Problem Set 7c

Released: October 26, 2021

1. Show that a tree has at most one perfect matching.

   Hint: Use induction.

2. Show that a tree is a bipartite graph.

   Hint: Consider the distance of each node from a fixed node. The two parts correspond to even and odd distances. Where do you use the fact that the graph is a tree?

3. A *spanning tree* of a graph G is a subgraph G' which has all the nodes in G and is a tree. Show that every connected graph G has a spanning tree.

   Hint: Induct on the number of cycles in G. Use the previous problem.

4. Prove that for a graph with n vertices, any two of the following imply the third:

   (a) G is connected.
   (b) G is acyclic.
   (c) G has $n - 1$ edges.

5. What is the maximum size of $|S|$ such that there is a poset $(S, \preceq)$ of height h and width w? Construct such a poset.

6. Use Dilworth's theorem to show that any set of 5 natural numbers either contains numbers of the form x, xy and xyz, or contains 3 numbers which are mutually indivisible by each other?