A new proportional-plus-integral optimal linear quadratic state-estimate tracker for continuous-time non-minimum phase systems

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Abstract

In this article, at first a new optimal proportional-plus-integral (PI) filter-shaped linear quadratic PI state-feedback tracker for continuous-time square and/or non-square non-minimum phase plants is presented. Subsequently, a new PID filter-shaped optimal linear quadratic PI state estimator, based on the framework of tracker design methodology, is proposed. Finally, the new integrated optimal PI state estimate tracker is presented in this paper, so that the tracker, state estimator, and the state estimate tracker-based controlled systems respectively can achieve good minimum phase-like tracking performances for arbitrary command inputs.

Keywords: Optimal linear quadratic estimator, frequency shaping, PID controller, non-minimum phase system, control zeros.

1. Introduction

Control engineers are often encountered with practical systems exhibiting a non-minimum phase behavior, particularly for the sampled-data systems (MacFariance and Karcanias, 1976; Astrom et al., 1984; Schrader and Sain, 1989; Clarke, 1984; Ram, 1998). Non-minimum phase systems exhibit either inverse response (undershoot) or time-delay characteristics for some considerable time. The non-minimum phase behavior degrades the performances of traditional tracker designs. A number of literatures have investigated this issue from a predictive control point of view (Uren and Schoor, 2011), such as the Smith predictor and the internal model control (IMC) (Katebi and Moradi, 2001; Morari and Zafiriou, 1989; Tan, 2001), as well as the generalized predictive control (GPC) and the model-based predictive control (MPC) (Johnson and Moradi, 2005; Miller et al., 1999; Moradi et al., 2001; Sato T, 2010; Tan et al., 2000; Wang, 2009; Meadows and Badgwell, 1998; Shiu et al., 2014). These methods commonly utilize a plant model to predict the future output of the controlled system, from which a control law is developed to act immediately to the class of step-like command input to avoid instability and sluggish control.

It is well-known that the poles and zeros of a non-minimum phase system have a certain property in order for the system to be strongly stabilizable. For example, a system is strongly stabilizable if and only if it has an even number of real poles located between every pair of real zeros in Re $s \geq 0$, where the poles at $s = \infty$ are considered as the real zeros of the system (Doyle et al., 1990).

To ensure the non-minimum phase general system to be strongly stabilizable with desirable

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tracking performance, based on the PID filter-based frequency shaping approach, an optimal PI filter-based state-feedback linear quadratic tracker for the non-square non-minimum phase system has been presented in our previous work (Ebrahimzadeh et al., 2015). As having shown that a square non-minimum plant cannot be transformed into a minimum phase one through an input/output/state transformation. To the best of our knowledge, how to design a well-performed tracker for the general square non-minimum phase plant without modifying any component of the given plant (A, B, C), such as the actuator/sensor/in-built state, is still a challenging problem. In this paper, a fictitious input-to-output direct feed-through term D is well selected and added to the non-minimum phase plant, so that the fictitious plant (A, B, C, D) becomes minimum phase in the sense of multivariable control zero assignment (Latawiec et al., 2000). Then, based on the PI filter-based frequency shaping approach, a new optimal PI filter-based state-feedback linear quadratic tracker for the square and/or non-square non-minimum phase plant is developed, so that the controlled fictitious system achieves a good minimum phase-like tracking performance for arbitrary command inputs. Finally, the controlled fictitious system is realized by the original plant (A, B, C) with an equivalent two-degrees-of-freedom tracker.

In order to achieve the control specifications such as the gain margins of $(1/2, \infty)$ and the guaranteed 60-degree phase margin, or their multivariable equivalents, it is often required that the given plant is assumed to be a square and minimum phase one. Unfortunately, a square non-minimum plant is still a non-minimum phase one, even though by appending the PID filter(s)/controller(s) (without unstable open-loop pole-zero cancellation) at either the input terminal, output terminal, or both terminals. Nevertheless, the minimum phase property for the given system is usually required if the output measurements are utilized to recover the state through an estimator and to retain the robustness associated with the state feedback controller and the state estimator. It should call an attention that, even for the minimum phase plant, the robustness property of the pass-band filter in the frequency domain might be lost via the full-state feedback optimal linear quadratic tracker (LQT) with the state estimator (Anderson and Mingori, 1985; Moore et al., 1990; Thompson et al., 1987; Anderson and Moore, 1989; Tsai et al., 2011).

The input robustness properties of the full-state feedback controller may be recovered by the loop recovery techniques (Anderson and Moore, 1989), when a plant has the same number of inputs or more outputs than inputs. Loop recovery techniques can be applied to non-minimum phase plants with care. In the non-minimum phase plant case, classical control theory tells us that it may be difficult or impossible to design a robust controller (Doyle et al., 1990).

The loop recovery relies on pole-zero cancellations in the open loop. This explains why the plant must be minimum phase with left half-plane zeros. Even so, there are two possible roles for loop recovery ideas described in designing the state estimate feedback controllers for non-minimum phase plants. One approach to applying loop recovery techniques to non-minimum phase plants is to proceed tentatively as if the plant were minimum phase. Just increase fictitious noise variances until there is a maximum degree of robustness enhancement; however, only partial recovery of state feedback open-loop properties can be achieved in general. In this approach, frequency shaping could well permit adequate loop recovery in the frequency band of interest, as long as the non-minimum phase zeros are outside this band (Anderson and Moore, 1989; Zhang and Freudenberg, 1987). The other approach, termed by the factored plant model approach (Anderson and Moore, 1989), is to work with a plant factorization to have a non-minimum phase all-pass factor and a minimum phase factor. Since the designs do not achieve minimum phase open-loop transfer functions, they will not achieve the input robustness properties in terms of the return difference inequality of the linear quadratic designs.

When the plant is non-minimum phase, an optimization procedure known as H^{∞} / H^{2} optimization can be used to find the optimal filter (Anderson and Moore, 1989; Moore and Tay, 1989;

Shi 1999). As indeed for the optimal filter $Q_f(s)$ selections above, the controller order may be excessive, so that controller reduction via the techniques in (Anderson and Moore, 1989) could well be in order for a final design. Similarly, in this non-minimum phase plant case, only partial loop and sensitivity recovery can be achieved.

Loop transfer recovery (LTR) techniques are known to enhance the input or output robustness properties of linear quadratic Gaussian (LQG) designs. In the case of minimum phase plants, full or asymptotic loop and sensitivity recovery can be achieved and the H^∞/H^2 optimization is trivial. However, for non-minimum phase plants, only a partial loop recovery is possible under the full state feedback, and the H^∞/H^2 optimization seeks to achieve a "best" possible recovery property (Anderson and Moore, 1989; Moore and Tay, 1989). Also, full loop recovery is achieved for non-minimum phase plants with the restricted feedback of only the square minimum phase factor states (whereas the states associated with the stable all-pass factor are not), assuming the plant is factorized into a product of a stable all-pass factor and a minimum phase factor.

Notice that the above-mentioned factorization is straightforward in the scalar plant case, and more complex in the non-scalar case, as studied in (Chu and Doyle, 1984). We stress again that since the designs (Anderson and Moore, 1989; Moore and Tay, 1989) do not achieve minimum phase open-loop transfer functions, they will not achieve the input robustness properties in terms of the return difference inequality of the linear quadratic designs. To further improve the observer design for the non-minimum phase plants without involving the above-mentioned factorization and achieve minimum phase open-loop transfer functions, a new PID filter-shaped optimal linear quadratic PI state estimator, constructed based on the framework of tracker designs, is proposed in this paper, which differs from the concept of traditional observer designs as well as the framework of the regulator designs presented in (Anderson and Moore, 1989). To proceed it, in corporation with the new optimal PI state-feedback linear quadratic tracker for the square and/or non-square non-minimum phase plant proposed in this paper is suggested so that the observer itself and the observer-based controlled system respectively achieve good minimum phase-like tracking performances for arbitrary command input. As a result, the estimated system output approaches the actual system output, and the actual system output approach to the pre-specified command input simultaneously for the square and/or non-square non-minimum phase plants.

Here, we would like to point out that to achieve the desired goals mentioned above, the generalized optimal LQAT for the continuous-time system with an input-to-output direct-feedthrough term and known/estimated system disturbances or compensatory signals presented in our previous work (Ebrahimzadeh et al., 2015) is required, so that those new design approaches proposed in this paper can be performed. To the best of our knowledge, no literature addresses this kind of applications, so the proposed approaches for either tracker, observer, and/or the observer-based tracker are new.

The paper is organized as follows. Preliminary introduction on the generalized optimal linear quadratic analog tracker for the system with a direct-feedthrough term and known/estimated system disturbances or compensatory signals is given in Section 2. Section 3 presents a new optimal PI filter-shaped linear quadratic PI state-feedback tracker for square and/or non-square non-minimum phase systems. Section 4 presents the design procedure of the new PID filter-shaped optimal linear quadratic PI state estimator, constructed based on the framework of tracker design methodology. The new integrated proportional plus integral state estimate tracker is then presented in Section 5. An illustrative example is demonstrated in Section 6 to show the effectiveness of the proposed design methodologies, and conclusions are finally given in Section 7.

2. Preliminary introduction on the optimal linear quadratic analog tracker for the system with a direct-feedthrough term and known/estimated system disturbances or compensatory signals

This section briefly describes the steady-state optimal LQAT with pre-specified measurement output and control input trajectories for the continuous-time controllable, observable, and non-degenerate system, which has both an input-to-output direct-feedthrough term and known/estimated system disturbances or compensatory signals (Ebrahimzadeh et al., 2015).

Consider the above-metioned system with compensatory signals $d(t)$ and $s(t)$

$$\dot{x}_c(t) = Ax_c(t) + Bu_c(t) + d(t), \tag{1a}$$

$$y_c(t) = Cx_c(t) + Du_c(t) + s(t), \tag{1b}$$

where $A \in \Re^{n\times n}$, $B \in \Re^{n\times m}$, $C \in \Re^{p\times n}$ and $D \in \Re^{p\times m}$ are state, input, output, and direct feedthrough matrices, respectively. $x_c(t) \in \Re^n$ is the state vector, $u_c(t) \in \Re^m$ is the control input, and $y_c(t) \in \Re^p$ is the measurable output of system at time t. Let the quadratic performance index function for the tracking problem be

$$J(y_c, u_c) = \frac{1}{2}\left[y_c(t_f) - r(t_f)\right]^T S\left[y_c(t_f) - r(t_f)\right] + \frac{1}{2}\int_0^{t_f}\left\{\left[y_c(t) - r(t)\right]^T Q_c\left[y_c(t) - r(t)\right] + \left[u_c(t) - u_c^*(t)\right]^T R_c\left[u_c(t) - u_c^*(t)\right]\right\}dt, \tag{2}$$

where $r(t)$ denotes the pre-specified output trajectory, t_f is the final time duration, Q_c is a $p\times p$ positive definite or positive semi-definite real symmetric matrix, R_c is an $m\times m$ positive definite real symmetric matrix, S is a $p\times p$ semi-definite real symmetric matrix, and $u_c^*(t)$ is a pre-specified control input trajectory. Solving (2) yields to the continuous-time state-feedback control law as

$$u_c(t) = -K_c x_c(t) + E_c r(t) + C_c(t) + C_{u_c}^* u_c^*(t), \tag{3a}$$

where

$$K_c = \bar{R}_c^{-1}\left(B^T P + N^T\right), \tag{3b}$$

$$E_c = -\bar{R}_c^{-1}\left[\left(C - DK_c\right)\left(A - BK_c\right)^{-1} B - D\right]^T Q_c, \tag{3c}$$

$$C_c(t) = \bar{R}_c^{-1}\left[\left(C - DK_c\right)\left(A - BK_c\right)^{-1} B - D\right]^T Q_c s(t) + \bar{R}_c^{-1} B^T\left[\left(A - BK_c\right)^T\right]^{-1} Pd(t), \tag{3d}$$

$$C_{u_c}^* = \bar{R}_c^{-1}\left\{I_m + B^T\left[\left(A - BK_c\right)^T\right]^{-1} K_c^T\right\}R_c, \tag{3e}$$

$$\bar{R}_c = R_c + D^T Q_c D, \tag{3f}$$

$$N = C^T Q_c D, \tag{3g}$$

and P satisfies the algebraic Riccati equation

$$A^T P + PA - \left(PB + N\right)\bar{R}_c^{-1}\left(B^T P + N^T\right) + C^T Q_c C = 0. \tag{4}$$

3. A new optimal PI state-feedback linear quadratic analog tracker for square and/or non-square non-minimum phase systems: PI filter-based frequency shaping approach

For the square non-minimum phase plant, non-singular input/output/state transformation does not make the system to be minimum phase. Concerning this issue, this section presents a new optimal PI state-feedback LQAT for the linear continuous-time square and/or non-square non-minimum phase system (5), so that without involving any modification in the in-built plant and the above-mentioned transformations, the two-degree-of-freedom state/output-feedback tracker in terms of (K_c, E_c)-based controlled system can now demonstrate a good minimum phase-like tracking performance for any arbitrary command input $r_c(t)$.

Consider the continuous-time controllable, observable, non-degenerate square/non-square non-minimum phase system described by

$$\dot{x}_c(t) = Ax_c(t) + Bu_c(t), \;\; x_c(0) = x_0, \tag{5a}$$

$$y_c(t) = Cx_c(t), \tag{5b}$$

where $A \in \Re^{n\times n}$, $B \in \Re^{n\times m}$, and $C \in \Re^{p\times n}$ denote the system, input, and output matrices, respectively, and $x_c(t) \in \Re^n$, $u_c(t) \in \Re^m$, and $y_c(t) \in \Re^p$ represent the state, input, and output vectors, respectively. The design procedure of the new PI filter-based frequency shaping approach for determining the optimal PI state feedback linear quadratic tracker for square and/or non-square ($m \ge p$) non-minimum phase systems is described as follows:

Step1: Transform the non-minimum phase system to a fictitious minimum phase system.

i) Randomly select a direct feed-through term $D \in \Re^{p\times m}$ and determine a LQAT with the high-gain property in ii), so that the fictitious system

$$\dot{x}_a(t) = Ax_a(t) + Bu_a(t), \;\; x_a(0) = x_0, \tag{6a}$$

$$y_a(t) = Cx_a(t) + Du_a(t), \tag{6b}$$

is to be minimum phase and $y_a(t)$ well tracks any pre-specified fictitious reference input $r_a(t)$.

ii) Apply the algorithm proposed by Emami-Naeini and Dooren (Emami-Naeini and Dooren, 1982) (see syntax ‘tzero(sys)’ in MATLAB) for calculation of control zeros of the system (A, B, C, D) in terms of the square Rosenbrock system matrix (Ebrahimzadeh et al., 2015)

$$S_{ac}(z_i) = \begin{bmatrix} (A - BK_a) - z_i I_n & BE_a \\ C - DK_a & DE_a \end{bmatrix}, \tag{7a}$$

where

$$u_a(t) = -K_a x_a(t) + E_a r_a(t), \tag{7b}$$

$$K_a = \bar{R}_c^{-1}\left(B^T P + N^T\right), \tag{7c}$$

$$E_a = -\bar{R}_c^{-1}\left[\left(C - DK_a\right)\left(A - BK_a\right)^{-1} B - D\right]^T Q_c, \tag{7d}$$

$$\bar{R}_c = R_c + D^T Q_c D, \tag{7e}$$

$$N = C^T Q_c D, \tag{7f}$$

and P satisfies the algebraic Riccati equation

$$A^T P + PA - \left(PB + N\right)\bar{R}_c^{-1}\left(B^T P + N^T\right) + C^T Q_c C = 0. \tag{7g}$$

Check whether the system (A, B, C, D) is minimum phase or not. If it’s so, then go to Step 2; otherwise, go back to Step i).

Step 2: Assign some extra target zeros (without open-loop pole-zero cancellation) to attract some of closed-loop poles in a closed-loop design.

A scalar PI filter $(k_p + k_i s^{-1})$ has a zero at a specified location and a pole at the origin. The PI filter gains $\{k_p,\, k_i\}$ are selected to achieve some extra minimum phase target zeros intended to attract some closed-loop poles in a closed-loop design.

i) Append the PI filter to each element of $\left[y_a(t)-r_a(t)\right]$ as shown in Figure 1. The output of the PI filter is given by

$$Y_f(s)=\begin{bmatrix} k_p^{(1)}+k_i^{(1)}s^{-1} & & 0 \\ & \ddots & \\ 0 & & k_p^{(p)}+k_i^{(p)}s^{-1} \end{bmatrix}\left[Y_a(s)-R_a(s)\right]. \tag{8}$$

The corresponding magnitude frequency response in dB is

$$20\log\left|Y_f(j\omega)\right| = 20\log\left|K_P - jK_I/\omega\right| + 20\log\left|Y_a(j\omega)-R_a(j\omega)\right|. \tag{9}$$

At low frequency, when ω approaches zero, (9) becomes

$$20\log\left|Y_f(j\omega)\right| = 20\log\left|-jK_I/\omega\right| + 20\log\left|Y_a(j\omega)-R_a(j\omega)\right|. \tag{10}$$

Thus, the integrator emphasizes low frequencies. Moreover, if the bandwidth is too low to achieve a practical design, then increase the proportional gain to tune the cross-over frequency.

ii) Choose the PI filter gains $\left\{ k_p^{(j)},\ k_i^{(j)}\ |\ j=1\sim p \right\}$ to assign some target zeros

$$\left\{ \frac{-k_i^{(j)}}{k_p^{(j)}}\ \middle|\ j=1\sim p \right\}. \tag{11}$$

Step 3: Construct the augmented plant.

i) Define the integral of $\left[y_a(t)-r_a(t)\right]$ as another state variable

$$x_I(t)=\int_0^t \left[y_a(\tau)-r_a(\tau)\right]d\tau. \tag{12}$$

ii) The state-space model of the augmented plant (shown in Figure 1) is given by

$$\begin{aligned}\begin{bmatrix}\dot{x}_c(t)\\ \dot{x}_I(t)\end{bmatrix} &= \begin{bmatrix} A & 0_{n\times p}\\ C & 0_{p\times p}\end{bmatrix}\begin{bmatrix}x_c(t)\\ x_I(t)\end{bmatrix}+\begin{bmatrix}B\\ D\end{bmatrix}u_c(t)+\begin{bmatrix}0_{n\times 1}\\ -r_a(t)\end{bmatrix}\\ &= A_{aug}x_{aug}(t)+B_{aug}u_c(t)+d_{aug}(t),\end{aligned} \tag{13a}$$

$$\begin{aligned} y_f(t) &= \left[K_P C \quad K_I\right]\begin{bmatrix}x_c(t)\\ x_I(t)\end{bmatrix}+K_P D u_c(t)-K_P r_a(t)\\ &= C_{aug}x_{aug}(t)+D_{aug}u_c(t)+s_{aug}(t).\end{aligned} \tag{13b}$$

Step 4: Formulate the performance index.

The first goal of this control problem is to minimize the performance index associated with the augmented plant with input $u_c(t)$ and output $y_f(t)$ as

$$J=\int_0^\infty \left\{ \left[y_f(t)-r_a(t)\right]^T Q_c \left[y_f(t)-r_a(t)\right] + u_c^T(t)R_c u_c(t) \right\} dt, \tag{14a}$$

so that $y_f(t)$ well tracks the command input $r_a(t)$. By Parseval's theorem, the performance index can be written in the frequency domain as

$$\begin{aligned} J=\frac{1}{2\pi}\int_{-\infty}^{\infty} \Big\{ &\left[\left(K_P+K_I s^{-1}\right)\left(Y_a(s)-R_a(s)\right)-R_a(s)\right]^* Q_c \\ &\times\left[\left(K_P+K_I s^{-1}\right)\left(Y_a(s)-R_a(s)\right)-R_a(s)\right]+U_c^*(s)R_c U_c(s)\Big\} d\omega,\end{aligned} \tag{14b}$$

where the superscript $*$ denotes the complex conjugate transpose and $s=j\omega$. The second goal of this control problem is to minimize the other performance index as

$$J' = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left\{\left(Y_a(s)-R_a(s)\right)^*\left[\left(K_P+K_I s^{-1}\right)^* Q_c\left(K_P+K_I s^{-1}\right)\right]\left(Y_a(s)-R_a(s)\right)+U_c^*(s)R_c U_c(s)\right\}d\omega, \quad (15)$$

so that $y_a(t)$ well tracks the command input $r_a(t)$. Some notable remarks are listed in the following.

i) From (14b), one observes that

$$\begin{aligned}
&\left(K_P+K_I s^{-1}\right)\left[Y_a(s)-R_a(s)\right]-R_a(s)\\
&=\left(K_P+K_I s^{-1}\right)\left\{Y_a(s)-\left[I_m+\left(K_P+K_I s^{-1}\right)^{-1}\right]R_a(s)\right\}\\
&=\left(K_P+K_I s^{-1}\right)\left\{Y_a(s)-\begin{bmatrix}\dfrac{(k_p^{(1)}+1)s+k_i^{(1)}}{k_p^{(1)}s+k_i^{(1)}} & & 0\\ & \ddots & \\ 0 & & \dfrac{(k_p^{(p)}+1)s+k_i^{(p)}}{k_p^{(p)}s+k_i^{(p)}}\end{bmatrix}R_a(s)\right\}\\
&\to\left(K_P+K_I s^{-1}\right)\left[Y_a(s)-R_a(s)\right] \text{ as } k_p^{(j)}\approx k_p^{(j)}+1 \text{ or } k_p^{(j)}\to\infty \text{ for indices } j=1,\ 2,\cdots,\ p,
\end{aligned}$$

or $\dfrac{(k_p^{(j)}+1)s+k_i^{(j)}}{k_p^{(j)}s+k_i^{(j)}}\to 1$ for $j=1,\ 2,\cdots,\ p,$ in general, which reveals a criterion for the selection of PI filters so that the performance index in (14b) approaches to the one in (15).

ii) Performance indices (14) and (15) show that $y_f(t)\to r_a(t)$ and $y_a(t)\to r_a(t)$, respectively, provided that $\dfrac{(k_p^{(j)}+1)s+k_i^{(j)}}{k_p^{(j)}s+k_i^{(j)}}\to 1$ for $j=1,\ 2,\cdots,\ p,$ as well as the weighting function pair $\left\{\left(K_P+K_I s^{-1}\right)^* Q_c\left(K_P+K_I s^{-1}\right),\ R_c\right\}$ has the high-gain property as mentioned above.

Step 5: Perform the linear quadratic PI state-feedback tracker design.

Use (3) and (4) with an appropriate weighting matrix pair $\{Q_c,\ R_c\}$ (not necessary $Q_c >> R_c$) as well as the PI filter gains to have the optimal control law

$$\begin{aligned}
u_c(t) &= -K_c x_{aug}(t)+E_c r_a(t)+C_c(t)\\
&= -\left[K_{c1} \;\vdots\; K_{c2}\right]\begin{bmatrix}x_c(t)\\ x_I(t)\end{bmatrix}+E_c r_a(t)+C_c(t), \qquad (16a)
\end{aligned}$$

for the augmented system model in (13), where

$$K_c = \bar{R}_{aug}^{-1}\left(B_{aug}^T P_{aug}+N_{aug}^T\right), \qquad (16b)$$

$$E_c = -\bar{R}_{aug}^{-1}\left[\left(C_{aug}-D_{aug}K_c\right)\left(A_{aug}-B_{aug}K_c\right)^{-1}B_{aug}-D_{aug}\right]^T Q_c, \qquad (16c)$$

$$\begin{aligned}
C_c(t) &= \bar{R}_{aug}^{-1}\left[\left(C_{aug}-D_{aug}K_c\right)\left(A_{aug}-B_{aug}K_c\right)^{-1}B_{aug}-D_{aug}\right]^T Q_c s_{aug}(t)\\
&\quad+\bar{R}_{aug}^{-1}B_{aug}^T\left[\left(A_{aug}-B_{aug}K_c\right)^T\right]^{-1}P_{aug}\,d_{aug}(t) \qquad (16d)\\
&= -E_c s_{aug}(t)+\bar{R}_{aug}^{-1}B_{aug}^T\left[\left(A_{aug}-B_{aug}K_c\right)^T\right]^{-1}P_{aug}\,d_{aug}(t),
\end{aligned}$$

in which $\bar{R}_{aug} = R_c + D_{aug}^T Q_c D_{aug}$ and $N_{aug} = C_{aug}^T Q_c D_{aug}$, and P_{aug} satisfies the algebraic Riccati equation

$$A_{aug}^T P_{aug} + P_{aug} A_{aug} - \left(B_{aug}^T P_{aug} + N_{aug}^T\right)^T \bar{R}_{aug}^{-1} \left(B_{aug}^T P_{aug} + N_{aug}^T\right) + C_{aug}^T Q_c C_{aug} = 0.$$

The architecture of the linear quadratic PI state-feedback tracker for the square and/or non-square ($m \geq p$) non-minimum phase system is shown in Figure 1.

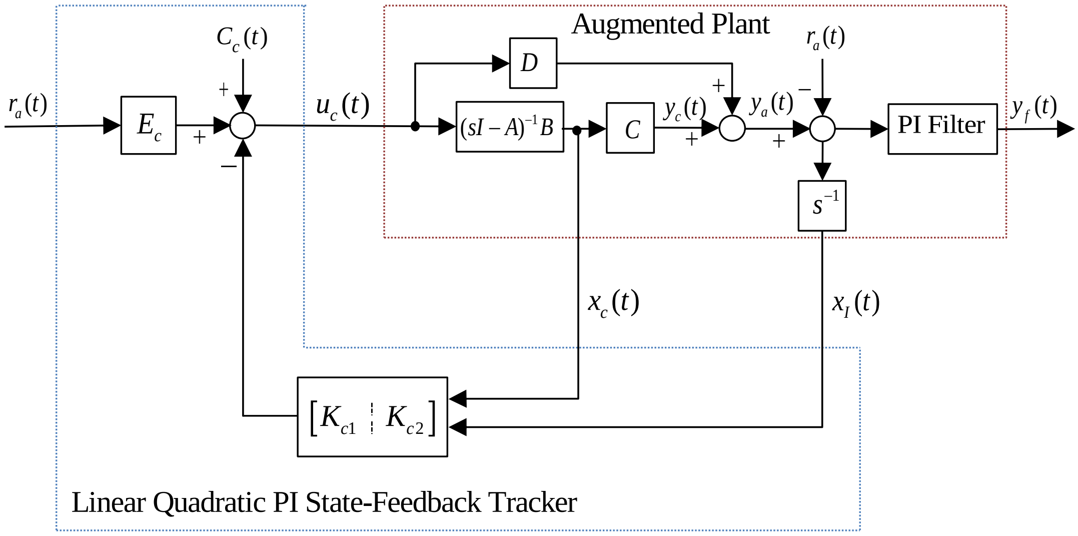


Figure 1. Linear quadratic PI state-feedback tracker.

Step 6: Realize the controlled fictitious system via implementing the modified PI-state feedback tracker for the original direct feed-through-term-free system.
To have $y_c(t) \to r_c(t)$, let $r_a(t)$ shown in Figure 2 be

$$r_a(t) = Du_c(t) + r_c(t), \tag{17}$$

where $u_c(t)$ is to be determined later. From (16a) and (16d), one observes

$$u_c(t) = -K_c x_{aug}(t) + E_c r_a(t) - E_c s_{aug}(t) + \bar{R}_{aug}^{-1} B_{aug}^T \left[\left(A_{aug} - B_{aug} K_c\right)^T\right]^{-1} P_{aug}\, d_{aug}(t), \tag{18}$$

where, from (13),

$$-E_c s_{aug}(t) = -E_c(-K_P r_a(t)) = E_c K_P r_a(t), \tag{19}$$

$$\bar{R}_{aug}^{-1} B_{aug}^T \left[\left(A_{aug} - B_{aug} K_c\right)^T\right]^{-1} P_{aug}\, d_{aug}(t) \equiv \left[Z_{1\, m\times n} \quad Z_{2\, m\times p}\right] \begin{bmatrix} 0_{n\times 1} \\ -r_a(t) \end{bmatrix} = -Z_2 r_a(t). \tag{20}$$

Substituting (19)-(20) and (17) into (18) yields

$$\begin{aligned} u_c(t) &= -K_c x_{aug}(t) + \left(E_c + E_c K_P - Z_2\right) r_a(t) \\ &= -K_c x_{aug}(t) + \left(E_c + E_c K_P - Z_2\right)\left(r_c(t) + Du_c(t)\right), \end{aligned} \tag{21a}$$

which implies

$$\begin{aligned} u_c(t) = &-\left[I_m - \left(E_c + E_c K_P - Z_2\right) D\right]^{-1} K_c x_{aug}(t) \\ &+ \left[I_m - \left(E_c + E_c K_P - Z_2\right) D\right]^{-1} \left(E_c + E_c K_P - Z_2\right) r_c(t), \end{aligned} \tag{21b}$$

where

$$x_{aug} = \begin{bmatrix} x_c(t) \\ x_I(t) \end{bmatrix}, \tag{22a}$$

and

$$\begin{aligned} x_I(t) &= \int_0^t [y_a(\tau) - r_a(\tau)]\,d\tau = \int_0^t \{[Cx_c(\tau) + Du_c(\tau)] - [r_c(\tau) + Du_c(\tau)]\}\,d\tau \\ &= \int_0^t [Cx_c(\tau) - r_c(\tau)]\,d\tau = \int_0^t [y_c(\tau) - r_c(\tau)]\,d\tau. \end{aligned} \tag{22b}$$

Thus, the realized tracker becomes

$$u_c(t) = -\begin{bmatrix} \bar{K}_{c1} & \bar{K}_{c2} \end{bmatrix} \begin{bmatrix} x_c(t) \\ x_I(t) \end{bmatrix} + \bar{E}_c r_c(t), \tag{23a}$$

where

$$\begin{bmatrix} \bar{K}_{c1\ m\times n} & \bar{K}_{c2\ m\times p} \end{bmatrix} = \left[I_m - (E_c + E_c K_P - Z_2)D\right]^{-1} K_c, \tag{23b}$$

$$\bar{E}_c = \left[I_m - (E_c + E_c K_P - Z_2)D\right]^{-1} (E_c + E_c K_P - Z_2). \tag{23c}$$

The configurations of the proposed tracker are shown in Figures 2 and 3. Notice that these two configurations are equivalent theoretically.

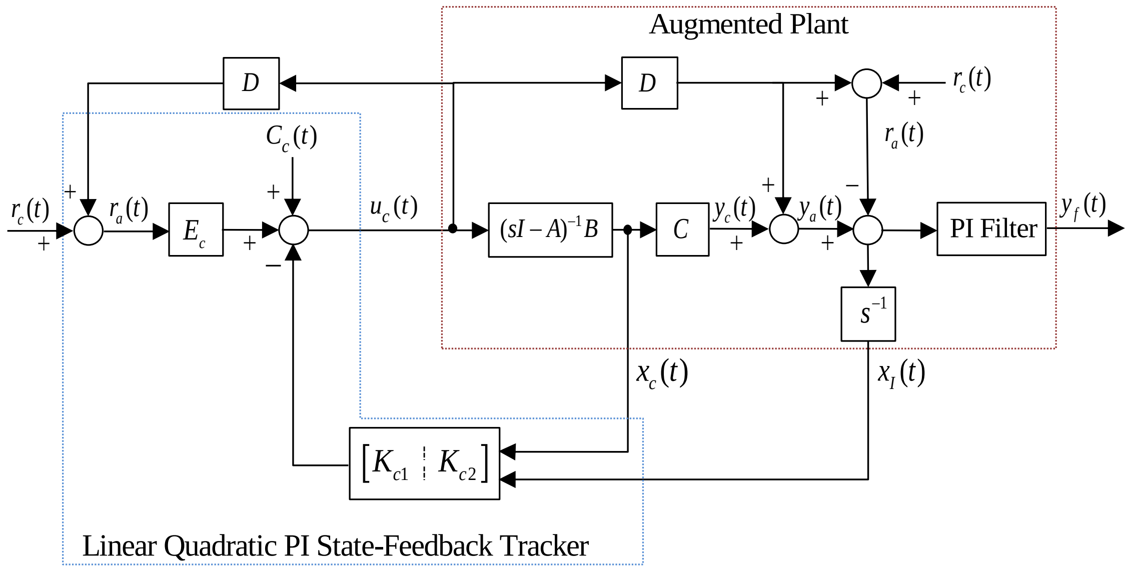


Figure 2. Configuration for the linear quadratic PI state-feedback tracker-based fictitious system.

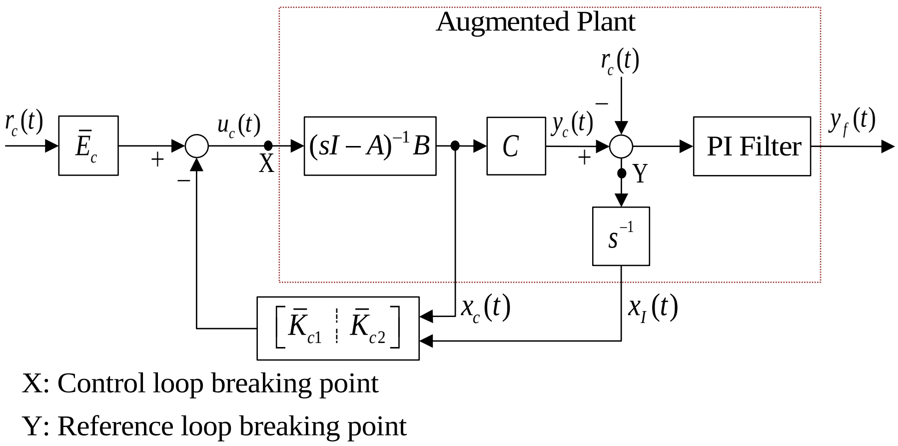


Figure 3. Configuration for the modified linear quadratic PI state-feedback tracker-based realized system.

Step 7: Examine the open-loop frequency response and adjust weights.
Examine the open-loop frequency response and adjust weights, as follows. If this bandwidth of the control loop is too high (low) to achieve a practical design, then increase (decrease) the first component of the diagonal matrix R_c. Repeat for all inputs. Likewise, when considering response to reference signals $r_c(t)$, the reference loop, the following rule is applied. Increase (decrease) the *i*th diagonal element of Q_c, if the reference loop bandwidth is too low (high) for a practical design. Repeat for all *i*. The control loop and reference loop can be respectively derived as follows

$$L_X(s) = \left(\bar{K}_{c1} + \bar{K}_{c2}\tilde{C}s^{-1}\right)\left(sI_n - A\right)^{-1} B \tag{24}$$

and

$$L_Y(s) = \tilde{C}\left[sI_n - \left(A - B\bar{K}_{c1}\right)\right]^{-1} B\bar{K}_{c2}s^{-1}. \tag{25}$$

Remark 1: To have a good minimum phase-like tracking performance $y_a(t) = Du_c(t) + y_c(t) \to r_a(t) = Du_c(t) + r_c(t)$ as well as $y_c(t) = Cx_c(t) \to r_c(t),$ there is no necessary to have the high-gain property weighting matrix pair $\{Q_c,\ R_c\},$ provided that $\dfrac{(k_p^{(j)}+1)s + k_i^{(j)}}{k_p^{(j)}s + k_i^{(j)}} \to 1$ for $j = 1,\ 2, \cdots, m,$ as well as the weighting function pair $\left\{\left(K_P + K_I s^{-1}\right)^* Q_c \left(K_P + K_I s^{-1}\right),\ R_c\right\}$ has the high-gain property in the interested frequency band. From the practical implementation point of view, it's suggested to select the low-gain property weighting matrix pair $\{Q_c,\ R_c\}$ to include a small control effort.

Remark 2: When the output number is greater than the input number, a pre-process is performed as follows. Transform the non-square non-minimum phase system to a square minimum phase system.

i) Define the new output variables, being a linear combination of the original output measurements, as

$$y_\xi(t) = \xi y_c(t) = \xi\left[Cx_c(t)\right] = \tilde{C}x_c(t), \tag{26}$$

where $\xi \in \Re^{m\times p}$ is a transformation matrix so that $(A,\ B,\ \tilde{C})$ is observable, in which $\tilde{C} = \xi C$.

ii) Randomly choose an above-mentioned $m\times p$ transformation matrix ξ to ensure that the system $(A - BK_c,\ BE_c,\ \tilde{C})$ is square minimum phase, where a control zero-related two degrees of freedom LQAT in terms of (K_c, E_c) with the high-gain property is used.

Then, applying the design procedure mentioned in this section results in the complete design methodology for the square and/or non-square non-minimum phase systems. Notice that when $(A,\ B,\ \tilde{C})$ is square minimum phase, introducing the feedthrough term D to construct the fictitious system in (6) becomes trivial. However, via this approach, it induces a small control effort as mentioned in Remark 2.

4. New optimal PI state-feedback linear quadratic analog estimators for non-minimum phase systems: PID filter-based frequency shaping approach

In this section we present procedures dual to those in Sec.3 for state estimator design. Here we propose a PID filter-based frequency-shaped estimator having proportional plus integral gains rather than just a proportional gain as in the standard case. These result in state estimators with proportional plus integral gains and asymptotic rejection in the state estimation of certain constant plant disturbances entering with the process noise. Such estimators are then combined with the proportional plus integral tracker designs presented in the previous section to achieve frequency-shaped state estimate tracker (shown in Sec. 5).

In general, a proportional plus integral estimator has several imperative properties such as: i) By virtue of the integrator in the loop and nonsingularity conditions, the estimator loop gain is high at low frequencies. It is optimum for a plant process noise which is high in intensity at low frequencies, relative to the plant sensor noise. We conclude that the estimator relies on sensor data more at low frequencies and the model more at high frequencies. This is a desirable property for an estimator in practice, since frequently sensor information is unreliable at high frequencies. Obviously, one could attempt to accomplish similar estimator properties with a signal model which involves a differentiation in a model for the sensor noise, and a nonfrequency-shaped plant process noise; however, such an approach with differentiation of white noise in the model is not a well-defined filtering task; ii) An essential property for robustness to low-frequency disturbances or sensors in modeling at low frequencies is the steady-state response of the estimator. The key property is that with estimator integral gain K_{e2} and $\tilde{C}A^{-1}D_e$ nonsingular then $x_e(t) \to x(t)$, irrespective of the presence of an extra constant disturbance entering the plant through a gain D_e. This disturbance introduces a constant error $-\tilde{C}A^{-1}D_e b$ into $y_\xi(t)$ and $-A^{-1}D_e$ into $x(t)$; the stability of the estimation loop ensures that there will be a corresponding $-\tilde{C}A^{-1}D_e b$ part of $y_{\xi e}(t)$. As a consequence, the input to the integrator in the estimator will not rely on b, while the integrator output will have a DC value b in order to cause $x_e(t) = A^{-1}D_e b$. This is an attractive robustness property not achieved in a standard estimator.

Consider a stochastic signal model based on the deterministic model (5) as

$$\dot{x}_e(t) = Ax_e(t) + Bu_c(t) + D_e v_f(t), \tag{27a}$$

$$y_e(t) = Cx_e(t), \tag{27b}$$

where $x_e(t) \in \Re^n$ and $y_e(t) \in \Re^p$ represent the estimated state and estimated output vectors, respectively. In this model, we include a fictitious colored process disturbance $D_e v_f(t)$. The design procedure of the new PID filter-based optimal linear quadratic state estimator, constructed based on the framework of tracker design, is described as follows, which differs from the concept of traditional observer designs.

Case 1: The number of inputs is less than the number of outputs

Step 1: Transform the non-square non-minimum phase system to a square minimum phase system.

i) Square the signal process model (27) via

$$y_{\xi e}(t) = \xi y_e(t) = \xi\left[Cx_e(t)\right] = \tilde{C}x_e(t), \tag{28}$$

where $\xi \in \Re^{m\times p}$ is a random transformation matrix such that $(A, \tilde{C})$ is an observable pair, in which $\tilde{C} = \xi C$.

ii) Randomly choose a $D_e \in \Re^{n\times m}$ so that the transfer function $P_{e1}(s) = \tilde{C}(sI_n - A)^{-1}D_e$ is square and desirably minimum phase, as well as (A, D_e) is a controllable pair. In this regard, to ensure that the system $(A - D_e K_c,\ D_e E_c,\ \tilde{C})$ is square minimum phase, a control-zero-related two degrees of freedom LQAT in terms of (K_c, E_c) with the high-gain property is used (Ebrahimzadeh *et al.*, 2015).

Remark 3: When A is nonsingular, $\tilde{C}A^{-1}D_e$ is nonsingular. That is, the poles of the integrator (at the origin) must not be cancelled by zeros in the transfer function $\tilde{C}(sI-A)^{-1}D_e$. The non-singularity of $\tilde{C}A^{-1}D_e$ permits infinite loop gains at the origin with use of integral feedback. Whenever A is singular, the plant itself contains a pure integration, and it may not be necessary to use the full integral feedback (Anderson and Moore, 1989).

Step 2: Assign some extra target zeros (without open-loop pole-zero cancellation) to attract some closed-loop poles of the signal process model in a state estimator design.

A PID filter $(k_{ep}+k_{ei}s^{-1}+k_{ed}s)$ has a pair of zeros at specified locations and a pole at the origin. The PID filter gains $\{k_{ep}, k_{ei}, k_{ed}\}$ are selected to achieve some minimum phase target zeros intended to attract some closed-loop poles of the signal process model in a state estimator design. This means that the zero assignments are approximate closed-loop pole assignments.

i) Append the PID filter at the process noise input point as shown in Figure 4. The augmented signal model transfer function of the PID filter-shaped signal process model is given by

$$P_{e2}(s)=\tilde{C}(sI_n-A)^{-1}D_e(K_{eP}+K_{eI}s^{-1}+K_{eD}s). \tag{29}$$

The output of the PID filter can be described as

$$\begin{aligned} V_f(s) &= (K_{eP}+K_{eI}s^{-1}+K_{eD}s)V_2(s) \\ &= \begin{bmatrix} k_{ep}^{(1)}+k_{ei}^{(1)}s^{-1}+k_{ed}^{(1)}s & & 0 \\ & \ddots & \\ 0 & & k_{ep}^{(m)}+k_{ei}^{(m)}s^{-1}+k_{ed}^{(m)}s \end{bmatrix}\left[Y_\xi(s)-Y_{\xi e}(s)\right]. \end{aligned} \tag{30}$$

The corresponding magnitude frequency response in dB is

$$20\log\left|V_f(j\omega)\right| = 20\log\left|K_{eP}-jK_{eI}/\omega+K_{eD}j\omega\right|+20\log\left|Y_\xi(j\omega)-Y_{\xi e}(j\omega)\right|. \tag{31}$$

At low frequency, when ω approaches zero, (31) becomes

$$20\log\left|V_f(j\omega)\right| = 20\log\left|-jK_{eI}/\omega\right|+20\log\left|Y_\xi(j\omega)-Y_{\xi e}(j\omega)\right|. \tag{32}$$

Thus, the integrator emphasizes low frequencies. One can select a large value of integral gain to improve the performance. Similarly, at high frequency, when ω approaches infinity, (31) becomes

$$20\log\left|V_f(j\omega)\right| = 20\log\left|K_{eD}j\omega\right|+20\log\left|Y_\xi(j\omega)-Y_{\xi e}(j\omega)\right|. \tag{33}$$

Thus, the differentiator emphasizes high frequencies. One can select a small value of derivative gain to improve the robustness. Moreover, if the bandwidth is too low to achieve a practical design, then increase the proportional gain to tune the cross-over frequency.

ii) Choose the PID filter gains $\left\{k_{ep}^{(j)}, k_{ei}^{(j)}, k_{ed}^{(j)} \mid j=1\sim m\right\}$ to assign some extra target zeros

$$\left\{\frac{-k_{ep}^{(j)}\pm\sqrt{(k_{ep}^{(j)})^2-4k_{ed}^{(j)}k_{ei}^{(j)}}}{2k_{ed}^{(j)}} \;\middle|\; j=1\sim m\right\}. \tag{34}$$

On the other hand, the numerator polynomial of the scalar PID filter can be written as

$$s^2+\frac{k_{ep}^{(j)}}{k_{ed}^{(j)}}s+\frac{k_{ei}^{(j)}}{k_{ed}^{(j)}}=(s-z_1^{(j)})(s-z_2^{(j)})=s^2-(z_1^{(j)}+z_2^{(j)})s+z_1^{(j)}z_2^{(j)}. \tag{35}$$

Thus, the sum and product of target zeros can be expressed in terms of the PID filter gains

$\{ k_{ep}^{(j)},\ k_{ei}^{(j)},\ k_{ed}^{(j)} \}$ as $z_1^{(j)} + z_2^{(j)} = -\dfrac{k_{ep}^{(j)}}{k_{ed}^{(j)}}$ and $z_1^{(j)} z_2^{(j)} = \dfrac{k_{ei}^{(j)}}{k_{ed}^{(j)}}$, respectively. Designer can assign target zeros at desired location to obtain the PID filter gains.

Step 3: Construct the augmented plant.

i) Define the integral of $\left[y_\xi(t) - y_{\xi e}(t) \right]$ as another state variable

$$x_{eI}(t) = \int_0^t \left[y_\xi(\tau) - y_{\xi e}(\tau) \right] d\tau. \tag{36}$$

ii) The state-space model of the augmented estimator is given by

$$\begin{bmatrix} \dot{x}_e(t) \\ \dot{x}_{eI}(t) \end{bmatrix} = \begin{bmatrix} A & 0_{n\times m} \\ (-\xi C) & 0_{m\times m} \end{bmatrix} \begin{bmatrix} x_e(t) \\ x_{eI}(t) \end{bmatrix} + \begin{bmatrix} D_e \\ 0_{m\times m} \end{bmatrix} v_f(t) + \begin{bmatrix} 0_{n\times 1} \\ y_\xi(t) \end{bmatrix}, \tag{37}$$

where

$$\begin{aligned} v_f(t) &= (K_{eP} + K_{eI} s^{-1} + K_{eD} s) v_2(t) = (K_{eP} + K_{eI} s^{-1} + K_{eD} s) \left[y_\xi(t) - y_{\xi e}(t) \right] \\ &= K_{eP} v_2(t) + K_{eI} x_{eI}(t) + K_{eD} \left[\dot{y}_\xi(t) - (\xi C) \dot{x}_e(t) \right], \end{aligned} \tag{38a}$$

in which

$$\dot{x}_e(t) = A x_e(t) + D_e v_f(t). \tag{38b}$$

The traditional observer design shows that the $Bu_c(t)$ term does not affect the determination of the observer gain, so it does not appear in (38b). Thus, substituting (38b) into (38a) yields

$$v_f(t) = K_{eP} v_2(t) + K_{eI} x_{eI}(t) - K_{eD}\ (\xi C) \left[A x_e(t) + D_e v_f(t) \right] + K_{eD} \dot{y}_\xi(t), \tag{38c}$$

which implies

$$\begin{aligned} v_f(t) &= \left[I_m + K_{eD}\ (\xi C)\ D_e \right]^{-1} K_{eP} v_2(t) + \left[I_m + K_{eD}\ (\xi C)\ D_e \right]^{-1} K_{eI} x_{eI}(t) \\ &\quad - \left[I_m + K_{eD}\ (\xi C)\ D_e \right]^{-1} K_{eD}\ (\xi C) A x_e(t) + \left[I_m + K_{eD}\ (\xi C)\ D_e \right]^{-1} K_{eD} \dot{y}_\xi(t). \end{aligned} \tag{38d}$$

Substituting (38d) into (37) yields the augmented estimator model (shown in Figure 4) as follows

$$\begin{aligned} \begin{bmatrix} \dot{x}_e(t) \\ \dot{x}_{eI}(t) \end{bmatrix} &= \begin{bmatrix} A - D_e \left[I_m + K_{eD}\ (\xi C)\ D_e \right]^{-1} K_{eD}\ (\xi C) A & D_e \left[I_m + K_{eD}\ (\xi C)\ D_e \right]^{-1} K_{eI} \\ (-\xi C) & 0_{m\times m} \end{bmatrix} \begin{bmatrix} x_e(t) \\ x_{eI}(t) \end{bmatrix} \\ &\quad + \begin{bmatrix} D_e \left[I_m + K_{eD}\ (\xi C)\ D_e \right]^{-1} K_{eP} \\ 0_{m\times m} \end{bmatrix} v_2(t) + \begin{bmatrix} D_e \left[I_m + K_{eD}\ (\xi C)\ D_e \right]^{-1} K_{eD} \dot{y}_\xi(t) \\ y_\xi(t) \end{bmatrix} \\ &= A_{e,aug}\, x_{e,aug}(t) + B_{e,aug}\, v_2(t) + d_{e,aug}(t), \end{aligned} \tag{39a}$$

$$\begin{aligned} y_{\xi e}(t) &= \begin{bmatrix} \xi C & 0_{m\times m} \end{bmatrix} \begin{bmatrix} x_e(t) \\ x_{eI}(t) \end{bmatrix} \\ &= C_{e,aug}\, x_{e,aug}(t), \end{aligned} \tag{39b}$$

$$\begin{bmatrix} x_e(0) \\ x_{eI}(0) \end{bmatrix} = C_{e,aug}^{\dagger}\, y_\xi(0),$$

where '†' denotes the pseudoinverse operator, and $\dot{y}_\xi(t)$ in $d_{e,aug}(t)$ can be approximated by

$$\dot{y}_\xi(t) \approx \xi \dot{y}_c(t) = \dot{r}_\xi(t) = \dot{y}_{r_\xi}(t) = C_{r_\xi} \dot{x}_{r_\xi}(t) = C_{r_\xi} \left[A_{r_\xi} x_{r_\xi}(t) + B_{r_\xi} u_{r_\xi}(t) \right], \tag{40}$$

where

$$u_{r_\xi}(t) = -K_{r_\xi} x_{r_\xi}(t) + E_{r_\xi} r_\xi(t), \tag{41a}$$

in which

$$K_{r_\xi} = R_{r_\xi}^{-1} B_{r_\xi}^T P_{r_\xi}, \tag{41b}$$

$$E_{r_\xi} = -R_{r_\xi}^{-1} B_{r_\xi}^T [(A_{r_\xi} - B_{r_\xi} K_{r_\xi})^{-1}]^T C_{r_\xi}^T Q_{r_\xi}, \tag{41c}$$

with the high-gain property, and P_{r_ξ} satisfies the algebraic Riccati equation

$$A_{r_\xi}^T P_{r_\xi} + P_{r_\xi} A_{r_\xi} - P_{r_\xi} B_{r_\xi} R_{r_\xi}^{-1} B_{r_\xi}^T P_{r_\xi} + C_{r_\xi}^T Q_{r_\xi} C_{r_\xi} = 0. \tag{41d}$$

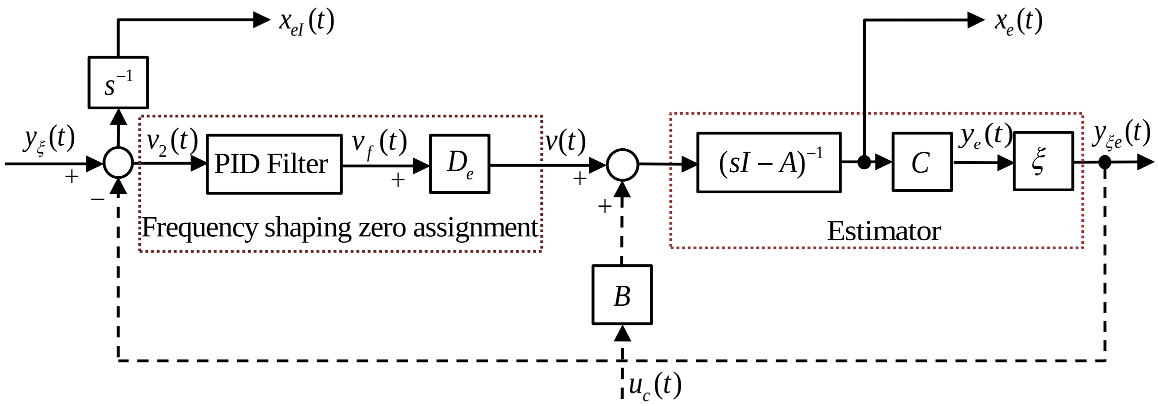


Fig. 4. The augmented estimator model.

Step 4: Formulate the performance index.

The goal of this control problem is to minimize the performance index associated with the augmented estimator model with input $v_f(t)$ and output $y_{\xi e}(t)$ as

$$J = \int_0^\infty \left\{ \left[y_{\xi e}(t) - y_\xi(t) \right]^T Q_e \left[y_{\xi e}(t) - y_\xi(t) \right] + v_f^T(t) R_e v_f(t) \right\} dt, \tag{42a}$$

so that $y_{\xi e}(t)$ well approaches the actual system output $y_\xi(t)$. By Parseval's theorem, the performance index can be written in the frequency domain as

$$\begin{aligned} J &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \left\{ \left[Y_{\xi e}(s) - Y_\xi(s) \right]^* Q_e \left[Y_{\xi e}(s) - Y_\xi(s) \right] \right. \\ &\quad \left. + \left[\left(K_{eP} + K_{eI} s^{-1} + K_{eD} s \right) V_2(s) \right]^* R_e \left[\left(K_{eP} + K_{eI} s^{-1} + K_{eD} s \right) V_2(s) \right] \right\} d\omega \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \left\{ \left[Y_{\xi e}(s) - Y_\xi(s) \right]^* Q_e \left[Y_{\xi e}(s) - Y_\xi(s) \right] \right. \\ &\quad \left. + V_2(s)^* \left[\left(K_{eP} + K_{eI} s^{-1} + K_{eD} s \right)^* R_e \left(K_{eP} + K_{eI} s^{-1} + K_{eD} s \right) \right] V_2(s) \right\} d\omega, \end{aligned} \tag{42b}$$

where the superscript $*$ denotes the complex conjugate transpose and $s = j\omega$. From (42b), the condition $Y_{\xi e}(s) \to Y_\xi(s)$ holds, provided that the weighting function pair $\left\{ Q_e,\ \left(K_{eP} + K_{eI} s^{-1} + K_{eD} s \right)^* R_e \left(K_{eP} + K_{eI} s^{-1} + K_{eD} s \right) \right\}$ has the high-gain property in the interested frequency range in general. Notice that if the weighting matrix pair $\left\{ Q_e, R_e \right\}$ does not have the high-gain property, by appropriately choosing small PID gains, the high-gain property of the above-mentioned weighting function pair can still be satisfied. For the PID filter-shaping-free case, the high-gain property weighting function pair $\left\{ Q_e,\ \left(K_{eP} + K_{eI} s^{-1} + K_{eD} s \right)^* R_e \left(K_{eP} + K_{eI} s^{-1} + K_{eD} s \right) \right\}$ reduces to the required high-gain property weighting matrix pair $\left\{ Q_e, R_e \right\}$ (see Lemma 1 to appear at the end of Sec. 4).

Step 5: Perform the linear quadratic PI state-feedback estimator design.
Use (3) and (4) with an appropriate weighting matrix pair $\{Q_e, R_e\}$ (such as $Q_e >> R_e$) as well as the PID filter gains to have the optimal control law

$$v_2(t) = -K_e x_{e,aug}(t) + E_{e,aug} y_\xi(t) + C_e(t)$$
$$= -\left[K_{e1} \;\vdots\; K_{e2}\right]\begin{bmatrix} x_e(t) \\ x_{eI}(t) \end{bmatrix} + E_{e,aug} y_\xi(t) + C_e(t), \tag{43a}$$

for the augmented estimator model in (39), where

$$K_e = R_e^{-1} B_{e,aug}^T P_{e,aug}, \tag{43b}$$

$$E_{e,aug} = -R_e^{-1}\left[C_{e,aug}\left(A_{e,aug} - B_{e,aug}K_e\right)^{-1} B_{e,aug}\right]^T Q_e, \tag{43c}$$

$$C_e(t) = R_e^{-1} B_{e,aug}^T \left[\left(A_{e,aug} - B_{e,aug}K_e\right)^T\right]^{-1} P_{e,aug}\, d_{e,aug}(t), \tag{43d}$$

in which $P_{e,aug}$ satisfies the algebraic Riccati equation

$$A_{e,aug}^T P_{e,aug} + P_{e,aug} A_{e,aug} - P_{e,aug}^T B_{aug} R_e^{-1} B_{aug}^T P_{aug} + C_{e,aug}^T Q_e C_{e,aug} = 0. \tag{43e}$$

The architecture of the linear quadratic PI state-feedback estimator for the non-square non-minimum phase system is shown in Figure 5.

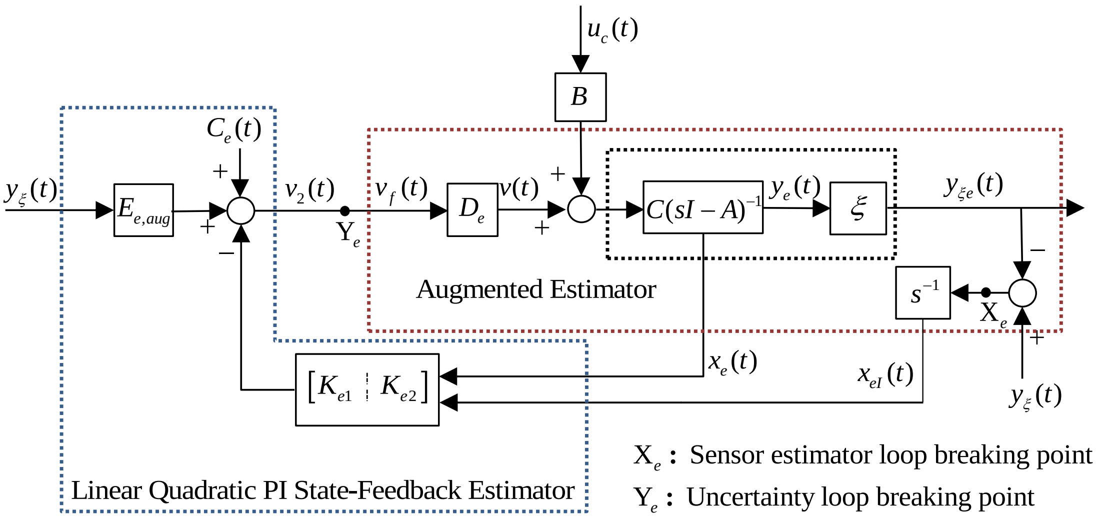


Figure 5. Linear quadratic PI state-feedback estimator.

Step 6: Examine the open-loop frequency response and adjust weights.
Open the sensor loop at point X_e shown in Figure 5 at the first sensor and examine the sensor (estimator) cross-over frequency, which we recall approximates the closed-loop estimator bandwidth. If this bandwidth is too high (low) to achieve a practical design, then increase (decrease) the first component of the diagonal matrix Q_e. Repeat for all sensors. Likewise when the uncertainty loop is opened at point Y_e shown in Figure 5, and the following rule applied. Increase (decrease) the ith diagonal element of R_e, if the uncertainty loop bandwidth is too low (high) for a practical design.

Repeat for all *i*. The uncertainty loop and sensor estimator loop can be respectively derived as

$$L_{\mathrm{Ye}}(s)=\left(K_{e1}+K_{e2}\tilde{C}s^{-1}\right)\left(sI_n-A\right)^{-1}D_e \tag{44}$$

and

$$L_{\mathrm{Xe}}(s)=\tilde{C}\left[sI_n-\left(A-D_eK_{e1}\right)\right]^{-1}D_eK_{e2}s^{-1}. \tag{45}$$

Case 2: The number of inputs is greater than or equal to the number of outputs

Step 1: Transform the non-square non-minimum phase system to a square minimum phase system.

i) Modify the signal process model (27) via

$$y_{\xi e}(t)=\xi y_e(t)=\xi\left[Cx_e(t)\right]=\tilde{C}x_e(t), \tag{46}$$

where $\xi\in\Re^{p\times p}$ is a non-singular transformation matrix such that $(A,\tilde{C})$ is an observable pair, in which $\tilde{C}=\xi C$.

ii) Randomly choose a $D_e\in\Re^{n\times p}$ so that the transfer function $P_{e1}(s)=\tilde{C}(sI_n-A)^{-1}D_e$ is square and desirably minimum phase, as well as (A,D_e) is a controllable pair. In this regard, to ensure that the system $(A-D_eK_c,\ D_eE_c,\ \tilde{C})$ is square minimum phase, a control-zero-related two degrees of freedom LQAT in terms of (K_c,E_c) with the high-gain property is used (Ebrahimzadeh et al., 2015). Set $\tilde{C}=C$, i.e. $\xi=I_p$, in Steps i) and ii), if $C(sI_n-A)^{-1}D_e$ is desirably minimum phase.

Step 2: Assign some extra target zeros (without open-loop pole-zero cancellation) to attract some closed-loop poles of the signal process model in a state estimator design.

i) Append the PID filter at the process noise input point. The output of the PID filter is given by

$$\begin{aligned}V_f(s)&=(K_{eP}+K_{eI}s^{-1}+K_{eD}s)V_2(s)\\&=\begin{bmatrix}k_{ep}^{(1)}+k_{ei}^{(1)}s^{-1}+k_{ed}^{(1)}s & & 0\\ & \ddots & \\ 0 & & k_{ep}^{(p)}+k_{ei}^{(p)}s^{-1}+k_{ed}^{(p)}s\end{bmatrix}\left[Y_\xi(s)-Y_{\xi e}(s)\right].\end{aligned} \tag{47}$$

ii) Choose the PID filter gains $\left\{\ k_{ep}^{(j)},\ k_{ei}^{(j)},\ k_{ed}^{(j)}\ |\ j=1\sim p\ \right\}$ to assign target zeros

$$\left\{\frac{-k_{ep}^{(j)}\pm\sqrt{(k_{ep}^{(j)})^2-4k_{ed}^{(j)}k_{ei}^{(j)}}}{2k_{ed}^{(j)}}\ \Big|\ j=1\sim p\right\}. \tag{48}$$

Step 3: Construct the augmented plant.

i) Define the integral of $\left[y_\xi(t)-y_{\xi e}(t)\right]$ as another state variable

$$x_{eI}(t)=\int_0^t\left[y_\xi(\tau)-y_{\xi e}(\tau)\right]d\tau. \tag{49}$$

ii) The state-space model of the augmented estimator is given by

$$\begin{bmatrix}\dot{x}_e(t)\\ \dot{x}_{eI}(t)\end{bmatrix}=\begin{bmatrix}A & 0_{n\times p}\\ (-\xi C) & 0_{p\times p}\end{bmatrix}\begin{bmatrix}x_e(t)\\ x_{eI}(t)\end{bmatrix}+\begin{bmatrix}D_e\\ 0_{p\times p}\end{bmatrix}v_f(t)+\begin{bmatrix}0_{n\times 1}\\ y_\xi(t)\end{bmatrix}. \tag{50}$$

Similar to Case 1, it can be described as

$$\begin{bmatrix} \dot{x}_e(t) \\ \dot{x}_{eI}(t) \end{bmatrix} = \begin{bmatrix} A - D_e\left[I_p + K_{eD}\ (\xi C)\ D_e\right]^{-1} K_{eD}\ (\xi C)A & D_e\left[I_p + K_{eD}\ (\xi C)\ D_e\right]^{-1} K_{eI} \\ (-\xi C) & 0_{p\times p} \end{bmatrix} \begin{bmatrix} x_e(t) \\ x_{eI}(t) \end{bmatrix}$$

$$+\begin{bmatrix} D_e\left[I_p + K_{eD}\ (\xi C)\ D_e\right]^{-1} K_{eP} \\ 0_{p\times p} \end{bmatrix} v_2(t) + \begin{bmatrix} D_e\left[I_p + K_{eD}\ (\xi C)\ D_e\right]^{-1} K_{eD}\dot{y}_\xi(t) \\ y_\xi(t) \end{bmatrix} \tag{51a}$$

$$= A_{e,aug}\, x_{e,aug}(t) + B_{e,aug}\, v_2(t) + d_{e,aug}(t),$$

$$y_{\xi e}(t) = \begin{bmatrix} \xi C & 0_{p\times p} \end{bmatrix} \begin{bmatrix} x_e(t) \\ x_{eI}(t) \end{bmatrix}$$

$$= C_{e,aug}\, x_{e,aug}(t),$$

$$\tag{51b}$$

$$\begin{bmatrix} x_e(0) \\ x_{eI}(0) \end{bmatrix} = C^{\dagger}_{e,aug}\, y_\xi(0).$$

Step 4: Formulate the performance index.
The goal of this control problem is to minimize the performance index associated with the augmented estimator with input $v_f(t)$ and output $y_{\xi e}(t)$ as

$$J = \int_0^\infty \left\{ \left[y_{\xi e}(t) - y_\xi(t)\right]^T Q_e \left[y_{\xi e}(t) - y_\xi(t)\right] + v_f^T(t) R_e v_f(t) \right\} dt, \tag{52a}$$

so that $y_{\xi e}(t)$ well approaches the actual system output $y_\xi(t)$. By Parseval's theorem, the performance index can be written in the frequency domain as

$$\begin{aligned} J &= \frac{1}{2\pi}\int_{-\infty}^{\infty} \Big\{ \left[Y_{\xi e}(s) - Y_\xi(s)\right]^* Q_e \left[Y_{\xi e}(s) - Y_\xi(s)\right] \\ &\quad + \left[\left(K_{eP} + K_{eI}s^{-1} + K_{eD}s\right)V_2(s)\right]^* R_e \left[\left(K_{eP} + K_{eI}s^{-1} + K_{eD}s\right)V_2(s)\right] \Big\} d\omega \\ &= \frac{1}{2\pi}\int_{-\infty}^{\infty} \Big\{ \left[Y_{\xi e}(s) - Y_\xi(s)\right]^* Q_e \left[Y_{\xi e}(s) - Y_\xi(s)\right] \\ &\quad + V_2(s)^* \left[\left(K_{eP} + K_{eI}s^{-1} + K_{eD}s\right)^* R_e \left(K_{eP} + K_{eI}s^{-1} + K_{eD}s\right)\right] V_2(s) \Big\} d\omega. \end{aligned} \tag{52b}$$

Similarly, the condition $Y_{\xi e}(s) \to Y_\xi(s)$ holds, provided that the weighting function pair $\left\{Q_e,\ \left(K_{eP} + K_{eI}s^{-1} + K_{eD}s\right)^* R_e \left(K_{eP} + K_{eI}s^{-1} + K_{eD}s\right)\right\}$ has the high-gain property in the interested frequency range in general. Besides $Y_{\xi e}(s) \to Y_\xi(s)$ implies $Y_e(s) \to Y_c(s)$, since ξ is a square non-singular transformation matrix.

Step 5: Perform the linear quadratic PI state-feedback estimator design.
Repeat Step 5 in Case 1 to have the optimal control law

$$v_2(t) = -K_e x_{e,aug}(t) + E_{e,aug}\, y_\xi(t) + C_e(t). \tag{53}$$

Step 6: Examine the open-loop frequency response and adjust weights.
Repeat Step 6 in Case 1. Likewise, the sensor estimator loop and uncertainty loop can be respectively derived as

$$L_{\mathrm{Ye}}(s) = \left(K_{e1} + K_{e2}\tilde{C}s^{-1}\right)\left(sI_n - A\right)^{-1} D_e \tag{54}$$

and

$$L_{\mathrm{Xe}}(s)=\tilde{C}\left[sI_n-\left(A-D_eK_{e1}\right)\right]^{-1}D_eK_{e2}s^{-1}. \tag{55}$$

The quantitative statement for the high-gain property controller mentioned in Step 4 is given as follows.

Lemma 1 (Tsai et al., 2014b) Given the analog system in terms of the pair of system matrices $\{A,B,C,D\}$, let a pair of weighting matrices $\{Q,R\}$ be given as diagonal matrices $Q=qI_P>>R$ and $R=rI_m>0$. There exists the lower bound of the weighting matrix pair $\{Q^*,R^*\}$, i.e. $Q^*=q^*I_p$ and $R^*=r^*I_m$, determined by

$$\kappa^*=\sqrt{\frac{\|B^TB\|\|C^TC\|}{\|A^TA\|}\left(\frac{q^*}{r^*}\right)},$$

as long as the property of the high-gain control still holds, that is $P_2\approx P_1$ for $\zeta=\kappa_2/\kappa_1=\sqrt{\frac{\|B^TB\|\|C^TC\|}{\|A^TA\|}\left(\frac{q_2}{r_2}\right)}\Big/\sqrt{\frac{\|B^TB\|\|C^TC\|}{\|A^TA\|}\left(\frac{q_1}{r_1}\right)}=\sqrt{\frac{q_2}{r_2}}\Big/\sqrt{\frac{q_1}{r_1}}$ and $\kappa_2>\kappa_1\geq\kappa^*$, where P_1 and P_2 are the symmetric positive-definite solutions of the following Riccati equations, respectively,

$$A^TP_1+P_1A+C^TQ_1C-(P_1B+C^TQ_1D)(R_1+D^TQ_1D)^{-1}(B^TP_1+D^TQ_1C)=0, \tag{56}$$

$$A^TP_2+P_2A+C^TQ_2C-(P_2B+C^TQ_2D)(R_2+D^TQ_2D)^{-1}(B^TP_2+D^TQ_2C)=0. \tag{57}$$

It is remarkable to notice that the high-gain property $P_2\approx P_1$ for the proper system $\{A,B,C,D\}$ shall be revised as $P_2\approx\kappa_2P_1$ for the strictly proper system $\{A,B,C\}$ (Tsai et al., 2011; Tsai et al., 2014a). □

5. Proportional plus integral state estimate tracker

The frequency-shaped design approach of this paper is to use proportional plus integral estimators in conjunction with proportional plus integral state feedback tracker designs, replacing states in the control law with state estimates. There results what we term proportional plus integral LQG trackers. Clearly they are a subset of the class of all stabilizing controllers.

The eigenvalue separation property holds for the proportional plus integral LQG designs, being a separation of the eigenvalue separation property of more general frequency-shaped design. By assigning the frequency-shaping minimum phase zeros, there is a crude assignment of those closed-loop poles attracted to those zeros in a closed-loop design. By adjustment of the weights in the individual state feedback tracker and estimator design, there is achieved a compromise in term of control loop and estimator loop bandwidths. In this regard, a significant property is that these sensor, reference, and uncertainty loop gains associated with the estimator and state feedback tracker remain invariant upon switching to state estimator feedback. Therefore, from (43) we derive the state estimated optimal tracker as

$$v_2(t) = -\begin{bmatrix} K_{e1} & K_{e2} \end{bmatrix}\begin{bmatrix} x_e(t) \\ x_{eI}(t) \end{bmatrix} + E_{e,aug}\, y_\xi(t)$$
$$+R_e^{-1}B_{e,aug}^T \left[\left(A_{e,aug} - B_{e,aug}K_e \right)^T \right]^{-1} P_{e,aug} \begin{bmatrix} D_e \left[I_m + K_{eD}\ (\xi C)\ D_e \right]^{-1} K_{eD}\dot{y}_\xi(t) \\ y_\xi(t) \end{bmatrix}, \tag{58}$$

where

$$\dot{y}_\xi(t) = C_{r_\xi}\left[A_{r_\xi} x_{r_\xi}(t) + B_{r_\xi} u_{r_\xi}(t) \right],$$

and

$$u_{r_\xi}(t) = -K_{r_\xi} x_{r_\xi}(t) + E_{r_\xi} r_\xi(t).$$

Substituting (58) into the plant $P_{e1}(s)$ (integrated with $Bu_c(t)$) yields

$$\dot{x}_e(t) = (A - D_e K_{e1})x_e(t) + Bu_c(t) - D_e K_{e2}\int_0^t \left[y_\xi(\tau) - y_{\xi e}(\tau) \right] d\tau + D_e E_{e,aug} y_\xi(t) + D_e C_e(t). \tag{59}$$

Notice that (52b) shows the determined signal $v_2(t)$ results in $y_{\xi e}(t) \to y_\xi(t)$ and $y_e(t) \to y_c(t)$ for the selected signal process model $(A,\ D_e,\ C)$, provided that the frequency-domain weighting function pair $\{Q_e, \tilde{R}_e(s)\} = \left\{Q_e,\ \left(K_{eP} + K_{eI}s^{-1} + K_{eD}s\right)^* R_e\left(K_{eP} + K_{eI}s^{-1} + K_{eD}s\right)\right\}$ has the high-gain property in the interested frequency band. From another point of view, the corresponding observer signal $v_f(t) = \left(K_{eP} + K_{eI}s^{-1} + K_{eD}s\right)v_2(t)$ shown in Figure 5 and (52a) also results in $y_{\xi e}(t) \to y_\xi(t)$ and $y_e(t) \to y_c(t)$ for the signal process model $(A,\ D_e,\ C)$.

Consequently, we perform the universal PI observer-based LQAT presented in Sec. 3 for the square and/or non-square non-minimum phase plants in the following way. According to (23), the proportional plus integral observer-based optimal linear quadratic tracker for the non-minimum phase plants can be represented as follows

$$u_c(t) = -\bar{K}_c x_{aug}(t) + \bar{E}_c r_c(t). \tag{60}$$

The architecture of the linear quadratic PI state estimate feedback for the square and/or non-square non-minimum phase system is shown in Figure 6.

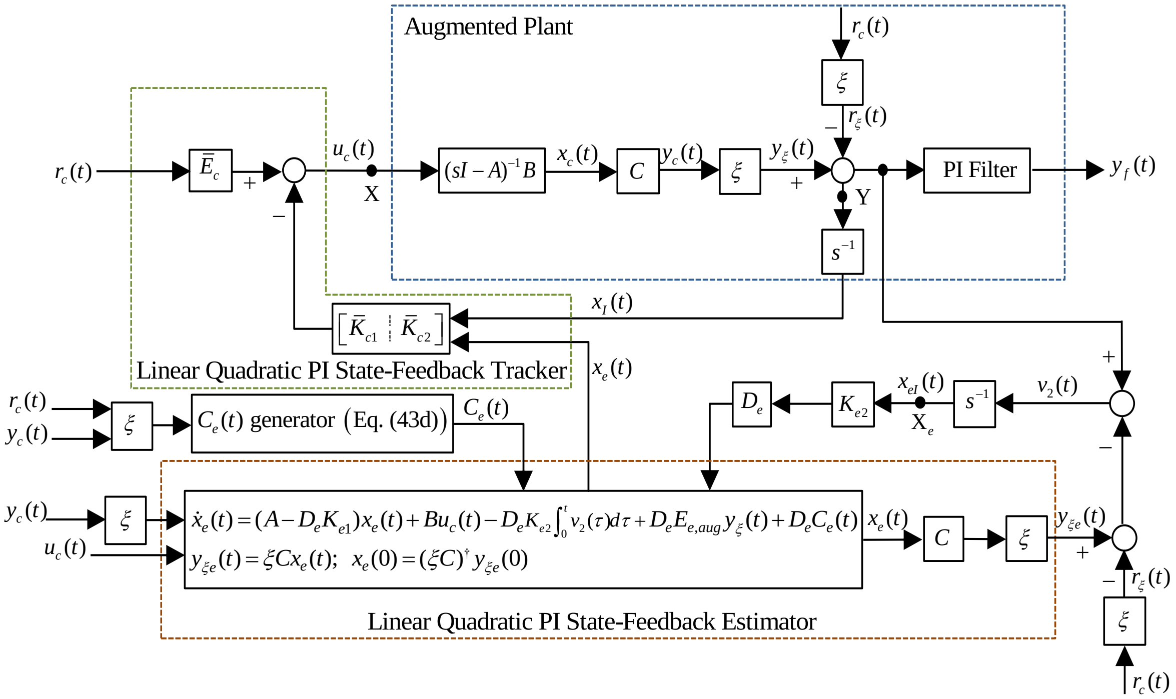


Figure 6. Proportional plus integral state estimate feedback control system.

6. Illustrative example

In this section, some numerical simulations are given to illustrate the proposed proportional plus integral optimal linear quadratic state estimate tracker for the non-minimum phase systems.

Consider the following square non-minimum phase system

$$\dot{x}_c(t) = \begin{bmatrix} -1.4158 & -1.3610 & 1.0212 & 0.3493 \\ 0.0596 & 0.7796 & -0.8740 & -0.7292 \\ -0.4113 & 0.4394 & 0.4147 & 0.3268 \\ -0.3680 & -0.0896 & 0.3484 & -0.5149 \end{bmatrix} x_c(t) + \begin{bmatrix} -0.8964 & -0.1729 \\ -1.2033 & -1.2087 \\ 1.0378 & -0.2971 \\ -0.8459 & -3.2320 \end{bmatrix} u_c(t),$$

$$y_c(t) = \begin{bmatrix} -1.0870 & -1.0145 & -0.3253 & -0.5718 \\ -1.4262 & -0.2133 & 1.9444 & -0.2500 \end{bmatrix} x_c(t),$$

$$x_c(0) = \begin{bmatrix} 0.01 & 0.03 & -0.02 & 0.02 \end{bmatrix}^T,$$

with system poles $\{0.5857 \pm 0.8567i, -1.3675, -0.5403\}$ and control zeros of $(A - BK_a, BE_a, C)$ are $\{1.5716, -0.9930, -2.0999\times10^4, -5.2025\times10^4\}$ for $Q_c = 10^6 I_2$ and $R_c = I_2$. It's desired to determine an optimal LQAT, so that the compensated non-minimum phase system (NMP) system demonstrates a good minimum phase-like controlled system to track the reference trajectory $r_c(t) = \begin{bmatrix} r_1(t) & r_2(t) \end{bmatrix}^T$, where

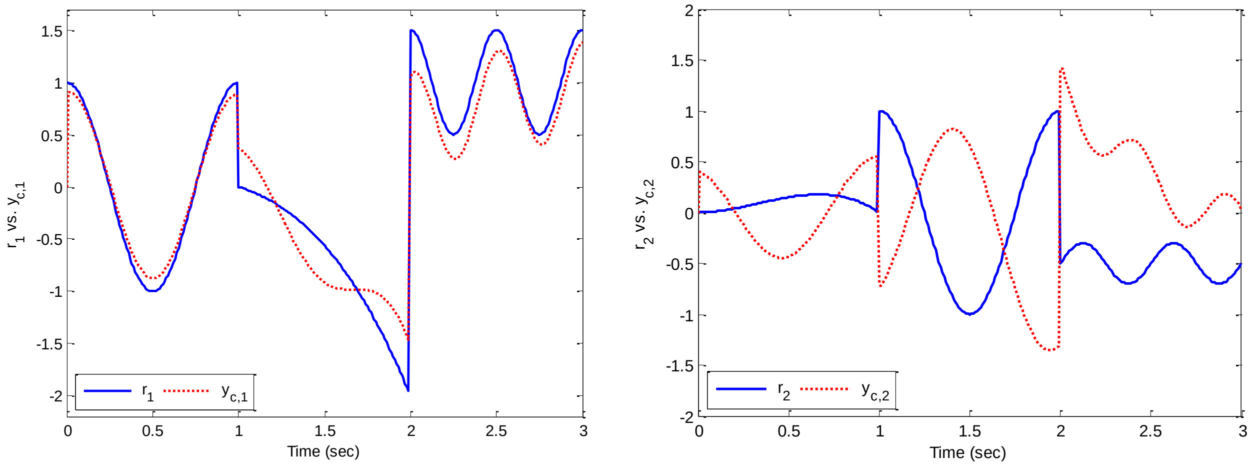
$$r_1(t) = \begin{cases} \cos(2\pi t) & , 0 \le t < 1 \text{ sec} \\ 0.5\, t^2(1-t) & , 1 \le t < 2 \text{ sec} \\ 0.5\, \cos(4\pi t) + 1 & , 2 \le t \le 3 \text{ sec} \end{cases} \text{ and } r_2(t) = \begin{cases} 1.2\, t^2(1-t) & , 0 \le t < 1 \text{ sec} \\ \cos(2\pi t) & , 1 \le t < 2 \text{ sec} \\ 0.2\, \sin(4\pi t) - 0.5 & , 2 \le t \le 3 \text{ sec} \end{cases}.$$

For comparison, apply the traditional optimal LQAT (Lewis, 1992; Lewis and Syrmos, 1995) to the NMP system, where

$$u_c(t) = -K_c x_c(t) + E_c r(t),$$

$$K_c = 10^3 \times \begin{bmatrix} -0.5171 & -3.2610 & 4.2664 & 0.7582 \\ -1.2593 & 0.3621 & -0.1007 & -1.0227 \end{bmatrix}, \; E_c = 10^2 \begin{bmatrix} 5.9891 & -8.0082 \\ 8.0082 & 5.9891 \end{bmatrix},$$

and P satisfies the Riccati equation (4) with $Q_c = 10^6 I_2$ and $R_c = I_2$. The resulting closed-loop eigenvalues are $\sigma(A - BK_c) = \{-1.5716, -0.9930, -2.0999\times10^3, -5.2025\times10^3\} \rightarrow \{-z_1, z_2, z_3, z_4\}$. Simulation results given in Figure 7 demonstrate a poor tracking performance due to the nature of a NMP system, where the serious undershoot causes a delay in the system response, and it can't be improved just by the traditional state-feedback/output feedback control.



(a) (b)

Figure 7. Tracking responses of the traditional optimal LQT-based controlled NMP system (a) $r_1(t)$ vs. $y_{c1}(t)$, (b) $r_2(t)$ vs. $y_{c2}(t)$.

To demonstrate the effectiveness of the proposed design methodology, here we consider the following steps presented in Sec. 3.

Step 1: Transform the non-minimum phase system to a fictitious minimum phase system.

Let the selected 2×2 direct feed-through term be $D=\begin{bmatrix} -3.4354 & 1.1625 \\ -2.0602 & 0.6860 \end{bmatrix}$ to ensure that the fictitious system $(A,\ B,\ C,\ D)$ is minimum phase. The resulting minimum phase control zeros of $(A-BK_a,\ BE_a,\ C-DK_a,\ DE_a)$ for $Q_c=10^6 I_2$ and $R_c=I_2$ are $\{-0.2208, -0.8679, -1.7895, -40.4090\}$.

Step 2: Assign some target zeros to attract some closed-loop poles in a closed-loop design.
Let extra target zeros be $\{-0.01,\ -0.01\}$, then one can select the PI filter gains as

$$K_P=\begin{bmatrix} 5\times 10^4 & 0 \\ 0 & 5\times 10^4 \end{bmatrix} \text{ and } K_I=\begin{bmatrix} 500 & 0 \\ 0 & 500 \end{bmatrix}.$$

Step 3: Perform the linear quadratic PI state-feedback tracker design.
In the LQAT design, choose an appropriate weighting matrix pair $\{\ Q_c,\ R_c\ \}=\{\ I_2,\ 10^3 I_2\ \}$ to have the optimal control law

$$u_c(t)=-K_c x_{aug}(t)+E_c r_a(t)+C_c(t),$$

where the PI state-feedback and forward gains are

$$K_c=\left[K_{c1} \;\vdots\; K_{c2}\right]$$
$$=10^3\times\left[\begin{array}{cccc:cc} 0.0238 & -0.0117 & -0.0648 & -0.0026 & 0.0002 & -0.0003 \\ 0.0695 & -0.0354 & -0.1919 & -0.0083 & 0.0005 & -0.0009 \end{array}\right],$$

$$E_c=10^{-4}\times\begin{bmatrix} -3.58 & -6.07 \\ 10.76 & -17.94 \end{bmatrix}.$$

The tracking response is shown in Figure 8, and the resulting closed-loop poles of the augmented system, $(A_{aug}, B_{aug}, C_{aug}, D_{aug})$,
$\sigma(A_{aug} - B_{aug}K_c) = \{-0.2208, -0.8679, -1.7895, -40.3102, -0.01, -0.01\}$ approach to the open-loop zeros of the augmented system $\{z_1, z_2, z_3, z_4, z_5, z_6\} = \{-0.2208, -0.8679, -1.7895, -40.4090, -0.01, -0.01\}$. It shows that $y_c(t) \to r_c(t)$ as expected.

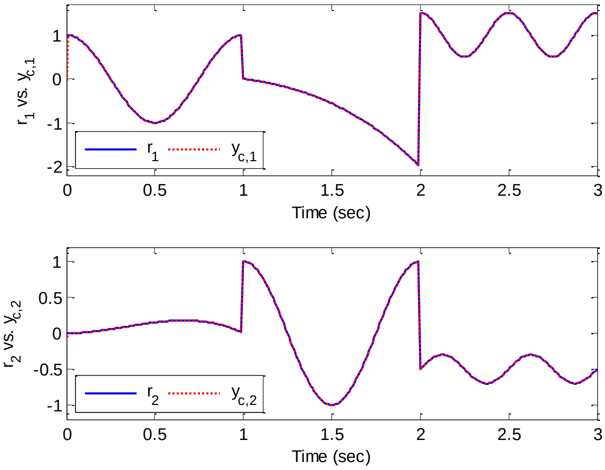


Figure 8. Tracking responses of the proposed optimal PI state-feedback controlled NMP system.

Open loop frequency responses for $Q_c = I_2$ and $R_c = 10^3 I_2$ are shown in Figure 9. At low frequency, the control loop has quite high dc gain, so the closed-loop system has good external constant disturbance rejection. Moreover, the bandwidth of the reference loop is very low, so the closed-loop system has good sensor noise rejection.

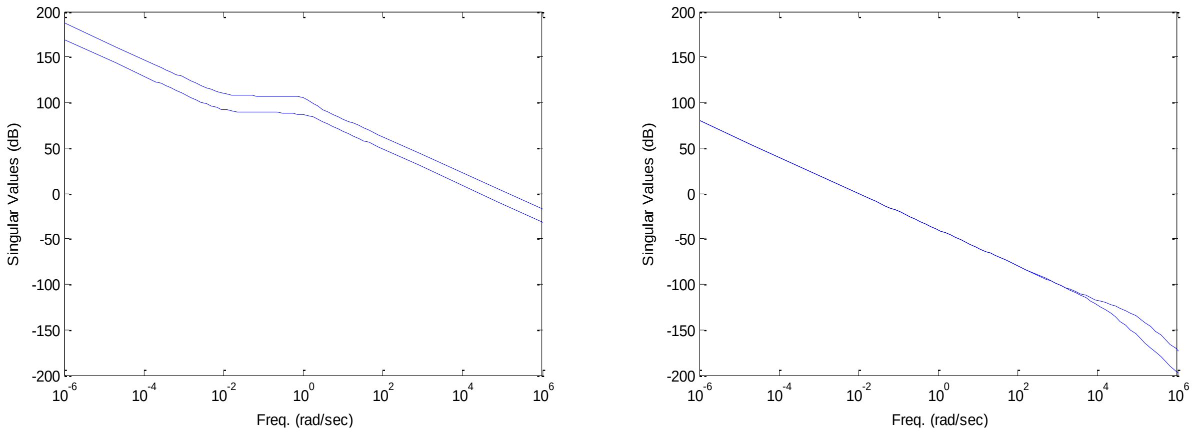


Figure 9. Open-loop frequency responses: (a) control loop in (24), (b) reference loop in (25).

- **Estimator design**

Step 4: Transform the non-minimum phase signal process model to a minimum phase signal process model.

Let us select matrix $D_e = \begin{bmatrix} -1.3826 & -0.8710 & 0.5499 & 0.1576 \\ 1.3846 & -0.6101 & -0.2537 & 1.0291 \end{bmatrix}^T$ to ensure that the transfer function $P_{e1}(s)=C(sI-A)^{-1}D_e$ is desirably minimum phase. The resulting minimum phase control zeros are $\{z_{e1},\ z_{e2},\ z_{e3},\ z_{e4}\}=\{-3.7549,\ -1.8193,\ -4.824\times 10^4,\ -2.188\times 10^3\}$.

Step 5: Assign target zeros to attract the closed-loop poles in a closed-loop design.
Let extra target zeros be $\{z_{e5},\ z_{e6},\ z_{e7},\ z_{e8}\}=\{-10^{-2},\ -10^{-2},\ -10^5,\ -10^5\}$, then one can select the PID filter gains as

$$K_{eP}=\begin{bmatrix}100 & 0\\ 0 & 100\end{bmatrix},\ K_{eI}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},\ \text{and}\ \ K_{eD}=\begin{bmatrix}0.001 & 0\\ 0 & 0.001\end{bmatrix}.$$

Step 6: Perform the linear quadratic PI state-feedback estimator design.
Choose an appropriate weighting matrix pair $\{\ Q_e,\ R_e\ \}=\{\ 10^8 I_2,\ I_2\ \}$ to have the optimal control law

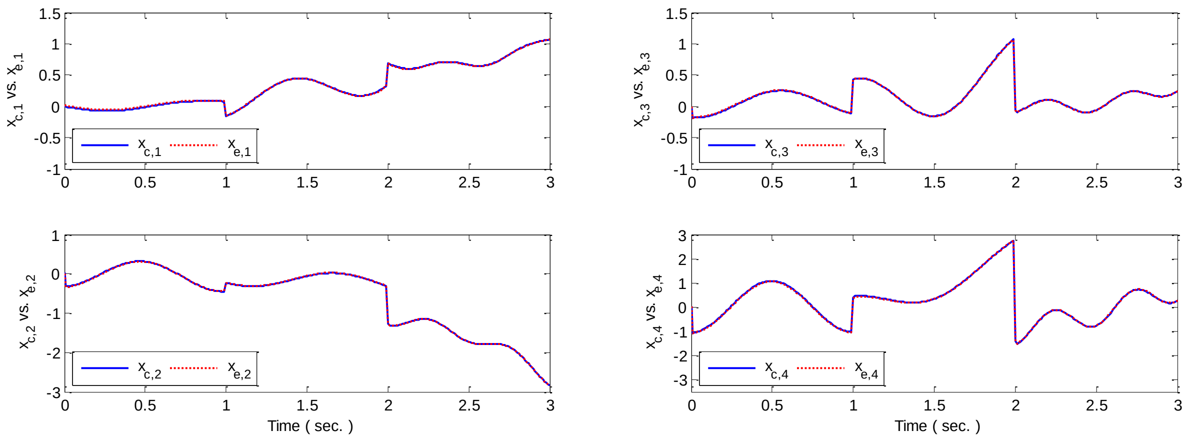
$$v_2(t)=-K_e x_{e,aug}(t)+E_{e,aug}y_\xi(t)+C_{e,aug}(t),$$

where

$$K_e=\left[K_{e1}\ \vdots\ K_{e2}\right]=10^4\times\left[\begin{array}{cccc:cc}-1.5237 & -1.0240 & 0.3884 & -0.6233 & 0.0000 & -0.0000\\ 0.9459 & -0.1619 & -1.9328 & 0.0300 & -0.0000 & 0.0000\end{array}\right],$$

$$E_{e,aug}=10^{-7}\times\begin{bmatrix}1.9280 & -11.8520\\ 4.8257 & 3.2164\end{bmatrix}.$$

Figure 5 and (59) show that the closed-loop poles of the estimator are $\sigma(A-D_eK_{e1})=\{-3.7687,-1.8176,-4.824\times 10^4,-2.188\times 10^3\}\rightarrow\{z_{e1},\ z_{e2},\ z_{e3},\ z_{e4}\}$. The performance of the estimator is shown in Figure 10. It shows that $x_e(t)\rightarrow x_c(t)$, which implies $y_e(t)=Cx_e(t)\rightarrow y_c(t)=Cx_c(t)$ also.



(a) (b)

Figure 10. The performance of the estimator: $x_c(t)$ vs. $x_e(t)$.

- **State estimate tracker design**

Step 7: Realize the linear quadratic PI state-estimate tracker design.
Realize the optimal control law

$$u_c(t)=-\bar{K}_c x_{aug}(t)+\bar{E}_c r_c(t),$$

where from (23) the PI state-feedback and feed-forward gains are

$$\bar{K}_c = \left[\bar{K}_{c1} \mid \bar{K}_{c2} \right]$$
$$= 10^5 \times \left[\begin{array}{cccc|cc} -0.1264 & -0.1832 & -0.1760 & -0.0957 & 0.0019 & -0.0006 \\ 0.0297 & -0.3374 & -0.6656 & -0.1542 & 0.0039 & -0.0028 \end{array} \right],$$
$$\bar{E}_c = 10^4 \times \begin{bmatrix} 1.9279 & -0.5827 \\ 3.9073 & -2.7694 \end{bmatrix}.$$

The tracking response and control input of the state-estimate closed-loop system are shown in Figure 11. Figures 10 and 11 show that $y_e(t) \to y_c(t) \to r_c(t)$, as expected, where $r_c(t)$ is an arbitrary command input. This also implies a full loop recovery is achieved by the proposed design methodology, since it holds for arbitrary command inputs $r_c(t)$. Such results constitute a powerful approach to loop recovery for reduced-order state estimator-based tracker designs. Such a standard application is the system identification-based state-space (reduced-order) estimator-based self-tuner designs for the unknown linear/nonlinear/time-delay systems (Tsai et al., 2014a;Lee et al., 2005; Wu et al., 2015). For the system identification, the (reduced-order) state estimator is constructed under the principle that the estimated output $y_e(kT)$ approaches to the actual output $y_c(kT)$ as possible, where T is the sampling period and k is the sampling index.

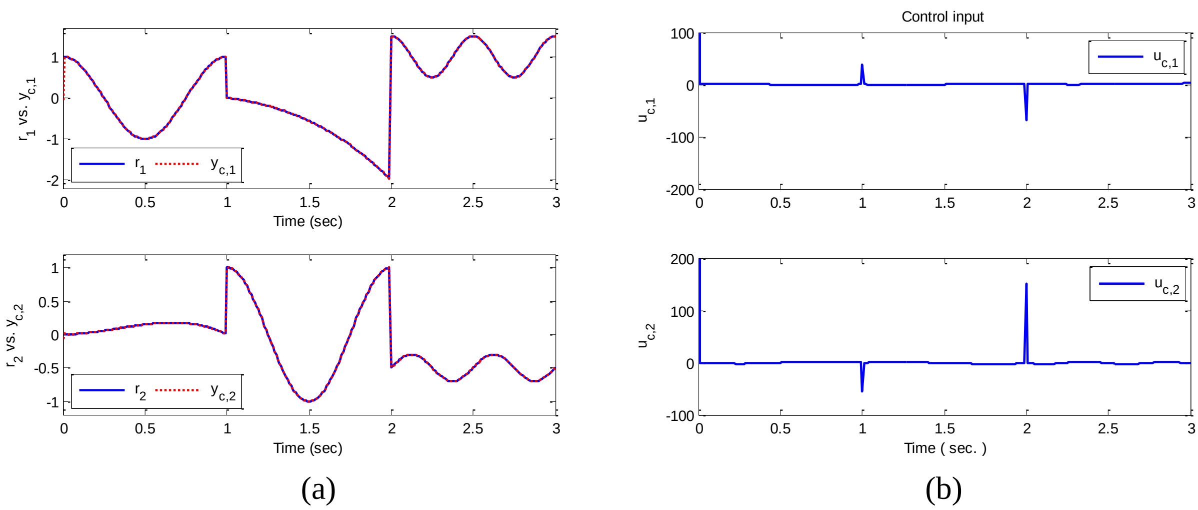
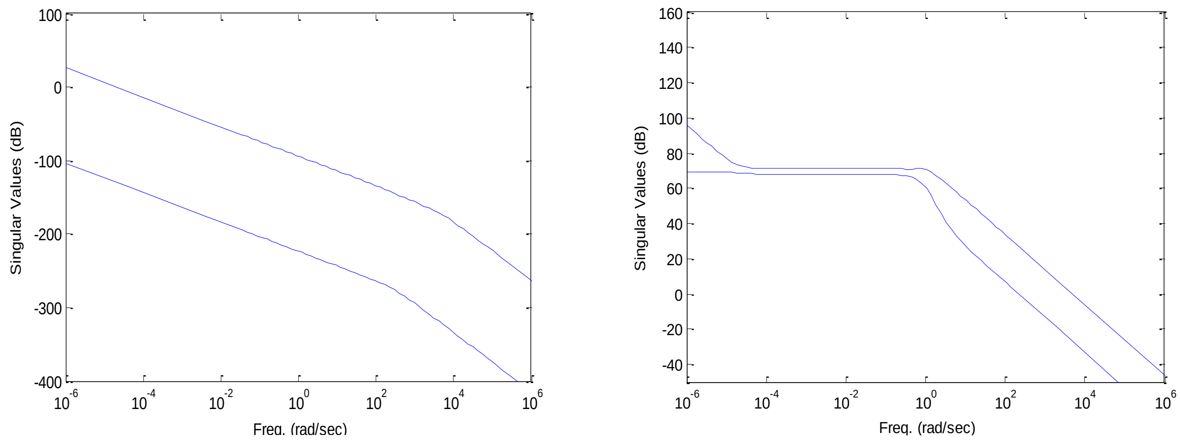


Figure 11. Tracking response and control input of the closed-loop system: (a) $r_c(t)$ vs. $y_c(t)$, (b) control input $u_c(t)$.

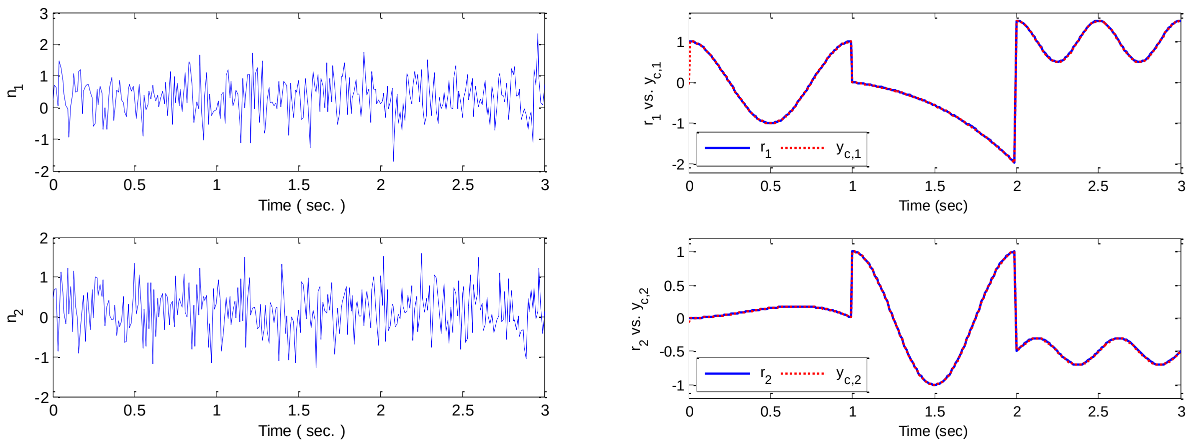
The open-loop frequency responses for $Q_e = 10^8 I_2$ and $R_e = I_2$ are shown in Figure 12. The bandwidth of the uncertainty loop is very low, so the closed-loop system has good sensor noise rejection. Moreover, at low frequency, the sensor estimator loop has quite high dc gain, so the closed-loop system has good external constant disturbance rejection.



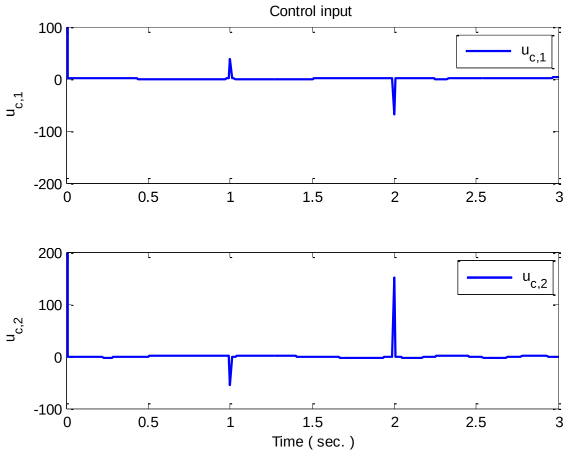
(a) (b)

Figure 12. Open-loop frequency responses: (a) uncertainty loop in (44), (b) sensor estimator loop in (45).

Step 8: Test for robustness of the linear quadratic PI state-estimate tracker design—A case study. Two normal distribution color noise sequences with means $[0.3\ \ 0.2]^T$ and standard deviations $[0.6\ \ 0.6]^T$ (shown in Figure 13 (a)) are artificially injected into the plant output terminal and the estimator input terminal, simultaneously, during the whole simulation process for the controlled system with a pre-specified command input $r_c(t)$ bounded around $[-1, 1]$. Simulation results shown in Figure 13 demonstrate that the reference loop and the uncertainty loop have good sensor noise rejections, respectively.



(a) (b)



(c)

Figure 13. Color noise disturbances, tracking responses and control inputs of the controlled closed-loop system with unknown disturbances: (a) color noise disturbance $n(t)$, (b) $r_c(t)$ vs. $y_c(t)$, (c) control input $u_c(t)$.

7. Conclusion

For the given non-minimum phase plants, classical control theory tells us that it may be difficult or impossible to design a well-performed tracker for arbitrary command inputs as well as robust state-estimate trackers. In this paper, a new optimal PI filter-shaped linear quadratic PI state-feedback tracker for the given continuous-time square and/or non-square non-minimum phase plants is presented. Here, a fictitious input-to-output direct feed-through term D is well selected and added to the non-minimum phase plant, so that the fictitious plant (A, B, C, D) becomes minimum phase in the sense of the closed-loop multivariable control zero assignment. Then, based on the PI filter-based frequency shaping approach, a new optimal PI filter-based state-feedback linear quadratic tracker for the non-minimum phase plant is developed, so that the controlled fictitious system achieves a good minimum phase-like tracking performance for arbitrary command inputs. Finally, the controlled fictitious system is realized by the original plant (A, B, C) with an equivalent two-degrees-of-freedom tracker.

This paper also proposes a new PID filter-shaped optimal linear quadratic PI state estimator, constructed based on the framework of tracker design methodology so that a minimum phase model can be easily constructed for the state estimator design. Finally, the new integrated optimal PI state estimate tracker is then presented in paper, so that the tracker, state estimator, and the state estimate tracker-based controlled systems respectively achieve good minimum phase-like tracking performances for arbitrary command inputs. Such results constitute a powerful approach to loop recovery for reduced-order state estimator-based tracker designs. Such a standard application is the system identification-based state-space (reduced-order) estimator-based self-tuner designs for the unknown linear/nonlinear/time-delay systems.

Acknowledgments

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