GREATER BOSTON MATHEMATICS LEAGUE

MEET 5 – MARCH 1999

ROUND 5 – Precalculus

1. ____________________

2. ____________________

3. ____________________

CALCULATORS ARE NOT ALLOWED ON THIS ROUND

1. Find the length of the line segment joining the center of circle C, which is $\left\{(x, y) \mid 2x^2 + 2y^2 - 4x + 12y + 4 = 0\right\}$ to the vertex of the parabola P, which is $\left\{(x, y) \mid y^2 = 8x + 4y + 12\right\}$

2. If $\sin\left(x + \frac{\pi}{4}\right) = \frac{1}{4}$, find the value for $\sin(2x)$.

3. Some of the solutions to $z^{12} = -16$, when plotted in the complex plane, are located in quadrant I. Find the product of these solutions and write the result in the form $a + bi$.

GREATER BOSTON MATHEMATICS LEAGUE

MEET 5 – MARCH 2000

ROUND 5 – Precalculus

1. ____________________

2. ____________________

3. ____________________

CALCULATORS ARE NOT ALLOWED ON THIS ROUND

1. Find the distance from F, the focus of the parabola, $\{(x, y) \mid y^2 - 8x + 16 = 0\}$ to circle C, $\{(x, y) \mid x^2 + y^2 + 12x + 11 = 0\}$.

2. Find the positive value for x satisfying the equation,

$$\cos(\text{Arctan } x) \cdot \tan\left(\text{Arccos}\left(\frac{2}{3}\right)\right) = \cos 660°.$$

Note: Arctan and Arccos are names for inverse trigonometric <u>functions</u>.

3. The equation, $-2z^3 = \left(1 - i\sqrt{3}\right)^4$ has complex solutions for z. Find **all** of these solutions in the polar form, $r\,cis\,\theta$ where $0° \le \theta < 360°$.

GREATER BOSTON MATHEMATICS LEAGUE

MEET 5 – MARCH 2001

ROUND 5 – Precalculus

1. ____________________

2. ____________________

3. ____________________

CALCULATORS ARE NOT ALLOWED ON THIS ROUND

1. Given the conic C, $\{(x,y)\,|\,x^2+y^2-x-4y-12=0\}$, find the length of the chord whose endpoints are where the y axis intersects C.

2. Given $\cos x=\frac{1}{7}, \frac{3\pi}{2}<x<2\pi$, find the value for $\cos\left(x+\frac{2\pi}{3}\right)$.

3. Given $z=c+di$, $c>0$, $d>0$, and $z^4=2-2i\sqrt{3}$, find the value for $\frac{2z}{1+i}$ in $a+bi$ form.

   Note: $i=\sqrt{-1}$.

GREATER BOSTON MATHEMATICS LEAGUE

MEET 5 – MARCH 2002

ROUND 5 – Precalculus
Problems submitted by Maimonides.

1. ____________________

2. ____________________

3. ____________________

CALCULATORS ARE NOT ALLOWED ON THIS ROUND

1. Find the radian measure for x given that $x = \text{Arctan}\left(\frac{1}{2}\right) + \text{Arctan}\left(\frac{1}{3}\right)$. Note that Arctan is the inverse tangent function.

2. In ΔABC, $AB = 1$, $AC = 5$, and the area of ΔABC = 2, find all possible values for the length of $\overline{BC}$.

3. The angle with measure $-30°$ is drawn in *standard position* in the coordinate plane. Find the point of intersection of the terminal side of this angle with the conic having vertices $\left(\pm\sqrt{6},0\right)$ and foci $(\pm 3,0)$.

GREATER BOSTON MATHEMATICS LEAGUE
MEET 5 – MARCH 2006

ROUND 5 – Pre-Calculus: Open

1. ____________________

2. (_____, _____, _____)

3. ____________________

CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

1. Find the shortest distance from the center of circle C_1: $x^2 + y^2 - 4x + 6y - 1 = 0$ to the circle C_2: $x^2 + y^2 + 12x - 6y + 41 = 0$.

2. Two conics have equations $4(x-2)^2 + y^2 = 64$ and $x^2 - 9(y+3)^2 = 36$. A parabola that is concave up has its vertex at the center of the ellipse and goes through the points $(4, m)$ and $(0, n)$ on the ellipse. The equation of the parabola can be written in the form $y = a(x-h)^2 + k$. Determine the ordered triple (a, h, k).

3. The transverse axis of a hyperbola coincides with the major axis of the ellipse $4x^2 + 9y^2 - 8x + 36y + 4 = 0$. The conjugate axis is twice the length of the focal chord of the ellipse. Find the equation of the asymptote with <u>positive</u> slope.
Express your answer in the simplified form $Ax + By + C = 0$, where A, B and C are integers and $A > 0$.

GREATER BOSTON MATHEMATICS LEAGUE
MEET 5– March 2007

ROUND 5 – Pre-Calculus: Open

1. (______) + (_______)i

2. ______________________

3. ______________________

CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

1. If $z = 3 - 4i$ and $w = 8 + 3i$, write in standard form $\overline{z^2 + 3w}$.

   Note: The notation $\bar{c}$ denotes the conjugate of the complex number c.

2. Given the parabola $(x-1)^2 = 16(y+4)$. Find the y-intercepts of the ellipse whose major axis is the focal chord (latus rectum) of the parabola and which has an endpoint of the other axis at the vertex of the parabola.

3. Find <u>all</u> values of θ, where $0 \le \theta < 360^\circ$ for which $\sqrt{3}\sec\theta - \csc\theta - 2\sqrt{3}\tan\theta + 2 = 0$

GREATER BOSTON MATHEMATICS LEAGUE

MEET 5 – FEBRUARY 2009

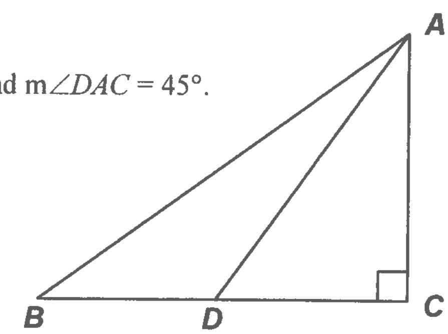
ROUND 5 – Pre-Calculus: Open

1. ____________________

2. ____________________

3. ____________________

CALCULATORS ARE NOT ALLOWED ON THIS ROUND.



1. In ΔABC with right angle at C, $AB = 18\sqrt{2}$, $m\angle B = 30°$ and $m\angle DAC = 45°$. Compute the ratio $\dfrac{BD}{AD}$.

2. Solve for θ, where $0 \le \theta < 360°$.

$$\sin^2(630° - \theta)\cos^2(450° + \theta) = \sin^2(240°) \cdot \cos^2(-420°)$$

3. Find the equation of all lines that are 4 units from the line $L_1 : \{(x, y) \mid 6x + 8y - 25 = 0\}$. Express your answers in the form $Ax + By + C = 0$, where A, B and C are integers and $A > 0$.

GREATER BOSTON MATHEMATICS LEAGUE

MEET 5 – FEBRUARY 2010

ROUND 5 – Pre-Calculus: Open

1. (__________ , __________)

2. ______________________________

3. ______________________________

CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

1. Two of the roots of the equation $Ax^4 + Bx^3 + Cx^2 + Dx + F = 0$, where the coefficients A, B, C, D and F are integers, are $1-i$ and $-1-i\sqrt{3}$ and the other two roots are r_3 and r_4. Determine the ordered pair (a, b) for which $(r_3 \cdot r_4)^6 = a + bi$.

2. Given: $f(x) = 2x - 1$
   If f^{-1} denotes inverse function and $f \circ f$ denotes composition of functions, determine all real values of x for which
   $$f \circ f(x) = \left(f^{-1}(x)\right)^{-1}$$

3. Given: $\csc x \sin 217° = -1$ for $90° < x < 270°$ and $\tan y \cot 203° = 1$ for $0° \leq y < 180°$
   Compute $\tan(x - y)$.

GREATER BOSTON MATHEMATICS LEAGUE

MEET 5 – MARCH 2011

ROUND 5 – Pre-Calculus: Open

1. ______________________

2. ______________________

3. ______________________

CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

1. A circle is inscribed in the region $\{(x, y) : 3|x| + 4|y| \le 12\}$. Compute its radius.

2. Given: $\cos\left(\text{Arcsin}\left(-\frac{1}{3}\right)\right) + \text{Tan}\left(\text{Arccos}\left(-\frac{1}{\sqrt{3}}\right)\right) = \tan X$

   Compute $\cos X$.

   Reminder: If necessary, denominators must be rationalized.

3. Given: $\log_{14}\left(\frac{1}{16}\right) = P$ and $\log_{\sqrt{14}}\left(\frac{343}{2}\right) = T$

   Find a <u>simplified</u> expression for T in terms of P.

'99

ROUND 5

1. $x^2 + y^2 - 2x + 6y - 2 = 0 \Rightarrow (x-1)^2 + (y+3)^2 = 12 \Rightarrow$ center $= (1, -3)$
$y^2 - 4y + 4 = 8x + 16 \Rightarrow (y-2)^2 = 8(x+2) \Rightarrow$ vertex $= (-2, 2) \Rightarrow$ distance $= \sqrt{34}$

2. $\sin\left(x + \frac{\pi}{4}\right) = \frac{1}{4} \Rightarrow \sin x \cdot \cos\frac{\pi}{4} + \cos x \cdot \sin\frac{\pi}{4} = \frac{1}{4} \Rightarrow \sin x \cdot \frac{\sqrt{2}}{2} + \cos x \cdot \frac{\sqrt{2}}{2} = \frac{1}{4} \Rightarrow$
$\frac{\sqrt{2}}{2}(\sin x + \cos x) = \frac{1}{4} \Rightarrow \frac{1}{2}(\sin x + \cos x)^2 = \frac{1}{16} \Rightarrow$
$\frac{1}{2}(\sin^2 x + 2\sin x \cdot \cos x + \cos^2 x) = \frac{1}{16} \Rightarrow \frac{1}{2}(1 + \sin 2x) = \frac{1}{16} \Rightarrow \sin 2x = -\frac{7}{8}$

3. $z^{12} = -16 = 16\ cis\ 180° \Rightarrow n = 0, 1, 2, \ldots 11 : z = 16^{\frac{1}{12}} cis\left(\frac{180°}{12} + 30°n\right) = 2^{\frac{1}{3}} cis(15° + 30°n)$
When $n = 0, 1,$ or 2: $z = 2^{\frac{1}{3}} cis\ 15°, 2^{\frac{1}{3}} cis\ 45°, 2^{\frac{1}{3}} cis\ 75°$ Their product $= 2\ cis\ 135° =$
$2\left(-\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right) = -\sqrt{2} + i\sqrt{2}$

'00

ROUND 5

1. $y^2 - 8x + 16 = 0 \Rightarrow y^2 = 8(x-2) \Rightarrow$ *vertex is* $(2,0)$ *and* $p = 2$; since the parabola opens to the right $\Rightarrow F = (2+2, 0) = (4,0)$; $x^2 + y^2 + 12x + 11 = 0 \Rightarrow x^2 + 12x + 36 + y^2 = 25 \Rightarrow$ $(x+6)^2 + y^2 = 5^2 \Rightarrow$ center of the circle is $(-6,0)$ and the radius is $5 \Rightarrow$ the closest point on the circle to $(4,0)$ is $(-1,0)$ and the distance is 5.

2. Let $\alpha = \text{Arctan}\, x \Rightarrow \tan\alpha = x \Rightarrow \cos\alpha = \frac{1}{\sqrt{1+x^2}}$; Let $\beta = \text{Arccos}\left(\frac{2}{3}\right) \Rightarrow \cos\beta = \frac{2}{3}$
$\Rightarrow \tan\beta = \frac{\sqrt{5}}{2}; \cos 660° = \cos 300° = \frac{1}{2}; \frac{1}{\sqrt{1+x^2}} \cdot \frac{\sqrt{5}}{2} = \frac{1}{2} \Rightarrow \sqrt{5} = \sqrt{1+x^2} \Rightarrow x = 2$

3. $-2 = 2\,cis\,180°; 1 - i\sqrt{3} = 2\,cis\,300° \Rightarrow (1 - i\sqrt{3})^4 = 16\,cis\,1200° = 16\,cis\,120°;$
$z^3 = \frac{16\,cis\,120°}{2\,cis\,180°} = 8\,cis(-60°) = 8\,cis\,300° \Rightarrow z = 2\,cis\,100°, 2\,cis\,220°, 2\,cis\,340°$
or $z = -2\,cis\,40°, -2\,cis\,160°, -2\,cis\,280°$

'01

ROUND 5

1. To find the endpoints of the chord on the circle set $x = 0$:
$y^2 - 4y - 12 = 0 \rightarrow (y-6)(y+2) = 0 \rightarrow y = -2, 6 \rightarrow$ length of chord $= 6 - (-2) = 8$.

2. $\cos x = \frac{1}{7}, \frac{3\pi}{2} < x < 2\pi \rightarrow \sin x = -\sqrt{1 - \left(\frac{1}{7}\right)^2} = -\sqrt{\frac{48}{49}} = -\frac{4\sqrt{3}}{7}$; $\cos\left(x + \frac{2\pi}{3}\right) =$
$\cos x \cdot \cos\left(\frac{2\pi}{3}\right) - \sin x \cdot \sin\left(\frac{2\pi}{3}\right) = \left(\frac{1}{7}\right)\left(-\frac{1}{2}\right) - \left(-\frac{4\sqrt{3}}{7}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{-1+12}{14} = \frac{11}{14}$.

3. $2 - 2i\sqrt{3} = 2(1 - i\sqrt{3}) = 2(2\text{cis}300°) = 4\text{cis}300°$;
since $z = c + di$, $c > 0$, $d > 0$, and $z^4 = 4\text{cis}300° \rightarrow z = 4^{1/4}\text{cis}75° = \sqrt{2}\text{cis}75°$;
$1 + i = \sqrt{2}\text{cis}45° \rightarrow \frac{2z}{1+i} = \frac{2\sqrt{2}\text{cis}75°}{\sqrt{2}\text{cis}45°} = 2\text{cis}30° = \sqrt{3} + i$

'02

ROUND 5 – Precalculus

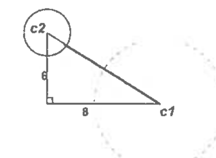
1. Since $x = \text{Arctan}\left(\frac{1}{2}\right) + \text{Arctan}\left(\frac{1}{3}\right) \Rightarrow \tan x = \frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2}\cdot\frac{1}{3}} = 1 \Rightarrow x = \frac{\pi}{4}$.

2. $\frac{1}{2}(1)(5)\sin A = 2 \Rightarrow \sin A = \frac{4}{5} \Rightarrow \cos A = \pm\frac{3}{5}$. By the Law of cosines,
$BC^2 = 1^2 + 5^2 - 2(1)(5)\left(\pm\frac{3}{5}\right) \Rightarrow BC^2 = 26 \pm 6 = 20, 32 \Rightarrow BC = 2\sqrt{5}, 4\sqrt{2}$.

3. The terminal side of the $-30°$ angle has slope $= \tan(-30°) = -\frac{\sqrt{3}}{3} \Rightarrow$ equation of the terminal side is $y = -\frac{\sqrt{3}}{3}x, x > 0$; for the hyperbola, $a = \sqrt{6}, c = 3 \Rightarrow b^2 = 3^2 - \sqrt{6}^2 = 3$.
The transverse axis is the x axis $\Rightarrow$ equation of the hyperbola is $\frac{x^2}{6} - \frac{y^2}{3} = 1 \Rightarrow$
$\frac{x^2}{6} - \frac{\left(-\frac{\sqrt{3}}{3}x\right)^2}{3} = 1 \Rightarrow \frac{x^2}{6} - \frac{x^2}{9} = 1 \Rightarrow x^2 = 18 \Rightarrow x = 3\sqrt{2} \Rightarrow y = -\frac{\sqrt{3}}{3}(3\sqrt{2}) = -\sqrt{6} \Rightarrow$
the point of intersection is $(3\sqrt{2}, -\sqrt{6})$.

'06

ROUND 5 – Pre-Calculus: Open



1. C_1: $(x^2 - 4x \underline{+ 4}) + (y^2 + 6y \underline{+ 9}) = 1 \underline{+ 4 + 9} = 14$
→ $(x-2)^2 + (y+3)^2 = 14$ → Center @ (2, -3)
C_2: $(x^2 + 12x \underline{+ 36}) + (y^2 - 6y \underline{+ 9}) = -41 \underline{+ 36 + 9} = 4$
→ $(x+6)^2 + (y-3)^2 = 4$ → Center @ (-6, 3), radius 2
Thus, the required distance = 10 – 2 = **8**.

2. $4(x-2)^2 + y^2 = 64$ → Ellipse: Center $(2, 0) = (h, k)$, vertex of parabola
$x^2 - 9(y+3)^2 = 36$ → Hyperbola – 2nd equation ignored
Parabola's equation: $y = a(x-2)^2$ Passes through $(4, m)$ → m = 4a
Since the parabola is concave up with vertex at (2, 0), both a and m must be positive.
Substituting in equation of ellipse to find m: $4(4-2)^2 + m^2 = 64$ → $m = 4\sqrt{3}$ → $a = \sqrt{3}$
Thus, $(a, h, k) = \underline{(\sqrt{3}, 2, 0)}$

3. Equation of the ellipse: $\frac{(x-1)^2}{9} + \frac{(y+2)^2}{4} = 1$ → ellipse is horizontal, center @ (1, -2),
$a = 3$, $b = 2$, length of focal chord $= \frac{2b^2}{a} = \frac{8}{3}$ and major axis connects (-2, -2) and (4, -2).
The hyperbola must be horizontal, $a = 3$, $b = \frac{8}{3}$ and the asymptote has a slope of $+\frac{8}{9}$.
$(y+2) = \frac{8}{9}(x-1)$ → $\underline{\mathbf{8x - 9y - 26 = 0}}$

'07

ROUND 5 – Pre-Calculus: Open

1. $z^2 = 9 - 24i - 16 = -7 - 24i$ and $3w = 24 + 9i \rightarrow$ sum $= 17 - 15i \rightarrow$ conjugate = $\underline{\mathbf{17 + 15}\boldsymbol{i}}$

2. $y = \frac{1}{16}(x-1)^2 - 4 \rightarrow$ vertex at (1, -4) $a = +4 \rightarrow$ focus at (1, 0) and endpoints of focal chord: (-7, 0), (9,0) Thus, for the horizontal ellipse $a = 8$, $b = 4$ and the center is at (1, 0) $\rightarrow$ the equation $\frac{(x-1)^2}{64} + \frac{y^2}{16} = 1$

To find the y-intercepts let $x = 0$: $1 + 4y^2 = 64 \rightarrow y^2 = \frac{63}{4} \rightarrow y = \underline{\pm\frac{3}{2}\sqrt{7}}$

3. $\sqrt{3}(\sec\theta - 2\tan\theta) - (\csc\theta - 2) = 0 \rightarrow \sqrt{3}\left(\frac{1-2\sin\theta}{\cos\theta}\right) - \left(\frac{1-2\sin\theta}{\sin\theta}\right) = 0$

$\rightarrow (1-2\sin\theta)\left(\frac{\sqrt{3}}{\cos\theta} - \frac{1}{\sin\theta}\right) = 0 \rightarrow (1-2\sin\theta)\left(\frac{\sqrt{3}\sin\theta - \cos\theta}{\sin\theta\cos\theta}\right) = 0$

$\rightarrow \sin\theta = ½ \rightarrow \theta = 30°, 150°$ Since $\sin\theta \neq 0$ and $\cos\theta \neq 0$, we restrict our attention to the numerator of the quotient. $\sqrt{3}\sin\theta - \cos\theta = 0 \rightarrow \sqrt{3}\tan\theta - 1 = 0 \rightarrow \tan\theta = \frac{1}{\sqrt{3}} \rightarrow \theta = 30°, 210°$

Thus, the solution set is **<u>30°, 150°, 210°</u>**.

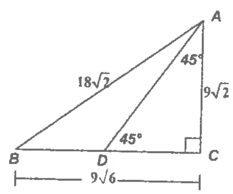
No '08

'09

ROUND 5

1. $AC = 9\sqrt{2}$, $BC = 9\sqrt{6}$ and $AD = 18$

Thus, $\frac{BD}{AD} = \frac{9\sqrt{6} - 9\sqrt{2}}{18} = \underline{\frac{\sqrt{6}-\sqrt{2}}{2}}$



2. $\sin^2(630° - \theta)\cos^2(450° + \theta) =$

$\sin^2(270 - \theta)\cos^2(90 + \theta) = (-\cos^2\theta)(-\sin^2\theta) = (\sin\theta\cos\theta)^2$

$\sin^2(240°)\cdot\cos^2(-420°) = \left(-\frac{\sqrt{3}}{2}\right)^2\left(\frac{1}{2}\right)^2 = \frac{3}{16}$

$(2\sin\theta\cos\theta)^2 = 4\left(\frac{3}{16}\right) \rightarrow \sin^2(2\theta) = \frac{3}{4} \rightarrow \sin 2\theta = \pm\frac{\sqrt{3}}{2} \rightarrow 2\theta = 60° + 180n$ or $120° + 180n$

$\rightarrow \theta = 30° + 90n$ or $60° + 90n \rightarrow \theta = 30, 120, 210, 300$ or $60, 150, 240, 330$

$\rightarrow \theta = \underline{\mathbf{30, 60, 120, 150, 210, 240, 300, 330}}$

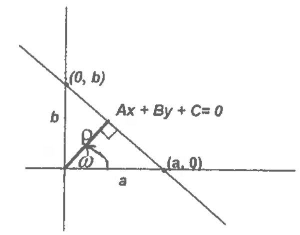
(continued...)

'08 cont.

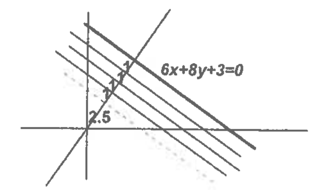
Detailed Solutions for GBML Meet 5 – FEBRUARY 2009

ROUND 5 - continued

3. (Here are 4 different solutions. – Look for the one you like.)
Solution #1 (Normalizing the line $x\cos\omega + y\sin\omega = \rho$)
To convert $Ax + By + C = 0$ to normal form,
divide by $\pm\sqrt{A^2 + B^2}$, where the sign of the radical is *<u>opposite</u>* of the sign of C ($C \neq 0$) and the *<u>same</u>* as the sign of B when $C = 0$.



We must divide thru by $+\sqrt{6^2 + 8^2} = +10$. $6x + 8y - 25 = 0$
$\rightarrow \frac{3}{5}x + \frac{4}{5}y - \frac{5}{2} = 0$
Since the lines we require are parallel to L_1,
C will change but the ratio of A to B must remain unchanged because the slope must remain the same.



Thus, the required equations are:
$10\left(\frac{3}{5}x + \frac{4}{5}y - \frac{5}{2} \pm 4 = 0\right) \rightarrow 6x + 8y - 25 \pm 40 = 0$
$\rightarrow \begin{cases} \mathbf{6x + 8y + 15 = 0} \\ \mathbf{6x + 8y - 65 = 0} \end{cases}$

Solution #2 (Point-to-Line Distance formula $\frac{|Ax + By + C|}{\sqrt{A^2 + B^2}}$)

Find a point on L_1 (any point will do!). There are no lattice points (integer coordinates), but we can find relatively simple coordinates by letting one coordinates be zero, e.g. $\left(\frac{25}{6}, 0\right)$.

We want the equations of parallel lines each 4 units from the given line.
The slope of L_1 is –3/4. Therefore, the equations we seek are of the form $3x + 4y + k = 0$

Applying the formula, $\frac{\left|3\left(\frac{25}{6}\right) + 4(0) + k\right|}{\sqrt{3^2 + 4^2}} = 4 \rightarrow \left|k + \frac{25}{2}\right| = 20 \rightarrow k + \frac{25}{2} = \pm 20 \rightarrow k = \frac{15}{2}, -\frac{65}{2}$

$3x + 4y \begin{cases} +15/2 \\ -65/2 \end{cases} = 0 \rightarrow \begin{cases} \mathbf{6x + 8y + 15 = 0} \\ \mathbf{6x + 8y - 65 = 0} \end{cases}$

'08 Cont.

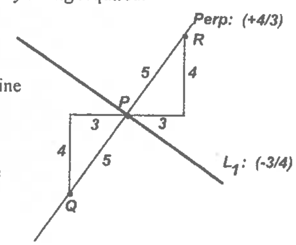
Detailed Solutions for GBML Meet 5 – FEBRUARY 2009

ROUND 5 - continued

Solution #3 (Find points 4 units from the given line using slope of perpendicular lines.)
Computationally, a little messy, but no fancy formulas or heavy lifting required.

Find any specific point P on L_1. Solving for y, $y = \frac{25-6x}{8}$.

If $x = 0.5$, then $y = 2.75 \rightarrow P(0.5, 2.75)$
Since the slope of $L_1 = -3/4$, the slope of any perpendicular line is $+4/3$. Thus, starting at any point on L_1, we arrive at a point (5 units away) on a perpendicular line by moving horizontally 3 units and then vertically 4 units. Adjusting by 80%, we change (3 – 4 – 5) to (2.4, 3.2, 4) and we will locate points Q and R, 4 units from L_1.
Specifically,
$Q(0.5-2.4, 2.75-3.2) = (-1.9, -0.45)$ and
$R(0.5+2.4, 2.75+3.2) = (2.9, 5.95)$.



Using point-slope form [$y - y_1 = m(x - x_1)$], we have the required equations.

Q: $y + 0.45 = \frac{-3}{4}(x+1.9) \rightarrow 4y + 1.8 = -3x - 5.7 \rightarrow 3x + 4y + 7.5 = 0 \rightarrow \underline{6x+8y+15=0}$

R: $y - 5.95 = \frac{-3}{4}(x-2.9) \rightarrow 4y - 23.8 = -3x + 8.7 \rightarrow 3x + 4y - 32.5 = 0 \rightarrow \underline{6x+8y-65=0}$

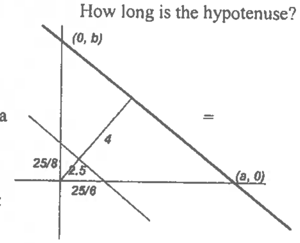
Solution #4 (3 – 4 – 5 magic and similar triangles)
Again no advanced techniques, only computational challenges.
L_1 and the coordinate axes form a triangle similar to the magical 3 – 4 – 5.

The x-intercept is at $\left(\frac{25}{6}, 0\right)$ and the y-intercept is at $\left(0, \frac{25}{8}\right)$. How long is the hypotenuse?

$\left(\frac{25}{8}, \frac{25}{6}, c\right) = 25\left(\frac{1}{8}, \frac{1}{6}, ?\right) = 25\left(\frac{3}{24}, \frac{4}{24}, ?\right) = \frac{25}{24}(3, 4, 5)$

$\rightarrow c = \frac{125}{24}$ If h denotes the altitude to the hypotenuse, then area

$\frac{1}{2}ab = \frac{1}{2}hc \Rightarrow h = ab \cdot \frac{1}{c}$ Thus, $h = \frac{25}{8} \cdot \frac{25}{6} \cdot \frac{24}{125} = \frac{5}{2} = 2.5$



Using similar triangles and proportions of corresponding sides:
$\frac{2.5}{6.5} = \frac{25/6}{a} \Rightarrow \frac{5}{13} = \frac{25}{6a} \Rightarrow a = \frac{65}{6}$

Similarly, $\frac{2.5}{6.5} = \frac{25/8}{b} \Rightarrow \frac{5}{13} = \frac{25}{8b} \Rightarrow b = \frac{65}{8}$

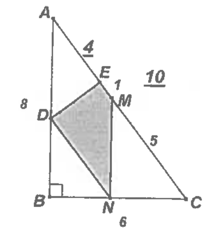
Using the intercept-intercept form of the linear equation $\left[\frac{x}{a} + \frac{y}{b} = 1\right]$, we have

$\frac{x}{65/6} + \frac{y}{65/8} = 1 \rightarrow \underline{6x + 8y - 65 = 0}$ It's left to you to derive the other equation.

'10

ROUND 5

1. If $(1 - i)$ and $\left(-1 - i\sqrt{3}\right)$ are roots and the equation has integral coefficients then $r_3 = (1 + i)$ and $r_4 = \left(-1 + i\sqrt{3}\right)$ are the additional roots. $(r_3 \cdot r_4) = (1+i)^6 \cdot \left(-1+i\sqrt{3}\right)^6 = (2i)^3 \cdot (2cis120°)^6$
$\rightarrow 8i^3 \cdot 2^6 cis(120 \cdot 6) = -8i \cdot 64cis720 = -512i(cis0°) = -512i(1+0i)$
$= 0 - 512i \rightarrow \underline{(0, -512)}$



2. $f(x) = 2x - 1 \rightarrow f^{-1}(x) = \frac{x+1}{2}$ and $f \circ f(x) = 2(2x-1) - 1 = 4x - 3$

Thus, we require x such that $4x - 3 = \frac{2}{x+1}$. Cross multiply, simplify and factor.

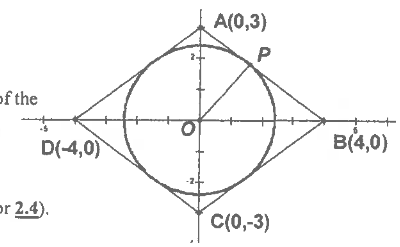
$4x^2 + x - 5 = (4x+5)(x-1) = 0 \rightarrow x = \underline{-\frac{5}{4}, 1}$

3. $\csc x \sin 217° = -1 \rightarrow \csc x = -\frac{1}{\sin 217°} = -\csc 217° > 0$ (quadrant 1, 2 / Reference value: 37°)
$90° < x < 270°$ (quadrant 2, 3) $\rightarrow x = 180 - 37 = 143°$
$\tan y \cot 203° = 1 \rightarrow \tan y = \frac{1}{\cot 203°} = \tan 203° > 0$ (quadrant 1, 3 / Reference value: 23°)
$0° \le y < 180°$ (quadrant 1, 2) $\rightarrow y = 23°$
Therefore, $\tan(x - y) = \tan(143° - 23°) = \tan(120°) = \underline{-\sqrt{3}}$

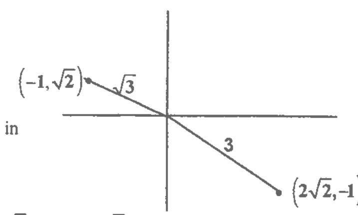
'11

ROUND 5

1. The graph of the given region is:
Note that $ABCD$ is a rhombus with side of length 5.
Let $\overline{OP}$ be the radius drawn to the point of contact of the tangent line $\overline{AB}$. ($\overline{OP} \perp \overline{AB}$)
The area of ΔAOB is given by $\frac{1}{2}AO \cdot OB$ and
$\frac{1}{2}AB \cdot OP$. Equating, $3 \cdot 4 = 5 \cdot OP \rightarrow OP = \underline{\frac{12}{5}}$ (or $\underline{2.4}$).



2. $Arc\sin\left(-\frac{1}{3}\right)$ denotes an angle in quadrant 4
whose sine is $-\frac{1}{3}$. $Arc\cos\left(-\frac{1}{\sqrt{3}}\right)$ denotes an angle in
quadrant 2 whose cosine is $-\frac{1}{\sqrt{3}}$. $\left[x^2 + y^2 = r^2\right]$



Thus, $\cos\left(Arc\sin\left(-\frac{1}{3}\right)\right) + Tan\left(Arc\cos\left(-\frac{1}{\sqrt{3}}\right)\right) = \frac{2\sqrt{2}}{3} - \sqrt{2} = -\frac{\sqrt{2}}{3} = \tan X$
Since $\tan X < 0$, X lies in quadrants 2 or 4.
$\rightarrow \cos X = \underline{-\frac{3\sqrt{11}}{11}}$ or $\underline{+\frac{3\sqrt{11}}{3}}$. Both answers required.

3. $\log_{14}\left(\frac{1}{16}\right)=P \rightarrow P=-4\log_{14}2=\frac{-4}{\log_2 14}=\frac{-4}{\log_2(2\cdot 7)}=\frac{-4}{1+\log_2 7}$

Thus, $\log_{14}2=-\frac{P}{4}$ and $\log_2 7=\frac{-(4+P)}{P}$ or $\log_7 2=-\frac{P}{P+4}$.

$\log_{\sqrt{14}}\left(\frac{343}{2}\right)=T \rightarrow T=\log_{14}\left(\frac{(7^3)^2}{2^2}\right)=6\log_{14}7-2\log_{14}2$

$\rightarrow T=\frac{6}{\log_7 14}+\frac{P}{2}=\frac{6}{\log_7 2+1}+\frac{P}{2}=\frac{6}{-\frac{P}{P+4}+1}+\frac{P}{2}=$

$=\frac{6(P+4)}{-P+P+4}+\frac{P}{2}=\frac{6P+24+2p}{4}=\underline{\mathbf{2P+6}}$ or $\underline{\mathbf{2(P+3)}}$