Linear Algebra-A
Assignments - Week 2

Assignments from the Textbook (*Hardcover*)

Section 1.4: 2,4,5,7,9,10,12,13,15,18,22,34,42.

Section 1.6（注意！此为教材 **1.6 节**课后作业）：1,7,11,12.

　　Note: tridiagonal(三对角)：All nonzero entries lie on the main diagonal and the two adjacent diagonals.

Supplementary Problem Set

1. Let A be an $n \times n$ skew-symmetric matrix. Please find the value of $x^{\mathrm{T}}Ax$ for any vector $x = [x_1 \quad x_2 \quad \cdots \quad x_n]^{\mathrm{T}}$.

2. (1) Let $A = \begin{bmatrix} 1 & \\ \lambda & 1 \end{bmatrix}$. Find A^k, where k is a positive integer.

   (2) Let $A = \begin{bmatrix} \lambda & 1 & \\ & \lambda & 1 \\ & & \lambda \end{bmatrix}$, find A^4.

3. Suppose that a square matrix A satisfies the equation $A^2 = 2A$. Please show that both the matrices $A - I$ and $A + 2I$ are invertible and find their inverses.

4. Please compute:
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}^{2020} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}^{2019}.$$

5. Suppose that D is an $n \times n$ diagonal matrix with 0 or 1 on the main diagonal, and X is a nonsingular matrix. Prove that $A = XDX^{-1}$ is an idempotent matrix.