1. Preliminary introduction on the optimal linear quadratic analog tracker for the system with a direct-feedthrough term and known/estimated system disturbances or compensatory signals

This section briefly describes the steady-state optimal LQAT with pre-specified measurement output and control input trajectories for the continuous-time controllable, observable, and non-degenerate system, which has both an input-to-output direct-feedthrough term and known/estimated system disturbances or compensatory signals [16].

Consider the above-metioned system with compensatory signals $d(t)$ and $s(t)$

$$\dot{x}_c(t) = Ax_c(t) + Bu_c(t) + d(t), \tag{1a}$$

$$y_c(t) = Cx_c(t) + Du_c(t) + s(t), \tag{1b}$$

where $A \in \Re^{n\times n}$, $B \in \Re^{n\times m}$, $C \in \Re^{p\times n}$ and $D \in \Re^{p\times m}$ are state, input, output, and direct feedthrough matrices, respectively. $x_c(t) \in \Re^n$ is the state vector, $u_c(t) \in \Re^m$ is the control input, and $y_c(t) \in \Re^p$ is the measurable output of system at time t. Let the quadratic performance index function for the tracking problem be

$$\begin{aligned} J(y_c, u_c) = &\frac{1}{2}\left[y_c(t_f) - r(t_f)\right]^T S \left[y_c(t_f) - r(t_f)\right] \\ &+ \frac{1}{2}\int_0^{t_f} \left\{\left[y_c(t) - r(t)\right]^T Q_c \left[y_c(t) - r(t)\right] + \left[u_c(t) - u_c^*(t)\right]^T R_c \left[u_c(t) - u_c^*(t)\right]\right\} dt, \end{aligned} \tag{2}$$

where $r(t)$ denotes the pre-specified output trajectory, t_f is the final time duration, Q_c is a $p \times p$ positive definite or positive semi-definite real symmetric matrix, R_c is an $m \times m$ positive definite real symmetric matrix, S is a $p \times p$ semi-definite real symmetric matrix, and $u_c^*(t)$ is a pre-specified control input trajectory. Solving (2) yields to the continuous-time state-feedback control law as

$$u_c(t) = -K_c x_c(t) + E_c r(t) + C_c(t) + C_{u_c}^* u_c^*(t), \tag{3a}$$

where

$$K_c = \bar{R}_c^{-1}\left(B^T P + N^T\right), \tag{3b}$$

$$E_c = -\bar{R}_c^{-1}\left[\left(C - DK_c\right)\left(A - BK_c\right)^{-1} B - D\right]^T Q_c, \tag{3c}$$

$$C_c(t) = \bar{R}_c^{-1}\left[\left(C - DK_c\right)\left(A - BK_c\right)^{-1} B - D\right]^T Q_c s(t) + \bar{R}_c^{-1} B^T \left[\left(A - BK_c\right)^T\right]^{-1} Pd(t), \tag{3d}$$

$$C_{u_c}^* = \bar{R}_c^{-1}\left\{I_m + B^T\left[\left(A - BK_c\right)^T\right]^{-1} K_c^T\right\} R_c, \tag{3e}$$

$$\bar{R}_c = R_c + D^T Q_c D, \tag{3f}$$

$$N = C^T Q_c D, \tag{3g}$$

and P satisfies the algebraic Riccati equation

$$A^T P + PA - \left(PB + N\right)\bar{R}_c^{-1}\left(B^T P + N^T\right) + C^T Q_c C = 0. \tag{4}$$

2. Five-DOF AMB system

2-1. Simulink model

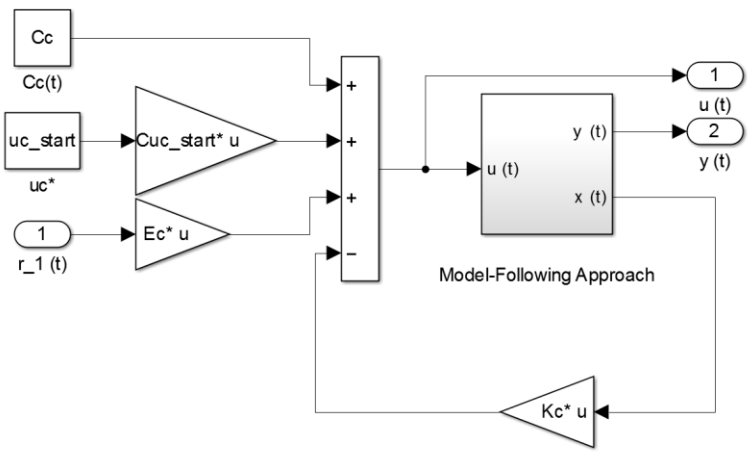
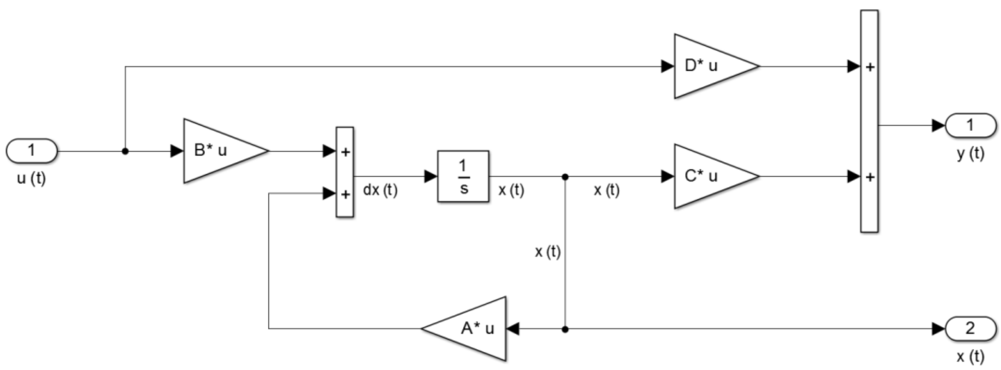


Figure1. Close loop system



Figute2. Plant diagram

2-2. Open loop System

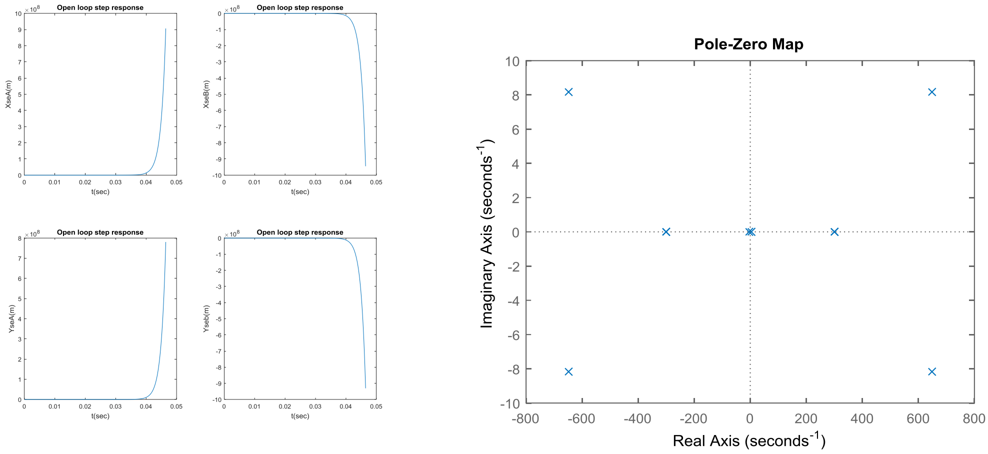


Figure3. Open loop step response and pole zero mapping

$$\lambda_{1,2} = -648.346396993605 \pm 8.16879691481991i$$
$$\lambda_{3,4} = 648.346396993605 \pm 8.16879691482052i$$
$$\lambda_{5,6} = \pm 300.266332703738$$
$$\lambda_{7,8} = \pm 300.443154630244$$
$$\lambda_{9,10} = \pm 3.94916188179442$$

2-3. Close loop System

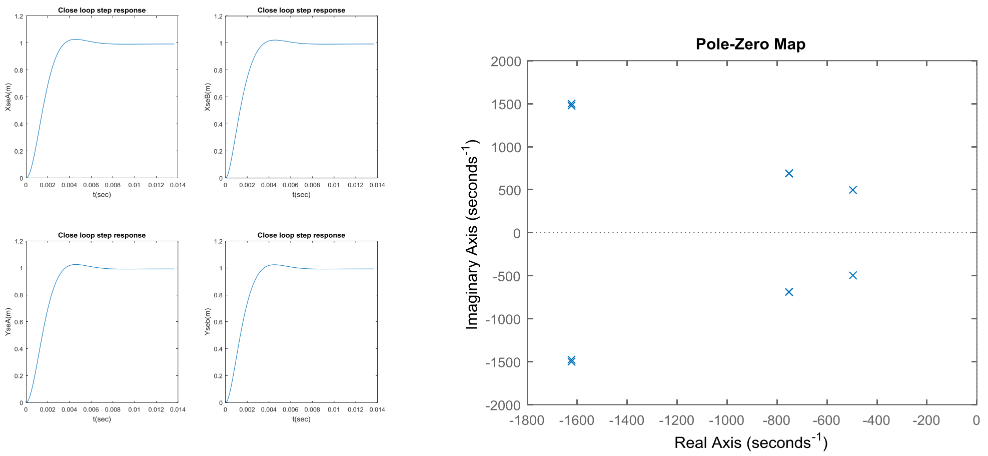


Figure4. Close loop step response and pole zero mapping

$$\lambda_{1,2} = -1623.52979980164 \pm 1500.10084447332i$$
$$\lambda_{3,4} = -1623.52957042486 \pm 1477.03662577893i$$
$$\lambda_{5,6} = -752.395940125403 \pm 688.673950037256i$$
$$\lambda_{7,8} = -752.353179414280 \pm 690.378218815862i$$
$$\lambda_{9,10} = -496.588668633977 \pm 496.572965369741i$$

2-4. Rotor orbit

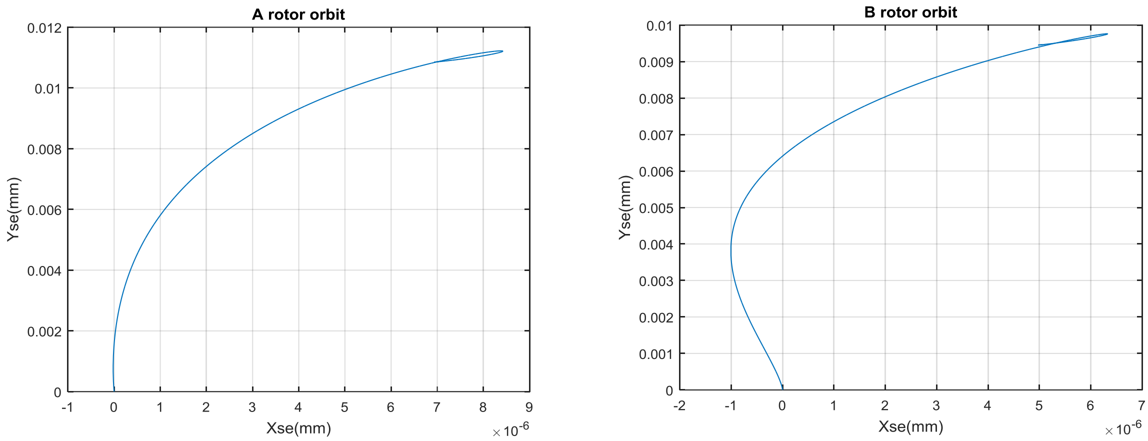


Figure5. Rotor orbit

2-5 Current

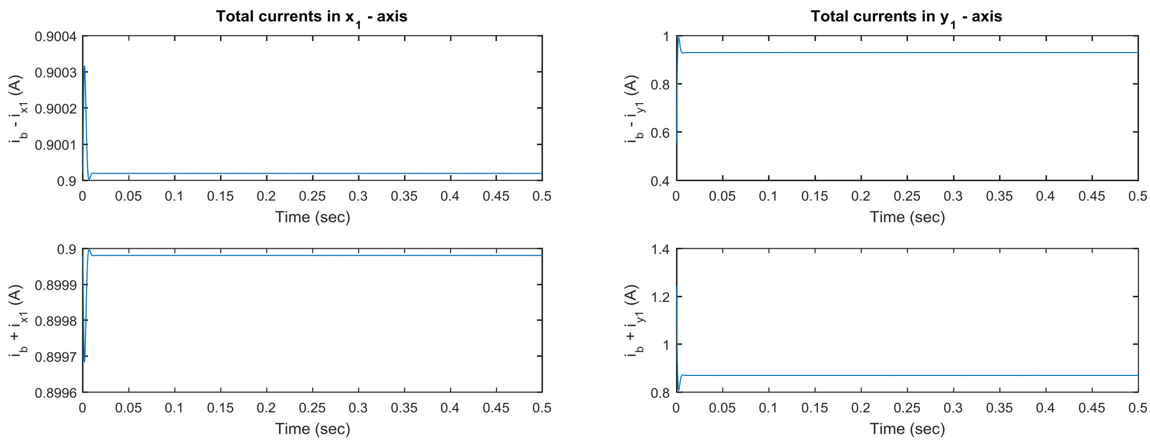


Figure6. Total currents in x1 - axis and y1 – axis

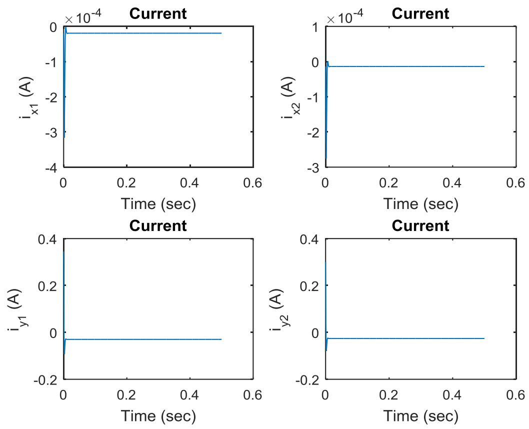


Figure7. Control currents in each axis