Linear Algebra-A

Assignments - Week 9

(Class 3 - 线性代数 A 中英双语 3 班)

Assignments from the Textbook (*Hardcover*)

Section 4.2: 2,4,6,7,10,12,14,23,28,35.
Section 4.3: 3,4,5,6,10,16,22,26,30.

Supplementary Problem Set

1. Calculate the following determinants:

   a) $D_1 = \begin{vmatrix} -1 & 1 & 1 & 1 \\ 2 & -1 & 3 & -2 \\ 4 & 1 & -1 & -2 \\ -3 & 1 & 5 & -1 \end{vmatrix}$.

   b) $D_2 = \begin{vmatrix} 0 & a_1 & b_1 & 0 \\ a_2 & 0 & 0 & b_2 \\ a_3 & 0 & 0 & b_3 \\ x & a_4 & b_4 & y \end{vmatrix}$.

   c) $D_3 = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & 0 & \cdots & 0 \end{vmatrix}$. ("reverse-triangular" matrix)

   d) $D_4 = \begin{vmatrix} 1+x_1y_1 & 1+x_1y_2 & \cdots & 1+x_1y_n \\ 1+x_2y_1 & 1+x_2y_2 & \cdots & 1+x_2y_n \\ \vdots & \vdots & \cdots & \vdots \\ 1+x_ny_1 & 1+x_ny_2 & \cdots & 1+x_ny_n \end{vmatrix}$.

2. If $\begin{vmatrix} 1+x & 2 & 3 \\ 2 & 1+x & 2 \\ 3 & 3 & 1+x \end{vmatrix} = 0$, find x.

3. Calculate

$$D_n = \begin{vmatrix} 1 & 1 & \cdots & 1 \\ x_1 & x_2 & \cdots & x_n \\ x_1^2 & x_2^2 & \cdots & x_n^2 \\ \vdots & \vdots & & \vdots \\ x_1^{n-2} & x_2^{n-2} & \cdots & x_n^{n-2} \\ x_1^n & x_2^n & \cdots & x_n^n \end{vmatrix}.$$

   [**Note**: This is ***not*** "Vandermonde determinant". But you can get some idea from that determinant.]

4. Calculate

$$D_n=\begin{vmatrix}\cos\theta & 1 & & & \\ 1 & 2\cos\theta & 1 & & \\ & \ddots & \ddots & \ddots & \\ & & 1 & 2\cos\theta & 1 \\ & & & 1 & 2\cos\theta\end{vmatrix}.$$

5. Please show that: $rank(\boldsymbol{A})=r$ <u>if and only if</u> the highest order of the nonzero minors of $\boldsymbol{A}$ is r. (This is an equivalent definition of ***the rank of a matrix***.)

即证明：$rank(\boldsymbol{A})=r$的<u>充要条件</u>是$\boldsymbol{A}$的非零子式的最高阶数为r. (这也可以作为<u>**矩阵的秩**</u>的等价定义.)

注：<u>定义</u>　对于矩阵$\boldsymbol{A}=\left[a_{ij}\right]_{m\times n}$，取其任意$k$行（第$i_1,i_2,\cdots,i_k$行）和任意$k$列（第$j_1,j_2,\cdots,j_k$列），其中$k\le n$，将这些行与列交叉处的$k^2$个元素按原来相对位置构成的$k$阶行列式

$$\begin{vmatrix}a_{i_1j_1} & a_{i_1j_2} & \cdots & a_{i_1j_k}\\ a_{i_2j_1} & a_{i_2j_2} & \cdots & a_{i_2j_k}\\ \vdots & \vdots & & \vdots\\ a_{i_kj_1} & a_{i_kj_2} & \cdots & a_{i_kj_k}\end{vmatrix}$$

称为$\boldsymbol{A}$的一个**k阶子行列式，简称<u>k阶子式</u>**(the minor of the k-th order). 当上述行列式等于零（不等于零）时，称为k阶零子式（非零子式）.

显然,如果矩阵$\boldsymbol{A}$<u>存在</u>r阶非零子式,而<u>所有</u>$r+1$阶子式（如果有$r+1$阶子式）都等于零，则矩阵$\boldsymbol{A}$的非零子式的最高阶数为r,因为由<u>所有</u>$r+1$阶子式都等于零可推出所有更高阶的子式都等于零.