Assignment - 4

1 a) $f(x,y) = x^2 - y^2$ subject to $x^2 + y^2 = 4$

$$2x = 2\lambda x$$
$$-2y = 2\lambda y$$

$$2x - 2\lambda x = 0$$
$$2x(1-\lambda) = 0$$
$$x = 0 \quad \text{or} \quad \lambda = 1$$
$$y = \pm 2$$

If $\lambda = 1$ |||ly

$\cancel{-2y}$
$$2\lambda y + 2y = 0$$
$$2y(\lambda + 1) = 0$$
$$y = 0 \quad \text{or} \quad \lambda = -1$$
$$x = \pm 2$$

The Points are $(0,2)$ $(0,-2)$ $(2,0)$ and $(-2,0)$

$$f(0,0) = 0$$
$$\left.\begin{aligned} f(0,-2) &= -4 \\ f(0,2) &= -4 \end{aligned}\right\} \rightarrow \text{Minimum}$$
$$\left.\begin{aligned} f(2,0) &= 4 \\ f(-2,0) &= 4 \end{aligned}\right\} \quad \text{Maximum}$$

The function minimum are at points $(0,-2)$ and $(0,2)$ and maximum at $(2,0)$ and $(-2,0)$.

b) ~~$F(x,y) = x^2y - \ln(x)$ subject to $g(x,y) = 8x+3y$~~

Alternate Method

$$y^2 = 4 - x^2$$

$$f(x,x) = x^2 - 4 + x^2$$
$$= 2x^2 - 4$$

$$f'(x) = 4x$$

Substituting $f'(x) = 0$ we get

$$4x = 0$$
$$x = 0$$

when $x = 0$, $y = \pm 2$

$$F''(x) = 4 > 0$$

Function maximum is 4 at $(2, 0)$ and $(-2, 0)$.

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b) $f(x,y) = x^2y - \log x$ subject to $x + 2y = 0$

$y = -\frac{x}{2}$

$f\left(x, -\frac{x}{2}\right) = -\frac{x^3}{2} - \log x$

$f'(x) = -\frac{3x^2}{2} - \frac{1}{x}$

~~Sett~~ Equating $f'(x) = 0$

$-3x^3 - 2 = 0$

$-3x^3 = 2$

$x^3 = -\frac{2}{3}$ or $x = -\sqrt[3]{\frac{2}{3}}$

$f''(x) = -3x + \frac{1}{x^2}$

$= -\frac{3x^3 + 1}{x^2}$ $\cancel{= 3.434 > 0}$

$= \frac{3}{0.7631}$

$f''\left(-\sqrt[3]{\frac{2}{3}}\right) = 3.931 > 0$

It is not possible to get value for log x if x is negative hence there is no solution for this function.

c) $f(x,y) = x^2 + 2xy + y^2$

subject to $x^2 - y^2 + 1 = 0$

$f(x,y,z) = x^2 + 2xy + y^2 - 2x - \lambda x^2 + \lambda y^2 - \lambda$

$f'(x) = 2x + 2y - 2 - 2\lambda x = 0$

$x + y - \lambda x - 1 = 0$

$y = 1 - x + x\lambda$ ——— (1)

$f'(y) = 2x + 2y + 2\lambda y = 0$

$x + y + \lambda y = 0$

$\cancel{x =}$

$x + (1 - x + x\lambda) + \lambda(1 - x + x\lambda) = 0$

$\cancel{x} + 1 - \cancel{x} + \cancel{x\lambda} + \lambda - \cancel{x\lambda} + x\lambda^2 = 0$

$1 + \lambda = -\lambda^2 x$

$x = -\frac{(1+\lambda)}{\lambda^2}$

Substituting x in (1)

$y = 1 - x(1-\lambda)$

$= 1 + \frac{(1+\lambda)(1-\lambda)}{\lambda^2}$

$= \frac{\lambda^2 + 1 - \lambda^2}{\lambda^2}$

$y = \frac{1}{\lambda^2}$

Substituting x and y values in $x^2 - y^2 = -1$

$\cancel{1} + 2\lambda + \lambda^2 - \cancel{1} + \lambda^4 = 0$

$\lambda^4 + \lambda^2 + 2\lambda = 0$

$\lambda(\lambda^3 + \lambda + 2) = 0$

$\lambda^3 + \lambda + 2 = 0$ which can also be written as

$$(\lambda^2 - \lambda + 2)(\lambda + 1) = 0$$

Hence $\lambda = -1$

Then $y = 1$ and $x = 0$

Hence Maximum of Function is $\underline{1}$.