GREATER BOSTON MATHEMATICS LEAGUE

MEET 5 – MARCH 1999

ROUND 3 – Geometry

1. ____________________

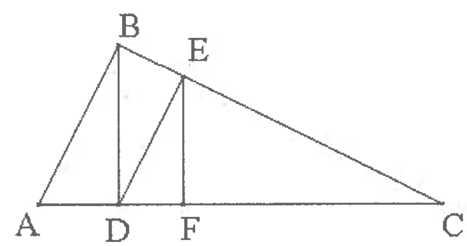
2. ____________________

3. ____________________

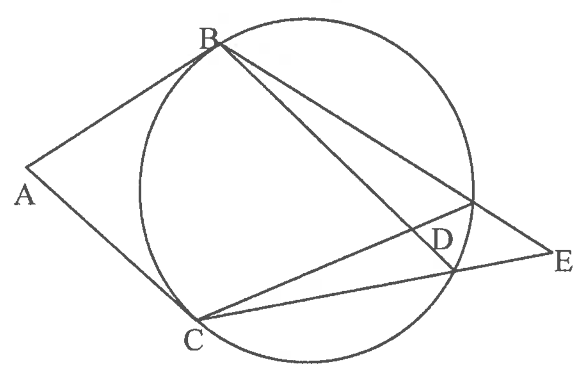
DIAGRAMS ARE NOT NECESSARILY DRAWN TO SCALE.
CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

1. A square and an equilateral triangle have the same perimeter. If the triangle has an altitude of 6 units, how many units long is a diagonal of the square?

2. Given AD = 3, DC = 12, $\angle$ ABC, $\angle$ ADB, $\angle$ BED, and $\angle$ EFC are right angles, find the length of $\overline{EF}$.



3. Given $\overline{AB}$ and $\overline{AC}$ are tangent to the circle, m $\angle$ E = 42°, and m $\angle$ BDC = 66°, find the measure of $\angle$ A in degrees.



GREATER BOSTON MATHEMATICS LEAGUE

MEET 5 – MARCH 2000

ROUND 3 – Geometry

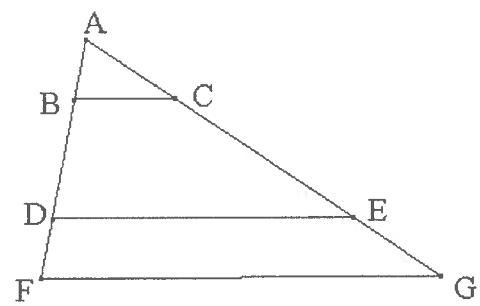
1. ______________

2. ______________

3. ______________

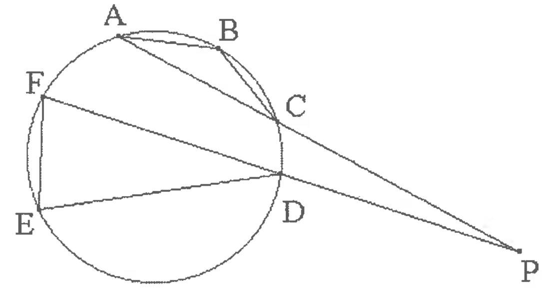
DIAGRAMS ARE NOT NECESSARILY DRAWN TO SCALE.
CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

1. Given $\overline{BC} \parallel \overline{DE} \parallel \overline{FG}$, area ($\Delta$ ABC) = 4, area (BCED) = 32, and area (DEGF) = 28, find the ratio of DE to FG in simplified form.



2. Given a triangle all of whose sides are of integral lengths, with these three lengths equaling $2x$, $3x + 95$, and $6x + 19$, find how many distinct triangles can satisfy these conditions.

3. Given point A, B, C, D, E, and F on a circle such that m $\angle$ B = 135°, m $\angle$ E = 80°, and m $\angle$ P = 10°, find the ratio of $m\,\overset{\frown}{CD}$ to $m\,\overset{\frown}{AF}$ in simplified form.



GREATER BOSTON MATHEMATICS LEAGUE

MEET 5 – MARCH 2001

ROUND 3 – Geometry

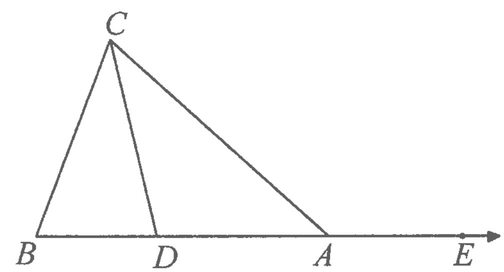
1. ____________________

2. ____________________

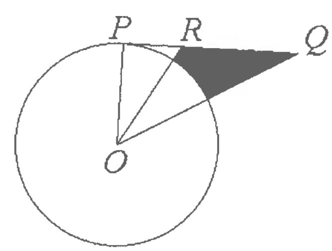
3. ____________________

DIAGRAMS ARE NOT NECESSARILY DRAWN TO SCALE.
CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

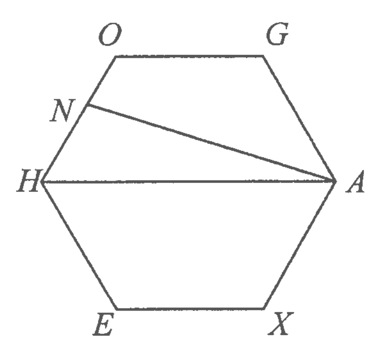
1. Given $AB = AC$, $\overline{BDAE}$, and $\overline{CD}$ bisects $\angle ACB$. If $m\angle CDE + m\angle CAE = 245°$, find the number of degrees in $m\angle B$.



2. Given circle O with radius of length 6 cm, $\overline{PRQ}$, $\overline{PQ}$ tangent to circle O at point P, $\overline{OR}$ bisects $\angle POQ$, and $OR = RQ$, find the number of square centimeters in the shaded area indicated on the diagram.



3. Given $HEXAGO$ is a regular hexagon, and $ON : NH = 3 : 5$. If the area of the regular hexagon is $24\sqrt{3}$ square inches, find the number of square inches in the area of quadrilateral $AGON$.



GREATER BOSTON MATHEMATICS LEAGUE

MEET 5 – MARCH 2002

ROUND 3 – Geometry
Problems submitted by Maimonides.

1. ____________________

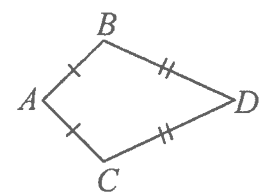
2. ____________________

3. ____________________

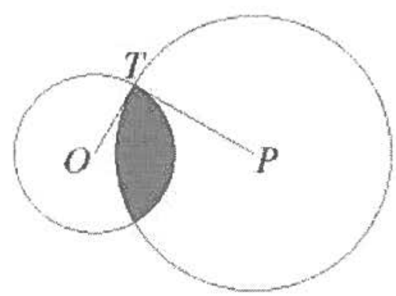
DIAGRAMS ARE NOT NECESSARILY DRAWN TO SCALE.
CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

1. An isosceles triangle has a perimeter of 111 cm and all of its sides are of integral length. There are N of this type of triangles. Find N.

2. A kite (See the figure on the right with indicated equal sides.) has diagonals whose lengths are in the ratio of 5:2. The area of the kite is 4 square centimeters. If a circle can be circumscribed about this kite, find the number of square centimeters in the area of the circle.



3. Given circles O and P with radii of length $\sqrt{6}$ and $3\sqrt{2}$ inches respectively. If T is a point of intersection of the two circles, and $\overline{PT}$ is tangent to circle O, find the number of square inches in the shaded area, which is the area common to both circles.



GREATER BOSTON MATHEMATICS LEAGUE
MEET 5 – MARCH 2006

ROUND 3 – Geometry: Open

1. ________________ **cm^2**

2. ________________ **units^2**

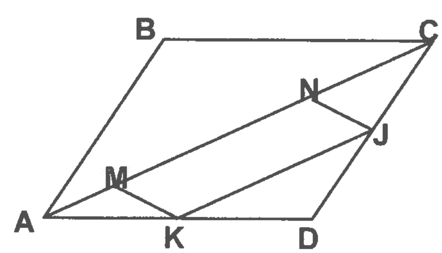
3. _________ **:** _________

CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

1. A regular hexagon has a perimeter of 48 cm. The midpoints of each side are connected in order. What is the number of square centimeters in the area of the region between the two hexagons?

2. Isosceles ΔABC has a perimeter of 24 units and its altitude drawn from vertex A to base $\overline{BC}$ is 6 units. Find the area of ΔABC.

3. In rhombus $ABCD$, points M and N lie on diagonal $\overline{AC}$ such that $AM : MN = 1{:}5$ and $AN : NC = 3{:}1$. J and K are midpoints of $\overline{CD}$ and $\overline{DA}$ respectively. Find the ratio of the area of $MNJK$ to the area of rhombus $ABCD$.



GREATER BOSTON MATHEMATICS LEAGUE
MEET 5 – March 2007

ROUND 3 – Geometry: Open

1. ______________

2. (______ , ______)

3. ______________

CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

1. A cube of volume $192\sqrt{3}$ cm^3 is inscribed in a sphere. The surface area of the sphere is k cm^2. Find the exact value of k.

2. ΔABC has vertex $A(-3,-2)$ and midpoints of sides $\overline{AB}$ and $\overline{AC}$ are $(2, w)$ and $(2w, 12)$ respectively. If $BC = 20$ and C has integer coordinates (h, k), determine (h, k).

3. $PQRS$ (P and R are opposite vertices) is a rectangle inscribed in a circle so that the degree measure of arc $\overparen{PQ}$ is greater than the degree measure of arc $\overparen{QR}$. The measure of arc $\overparen{PQ}$ is $A°$. The tangent to the circle at S meets ray $\overrightarrow{RP}$ at point T. Find the measure of $\angle STP$ in terms of A.

GREATER BOSTON MATHEMATICS LEAGUE

MEET 5 – MARCH 2008

ROUND 3 – Geometry: Open

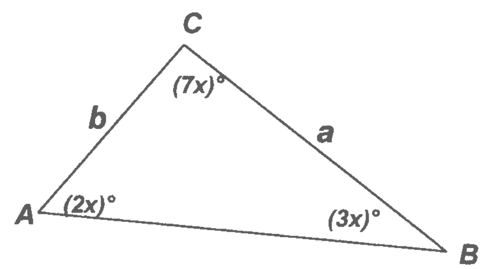
1. ____________ : ____________

2. ________________________

3. ________________________

DIAGRAMS ARE NOT NECESSARILY DRAWN TO SCALE.
CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

1. Find the numerical value of the ratio $b^2 : a^2$.



2.

3. An equilateral triangle whose sides have length 8 has 3 congruent equilateral triangles cut off the corners. If the area of the remaining polygon to the area of the 3 equilateral triangles is 5 : 3, what is the length of one of the sides of the triangles cut off?

GREATER BOSTON MATHEMATICS LEAGUE

MEET 5 – FEBRUARY 2009

ROUND 3 – Geometry: Open

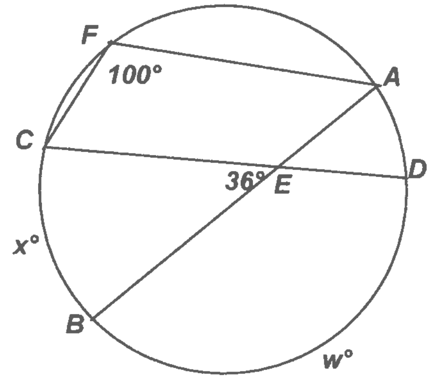
1. ______________________

2. ______________________

3. ______________________

DIAGRAMS ARE NOT NECESSARILY DRAWN TO SCALE.
CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

1. In the circle at the right, the angle measures are as indicated and the ratio of the lengths of the minor arcs $\overset{\frown}{BC}$ and $\overset{\frown}{AD}$ is 5 : 1. Determine the simplified ratio $x : w$.



2. From a regular hexagon H whose side has length s, two regular hexagons are formed, namely H_x and H_g. The longest diagonal of H is a shortest diagonal of H_x, while the shortest diagonal of H is the longest diagonal of H_g.
Find the ratio of the area of H_x to the area of H_g.

3. A frustum F of a regular square pyramid P has an altitude of 12 and its upper and lower bases have edges of lengths 10 cm and 20 cm respectively. Find the numerical ratio of the volume of pyramid P to the volume of the frustum F to the lateral area of the frustum F.

GREATER BOSTON MATHEMATICS LEAGUE

MEET 5 – FEBRUARY 2010

ROUND 3 – Geometry: Open

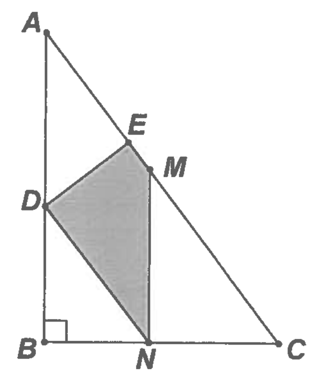
1. ______________________________

2. ______________________________

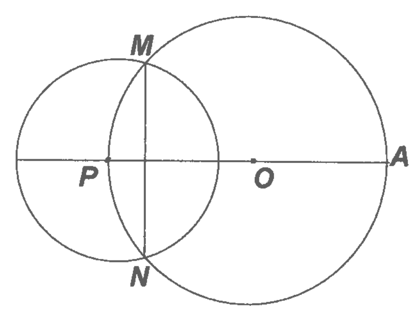
3. __________ : __________

DIAGRAMS ARE NOT NECESSARILY DRAWN TO SCALE.
CALCULATOR ARE NOT ALLOWED ON THIS ROUND.

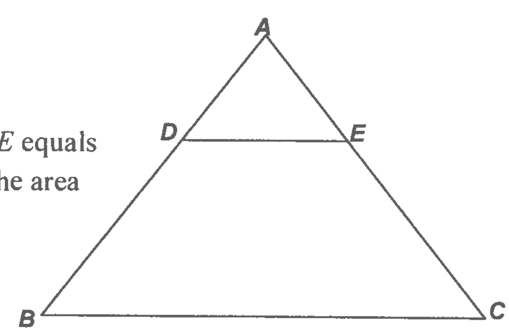
1. Compute the area of the shaded region in right triangle ABC, given that $\overline{DE} \perp \overline{AC}$, $\overline{MN} \perp \overline{BC}$, $AB = 8$, $BC = 6$, $MC = 5$ and $EM = 1$.



2. Circle O of radius 6 passes through the center of circle P of radius 4. If M and N are the points of intersection of the two circles, determine the length of chord $\overline{AM}$.



3. ΔABC is equilateral, $\overline{DE} \parallel \overline{BC}$ and the perimeter of ΔADE equals the perimeter of trapezoid $DECB$. Compute the ratio of the area of ΔADE to the area of trapezoid $DECB$.



GREATER BOSTON MATHEMATICS LEAGUE

MEET 5 – MARCH 2011

ROUND 3 – Geometry: Open

1. ______________________

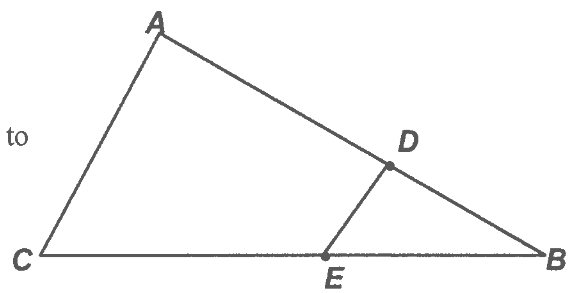
2. ____________ : ____________

3. ____________ : ____________

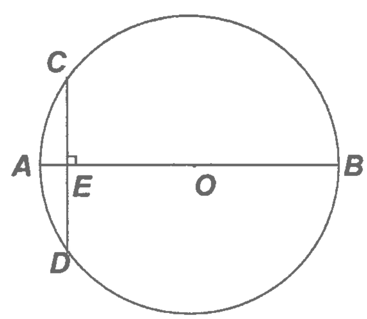
DIAGRAMS ARE NOT NECESSARILY DRAWN TO SCALE.
CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

1. The complement of $\frac{1}{3}$ of an angle is equal to three times the complement of the supplement of the angle. Compute the degree-measure of this angle.

2. In ΔABC, $AD : DB = 3 : 2$ and $CE : CB = 5 : 8$. Compute the ratio of the area of quadrilateral $ADEC$ to the area of ΔABC.



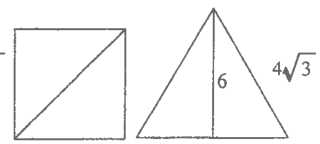
3. $\overline{AB}$ is a diameter of circle O, chord $\overline{CD}$ is perpendicular to $\overline{AB}$ and $CD = \frac{1}{3}AB$. If $y = AE$ and r is the radius of circle O, compute the numerical value of the simplified ratio $y : r$.



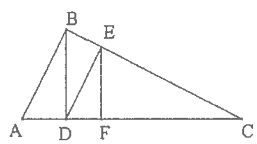
'99

ROUND 3

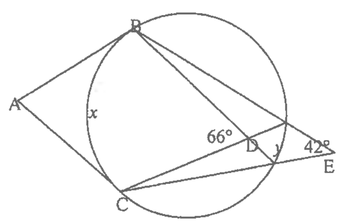
1. side of equilateral triangle = $\frac{12}{\sqrt{3}} = 4\sqrt{3}$ $\quad 3\sqrt{3}$
Perimeter of triangle = $12\sqrt{3}$ = Perimeter of of square $\Rightarrow$ side of square = $3\sqrt{3}$ $\Rightarrow$ diagonal of square = $3\sqrt{3}\sqrt{2} = \mathbf{3\sqrt{6}}$



2. $BD^2 = AD \times DC \Rightarrow BD = 6 \Rightarrow AB = 3\sqrt{5} \Rightarrow$
$6 : 3\sqrt{5}$ = DE: 6 $\Rightarrow$ DE = $2.4\sqrt{5} \Rightarrow$
EF: $2.4\sqrt{5}$ = 6: $3\sqrt{5}$ $\Rightarrow$ EF = **4.8**



3. $\frac{x+y}{2} = 66; \frac{x-y}{2} = 42; \Rightarrow$
$x = 108$; m $\angle$ A = 180° – 108° = **72°**



'00

ROUND 3

1. area (Δ ADE) = 4 + 32 = 36; area (Δ AFG) = 4 + 32 + 28 = 64;
area (Δ ADE): area (Δ AFG) = 36: 64 $\Rightarrow$ DE:FG = 6:8 = 3:4

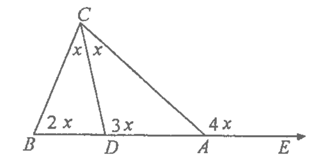
2. Applying the triangle inequality theorem three times:
$2x+3x+95>6x+19 \Rightarrow x<76$; $2x+6x+19>3x+95 \Rightarrow 5x>76 \Rightarrow x>15.2$, and
$9x+114>2x \Rightarrow 7x>-114 \Rightarrow x>-16\frac{2}{7}$; the intersection of these three inequalities is:
$15.2<x<76 \Rightarrow x=16,17,\ldots 75$ which are 60 distinct cases.

3. Let $m\widehat{AF} = x$ and $m\widehat{CD} = y$; Since $\angle$ P is a *secant-secant* angle, $\frac{x-y}{2} = 10 \Rightarrow x-y=20$
$\angle$ B and $\angle$ E are inscribed $\Rightarrow$ $m\widehat{AFC} = 270°$ and $m\widehat{FAD} = 160°$; the sum of these arcs include $\widehat{AF}$ and $\widehat{CD}$ twice $\Rightarrow 430 = 360 + x + y \Rightarrow x+y=70$; Solving the two equations for x and $y \Rightarrow x=45$ and $y=25 \Rightarrow y:x=5:9$

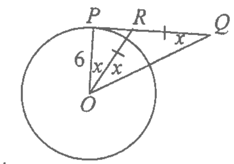
'01

ROUND 3

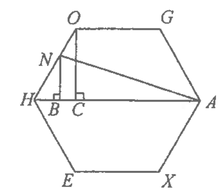
1. Call $m\angle BCD = x \rightarrow m\angle ACD = x \rightarrow m\angle B = 2x$
$\rightarrow m\angle DCA = 3x$ and $m\angle CAE = 4x \rightarrow$
$3x+4x=245 \rightarrow 7x=245 \rightarrow x=35 \rightarrow 2x=70$



2. $3x=90 \rightarrow x=30 \rightarrow PQ=6\sqrt{3}$ and $PR=2\sqrt{3} \rightarrow$
$RQ=4\sqrt{3}$; shaded area = area of ΔORQ – area of
30° sector = $\frac{1}{2}(4\sqrt{3})(6) - \frac{30}{360}\pi \cdot 6^2 = 12\sqrt{3} - 3\pi$.



3. Let s = side of hexagon: $\frac{3}{2}s^2\sqrt{3} = 24\sqrt{3} \rightarrow s^2 = 16 \rightarrow s = 4$.
Draw perpendiculars from O and N to $\overline{AH}$.
$s=4 \rightarrow AH=8$, $OC=2\sqrt{3} \rightarrow NB=\frac{5}{8}(2\sqrt{3})=\frac{5\sqrt{3}}{4}$.
Area of $AGON$ = area of $AGOH$ – area of ΔAHN
$=\frac{1}{2}(24\sqrt{3}) - \frac{1}{2}(8)\left(\frac{5\sqrt{3}}{4}\right) = 12\sqrt{3} - 5\sqrt{3} = 7\sqrt{3}$.

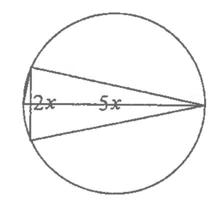


'02

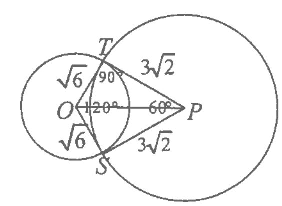
ROUND 3 – Geometry

1. The base must be an odd integer. Call its length $2x+1$. The sum of the lengths of the legs must then = $111-(2x+1)=110-2x$. By the triangle inequality theorem, $110-2x>2x+1 \Rightarrow 4x<109 \Rightarrow x<27\frac{1}{4}$. This means x can be any whole number from 0 to 27 $\Rightarrow$ 28 possibilities.

2. Let the length of the diagonals = $5x$ and $2x \Rightarrow$
$\frac{1}{2}(2x)(5x)=4 \Rightarrow x^2=\frac{4}{5} \Rightarrow x=\frac{2}{\sqrt{5}} \Rightarrow 5x=\frac{10}{\sqrt{5}}=2\sqrt{5}$.
Since a circle can be circumscribed around the kite, the angles opposite the longer diagonal are supplementary. Since they are also equal, then they are right and so the diameter of the circle = $2\sqrt{5} \Rightarrow A=\pi(\sqrt{5})^2=5\pi$.

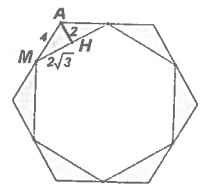


3. $\angle T$ is right and $\frac{PT}{OT}=\frac{3\sqrt{2}}{\sqrt{6}}=\sqrt{3} \Rightarrow \Delta\, OPT$ is a 30-60-90° triangle $\Rightarrow m\angle SOT=120°$ and $m\angle SPT=60°$
The common area of the two circles is a 120° and 60° segment of circles O and P respectively $\Rightarrow$ Common area
$=\left(\frac{1}{3}\pi(\sqrt{6})^2-\frac{(\sqrt{6})^2\sqrt{3}}{4}\right)+\left(\frac{1}{6}\pi(3\sqrt{2})^2-\frac{(3\sqrt{2})^2\sqrt{3}}{4}\right)=$
$\left(2\pi-\frac{3\sqrt{3}}{2}\right)+\left(3\pi-\frac{9\sqrt{3}}{2}\right)=5\pi-6\sqrt{3}$.

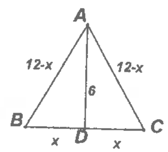


'06

ROUND 3 – Geometry: Open



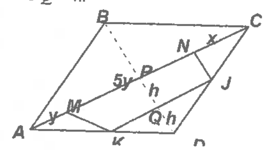
1. $Per_{orig\ hex} = 48 \rightarrow$ side $= 8 \rightarrow$ Area $= 6$(Area(Eq. Δ w/ side = 8))
$= 6\dfrac{8^2\sqrt{3}}{4} = 96\sqrt{3}$
$Side_{inner\ hex} = 4\sqrt{3} \rightarrow A = 72\sqrt{3}$.
Thus, the area of the shaded region is $\underline{24\sqrt{3}}$



2. Let $2x$ be the length of the base of the isosceles triangle.
Using the Pythagorean Theorem,
$6^2 + x^2 = (12 - x)^2 \rightarrow 36 = 144 - 24x \rightarrow x = 9/2$
Thus, the area = (1/2)(9)(6) = **27**

3. $AN = 3x = 6y \rightarrow x = 2y \rightarrow AC = x + 6y = 8y$
$\overline{KJ} \parallel \overline{AC}$ and $KJ = \dfrac{AC}{2} = 4y$, since $\Delta DKJ \sim \Delta DAC$ (by SAS)
$MNJK$ must actually be a trapezoid. Let $BD = 4h \rightarrow QD = PQ = h$.
Since the diagonals of a rhombus are perpendicular,

$$\frac{\text{Area}(MNJK)}{\text{Area}(ABCD)} = \frac{\frac{1}{2}h(4y+5y)}{\frac{1}{2}(8y)(4h)} = \underline{\frac{9}{32}}$$

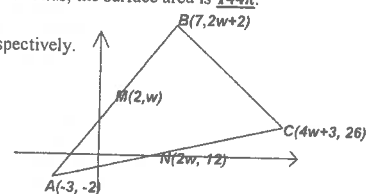


'07

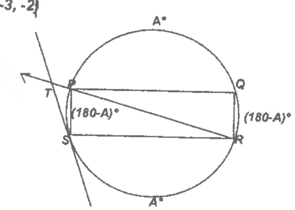
ROUND 3 – Geometry: Open

1. If x denotes the length of the edge of the cube, then $x\sqrt{3}$ denotes the diagonal of the cube, which is also the diameter of the sphere. The surface area of the sphere is $4\pi r^2 = 4\pi(\frac{x\sqrt{3}}{2})^2 = 3\pi x^2$
$V_{cube} = x^3 = 192\sqrt{3} = 2^6 3^{3/2} \rightarrow x^2 = (2^6 3^{3/2})^{2/3} = 2^4 3 = 48$ Thus, the surface area is **144π**.

2. Let M and N denote the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively.
Then $BC^2 = (4w - 4)^2 + (24 - 2w)^2 = 20^2$
$\rightarrow 16(w - 1)^2 + 4(12 - w)^2 = 400$
$\rightarrow 4(w - 1)^2 + (12 - w)^2 = 100$
$\rightarrow 5w^2 - 32w + 48 = (5w - 12)(w - 4) = 0$
$\rightarrow w = 12/5$ or $w = 4$
Only the latter produces integer coordinates for point C**(19, 26)**



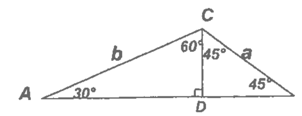
3. Since T is an angle formed by a tangent and a secant line, its measure is determined by half the difference between its intercepted arcs. $m\angle T = \dfrac{1}{2}(A - (180 - A)) = \underline{(A - 90)°}$



'08

ROUND 3

1. $12x = 180 \rightarrow x = 15$
Drop a perpendicular from C to $\overline{AB}$ and we have:
Thus, $CD = b/2$ and $a = CD\sqrt{2}$
$\rightarrow b^2 = 4CD$ and $a^2 = 2CD \rightarrow b^2 : a^2 = \underline{\mathbf{2:1}}$



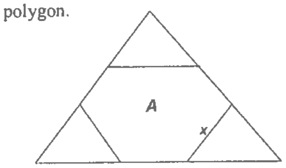
2.

3. Let x denote the length we must determine and A the area of the polygon. Then:

$$\frac{A}{3\left(\frac{x^2\sqrt{3}}{4}\right)} = \frac{5}{3} \rightarrow A = \frac{5x^2\sqrt{3}}{4} \rightarrow$$

$$\frac{5x^2\sqrt{3}}{4} + 3\left(\frac{x^2\sqrt{3}}{4}\right) = \frac{8^2\sqrt{3}}{4} = 16\sqrt{3} \rightarrow x^2 = \frac{16 \cdot 4}{8} = 8$$

$\rightarrow x = \mathbf{2\sqrt{2}}$

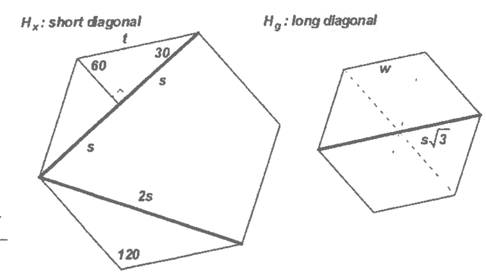


'09

ROUND 3

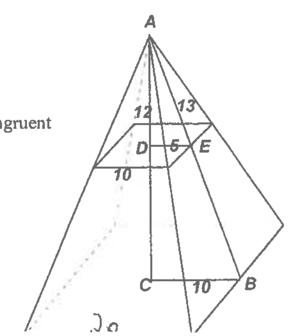
1. Let $\widehat{BC} = 5d$ and $\widehat{AD} = d$. Then $6d = 2(36) \rightarrow d = 12, x = 60$
$\widehat{CBA} = x + w + d = 200 \rightarrow w = 200 - 72 = 128$ Thus $x : w = \dfrac{60}{128} = \underline{\dfrac{15}{32}}$

2. H_x: $\dfrac{s}{t} = \dfrac{\sqrt{3}}{2} \rightarrow t = \dfrac{2s}{\sqrt{3}}$
H_g: $2w = s\sqrt{3} \rightarrow w = \dfrac{s\sqrt{3}}{2}$
Thus, $\dfrac{t}{w} = \dfrac{\frac{2s}{\sqrt{3}}}{\frac{s\sqrt{3}}{2}} = \dfrac{2s}{\sqrt{3}} \cdot \dfrac{2}{s\sqrt{3}} = \dfrac{4}{3}$
and the ratio of the areas is $\dfrac{t^2}{w^2} = \underline{\dfrac{16}{9}}$



3. The altitude from A passes through the centers of both bases of the frustum and, therefore, we know that $DE = 5$ and $CB = 10$. $\Delta ADE \sim \Delta ACB$ and $DC = 12 \rightarrow AD = 12$
Thus, ΔADE has sides of 5, 12 and 13.

Vol(P) $= \dfrac{1}{3} \cdot 20^2 \cdot 24 = 3200$
Vol(F) $= \dfrac{1}{3} \cdot 20^2 \cdot 24 - \dfrac{1}{3} \cdot 10^2 \cdot 12 = 3200 - 400 = 2800$
The lateral surface area of the frustum F consists of four congruent isosceles trapezoids.
Thus, LSA(F) $= 4\left(\dfrac{1}{2} \cdot 13 \cdot (10 + 20)\right) = 780$
3200 : 2800 : 780 = **160 : 140 : 39**

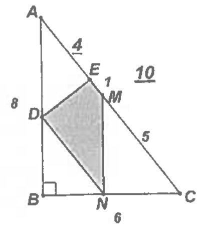


20

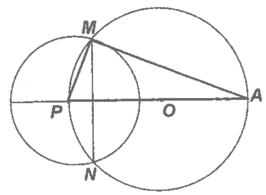
10

ROUND 3

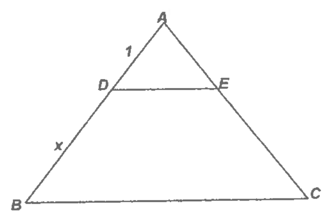
1. $AC = 10$, $AE = 4$ and $\Delta MCN \sim \Delta ACB$
   $\rightarrow \frac{MC}{AC} = \frac{MN}{AB} = \frac{CN}{CB} \rightarrow \frac{5}{10} = \frac{MN}{8} = \frac{CN}{6}$
   $\rightarrow MN = 4$, $CN = BN = 3$
   $\Delta ADE \sim \Delta ACB \rightarrow AD = 5$ and $DE = BD = 3$
   Thus, the area of the shaded region is
   $\frac{1}{2}\cdot 6\cdot 8 - 2\left(\frac{1}{2}\cdot 3\cdot 4\right) - \frac{1}{2}\cdot 3\cdot 3 = 24 - 12 - 4.5 = \underline{7.5}$



2. $PM = 4$, $AP = 12$ and since $\angle PMA$ is inscribed in a semicircle, ΔPMA is a right triangle.
   Therefore, $AM^2 = 12^2 - 4^2 = 128 = 64(2) \rightarrow AM = \underline{8\sqrt{2}}$



3. Let $AD = 1$ and $BD = x$. Then:
   $BC = x + 1$, Per(ΔADE) = 3, Per(trap $DECB$) = $3x + 2$
   $3x + 2 = 3 \rightarrow x = 1/3$
   $\Delta ADE \sim \Delta ABC \rightarrow \frac{area(\Delta ADE)}{area(\Delta ABC)} = \frac{1^2}{(x+1)^2}$
   $\rightarrow \frac{area(\Delta ADE)}{area(DECB)} = \frac{1}{(x+1)^2 - 1} = \frac{1}{x^2 + 2x} = \frac{1}{x(x+2)}$
   Substituting, $\frac{1}{\frac{1}{3}\cdot\frac{7}{3}} = \underline{9:7}$

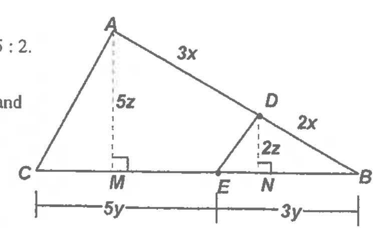


11

ROUND 3

1. $3[90 - (180 - x)] = 90 - \frac{x}{3} \rightarrow 3(x - 90) = 90 - \frac{x}{3} \rightarrow 9x - 810 = 270 - x \rightarrow x = \underline{108}$
   Check: $\frac{1}{3}(108) = 36$, compl. of 36 = $\underline{54}$; suppl of 108 = 72, compl of 72 = 18, 3(18) = $\underline{54}$.

2. Drop perpendiculars from A and D to $\overline{BC}$.
   $\Delta DNB \sim \Delta AMB$ and $AD : DB = 3 : 2 \rightarrow AM : DN = 5 : 2$.
   $CE : CB = 5 : 8 \rightarrow CE : BE = 5 : 3$.
   The areas of ΔDEB and ΔABC are $\frac{1}{2}(2z)(3y) = 3yz$ and $\frac{1}{2}(5z)(8y) = 20yz$ respectively. Therefore, the required ratio is $\frac{20yz - 3yz}{20yz} = \underline{17:20}$



3. Let $x = CE = DE$ and $y = AE$. Then:
   Applying the Pythagorean Theorem to ΔCEO, $x^2 + (r - y)^2 = r^2$.
   $CD = \frac{1}{3}AB \rightarrow 2x = \frac{1}{3}2r \rightarrow x = \frac{r}{3}$
   Transposing terms and substituting for x, $(r-y)^2 = r^2 - \frac{r^2}{9} = \frac{8r^2}{9}$
   Taking the square root, $r - y = +\frac{2\sqrt{2}r}{3}$ (since $r > y$)
   $\rightarrow y = r\left(1 - \frac{2\sqrt{2}}{3}\right) \rightarrow \frac{y}{r} = \underline{\frac{3-2\sqrt{2}}{3}}$

   Alternately, let $OB = 3t$, $CE = t$. Applying the Pythagorean Theorem on ΔCEO,
   $t^2 + CE^2 = (3t)^2 \rightarrow CE = \sqrt{8t^2} = \left(2\sqrt{2}\right)t \rightarrow AE = \left(3 - 2\sqrt{2}\right)t$
   and we have $\frac{AE}{OB} = \frac{y}{r} = \frac{\left(3-2\sqrt{2}\right)t}{3t} = \underline{\frac{3-2\sqrt{2}}{3}}$