Round 3 Algebra 1

Exponents and Radicals; Equations involving them

GREATER BOSTON MATHEMATICS LEAGUE

MEET 1 – OCTOBER 1998

ROUND 3 – Algebra 1 – Exponents and Radicals

1. ____________________

2. ____________________

3. ____________________

CALCULATORS ARE NOT ALLOWED ON THIS ROUND

1. Write in simplest radical form: $5\left(20^{-1/2}\right)+\left(3+\sqrt{5}\right)^{-1}-\left(1\,7/9\right)^{-1/2}$

2. Given $\dfrac{\sqrt{6}}{\sqrt{3}-\sqrt{2}}+\dfrac{\sqrt{12}}{\sqrt{6}+\sqrt{2}}=a\sqrt{2}+b\sqrt{3}+c\sqrt{6}$, where a, b, and c are rational, find the product abc.

3. Solve for x: $2^{x+1}+2^{x+2}=4^{19}-4^{18}$

GREATER BOSTON MATHEMATICS LEAGUE

MEET 1 – SEPTEMBER 1999

ROUND 3 – Algebra 1 – Exponents and Radicals

1. ____________________

2. ____________________

3. ____________________

CALCULATORS ARE NOT ALLOWED ON THIS ROUND

1. Compute the following:

$$\frac{\left(\sqrt[3]{5}\right)\left(\sqrt[4]{25}\right)}{\left(\sqrt[6]{0.2}\right)}$$

2. $\sqrt{44+16\sqrt{6}}$ in simplest radical form equals $a\sqrt{b}+c\sqrt{d}$, where a, b, c, and d are all positive integers. Compute the product $a\,b\,c\,d$.

3. Compute all real solutions to the equation, $\sqrt{5x-4}-\sqrt{x+8}=2$.

GREATER BOSTON MATHEMATICS LEAGUE

MEET 1 – SEPTEMBER 2000

ROUND 3 – Algebra 1 – Exponents and Radicals

1. ____________________

2. ____________________

3. ____________________

CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

1. Find the value of the expression below in the form $\frac{a}{b}$ where a and b are relatively prime positive integers.

$$\sqrt{7\tfrac{1}{9}}-\sqrt{\frac{1}{9}+\frac{1}{16}}+\left(3+3^{-1}\right)^2$$

2. Solve the following equation for x, where $x>0$:

$$\frac{\sqrt[3]{\sqrt[4]{x^6}}}{\sqrt{\sqrt[6]{x^4}}}=\sqrt{3}\cdot\sqrt[3]{2}$$

3. Simplify the following expression:

$$\left(\frac{2^{-1}}{\sqrt[3]{2}-1}\right)\left(1-2^{1/6}\right)\left(1+2^{1/6}\right)-2^{-2}$$

GREATER BOSTON MATHEMATICS LEAGUE

MEET 1 – OCTOBER 2001

ROUND 3 – Algebra 1 – Exponents and Radicals

1. ____________________

2. ____________________

3. ____________________

CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

1. Write the following expression in simplest radical form:

$$\left(\frac{\sqrt{2}}{1-\frac{\sqrt{2}}{\sqrt{3}}}\right)\left(\sqrt{6}-\frac{5}{\sqrt{6}}\right)$$

2. Find the value of the following expression:

$$\left(8^{-2/3}\right)\left(16^{-1/2}\right)+\left(2\tfrac{1}{4}\right)^{3/2}\left(\sqrt{3}\right)^{-2}$$

3. Solve the following equation for x:

$$\sqrt{4x+12}+11=x-\sqrt[4]{81x^2+486x+729}$$

GREATER BOSTON MATHEMATICS LEAGUE

MEET 1 – OCTOBER 2006

ROUND 3 – Algebra 1 – Exponents and Radicals

1. ______________ : ______________

2. ______________________________

3. (______________ , ______________)

CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

1. If $x = 9$, evaluate $\dfrac{x^{3/2} - (x-1)^{-2/3}}{x^{-1/2}}$ as a simplified ratio of integers.

2. Simplify the following expression: $\dfrac{3 - \dfrac{1}{\sqrt{6}}}{\sqrt{2}} - \dfrac{\sqrt{6}}{\sqrt{3}}$

3. Given that

$$\frac{a\sqrt{b} + b\sqrt{a}}{a\sqrt{b} - b\sqrt{a}} - \frac{a\sqrt{b} - b\sqrt{a}}{a\sqrt{b} + b\sqrt{a}} = \sqrt{ab} \qquad (a > 0,\ b > 0 \text{ and } a \neq b)$$

solve for a <u>explicitly</u> in terms of b,
i.e. determine the ordered pair of constants (m, n) for which $a = mb + n$.

GREATER BOSTON MATHEMATICS LEAGUE

MEET 1 – OCTOBER 2007

ROUND 3 – Algebra 1 – Exponents and Radicals

1. ______________________

2. ______________________

3. ______________________

CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

1. Find the value of A, if $x^{\frac{3}{4}(18-A)} \cdot x^{-\frac{2}{3}A} = x^5$.

2. Compute: $\sqrt{\frac{25}{16}+9} - 2^{-\frac{1}{2}} - \left(\frac{1-\sqrt{2}}{2}\right)^2$

3. Simplify

$$\sqrt{19+6\sqrt{10}} + \sqrt{19-6\sqrt{10}}$$

GREATER BOSTON MATHEMATICS LEAGUE

MEET 1 – OCTOBER 2008

ROUND 3 – Algebra 1 – Exponents and Radicals

1. ______________________

2. ______________________

3. ______________________

CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

1. When simplified, $\sqrt{216}-\left(2\sqrt{2}-\sqrt{3}\right)^2=a+b\sqrt{c}$. Find $a+b+c$.

2. Simplify: $\sqrt{6.25}+\sqrt[4]{64}-\sqrt{\frac{1}{4}+\frac{4}{9}}-\left(\sqrt{2}\right)^3$

3. Find all values of x, $x\in\Re$, which make the following statement true: $\sqrt{x}-\sqrt{x-1}=0.\overline{3}$

GREATER BOSTON MATHEMATICS LEAGUE

MEET 1 – OCTOBER 2009

ROUND 3 – Algebra 1 – Exponents and Radicals

1. ________________________

2. ________________________

3. ________________________

CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

1. Find all values of x such that $\sqrt{x^2+x^2+x^2+x^2}=\sqrt{\frac{4}{9}+\frac{1}{4}}+\sqrt{7\frac{1}{9}}$.

2. Find all values of x such that $9^x-3^{x+1}-3^x+3=0$

3. In simplified form, $\dfrac{\sqrt{17+12\sqrt{2}}}{\sqrt{3-2\sqrt{2}}}=a+b\sqrt{c}$, where a, b and c are positive integers. Find the product abc.

GREATER BOSTON MATHEMATICS LEAGUE

MEET 1 – OCTOBER 2010

ROUND 3 – Algebra 1 – Exponents and Radicals

1. ______________________

2. ______________________

3. ______________________

CALCULATORS ARE NOT ALLOWED ON THIS ROUND.

1. Compute: $\sqrt{\dfrac{9^{x+1}\cdot 8^{2y}}{3^{2x-2}\cdot 4^{3y-3}}}$

2. Compute: $\left(\sqrt{\sqrt{1-\left(\dfrac{7}{25}\right)^2}}\right)\left(\dfrac{\sqrt[4]{576}}{2}\right)\left(\sqrt{\dfrac{1}{9}+\dfrac{1}{16}}\right)$

3. Compute: $\sqrt{\dfrac{\left(\sqrt{2}+\sqrt{14}\right)^2}{4}\left(1-\dfrac{\sqrt{63}}{12}\right)}$

MASSACHUSETTS MATHEMATICS LEAGUE
MARCH 2004
ROUND 2: EXPONENTS & RADCALS

ANSWERS

A)_______________

B)_______________

C)_______________

A) Find the exact value of x: $\sqrt{4+\frac{1}{4}+\frac{4}{9}} = 2+\frac{1}{2}+\frac{x}{3}$

B) Convert to simplified radical form: $\frac{\sqrt{6}-\sqrt{2}}{\sqrt{3}+1}+\frac{2\sqrt{3}}{\sqrt{2}}$

C) Solve for x. $8^{\frac{x+2}{x}} = 16^{\frac{x+2}{4}}$

MASSACHUSETTS MATHEMATICS LEAGUE
MARCH 2005
ROUND 2: ALGEBRA ONE RATIONAL EXPONENTS/RADICALS

ANSWERS

A)_______________

B)_______________

C)_______________

A) Find the value of x if $\sqrt{1+\frac{4}{9}+\frac{9}{16}}=1+\frac{2}{3}+\frac{x}{4}$

B) Simplify $\left(1+2\sqrt{3}\right)^2+\sqrt{\frac{4}{27}}-\left(\sqrt{3}\right)^3+\frac{7}{3\sqrt{3}}$

C) Simplify $\frac{3^{n+4}+3\cdot 3^{n+1}}{3^{n+6}}$ Express your answer as a simplified fraction.

MASSACHUSETTS MATHEMATICS LEAGUE
MARCH 2006
ROUND 2 ALGEBRA 1: RATIONAL EXPONENTS/RADICALS

ANSWERS

A)________________

B)________________

C)________________

A) If $\sqrt{2^{3^2}}+\sqrt{2^{2^3}}=a+8\sqrt{b}$, find the ordered pair (a, b).

B) Express the sum below as a simplified radical:

$$\frac{2}{2\sqrt{2}+\sqrt{7}}+\frac{2}{\sqrt{7}+\sqrt{6}}+\frac{2}{\sqrt{6}+\sqrt{5}}+\frac{2}{\sqrt{5}+2}+\frac{2}{2+\sqrt{3}}+\frac{2}{\sqrt{3}+\sqrt{2}}$$

C) Solve for x: $\dfrac{(1/4)^{3x}}{2(4)^7}=(8^{x+4})^x$

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 – MARCH 2007
ROUND 2 ALG 1: EXPONENTS AND RADICALS

ANSWERS

A) ______________________________

B) $x =$ ___________ , $y =$ ___________

C) $a =$ ______ , $b =$ ______ , $c =$ ______

A) The variables a, b, c and d have distinct values of 1, -2, 3 and -4, but not necessarily in that order. Determine the maximum possible value of the expression $a^b - c^d$.

B) Solve for x and y, if $\dfrac{4^{2x}}{2^{2y}} = \dfrac{8^{8x}}{64^{y}}$ and $\left(\dfrac{1}{3}\right)^{y-x} = 81$.

C) $\sqrt{48-24\sqrt{3}}$ in a simplified form can be written as $a+b\sqrt{c}$ where a, b and c are integers and c is square-free, i.e. contains no factors which are perfect squares (other than 1). Find a, b and c.

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 - MARCH 2008
ROUND 2 ALG1: EXPONENTS AND RADICALS

ANSWERS

A) ______________________

B) ______________________

C) (____________ , ____________)

A) Given: $N = 2x^{-2/3}$
If $x = 64$, find y, where $N = 4^y$.

B) Simply $\sqrt{4+\left(x-\frac{1}{x}\right)^2}\cdot\left(\frac{x}{3}+\frac{1}{3x}\right)^{-1}$ so that your answer is free of radicals and/or negative exponents.

C) Determine the ordered pair of positive integers (A, B) for which the quotient $\dfrac{\sqrt{49-8\sqrt{3}}}{\sqrt{21+12\sqrt{3}}}$ may be expressed as $\dfrac{A-B\sqrt{3}}{3}$.

GBML 1998

ROUND 3

1. $5\left(20^{-1/2}\right)+\left(3+\sqrt{5}\right)^{-1}-\left(1\,7/9\right)^{-1/2}=5\left(\frac{1}{\sqrt{20}}\right)+\frac{1}{3+\sqrt{5}}-\left(\frac{9}{16}\right)^{1/2}=\frac{\sqrt{5}}{2}+\frac{3-\sqrt{5}}{4}-\frac{3}{4}=\frac{\sqrt{5}}{4}$

2. $\frac{\sqrt{6}}{\sqrt{3}-\sqrt{2}}+\frac{\sqrt{12}}{\sqrt{6}+\sqrt{2}}=\sqrt{6}\left(\sqrt{3}+\sqrt{2}\right)+\frac{2\sqrt{3}\left(\sqrt{6}-\sqrt{2}\right)}{4}=$
$3\sqrt{2}+2\sqrt{3}+\frac{3}{2}\sqrt{2}-\frac{1}{2}\sqrt{6}=\frac{9}{2}\sqrt{2}+2\sqrt{3}-\frac{1}{2}\sqrt{6}\Rightarrow abc=-\frac{9}{2}$ or -4.5

3. $2^{x+1}+2^{x+2}=4^{19}-4^{18}\Rightarrow 2^{x+1}\left(1+2\right)=4^{18}\left(4-1\right)\Rightarrow 2^{x+1}=4^{18}\Rightarrow 2^{x+1}=2^{36}\Rightarrow x=35$

GBML 1999

ROUND 3

1. $\frac{\left(\sqrt[3]{5}\right)\left(\sqrt[4]{25}\right)}{\left(\sqrt[6]{0.2}\right)}=\frac{\left(5^{1/3}\right)\left(25^{1/4}\right)}{\left(\frac{1}{5}\right)^{1/6}}=\frac{5^{1/3}\cdot 5^{1/2}}{5^{-1/6}}=5^{1/3+1/2+1/6}=\mathbf{5}$

2. $\sqrt{44+16\sqrt{6}}=\sqrt{4\left(11+4\sqrt{6}\right)}=2\cdot\sqrt{11+4\sqrt{6}}$;
$\left(a'\sqrt{b'}+c'\sqrt{d'}\right)^2=11+4\sqrt{6}\Rightarrow a'^2b'+c'^2d'=11$ and $a'c'\sqrt{b'd'}=2\sqrt{6}\Rightarrow$
$b'=2,\ d'=3,\ a'=2,\ c'=1\Rightarrow\sqrt{44+16\sqrt{6}}=4\sqrt{2}+2\sqrt{3}\Rightarrow a\cdot b\cdot c\cdot d=\mathbf{48}$

3. $\sqrt{5x-4}-\sqrt{x+8}=2\Rightarrow\sqrt{5x-4}=2+\sqrt{x+8}\Rightarrow 5x-4=x+12+4\sqrt{x+8}\Rightarrow$
$4x-16=4\sqrt{x+8}\Rightarrow x-4=\sqrt{x+8}\Rightarrow x^2-8x+16=x+8\Rightarrow$
$x^2-9x+8=0\Rightarrow(x-1)(x-8)=0\Rightarrow$ Since $x=1$ is extraneous, $x=\mathbf{8}$.

GBML 2000

ROUND 3

1. $\sqrt{7\tfrac{1}{9}}-\sqrt{\frac{1}{9}+\frac{1}{16}}+\left(3+3^{-1}\right)^2=$
$\sqrt{\frac{64}{9}}-\sqrt{\frac{25}{9\cdot 16}}+\left(\frac{10}{3}\right)^2=\frac{8}{3}-\frac{5}{12}+\frac{100}{9}=\frac{96-15+400}{36}=\frac{481}{36}$

2. $\frac{\sqrt[3]{\sqrt[4]{x^6}}}{\sqrt{\sqrt[6]{x^4}}}=\sqrt{3}\cdot\sqrt[3]{2}\rightarrow\frac{x^{\frac{6}{12}}}{x^{\frac{4}{12}}}=3^{\frac{1}{2}}\cdot 2^{\frac{1}{3}}\rightarrow x^{\frac{1}{6}}=3^{\frac{1}{2}}\cdot 2^{\frac{1}{3}}\rightarrow x=3^3\cdot 2^2=108$

3. $\left(\frac{2^{-1}}{\sqrt[3]{2}-1}\right)\left(1-2^{1/6}\right)\left(1+2^{1/6}\right)-2^{-2}=$
$\left(\frac{2^{-1}}{\sqrt[3]{2}-1}\right)\left(1-2^{1/3}\right)-\frac{1}{4}=\left(\frac{2^{-1}}{\sqrt[3]{2}-1}\right)\left(1-\sqrt[3]{2}\right)-\frac{1}{4}=-\frac{1}{2}-\frac{1}{4}=-\frac{3}{4}$

GBML 2001

ROUND 3

1. $\left(\frac{\sqrt{2}}{1-\frac{\sqrt{2}}{\sqrt{3}}}\right)\left(\sqrt{6}-\frac{5}{\sqrt{6}}\right)=\left(\frac{\sqrt{2}}{\frac{\sqrt{3}}{\sqrt{3}}-\frac{\sqrt{2}}{\sqrt{3}}}\right)\left(\frac{6}{\sqrt{6}}-\frac{5}{\sqrt{6}}\right)=\left(\frac{\sqrt{6}}{\sqrt{3}-\sqrt{2}}\right)\left(\frac{1}{\sqrt{6}}\right)=$
$\frac{1}{\sqrt{3}-\sqrt{2}}=\sqrt{3}+\sqrt{2}$

2. $\left(8^{-2/3}\right)\left(16^{-1/2}\right)+\left(2\tfrac{1}{4}\right)^{3/2}\left(\sqrt{3}\right)^{-2}=\left(\frac{1}{4}\right)\left(\frac{1}{4}\right)+\left(\frac{9}{4}\right)^{3/2}\left(\frac{1}{3}\right)=\frac{1}{16}+\left(\frac{27}{8}\right)\left(\frac{1}{3}\right)=\frac{1}{16}+\frac{9}{8}=\frac{19}{16}$

3. $\sqrt{4x+12}+11=x-\sqrt[4]{81x^2+486x+729}\Rightarrow\sqrt{4(x+3)}+11=x-\sqrt[4]{81\left(x^2+6x+9\right)}\Rightarrow$
$2\sqrt{x+3}+11=x-3\sqrt[4]{(x+3)^2}\Rightarrow 5\sqrt{x+3}=x-11\Rightarrow 25(x+3)=x^2-22x+121\Rightarrow$
$x^2-47x+46=0\Rightarrow(x-46)(x-1)=0\Rightarrow x=46$. ($x=1$ is an extraneous solution.)

GBML 2006

ROUND 3

1.

$$\frac{9^{3/2}-(9-1)^{-2/3}}{9^{-1/2}}=\frac{3^3-2^{-2}}{3^{-1}}=\frac{27-\frac{1}{4}}{\frac{1}{3}}=\frac{12(27)-3}{4}=\frac{321}{4}\rightarrow \underline{\mathbf{321:4}}$$

2.

$$\frac{3-\frac{1}{\sqrt{6}}}{\sqrt{2}}-\frac{\sqrt{6}}{\sqrt{3}}=\frac{3-\frac{1}{\sqrt{6}}}{\sqrt{2}}-\sqrt{2}=\frac{3-\frac{1}{\sqrt{6}}-2}{\sqrt{2}}=\frac{1-\frac{1}{\sqrt{6}}}{\sqrt{2}}=\frac{\frac{\sqrt{6}-1}{\sqrt{6}}}{\sqrt{2}}=\frac{\sqrt{6}-1}{2\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\underline{\frac{3\sqrt{2}-\sqrt{3}}{6}}$$

3.

Simplifying the left hand side of the equation we have $\frac{(a\sqrt{b}+b\sqrt{a})^2-(a\sqrt{b}-b\sqrt{a})^2}{a^2b-b^2a}=\sqrt{ab}$

$\rightarrow \cancel{a^2b}+2ab\sqrt{ab}+\cancel{b^2a}-\cancel{a^2b}+2ab\sqrt{ab}-\cancel{b^2a}=4ab\sqrt{ab}=\sqrt{ab}(a^2b-b^2a)$

$\rightarrow 4ab=a^2b-b^2a=ab(a-b)\rightarrow a-b=4\rightarrow a=b+4\rightarrow (m,n)=\underline{\mathbf{(1,4)}}$

GBML 2007

ROUND 3

1. $\frac{3}{4}(18-A)+\frac{2}{3}A=5\rightarrow 9(18-A)-8A=60\rightarrow 162-60=102=17A\rightarrow A=\underline{6}$

2. $\sqrt{\frac{25}{16}+9-2^{-\frac{1}{2}}}-\left(\frac{1-\sqrt{2}}{2}\right)^2=\sqrt{\frac{169}{16}}-\frac{1}{\sqrt{2}}-\frac{1-2\sqrt{2}+2}{4}=\frac{13}{4}-\frac{\sqrt{2}}{2}-\frac{3}{4}+\frac{\sqrt{2}}{2}=\frac{10}{4}=\underline{\mathbf{\frac{5}{2}}}$

3. Let x denote the sum of the two radicals. Then:

$$x^2=\left(\sqrt{19+6\sqrt{10}}+\sqrt{19-6\sqrt{10}}\right)^2=19+6\sqrt{10}+19-6\sqrt{10}+2\sqrt{361-360}=40\rightarrow x=\underline{\mathbf{2\sqrt{10}}}$$

GBML 2008

ROUND 3

1. $6\sqrt{6}-\left(8-4\sqrt{6}+3\right)=-11+10\sqrt{6}\rightarrow -11+10+6=5$

2. $\sqrt{\frac{25}{4}}+\sqrt{8}-\sqrt{\frac{25}{26}}-2\sqrt{2}=\frac{5}{2}+2\sqrt{2}-\frac{5}{6}-2\sqrt{2}=\frac{5}{3}$

3. $\sqrt{x}-\frac{1}{3}=\sqrt{x-1}\rightarrow 3\sqrt{x}-1=3\sqrt{x-1}\rightarrow 9x-6\sqrt{x}+1=9(x-1)\rightarrow x=\frac{25}{9}$

GBML 2009

ROUND 3

1. $\sqrt{x^2+x^2+x^2+x^2}=\sqrt{\frac{4}{9}+\frac{1}{4}}+\sqrt{7\frac{1}{9}}\rightarrow +\sqrt{4x^2}=\sqrt{\frac{25}{36}}+\sqrt{\frac{64}{9}}\rightarrow 2|x|=\frac{5}{6}+\frac{8}{3}=\frac{21}{6}=\frac{7}{2}$

$\rightarrow x=\underline{\pm\frac{7}{4}}$

2. $9^x-3^{x+1}-3^x+3=0\rightarrow \left(3^x\right)^2-3^x\cdot 3-3^x+3=0\rightarrow \left(3^x\right)^2-4\cdot 3^x\cdot 3=\left(3^x-1\right)\left(3^x-3\right)=0$

$\rightarrow x=\underline{\mathbf{0,1}}$

3. $\frac{\sqrt{17+12\sqrt{2}}}{\sqrt{3-2\sqrt{2}}}=\sqrt{\frac{17+12\sqrt{2}}{3-2\sqrt{2}}\cdot\frac{3+2\sqrt{2}}{3+2\sqrt{2}}}=\sqrt{\frac{51+70\sqrt{2}+48}{1}}\rightarrow 99+70\sqrt{2}=\left(a^2+b^2c\right)+2ab\sqrt{c}$

$\rightarrow c=2,\ ab=35$ and $a^2+2b^2=99\rightarrow (a,b,c)=(7,5,2)\rightarrow abc=\underline{\mathbf{70}}$

GBML 2010

ROUND 3

1. $\sqrt{\frac{9^{x+1}\cdot 8^{2y}}{3^{2x-2}\cdot 4^{3y-3}}}=\sqrt{\frac{3^{2x+2}\cdot 2^{6y}}{3^{2x-2}\cdot 2^{6y-6}}}=\sqrt{3^4\cdot 2^6}=3^2\cdot 2^3=\underline{\mathbf{72}}$

2. $\left(\sqrt{\sqrt{1-\left(\frac{7}{25}\right)^2}}\right)\left(\frac{\sqrt[4]{576}}{2}\right)\left(\sqrt{\frac{1}{9}+\frac{1}{16}}\right)$

Note: (7, 24, 25) is a Pythagorean triple and $576=24^2$.

$$\left(\sqrt{\sqrt{\frac{25^2-7^2}{25^2}}}\right)\left(\frac{\sqrt[4]{24^2}}{2}\right)\left(\sqrt{\frac{16+9}{16\cdot 9}}\right)=\left(\sqrt{\frac{24}{25}}\right)\left(\frac{\sqrt{24}}{2}\right)\left(\frac{5}{12}\right)=\frac{24(5)}{5(24)}=\underline{\mathbf{1}}$$

3. $\sqrt{\frac{\left(\sqrt{2}+\sqrt{14}\right)^2}{4}\left(1-\frac{\sqrt{63}}{12}\right)}=\sqrt{\left(\frac{16+4\sqrt{7}}{4}\right)\left(\frac{12-3\sqrt{7}}{12}\right)}=\sqrt{\left(4+\sqrt{7}\right)\left(\frac{4-\sqrt{7}}{4}\right)}=\sqrt{\frac{16-7}{4}}=\underline{\mathbf{\frac{3}{2}}}$

MML 3/04

A) Find the exact value of x: $\sqrt{4+\frac{1}{4}+\frac{4}{9}} = 2+\frac{1}{2}+\frac{x}{3}$

$\sqrt{\frac{144+9+16}{36}} = \sqrt{\frac{169}{36}} = \frac{13}{6} = \frac{5}{2}+\frac{x}{3}, \quad \frac{13-15}{6} = \frac{x}{3}$

$\frac{x}{3} = -\frac{2}{6} = -\frac{1}{3}, \quad \boxed{x=-1}$

B) Convert to simplified radical form $\frac{\sqrt6-\sqrt2}{\sqrt3+1}+\frac{2\sqrt3}{\sqrt2}$

$\frac{(\sqrt6-\sqrt2)(\sqrt3-1)}{(\sqrt3+1)(\sqrt3-1)} + \sqrt2\sqrt3 = \frac{3\sqrt2-2\sqrt6+\sqrt2+2\sqrt6}{2} =$

$\frac{4\sqrt2}{2} = \boxed{2\sqrt2}$

C) Solve for x $8^{\frac{x+2}{x}} = 16^{\frac{x+2}{4}}$

I. $2^{\frac{3(x+2)}{x}} = 2^{\frac{4(x+2)}{4}} \quad 3x+6 = x(x+2), \quad 3x+6 = x^2+2x,$

$x^2-x-6=0, \quad (x+2)(x-3)=0, \quad x=-2,3.$

II $x=-2$ since $8^0=16^0=1$, Then $8^{\frac{1}{x}} = 16^{\frac{1}{4}} = 2, \quad x=3$

III, $8^{\frac{x+2}{x}} = 8^{\frac{4}{3}\cdot\frac{x+2}{4}} = 8^{\frac{x+2}{3}} \quad \boxed{x=-2 \text{ or } 3}$

MML 3/05

Round Two:

A. $\sqrt{\frac{144+64+81}{144}} = \frac{12+8+3x}{12}$ so $\sqrt{289} = 17 = 20 = 3x$ so $x=-1$.

B. $1+4\sqrt3+12+\frac{2\sqrt3}{9}-3\sqrt3+\frac{7\sqrt3}{9} =$

$13+\frac{36+2-27+7}{9}\sqrt3 = 13+2\sqrt3$

C. $\frac{3^{n+4}}{3^{n+6}}+\frac{3^{n+2}}{3^{n+6}} = \frac{1}{3^2}+\frac{1}{3^4} = \frac{3^2+1}{3^4} = \frac{10}{81}$

MML 3/06

Round Two:

A. $\sqrt{2^9}+\sqrt{2^8} = 2^{4.5}+2^4 = 16+2^3 2^{1.5} = 16+8\sqrt8$ so $(a, b) = (16, 8)$.

B. Replace $2\sqrt2$ .with $\sqrt8$. Note $\frac{1}{\sqrt{x+1}+\sqrt{x}}\left(\frac{\sqrt{x+1}-\sqrt{x}}{\sqrt{x+1}-\sqrt{x}}\right) = \sqrt{x+1}-\sqrt{x}$ so

$2(\sqrt8-\sqrt7+\sqrt7-\sqrt6+\sqrt6-\sqrt5+\ldots+\sqrt3-\sqrt2) = 2(\sqrt8-\sqrt2) = 2(2\sqrt2-\sqrt2)$

C. 2^(-6x) / 2^(15) = 2^(3(x + 4)x) so $-6x-15 = 3x^2+12x$ etc.

MML 3/07

Round 2

A) Trial and error → $(a, b, c, d) = (1, -2, -4, 3)$ → $(1)^{-2}-(-4)^3 = 1+64 = \underline{65}$

B) $x-y=4$ and $2^{4x-2y} = 2^{24x-6y}$ → $2^{20x-4y} = 1$ → $20x-4y=0$ or $5x-y=0$
Solving simultaneously, $4x=-4$ → $\underline{x=-1}$. Substituting back, $\underline{y=-5}$.

C) The radicand must represent a perfect square, call it $(a+b\sqrt3)^2 = a^2+3b^2+2ab\sqrt3$
For integer values of a and b, a^2+3b^2 must represent an integer and $2ab\sqrt3$ a multiple of $\sqrt3$. Thus, $a^2+3b^2=48$ and $ab=-12$. Clearly, a and b have opposite signs and checking out factors of 12 in the first equation produces either $(6, -2)$ → $\boxed{6-2\sqrt3}$ which is positive or $(-6, 2)$ → $-6+2\sqrt3$ which is a negative value and must be rejected. Thus, $\underline{a=6, b=-2, c=3}$

MML 3/08

Round 2

A) Suppose $N=4^y$.
$N = 2(64)^{-2/3} = 2(4)^{-2} = 2^{-3} = 4^y = 2^{2y}$ → $2y=-3$ → $y = \underline{-3/2}$

B) $= \left(\sqrt{4+x^2-2+\frac{1}{x^2}}\right)\left(\frac{x^2+1}{3x}\right)^{-1} = \left(\sqrt{x^2+2+\frac{1}{x^2}}\right)\left(\frac{3x}{x^2+1}\right) = \left(\sqrt{(x+x^{-1})^2}\right)\left(\frac{3x}{x^2+1}\right)$

$= \left(\sqrt{\frac{(x^2+1)^2}{x^2}}\right)\cdot\left(\frac{3x}{x^2+1}\right) = \frac{x^2+1}{|x|}\cdot\frac{3x}{x^2+1} = \frac{3x}{|x|} = \underline{\pm3}$

C) In order to extract the square roots in both the numerator and the denominator, the radicands must be perfect squares. Therefore, let $49-8\sqrt3 = (A+B\sqrt3)^2$ and $21+12\sqrt3 = (C+D\sqrt3)^2$.
Multiplying out and equating rational and irrational parts we have,
$\begin{cases} A^2+3B^2=49 \\ AB=4 \end{cases}$ → $(A, B) = (-1, 4)$ and $\begin{cases} C^2+3D^2=21 \\ CD=6 \end{cases}$ → $(C, D) = (3, 2)$

Thus, $\frac{\sqrt{49-8\sqrt3}}{\sqrt{21+12\sqrt3}} = \frac{-1+4\sqrt3}{3+2\sqrt3}\cdot\frac{3-2\sqrt3}{3-2\sqrt3} = \frac{-3+2\sqrt3+12\sqrt3-24}{-3} = \frac{27-14\sqrt3}{3}$ → $\underline{(27, 14)}$