Opinion formation on multilayer social systems: a numerical simulation study

Yang Siwei, Allen
School of Physical and Mathematical Sciences

Associate Professor Xiao Gaoxi
Dr Nguyen Xuan Vu
School of Electrical and Electronic Engineering

Abstract - In modern society, social systems play an ever-prevalent role. The study of opinion formation in such systems via temporal processes in networks is thus of large significance. In this study, we extend the Deffaunt Model to a network of three layers, where there exist tolerance ranges respective to each layer and degrees of overlap between layers.

It was found that despite two layers having a low degree of overlap, if there exists a layer in-between that has significant overlap with both of two, then an increase in tolerance range on the first layer will reduce opinion diversity on the other. That is, it was found that effects could be propagated through an intermediary layer. Furthermore, opinion diversity on the third layer was not only affected by tolerance range on the first layer but also the degree of overlap between first and third layers, where increased overlap decreases opinion diversity. Evidence also suggests that the intermediary layer's tolerance has a larger impact on the third layer's cluster count compared to the effects of the tolerance from the first layer. In comparison to the two-layer case, it was found that unlike the two-layer case, having the sum of the tolerance ranges being greater than or equal to a constant does not guarantee convergence of opinion to a consensus across all layers. More precisely, such a threshold would differ based on the degree of overlap between layers.

1 INTRODUCTION

Opinion formation is an expanding area of research, which has relevance in areas such as politics [2] and social media analysis [5]. The relevance is due to its use in the prediction of opinion distributions in populations via temporal processes which would represent social interaction [4,7,9].

A well-known bounded confidence model would be the Deffaunt model [6]. After assigning continuous opinions x_i to a graph of N nodes, with each node labelled as i, opinions of two connected nodes are readjusted by a proportion of their difference (μ) if opinion values are within a threshold d. On a graph, random pairs of nodes are selected, and the model is applied. It is found that μ only affects the rate of convergence of opinion across the graph and that there exists a minimum threshold value of d, where d=0.5 approximately, above which all opinions converge towards a single value, forming a single cluster.

Recently, Vu et al [9] extended the model to a two-layer network. This form of network falls under the class of multilayer networks [8]. In the paper, correlation between layers is defined as the number of overlapping edges between nodes on each layer. It is found that on loosely correlated layers, largely independent opinion evolution is observed. On strongly correlated layers, increased tolerance increases opinion diversity. A "one-sum rule" is also established, which means that if the sum of tolerance ranges is greater than a constant, opinion consensus would be achieved across each layer.

However, it is not certain that these results would extend to a three-layer case. A third intermediary layer might disrupt the opinion evolution on the other two. Similarly, a "one sum rule" might not be able to be established.

In this study, we extend the Deffaunt model [6] to a three-layer network. As per the Deffaunt model, each node represents an individual and each edge, a social connection. Each layer represents the population and the opinion assigned to nodes on that layer is regarding a topic specific to that layer. We then proceed to investigate effects of opinion formation between two layers via a third intermediary layer. Characteristics such as the existence of the one-sum rule are also compared with those of the two-layer case [9].

2 MODEL

2.1 SET-UP

In the model, uncorrelated random scale-free networks [3] are used, where each layer is a Barabasi-Albert scale-free network [1]. This disallows duplicate edges within a layer and emulates Vu et al [9], for a better comparison of results. The network size of each layer is set to 5000. Nodal degree distribution on each layer follows a "power-law" distribution [3], with minimum degree being 5 and maximum being 80. We create a first network layer via the generative model [3] and label it as G_1. We then proceed to further initialize two other layers, G_2 and G_3, such that they are identical to G_1. That is, $G_1 \equiv G_2 \equiv G_3$.

We also proceed to define overlap/correlation between the layers. We define E_1, E_2 and E_3 to be the edge sets of G_1, G_2 and G_3 respectively. From this, α, β, γ are defined.

$$\alpha = \frac{|E_1 \cap E_2|}{|E_1|} \tag{1}$$

where $\alpha \in [0,1]$ is defined as the correlation between G_1 and G_2.

$$\beta = \frac{|E_2 \cap E_3|}{|E_1|} \tag{2}$$

where $\beta \in [0,1]$ is defined as the correlation between G_2 and G_3.

$$\gamma = \frac{|E_1 \cap E_3|}{|E_1|} \tag{3}$$

where $\gamma \in [0,1]$ is defined as the correlation between G_1 and G_3.

2.2 REWIRING ALGORTHM

Upon generation of three identical layers as mentioned in the set-up, we proceed to rewire the edges on G_2 and G_3. This is to attain graphs corresponding to the various values of α, β and γ to be used in the study.

We first rewire G_2 from G_1 via the rewiring process specified in Vu et al [9]. If $\alpha = 1$, the two layers are comprised of the same topology and hence we stop. Otherwise nodes A, B, C, D are chosen randomly where edges $\{A,B\} \in E_2$, $\{C,D\} \in E_2$ and $\{A,C\} \notin E_2$, $\{B,D\} \notin E_2$. $\{A,B\}$, $\{C,D\}$ are then removed from E_2 and $\{A,C\}$, $\{B,D\}$ added into E_2. This process is repeated until the corresponding value of α is obtained.

We then proceed to rewire G_3, to attain corresponding values of β and γ. G_3 is first set to be identical to the rewired G_2 ($G_3 \equiv G_2$). The rewiring comprises of two phases, one to obtain γ, and the other, β.

In the first phase, U is defined as the set of all possible edges. Edges of G_3 in $E_1 \cap E_2$ are then selected and mapped into $U-(E_1 \cup E_2)$. More precisely, nodes A, B, C, D are randomly chosen, where edges $\{A,B\} \in E_1 \cap E_2$ and $\{C,D\} \in E_1 \cap E_2$. It is also required that $\{A,C\} \notin E_1 \cup E_2 \cup E_3$ and $\{B,D\} \notin E_1 \cup E_2 \cup E_3$. Edges $\{A,B\}$, $\{C,D\}$ in G_3 are then replaced with $\{A,C\}$, $\{B,D\}$. This is repeated until the desired value of γ is attained.

For the second phase, edges of G_3 in $E_2 - E_1$ are mapped into $U-(E_1 \cup E_2)$. Nodes A, B, C, D are chosen, where edges $\{A,B\}$, $\{C,D\}$ in E_2 but not E_1. It is also required that $\{A,C\} \notin E_1 \cup E_2 \cup E_3$ and $\{B,D\} \notin E_1 \cup E_2 \cup E_3$. Edges $\{A,B\}$, $\{C,D\}$ in G_3 are then replaced with $\{A,C\}$, $\{B,D\}$. This is repeated until the desired value of β is obtained.

The algorithm preserves nodal degree across all layers. That is, for any given node on a layer, its corresponding node on the other two layers will have the same nodal degree. The algorithm also preserves the number of edges on each layer, where $|E_1| = |E_2| = |E_3|$. It is to be also noted that the correlations are subject to constraint $\alpha + \beta - \gamma \leq 1$.

2.3 PAIRWISE INTERACTION

After rewiring, nodes are assigned a continuous-valued opinion from uniform distribution $[0,1]$.

We now extend the Deffaunt Model [6] to three layers. First, tolerance ranges, $d_p \in [0,1]$ are assigned to each layer G_p. Opinion values are denoted by $f_i(u_j,t) \in [0,1]$ for each node u_j on each layer i at time t. A convergence parameter $\mu \in (0,0.5]$ is also introduced.

We proceed to define the process of pairwise interaction. At timestep t, a random layer i is selected. A pair of nodes, u_1 and u_2, which are connected on layer , is selected at random. If

$$\sum_{k \in K} |f_k(u_1,t) - f_k(u_2,t)| \leq \sum_{k \in K} d_k \tag{4}$$

where K is the set of layer indices such that $\{u_1,u_2\} \in E_k$ and d_k is the tolerance on layer k, then readjustment of the opinion values is as follows:

$$f_i(u_1, t+1) = f_i(u_1,t) + \mu\big(f_i(u_2,t) - f_i(u_1,t)\big) \tag{5}$$

$$f_i(u_2, t+1) = f_i(u_2,t) + \mu\big(f_i(u_1,t) - f_i(u_2,t)\big) \tag{6}$$

If there is only a connection on the chosen layer, readjustment reduces to the single-layer case where there is only one value of k. Similarly, if there is only a connection on two of the three layers, readjustment reduces to the two-layer scenario where there are only 2 values of k.

In the model, pairwise interactions and opinion readjustment occur until a quasi-steady state is achieved for all nodes. That is, the opinion value of all nodes fluctuates less than 10^{-5} between consecutive timesteps.

3 RESULTS

In our study, two main hypotheses are investigated. The first hypothesis is that there exists a propagation of effects from the first layer to the third via the intermediary second layer. The second is that the one-sum rule of the two-layer case introduced by Vu et al [9] would not extend to the three-layer scenario.

In all simulations, convergence parameter μ is set to 0.5. This is because μ was found to only affect the rate of convergence [6,9]. To compute the number of clusters after all nodes are readjusted to a quasi-steady state, edges between any two nodes u_1 and u_2 are removed if

$$\sum_{k \in K} \left| f_k(u_1,t) - f_k(u_2,t) \right| > \sum_{k \in K} d_k \tag{7}$$

where K is the set of layer indices such that $\{u_1,u_2\} \in E_k$ and d_k is the tolerance on layer k. Each set of connected nodes on each layer is then defined as a cluster.

3.1 PROPAGATION

To verify that effects are propagated through intermediary layers, we verify that opinion diversity can be reduced via an intermediary layer. We proceed to generate and rewire graphs G_1, G_2 and G_3 as described in the previous section. Values of α and β are kept significant, i.e. greater than 0.5, and constant. Values of γ are kept low, i.e. less than or equal to α and β, and constant. Tolerance ranges d_2, d_3 are kept constant with d_2=d_3=0.2. Tolerance range d_1 is then varied. To capture the effects of d_1 on G_3, we first observe the number of clusters on G_3 as d_1 is varied. If the number of clusters on G_3 decreases as d_1 increases, this would be evidence that an increased number of nodes have come to a consensus as an effect from the increased tolerance of G_1.

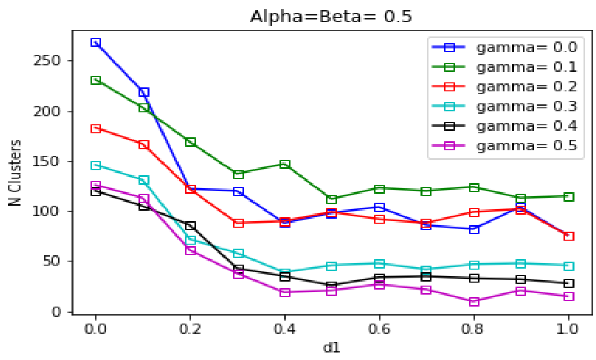

Figure 1: Number of clusters on G_3 at steady-state against d_1 with α=β=0.5, d_2=d_3=0.2.

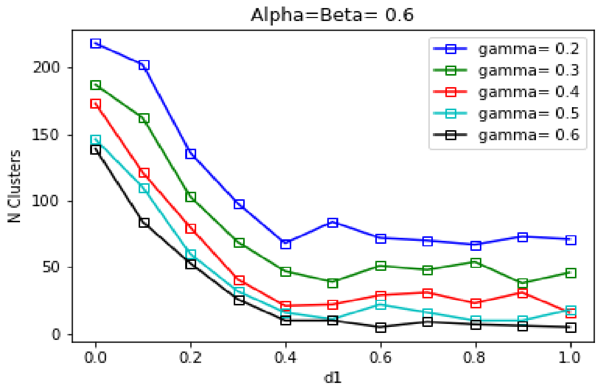

Figure 2: Number of clusters on G_3 at steady-state against d_1 with α=β=0.6, d_2=d_3=0.2.

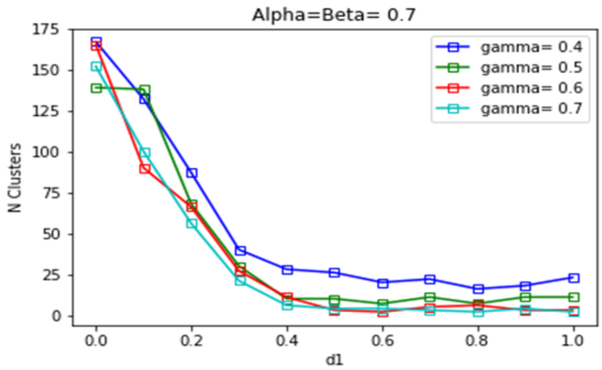

Figure 3: Number of clusters on G_3 at steady-state against d_1 with α=β=0.7, d_2=d_3=0.2.

As observed from Fig. 1, Fig. 2 and Fig. 3, the number of clusters at a steady-state does fall as d_1 is increasing. We then proceed to examine directly the changes in opinion distribution as d_1 is varied.

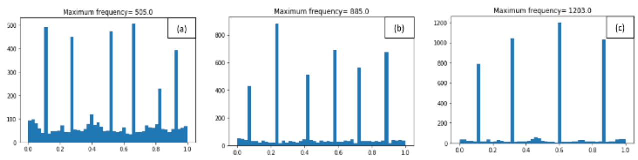

Figure 4: Opinion Distribution on G_3 with d_2=d_3=0.05, α=β=0.5, γ=0.0, d_1=0.0 (a), d_1=0.2 (b), d_1=0.4 (c).

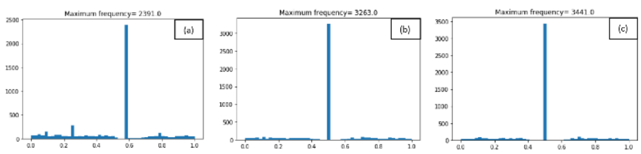

Figure 5: Opinion Distribution on G_3 with d_2=d_3=0.05, α=β=0.5, γ=0.1, d_1=0.0 (a), d_1=0.2 (b), d_1=0.4 (c).

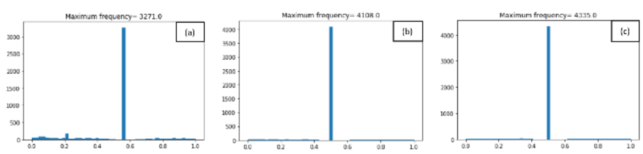

Figure 6: Opinion Distribution on G_3 with d_2=d_3=0.05, α=β=0.5, γ=0.2, d_1=0.0 (a), d_1=0.2 (b), d_1=0.4 (c).

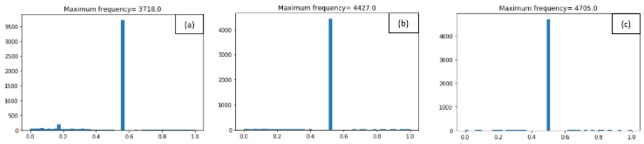

Figure 7: Opinion Distribution on G_3 with d_2=d_3=0.05, α=β=0.5, γ=0.3, d_1=0.0 (a), d_1=0.2 (b), d_1=0.4 (c).

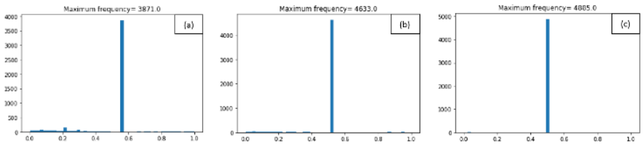

Figure 8: Opinion Distribution on G_3 with d_2=d_3=0.05, α=β=0.5, γ=0.4, d_1=0.0 (a), d_1=0.2 (b), d_1=0.4 (c).

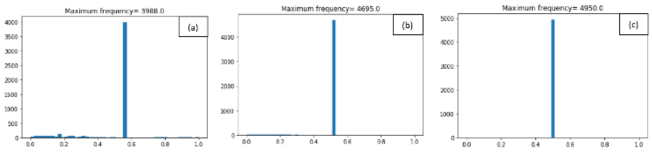

Figure 9: Opinion Distribution on G_3 with d_2=d_3=0.05, α=β=0.5, γ=0.5, d_1=0.0 (a), d_1=0.2 (b), d_1=0.4 (c).

As can be observed from Figs. 4 to 9, opinion distribution on G_3 proceeds to become sparser as d_1 increases. It can hence be concluded that opinion diversity is reduced on G_3 as d_1 increases and that effects are propagated through an intermediary layer.

It is further observed from Figs. 1 through 9 that opinion diversity on G_3 is primarily affected by γ and d_1. An increase in γ causes a decrease in opinion diversity on G_3 and an increase in d_1 also causes a decrease in opinion diversity on G_3.

3.2 ONE SUM RULE

We proceed to examine the applicability of a “one sum rule” for the three-layer case which would be similar to that for the two-layer networks. That is, to have a constant value k, such that $d_1+d_2+d_3=k$ would imply that global consensus is attained. In this case, global consensus is defined as that all agents in the population finally share a single opinion forming into a single cluster. Furthermore, it is not possible to attain global consensus if $d_1+d_2+d_3<k$ for any possible combination of d_1, d_2 and d_3.

From Figs. 1, 2 and 3, cluster count tends to stabilise at d_1=0.4. From this, it might be hypothesized that a sum rule might have a value of k=0.8. However, upon comparison of Fig. 4(c) and Fig. 9(c), it can be observed that global consensus is close to being attained in Fig. 9(c) but not Fig. 4(c). It is thus inferred that it is not possible to produce a sum rule that satisfies both criteria in the paragraph above for all possible values of γ.

4 CONCLUSION

We have extended the Deffaunt Model [6] to three-layer scale-free networks. In investigating inter-layer effects, it was found that increasing tolerance on one layer decreases opinion diversity on another via an intermediary layer. Specifically, both γ and d_1 had a negative correlation with opinion diversity. This is evidence for the propagation of effects through intermediary layers. Further studies to quantify the effects of propagation via intermediary layers could be conducted. It may be hypothesised that an increase in number of intermediary layers decreases the effects of the propagation.

It was also found that the one sum rule could not be extended to the three-layer case. However, the possibility of sum rules specific to values of γ is not excluded. Further investigations into sum rules for specified degrees of correlation between layers may be performed.

ACKNOWLEDGMENT

I would like to thank Associate Professor Xiao Gaoxi for providing the opportunity to embark on this project as well as the ideas and guidance he has offered. I would also like to thank Dr Nguyen Xuan Vu for his technical guidance in the reproduction of past results and simulations via code.

I would like to acknowledge the funding support from Nanyang Technological University – URECA Undergraduate Research Programme for this research project.

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