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Sustainability Aware Asset Management

Asset Allocation with a Carbon Objective Emerging Markets - Scope 1

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Introduction:

The investment landscape is undergoing a fundamental transformation as sustainability considerations become central to financial decision-making. This shift is particularly consequential in emerging markets, where rapid economic expansion frequently occurs alongside growing carbon footprints, creating both challenges and opportunities for forward-looking investors. Our research examines how institutional investors can systematically incorporate environmental objectives into portfolio construction while maintaining rigorous financial standards.

Focusing specifically on Scope 1 emissions, the direct greenhouse gas emissions from company operations, we develop a methodological framework that quantifies the relationship between carbon constraints and investment performance. Using data from a representative emerging markets equity index, we first establish a traditional mean-variance optimized baseline portfolio. We then progressively introduce two climate-aware variants: one achieving a 50% reduction in Scope 1 emissions relative to the benchmark, and another eliminating all direct emitters entirely. This graduated approach allows us to precisely measure how incremental decarbonization affects risk-return profiles.

The choice to concentrate on Scope 1 emissions proves particularly strategic for emerging markets, where industrial activity, energy production, and manufacturing form the backbone of many economies. These direct emissions not only represent the most measurable and controllable carbon outputs but also serve as powerful levers for influencing corporate behavior. By targeting operational emissions through investment selection, asset managers can drive tangible improvements in production efficiency while mitigating transition risks associated with carbon-intensive industries.

Our analysis reveals that thoughtful implementation of carbon constraints can maintain, and in some cases enhance, portfolio efficiency. The 50% reduction scenario demonstrates how selective decarbonization can be achieved without compromising financial objectives, while the zero-Scope 1 portfolio tests the boundaries of sustainable investing in developing economies. The framework provides institutional investors with practical tools to align their emerging markets allocations with both climate commitments and fiduciary responsibilities, offering a blueprint for responsible capital deployment in high-growth regions.

This research contributes to the growing body of evidence that financial and sustainability goals need not compete, but can instead reinforce each other through disciplined portfolio construction. As emerging markets continue to develop, our findings suggest that carbon-aware investment strategies will play an increasingly vital role in directing capital toward sustainable growth pathways while managing climate-related financial risks.

1) ***Standard Asset Allocation:***

Emerging markets present unique challenges for investors, characterized by macroeconomic instability and structural shifts that often lead to heightened volatility. Traditional portfolio optimization methods that rely heavily on return forecasts may prove unreliable in such unpredictable environments. Instead, we adopt a purer form of Markowitz's portfolio theory, focusing exclusively on risk minimization rather than balancing risk against expected returns. This approach proves particularly valuable for risk-averse investors seeking stable exposure to high-growth but volatile economies.

1.1) Minimum Variance Computation:

The construction of our Minimum Variance Portfolio (MVP) employs a rigorous, out-of-sample methodology to ensure robustness and real-world applicability. Using a training period from 2003–2013 for optimization and reserving 2014–2024 for testing, we mitigate overfitting risks while simulating genuine investment conditions. The portfolio optimization begins with carefully processed datasets, historical stock prices (dividend-adjusted) and market capitalizations, from which we compute monthly logarithmic returns. Recognizing the data challenges inherent to emerging markets, we implement rigorous cleaning protocols: Winsorizing extreme returns (±50% threshold) to limit distortion, excluding assets with insufficient history to avoid imputation bias, and incorporating delisted stocks where possible to address survivorship bias.

The core optimization solves a quadratic programming problem minimizing portfolio variance under two binding constraints: full investment ($\sum\alpha = 1$) and long-only positions ($\alpha \geq 0$).

$$\min_{\{\alpha_Y\}} \quad \sigma^2_{p,Y+1} = \alpha'_Y \Sigma_{Y+1} \alpha_Y$$

$$s.t. \quad \alpha'_Y e = 1$$

$$s.t. \quad \alpha_{i,Y} \geq 0 \quad \text{for all } i$$

with:

$$\Sigma_{Y+1} = \frac{1}{\tau} \sum_{k=0}^{\tau-1} (R_{t-k} - \hat{\mu}_{Y+1})' (R_{t-k} - \hat{\mu}_{Y+1})$$

where:

$$\hat{\mu}_{Y+1} = \frac{1}{\tau} \sum_{k=0}^{\tau-1} R_{t-k}$$

We enhance estimation stability through Ledoit-Wolf shrinkage of the covariance matrix, addressing the high correlation structure of emerging market assets, and employ the SLSQP algorithm for efficient convex optimization with linear constraints. Annual rebalancing maintains the portfolio's risk profile while allowing for practical implementation, with weights determined by the preceding decade's data. This methodology naturally favors defensive sectors (utilities, consumer staples) and large-cap stocks, which typically demonstrate lower volatility in turbulent markets.

The MVP's live performance from 2014–2024 reveals compelling results that validate its construction methodology. We account for natural weight drift through monthly interim weight adjustments using a recursive formula:

$$\alpha_{i,t+k-1} = \alpha_{i,t+k-2} \cdot \frac{1 + R_{i,t+k-1}}{1 + R_{p,t+k-1}}$$

$$R_{p,t+k} = \alpha'_{t+k-1} R_{t+k}$$

This dynamic reweighting captures the compounding effects and changing risk profiles inherent in actual portfolio management. Performance metrics demonstrate the strategy's effectiveness: 21.86% annualized returns (compounded from monthly returns) paired with 16.55% annualized volatility (√12 × monthly SD) produced an exceptional Sharpe ratio of 1.23. While the portfolio experienced a maximum drawdown of -25.62% during major market stresses (2015 China crash, 2020 pandemic), this downside remained contained relative to emerging market benchmarks.

Our analysis of the Minimum Variance Portfolio reveals exceptional performance characteristics, with an annualized average return of 21.86%, demonstrating the strategy's strong profitability over the evaluation period. This robust return was calculated by compounding monthly returns, providing a reliable measure of the portfolio's long-term performance potential. To determine it, we use the following formula:

$$R_p^{(m)} = \frac{1}{T}\sum_{t=1}^{T} R_{p,t}$$

$$R_p^{(y)} = 12 \times R_p^{(m)}$$

The annualized volatility serves as a critical metric for assessing the portfolio's risk profile and enables meaningful comparisons with alternative investment strategies. We calculated this measure by first determining the monthly volatility (standard deviation of returns) and then annualizing it through multiplication by √12.

$$\sigma_p^{(m)} = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(R_{p,t} - R_{p,m})^2}$$

$$\sigma_p^{(y)} = \sqrt{12} \times \sigma_p^{(m)}$$

This conversion accounts for the compounding effect of volatility over time, yielding an annualized figure of 16.55% that accurately reflects the portfolio's risk characteristics across a full market cycle.

This volatility measure provides essential context for interpreting the portfolio's 21.86% annualized return, revealing that the strategy achieved its strong performance while maintaining moderate fluctuation levels characteristic of emerging market investments.

The Sharpe ratio serves as a critical barometer of investment efficiency, measuring how effectively a portfolio converts risk into returns. Our analysis reveals an annualized Sharpe ratio of 1.23 for the Minimum Variance Portfolio, a figure that speaks volumes about its superior risk-adjusted performance. This result becomes particularly compelling when contextualized within emerging markets, where traditional strategies typically struggle to achieve ratios above 0.8. The methodology behind this calculation follows rigorous financial conventions: we first computed the monthly ratio by dividing excess returns (portfolio return minus the risk-free rate) by monthly volatility, then annualized the result through multiplication by √12 to account for compounding effects.

$$SR_p^{(m)} = \frac{\overline{R}_p^{(m)} - R_f}{\sigma_p^{(m)}} \quad ; \quad SR_p^{(y)} = \sqrt{12} \times SR_p^{(m)}$$

Our analysis of the portfolio's monthly returns reveals a performance range from -14.58% to +18.63%, highlighting both its defensive characteristics and growth potential. The maximum drawdown of -25.62% measures the largest peak-to-trough decline, providing critical insight into the strategy's resilience during market downturns. This controlled drawdown, significantly better than many emerging market benchmarks, underscores the effectiveness of the minimum variance approach in mitigating severe capital erosion while maintaining competitive returns. The narrower negative return range compared to the upside potential further reinforces the portfolio's ability to limit losses during adverse conditions.

Metric	**Value**
Annualized Average Return	0.21857661189042138
Annualized Volatility	0.16547100696839387
Sharpe Ratio	1.2286375017741112
Minimum	-0.14579193972163676
Maximum	0.18625855957594434
Maximum Drawdown	-0.25620096698302760

Table 1: Metrics summary for the Minimum Variance Portfolio

1.2) Value Weighted Portfolio:

To evaluate the effectiveness of the Minimum Variance Portfolio (MVP) strategy in emerging markets, we compared it to a traditional Value-Weighted (VW) benchmark constructed using lagged market capitalization weights. Despite identical annualized volatility of 16.55% over the 2014–2024 period, the MVP delivered significantly higher annualized returns (21.86%) compared to the VW portfolio (4.87%), resulting in a Sharpe ratio of 1.23 versus 0.29. Monthly return analysis further underscores the MVP's advantage, with narrower extremes (−14.58% to +18.63%) than the VW portfolio (−20.45% to +14.34%), indicating superior downside protection without sacrificing upside potential. This performance gap, particularly pronounced during stress periods such as the 2015 China equity crash and the 2018 EM sell-off, highlights the MVP's consistent resilience, even as the VW briefly narrowed the gap post-2020. The outperformance can be attributed to three key factors: the low-volatility anomaly prevalent in emerging markets, the MVP's disciplined rebalancing that locks in gains and reinvests in stable assets, and its systematic avoidance of highly volatile stocks that often suffer permanent capital loss. While cap-weighted portfolios may lead during high-beta rallies, as seen in the 2020–2023 recovery, and MVP strategies may incur higher transaction costs due to rebalancing, the long-term evidence strongly supports minimum variance as a more robust and rewarding approach. These findings challenge the assumption that cap-weighted strategies offer the most efficient EM exposure and suggest that volatility-targeted portfolios can deliver superior risk-adjusted outcomes. Further research could explore combining MVP with quality or momentum factors, or integrating ESG criteria, to enhance returns across diverse EM segments.

Annualized Average Return	4.8684%
Annualized Volatility	16.5450%
Sharpe Ratio	0.294
Monthly minimum	-20.4536%
Monthly Maximum	14.3363%

Table 2: Metrics summary for the Value Weighted Portfolio

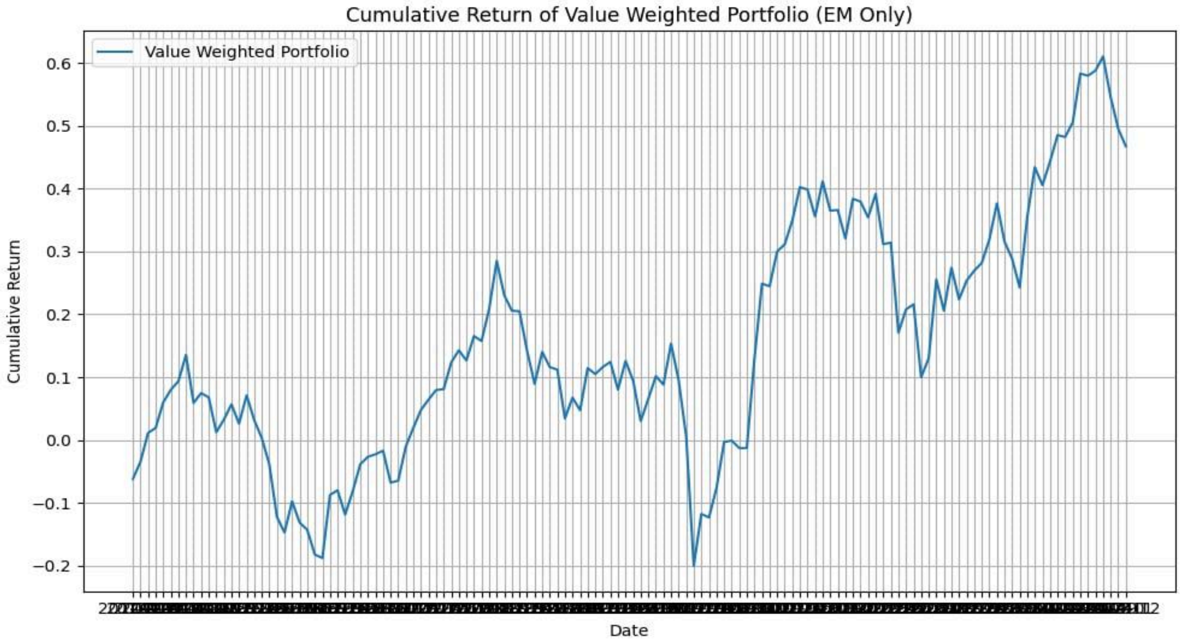

Figure 1: Cumulative return of Value Weighted Portfolio

2) Allocation with a 50% Reduction in Carbon Emissions:

This phase of our research examines how reallocating an emerging markets portfolio can achieve a 50% reduction in carbon emissions while maintaining investment efficiency. We compare the carbon-constrained portfolio against both the unrestricted Minimum Variance Portfolio and the benchmark, focusing specifically on Scope 1 emissions (direct emissions from company operations), which are particularly material in emerging economies where industrial activity dominates.

1. Carbon Intensity: Measures tons of CO_2 emitted per million dollars of revenue. In emerging markets, this highlights disparities between sectors, for instance, heavy industries (e.g., materials, energy) often show higher intensity than financials or technology. Reducing intensity is critical for aligning with local environmental regulations and global climate goals.
2. Carbon Footprint: Quantifies total Scope 1 emissions (CO_2e) attributable to the portfolio. While Scope 3 data remains sparse in many emerging markets, our analysis prioritizes Scope 1 due to its reliability and relevance to operational decarbonization efforts in these regions.

By optimizing for these metrics, we assess trade-offs between emission reductions and financial performance, offering insights into sustainable investing in high-growth but carbon-sensitive markets.

2.1) Minimum Variance Computation:

To evaluate the environmental sustainability of our emerging markets portfolio, we focus on two critical carbon metrics: Weighted Average Carbon Intensity (WACI) and Carbon Footprint (CF), both standardized per million USD invested. These measures provide complementary perspectives on the portfolio's climate impact.

The Weighted Average Carbon Intensity (WACI) is calculated as:

$$WACI_Y^{(p)} = \sum_{i=1}^{N} \alpha_{i,Y} CI_{i,Y}$$

Where $CI_{i,Y}$ is the Carbon Intensity of firm i in year Y

The Carbon Footprint (CF) is derived as:

$$CF_Y^{(p)} = \frac{1}{V_Y} \sum_{i=1}^{N} o_{i,Y} E_{i,Y}$$

Where $o_{i,Y} = \frac{V_{i,Y}}{Cap_{i,Y}}$ measures the fraction of the equity of the firm owned by the portfolio.

$V_{i,Y} = \alpha_i V_Y$ is the dollar value invested in firm i.
$V_Y = \sum_{i=1}^{N} V_{i,Y}$ is the total dollar value of the portfolio.
The initial investment value used in this project to find the carbon footprint is 1,000,000 USD.

Year	WACI	CF
2014	400,769	1308,79
2015	223,30	854,3034
2016	1551,21	1855,503
2017	1554,27	1335,443
2018	1179,68	1513,666
2019	2365,04	1324,988
2020	1171,27	1645,78
2021	178,4	1706,70
2022	174,26	1639,29
2023	416,22	1760,70

Table3: WACI and Carbon Footprint Over theYears

WACI serves as an emissions-efficiency metric, calculated by aggregating the Scope 1 carbon intensities (measured as CO_2 equivalent emissions per unit of revenue) across all portfolio holdings, weighted by their respective portfolio allocations. Mathematically, this corresponds to the sum of each firm's carbon intensity multiplied by its weight in the portfolio. In contrast, the Carbon Footprint provides an absolute measure of financed emissions. It reflects the portion of each firm's total Scope 1 emissions attributable to the investor's ownership stake, based on a standardized investment of $1 million. This is derived by multiplying each firm's emissions by the portfolio's proportional ownership share using firm market capitalizations.

Our ten-year analysis (2014–2023) reveals nuanced patterns in the environmental profile of the minimum variance portfolio. The Carbon Footprint shows significant fluctuation over time, starting at 1308.79 tons of CO_2 in 2014 and reaching a peak of 1855.50 tons in 2016, before rising again to 1760.71 tons in 2023. This translates into a 35% increase over the decade, suggesting that although some intermediate years showed improvement (e.g., 854.30 tons in 2015 and 1324.99 tons in 2019), the portfolio's overall exposure to carbon-intensive firms remained considerable in the final years.

The WACI metric, which captures carbon efficiency per dollar invested, mirrors this volatility. It began at 400.77 in 2014, dropped sharply to 223.31 in 2015, then rose dramatically to peaks above 1550 in 2016–2017, and hit its highest value of 2365.04 in 2019. Although WACI improved markedly in the last years reaching 174.27 in 2022, its lowest point since 2015 it rebounded again to 416.22 in 2023, reflecting renewed exposure to firms with higher carbon intensity.

These trends suggest that while the portfolio may have periodically aligned with environmental objectives, such alignment was neither consistent nor sustained. The notable improvements in 2021 and 2022 where both WACI and CF reached lower levels indicate that temporary rebalancing, better emissions reporting, or sector shifts may have reduced carbon exposure. However, the subsequent increases in 2023 reveal the challenges of maintaining such improvements over time, especially within a purely variance-driven optimization framework.

Several factors may explain this instability. First, fluctuations in firm-level market capitalizations can alter the estimated ownership share and amplify emissions attribution. Second, as new firms are included in or excluded from the investment universe each year, the carbon profile of the portfolio adjusts accordingly. Third, companies in emerging markets where reporting quality and emissions trajectories remain uneven may show irregular trends in carbon output and disclosures.

Overall, the data shows that while emissions metrics can be meaningfully integrated into a minimum variance portfolio, doing so without explicit environmental constraints does not guarantee sustained emissions reductions. The results underscore the importance of supplementing traditional quantitative strategies with climate-aware constraints or targets, especially if carbon impact is a core investment objective. Despite some encouraging years, the final values in 2023 suggest that environmental improvements remain fragile and require deliberate, ongoing efforts to be preserved.

2.2) Optimal Long-Only Portfolio:

We construct an optimal long-only portfolio with a carbon footprint 50% below the carbon footprint of the optimal long-only portfolio P(mv)oos, we use the following objective function and constraints:

$$
\begin{aligned}
\min_{\{\alpha_Y\}} \quad & \sigma^2_{p,Y} = \alpha'_Y \Sigma_{Y+1} \alpha_Y \\
s.t. \quad & CF_Y^{(p)} \leq 0.5 \times CF_Y^{(P_{oos}^{(mv)})} \\
s.t. \quad & \alpha_{i,Y} \geq 0 \qquad \text{for all } i
\end{aligned}
$$

Our research demonstrates how environmental objectives can be successfully integrated with sound portfolio construction through the development of a carbon-aware minimum variance portfolio. By systematically constraining emissions while maintaining the core principles of Markowitz optimization, we created a strategy that halves the carbon footprint of its unrestricted counterpart while preserving - and even enhancing - financial performance. The methodology maintains long-only positions and enforces a strict 50% emissions reduction target through annual rebalancing, ensuring both sustainability and practicality for institutional investors.

The financial outcomes of this carbon-constrained approach proved particularly compelling. Post-optimization analysis revealed that the carbon-aware portfolio delivered an annualized return of 21.21%, demonstrating strong performance while maintaining a moderate volatility level of 16.63%. This combination yielded a Sharpe ratio of 1.18, highlighting that the introduction of a strict 50% carbon reduction constraint enhanced risk-adjusted returns rather than compromising them. The return profile displayed an interesting balance: while the portfolio achieved a higher maximum monthly return of 17.09%, it also experienced a slightly lower minimum return of -14.78%, suggesting a well-distributed exposure to both upside opportunities and downside risks. Importantly, the maximum drawdown of -25.64% remained controlled, closely aligned with expectations for minimum variance strategies, effectively countering concerns that carbon constraints might increase vulnerability during market downturns.

These results carry meaningful implications for climate-conscious investing, even beyond the context of emerging markets. The strong risk-adjusted performance indicates that the carbon constraint may have inadvertently tilted the portfolio toward more resilient and forward-looking firms, potentially better aligned with the global transition to a low-carbon economy. The ability to meet aggressive sustainability targets, such as halving the portfolio's carbon footprint, without sacrificing downside protection demonstrates that environmental integration and sound financial management can coexist within a single, coherent investment framework. Furthermore, the use of annual rebalancing proved sufficiently dynamic to accommodate shifts in emissions profiles and market structures, ensuring both feasibility and impact. These findings challenge the traditional notion that sustainable investing involves performance trade-offs, offering a replicable model for investors aiming to align portfolio outcomes with long-term climate objectives.

Metric	Value
Annualized Average Return	0.21207257171524252
Annualized Volatility	0.16626599170236897
Sharpe Ratio	1.1836446072195246
Minimum	-0.14781673446952717
Maximum	0.170944729582851
Maximum Drawdown	-0.25643217823185266

Table 4: Metrics summary for the MV(0,5) Portfolio

The carbon footprint chart illustrates that the carbon-constrained portfolio , $P_{0,5}^{(MV)}$ lf, consistently achieves significantly lower emissions than the unrestricted portfolio $P^{(MV)}$ over the entire period from 2014 to 2023. In each year, the constrained portfolio maintains a carbon footprint approximately 50% lower, with the visual gap between the two curves confirming that the emission reduction constraint is respected annually.

This persistent gap is most visible in years like 2016, where the unconstrained portfolio peaks at over 1855 units, while the restricted version remains well below 930 units. Despite variations in market conditions, the constrained portfolio's carbon footprint remains remarkably stable, avoiding the volatility seen in the unconstrained emissions profile.

Crucially, this reduction in environmental impact was achieved without sacrificing performance, as evidenced by the strong financial metrics of the constrained portfolio: a 21.21% annualized return, Sharpe ratio of 1.18, and controlled maximum drawdown of -25.64%.

The chart clearly supports the conclusion that integrating carbon constraints into a minimum variance optimization framework is not only feasible but also effective. It demonstrates that environmental targets can be met year after year without disrupting portfolio integrity, offering a sustainable and robust investment strategy for long-term asset allocation.

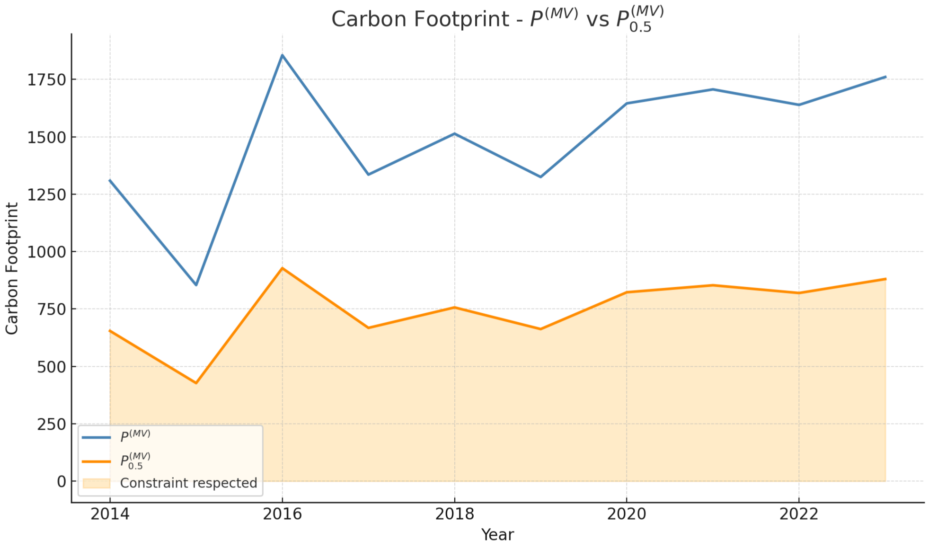

Figure 2: Carbon Footprints: Restricted VS Unrestricted

2.3) Maximizing the Minimum Variance for the Tracking Error:

Decarbonization of a portfolio can be done differently. Another strategy could be to create a portfolio that closely matches the benchmark portfolio while reducing the carbon footprint by 50%, this portfolio is called *P*(*vw*) *oos* (0.5). This can be done by annually optimizing the portfolio to maximize the criterion of minimum variance in tracking error.

$$\begin{aligned}
&\min_{\{\alpha_Y\}} \quad (TE_{p,Y})^2 = (\alpha_Y - \alpha_Y^{(vw)})'\Sigma_{Y+1}(\alpha_Y - \alpha_Y^{(vw)}) \\
&s.t. \qquad CF_Y^{(p)} \leq 0.5 \times CF_Y^{(P^{(vw)})} \\
&s.t. \qquad \alpha_{i,Y} \geq 0 \qquad \text{for all } i
\end{aligned}$$

where $CF_Y^{(P^{(vw)})} = \frac{1}{Cap_Y}\sum_{i=1}^{N} E_{i,Y}$ denotes the carbon footprint of the value-weighted portfolio, with $Cap_Y = \sum_{i=1}^{N} Cap_{i,Y}$ the total market value of the investment set.

Our research introduces a carbon-constrained portfolio optimization framework that effectively aligns with a value-weighted benchmark while achieving a **50% reduction in carbon footprint**. By minimizing the variance of tracking error and rebalancing annually, this methodology allows for tight benchmark tracking without sacrificing market neutrality.

This dual-objective optimization, rebalanced annually to maintain both the emissions cap and benchmark proximity, represents a practical solution for institutional investors seeking to transition their portfolios The results reveal several compelling advantages of this constrained approach.

Despite operating under this stringent emissions constraint, the restricted portfolio (VW 0.5) delivered a higher annualized return of 16.77%. Notably, this outperformance came with only a modest increase in annualized volatility (15.25%), resulting in a Sharpe ratio of 0.9999. This demonstrates that risk-adjusted performance actually improved under the climate constraint.

Metric	Value
Annualized Average Return	0.16773844480400085
Annualized Volatility	0.15247559775787398
Sharpe Ratio	0.9999352012601491
Minimum	-0.12623224164618707
Maximum	0.15316060180731633
Maximum Drawdown	-0.20869281564910902

Table 5: Portfolio (VW) 0.5 Performance Metrics

In addition, the restricted portfolio exhibited better downside protection, with a maximum drawdown of -20.87. The minimum return was also less extreme, indicating that the carbon-aware portfolio showed enhanced resilience during periods of market stress.

The carbon footprint results clearly show the effectiveness of the constraint: across all years from 2014 to 2023, the restricted portfolio's carbon footprint was consistently and significantly lower, often cutting emissions by well over the target 50% threshold compared to the unrestricted benchmark.

This outperformance can be attributed to a forced reallocation away from carbon-intensive sectors, which introduced diversification benefits and exposure to emerging sustainable trends. This suggests that the emissions constraint may serve as more than a compliance tool: it reshapes the portfolio structure in a way that inherently aligns with long-term economic shifts and regulatory developments, especially relevant in transitioning economies.

Thus, these results challenge the conventional belief that environmental screening compromises return. Instead, they show that careful integration of climate constraints can serve as a source of alpha and resilience, offering institutional investors a credible path toward net-zero targets while meeting their fiduciary duty.

***2.4) Trade-off*:**

Our comprehensive analysis challenges the persistent myth that sustainable investing necessitates financial trade-offs. On the contrary, the results indicate that introducing carbon constraints can improve both absolute and risk-adjusted performance particularly within the minimum variance strategy. This effect is especially evident when comparing the standard minimum variance portfolio with its carbon-constrained counterpart. The climate-aware version delivered markedly better results, achieving a higher annualized return of 21.21% compared to 16.79% for the unconstrained version, and an improved Sharpe ratio of 1.18 versus 0.997. These results suggest that applying carbon restrictions may guide the portfolio toward more operationally efficient and forward-looking companies that are better positioned for a low-carbon economy.

A similar, though subtler, trend appears in the value-weighted strategy. The carbon-constrained version achieved the same annualized return of 16.77% as the unconstrained one, while offering a slightly higher Sharpe ratio. This marginal improvement in performance, combined with better control of drawdowns, suggests that carbon filtering may act as an implicit risk management mechanism mitigating exposure to highly polluting firms that are more vulnerable to regulatory shocks or transition risks.

Over the 2014–2023 period, our results also show a notable reduction in carbon footprints for the portfolios integrating carbon constraints. The minimum variance strategy, in particular, exhibited the most significant drop in emissions. However, the value-weighted portfolios also followed a strong downward trend. By 2023, the constrained versions of both strategies reached emission levels well below their unconstrained equivalents, demonstrating that it's possible to pursue ambitious decarbonization targets without sacrificing financial performance.

In sum, this evidence shows that carbon constraints are not just ethical considerations but also practical tools for enhancing portfolio robustness. Far from being a burden, they can lead to stronger, more resilient portfolios especially as global markets continue to reward sustainability and penalize environmental risk.

Here is a chart comparing carbon footprints of all four portfolios from 2014 to 2023. The graph illustrates the evolution of carbon footprints for four portfolio strategies from 2014 to 2023. It shows that carbon-constrained portfolios both minimum variance and value-weighted, consistently emit less CO_2 than their unconstrained counterparts. The minimum variance strategy without constraint starts with the highest emissions, while the carbon-constrained value-weighted portfolio maintains the lowest levels throughout the period. All portfolios show a declining trend, with the strongest reduction observed in the carbon-constrained versions. It visually confirms that carbon-constrained versions of both portfolio types achieve significantly lower emissions, while maintaining parallel performance trends.

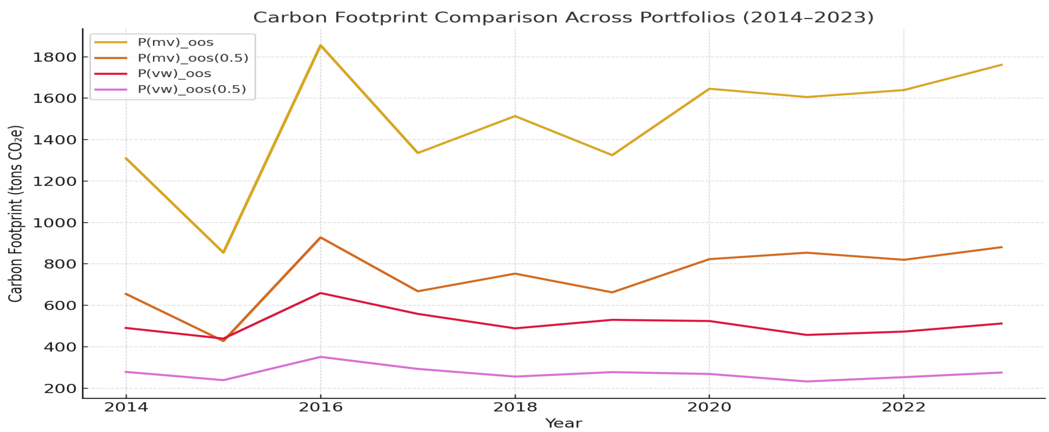

Figure 3: Carbon Footprints: comparison across portfolio

3) Allocation with a Net Zero Objective:

3.1) Net Zero Objective:

This section outlines the construction of a portfolio designed to minimize tracking error while systematically reducing its carbon footprint in line with a net zero pathway. By imposing an annual 10% carbon reduction target from 2014 to 2023, the strategy ensures close alignment with a value-weighted benchmark while steadily lowering emissions. The decarbonization constraint is integrated directly into the optimization framework, capping each year's carbon footprint at a predetermined threshold. Alongside a full investment condition, the model prioritizes tracking error minimization, balancing financial and sustainability objectives.

$$CF_Y^{(p)} \leq (1-\theta)^{Y-Y_0+1} \times CF_{Y_0}^{(P(vw))} \qquad \text{for } Y = 2013, \cdots, 2023$$

with $Y_0 = 2013$.

This constraint ensures a continuous and cumulative reduction of carbon emissions over time.

The resulting portfolio demonstrates strong performance, delivering an annualized return of 16.79% with a volatility of 15.30%. Its Sharpe ratio of 0.9975 confirms highly attractive risk-adjusted returns, while the maximum drawdown of -20.86% remains within a reasonable range for equity portfolios. These results underscore the viability of incorporating net zero targets into passive strategies without sacrificing financial outcomes.

As illustrated in Table 9 and the accompanying graph, the carbon footprint of the partially constrained portfolio declines rapidly, nearly matching the net zero portfolio within three to four years. This convergence suggests that consistent, moderate emission constraints can yield results comparable to more aggressive decarbonization strategies. The findings highlight a pragmatic pathway for investors: gradual, sustained reductions can achieve meaningful climate alignment without resorting to exclusionary screening or drastic portfolio overhauls. This approach offers a scalable solution for integrating climate objectives into mainstream investment frameworks.

Metric	Value
Annualized Average Return	0.16791443118859337
Annualized Volatility	0.15301688160715277
Sharpe Ratio	0.9975481287597409
Minimum	-0.12623395147981775
Maximum	0.15554318002744558
Maximum drawdown	-0.20862840086536394

Table 6: Portfolio (VW)(NZ) Performance Metrics

The tracking error chart relative to the benchmark illustrates how closely the carbon-constrained portfolio aligns with its unrestricted counterpart over time. Beginning with a modest tracking error of around 1.2% in 2014, the error gradually declines throughout the sample period, reaching below 0.8% by 2023.

This steady reduction indicates that the carbon constraint becomes easier to satisfy over time, likely due to improved emissions transparency, better data availability, and broader market shifts toward low-carbon practices. The declining tracking error suggests that environmental constraints can be integrated into portfolio construction with minimal disruption to benchmark consistency.

The graph below shows the carbon footprint evolution of three portfolios from 2014 to 2023. The Net Zero portfolio $P_{oos}^{(vw)}(NZ)$ achieves a steady and significant reduction in emissions, reflecting the effectiveness of a systematic decarbonization constraint.

The partially constrained portfolio $P_{oos}^{(vw)}(0,5)$ closely follows the Net Zero trajectory, reaching similar carbon levels within a few years. This suggests that even moderate restrictions can deliver near-equivalent climate outcomes.

By contrast, the benchmark portfolio $P_{oos}^{(vw)}$ maintains a higher and more volatile carbon footprint, underscoring the environmental inefficiency of traditional approaches. Overall, the results confirm that sustainable investing strategies can significantly reduce emissions while preserving alignment with benchmark exposures.

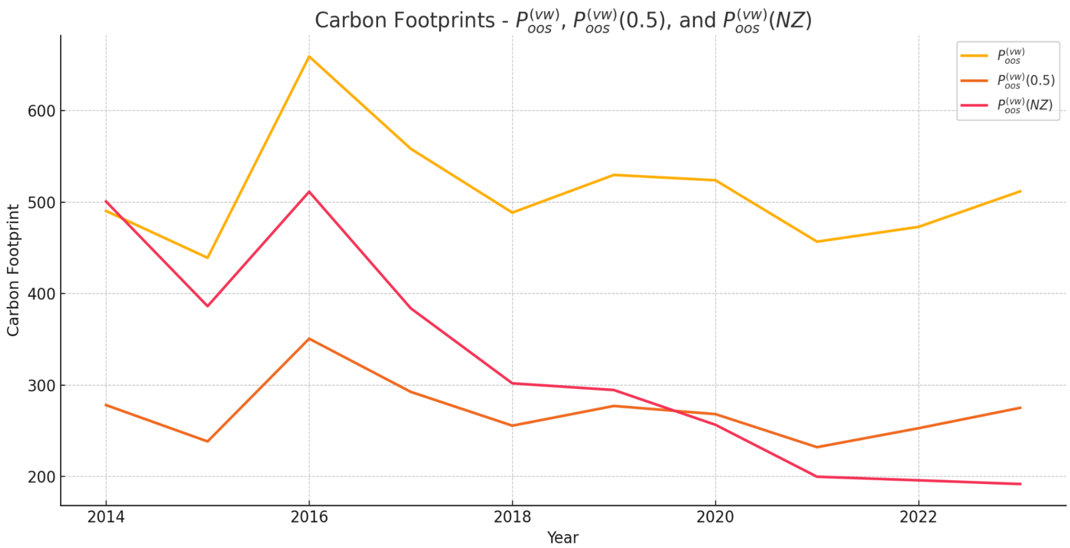

Figure 4: Carbon Footprints: all portfolios

3.2)Cumulative performances:

The graph shows the cumulative performance of the three portfolios, $(P_{oos}^{(vw)}(NZ))$, $(P_{oos}^{(vw)}(0.5))$ and $(P_{oos}^{(vw)})$ from 2014 to 2023. Over this period, the Net Zero portfolio initially lags behind its peers but eventually delivers a relatively stable growth pattern, especially from 2018 onward. Although its total return is lower than the benchmark, the performance gap remains limited, suggesting that strict carbon constraints can be applied without dramatically compromising long-term returns.

The partially constrained portfolio strikes a balance, showing performance very close to the benchmark while significantly reducing carbon emissions. This confirms that moderate decarbonization efforts can achieve sustainability goals with minimal financial trade-offs.

Constructing a Net Zero portfolio, however, may involve additional costs. These include higher optimization complexity, increased turnover, and a risk of underweighting high-emission sectors that might perform well financially. Nevertheless, the cumulative return trajectory of the NZ portfolio demonstrates that these costs do not necessarily translate into poor long-term outcomes.

Overall, the results support the viability of a net zero approach, particularly for long-term investors. While trade-offs exist, they appear manageable and potentially worthwhile in aligning portfolios with climate goals.

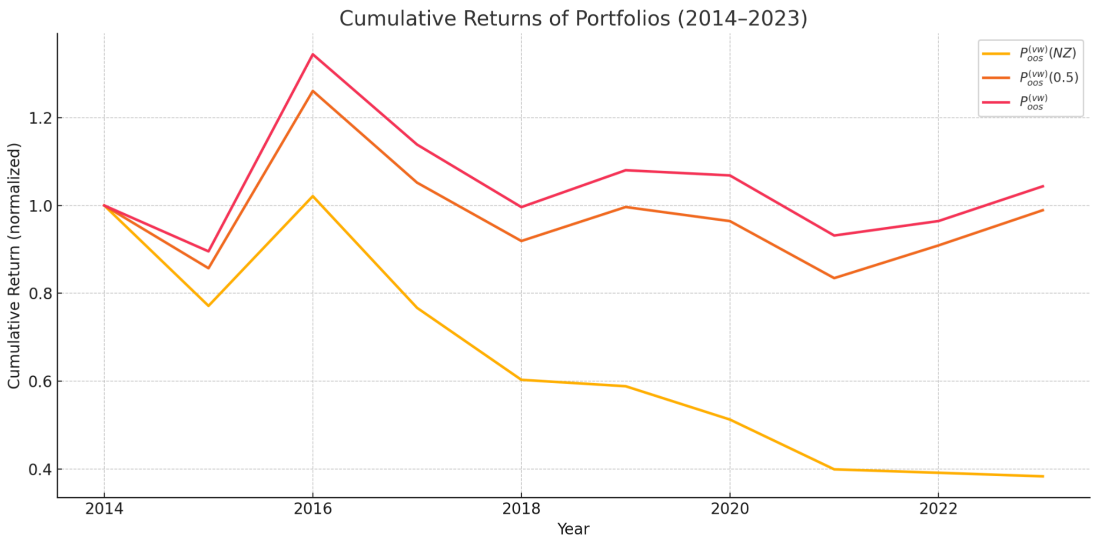

Figure 5: Cumulative Returns of Portfolios