~~Inverted~~

DSP Tut 3

Q2 (i) (a) $y[n] = \dfrac{x[n] + x[n-1]}{2}$

$x[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n]$

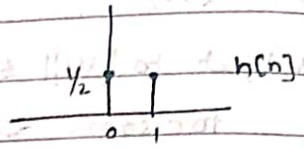
$\cancel{Y(e^{j\omega}) = \frac{1}{2} X(e^{j\omega})}$

$y[n] = h[n] * x[n]$

$= \sum_{-\infty}^{+\infty} x[k]\, h[n-k]$

$\rightarrow$ $x[n] = \delta[n]$

$y[n] = \dfrac{\delta[n] + \delta[n-1]}{2} \Big\} = h[n]$



(b) $h[n] = \dfrac{\delta[n] - \delta[n-1]}{2}$

(ii) (a) $H_a(e^{j\omega}) = \sum_{-\infty}^{+\infty} h[n] e^{-j\omega n} = \frac{1}{2} \sum \delta[n] + \delta[n-1]\, e^{-j\omega n}$

$H_a(e^{j\omega}) = \frac{1}{2}\left[1 + e^{-j\omega}\right]$

(b) $H_b(e^{j\omega}) = \frac{1}{2}\left(1 - e^{-j\omega}\right)$

(iii) $x[n] = \cos \omega_0 n \qquad \omega_0 \in [0, \pi]$

$y_a[n] = \dfrac{\cos \omega_0 n + \cos(\omega_0 n - \omega_0)}{2} = \cancel{\frac{1}{2}} \cos\left(\omega_0 n - \frac{\omega_0}{2}\right) \cos\left(\frac{\omega_0}{2}\right)$

$y_b[n] = \sin\left(\omega_0 n - \frac{\omega_0}{2}\right) \sin\left(\frac{-\omega_0}{2}\right)$

(iv) $y_a[n] = \cos\left(\frac{\omega_0}{2}\right) \cos\left(\omega_0\left(n - \frac{1}{2}\right)\right)$

$H_a(e^{j\omega}) = \frac{1}{2}(1 + e^{-j\omega})$

$\cos(\omega_0 n) \longrightarrow \boxed{\delta} \longrightarrow y_a[n]$

$h_a[n]$

$= \frac{1}{2}(1 + \cos\omega - j \sin\omega)$

$= \cos^2\left(\frac{\omega}{2}\right) - \frac{j \sin\omega}{2}$

$\dfrac{e^{j\omega_0 n} + e^{-j\omega_0 n}}{2} \xrightarrow[\text{eigenseq}]{\delta} \frac{1}{2}\left[e^{j\omega_0 n} H(e^{j\omega_0}) + e^{-j\omega_0 n} H(e^{-j\omega_0})\right]$

$\frac{1}{2}\left[\frac{1}{2}(1 + e^{-j\omega_0}) e^{j\omega_0 n} + e^{-j\omega_0 n}\frac{(1 + e^{j\omega_0})}{2}\right]$

$= \frac{1}{2}\left[\dfrac{e^{j\omega_0 n} + e^{-j\omega_0 n}}{2} + \dfrac{e^{j\omega_0(n-1)} + e^{-j\omega_0(n-1)}}{2}\right]$

$= \frac{1}{2}\left[\cos(\omega_0 n) + \cos(\omega_0(n-1))\right]$

$= y_a[n]$ HP

(v) $H_a(e^{j\omega}) = \frac{1}{2}(1+e^{-j\omega})$ $\qquad$ $H_b = \frac{1}{2}(1-z^{-1})$

$H(z) = \frac{1}{2}(1+z^{-1})$ $\qquad$ $h_b[n] = \frac{1}{2}(\delta[n]-\delta[n-1])$

$= \frac{1}{2} + \frac{z^{-1}}{2}$

$h_a[n] = \frac{\delta[n]}{2} + \frac{1}{2}\delta[n-1]$

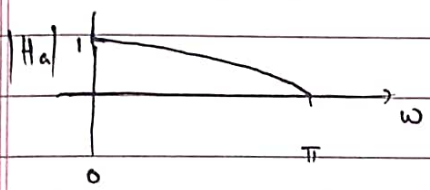
(vi) $H_a(e^{j\omega}) = \frac{1}{2}(1+e^{-j\omega})$ $\qquad$ $H(\omega=0) = 1$ $\qquad$ $H(\omega=\pi) = 0$

$= \frac{1}{2}(1+\cos\omega - j\sin\omega)$

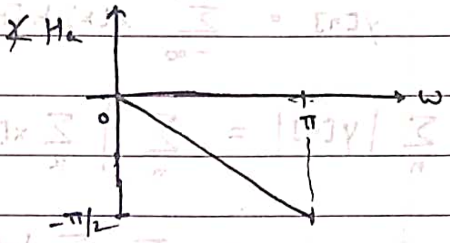
$|H_a| = \frac{1}{2}\sqrt{(1+\cos\omega)^2+\sin^2\omega}$ $\qquad$ $\measuredangle H_a = \tan^{-1}\left(\frac{-\sin\omega}{1+\cos\omega}\right) = \tan^{-1} - \frac{\sin(\omega/2)}{\cos^2 \omega/2}$

$= \frac{1}{2}\sqrt{1+1+2\cos\omega}$ $\qquad$ $= \tan^{-1}[-\tan(\omega/2)]$

$= \sqrt{\frac{1+\cos\omega}{2}} = \cos\left(\frac{\omega}{2}\right)$ $\qquad$ $= -\omega/2$



Low Pass Filter



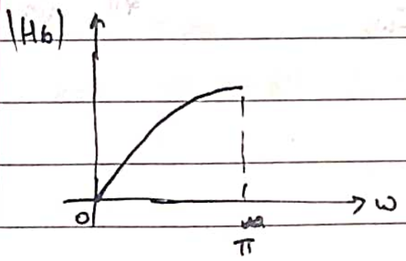
(b) $H_b(e^{j\omega}) = \frac{1}{2}(1-e^{-j\omega})$ $\qquad$ $\measuredangle H_b = \tan^{-1}\frac{\sin\omega}{1-\cos\omega}$

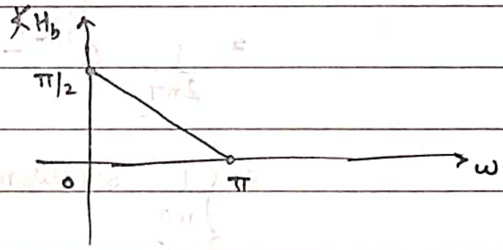
$|H_b| = \frac{1}{2}|1-\cos\omega + j\sin\omega|$ $\qquad$ $= \tan^{-1}\frac{\cancel{\sin(\omega/2)}\cos(\omega/2)}{\sin(\omega/2)}$

$= \frac{1}{2}\sqrt{2-2\cos\omega}$ $\qquad$ $= \tan^{-1}\cot(\omega/2)$

$= \sqrt{\frac{1-\cos\omega}{2}} = \sin(\omega/2)$ $\qquad$ $= \cancel{\cot} \; \pi/2 - \omega/2$

$= \frac{\pi-\omega}{2}$





High Pass

Q1 (a) $\sum_n x[n]$, $\sum_n h[n] < \infty$

$\hookrightarrow \Sigma_x \quad \hookrightarrow \Sigma_h$

$x[n] \xrightarrow{h[n]} y[n]$

$$\sum_{n=-\infty}^{+\infty} y[n] = \sum_{n=-\infty}^{+\infty} \sum_{k=-\infty}^{+\infty} x[k]\, h[n-k]$$

$$= \sum_n \sum_k x[k]\, h[n-k]$$

Eliminate n :
$n-k = m$
$n = m+k$

$$= \sum_m \sum_k x[k]\, h[m]$$

$$= \left(\sum_k x[k]\right)\left(\sum_m h[m]\right)$$

(b) $\sum_q |x[n]| = X_0 \qquad \sum_n |h[n]| = H_0$

$$y[n] = \sum_{-\infty}^{+\infty} x[k]\, h[n-k]$$

$$\sum_n |y[n]| = \sum_n \left| \sum_k x[k]\, h[n-k] \right|$$

$$\leq \sum_n \sum_k |x[k]|\cdot|h[n-k]|$$

$\to m$

$$= \sum_m \sum_k |x[k]|\,|h[m]| = X_0 H_0$$

Q3 (a)

$$H(e^{j\omega}) = \begin{cases} 1 & 0 \leq \omega \leq \omega_c \\ 0 & \omega_c < \omega \leq \pi \end{cases}$$

($\because h[n]$ real) assumed

$$h[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H(e^{j\omega})\, e^{j\omega n}\, d\omega = \frac{1}{2\pi}\int_{-\omega_c}^{+\omega_c} e^{j\omega n}\, d\omega = \frac{1}{2\pi} \left.\frac{e^{j\omega n}}{jn}\right|_{-\omega_c}^{+\omega_c}$$

$$= \frac{1}{2n\pi j}\, e^{j\omega_c n} - e^{-j\omega_c n}$$

$$= \begin{cases} \frac{1}{n\pi}\sin(\omega_c n) & n \neq 0 \\ \omega_c/\pi & n = 0 \end{cases}$$

(b) Highpass

$$h[n] \xrightarrow{DTFT} H(e^{jw})$$

$\cancel{H(e^{jw})} =$ for $0, \pi$ $\longrightarrow$ $\frac{1}{2\pi}\int_{-\pi}^{\pi} 1\, e^{jwn}\, dw = \frac{1}{2\pi} \frac{e^{jwn}}{jn}\Big|_{-\pi}^{\pi} = \frac{((-1)^n - (-1)^n)}{2\pi n j}$

$= 0$

HP + LP = Total

$$HP = \begin{cases} -\frac{1}{n\pi}\sin(\omega_c n) & n \neq 0 \\ \omega_c/\pi & n = 0 \end{cases}$$

(c) $h_{BP}[n] + h_{LP}[n] = h'_{LP}[n]$

$$(\quad) + \begin{cases} \frac{1}{n\pi}\sin(\omega_{c_1} n) \\ \omega_{c_1}/n \end{cases} = \begin{cases} \frac{1}{n\pi}\sin(\omega_{c_2} n) \\ \omega_{c_2}/n \end{cases}$$

$$h_{BP}[n] = \begin{cases} \frac{1}{n\pi}(\sin(\omega_{c_2} n) - \sin(\omega_{c_1} n)) & n \neq 0 \\ \frac{\omega_{c_2} - \omega_{c_1}}{n} & n = 0 \end{cases}$$

(d) $h_{BS}[n] = 1 - h_{BP}[n]$

$= 0 - h_{BP}[n]$