

Linear Algebra-A

Assignments - Week 3

Assignments from the Textbook (*Hardcover*)

Section 1.4: 49.

Section 1.5: 5,8,9,14,24,25,30,41,45.

Section 1.6: 2,6,10,17,19,37,43,48,54.

Supplementary Problem Set

1. Let $\mathbf{B} = \begin{bmatrix} 1 & 2 & -3 & -2 \\ 0 & 1 & 2 & -3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, $\mathbf{C} = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, and $(2\mathbf{I} - \mathbf{C}^{-1}\mathbf{B})\mathbf{A}^T = \mathbf{C}^{-1}$.

Please find \mathbf{A} .

2. Let $2\mathbf{CA} - 2\mathbf{AB} = \mathbf{C} - \mathbf{B}$, where $\mathbf{A} = \begin{bmatrix} 2 & 1 & 0 \\ 2 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 1 & & \\ & -1 & \\ & & 2 \end{bmatrix}$.

Please calculate \mathbf{C}^5 .

3. If $\mathbf{A} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$, then find \mathbf{A}^n .

4. Please show that: for an invertible $n \times n$ upper (lower) triangular matrix \mathbf{A} , its inverse \mathbf{A}^{-1} is also an upper (lower) triangular matrix.

[Hint: by the technique of partitioned matrices and method of mathematical induction (用分块矩阵和数学归纳法)]

5. (1) Let $\mathbf{u}, \mathbf{v} \in \mathbf{R}^n$. When $\mathbf{I} - \mathbf{uv}^T$ is invertible, its inverse is $\mathbf{I} + \frac{1}{k}\mathbf{uv}^T$. What is k ?

- (2) If \mathbf{A}^{-1} is known, and $\mathbf{M} = \mathbf{A} - \mathbf{uv}^T$. What is \mathbf{M}^{-1} ?

(3) Let $\mathbf{A} = \begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix}$, please calculate the inverse of \mathbf{A} .