**All Experiments using Languerre Polynomial as Basis Functions**

**[[1. 1.09 1.08 1.34]**
**[1. 1.16 1.26 1.54]**
**[1. 1.22 1.07 1.03]**
**[1. 0.93 0.97 0.92]**
**[1. 1.11 1.56 1.52]**
**[1. 0.76 0.77 0.9 ]**
**[1. 0.92 0.84 1.01]**
**[1. 0.88 1.22 1.34]]**

**Cash Flow at time t = 3:**

Cash-flow matrix at time 3

| Path | $t = 1$ | $t = 2$ | $t = 3$ |
|---|---|---|---|
| 1 | — | — | .00 |
| 2 | — | — | .00 |
| 3 | — | — | .07 |
| 4 | — | — | .18 |
| 5 | — | — | .00 |
| 6 | — | — | .20 |
| 7 | — | — | .09 |
| 8 | — | — | .00 |

**At time t = 2**

**Coefficients = [-3094.53010697, 5713.89537929, -3773.67662635, 1223.18245248]**

| Paths | Exercise | Continuation | Stopping Rule | Value |
|---|---|---|---|---|
| 1 | 0.02 | 0.0133869 | 1. | 0.02 |
| 2 | 0.0 | 0. | 0. | 0. |
| 3 | 0.03 | 0.05010429 | 0. | 0.0659232 |
| 4 | 0.13 | 0.17319321 | 0. | 0.1695168 |
| 5 | 0.0 | 0. | 0. | 0. |
| 6 | 0.33 | 0.18910104 | 1. | 0.33 |
| 7 | 0.26 | 0.08276497 | 1. | 0.26 |
| 8 | 0.0 | 0. | 0. | 0. |

**Cash Flow matrix at time t=2**

| Path | t=1 | t=2 | t=3 |
|---|---|---|---|
| 1 | -- | 0.02 | 0.0 |
| 2 | -- | 0.0 | 0.0 |
| 3 | -- | 0.0 | 0.7 |
| 4 | -- | 0.0 | 0.0 |
| 5 | -- | 0.0 | 0.0 |
| 6 | -- | 0.33 | 0.0 |
| 7 | -- | 0.26 | 0.0 |
| 8 | -- | 0.0 | 0.0 |

**At time t = 1**

**Coefficients :**

**[ -9091.91659795, 16794.05861997, -11103.23289512, 3606.65973536]**

| Paths | Exercise | Continuation | Stopping Rule | Value |
|---|---|---|---|---|
| 1 | 0.01 | 0.01839949 | 0 | 0.0188352 |
| 2 | -- | 0. | 0 | 0. |
| 3 | -- | 0. | 0 | 0.06208383 |
| 4 | 0.17 | 0.21653033 | 0 | 0.15964414 |
| 5 | -- | 0. | 0 | 0. |
| 6 | 0.34 | 0.31010703 | 1 | 0.34 |
| 7 | 0.18 | 0.17401594 | 1 | 0.18 |
| 8 | 0.22 | 0.01506496 | 1 | 0.22 |

**Cash flow matrix at time t = 1**

| Path | t=1 | t=2 | t=3 |
|---|---|---|---|
| 1 | 0.0 | 0.02 | 0.0 |
| 2 | 0.0 | 0.0 | 0.0 |
| 3 | 0.0 | 0.0 | 0.7 |
| 4 | 0.0 | 0.0 | 0.0 |
| 5 | 0.0 | 0.0 | 0.0 |
| 6 | 0.34 | 0.0 | 0.0 |
| 7 | 0.18 | 0.0 | 0.0 |
| 8 | 0.22 | 0.0 | 0.0 |

**Final Stopping Rule :**

| Paths | Time = 1 | Time = 2 | Time = 3 |
|---|---|---|---|
| 1 | 0 | 1. | 0 |
| 2 | 0 | 0. | 0 |
| 3 | 0 | 0. | 1 |
| 4 | 0 | 0. | 1 |
| 5 | 0 | 0. | 0 |
| 6 | 1 | 0 | 0 |
| 7 | 1 | 0 | 0 |
| 8 | 1 | 0. | 0 |

**Optimal Price = 0.11543189689036804 (In simple basis function it was 0.1144)**

**Introducing the Gaussian Noise of Sigma = 0.2:**

**[[1. 1.11343978 1.13340296 1.36698591]**
**[1. 1.19068048 1.41736844 1.78457611]**
**[1. 1.23867356 1.17903336 1.04504003]**
**[1. 1.03560851 1.13502242 1.3634956 ]**
**[1. 1.1168677 1.85937596 1.70780791]**
**[1. 0.779318 0.82099096 1.15783301]**
**[1. 0.92590538 0.8445708 1.06621553]**
**[1. 0.90475507 1.26372151 1.34264561]]**

**Cash Flow matrix at time t = 3**

| Path | t=1 | t=2 | t=3 |
|---|---|---|---|
| 1 | -- | -- | 0.0 |
| 2 | -- | -- | 0.0 |
| 3 | -- | -- | 0.05175911 |
| 4 | -- | -- | 0.0 |
| 5 | -- | -- | 0.0 |
| 6 | -- | -- | 0.0 |
| 7 | -- | -- | 0.034 |
| 8 | -- | -- | 0.0 |

**At time t = 2**
**Coefficients = [-3094.53010697, 5713.89537929, -3773.67662635, 1223.18245248]**

| Paths | Exercise | Continuation | Stopping Rule | Value |
|---|---|---|---|---|
| 1 | 0.0 | 0.0 | 0 | 0. |
| 2 | 0.0 | 0.0 | 0 | 0. |
| 3 | 0.0 | 0.0 | 0 | 0.05175911 |
| 4 | 0.0 | 0.0 | 0 | 0. |
| 5 | 0.0 | 0.0 | 0 | 0. |
| 6 | 0.27900904 | 0.09065539 | 1 | 0.27900904 |
| 7 | 0.2554292 | 0.08260464 | 1 | 0.2554292 |
| 8 | 0.0 | 0.0 | 0 | 0. |

**Cash flow matrix at time t = 2:**

| Path | t=1 | t=2 | t=3 |
|---|---|---|---|
| 1 | -- | 0.0 | 0.0 |
| 2 | -- | 0.0 | 0.0 |
| 3 | -- | 0.0 | 0.05175911 |
| 4 | -- | 0.0 | 0.0 |
| 5 | -- | 0.0 | 0.0 |
| 6 | -- | 0.27900904 | 0.0 |
| 7 | -- | 0.2554292 | 0.0 |
| 8 | -- | 0.0 | 0.0 |

**At time t = 1**
**Coefficients : [ -9091.91659795, 16794.05861997, -11103.23289512, 3606.65973536]**

| Paths | Exercise | Continuation | Stopping Rule | Value |
|---|---|---|---|---|
| 1 | 0.0 | 0. | 0 | 0. |
| 2 | 0.0 | 0. | 0 | 0. |
| 3 | 0.0 | 0. | 0 | 0.04874466 |
| 4 | 0.065 | 0.38393835 | 0 | 0. |
| 5 | 0.0 | 0. | 0 | 0. |
| 6 | 0.320682 | 0.1255111 | 1 | 0.320682 |
| 7 | 0.175 | 0.19920065 | 0 | 0.240553 |
| 8 | 0.19524493 | 0.10950071 | 1 | 0.19524493 |

**Cash flow matrix at time t = 1:**

| Path | t=1 | t=2 | t=3 |
|---|---|---|---|
| 1 | 0.0 | 0.0 | 0.0 |
| 2 | 0.0 | 0.0 | 0.0 |
| 3 | 0.0 | 0.0 | 0.05175911 |
| 4 | 0.0 | 0.0 | 0.0 |
| 5 | 0.0 | 0.0 | 0.0 |
| 6 | 0.320682 | 0.0 | 0.0 |
| 7 | 0.0 | 0.2554292 | 0.0 |
| 8 | 0.19524493 | 0.0 | 0.0 |

**Final Stopping Rule :**

| Paths | Time = 1 | Time = 2 | Time = 3 |
|---|---|---|---|
| 1 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 |
| 3 | 0 | 0 | 1 |
| 4 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 |
| 6 | 1 | 0 | 0 |
| 7 | 0 | 0 | 1 |
| 8 | 1 | 0 | 0 |

**Manual Cross Checking : (Gamma = 0.94176)**

| **Paths** | **diff1** | **Gamma * diff2** | **Gamma^2 * diff3** | **Stoping Rule** |
|---|---|---|---|---|
| **1** | 0.0 | 0.0 | 0.0 | - |
| **2** | 0.0 | 0.0 | 0.0 | - |
| **3** | 0.0 | 0.0 | 0.04873608 | 3 |
| **4** | 0.0606417157 | 0.0 | 0.0 | 1 |
| **5** | 0.0 | 0.0 | 0.0 | - |
| **6** | 0.302022432 | 0.262760 | 0.0 | 1 |
| **7** | 0.163960 | 0.24109056 | 0.03201984 | 2 |
| **8** | 0.183873 | 0.0 | 0.0 | 1 |

Optimal Price = 0.10000000000000009

**Accuracy : 2 wrong prediction => 75 % accuracy**

**Gaussian Predicion with sigma = 0.6 :**

**[[1. 1.20993179 1.75371182 1.96063962]**
**[1. 1.26962438 2.2039869 1.87847896]**
**[1. 1.87851098 1.37150358 1.31889186]**
**[1. 1.17555113 1.44712196 1.55477263]**
**[1. 1.42239849 2.25786685 1.61419144]**
**[1. 1.21605533 0.89513254 2.00467478]**
**[1. 1.58719376 2.93110567 1.43612848]**
**[1. 1.23806832 1.38557408 1.66104208]]**

**Cash Flow matrix at time t = 3**

| Path | t=1 | t=2 | t=3 |
|---|---|---|---|
| 1 | -- | -- | 0.0 |
| 2 | -- | -- | 0.0 |
| 3 | -- | -- | 0.0 |
| 4 | -- | -- | 0.0 |
| 5 | -- | -- | 0.0 |
| 6 | -- | -- | 0.0 |
| 7 | -- | -- | 0.0 |
| 8 | -- | -- | 0.0 |

**At time t = 2**
**Coefficients = [-3094.53010697, 5713.89537929, -3773.67662635, 1223.18245248]**

| Paths | Exercise | Continuation | Stopping Rule | Value |
|---|---|---|---|---|
| 1 | 0.0 | 0.0 | 0 | 0 |
| 2 | 0.0 | 0.0 | 0 | 0 |
| 3 | 0.0 | 0.0 | 0 | 0 |
| 4 | 0.0 | 0.0 | 0 | 0 |
| 5 | 0.0 | 0.0 | 0 | 0 |
| 6 | 0.20486746 | 0.11109279 | 1 | 0.20486746 |
| 7 | 0.0 | 0.0 | 0 | 0 |
| 8 | 0.0 | 0.0 | 0 | 0 |

**Cash Flow Matrix at time t=2**

| Path | t=1 | t=2 | t=3 |
|---|---|---|---|
| 1 | -- | 0.0 | 0.0 |
| 2 | -- | 0.0 | 0.0 |
| 3 | -- | 0.0 | 0.0 |
| 4 | -- | 0.0 | 0.0 |
| 5 | -- | 0.0 | 0.0 |
| 6 | -- | 0.20486746 | 0.0 |
| 7 | -- | 0.0 | 0.0 |
| 8 | -- | 0.0 | 0.0 |

**At time t = 1**
**Coefficients : [ -9091.91659795, 16794.05861997, -11103.23289512, 3606.65973536]**

| Paths | Exercise | Continuation | Stopping Rule | Value |
|---|---|---|---|---|
| 1 | 0.0 | 0.0 | 0 | 0.0 |
| 2 | 0.0 | 0.0 | 0 | 0.0 |
| 3 | 0.0 | 0.0 | 0 | 0.0 |
| 4 | 0.0 | 0.0 | 0 | 0.0 |
| 5 | 0.0 | 0.0 | 0 | 0.0 |
| 6 | 0.0 | 0.0 | 0 | 0.19293598 |
| 7 | 0.0 | 0.0 | 0 | 0.0 |
| 8 | 0.0 | 0.0 | 0 | 0.0 |

**Cash Flow at time t = 1**

| Path | t=1 | t=2 | t=3 |
|---|---|---|---|
| 1 | 0.0 | 0.0 | 0.0 |
| 2 | 0.0 | 0.0 | 0.0 |
| 3 | 0.0 | 0.0 | 0.0 |
| 4 | 0.0 | 0.0 | 0.0 |
| 5 | 0.0 | 0.0 | 0.0 |
| 6 | 0.0 | 0.20486746 | 0.0 |
| 7 | 0.0 | 0.0 | 0.0 |
| 8 | 0.0 | 0.0 | 0.0 |

**Final Stopping Rule :**

| Paths | Time = 1 | Time = 2 | Time = 3 |
|---|---|---|---|
| 1 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 |
| 6 | 0 | 1 | 0 |
| 7 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 |

**Manual Cross Checking : (Gamma = 0.94176)**

| **Paths** | **diff1** | **Gamma * diff2** | **Gamma^2 * diff3** | **Stoping Rule** |
|---|---|---|---|---|
| **1** | 0.0 | 0.0 | 0.0 | - |
| **2** | 0.0 | 0.0 | 0.0 | - |
| **3** | 0.0 | 0.0 | 0.0 | - |
| **4** | 0.0 | 0.0 | 0.0 | - |
| **5** | 0.0 | 0.0 | 0.0 | - |
| **6** | 0.0 | 0.192966624 | 0.0 | 2 |
| **7** | 0.0 | 0.0 | 0.0 | - |
| **8** | 0.0 | 0.0 | 0.0 | - |

Optimal Price = 0.10000000000000009

**Accuracy : 0 wrong prediction => 100 % accuracy**

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**Gaussian Predicion with sigma = 1 :**

**[[1. 2.48151885 1.50961896 2.09427441]**
**[1. 2.05334654 1.60238537 3.52100738]**
**[1. 1.66599697 1.63350074 1.33373645]**
**[1. 1.92539639 1.98067561 0.96344852]**
**[1. 2.27563194 2.68070282 3.18124507]**
**[1. 1.44581574 1.7548213 1.76783921]**
**[1. 2.07006125 1.29450447 1.1010486 ]**
**[1. 1.41987375 5.48936929 3.22416844]]**

**Cash Flow matrix at time t = 3**

| Path | t=1 | t=2 | t=3 |
|---|---|---|---|
| 1 | -- | -- | 0.0 |
| 2 | -- | -- | 0.0 |
| 3 | -- | -- | 0.0 |
| 4 | -- | -- | 0.13655148 |
| 5 | -- | -- | 0.0 |
| 6 | -- | -- | 0.0 |
| 7 | -- | -- | 0.0 |
| 8 | -- | -- | 0.0 |

**At time t = 2**
**Coefficients = [-3094.53010697, 5713.89537929, -3773.67662635, 1223.18245248]**

| Paths | Exercise | Continuation | Stopping Rule | Value |
|---|---|---|---|---|
| 1 | 0.0 | 0.0 | 0 | 0 |
| 2 | 0.0 | 0.0 | 0 | 0 |
| 3 | 0.0 | 0.0 | 0 | 0 |
| 4 | 0.0 | 0.0 | 0 | 0.12859872 |
| 5 | 0.0 | 0.0 | 0 | 0 |
| 6 | 0.0 | 0.0 | 0 | 0.0 |
| 7 | 0.0 | 0.0 | 0 | 0 |
| 8 | 0.0 | 0.0 | 0 | 0 |

## Cash Flow Matrix at time t=2

| Path | t=1 | t=2 | t=3 |
|---|---|---|---|
| 1 | -- | 0.0 | 0.0 |
| 2 | -- | 0.0 | 0.0 |
| 3 | -- | 0.0 | 0.0 |
| 4 | -- | 0.0 | 0.12859872 |
| 5 | -- | 0.0 | 0.0 |
| 6 | -- | 0.0 | 0.0 |
| 7 | -- | 0.0 | 0.0 |
| 8 | -- | 0.0 | 0.0 |

**At time t = 1**
**Coefficients : [ -9091.91659795, 16794.05861997, -11103.23289512, 3606.65973536]**

| Paths | Exercise | Continuation | Stopping Rule | Value |
|---|---|---|---|---|
| 1 | 0.0 | 0.0 | 0 | 0.0 |
| 2 | 0.0 | 0.0 | 0 | 0.0 |
| 3 | 0.0 | 0.0 | 0 | 0.0 |
| 4 | 0.0 | 0.0 | 0 | 0.12110913 |
| 5 | 0.0 | 0.0 | 0 | 0.0 |
| 6 | 0.0 | 0.0 | 0 | 0.0 |
| 7 | 0.0 | 0.0 | 0 | 0.0 |
| 8 | 0.0 | 0.0 | 0 | 0.0 |

## Cash Flow Matrix at time t=1

| Path | t=1 | t=2 | t=3 |
|---|---|---|---|
| 1 | 0.0 | 0.0 | 0.0 |
| 2 | 0.0 | 0.0 | 0.0 |
| 3 | 0.0 | 0.0 | 0.0 |
| 4 | 0.0 | 0.0 | 0.12859872 |
| 5 | 0.0 | 0.0 | 0.0 |
| 6 | 0.0 | 0.0 | 0.0 |
| 7 | 0.0 | 0.0 | 0.0 |
| 8 | 0.0 | 0.0 | 0.0 |

**Final Stopping Rule :**

| Paths | Time = 1 | Time = 2 | Time = 3 |
|---|---|---|---|
| 1 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 |
| 4 | 0 | 0 | 1 |
| 5 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 |

**Manual Cross Checking : (Gamma = 0.94176)**

| **Paths** | **diff1** | **Gamma * diff2** | **Gamma^2 * diff3** | **Stoping Rule** |
|---|---|---|---|---|
| **1** | 0.0 | 0.0 | 0.0 | - |
| **2** | 0.0 | 0.0 | 0.0 | - |
| **3** | 0.0 | 0.0 | 0.0 | - |
| **4** | 0.0 | 0.0 | 0.121109132 | 3 |
| **5** | 0.0 | 0.0 | 0.0 | - |
| **6** | 0.0 | 0.0 | 0.0 | - |
| **7** | 0.0 | 0.0 | 0.0 | - |
| **8** | 0.0 | 0.0 | 0.0 | - |

Optimal Price = 0.10000000000000009

**Accuracy : 0 wrong prediction => 100 % accuracy**

**Increasing the number of Paths to 16 : (8 extra paths added by an randomly generated path with variance of 0.1)**

**Longstaff Paper :**

**[[1. 1.09 1.08 1.34 ]**
**[1. 1.16 1.26 1.54 ]**
**[1. 1.22 1.07 1.03 ]**
**[1. 0.93 0.97 0.92 ]**
**[1. 1.11 1.56 1.52 ]**
**[1. 0.76 0.77 0.9 ]**
**[1. 0.92 0.84 1.01 ]**
**[1. 0.88 1.22 1.34 ]**
**[1. 1.09972955 1.20984266 1.42786124]**
**[1. 1.30282465 1.33379354 1.54642148]**
**[1. 1.30811775 1.18827482 1.05821046]**
**[1. 1.00225371 1.22542088 1.06681238]**
**[1. 1.17592812 1.65280718 1.72744582]**
**[1. 0.89931269 0.85576233 0.95650906]**
**[1. 0.9397596 0.92159093 1.03515879]**
**[1. 0.90082674 1.26323588 1.41475168]]**

**A. Using Longstaff Coefficients:**

**Cash Flow Matrix at time t = 3:**

| Path | t=3 |
|---|---|
| 1 | 0.0 |
| 2 | 0.0 |
| 3 | 0.7 |
| 4 | 0.18 |
| 5 | 0.0 |
| 6 | 0.2 |
| 7 | 0.09 |
| 8 | 0.0 |
| 9 | 0.0 |
| 10 | 0.0 |
| 11 | 0.042 |
| 12 | 0.034 |
| 13 | 0.0 |

| | |
|---|---|
| 14 | 0.144 |
| 15 | 0.065 |
| 16 | 0.0 |

**For time t=2:**

**Coefficients: [-2239.87598024, 4136.69651566, -2733.09043231, 887.0782914 ]**

| Paths | Exercise | Continuation | Stopping Rule | Value |
|---|---|---|---|---|
| 1 | 0.02 | 0.0200694 | 0 | 0. |
| 2 | 0.0 | 0. | 0 | 0. |
| 3 | 0.03 | 0.04641799 | 0 | 0.0659232 |
| 4 | 0.13 | 0.13890187 | 0 | 0.1695168 |
| 5 | 0.0 | 0. | 0 | 0. |
| 6 | 0.33 | 0.19323406 | 1 | 0.33 |
| 7 | 0.26 | 0.09520494 | 1 | 0.26 |
| 8 | 0.0 | 0. | 0 | 0. |
| 9 | 0.0 | 0. | 0 | 0. |
| 10 | 0.0 | 0. | 0 | 0. |
| 11 | 0.0 | 0. | 0 | 0.03935572 |
| 12 | 0.0 | 0. | 0 | 0.03125477 |
| 13 | 0.0 | 0. | 0 | 0. |
| 14 | 0.25 | 0.09275029 | 1 | 0.24423767 |
| 15 | 0.18 | 0.11817074 | 1 | 0.17840907 |
| 16 | 0.0 | 0. | 0 | 0. |

**Cash Flow Matrix at time t = 2:**

| Paths | t=1 | t=2 | t=3 |
|---|---|---|---|
| 1 | -- | 0 | 0.0 |
| 2 | -- | 0 | 0.0 |
| 3 | -- | 0 | 0.7 |
| 4 | -- | 0 | 0.18 |
| 5 | -- | 0 | 0.0 |
| 6 | -- | 0.33 | 0.0 |

| | | | |
|---|---|---|---|
| 7 | -- | 0.26 | 0.0 |
| 8 | -- | 0 | 0.0 |
| 9 | -- | 0 | 0.0 |
| 10 | -- | 0 | 0.0 |
| 11 | -- | 0 | 0.042 |
| 12 | -- | 0 | 0.034 |
| 13 | -- | 0 | 0.0 |
| 14 | -- | 0.24423767 | 0.0 |
| 15 | -- | 0.17840907 | 0.0 |
| 16 | -- | 0 | 0.0 |

**At time t = 1:**

**Coefficients:[-1723.47923186, 3185.06486082, -2106.88623773, 686.66193793]**

| Paths | Exercise | Continuation | Stopping Rule | Value |
|---|---|---|---|---|
| 1 | 0.01 | 0.00482983 | 1 | 1.0e-02 |
| 2 | 0.0 | 0. | 0 | 0.0 |
| 3 | 0.0 | 0. | 0 | 6.20838328e-02 |
| 4 | 0.17 | 0.12381877 | 1 | 1.70000000e-01 |
| 5 | 0.0 | 0. | 0 | 0.0 |
| 6 | 0.34 | 0.30008027 | 1 | 3.40e-01 |
| 7 | 0.18 | 0.12288012 | 1 | 1.80e-01 |
| 8 | 0.22 | 0.12384664 | 1 | 2.20e-01 |
| 9 | 0.001 | -0.0200075 | 1 | 2.70450e-04 |
| 10 | 0.0 | 0. | 0 | 0. |
| 11 | 0.0 | 0. | 0 | 3.70636402e-02 |
| 12 | 0.1 | 0.11858584 | 0 | 2.94344950e-02 0.0 |
| 13 | 0.0 | 0. | 0 | 0. |
| 14 | 0.201 | 0.12198031 | 1 | 2.00687310e-01 |
| 15 | 0.16 | 0.12476732 | 1 | 1.60240400e-01 |
| 16 | 0.2 | 0.12196722 | 1 | 1.99173260e-01 |

**Cash Flow Matrix at time t = 1:**

| Paths | t=1 | t=2 | t=3 |
|---|---|---|---|
| 1 | 0.01 | 0 | 0.0 |
| 2 | 0.0 | 0 | 0.0 |
| 3 | 0.0 | 0 | 0.7 |
| 4 | 0.17 | 0 | 0.0 |
| 5 | 0.0 | 0 | 0.0 |
| 6 | 0.34 | 0.0 | 0.0 |
| 7 | 0.18 | 0.0 | 0.0 |
| 8 | 0.22 | 0 | 0.0 |
| 9 | 0.001 | 0 | 0.0 |
| 10 | 0.0 | 0 | 0.0 |
| 11 | 0.0 | 0 | 0.042 |
| 12 | 0.0 | 0 | 0.034 |
| 13 | 0.0 | 0 | 0.0 |
| 14 | 0.201 | 0.0 | 0.0 |
| 15 | 0.16 | 0.0 | 0.0 |
| 16 | 0.2 | 0 | 0.0 |

**Optimal Price = 0.10000000000000009**

**Final Stopping Rule :**

| Paths | t=1 | t=2 | t=3 |
|---|---|---|---|
| 1 | 1 | 0 | 0 |
| 2 | 0 | 0 | 0 |
| 3 | 0 | 0 | 1 |
| 4 | 1 | 0 | 1 |
| 5 | 0 | 0 | 0 |
| 6 | 1 | 1 | 1 |
| 7 | 1 | 1 | 1 |
| 8 | 1 | 0 | 0 |
| 9 | 1 | 0 | 0 |
| 10 | 0 | 0 | 0 |
| 11 | 0 | 0 | 1 |

| | | | |
|---|---|---|---|
| 12 | 0 | 0 | 1 |
| 13 | 0 | 0 | 0 |
| 14 | 1 | 1 | 1 |
| 15 | 1 | 1 | 1 |
| 16 | 1 | 0 | 0 |

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## Gaussian Paths Prediction : (Noise of sigma = 0.2)

**[[1. 1.23296184 1.09489692 1.69158642]**
**[1. 1.36941351 1.44845597 1.60094919]**
**[1. 1.226444 1.26601982 1.10929528]**
**[1. 1.17058925 1.13467595 1.23638033]**
**[1. 1.12599369 1.61979095 1.92744534]**
**[1. 0.9406163 0.85613236 0.95730729]**
**[1. 1.03080596 0.90414056 1.12997546]**
**[1. 1.09451694 1.23393434 1.36329816]**
**[1. 1.51070774 1.29510313 1.80364087]**
**[1. 1.39559217 1.78477954 1.57052059]**
**[1. 1.47210796 1.31940943 1.20264906]**
**[1. 1.08651136 1.33196722 1.36624595]**
**[1. 1.71249869 2.14646017 2.04578883]**
**[1. 1.13143053 0.866616 1.24396573]**
**[1. 1.05753806 1.11250837 1.09522624]**
**[1. 0.9610716 1.4232594 1.46694658]]**

**Cash Flow Matrix at time t = 3:**

| Path | t=3 |
|---|---|
| 1 | 0.0 |
| 2 | 0.0 |
| 3 | 0.0 |
| 4 | 0.0 |
| 5 | 0.0 |
| 6 | 0.143 |

| | |
|---|---|
| 7 | 0.0 |
| 8 | 0.0 |
| 9 | 0.0 |
| 10 | 0.0 |
| 11 | 0.0 |
| 12 | 0.0 |
| 13 | 0.0 |
| 14 | 0.0 |
| 15 | 0.00449574 |
| 16 | 0.0 |

**For time t=2:**

**Coefficients: [-2239.87598024, 4136.69651566, -2733.09043231, 887.0782914 ]**

| Paths | Exercise | Continuation | Stopping Rule | Value |
|---|---|---|---|---|
| 1 | 0.00510308 | -0.02670134 | 1 | 0.00510308 |
| 2 | 0.0 | 0. | 0 | 0. |
| 3 | 0.0 | 0. | 0 | 0. |
| 4 | 0.0 | 0. | 0 | 0. |
| 5 | 0.0 | 0. | 0 | 0. |
| 6 | 0.24386764 | 0.09275197 | 1 | 0.24386764 |
| 7 | 0.19585944 | 0.10850184 | 1 | 0.19585944 |
| 8 | 0.0 | 0. | 0 | 0. |
| 9 | 0.0 | 0. | 0 | 0. |
| 10 | 0.0 | 0. | 0 | 0. |
| 11 | 0.0 | 0. | 0 | 0. |
| 12 | 0.0 | 0. | 0 | 0. |
| 13 | 0.0 | 0. | 0 | 0. |
| 14 | 0.233384 | 0.09377458 | 1 | 0.233384 |
| 15 | 0.0 | 0. | 0 | 0.00449574 |
| 16 | 0.0 | 0. | 0 | 0. |

**Cash Flow at time t=2:**

| Path | t = 1 | t=2 | t=3 |
|---|---|---|---|
| **1** | - | 0.00510308 | 0.0 |
| **2** | - | 0.0 | 0.0 |
| **3** | - | 0.0 | 0.0 |
| **4** | - | 0.0 | 0.0 |
| **5** | - | 0.0 | 0.0 |
| **6** | - | 0.24386764 | 0.0 |
| **7** | - | 0.19585944 | 0.0 |
| **8** | - | 0.0 | 0.0 |
| 9 | - | 0.0 | 0.0 |
| 10 | - | 0.0 | 0.0 |
| 11 | - | 0.0 | 0.0 |
| 12 | - | 0.0 | 0.0 |
| 13 | - | 0.0 | 0.0 |
| 14 | - | 0.233384 | 0.0 |
| 15 | - | 0.0 | 0.00449574 |
| 16 | - | 0.0 | 0.0 |

**At time t = 1**

**Coefficients:[-1723.47923186, 3185.06486082, -2106.88623773, 686.66193793]**

| Paths | Exercise | Continuation | Stopping Rule | Value |
|---|---|---|---|---|
| 1 | 0.0 | 0.0 | 0 | 0.00480588 |
| 2 | 0.0 | 0.0 | 0 | 0.0 |
| 3 | 0.0 | 0.0 | 0 | 0.0 |
| 4 | 0.0 | 0.0 | 0 | 0.0 |
| 5 | 0.0 | 0.0 | 0 | 0.0 |
| 6 | 0.1593837 | 0.12484539 | 1 | 0.1593837 |
| 7 | 0.07 | 0.09990151 | 0 | 0.18445258 |
| 8 | 0.00548306 | -0.00633468 | 1 | 0.00548306 |
| 9 | 0.0 | 0.0 | 0 | 0.0 |
| 10 | 0.0 | 0.0 | 0 | 0.0 |

| | | | | |
|---|---|---|---|---|
| 11 | 0.0 | 0.0 | 0 | 0.0 |
| 12 | 0.01348864 | 0.0130286 | 1 | 0.01348864 |
| 13 | 0.0 | 0.0 | 0 | 0.0 |
| 14 | 0.0 | 0.0 | 0 | 0.21979172 |
| 15 | 0.05 | 0.06782495 | 0 | 0.00423391 |
| 16 | 0.1389284 | 0.12598221 | 1 | 0.1389284 |

**Cash Flow at time t=1:**

| Path | t = 1 | t=2 | t=3 |
|---|---|---|---|
| **1** | 0.0 | 0.00510308 | 0.0 |
| **2** | 0.0 | 0.0 | 0.0 |
| **3** | 0.0 | 0.0 | 0.0 |
| **4** | 0.0 | 0.0 | 0.0 |
| **5** | 0.0 | 0.0 | 0.0 |
| **6** | 0.1593837 | 0.0 | 0.0 |
| **7** | 0.0 | 0.19585944 | 0.0 |
| **8** | 0.00548306 | 0.0 | 0.0 |
| 9 | 0.0 | 0.0 | 0.0 |
| 10 | 0.0 | 0.0 | 0.0 |
| 11 | 0.0 | 0.0 | 0.0 |
| 12 | 0.01348864 | 0.0 | 0.0 |
| 13 | 0.0 | 0.0 | 0.0 |
| 14 | 0.0 | 0.233384 | 0.0 |
| 15 | 0.0 | 0.0 | 0.00449574 |
| 16 | 0.1389284 | 0.0 | 0.0 |

**Optimal Price Gaussian Noise : 0.10000000000000009**

**Final Stopping Rule :**

| Path | t = 1 | t=2 | t=3 |
|---|---|---|---|
| **1** | 0 | 1 | 0 |
| **2** | 0 | 0 | 0 |
| **3** | 0 | 0 | 0 |
| **4** | 0 | 0 | 0 |
| **5** | 0 | 0 | 0 |

| | | | |
|---|---|---|---|
| **6** | 1 | 0 | 0 |
| **7** | 0 | 1 | 0 |
| **8** | 1 | 0 | 0 |
| 9 | 0 | 0 | 0 |
| 10 | 0 | 0 | 0 |
| 11 | 0 | 0 | 0 |
| 12 | 1 | 0 | 0 |
| 13 | 0 | 0 | 0 |
| 14 | 0 | 1 | 0 |
| 15 | 0 | 0 | 1 |
| 16 | 1 | 0 | 0 |

**Manual Checking : (Gamma = 0.94176)**

| Paths | diff1 | Gamma * diff2 | Gamma^2 * diff3 | Stopping Rule |
|---|---|---|---|---|
| **1** | 0.0 | 0.004805877 | 0.0 | 2 |
| **2** | 0.0 | 0.0 | 0.0 | - |
| **3** | 0.0 | 0.0 | 0.0 | - |
| **4** | 0.0 | 0.0 | 0.0 | - |
| **5** | 0.0 | 0.0 | 0.0 | - |
| **6** | 0.1593837 | 0.229664789 | 0.126828401 | 2 |
| **7** | 0.07 | 0.184452586 | 0.0 | 2 |
| **8** | 0.00548306 | 0.0 | 0.0 | 1 |
| 9 | 0.0 | 0.0 | 0.0 | - |
| 10 | 0.0 | 0.0 | 0.0 | - |
| 11 | 0.0 | 0.0 | 0.0 | - |
| 12 | 0.01348864 | 0.0 | 0.0 | 1 |
| 13 | 0.0 | 0.0 | 0.0 | - |
| 14 | 0.0 | 0.219791716 | 0.0 | 2 |
| 15 | 0.05 | 0.0 | 0.0039 | 3 |
| 16 | 0.1389284 | 0.0 | 0.0 | 1 |

**Accuracy : 1 wrong prediction => 93.75 % accuracy**

**Conclusion : Languerre polynomial giving better accuracy than Simple basis function**