# CS736 Assignment 3

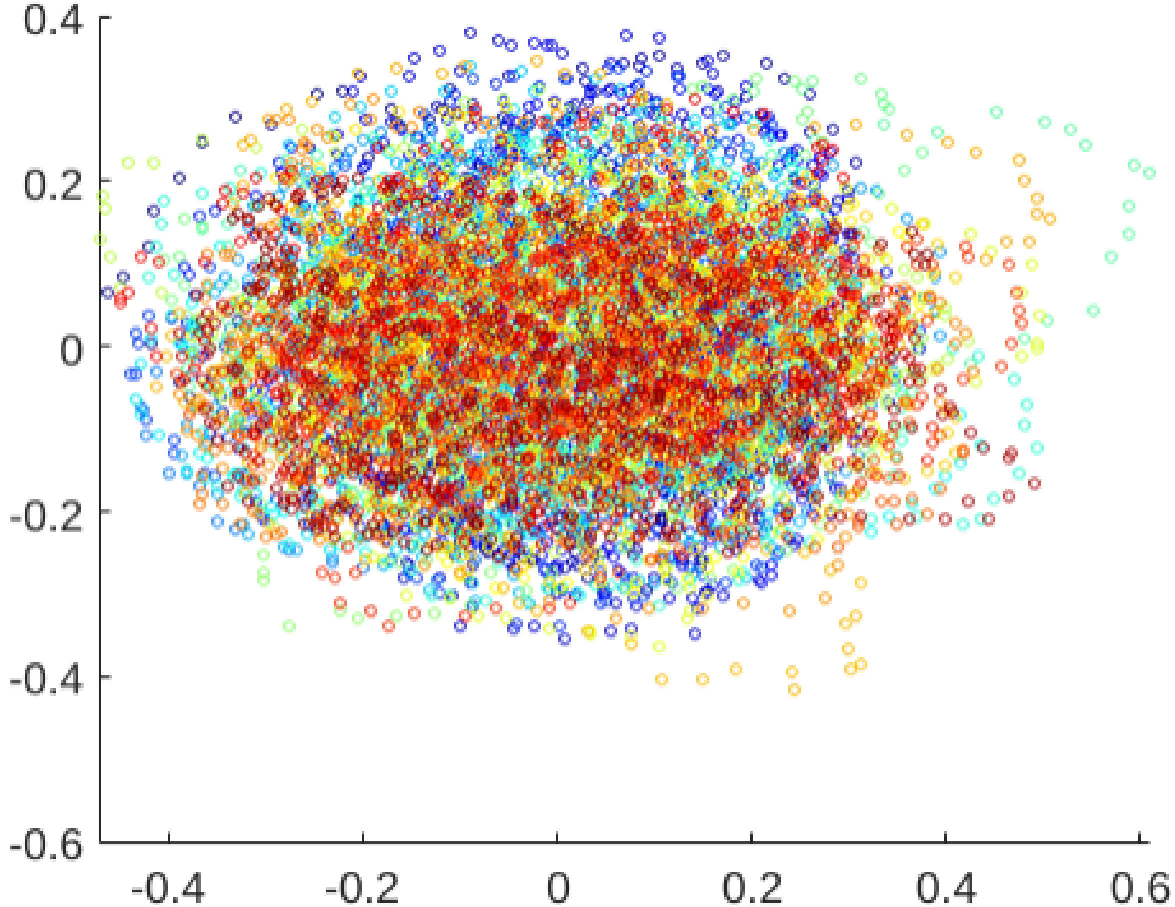
Hastyn Doshi, Kartik Gokhale & Sarthak Mittal

March 27, 2022

## Contents

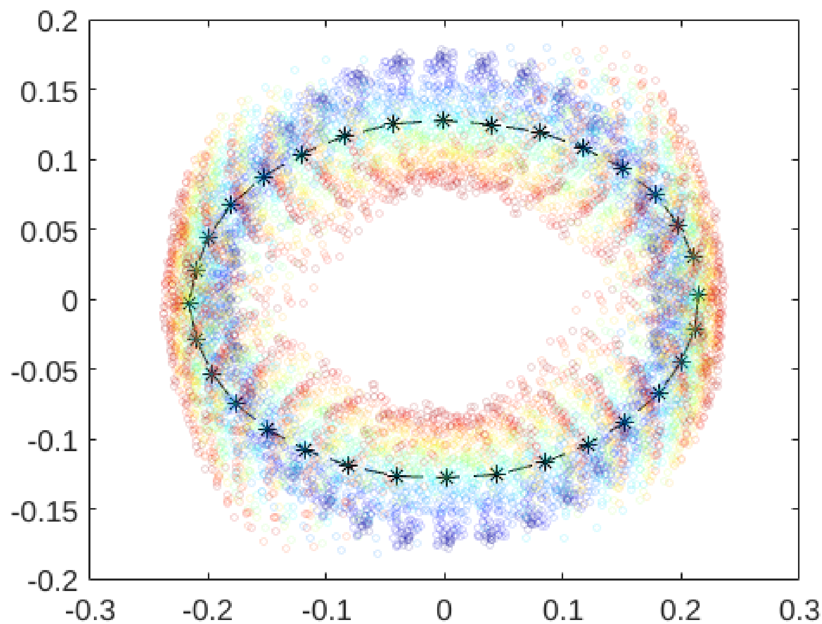
# 1 Shape Analysis on Simulated Shapes

## 1.1 Initial Pointsets

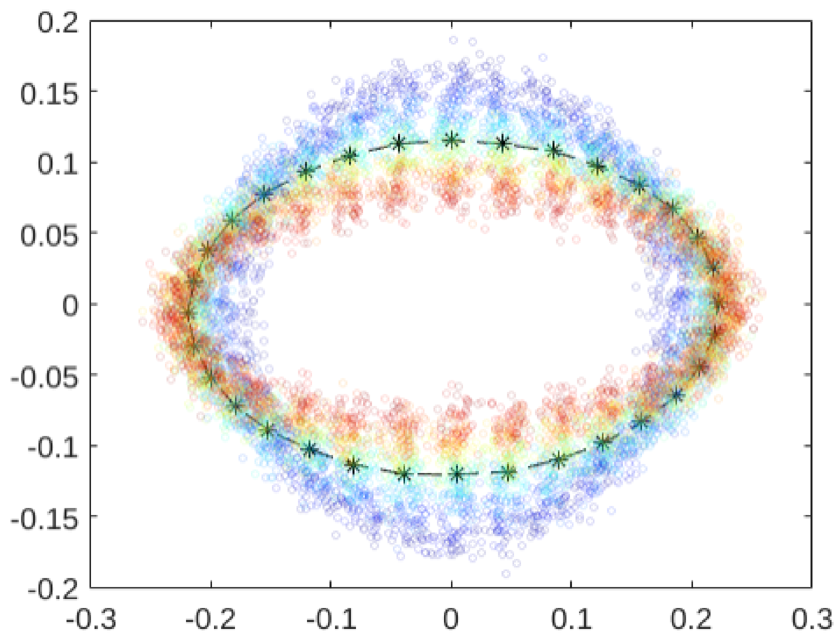


## 1.2 Mean along with Aligned Data

### 1.2.1 Case 1

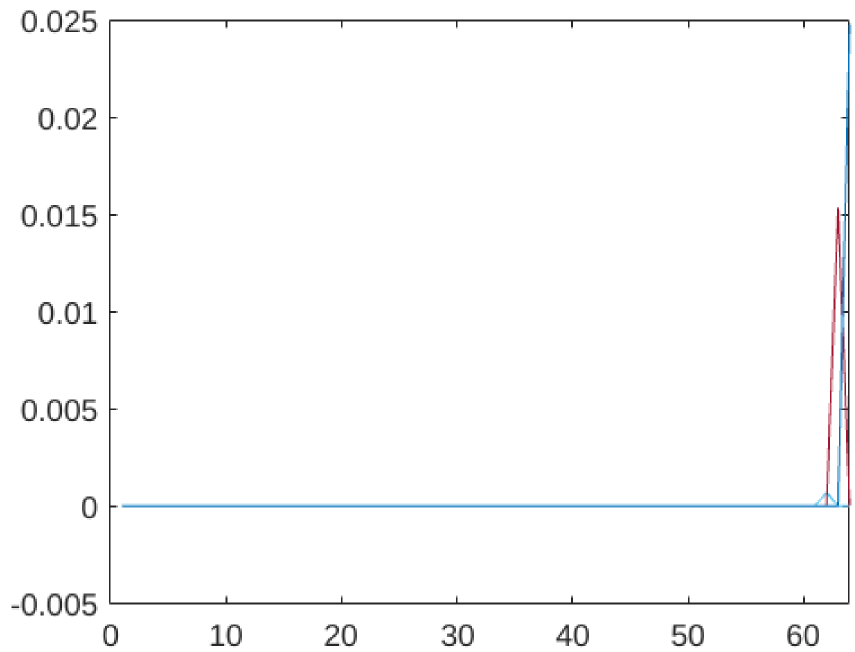



### 1.2.2 Case 2

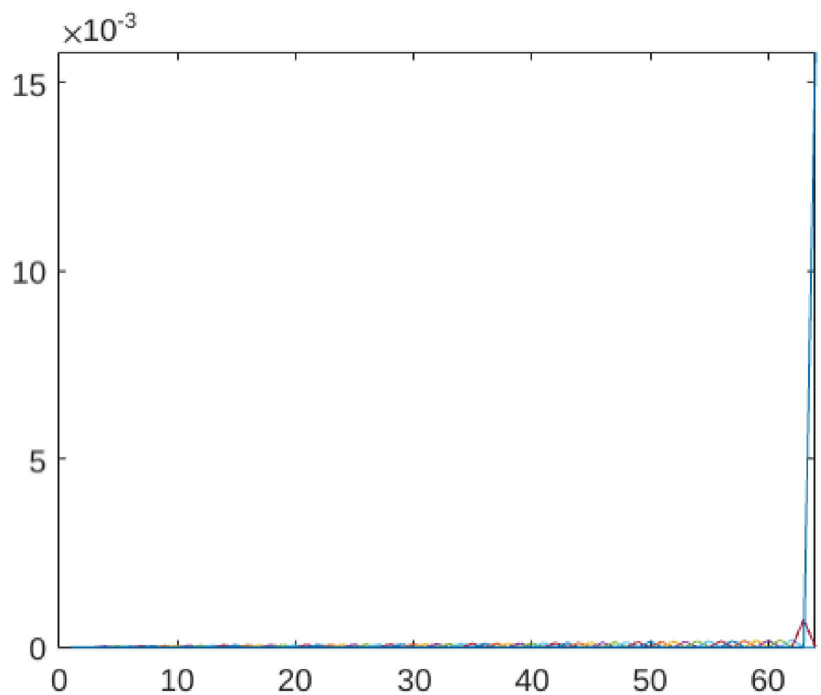


## 1.3 Eigenvalues

### 1.3.1 Case 1

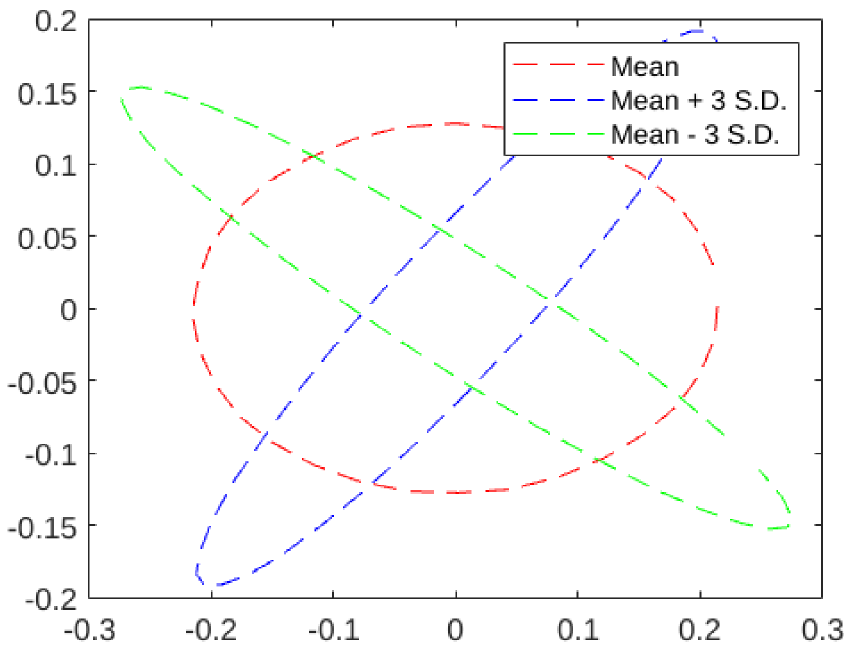



### 1.3.2 Case 2

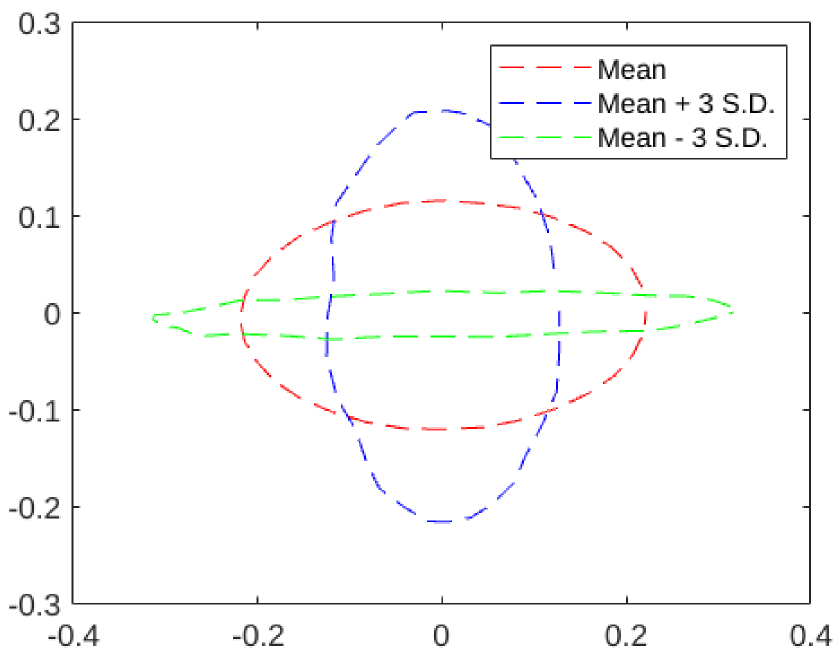


## 1.4 First Mode of Variation

### 1.4.1 Case 1

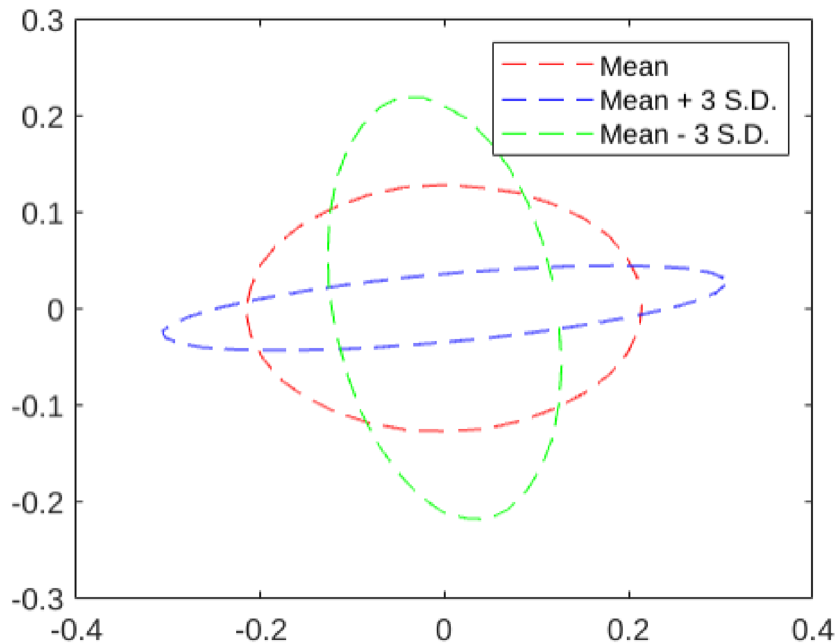



### 1.4.2 Case 2

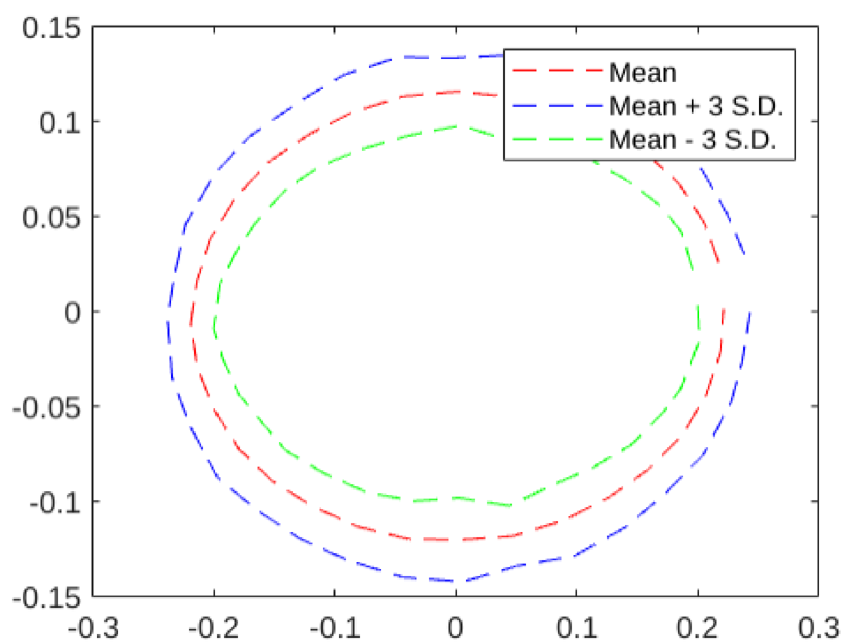


### 1.5 Second Mode of Variation

#### 1.5.1 Case 1

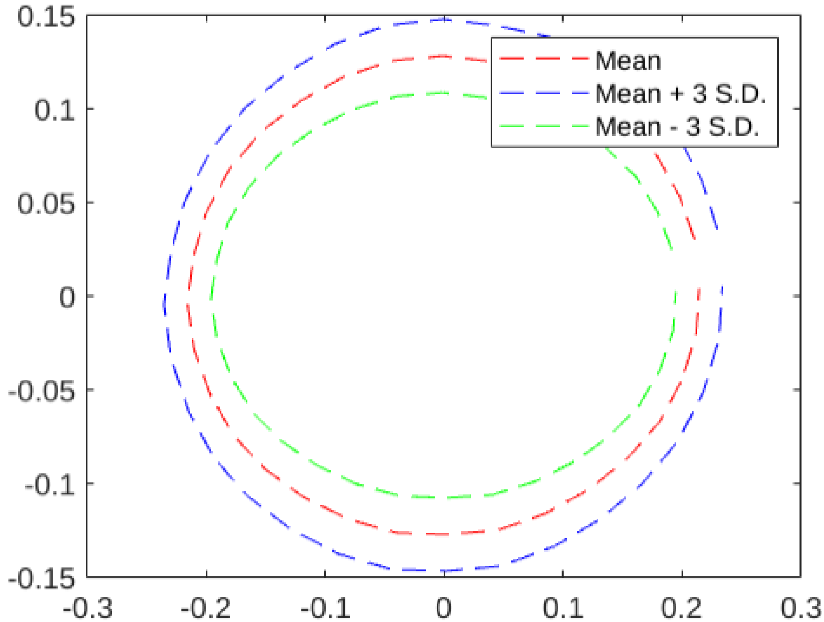



#### 1.5.2 Case 2

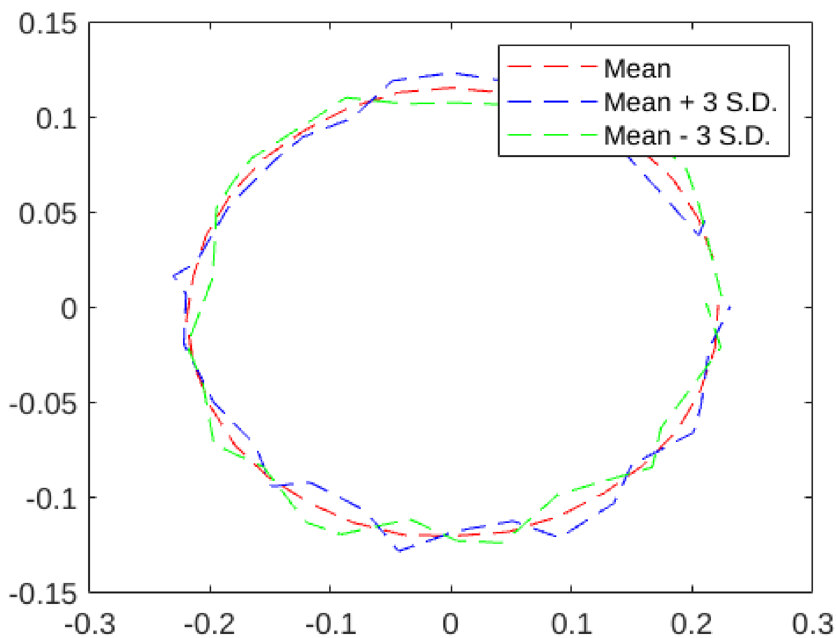


### 1.6 Third Mode of Variation

#### 1.6.1 Case 1

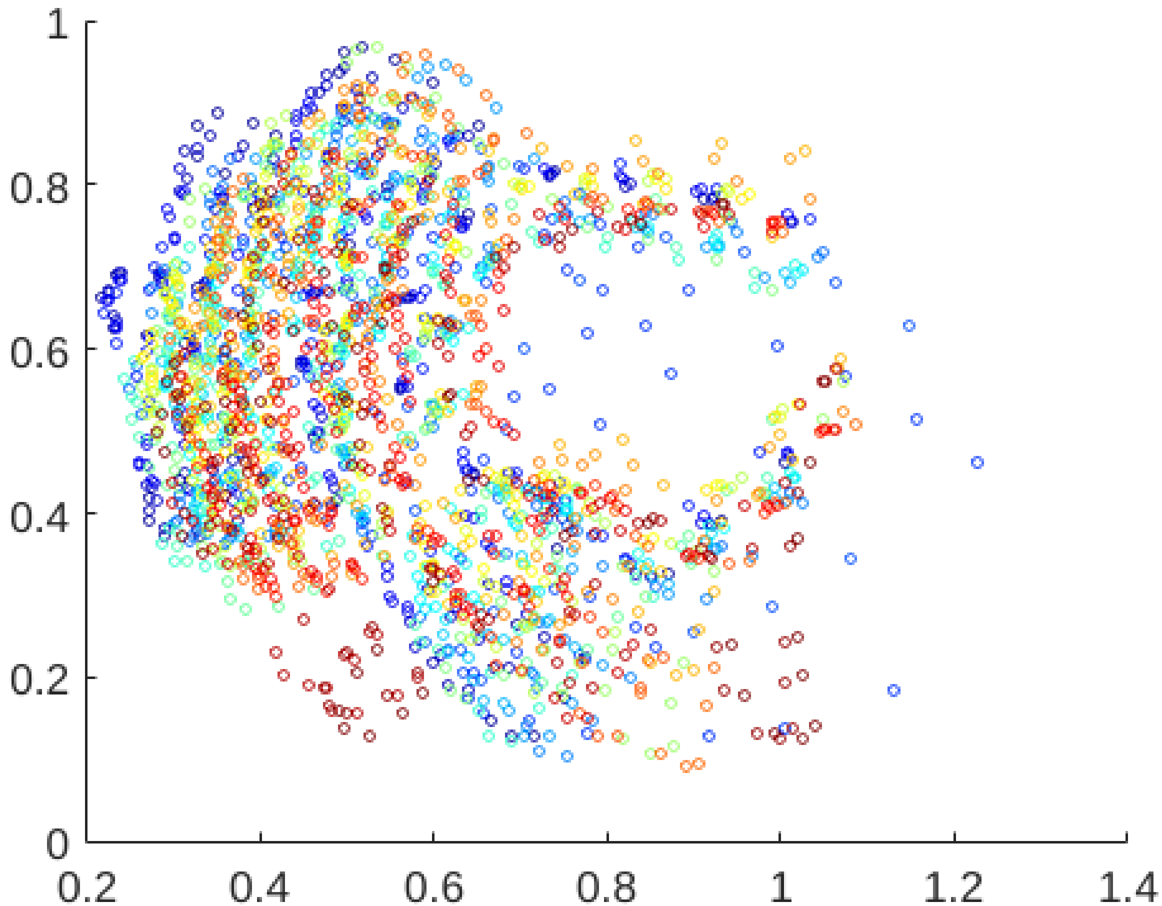



#### 1.6.2 Case 2



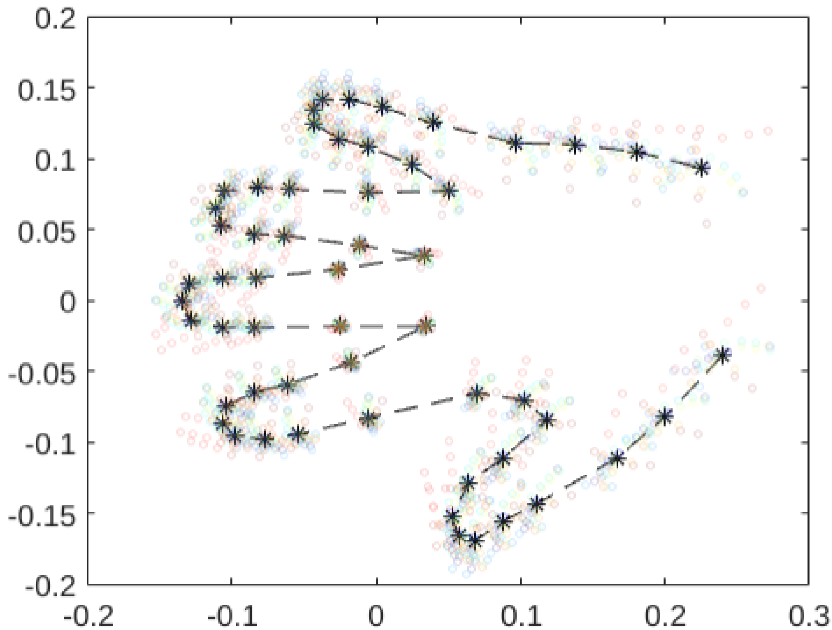
# 2 Shape Analysis on Human Hand Shapes

## 2.1 Initial Pointsets

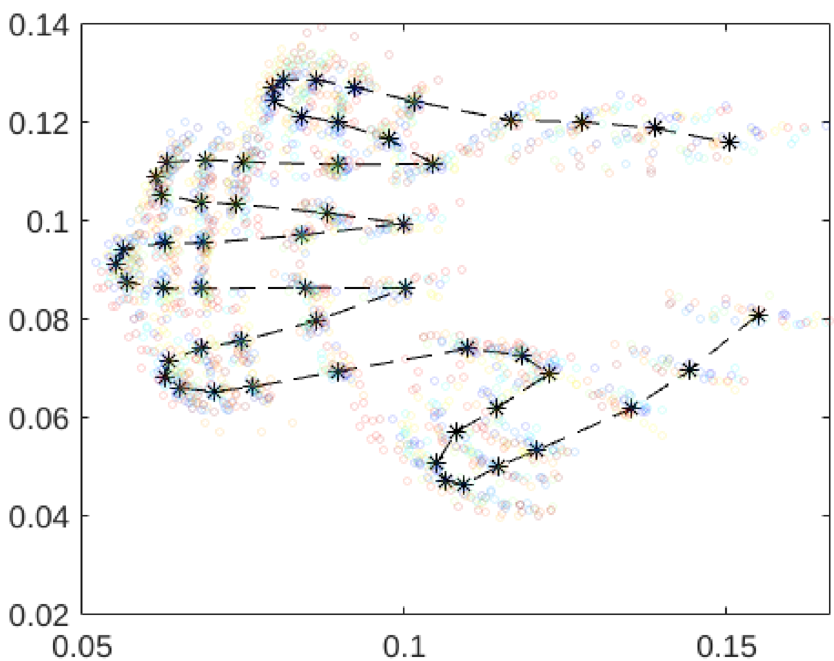


## 2.2 Mean along with Aligned Data

### 2.2.1 Case 1

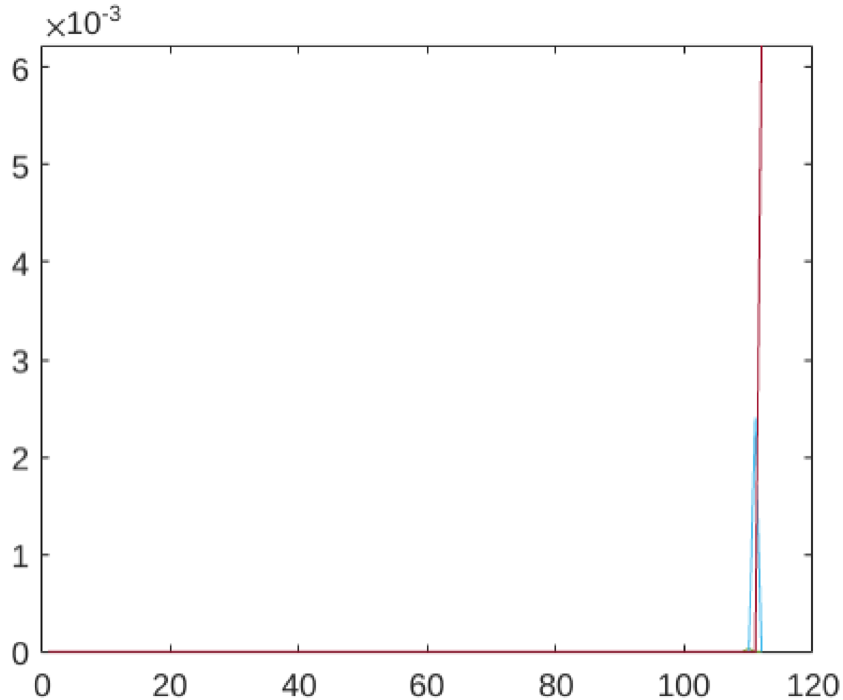



### 2.2.2 Case 2

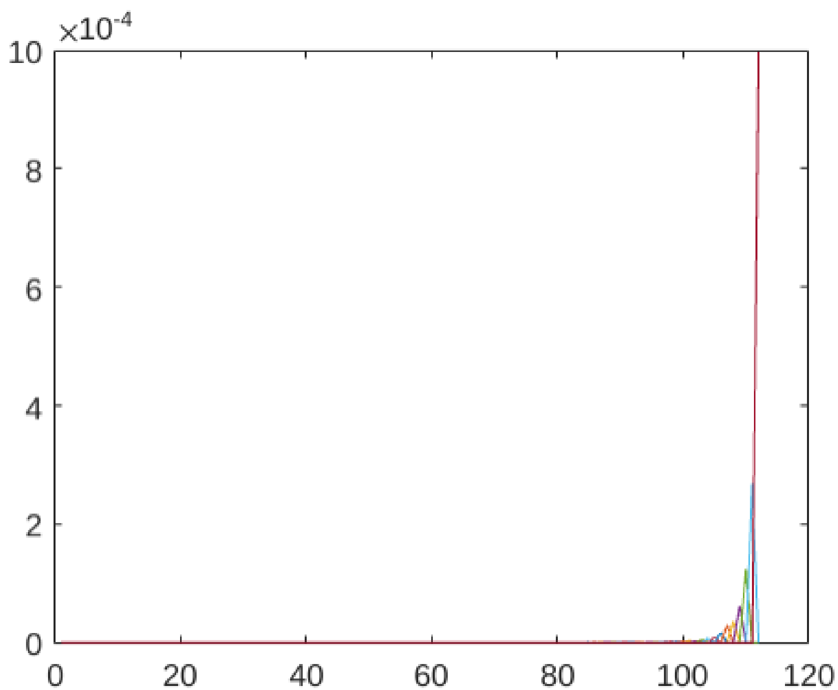


## 2.3 Eigenvalues

### 2.3.1 Case 1

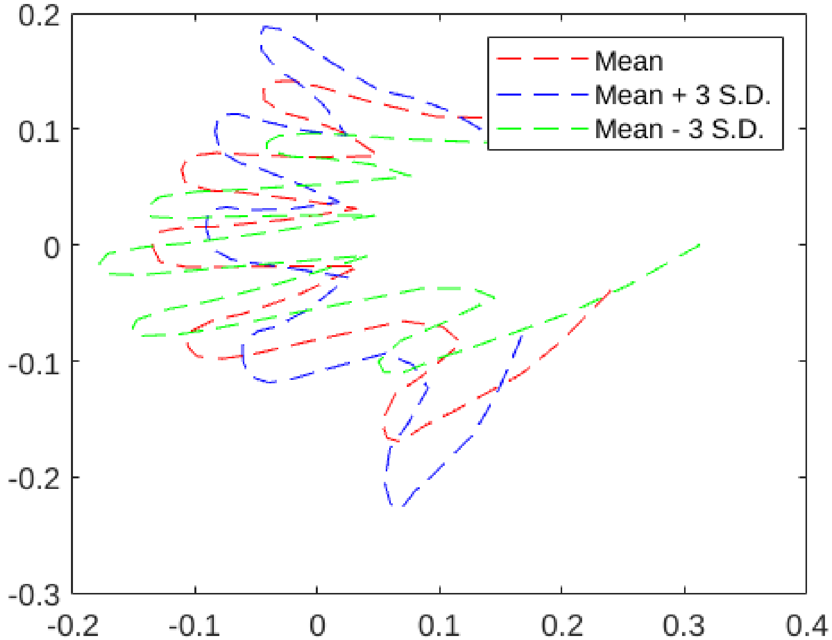



### 2.3.2 Case 2

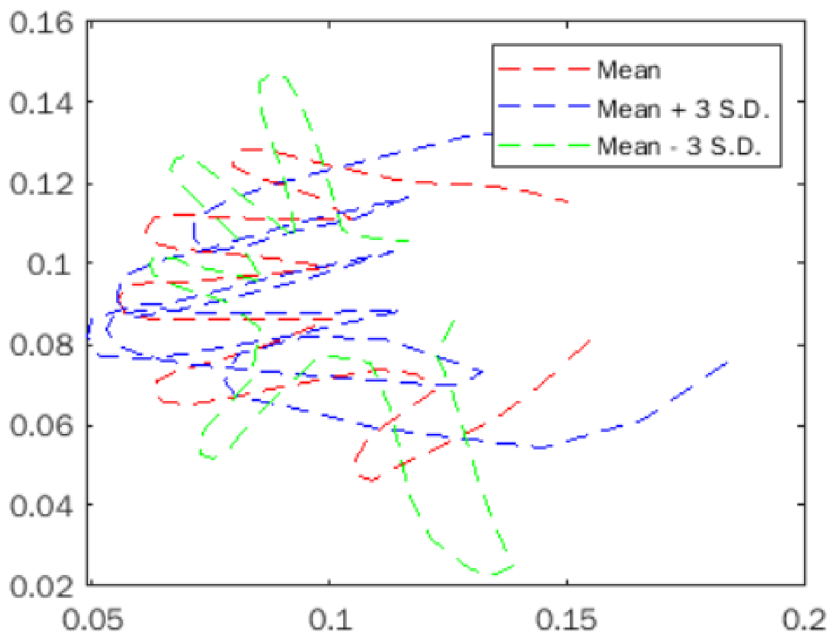


## 2.4 First Mode of Variation

### 2.4.1 Case 1

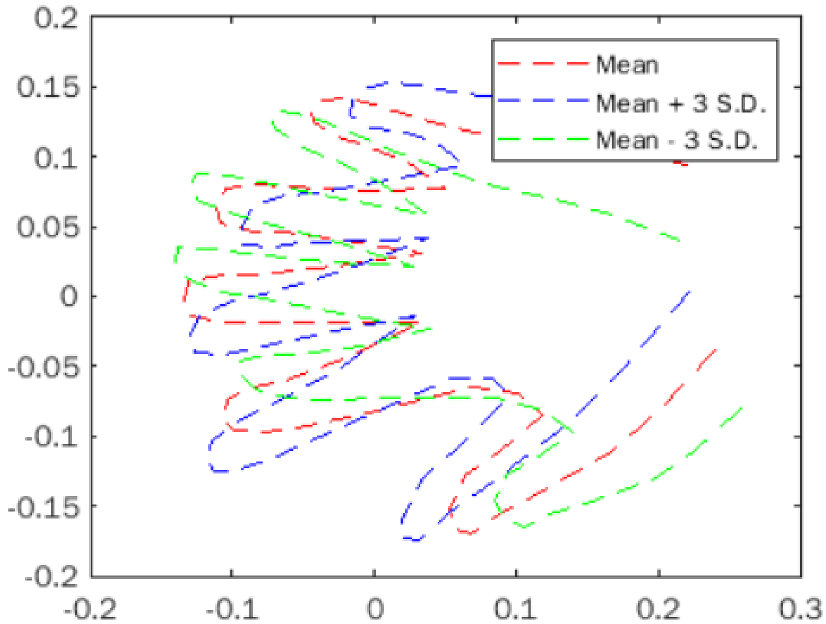



### 2.4.2 Case 2

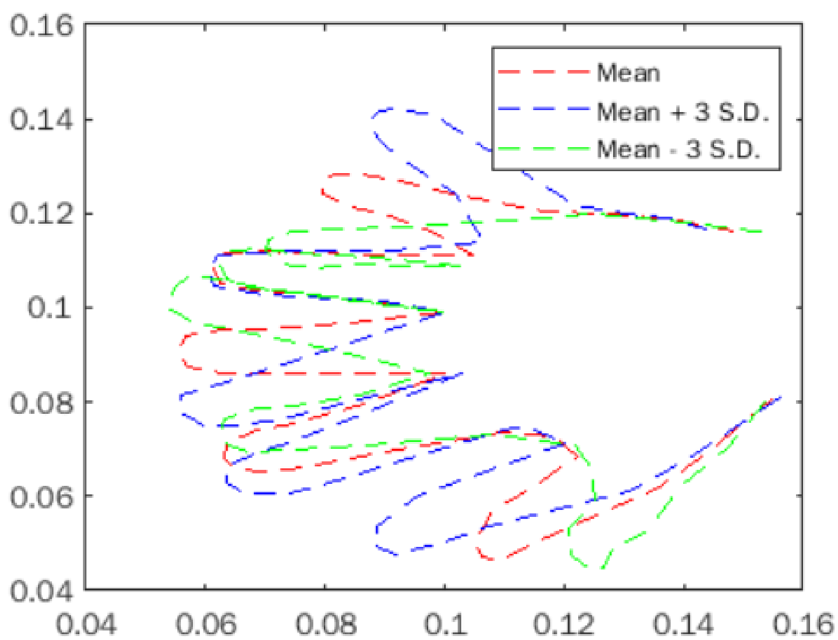


## 2.5 Second Mode of Variation

### 2.5.1 Case 1

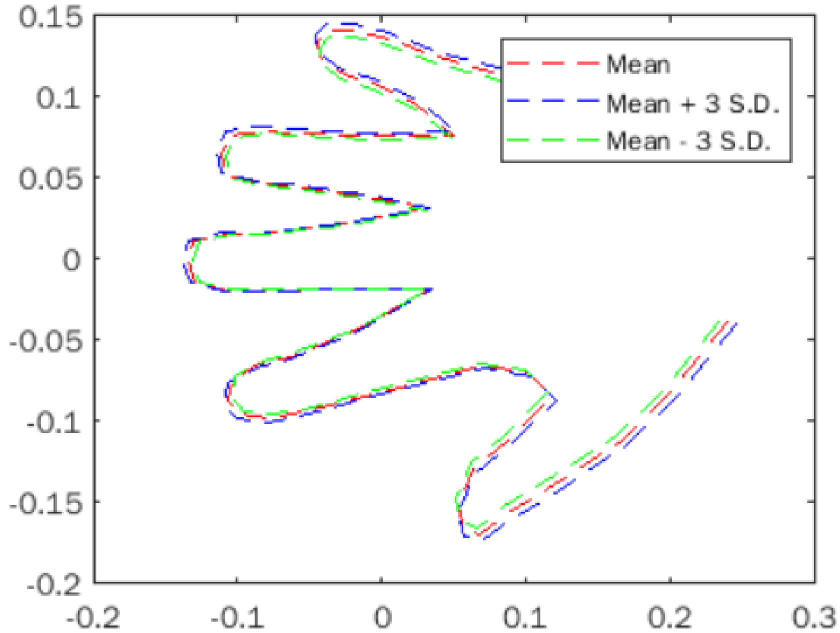



### 2.5.2 Case 2

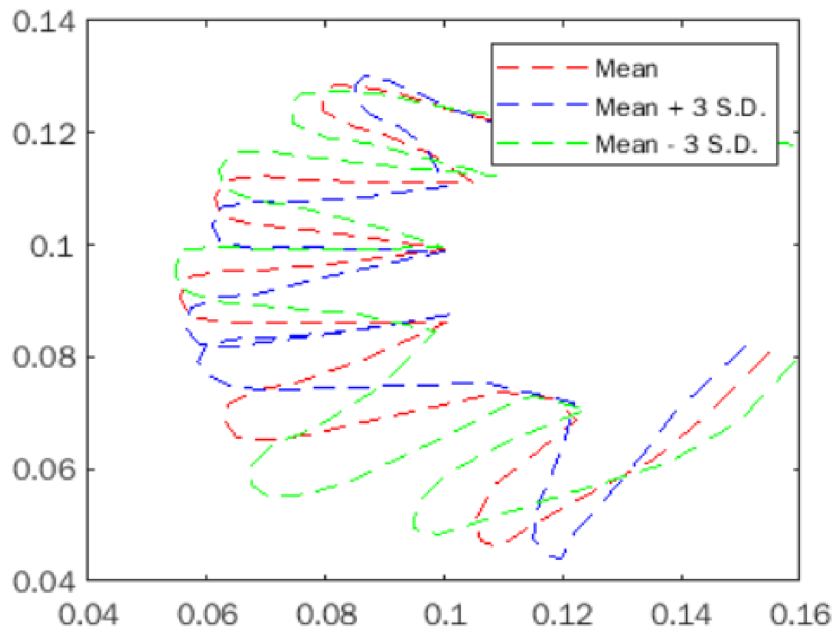


### 2.6 Third Mode of Variation

#### 2.6.1 Case 1




#### 2.6.2 Case 2



# 3 Clustering of 3-D Point Sets

## 3.1 Procrustes Distance

The Procrsutes distance between two shapes given as two pointsets $z_1$ and $z_2$ is given by,

$$d^2(z1, z2) = \min_{\theta,T,s} \sum_{n=1}^{N} \|z_{1n} - s \times M_\theta \times z_{2n} - T\|^2$$

where $\theta$ is the angle by which it is rotated, s is the scale factor, T is the translation factor and the norm is the L2 Euclidean norm.

## 3.2 Objective Function

As an objective function we could use the standard K-means objective function as our purpose is to cluster the shapes. The only modification we must make is the distance function. In our K-means problem we could just take the L2-norm of the two different vectors but in our case here we are given two pointsets representing shapes thus we must take into account transformations to which the shape is invariant. Thus we decided to use the Procrustes distance as defined above,

$$\sum_{i=1}^{K} \sum_{x_j \in S_i} d^2(x_j, \mu_i)$$

where $x_j$ is a pointset which we know is in cluster $i$, $\mu_i$ is the mean point for that cluster and $d^2(a, b)$ is the Procrustes distance as defined above.

## 3.3 Clustering Algorithm

### 3.3.1 Calculating Procrustes Distance

We first compute and store the Procrustes distances to use further in the algorithm. We apply similarity tranforms (scaling, rotation and translation) on second point-set to align it with the first point-set (we can make use of the implementation as done by Cootes et. al.) and then find the sum of squares of 2-norms for all corresponding pairs of points.

### 3.3.2 Initialisation

We have been told to reduce the sensitivity to outliers. Thus we choose K-means++ method of initialisation which reduces sensitivity to outliers. The way this is implemented in the way given below,

**Algorithm**

- Pick $x \in S$ (given set of pointsets) uniformly at random
- set T $\Longleftarrow \{x\}$
- while($|T| < K$)
- pick $x \in S$ at random with probability proportional to cost(x,T)=$\min_{z\in T} d^2(x, z)$
- $T \Longleftarrow T \cup \{x\}$

### 3.3.3 Main strategy

We shall use the standard K-means algorithm with a slight modification due to the Procrustes distance.

**Step 1**

In this step, given the representative points (or means) we must find the optimal classes to which the pointsets belong. This optimization is relatively straightforward as all we have to do is find the the Procrustes distance of a pointset with all the representative points and assign our pointset a class based on which representative point is the nearest (least distance). Implementation for Procrustes distance has been provided above and finding minimum is a trivial task.

**Step 2**

In this step, our problem is to find the best representative points give the classes to which these points belong to. So our problem boils down to find set of $u_i$ such that,

$$\sum_{i=1}^{K} \sum_{x_j \in S_i} \min_{\theta,T,s} \sum_{n=1}^{N} \|x_{jn} - s \times M_\theta \times u_{in} - T\|^2$$

Restricting to a particular class, we observe that this function is identical to the function that we minimised to calculate the mean shape during shape analysis! Thus to find the optimal representative for each cluster, all we have to do is find the mean shape of all the pointsets belonging to that cluster.

The algorithm for this as we have seen before is

- Given mean, find optimal transformations (Can be solved independently for each data shape)
- Given all transformations, find optimal mean pointset
    (a) Average all (aligned) pointsets similar to gradient descent
    (b) Take resulting pointset and rescale (divide) by the norm

#### 3.3.4 Stopping Criterion

We stop the updates when the relative decrease in the objective function falls below a pre-defined threshold value.