Tut 2 DSP

Q2 ~~(a)~~ $x[n] = \alpha^n u[n]$ $\qquad |\alpha|, |\beta| < 1$

$h[n] = \beta^n u[n]$ $\qquad y[n] = ?$

$$y[n] = x[n] * h[n]$$

$$= \sum_{-\infty}^{\infty} \alpha^k u[k]\, \beta^{n-k} u[n-k] = \sum_{0}^{n} \alpha^k \beta^{n-k}$$

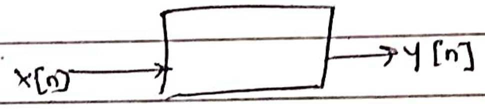
$$= \beta^n \sum_{0}^{n} (\alpha/\beta)^k$$

$$= \beta^n (1) \frac{(\alpha/\beta)^{n+1} - 1}{\alpha/\beta - 1}$$

$$= \cancel{\beta} \frac{\frac{\alpha^{n+1} - \beta^{n+1}}{\cancel{\beta}}}{\frac{\alpha - \beta}{\cancel{\beta}}} = \frac{\alpha^{n+1} - \beta^{n+1}}{\alpha - \beta}$$

Q3 $x[n] = 0 \ \forall n$

PT $y[n] = 0 \ \forall n$



(i) Additivity

$x[n] = 0 \longrightarrow y[n]$

$x[n] = 0 \longrightarrow y[n]$ $\qquad 2y[n] = y[n] \ \forall n$

$x[n] + x[n] \longrightarrow 2y[n]$ $\qquad y[n] = 0 \ \forall n$

(ii) Homo :

$x[n] \longrightarrow y[n]$

$c\,x[n] \longrightarrow c\,y[n]$ $\quad$ for $c \in \mathbb{R}$ & ~~c≠0~~ $c \neq 1$

$= x[n]$

$$c\,y[n] = y[n]$$

$$(c-1)\,y[n] = 0 \quad \forall n$$

$$y[n] = 0 \quad \forall n.$$

Q4 $x[n] \rightarrow$ Periodic

$x[n+N_0] = x[n]$

$x[n] \xrightarrow{LSI} y[n]$

$x[n+N_0] \longrightarrow y[n+N_0]$ $\quad (\because SI)$

$= x[n]$

$\therefore \quad y[n] = y[n+N_0]$

DATE

Q5 $x[n] \longrightarrow$ nonzero $\longrightarrow N_0 \le n \le N_1$

$h[n] \longrightarrow$ " $\longrightarrow N_2 \le n \le N_3$

$\left(\begin{array}{l} N_2 \le n-k \le N_3 \\ N_2 - n \le -k \le N_3 - n \end{array} \right)$

$y[n] = \sum_{k=-\infty}^{+\infty} x[k]\, h[n-k] = \sum_{N_0}^{N_1} x[k]\, h[n-k]$

$n - N_2 \ge k \ge n - N_3$

$\underbrace{n - N_2 \ge N_0}\ \underbrace{\ge n - N_3}$

$n \ge N_0 + N_2$ — ① $n \le N_0 + N_3$ — ②

N_1

$n \ge N_1 + N_2$ — ⑤

$n \le N_1 + N_3$ — ④

(Taking lineant condit^n)

① & ③ $n \ge N_0 + N_2$ (lineant)

② & ④ $n \le N_1 + N_3$ (—"—)

f/p

Eg. $x[n] \longrightarrow -2 \le n \le 3$

$h[n] \longrightarrow -8 \le n \le -1$

$\sum_{-2}^{3} x[k]\, h[n-k] = ~~\sum_{-2}^{-1} x[k] h[n-k]~~$

$\sum_{-2}^{3} x[k]\, h[n-k] =$ ~~x[0] h~~ $x[-2]\, h[n+2] + \ldots\ x[3]\, h[n-3]$

$-10 \le n \le -3$ $-5 \le n \le 2$ } for non zero

$-9 \le n \le -2$

→ At least one shd be non-zero

$\therefore -10 \le n \le 2$

$(-2)+(-8)$ $(3)+(-1)$

Q2 $x[n] = 1 \quad 2 \quad \underset{0}{\uparrow}{-1} \quad 4 \quad 3 \quad 5$ $h[n] = \underset{2}{\uparrow}1 \quad -1 \quad 2 \quad -2 \quad 3$

$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$

$h[-k] = 3 \quad -2 \quad \underset{0}{\uparrow}2 \quad -1 \quad \underset{2}{\uparrow}1$

$h[-(k-n)] = 3 \quad -2 \quad \underset{n}{\uparrow}2 \quad -1 \quad 1$

$h[n-k]$

$\overset{-2}{\downarrow}$ $1 \quad 2 \quad \underset{0}{\uparrow}{-1} \quad 4 \quad 3 \quad \overset{3}{\downarrow}5$

$3 \quad -2 \quad \underset{n}{\uparrow}2 \quad -1 \quad 1$

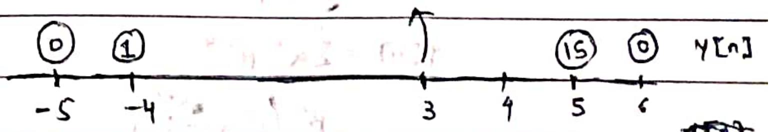
$3 \quad -2 \quad \overset{n}{\downarrow}2 \quad -1 \quad 1$

$y[5] = 15$ $y[n] = 0$ if $n > 5$

— ① — ②

$y[-4] = 1$ $y[n] = 0$ if $n < -4$

$(4)(3) + (3)(-2) + (5)(2)$



$(3)(3) + (5)(-2)$

y[n] ✓

deposited

Q6 $y[n] = x[n] + \frac{p_1}{100}x[n-1] + \frac{p_2}{100}x[n-2]$ ~~α~~

Curr month final balance

$$Y[n] = x[n] + \frac{p_1}{100}y[n-1] + \frac{p_2}{100}y[n-2]$$

SI?

$$\underset{\text{at } m}{x[n-n_0]} \longrightarrow Y_B = x[n-n_0] + \frac{p_1}{100}y[n-n_0-1] + \frac{p_2}{100}y[n-n_0-2]$$

$$= Y[n-n_0]$$

LSI if p_1, p_2 indep. of n

Q8
$$x[n] = \begin{cases} 0 & n<0 \\ \alpha^n & 0 \le n \le N_1 \quad (|\alpha|<1) \\ 0 & N_1 < n \le N_2 \\ \alpha^{n-N_2} & N_2 \le n \le N_1+N_2 \\ 0 & n > N_1+N_2 \end{cases}$$

$h[n] = \beta^n u[n]$ $|\beta|<1$

~~$Y[n] = \sum_{-\infty}^{+\infty} x[k]h[n-k]$~~

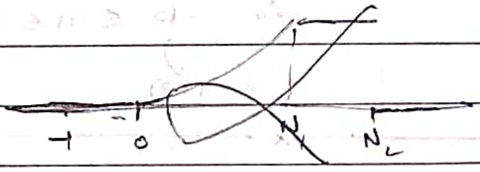
~~$y[n] = \sum_{0}^{\infty} x[k]\beta^{n-k}$~~

① $n<0$: ~~$Y[n] = 0 * h[n] = 0$~~

② ~~$0 \le n \le N_1$:~~ ~~$Y[n] = \sum_{-\infty}^{+\infty} x[k]h[n-k]$~~

~~① $n \le 0$: $Y[n] = \beta^n \sum_{k=0}^{\infty} \frac{x[k]}{\beta^k}$~~

~~$Y[n] =$~~

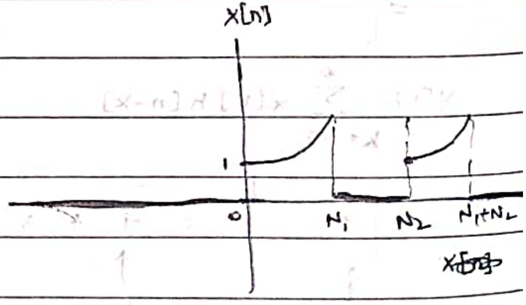


$$W[n] \xrightarrow[h[n]=\beta^n u[n]]{LSI} ?$$

$$= \begin{cases} 0 & n<0 \\ \alpha^n & 0 \le n \le N_1 \\ 0 & n > N_1 \end{cases}$$

$$Y[n] = \sum_{-\infty}^{\infty} W[k]h[n-k]$$

$$= \sum_{0}^{n} W[k]\beta^{n-k}$$



~~x[n]~~

$x[n] = W[n] + W[n-N_2]$

$y[n] + y[n-N_2]$

$0 \le n \le N_1$

$$Y[n] = \sum_{0}^{n}\alpha^k\beta^{n-k}$$

$$= \beta^n\left(\frac{(\alpha/\beta)^{n+1}-1}{\alpha/\beta - 1}\right)$$

$n > N_1$

$$Y[n] = \sum_{0}^{N_1}\alpha^k\beta^{n-k}$$

$$= \beta^n\left(\frac{(\alpha/\beta)^{N_1+1}-1}{\alpha/\beta - 1}\right)$$