Experiments on introducing Gaussian noises with zero mean in the Longstaff papers's Array:

Experiment 1: Sigma = 0.2

```
[[1.         1.18965555   1.285144     1.51017878]
 [1.         1.36905235   1.66115282   1.82867055]
 [1.         1.4257547    1.07111136   1.0869511 ]
 [1.         1.07246552   1.21498107   1.00462678]
 [1.         1.2636681    1.91049475   1.75961231]
 [1.         0.80814366   0.79208179   0.97497149]
 [1.         1.18497278   0.87552334   1.03201944]
 [1.         1.08587804   1.22304272   1.40172305]]
```

A. Using Longstaff Coefficients:

Cash Flow at time t = 3:

Path	t=1	t=2	t=3
1	--	--	0.0
2	--	--	0.0
3	--	--	0.0131
4	--	--	0.09538
5	--	--	0.0
6	--	--	0.1251
7	--	--	0.08
8	--	--	0.0

For time t=2:

Coefficients : [-1.0699825 , 2.98339626, -1.81356745]

Paths	Exercise	Continuation	Stopping Rule	Value
1	0.0	0.	0	0.
2	0.0	0.0	0	0.
3	0.02	0.04489829	0	0.01228893
4	0.096	0.	0	0.08981868
5	0.0	0.	0	0.
6	0.30791821	0.1552908	1	0.30791821

7	0.22447666	0.15187653	1	0.22447666
8	0.0	0.	0	0.

Cash Flow Matrix at time t = 2:

Path	t=1	t=2	t=3
1	--	0.0	0.0
2	--	0.0	0.0
3	--	0.0	0.0131
4	--	0.0	0.09538
5	--	0.0	0.0
6	--	0.30791821	0.0
7	--	0.22447666	0.0
8	--	0.0	0.0

For time t=1:

Coefficients : [2.03750253, -3.33542735, 1.35645006]

Paths	Exercise	Continuation	Stopping Rule	Value
1	0.0	0.0	0	0.
2	0.0	0.0	0	0.
3	0.0	0.0	0	0.01157322
4	0.02753448	0.02053655	1	0.02753448
5	0.0	0.0	0	0.0
6	0.29185634	0.22789041	1	0.29185634
7	0	0.0	0	0.21140314
8	0.0141	0.0150677	0	0.0

Cash Flow Matrix at time t=1:

Path	t=1	t=2	t=3
1	0.0	0.0	0.0
2	0.0	0.0	0.0
3	0.0	0.0	0.0131
4	0.02753448	0.0	0.09538
5	0.0	0.0	0.0

6	0.29185634	0.0	0.0
7	0.0	0.22447666	0.0
8	0.0	0.0	0.0

Optimal Price Gaussian Noise : 0.10000000000

Paths	Time = 1	Time = 2	Time = 3
1	0	0	0
2	0	0	0
3	0	0	1
4	1	0	1
5	0	0	0
6	1	1	1
7	0	1	1
8	0	0	0

Finally the above rule becomes as below

Paths	Time = 1	Time = 2	Time = 3
1	0	0	0
2	0	0	0
3	0	0	1
4	1	0	0
5	0	0	0
6	1	0	0
7	0	1	0
8	0	0	0

Testing the results :

Paths	diff1	Gamma * diff2	Gamma^2 * diff3	Stoping Rule
1	0.0	0.0	0.0	-
2	0.0	0.0	0.0	-
3	0.0	0.0188349	0.0118224388	2
4	0.03	0.09040838	0.0845936569	2
5	0.0	0.0	0.0	-

6	0.3	0.12242875	0.110889273	1
7	0	0.0659231651	0.0602927675	2
8	0.0141	0.0	0.0	1

Accuracy : 3 wrong prediction => 62.5 % accuracy

Experiment 2: Sigma = 0.6

```
[[1.         1.10158462   1.19947471   2.12124473]
 [1.         1.17047468   1.86222175   1.75820157]
 [1.         1.30688774   1.30316867   1.12466611]
 [1.         1.89470367   0.991329     1.91725458]
 [1.         1.45091855   1.72820445   2.01861799]
 [1.         1.250222     0.82885862   1.09986139]
 [1.         1.31574127   0.8833223    2.21511899]
 [1.         1.08345237   2.30763487   1.47271381]]
```

A. Using Longstaff Coefficients:

Cash Flow at time t = 3:

Path	t=1	t=2	t=3
1	--	--	0.0
2	--	--	0.0
3	--	--	0.0
4	--	--	0.0
5	--	--	0.0
6	--	--	0.00014
7	--	--	0.0
8	--	--	0.0

For time t=2:

Coefficients : [-1.0699825 , 2.98339626, -1.81356745]

Paths	Exercise	Continuation	Stopping Rule	Value
1	0.0	0.	0.	0.
2	0.0	0.	0.	0.
3	0.0	0.	0.	0.
4	0.108671	0.10529181	1.	0.108671
5	0.0	0.	0.	0.
6	0.27114138	0.15689837	1.	0.27114138
7	0.2166777	0.15026691	1.	0.2166777
8	0.0	0.	0.	0.

Cash Flow Matrix at time t = 2:

Path	t=1	t=2	t=3
1	--	0.0	0.0
2	--	0.0	0.0
3	--	0.0	0.0
4	--	0.108671	0.0
5	--	0.0	0.0
6	--	0.27114138	0.0
7	--	0.2166777	0.0
8	--	0.0	0.0

For time t=1:

Coefficients : [2.03750253, -3.33542735, 1.35645006]

Paths	Exercise	Continuation	Stopping Rule	Value
1	0.0	0.0	0	0.
2	0.0	0.0	0	0.
3	0.0	0.0	0	0.
4	0.0	0.0	0	0.102342
5	0.0	0.0	0	0.
6	0.0	0.0	0	0.25535011
7	0.0	0.0	0	0.20405839
8	0.01654763	0.0160206	1	0.01654763

Cash Flow Matrix at time t = 2:

Path	t=1	t=2	t=3
1	0.0	0.0	0.0
2	0.0	0.0	0.0
3	0.0	0.0	0.0
4	0.0	0.108671	0.0
5	0.0	0.0	0.0
6	0.0	0.27114138	0.0
7	0.0	0.2166777	0.0
8	0.01654763	0.0	0.0

Optimal Price : 0.10000000009

Paths	Time = 1	Time = 2	Time = 3
1	0	0.	0
2	0	0.	0
3	0	0.	0
4	0	1.	0
5	0	0.	0
6	0	1.	0
7	0	1.	0
8	1	0.	0

Testing the results : (Gamma = 0.94176)

Paths	diff1	Gamma * diff2	Gamma^2 * diff3	Stopping Rule
1	0.0	0.0	0.0	-
2	0.0	0.0	0.0	-
3	0.0	0.0	0.0	-
4	0.0	0.102342001	0.0	2
5	0.0	0.0	0.0	-
6	0.0	0.255350106	0.00012293	2
7	0.0	0.204058391	0.0	2
8	0.01654763	0.0	0.0	1

Accuracy = 0 wrong prediction => 100 % accuracy

Experiment 3: Sigma = 1.0

```
[[1.         2.095729     1.89726891   1.72558459]
 [1.         1.94745364   2.09365109   1.59982323]
 [1.         1.73874069   2.08775799   1.47553463]
 [1.         1.47751593   1.9220144    2.02306972]
 [1.         3.28720165   2.73217192   2.30113517]
 [1.         1.39089225   2.0327277    1.05647165]
 [1.         0.92000938   1.82602416   1.26053131]
 [1.         1.45919652   1.73545623   1.43601632]]
```

A. Using Longstaff Coefficients:

Cash Flow at time t = 3:

Path	t=1	t=2	t=3
1	--	--	0.0
2	--	--	0.0
3	--	--	0.0
4	--	--	0.0
5	--	--	0.0
6	--	--	0.04099326
7	--	--	0.0
8	--	--	0.0

For time t=2:

Coefficients : [-1.0699825 , 2.98339626, -1.81356745]

Paths	Exercise	Continuation	Stopping Rule	Value
1	0.0	0.0	0	0.0
2	0.0	0.0	0	0.0
3	0.0	0.0	0	0.0
4	0.0	0.0	0	0.0
5	0.0	0.0	0	0.0
6	0.0	0.0	0	0.04099326
7	0.0	0.0	0	0.0
8	0.0	0.0	0	0.0

Cash Flow Matrix at time t = 2:

Path	t=1	t=2	t=3
1	--	0.0	0.0
2	--	0.0	0.0
3	--	0.0	0.0
4	--	0.0	0.0
5	--	0.0	0.0
6	--	0.0	0.04099326
7	--	0.0	0.0
8	--	0.0	0.0

For time t=1:

Coefficients : [2.03750253, -3.33542735, 1.35645006]

Paths	Exercise	Continuation	Stopping Rule	Value
1	0.0	0.0	0	0.0
2	0.0	0.0	0	0.0
3	0.0	0.0	0	0.0
4	0.0	0.0	0	0.0
5	0.0	0.0	0	0.0
6	0.0	0.0	0	0.03860582
7	0.17999062	0.11700083	1	0.17999062
8	0.0	0.0	0	0.0

Cash Flow Matrix at time t = 1:

Path	t=1	t=2	t=3
1	0.0	0.0	0.0
2	0.0	0.0	0.0
3	0.0	0.0	0.0
4	0.0	0.0	0.0
5	0.0	0.0	0.0
6	0.0	0.0	0.04099326
7	0.17999062	0.0	0.0
8	0.0	0.0	0.0

Optimal Price Gaussian Noise : 0.10000000009

Paths	Time = 1	Time = 2	Time = 3
1	0	0	0
2	0	0	0
3	0	0	0
4	0	0	0
5	0	0	0
6	0	0	1
7	1	0	0
8	0	0	0

Testing the results : (Gamma = 0.94176)

Paths	**diff1**	**Gamma * diff2**	**Gamma^2 * diff3**	**Stopping Rule**
1	0.0	0.0	0.0	-
2	0.0	0.0	0.0	-
3	0.0	0.0	0.0	-
4	0.0	0.0	0.0	-
5	0.0	0.0	0.0	-
6	0.0	0.0	0.0386058115	3
7	0.17999062	0.0	0.0	1
8	0.0	0.0	0.0	-

Accuracy = 0 wrong prediction => 100 % accuracy

Increasing the number of Paths to 16 : (8 extra paths added by an randomly generated path with variance of 0.1)

Longstaff Paper :

[[1. 1.09 1.08 1.34]
[1. 1.16 1.26 1.54]
[1. 1.22 1.07 1.03]
[1. 0.93 0.97 0.92]
[1. 1.11 1.56 1.52]
[1. 0.76 0.77 0.9]
[1. 0.92 0.84 1.01]
[1. 0.88 1.22 1.34]
[1. 1.09972955 1.20984266 1.42786124]
[1. 1.30282465 1.33379354 1.54642148]
[1. 1.30811775 1.18827482 1.05821046]
[1. 1.00225371 1.22542088 1.06681238]
[1. 1.17592812 1.65280718 1.72744582]
[1. 0.89931269 0.85576233 0.95650906]
[1. 0.9397596 0.92159093 1.03515879]
[1. 0.90082674 1.26323588 1.41475168]]

A. Using Longstaff Coefficients:

Cash Flow Matrix at time t = 3:

Path	t=3
1	0.0
2	0.0
3	0.7
4	0.18
5	0.0
6	0.2
7	0.09
8	0.0
9	0.0
10	0.0
11	0.042
12	0.034

13	0.0
14	0.144
15	0.065
16	0.0

For time t=2:

Coefficients : [0.25244901, 0.06261813, -0.23969113]

Paths	Exercise	Continuation	Stopping Rule	Value
1	0.02	0.04050085	0	0.
2	0.0	0.	0	0.
3	0.03	0.04502803	0	0.0659232
4	0.13	0.08766321	1	0.13
5	0.0	0.	0	0.
6	0.33	0.1585521	1	0.33
7	0.26	0.13592218	1	0.26
8	0.0	0.	0	0.
9	0.0	0.	0	0.
10	0.0	0.	0	0.
11	0.0	0.	0	0.03935572
12	0.0	0.	0	0.03125477
13	0.0	0.	0	0.
14	0.25	0.13050244	1	0.24423767
15	0.18	0.10658048	1	0.17840907
16	0.0	0.	0	0.

Cash Flow Matrix at time t = 2:

Paths	t=1	t=2	t=3
1	--	0	0.0
2	--	0	0.0
3	--	0	0.7
4	--	0.13	0.0
5	--	0	0.0
6	--	0.33	0.0
7	--	0.26	0.0
8	--	0	0.0

9	--	0	0.0
10	--	0	0.0
11	--	0	0.042
12	--	0	0.034
13	--	0	0.0
14	--	0.24423767	0.0
15	--	0.17840907	0.0
16	--	0	0.0

For time t=1:

Coefficients : [1.8656825 , -2.94938448, 1.14146326]

Paths	Exercise	Continuation	Stopping Rule	Value
1	0.01	0.00702592	1.	0.01
2	--	0.	0.	0.
3	--	0.	0.	0.06208383
4	0.17	0.11000651	1.	0.17
5	--	0.	0.	0.
6	0.34	0.28345948	1.	0.34
7	0.18	0.11838328	1.	0.18
8	0.22	0.15417331	1.	0.22
9	0.001	0.0026487	0.	0.
10	0.0	0.	0.	0.
11	0.0	0.	0.	0.03706364
12	0.1	0.05626508	1.	0.09774629
13	0.0	0.	0.	0.
14	0.201	0.13643722	1.	0.20068731
15	0.16	0.10205124	1.	0.1602404
16	0.2	0.13508276	1.	0.19917326

Cash Flow Matrix at time t = 2:

Paths	t=1	t=2	t=3
1	0.01	0	0.0
2	0.	0	0.0
3	0.0	0	0.7

4	0.17	0.13	0.0
5	0.	0	0.0
6	0.34	0.33	0.0
7	0.18	0.26	0.0
8	0.22	0	0.0
9	0.	0	0.0
10	0.	0	0.0
11	0.0	0	0.042
12	0.09774629	0	0.034
13	0.	0	0.0
14	0.20068731	0.24423767	0.0
15	0.1602404	0.17840907	0.0
16	0.19917326	0	0.0

Optimal Price : 0.10000000000000009

Paths	Time = 1	Time = 2	Time = 3
1	1.	0	0
2	0.	0	0
3	0.	0	1
4	1.	0	0
5	0.	0	0
6	1.	0	0
7	1.	0	0
8	1.	0	0
9	0.	0	0
10	0.	0	0
11	0.	0	1
12	1.	0	0
13	0.	0	0
14	1.	1	0
15	1.	1	0
16	1.	0	0

Gaussian Paths Prediction : (Noise of sigma = 0.2)

[[1. 1.18118243 1.26398486 1.38828462]
[1. 1.2036367 1.33411664 1.93452951]
[1. 1.26285978 1.1504606 1.1284724]
[1. 1.20997748 1.37018284 1.06008263]
[1. 1.23726994 1.7293559 1.77985815]
[1. 0.96790023 0.81203459 0.95757214]
[1. 1.18962372 1.03696775 1.1394861]
[1. 0.96744056 1.31130084 1.57966712]
[1. 1.15186597 1.41572591 1.96591756]
[1. 1.68872173 1.7660047 1.72725647]
[1. 1.41209986 1.40199992 1.12535379]
[1. 1.04094303 1.23585136 1.09625934]
[1. 1.18662226 1.89871525 1.95740187]
[1. 1.00879505 1.12246048 1.29914887]
[1. 1.00016948 1.0629361 1.45185396]
[1. 0.96770218 1.27957512 1.69772665]]

Cash Flow Matrix at time t = 3:

Path	t=3
1	0.0
2	0.0
3	0.0
4	0.03991
5	0.0
6	0.142428
7	0.0
8	0.0
9	0.0
10	0.0
11	0.0
12	0.013741
13	0.0
14	0.0
15	0.0
16	0.0

For time t=2:

Coefficients : [0.25244901, 0.06261813, -0.23969113]

Paths	Exercise	Continuation	Stopping Rule	Value
1	0.0	0.	0	0.
2	0.0	0.	0	0.
3	0.0	0.	0	0.
4	0.0	0.	0	0.03759258
5	0.0	0.	0	0.
6	0.28796541	0.14524472	1	0.28796541
7	0.06303225	0.05964161	1	0.06303225
8	0.0	0.	0	0.
9	0.0	0.0	0	0.
10	0.0	0.0	0	0.
11	0.0	0.0	0	0.
12	0.0	0.0	0	0.0035228
13	0.0	0.0	0	0.
14	0.0	0.0	0	0.
15	0.04	0.04819709	0	0.
16	0.0	0.0	0	0.

Cash Flow at time t=2:

Path	**t = 1**	**t=2**	**t=3**
1	-	0.0	0.0
2	-	0.0	0.0
3	-	0.0	0.0
4	-	0.0	0.03991
5	-	0.0	0.0
6	-	0.28796541	0.142428
7	-	0.06303225	0.0
8	-	0.0	0.0
9	-	0.0	0.0
10	-	0.0	0.0

11	-	0.0	0.0
12	-	0.0	0.013741
13	-	0.0	0.0
14	-	0.0	0.0
15	-	0.0	0.0
16	-	0.0	0.0

For time t=1:

Coefficients : [1.8656825 , -2.94938448, 1.14146326]

Paths	Exercise	Continuation	Stopping Rule	Value
1	0.0	0.	0	0.
2	0.0	0.	0	0.
3	0.0	0.	0	0.
4	0.0	0.	0	0.03540319
5	0.14	0.	0	0.
6	0.13209977	0.08033058	1	0.13209977
7	0.14	0.	0	0.05936125
8	0.13255944	0.08067087	1	0.13255944
9	0.0	0.	0	0.
10	0.0	0.	0	0.
11	0.0	0.	0	0.
12	0.05905697	0.03238794	1	0.05905697
13	0.0	0.	0	0.
14	0.09120495	0.05198805	1	0.09120495
15	0.09983052	0.05764836	1	0.09983052
16	0.13229782	0.08047714	1	0.13229782

Cash Flow at time t=2:

Path	t = 1	t=2	t=3
1	0.0	0.0	0.0
2	0.0	0.0	0.0
3	0.0	0.0	0.0

4	0.0	0.0	0.03991
5	0.0	0.0	0.0
6	0.13209977	0.0	0.0
7	0.0	0.0	0.0
8	0.13255944	0.0	0.0
9	0.0	0.0	0.0
10	0.0	0.0	0.0
11	0.0	0.0	0.0
12	0.05905697	0.0	0.0
13	0.0	0.0	0.0
14	0.09120495	0.0	0.0
15	0.09983052	0.0	0.0
16	0.13229782	0.0	0.0

Optimal Price Gaussian Noise : 0.10000000000000009

Stopping Rule :

Paths	Time = 1	Time = 2	Time = 3
1	0	0	0
2	0	0	0
3	0	0	0
4	0	0	1
5	0	0	0
6	1	0	0
7	0	1	0
8	1	0	0
9	0	0	0
10	0	0	0
11	0	0	0
12	1	0	0
13	0	0	0
14	1	0	0
15	1	0	0
16	1	0	0

Testing the results : (Gamma = 0.94176)

Paths	diff1	Gamma * diff2	Gamma^2 * diff3	Stopping Rule
1	0.0	0.0	0.0	-
2	0.0	0.0	0.0	-
3	0.0	0.0	0.0	-
4	0.0	0.0	0.0355	3
5	0.0	0.0	0.0	-
6	0.14	0.282528	0.044346	2
7	0.0	0.094176	0.0	1
8	0.14	0.0	0.0	1
9	0.0	0.0	0.0	-
10	0.0	0.0	0.0	-
11	0.0	0.0	0.0	-
12	0.06	0.0	0.00887	1
13	0.0	0.0	0.0	-
14	0.1	0.0	0.0	1
15	0.1	0.0376704	0.0	1
16	0.14	0.0	0.0	1

Accuracy = 4 wrong prediction => 87.5 % accuracy
with 8 paths it was 62.5 %. Therefore, accuracy did improved.