CS736 Assignment 1

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February 3, 2022

Contents

1 Denoising a Phantom Magnetic Resonance Image

1.1 RRMSE

The RRMSE between the noisy and noiseless images is 0.2986.

1.2 Optimal Values

1.2.1 Quadratic Prior

Parameter	Value
α	0.12
RRMSE(α)	0.2809
RRMSE(1.2α)	0.2811
RRMSE(0.8α)	0.2818

1.2.2 Huber Prior

Parameter	Value
α	0.99999
γ	0.03
RRMSE(α,γ)	0.2391
RRMSE(1.2α,γ)	-
RRMSE(0.8α,γ)	0.2435
RRMSE(α,1.2γ)	0.2396
RRMSE(α,0.8γ)	0.2393

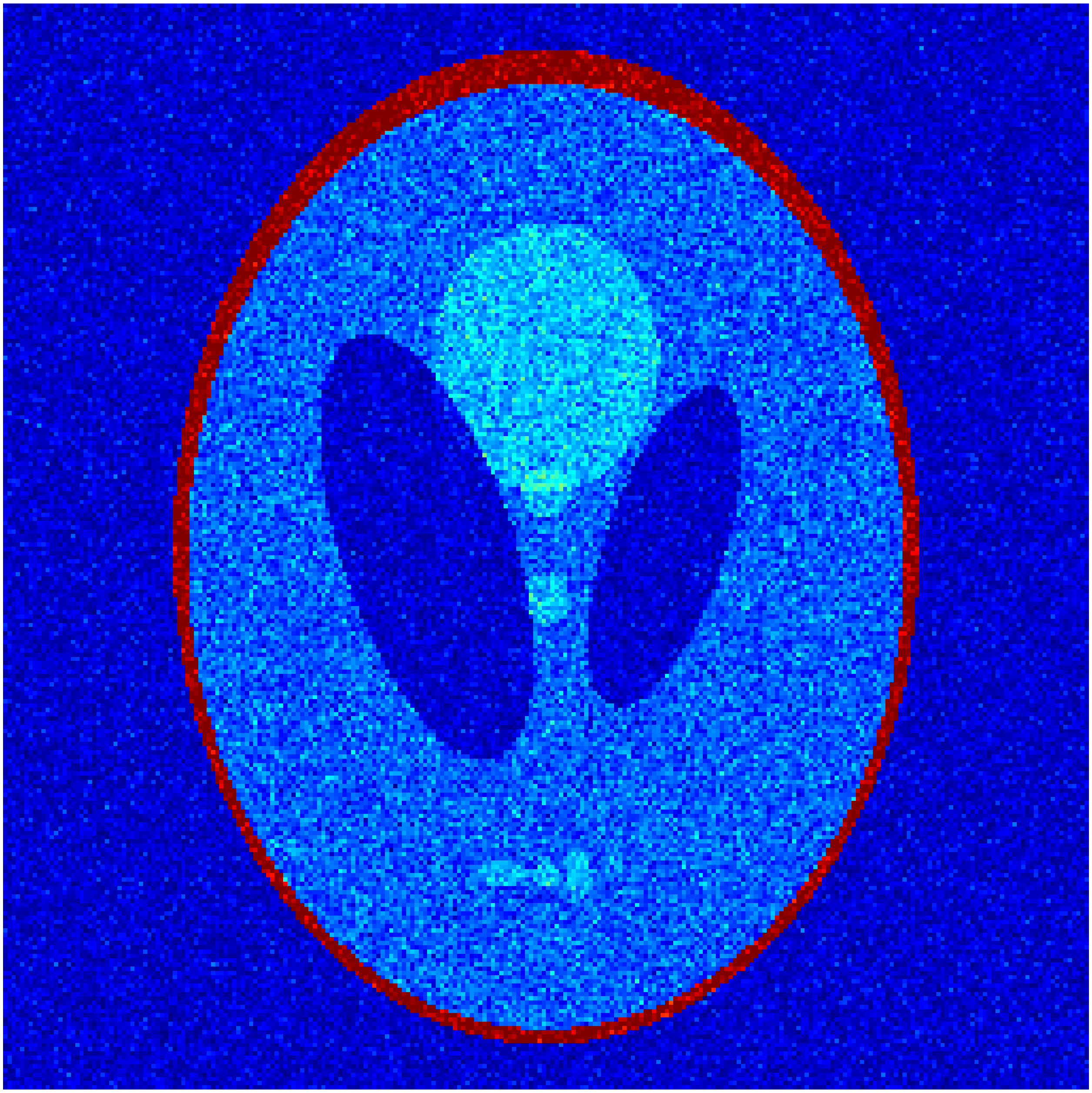
1.2.3 Discontinuity-Adaptive Prior

Parameter	Value
α	0.9999999
γ	0.008
RRMSE(α,γ)	0.2385
RRMSE(1.2α,γ)	-
RRMSE(0.8α,γ)	0.2588
RRMSE(α,1.2γ)	0.2389
RRMSE(α,0.8γ)	0.2388

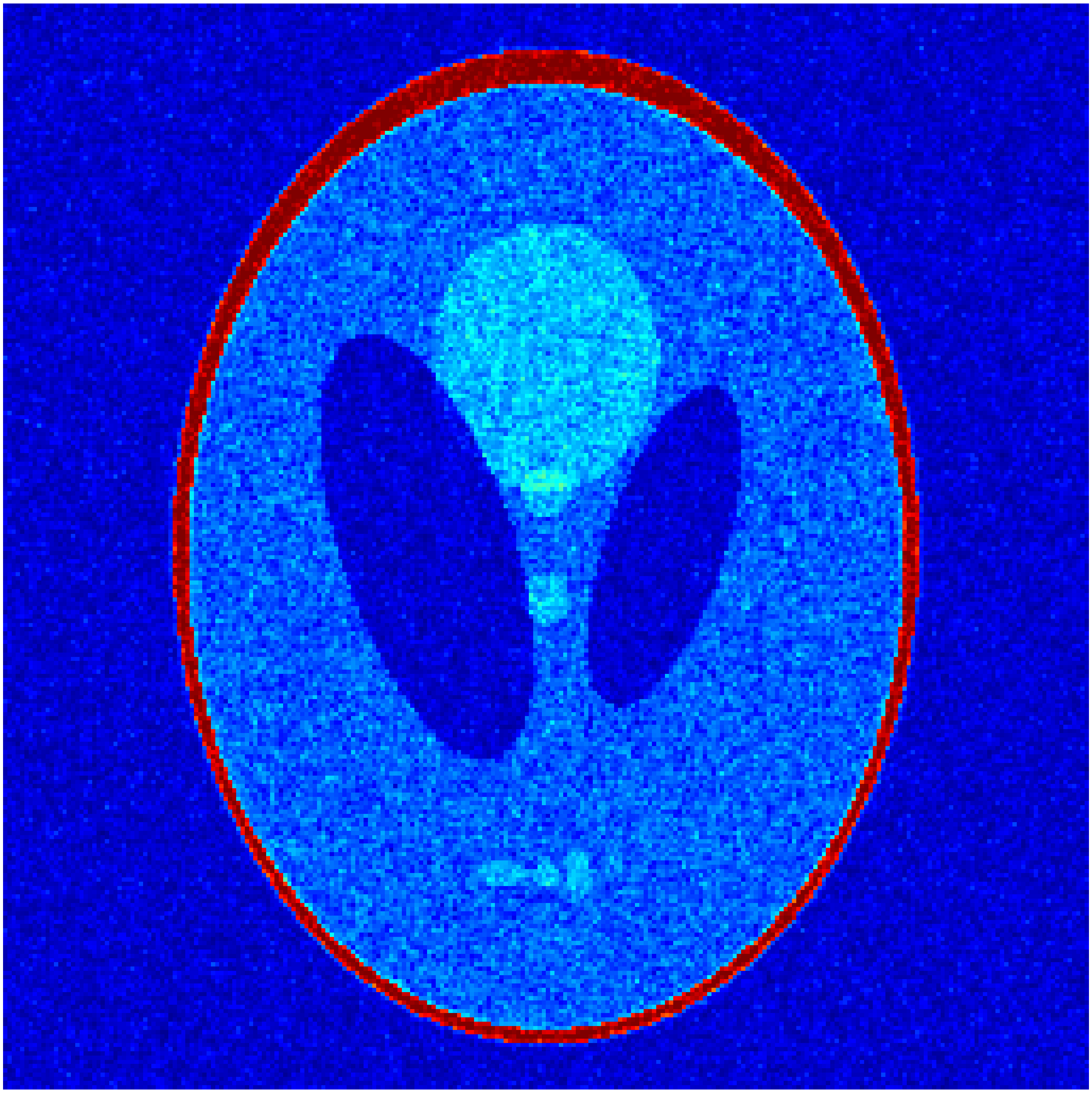
1.3 Images

1.3.1 Noiseless Image

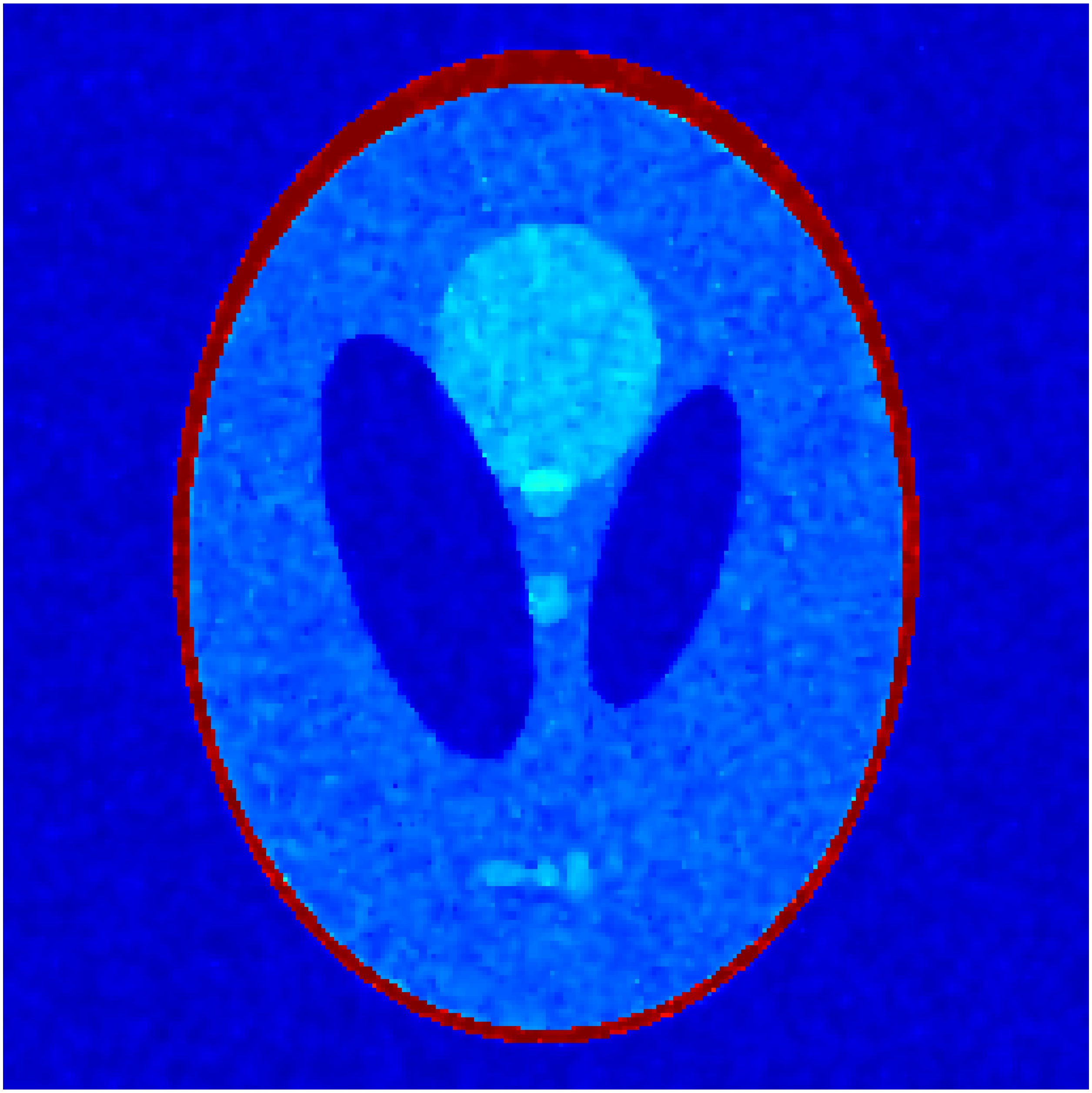
1.3.2 Noisy Image



1.3.3 Denoised (Quadratic Prior)



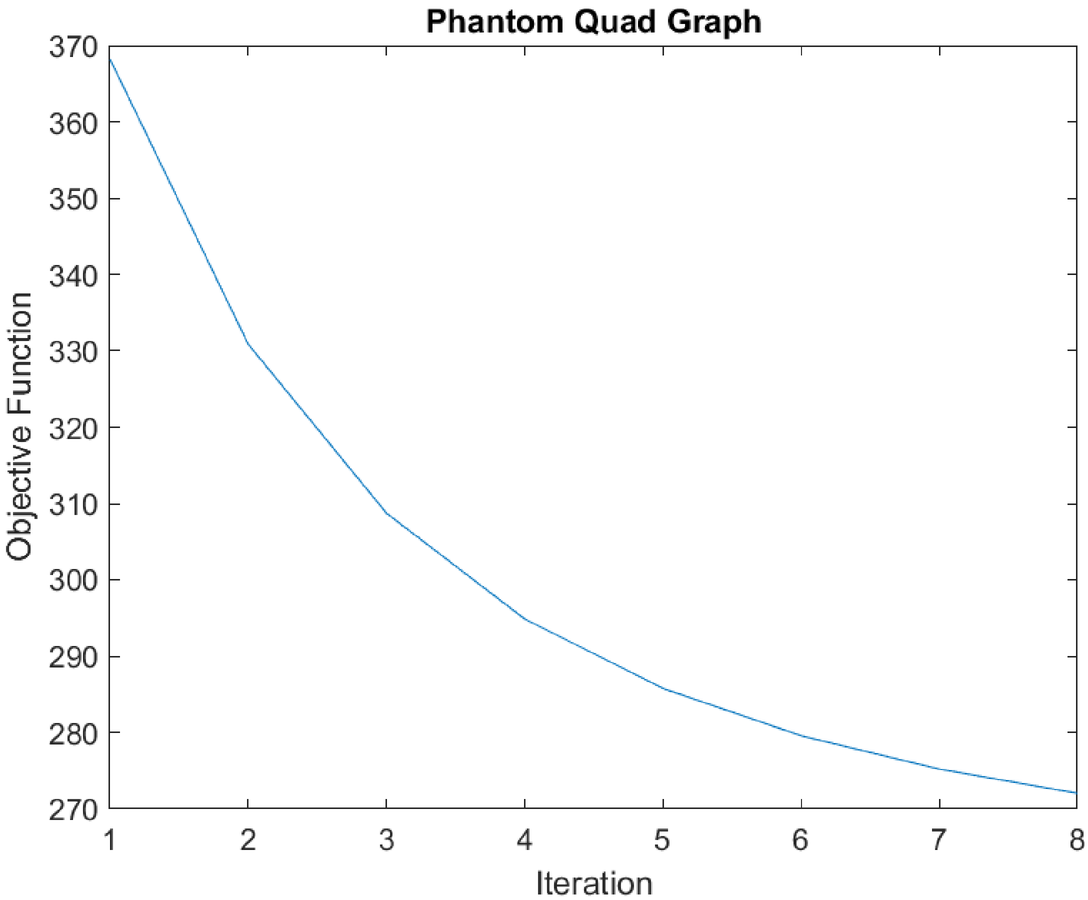
1.3.4 Denoised (Huber Prior)



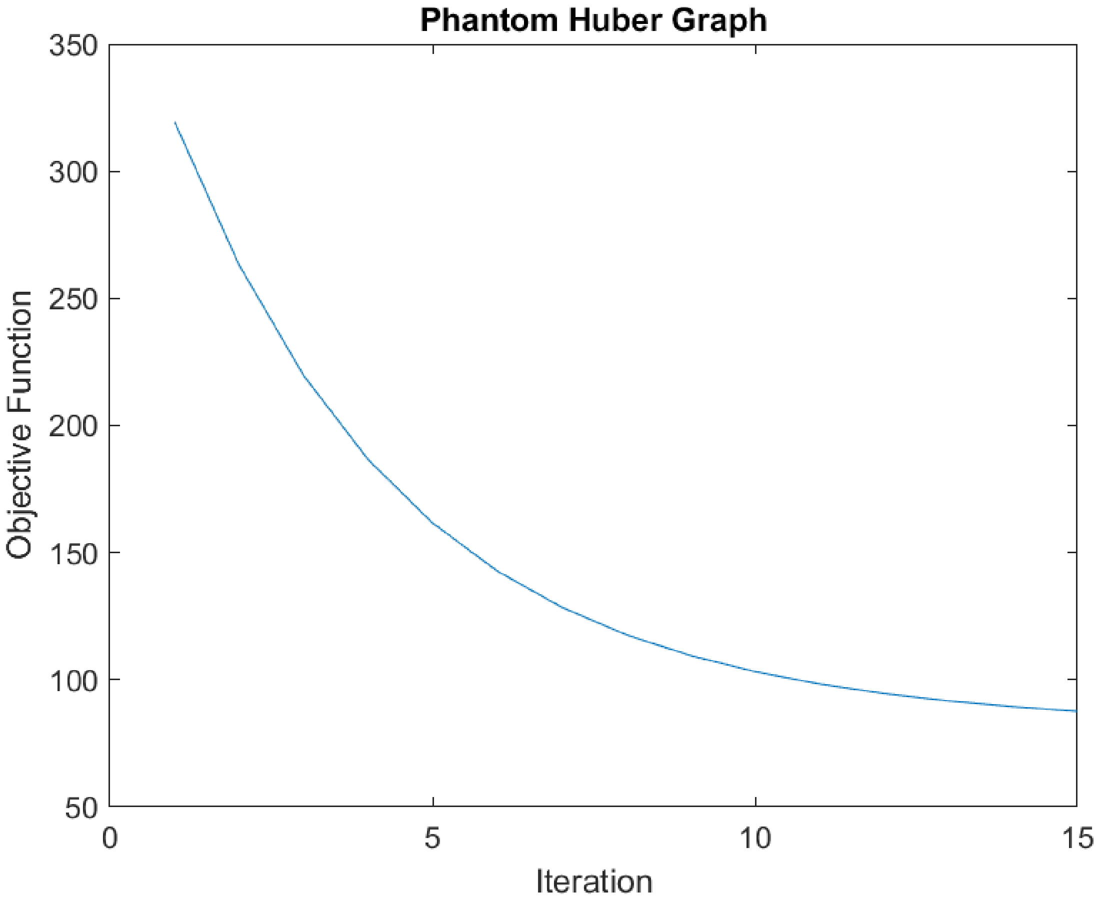
1.3.5 Denoised (Discontinuity-Adaptive Prior)

1.4 Plots

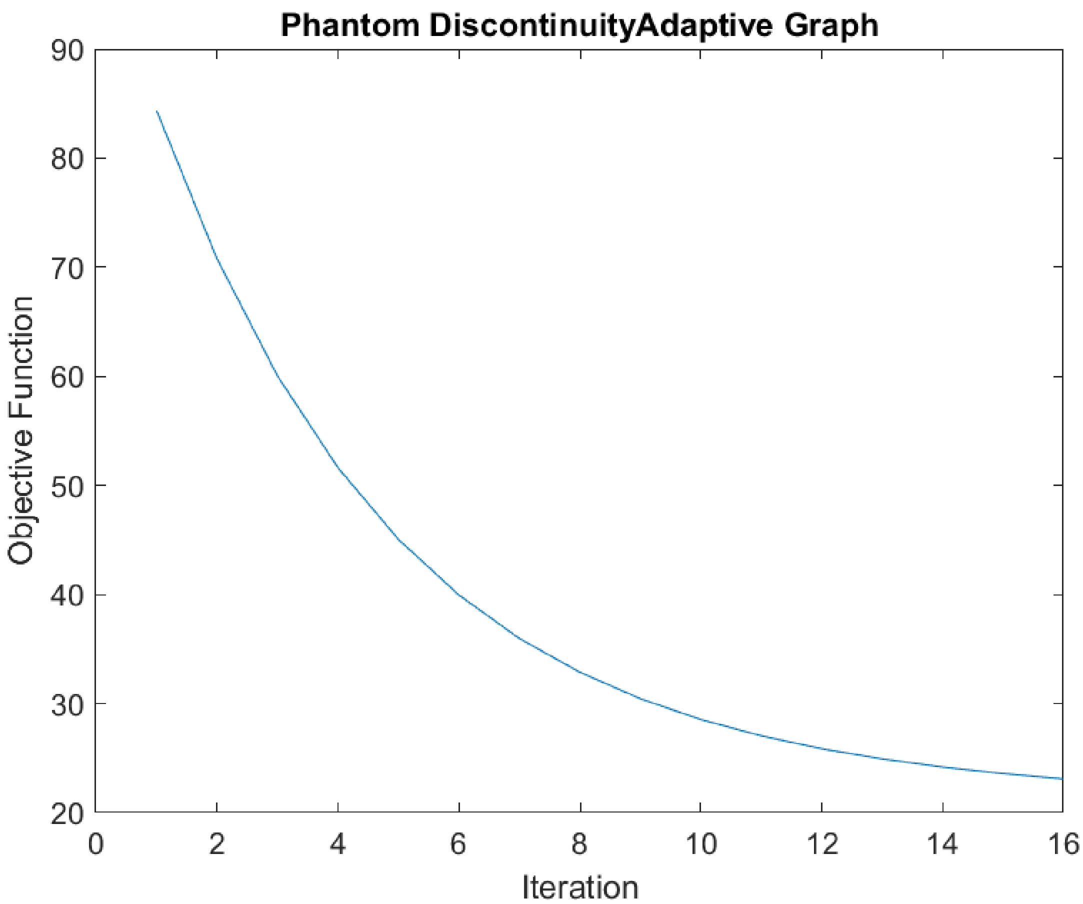
1.4.1 Quadratic Prior



1.4.2 Huber Prior



1.4.3 Discontinuity-Adaptive Prior



2 Denoising a Magnetic Resonance Image of the Brain

2.1 RRMSE

The RRMSE between the noisy and noiseless images is 0.1424.

2.2 Optimal Values

2.2.1 Quadratic Prior

Parameter	**Value**
α	0.15
RRMSE(α)	0.1217
RRMSE(1.2α)	0.1219
RRMSE(0.8α)	0.1228

2.2.2 Huber Prior

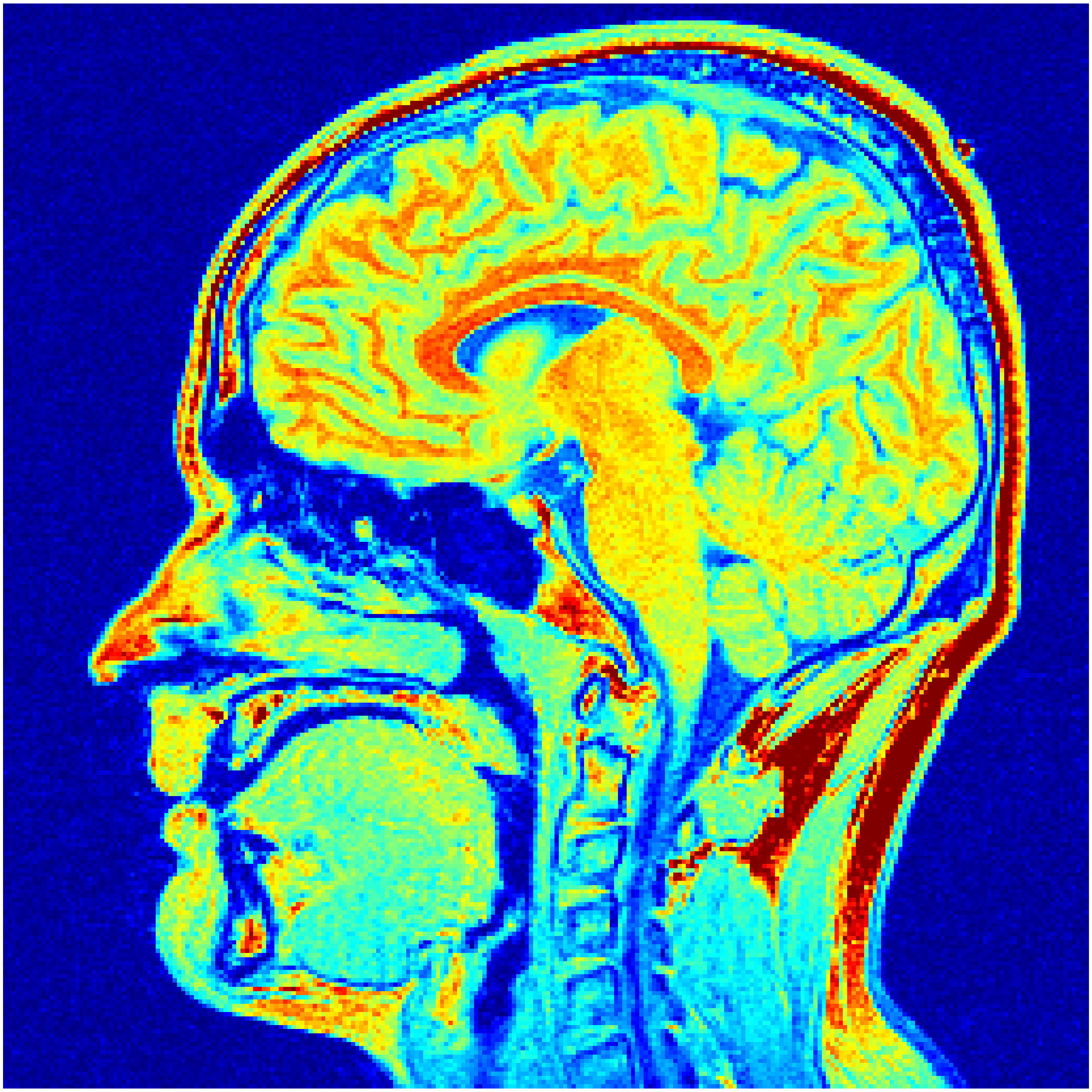
Parameter	**Value**
α	0.936
γ	0.016
RRMSE(α,γ)	0.1134
RRMSE(1.2α,γ)	-
RRMSE(0.8α,γ)	0.1208
RRMSE(α,1.2γ)	0.1134
RRMSE(α,0.8γ)	0.1152

2.2.3 Discontinuity-Adaptive Prior

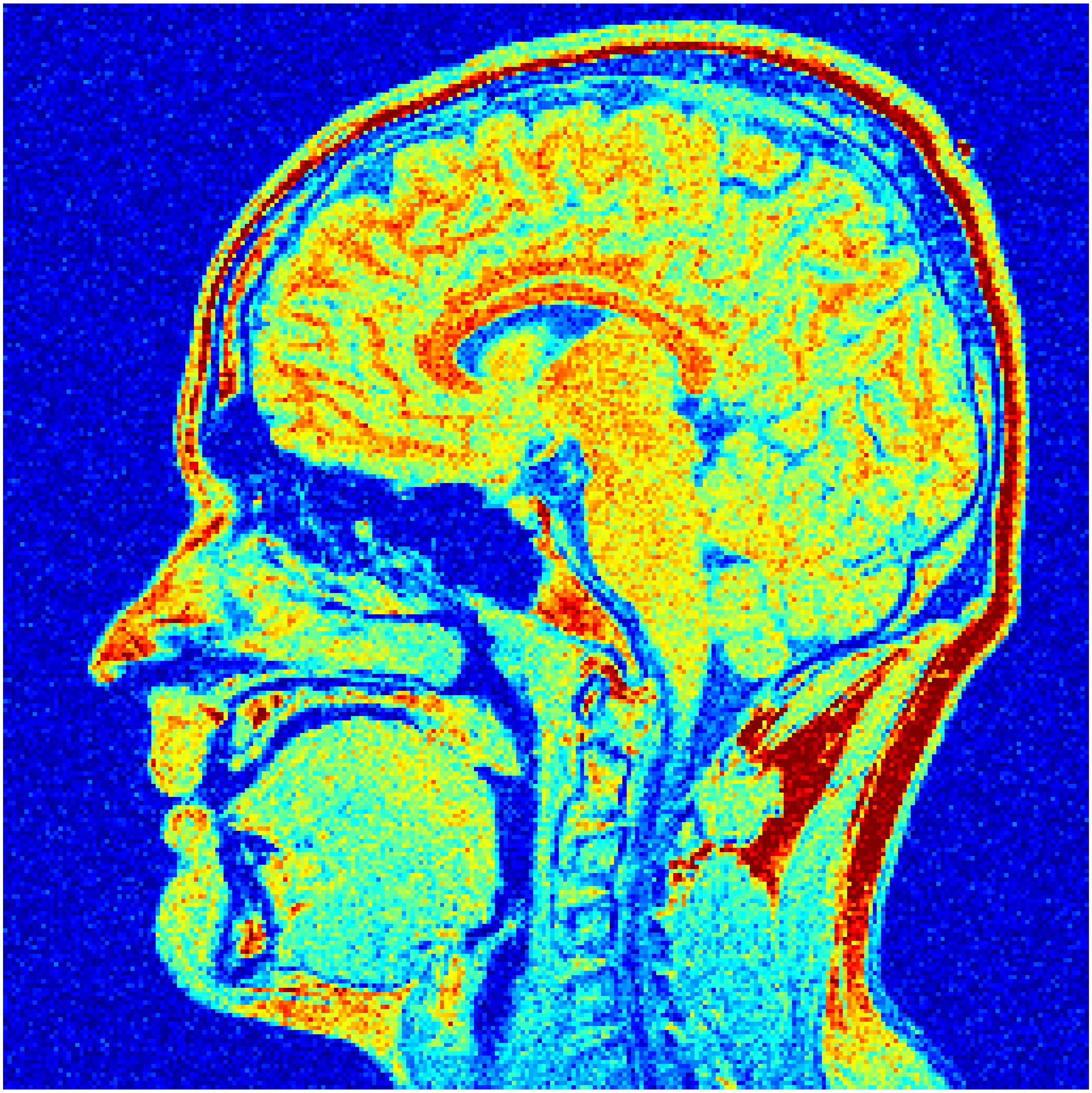
Parameter	**Value**
α	0.7
γ	0.05
RRMSE(α,γ)	0.1144
RRMSE(1.2α,γ)	0.1164
RRMSE(0.8α,γ)	0.1186
RRMSE(α,1.2γ)	0.1148
RRMSE(α,0.8γ)	0.1154

2.3 Images

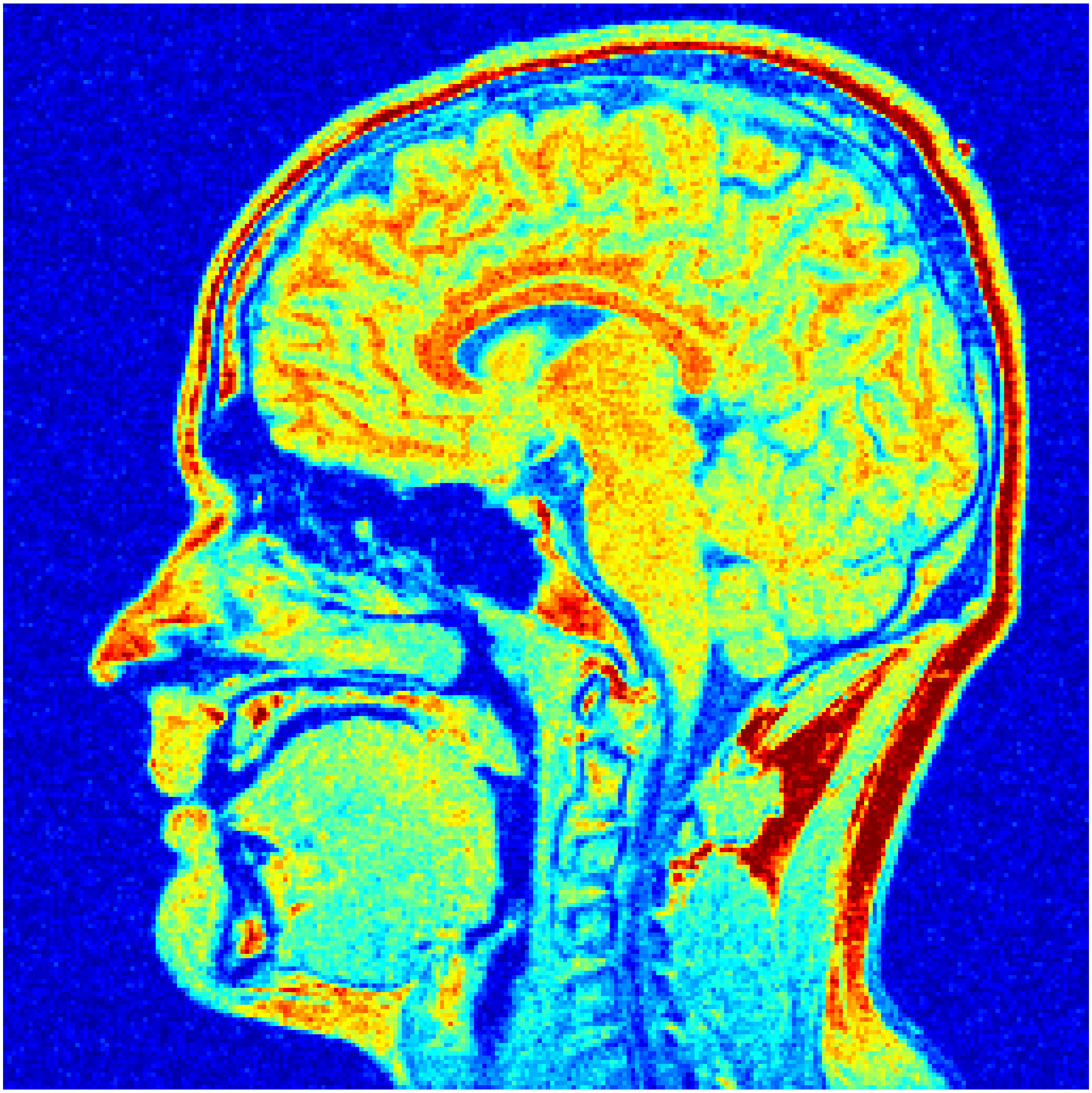
2.3.1 Noiseless Image



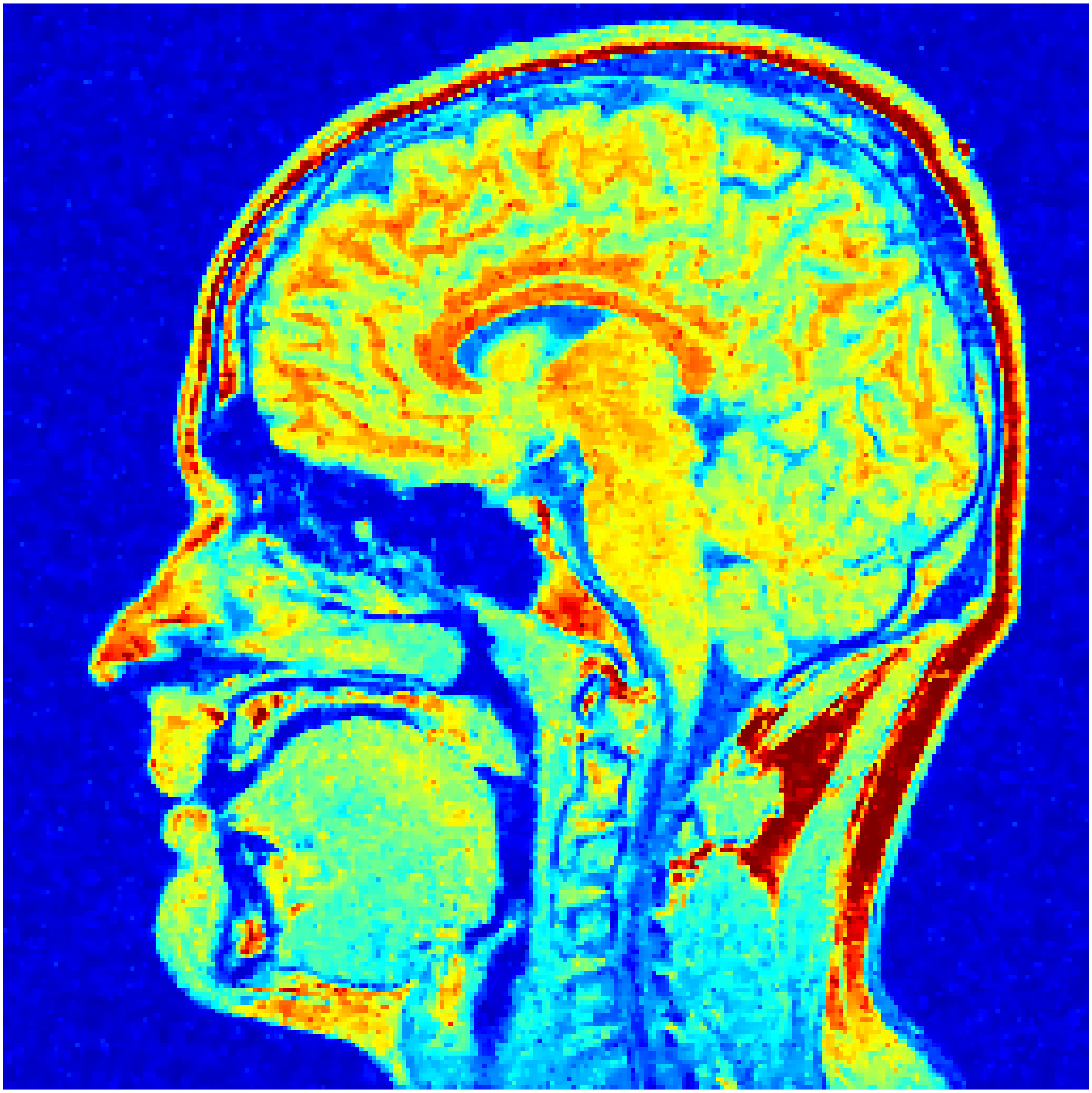
2.3.2 Noisy Image



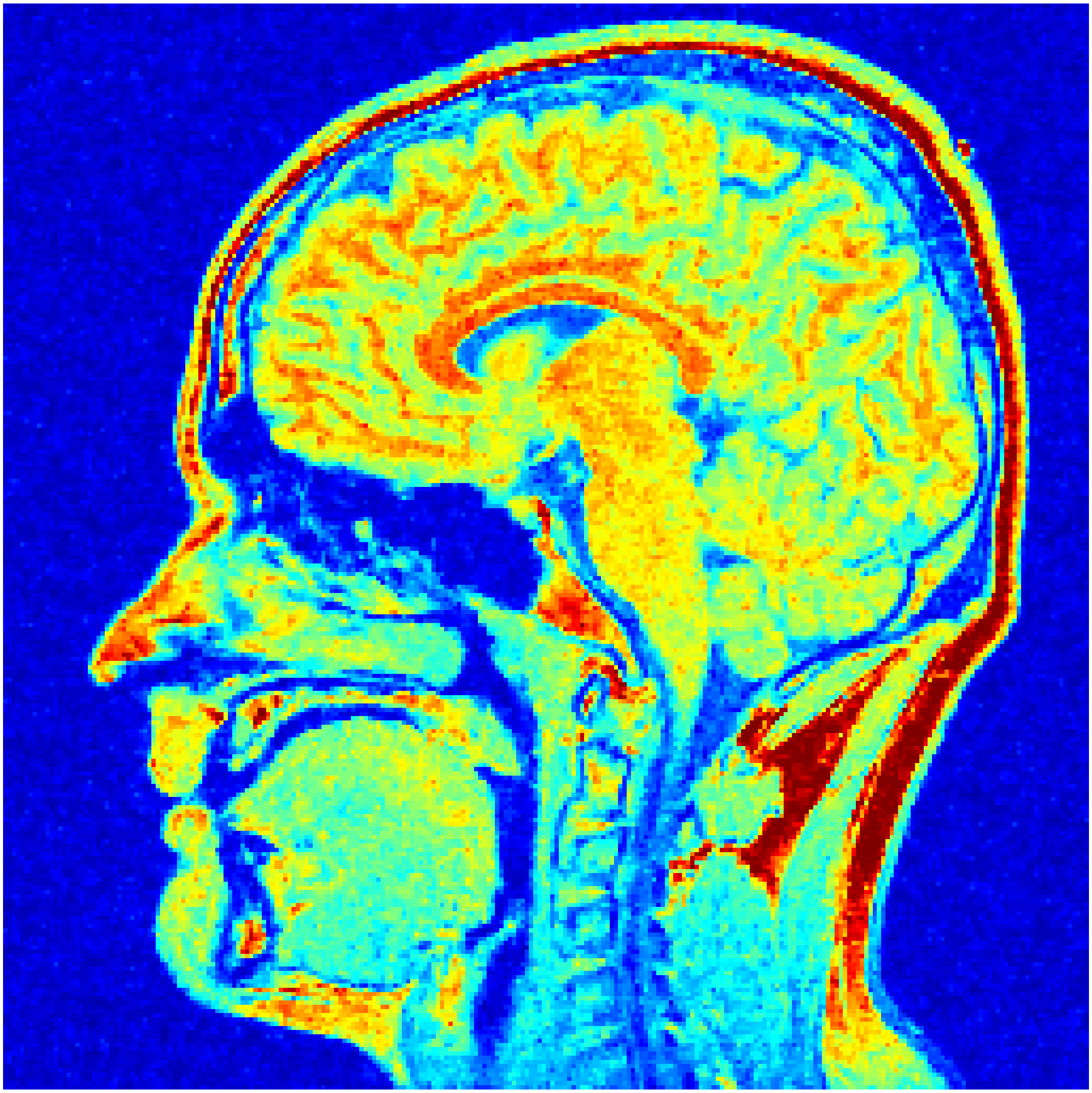
2.3.3 Denoised (Quadratic Prior)



2.3.4 Denoised (Huber Prior)

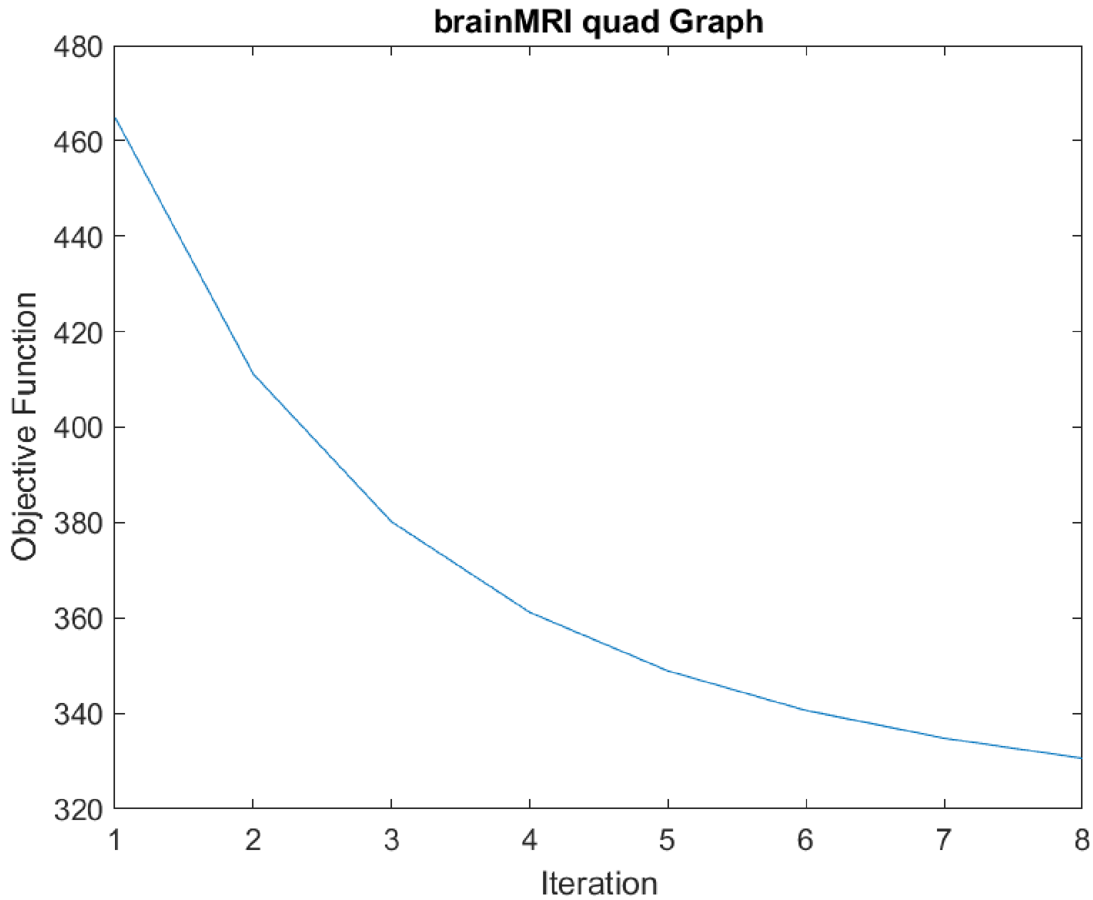


2.3.5 Denoised (Discontinuity-Adaptive Prior)

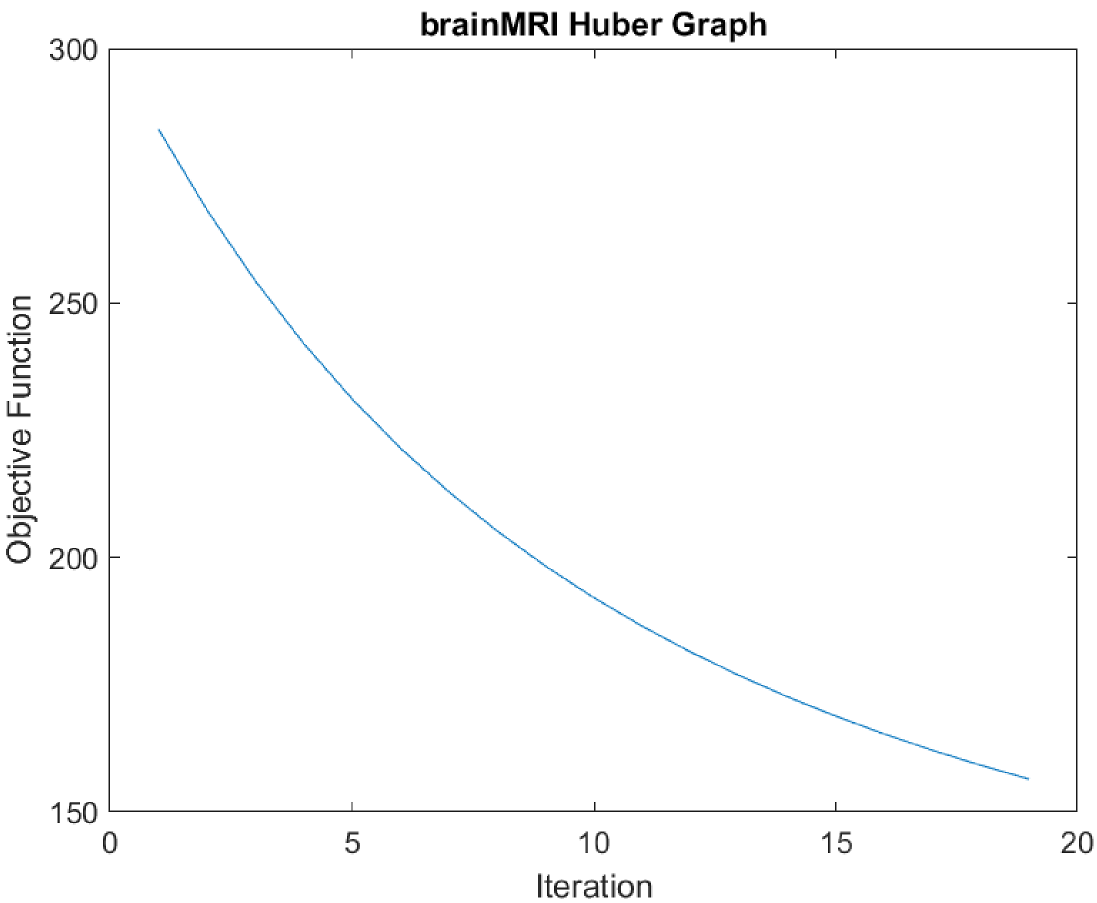


2.4 Plots

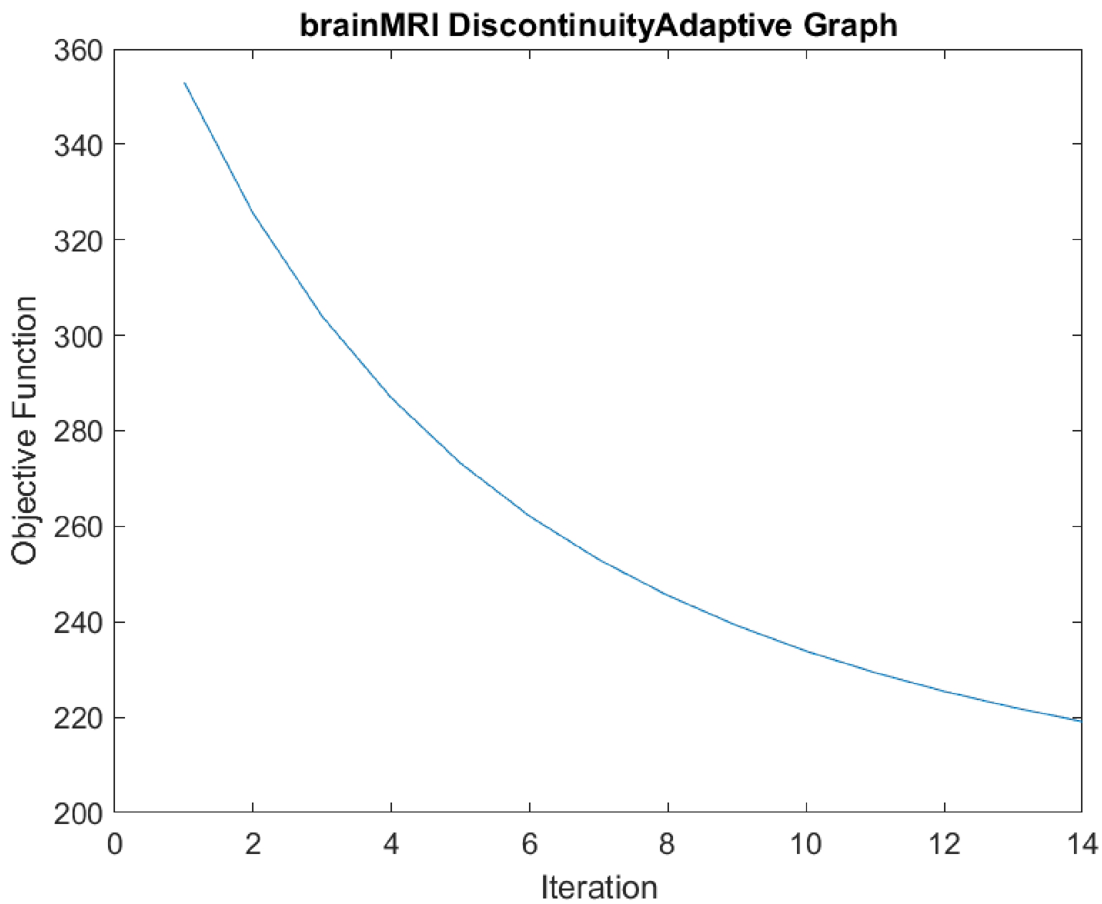
2.4.1 Quadratic Prior



2.4.2 Huber Prior



2.4.3 Discontinuity-Adaptive Prior



3 Bayesian Denoising Formulation for RGB Microscopy

3.1 MRF Prior Model

To take the statistical dependencies within a spatial neighborhood into consideration, we use 3×3 cliques to model the neighborhood intensities. Although, for a colored image, we need to consider the 3 color channels (red, green and blue) as well. We treat each pixel as a 3×1 column vector having values corresponding to red, green and blue channels. Using the $2-$norm of the difference between 2 neighboring pixels, we define the relative potential function for a point x_i with respect to an arbitrary point c in the 3×3 clique C_i as,

$$V_c(x_i) = g\left(u = \sqrt{(c_R - x_{iR})^2 + (c_G - x_{iG})^2 + (c_B - x_{iB})^2}; \gamma\right)$$

where $g(u;\gamma)$ is the Huber function with a suitably chosen parameter γ. This potential function will assign low values to good combinations, as it favours smoothness. Since it is heavy-tailed, it will prevent boundaries from getting low probabilities as well.

The global energy is minimized by a maximally smooth image, and corresponds to the sum of all maximal cliques energies, which reduces to sum over neighborhood cliques for an MRF. The most general form for the MRF prior is then given by,

$$P(x_i) = \frac{1}{Z(\beta)} \exp\left(-\beta \frac{1}{T} U(x_i)\right)$$

where the energy function and normalization constant are given as,

$$U(x_i) = \sum_{c \in C_i} V_c(x_i) \qquad\qquad Z(\beta) = \sum_{x_i} \exp\left(-\beta \frac{1}{T} U(x_i)\right)$$

and $\beta \in [0, 1]$ is the denoising weight parameter.

3.2 Noise Model

Measurements in light microscopy are generally done by accumulation of photons over a detector. We can also assume that noise will be random, independent at each pixel and follows a particular distribution. Based on this, and given the average number of captured photons $\lambda \in \mathbb{R}$ during a certain duration of imaging, the preferred noise model would be a Poisson distribution with parameter λ, since 'Photon noise' is generally the more important factor, as compared to 'Read noise'.

The probability of k captured photons (as part of the noise) is given by,

$$P(X = k; \lambda) = \frac{\lambda^k \exp(-\lambda)}{k!}$$

3.3 Bayesian Denoising Formulation

Given prior data x and noisy data y, we will maximize the posterior distribution based on the prior and the noise model described. The condition is equivalent to minimizing the negative logarithm, and thus, the condition is,

$$\max_{x_i} P(x|y,\theta) = \min_{x_i} \left((1-\beta)\left(\lambda - (y_i - x_i)\log\lambda + \log((y_i - x_i)!)\right) + \beta \frac{1}{T} \sum_{a \in C_i} V_a(x_i) \right)$$