Indian Academy of Sciences, Bengaluru
Indian National Science Academy, New Delhi
The National Academy of Sciences India, Prayagraj

SUMMER RESEARCH FELLOWSHIPS — 2025

Format for the four-week Report*,^,@

Name of the candidate : AYUSH DUTTA

Application Registration no. : ENG S 307

Date of joining : 13/05/2025

Name of the guide : Dr. NAVANEETHA KRISHNAN RAVICHANDRAN

Guide's institution : IISc, BENGALURU

Place of stay during the tenure of the fellowship : Hostel provided by Guide Own arrangement Other (Specify)

Indian Academy of Science Fellows Residency

NA

Ayush Dutta
Signature of the candidate
Date: 09/06/2025

R. Navaneeth
Signature of the guide
Date: 09/06/2025

INSPIRE/KVPY FELLOWSHIP (please fill this box)#			
1.	I am currently a recipient of	INSPIRE FELLOWSHIP	☐ Yes / ☑ No
		KVPY FELLOWSHIP	☐ Yes / ☑ No
		If, YES, fill cols. 2, 3 & 4	
2.	INSPIRE/KVPY Fellowship is from ______[month]/______[yr] to ______[month]/______[yr]		
3.	I receive a monthly fellowship of Rs. __________ from INSPIRE/KVPY towards my living expenses		
4.	I also receive towards contingencies a sum of Rs. __________ per year		
	I affirm that the information given above is correct.		Ayush Dutta Signature of the candidate

IMPORTANT NOTES:

* The four-week report could be between 300 and 350 words.
^ This format should be the first page of the report and should be stapled with the main report.
# Mandatory to fill this section, this should be filled and signed by you even if you are not an INSPIRE/KVPY Fellow. Otherwise release of fellowship amount will be withheld.
@ The hard copy of the duly signed report should reach the Academy office within 10 days of completing the first month fellowship. If delayed the fellowship amount will not be paid.

(For office use only; do not fill/tear)

Candidate's name:	Fellowship amount:
Student: Teacher:	Deduction:
Guide's name:	Amount to be paid:
KVPY Fellow: INSPIRE Fellow:	A/c holder's name:
PFMS Unique Code:	
Others	

Indian Academy of Sciences, Bengaluru
Indian National Science Academy, New Delhi
The National Academy of Sciences, India, Allahabad

SUMMER RESEARCH FELLOWSHIP PROGRAMME 2025

FOUR WEEK REPORT

Adaptive Neural Galerkin Technique for High-Dimensional Evolutionary Systems

Under the supervision of -

Dr. Navaneetha Krishnan Ravichandran

Assistant Professor

Department of Mechanical Engineering

IISc Bangalore

Submitted By -

Ayush Dutta

Summer Research Fellow

Application No - ENGS307

IIEST Shibpur

Solving partial differential equations (PDEs) is a major task in science and engineering. These equations basically help in describing the dynamics of many physical or mechanical phenomenon which change over time and space like heat, waves, interaction between particles, etc.

Tradition methods like the finite difference method and several other classical tools such as the classical Galerkin method are used generally used for solving PDEs and have worked well with many problems specially in low dimensions but once the number of variables increase say to 10 or 15 dimensions – they begin to fail.

This is something known as the curse of dimensionality, where the computational requirements as well as the memory requirements grow exponentially.

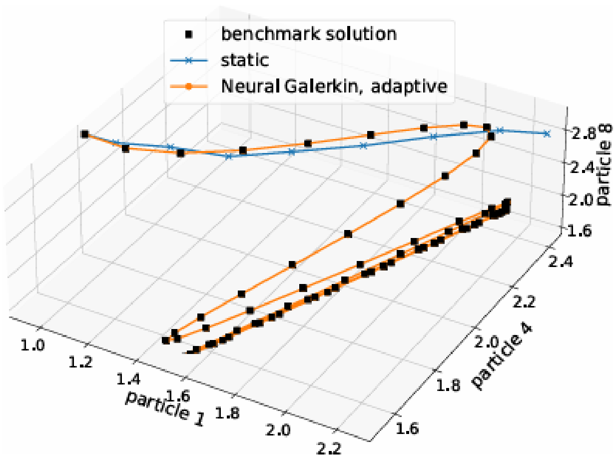

Fig. Particle in a harmonic Trap (An example of High Dimensional PDE)

That's where the Neural Galerkin approach comes into picture

The Neural Galerkin method is a contemporary technique that integrates deep learning—specifically, neural networks—with the Galerkin principle. Simply put, it substitutes a trainable neural network that can recognize the shape of the solution as it changes over time for the fixed basis functions (such as sin, cos, or polynomials) used in traditional Galerkin methods.

The Neural Galerkin method uses a neural network with weights and biases (parameters) that vary over time, rather than fixed basis functions. Under the guidance of a physical principle known as the Dirac-Frenkel variational principle, these parameters are not optimized once but rather evolve gradually over time.

The other thing which was found to be beneficial and can be integrated is time steeping using neural network rather than trying the solve everything at once and also the use of adaptive sampling which makes the method faster and smarter.

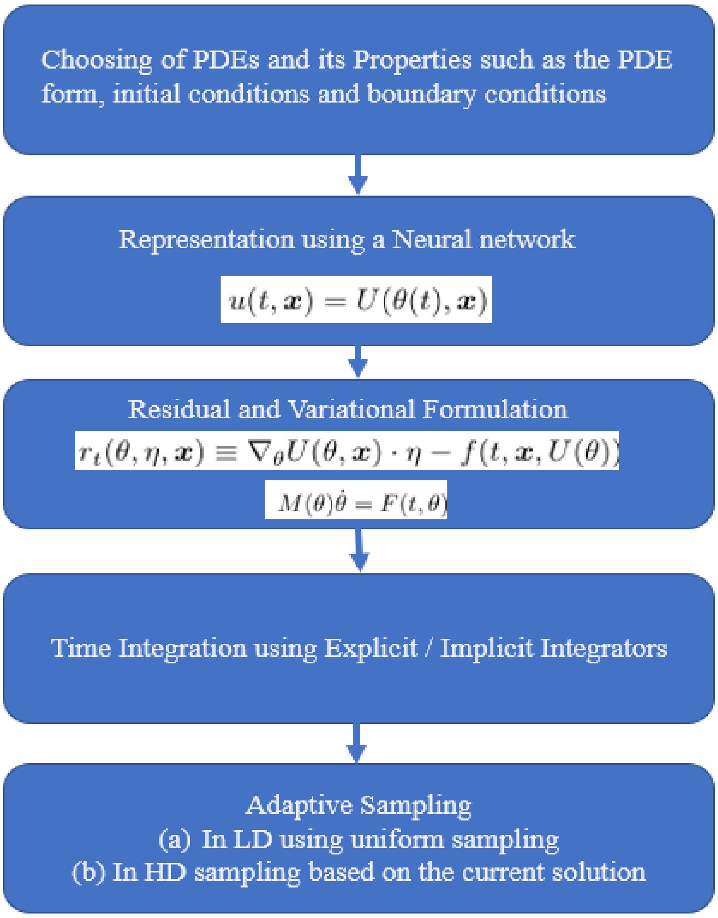

Fig. Steps to Implement the Neural Galerkin Approach using adaptive sampling and Time Integration to solve PDEs

I learned two network architectures: a shallow network using Gaussian or periodic activations, and a deep network with sine-based tanh layers.

Shallow network using Gaussian Activation-

$$\varphi_G(x, w, b) = \exp\left(-w^2 \|x - b\|^2\right)$$

$$\varphi_G^L(x, w, b) = \exp\left(-w^2 \left|\sin\left(\frac{\pi(x-b)}{L}\right)\right|^2\right)$$

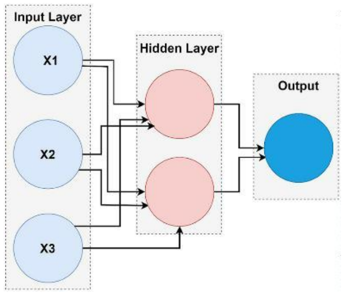

Fig. Shallow Network Architecture

Deep network with sine-based tanh layers -

$$U(\theta, x) = c(t)^T \tanh\left(\mathbf{W}_\ell \tanh\left(\mathbf{W}_{\ell-1} \cdots \varphi^L_{\tanh}(x, \mathbf{W}_1, \mathbf{b}_1) \cdots + \mathbf{b}_{\ell-1}\right) + \mathbf{b}_\ell\right)$$

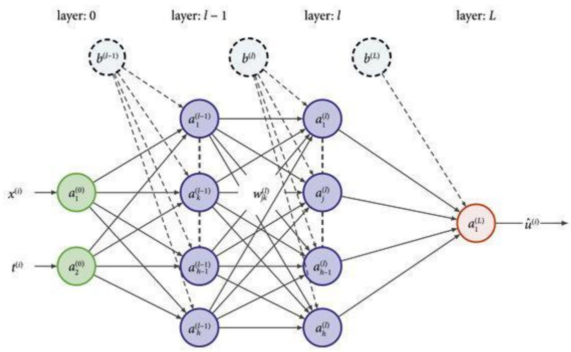

Fig. Deep network with sine-based tanh layers Architecture

I have so far worked on the advection equation, which depicts the movement of quantities through a medium; the Allen–Cahn equation, which describes the separation of two distinct phases; and the KdV (Korteweg–de Vries) equation, which models soliton waves that maintain their shape over time. I represented the solutions using the Neural Galerkin method in JAX by using neural networks with time-varying parameters. I calculated the residuals at each stage to update the network and allow it to precisely follow the PDE dynamics. I managed to capture complex behaviors such as interface evolution in Allen–Cahn, sharp fronts in advection, and wave interactions in KdV. To more accurately estimate the dynamics of energy transfer, I will apply what I've learned to solve larger classes of dynamical PDEs.