]}$ to be real, $[x^2] \le 1 \rightarrow [x^2] = 0,1 \rightarrow 0 \le x^2 < 2 \rightarrow -\sqrt{2} < x < \sqrt{2}$.

3. Graphing $y = \frac{x-1}{x}$ and $y = \frac{x}{x-1}$ shows us that the domain of the inequality is $x < 0$ or $x > 1$. It is also shows that the solution set is $x < 0$.



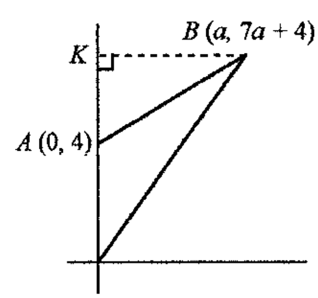
Alternate solution: Squaring both sides we obtain:
$\frac{x-1}{x} > \frac{x}{x-1} \rightarrow \frac{x-1}{x} - \frac{x}{x-1} > 0 \rightarrow \frac{1-2x}{x(x-1)} > 0$.
Solving using the sign graph we obtain $x < 0$ or $1/2 < x < 1$, but reject the latter set since it violates the domain of the inequality.



Round 5

1. A line passing through the center of a square bisects the square. The center of $ABCD$ is $M\left(\frac{15}{2},\frac{21}{2}\right)$ and the slope of ℓ is $\frac{21/2-0}{15/2-0}=\frac{7}{5}$.

2. Let the coordinates of B be $(a, 7a+4)$. Then
$\frac{1}{2}\cdot AC\cdot BK = slope \rightarrow \frac{1}{2}\cdot 4a = \frac{7a+4}{a} \rightarrow$
$2a^2-7a-4=0 \rightarrow a = 4$ gives $B(4, 32)$.



3. By the Triangle Angle Bisector Theorem, if $\overline{AP}$ bisects $\angle OAB$ then $\dfrac{OA}{AB}=\dfrac{OP}{PB}$.

Since P is a trisection point then $\dfrac{OP}{PB}=\dfrac{1}{2}\rightarrow\dfrac{OA}{AB}=\dfrac{1}{2}$. Thus, ΔOAB is a 30-60-90 right triangle making $AB = 12$ and $OB = 6\sqrt{3}$. Hence, the x-coordinate of P is $2\sqrt{3}$, making the slope of $\overline{AP}$ equal $\dfrac{6-0}{0-2\sqrt{3}}=-\dfrac{3}{\sqrt{3}}=-\sqrt{3}$.

Round 6

1. $\cos(127.5)\cos(7.5)-\cos(37.5)\sin(187.5)=-\sin(37.5)\cos(7.5)-\cos(37.5)[-\sin(7.5)]$
$=-(\sin37.5\cos7.5-\cos37.5\sin7.5)= -\sin 30 = -\dfrac{1}{2}$.

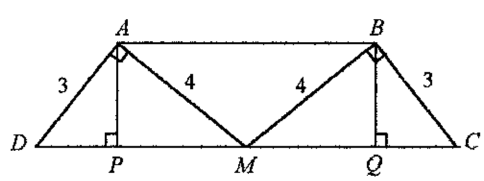
2. $\dfrac{\sin4x}{1+\cos4x}=\dfrac{2\sin2x\cos2x}{1+2\cos^2 2x-1}=\dfrac{\sin2x}{\cos2x}=\tan2x=\dfrac{2\tan x}{1-\tan^2 x}$. Thus, $f(y)=\dfrac{2y}{1-y^2}$.

3. Group the first 2004 terms by 4's obtaining
$\left(i+2i^2+3i^3+4i^4\right)+\cdots+\left(2001^{2001}+2002^{2002}+2003^{2003}+2004^{2004}\right)=$
$\left(i-2-3i+4\right)+\cdots+\left(2001-2002-2003+2004\right) = \left(-2i+2\right)+\cdots+\left(-2i+2\right)$. There are 501 such terms with a sum total of $-1002i + 1002$. Grouping the next 2004 terms in the same way we obtain 501 terms of $2i + 2$ for a total of $1002i + 1002$. The sum of both groups is 2004. $2004 = 2^2\cdot3\cdot167$

Solutions – Team Round

1. The minimum occurs when B bisects $\widehat{AC}$. The area of $\Delta AOC = \dfrac{1}{2}\left(3\sqrt{2}\right)^2$. The area of ΔABC is $\dfrac{1}{2}(6)\left(3\sqrt{2}-2\right)$.

2. Both ΔADP and ΔBOQ are 3-4-5 triangles so $AD = 3 = 5x$ gives $x = 3/5$. Then, $DP = 3x = 9/5$ and $\overline{AP} = 4x = 12/5$. The length of base $\overline{AB} = 10-2(9/5) = 32/5$. The area of $ABCD$ equals $\dfrac{1}{2}\cdot\dfrac{12}{5}\left(10+\dfrac{32}{5}\right)=\dfrac{492}{25}$.



3. $\sin^3 x - \cos^3 x = (\sin x - \cos x)(\sin^2 x + \sin x\cos x + \cos^2 x) = n(1 + \sin x\cos x)$. From $(\sin x - \cos x)^2 = n^2$ we obtain $\sin^2 x - 2\sin x\cos x + \cos^2 x = n^2 \rightarrow$ $\sin x\cos x = \frac{n^2 - 1}{-2} = \frac{1 - n^2}{2}$. Thus, $\sin^3 x - \cos^3 x = n\left(1 + \frac{1 - n^2}{2}\right) = n\left(\frac{3 - n^2}{2}\right)$.

4. $S_1 = 1, S_2 = -1, S_3 = -4, S_4 = 0, S_5 = 5, S_6 = -1, S_7 = -8, S_8 = 0, S_9 = 9, S_{10} = -1$, Thus, $S_{2+4k} = -1$. Starting with $k = 0$, The 2004$^{\text{th}}$ term occurs when $k = 2003$, giving $n = 2 + 4(2003) = 8014$.

5. The problem implies an invariant result. The following system leads to the answer:

$$\begin{aligned} x + 5y &= 13 \\ 29x + 61y &= 125 \end{aligned}$$ Solving gives $x = -2$ and $y = 3$.

More generally, starting with a_1 we obtain in turn $a_2 = 2a_1, a_3 = 4a_1 + 9$, . $a_4 = 8a_1 + 21, a_5 = 16a_1 + 45, and\ a_6 = 32a_1 + 93$ Using determinants, we have

$$x = \frac{\begin{vmatrix} a_3 & a_2 \\ a_1 & a_5 \end{vmatrix}}{\begin{vmatrix} a_1 & a_2 \\ a_4 & a_5 \end{vmatrix}} = \frac{(4a+9)(16a+45) - (32a+93)(2a+3)}{a(16a+45) - (2a+3)(8a+21)} = \frac{42a(a+3)}{-21a(a+3)} = -2.$$

$$y = \frac{\begin{vmatrix} a_1 & a_3 \\ a_4 & a_6 \end{vmatrix}}{-21a(a+3)} = \frac{-63a(a+3)}{-21a(a+3)} = 3.$$

6. In the 1$^{\text{st}}$ quadrant the system becomes $x + xy = 16$ and $x + xy = -9$. That system has no solution. In the 2$^{\text{nd}}$ quadrant we have $x + xy = 16$ and $x - xy = -9$. Solving, we obtain $x = \frac{7}{2}$ which lies outside the quadrant. In the 3$^{\text{rd}}$ quadrant we obtain $x - xy = 16$ and $x - xy = -9$ which has no solution. Finally, in the 4$^{\text{th}}$ quadrant the system becomes $x - xy = 16$ and $x + xy = -9$. Solving, we obtain $x = \frac{7}{2}$ and $y = -\frac{25}{7}$ yielding the answer $\left(\frac{7}{2}, -\frac{25}{7}\right)$.

N.E.A.M.L. PLAYOFFS - 1997

Round 1 - Arithmetic, Number Theory

1. ______________________

2. ______________________

3. ______________________

1. A man paid cash for a book and a shirt. He gave the first clerk \$20.00 for the book and received \$7.50 change. He gave the second clerk \$25.00 for the shirt and received \$2.50 change. After that he had \$8.00 left. How much money did the man have before buying these items?

2. Let n be a positive integer. If there are 1996 integers between n^2 and $(n+1)^2$, not including n^2 and $(n+1)^2$, determine the value of n.

3. If digits A and B are not necessarily distinct, determine how many 5-digit numbers of the form 2A1B2 are divisible by 11. (Note: A number is divisible by 11 if the sum of the digits in the odd positions minus the sum of the digits in the even positions is divisible by 11.)

N.E.A.M.L. PLAYOFFS - 1997

Round 2 - Algebra 1

1. ______________________

2. ______________________

3. ______________________

1. $\sqrt{\left(\frac{25}{4}\right)^{-1}} + \left(\frac{8}{27}\right)^{\frac{2}{3}}$

2. Paul left his house at 8 a.m. and traveled for 10 hours at 3 mph. The next day he returned home, leaving at 8 a.m. and walking at 5 mph. He passed by the same mill at the same time both days. To the minute, what time of day was that?

3. What are the real value(s) of x which satisfy the statement to the right? $\frac{|x|}{x} < x$

N.E.A.M.L. PLAYOFFS - 1997

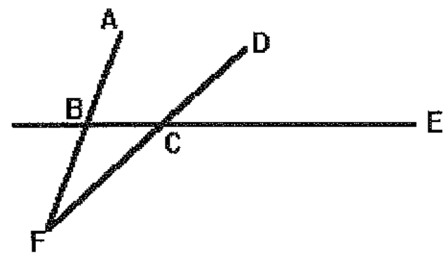
Round 3 - Geometry

1. ______________________

2. ______________________

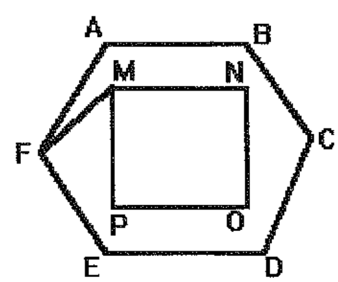
3. ______________________

1. If m∠ABC is 6° greater than m∠DCE, how many degrees are in m∠F?



2. A chord of length 20 cm passes through a trisection point of a diameter. The segments of the chord have lengths in the ratio 2 to 3. Exactly how many centimeters are in the radius of the circle?

3. Regular hexagon ABCDEF and square MNOP have the same center and sides of length 1. A, M, P, E are collinear. If $\overline{AB} // \overline{MN}$, determine the number of degrees in ∠MFE.



N.E.A.M.L. PLAYOFFS - 1997

Round 4 - Algebra II

1. ____________________

2. ____________________

3. ____________________

1. Reduce to simplest form: $\dfrac{x^4 - y^4}{\left(1 - \dfrac{y^2}{x^2}\right)\left(1 + \dfrac{y^2}{x^2}\right)}$

2. If $f(x) = 7x + b, g(x) = 3x + d,$ and $f(g(x)) = g(f(x))$, determine the value of $\dfrac{b}{d}$.

3. For A, B, C, D defined as shown, if $1 < x < y$, then write in order from left to right, smallest to largest:

   $A = \log_x y$, $B = \log_{2x}(2y)$, $C = \log_{3x}(3y)$, $D = \log_{4x}(4y)$

N.E.A.M.L. PLAYOFFS - 1997

Round 5 - Analytic Geometry

1. ____________________

2. ____________________

3. ____________________

1. For $f(x) = 3x^2 - 6x + 7$, how many units are in the length of the horizontal chord one unit from the vertex?

2. One diagonal of a rhombus connects points A(2, 1) and B(6, 9). If one vertex of the rhombus is on the x-axis, determine the number of square units in the area of the rhombus.

3. Let the coordinates of point A be $(k, 4k^2)$ with $k > 0$. A line through A is perpendicular to the x-axis at B. Point B is reflected across the line connecting A with the origin. If the reflection lies on the y-axis, determine the value of k.

N.E.A.M.L. PLAYOFFS - 1997

Round 6 - Complex Numbers, Trigonometry

1. ______________________

2. $a =$ ________ $b =$ ________

3. ______________________

NO CALCULATORS ALLOWED ON THIS ROUND

IRRATIONAL ANSWERS MUST BE IN EXACT FORM WITH RATIONALIZED DENOMINATORS

1. If $\tan A = -\frac{3}{7}$ and $\csc A < 0$, find $\sin 2A$.

2. The expression $\frac{i^{17} + i^{16} + \ldots + i + 1}{i^{95} + i^{94} + \ldots + i}$ can be written in the form $a + bi$ where a and b are real. Find the values of a and b.

3. In square ABCD, let M be the midpoint of $\overline{AB}$. Determine the sine of $\angle MDB$.

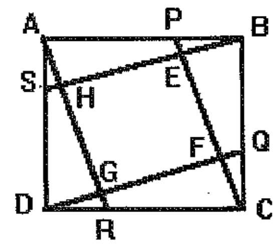
N.E.A.M.L. PLAYOFFS - 1997-TEAM ROUND

Large Schools: 4 points each | Medium and Small Schools: 3 points each

1. ____________________
2. ____________________
3. ____________________
4. ____________________
5. ____________________
6. ____________________

SAT APPROVED CALCULATORS ALLOWED ON THIS ROUND!

1. ABCD is a square. Points P, Q, R and S are the trisection points of their respective sides closest to the vertex indicated on the diagram. Determine the ratio of the area of ΔBEC to the area of EFGH



2. The center of circle P is (h, k) with $h, k > 0$. If the radius of P were were doubled, the circle would be tangent to the x-axis; if the radius were tripled, the circle would pass through the origin. Determine the ratio of h to k.

3. For a and b positive integers less than or equal to 100, determine the number of distinct ordered pair solutions (a, b) to the equation $ab = a^b$.

4. Determine all x satisfying $0 \le x \le 2\pi$ such that $-1 \le \cos x + \sin x \le 1$

5. Find all $z = a + bi$ such that $z^3 + (\bar{z})^3 = 128$ and $|z| = 4$

6. Compute the sum:

$$1 - 2 - 3 + 4 - 5 - 6 + 7 - 8 - 9 + 10 \cdots + 1993 - 1994 - 1995 + 1996 - 1997$$

N.E.A.M.L. PLAYOFFS - 1997

ANSWER SHEET

ROUND 1

1. \$43.00

2. 998

3. 9

ROUND 2

1. $\frac{38}{45}$

2. 11:45 am

3. $-1 < x < 0$ or $x > 1$ or $(-1, 0) \cup (1, +\infty)$

ROUND 3

1. 6

2. $6\sqrt{3}$

3. 105

ROUND 4

1. x^4

2. 3

3. D, C, B, A

ROUND 5

1. $\frac{2\sqrt{3}}{3}$

2. 100

3. $\frac{1}{4}$

ROUND 6

1. $-\frac{21}{29}$

2. $a = -1,\ b = -1$

3. $\frac{\sqrt{10}}{10}$

TEAM ROUND

1. 3:8 or $\frac{3}{8}$

2. $\sqrt{5}:2$ or $\frac{\sqrt{5}}{2}$

3. 101

4. $0,\ \frac{\pi}{2} \le x \le \pi,\ \frac{3\pi}{2} \le x \le 2\pi$

   or $\{0\} \cup [\frac{\pi}{2}, \pi] \cup [\frac{3\pi}{3}, 2\pi]$

5. 4 (or $4 + 0i$), $-2 + 2i\sqrt{3}$, $-2 - 2i\sqrt{3}$

6. $-665{,}001$

NEAML 1997 SOLUTIONS OUTLINE

ROUND 1

1. Book 12.50
   Shirt 22.50
   Charge 8.00
   43.00

2. $(n+1)^2 - n^2 = 1996 + 1$

3. $(2+1+2)-(A+B) = 11, 0, \text{or } -11$
   $A+B=-11$ no solutions
   $A+B = 5$ 6 solutions
   $A+B = 16$ 3 solutions

ROUND 2

1. $\frac{2}{5} + \frac{4}{9}$

2. walks $3t$ first day, $5t$ second day
   $3t = 30 - 5t$

3. Inspect graph of $y=x$ and $y=\frac{|x|}{x}$

Round 3

1. let $m\angle DCE = x$, then $m\angle ABC = x+6$
   $m\angle FBC = 174 - x$
   $m\angle F + (174-x) + x = 180$

2. segments of chord 8 and 12
   $y \cdot 2y = 96$ $y = 4\sqrt{3}$ $D = 12\sqrt{3}$

3. $m\angle FAM = 30$ $\triangle AFT$ is 30-60-90
   $AF = 1$ $FT = \frac{1}{2}$ $m\angle MFT = 45$
   $m\angle MFE = 105$

Round 4

1. $$\frac{(x^2-y^2)(x^2+y^2)}{\left(\frac{x^2-y^2}{x^2}\right)\left(\frac{x^2+y^2}{x^2}\right)}$$

2. $7(3x+d)+b = 3(7x+b)+d$

3. Suppose $\log_{ax} ay \le \log_{(a+1)x} (a+1)y$

   $$\frac{\ln a + \ln y}{\ln a + \ln x} \le \frac{\ln(a+1) + \ln y}{\ln(a+1) + \ln x}$$

   leads to $\ln y \le \ln x$
   contradiction

Round 5

1. $V(1,4)$ chord through $(1,5)$
   $3x^2 - 6x + 7 = 5$ $x = \frac{3 \pm \sqrt{3}}{3}$

2. $\text{mdpt}(A,B) = (4,5)$ slope $= 2$
   eq. of other diag $y-5 = -\frac{1}{2}(x-4)$
   if $y=0$ $x=14$ $C(14,0)$
   $CM = \sqrt{125}$ $\frac{1}{2}(4\sqrt{5})(2\sqrt{125}) = 100$

3. $B(k,0)$ line OA is $y=x$
   $\therefore k = 4k^2$

Round 6

1. A in quad 4 $r = \sqrt{58}$
   $2\sin A \cos A = 2\left(-\frac{3}{\sqrt{58}}\right)\left(\frac{7}{\sqrt{58}}\right)$

2. $i^{16} + i^{15} + \cdots + i + 1 = 0$ $\frac{i^{17} + i^{16}}{-1} = \frac{1+i}{-1}$
   $i^{95} + i^{94} + \cdots + i + 1 = 0$

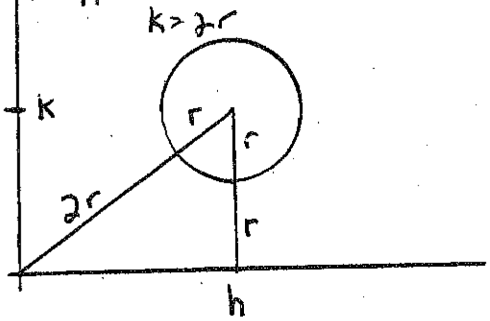
3. Let $AB = 1$ Area $\triangle MDB = \frac{1}{4}$
   $MD = \frac{\sqrt{5}}{2}$ $BD = \sqrt{2}$
   $\frac{1}{2}ab\sin C = \frac{1}{2}\,\frac{\sqrt{5}}{2}\sqrt{2}\sin\angle MDB = \frac{1}{4}$

NEAML '97 TEAM ROUND SOLUTIONS OUTLINE

1. Let $AB = K$ $DQ \parallel BS$ $CQ : CB = 1:3$ $\frac{a\triangle CFQ}{a\triangle CBE} = \frac{1}{9}$ $a\triangle PBC = \frac{1}{2}\frac{k}{3}k$

$= \frac{10}{9} a_\triangle BEC$ $a\triangle BEC = \frac{3k^2}{20}$ $aEFGH + 4\,\frac{3k^2}{20} = k^2$

2. $h^2 + 4r^2 = 9r^2$



3. $b = a^{b-1}$ If $b=1$, a can be any positive integer ≤ 100

If $b = 2$ then $a = 2$

If $b \geq 2$, no integer solutions

4. Let $\cos x + \sin x = k \rightarrow \sin 2x = k^2 - 1$

since $0 \leq k^2 \leq 1$ then $-1 \leq \sin 2x \leq 0$

5. $(a+bi)^3 + (a-bi)^3 = 128 \longrightarrow a^3 - 3ab^2 = 64$

$a^2 + b^2 = 16 \rightarrow b^2 = 16 - a^2$ substitute into above

$a^3 - 12a - 16 = 0$ By synthetic substitution 4 is a root

$(a-4)(a^2+4a+4) = 0$

6. Group by three's

$(1-2-3) + (4-5-6) + \cdots + (1993 - 1994 - 1995) + 1996 - 1997$

$n - (n+1) - (n+2) = -n-3$

$(-4 - 7 - 10 - \cdots - 1996) + 1996 - 1997$

Inside ( ) is arith. prog w/common diff $= -3$

$\left(\frac{-8 + 664(-3)}{2}\right) 665 - 1$

NEAML PLAYOFFS - 1987

ROUND 1 - Arithmetic

1. ____________________

2. ____________________

3. ____________________

1. Determine the largest number of boxes of dimensions 2x2x3 which can be placed inside a box 3x4x5.

(1 point)

2. Miss Gray gives three one-hour exams during each quarter term. The second exam has twice as many questions as the first, and the third has four times as many questions as the first. Wesley Wise got 70% of the questions correct on the first exam, 81% correct on the second exam, and 89% correct on the third exam. What percent of Miss Gray's questions did Wesley correctly answer for that quarter term?

(2 points)

3. In decimal notation, what is the sum of the tens and units digit of the integer

$2! + 4! + 6! + 8! + 10! + 12! + 14! + 16!$ (Note - e.g., $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$)

(3 points)

NEAML PLAYOFFS - 1987

ROUND 2 - Algebra I

1. ______________________

2. ______________________

3. ______________________

1. For what value(s) of x is the following statement true?

$$x = 11 + 2\sqrt{x+4}$$

(1 point)

2. A teacher determined she could change the make-up of her class so that the ratio of girls to boys would be 3:1 in two ways; either by adding 21 girls to the class or by asking x boys to leave the class. For what value of x is this true?

(2 points)

3. Factor completely:

$$2^{4x+1} - 4^{x}3^{2y} - 9^{2y}$$

(3 points)

NEAML PLAYOFFS - 1987

ROUND 3 - Geometry

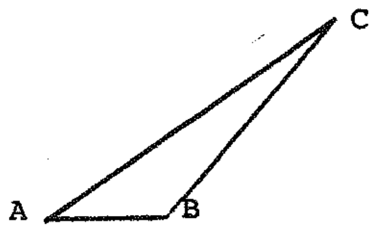
1. ____________________

2. __________ : __________

3. ____________________

1. Given the figure, $\triangle ABC$, find the length of the altitude drawn from vertex C. AC = 15 BC = 13 AB = 4

(1 point)



2. A regular polygon of n sides has 20 diagonals. Find the ratio of the length of the shortest diagonal to the length of the largest diagonal that can be drawn in this regular polygon.

(2 points)

3. $\triangle PQR$ is inscribed in a semi-circle of radius 4. Chord $\overline{QN}$ the angle bisector of $\angle PQR$ intersects the diameter $\overline{PR}$ at M. Given the measure of $\angle QPR$ is 30°, find the length of PM.

(3 points)

NEAML PLAYOFFS - 1987

ROUND 4 - Algebra II

1. ______________________

2. ______________________

3. ______________________

1. The decimal number t89R is divisible by 45. Find all possible ordered pairs (t,R) of decimal digits.
(1 point)

2. Find all the values of $\frac{x}{y}$ if $\frac{3}{\sqrt{y}} - \frac{1}{\sqrt{x}} = \frac{2}{\sqrt{x} + \sqrt{y}}$
(2 points)

3. A radiator is $\frac{2}{3}$ full of an 8% antifreeze solution. 1 gallon of pure antifreeze is added to the radiator, and then $\frac{2}{3}$ of a gallon of water is added to fill the radiator. What percent of the solution now in the radiator is antifreeze?
(3 points)

NEAML PLAYOFFS - 1987

ROUND 5 - Coordinate Geometry

1. ____________________

2. ____________________

3. ____________________

1. Find, in the form $y = mx+b$, the equation of the straight line passing through the center and y-intercept of the hyperbola $(x-2)y = 4$. (1 point)

2. Find the shortest distance between a point on the line $2x+y-6 = 0$ and a point on the circle $x^2+2x+y^2+4y = 0$. (2 points)

3. The equation of a circle is $x^2 + y^2 = 25$. From $P(9,0)$ a line is drawn intersecting the circle at $A(-3,4)$ and at point B. Determine the length of $\overline{AB}$. (3 points)

NEAML PLAYOFFS - 1987

ROUND 6 - Trig., Complex Numbers

1. ______________________

2. ( ) + ( )i

3. ______________________

1. Find all values of x, $0 \leq x < 180°$ such that $\cot\left(\frac{\pi}{2} - 3x\right) = -1$.

(1 point)

2. If $z_1 = -4+4i$ and $z_2 = 3 \operatorname{cis} \frac{2\pi}{3}$, find $z_1^2 \cdot z_2^3$

in $a + bi$ form where a and b are real numbers.

Note: $\operatorname{cis} \theta = \cos \theta + i \sin \theta$.

(2 points)

3. If $\tan A = \frac{5}{6}$ and $\cot B = \frac{2}{3}$, find $\sin(2A+B)$ given that A and B are both in the first quadrant.

(3 points)

NEAML PLAYOFFS - 1987

TEAM ROUND - (Page 1)

(Large Schools 4 points each Medium and Small Schools 3 points each)

1. ______________________ 4. ______________________

2. x = y = ______________ 5. ______________________

3. ______________________ 6. ______________________

1. Let m and n be elements of A = $\{2,3,4,5,6,7,8,9\}$. Determine all ordered pairs (m,n) such that 20m + n and 45m + n are both prime.

2. Team A has won x games out of 11 and B has won y games out of 8. Team A is presently in first place with the better winning percentage. However, if both teams win their next two games, Team B will be in first place with the better percentage. Find the values of x and y for which this is true.

3. Suppose the "distynce" between points $S(x_1,y_1)$ and $T(x_2,y_2)$ is defined as $S*T = |x_2-x_1| + |y_2-y_1|$. Given A(-2,0), B(4,0), and P(x,y), find the area of the region bounded by the locus of all points P such that $P*A = 2(P*B)$.

4. In trapezoid ABCD, base AB = 10 and base DC = 18. Point E lies on $\overline{AD}$ such that AE:ED = 3:1. Find the ratio of the area of traingle BEC to the area of trapezoid ABCD.

5. In triangle ABC, BC = 4, AC = 8, and AB is an integer k. If $m \leq \cos A \leq n$, then determine (m,n).

6. Given $\log_{10} 12 = a$ and $\log_{100} 15 = b$, find, in terms of a and b, the value of x when $2^x = 45$.

NEAML PLAYOFFS - 1987

ANSWER SHEET

ROUND 1

1. 4

2. 84%

3. 12

ROUND 2

1. 21

2. 7

3. $(2^x+3^y)(2^x-3^y)(2\cdot4^x+9^y)$

ROUND 3

1. 12

2. $\sqrt{2} : 2$ or $1 : \sqrt{2}$

3. $12 - 4\sqrt{3}$

ROUND 4

1. (5,5) (1,0)

2. $\frac{1}{3}$

3. $25\frac{1}{3}$

ROUND 5

1. $y = x - 2$

2. $\sqrt{5}$

3. $\frac{13\sqrt{10}}{5}$

ROUND 6

1. 45, 105, 165

2. $0 - 864i$

3. $\frac{153\sqrt{13}}{793}$

TEAM ROUND

1. (2,7), (6,7), (8,7)

2. $x = 7$ $y = 5$

3. 48

4. 4/7

5. (97/112, 169/176)

6. $\frac{a + 10b - 2}{a - 2b + 1}$

NEAML Playoffs - 1987 - Solutions Outline

Round 1

1. Divide volumes $60 \div 12 = 5$
   Can't arrange 5 in space provided, max is 4.

2. Assume 100 questions on first exam
   $\frac{70}{100} + \frac{162}{200} + \frac{356}{400} = \frac{588}{700}$ 84%

3. $2 + 24 + 720 + 40320 +$ (numbers ending in at least 2 0's)
   $\_\_\_66$ $6 + 6 = 12$

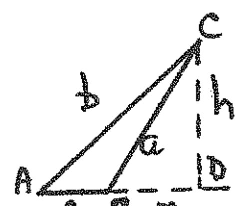
Round 2

1. $x - 11 = 2\sqrt{x+4}$ $x^2 - 22x + 121 = 4x + 16$
   $x^2 - 26x + 105 = 0$ $(x-21)(x-5)$

2. $\frac{9+21}{b} = 3$ $\frac{9}{b-x} = 3$ $3b - 21 = 3b - 3x$

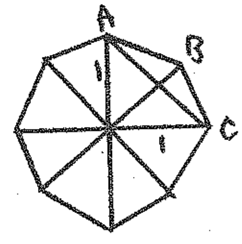
3. $2 \cdot 2^{4x} - 4^x 9^y - 3^{4y} = 2 \cdot 4^{2x} - 4^x 9^y - 9^{2y}$
   $= (4^x - 9^y)(2 \cdot 4^x + 9^y)$

Round 3

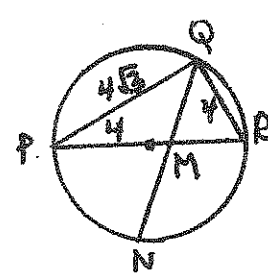


1. $m^2 + h^2 = 169$ $(4+m)^2 + h^2 = 225$

2. $\frac{n(n-3)}{2} = 20$ $n = 8$ $AC = \sqrt{2}$



3. $\frac{4\sqrt{3}}{4} = \frac{x}{8-x}$



$PM = x$
$MR = 8 - x$

Round 4

1. Div by 45 $\rightarrow$ Div by 3, 5, 9

   $R = 0$ or 5 $\quad (1,0) \quad (5,5)$

2. $\dfrac{3\sqrt{x} - \sqrt{y}}{\sqrt{xy}} = \dfrac{2}{\sqrt{x}+\sqrt{y}}$ $\quad 3x + 2\sqrt{xy} - y = 2\sqrt{xy}$

3. x = amt soln. $\quad .08x$ = amt ant. freeze

   $\dfrac{.08x + 1}{x + 1 + 2/3}$ $\quad \frac{1}{2}x = 1 + \frac{2}{3} = \frac{5}{3}$

   $x = 10/3$

Round 5

1. $C(2,0)$ $\quad$ y-int $= -2$
2. $(x+1)^2 + (y+2)^2 = 5$ $\quad C(-1,-2)$ $\quad m_\perp = \frac{1}{2}$
3. $PA \cdot PB = PC \cdot PD = 4 \cdot 14 = 56$ $\quad PB = \sqrt{144+16} = 4\sqrt{10}$

Round 6

1. $\tan 3x = -1$ $\quad 3x = 135, 315, 495$
2. $z_1 = 4\sqrt{2} \text{ cis } \frac{3\pi}{4}$ $\quad z_1^2 z_2^3 = \left(32 \text{ cis } \frac{3\pi}{2}\right)\left(27 \text{ cis } 2\pi\right)$
3. $\sin A = \dfrac{5}{\sqrt{61}}$ $\quad \cos A = \dfrac{6}{\sqrt{61}}$ $\quad \sin B = \dfrac{3}{\sqrt{13}}$ $\quad \cos B = \dfrac{2}{\sqrt{13}}$

$$\sin(2A+B) = \sin 2A \cos B + \cos 2A \sin B$$
$$= 2 \sin A \cos A \cos B + (\cos^2 A - \sin^2 A) \sin B$$

Team Round

1. n cannot have factors in common with 20 and 45, $\therefore n = 7$

m	$20m+n$	$45m+n$
		97
2	47	142
3	67	

etc.

2. $\frac{x}{11} > \frac{y}{8} \rightarrow x > \frac{11y}{8}$ $\quad \frac{x+12}{13} < \frac{y+2}{10}$ $\quad x < \frac{13y+6}{10}$

3. $P*A = |x+2| + |y|$ $\quad P*B = |x-4| + |y|$

$|x+2| = 2|x-4| + |y|$

Cases: $x \geq 4 \ \ y \geq 0$ $\quad x \geq 4 \ \ y < 0$

$-2 \leq x \leq 4 \ \ y \geq 0$ $\quad -2 \leq x \leq 4 \ \ y < 0$

4. Area trapezoid $\frac{1}{2}h(28) = 14h$

Area $\triangle EAB = \frac{1}{2}\left(\frac{3}{4}h\right)10 = \frac{15h}{4}$

Area $\triangle EDC = \frac{1}{2}\left(\frac{1}{4}h\right)18 = \frac{9h}{4}$

5. $4^2 = 8^2 + k^2 - 2 \cdot 8 \cdot k \cos A \rightarrow \cos A = \frac{3}{k} + \frac{k}{16}$

$k = 11$ maximizes $\cos A$

largest $\angle A$ minimizes $\cos A \rightarrow \overline{BC} \perp \overline{AB}$ $\quad AB = 4\sqrt{3}$

let $AB = 7$

6. $2\log 2 + \log 3 = a$ $\quad \log 5 + \log 3 = 2b$

$2\log 2 - \log 5 = a - 2b$

$2\log 2 - (1 - \log 2) = a - 2b$