

# Databases and Information Systems

## CS303

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Normalization  
07-09-2023

# Recap

- First Normal Form
- Functional Dependency
- Boyce Codd Normal Form
- Third Normal Form

# Functional Dependency

- Let  $R$  be the set of all attributes of the relational schema  $r(R)$  and let  $\alpha \subseteq R$  and  $\beta \subseteq R$ . Given an instance of  $r$ , we say  $\alpha \rightarrow \beta$  if for any two tuples  $t_1$  and  $t_2$  if  $t_1[\alpha] = t_2[\alpha]$  then  $t_1[\beta] = t_2[\beta]$
- Example: `inst_dept` (ID, name, salary, dept name, building, budget)
  - `dept_name`  $\rightarrow$  building, budget
  - ID, dept\_name  $\rightarrow$  name, salary, building, budget

# Boyce Codd Normal Form

- A relation schema  $R$  is in BCNF with respect to a set  $F$  of functional dependencies if, for all functional dependencies in  $F^+$  of the form  $\alpha \rightarrow \beta$ , where  $\alpha \subseteq R$  and  $\beta \subseteq R$  : at least one of the following holds:
  - $\alpha \rightarrow \beta$  is a trivial functional dependency (that is,  $\beta \subseteq \alpha$  )
  - $\alpha$  is a superkey for schema  $R$
- A database design is in BCNF if every relational schema of the design is in BCNF

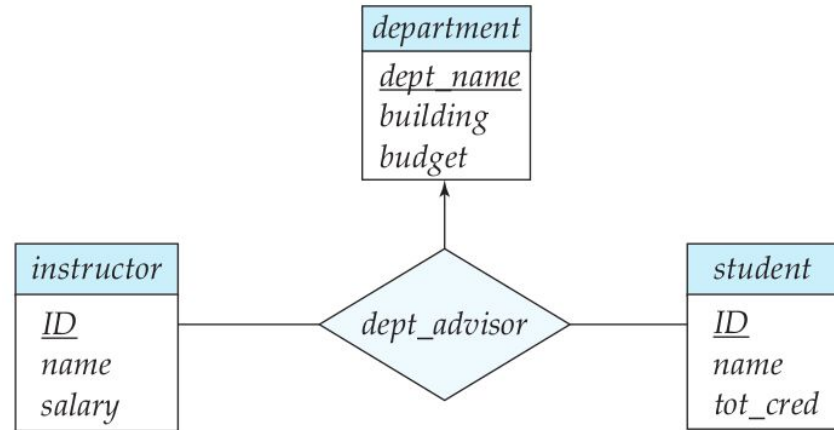
# Boyce Codd Normal Form : General decomposition Rule

- Let  $r(R)$  be a schema that is not in BCNF  
(There is a non-trivial functional dependency  $\alpha \rightarrow \beta$  where  $\alpha$  is not a superkey)
- Replace  $r(R)$  with two new schemas
  - $\alpha \cup \beta$
  - $(R - (\beta - \alpha))$
- Example:  $inst\_dept (ID, name, salary, dept\_name, building, budget)$ 
  - $dept\_name \rightarrow building, budget$
- Application of the rule might result in smaller relations that are not in BCNF
  - Repeat for procedure for smaller relations till the resulting schema is in BCNF

# Boyce Codd Normal Form : Dependency Preservation

- dept\_advisor (s\_ID, i\_ID, dept\_name)

- s\_ID, dept\_name → i\_ID
- i\_ID → dept\_name  
( every instructor can be advisor for a single department )
- Not in BCNF because i\_ID is not superkey
- (s\_ID, i\_ID)
- (i\_ID, dept\_name)
- s\_ID, dept\_name → i\_ID  
decomposition makes it computationally hard to enforce this functional dependency



- BCNF is not dependency preserving

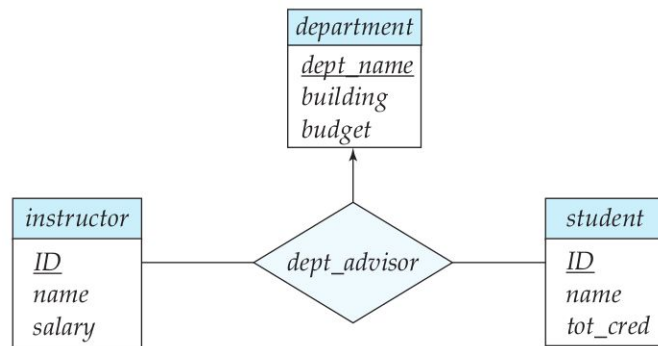
- 3NF (weaker than BCNF) is always dependency preserving

# Third Normal Form

- A relation schema  $r(R)$  is in Third Normal Form (3NF) with respect to a set  $F$  of functional dependencies if, for all functional dependencies in  $F^+$  of the form  $\alpha \rightarrow \beta$ , where  $\alpha \subseteq R$  and  $\beta \subseteq R$  : at least one of the following holds:
  - $\alpha \rightarrow \beta$  is a trivial functional dependency (that is,  $\beta \subseteq \alpha$  ).
  - $\alpha$  is a superkey for schema  $R$ .
  - Each attribute  $A$  in  $\beta - \alpha$  is contained in some candidate key of  $R$
- Each such  $A$  can be part of different candidate keys
- Any schema that is in BCNF is also in 3NF

# Third Normal Form

- A relation schema  $r(R)$  is in Third Normal Form (3NF) with respect to a set  $F$  of functional dependencies if, for all functional dependencies in  $F^+$  of the form  $\alpha \rightarrow \beta$ , where  $\alpha \subseteq R$  and  $\beta \subseteq R$  : at least one of the following holds:
  - $\alpha \rightarrow \beta$  is a trivial functional dependency (that is,  $\beta \subseteq \alpha$  ).
  - $\alpha$  is a superkey for schema  $R$ .
  - Each attribute  $A$  in  $\beta - \alpha$  is contained in some candidate key of  $R$
- dept\_advisor (s\_ID, i\_ID, dept\_name) is in 3NF
  - s\_ID, dept\_name  $\rightarrow$  i\_ID
  - i\_ID  $\rightarrow$  dept\_name
  - ( every instructor can be advisor for a single department )





# Decomposition Algorithms

- Real life schema diagrams are typically Huge
- Cannot be done manually
- We need Algorithms that take a schema as an input and decompose them into BCNF or 3NF
- For that we need subroutines to compute  $F^+$ , Check if a set of attributes is a superkey

# Computing $F^+$

- Testing BCNF and 3NF needs  $F^+$  obtained from the initial set of functional dependencies  $F$
- Suppose  $r(A, B, C, G, H, I)$  is a relational schema

$$A \rightarrow B$$

$$A \rightarrow C$$

$$CG \rightarrow H$$

$$CG \rightarrow I$$

$$B \rightarrow H$$

$$A \rightarrow H$$

# Computing $F^+$

- Use the following **Axioms to compute  $F^+$**  (Armstrong's axioms)
  - Reflexivity Rule : If  $\beta \subseteq \alpha$  then  $\alpha \rightarrow \beta$
  - Augmentation Rule : If  $\alpha \rightarrow \beta$  and  $\gamma$  is a set of attributes then  $\gamma\alpha \rightarrow \gamma\beta$
  - Transitive Rule : If  $\alpha \rightarrow \beta$  and  $\beta \rightarrow \gamma$  then  $\alpha \rightarrow \gamma$
- Armstrong's axioms are **sound**  
(They do not generate any incorrect functional dependencies)
- Armstrong's axioms are **complete**  
(All functional dependencies can be deduced using these rules)

# Computing $F^+$

- Use the following Axioms to compute  $F^+$  (Armstrong's axioms)
  - Reflexivity Rule : If  $\beta \subseteq \alpha$  then  $\alpha \rightarrow \beta$
  - Augmentation Rule : If  $\alpha \rightarrow \beta$  and  $\gamma$  is a set of attributes then  $\gamma\alpha \rightarrow \gamma\beta$
  - Transitive Rule : If  $\alpha \rightarrow \beta$  and  $\beta \rightarrow \gamma$  then  $\alpha \rightarrow \gamma$

$F^+ = F$

**repeat**

**for each** functional dependency  $f$  in  $F^+$

        apply reflexivity and augmentation rules on  $f$

        add the resulting functional dependencies to  $F^+$

**for each** pair of functional dependencies  $f_1$  and  $f_2$  in  $F^+$

**if**  $f_1$  and  $f_2$  can be combined using transitivity

            add the resulting functional dependency to  $F^+$

**until**  $F^+$  does not change any further

- Correctness of the algorithm ?
- Running time of the algorithm ?

# Computing $F^+$

- Use the following Axioms to compute  $F^+$  (Armstrong's axioms)
  - Reflexivity Rule : If  $\beta \subseteq \alpha$  then  $\alpha \rightarrow \beta$
  - Augmentation Rule : If  $\alpha \rightarrow \beta$  and  $\gamma$  is a set of attributes then  $\gamma\alpha \rightarrow \gamma\beta$
  - Transitive Rule : If  $\alpha \rightarrow \beta$  and  $\beta \rightarrow \gamma$  then  $\alpha \rightarrow \gamma$
- Other useful rules (Can be derived using the above axioms)
  - Union Rule : If  $\alpha \rightarrow \beta$  and  $\alpha \rightarrow \gamma$  then  $\alpha \rightarrow \beta\gamma$
  - Decomposition Rule : If  $\alpha \rightarrow \beta\gamma$  then  $\alpha \rightarrow \beta$  and  $\alpha \rightarrow \gamma$  holds
  - Pseudotransitivity Rule: If  $\alpha \rightarrow \beta$  and  $\beta\gamma \rightarrow \delta$  then  $\alpha\gamma \rightarrow \delta$

# Closure of Attribute set

- An Attribute B is functionally dependent on  $\alpha$  if  $\alpha \rightarrow B$
- To check if  $\alpha$  is a superkey, we should compute the set of all attributes functionally dependant on  $\alpha$
- One way to do this is to compute  $F^+$ , take all FDs with  $\alpha$  as left-hand side and right-hand side will be the union of the right-hand side of all such FDs
  - This is expensive

# Closure of Attribute set

- There is an efficient algorithm

```
result :=  $\alpha$ ;  
repeat  
  for each functional dependency  $\beta \rightarrow \gamma$  in  $F$  do  
    begin  
      if  $\beta \subseteq \text{result}$  then  $\text{result} := \text{result} \cup \gamma$ ;  
    end  
until (result does not change)
```

$A \rightarrow B$

$A \rightarrow C$

$CG \rightarrow H$

$CG \rightarrow I$

$B \rightarrow H$

- Over schema  $r(A,B,C,G,H,I)$  Compute  $(AG)^+$ 
  - $\text{result} = \{A,G\}$
  - $\text{result} = \{A,G,B\}$
  - $\text{result} = \{A,G,B,C\}$
  - $\text{result} = \{A,G,B,C,H\}$
  - $\text{result} = \{A,G,B,C,H,I\}$

# Closure of Attribute set

- There is an efficient algorithm

```
result :=  $\alpha$ ;  
repeat  
  for each functional dependency  $\beta \rightarrow \gamma$  in  $F$  do  
    begin  
      if  $\beta \subseteq \textit{result}$  then  $\textit{result} := \textit{result} \cup \gamma$ ;  
    end  
until (result does not change)
```

- Correctness of the Algorithm
  - Soundness and Completeness
- Running time of the Algorithm



# Closure of Attribute set : Uses

- To test if  $\alpha$  is a superkey
- Check if a functional dependency  $\alpha \rightarrow \beta$  holds
- Alternative way to compute  $F^+$

# Canonical Cover

- Suppose database has functional dependency set  $F$ :
- Whenever a database is updated, the system should ensure all functional dependencies are satisfied
- If we have a minimal set of functional dependencies  $F^*$  that imply all the functional dependencies of  $F$  then the check can be made faster

# Canonical Cover

- An attribute of a functional dependency is extraneous if we can remove it without changing the closure set of the functional dependencies.
- Consider a set of functional dependencies  $F$  and let  $\alpha \rightarrow \beta$  be in  $F$ 
  - Attribute  $A$  in  $\alpha$  is extraneous if  $F$  logically implies  $(F - \{\alpha \rightarrow \beta\}) \cup \{(\alpha - A) \rightarrow \beta\}$
  - Attribute  $A$  in  $\beta$  is extraneous if the set  $(F - \{\alpha \rightarrow \beta\}) \cup \{\alpha \rightarrow (\beta - A)\}$  logically implies  $F$

# Canonical Cover

- An attribute of a functional dependency is extraneous if we can remove it without changing the closure set of the functional dependencies.
- Consider a set of functional dependencies  $F$  and let  $\alpha \rightarrow \beta$  be in  $F$ 
  - Attribute  $A$  in  $\alpha$  is extraneous if  $F$  logically implies  $(F - \{\alpha \rightarrow \beta\}) \cup \{(\alpha - A) \rightarrow \beta\}$
  - Attribute  $A$  in  $\beta$  is extraneous if the set  $(F - \{\alpha \rightarrow \beta\}) \cup \{\alpha \rightarrow (\beta - A)\}$  logically implies  $F$
- Example:
  - In  $F = \{AB \rightarrow C, A \rightarrow C\}$ ,  $B$  is extraneous in  $AB \rightarrow C$
  - In  $F = \{AB \rightarrow CD, AB \rightarrow C\}$   $C$  is extraneous in  $AB \rightarrow CD$
- Note: In both cases the other direction of the implication trivially holds always

# Canonical Cover : Checking for extraneous attributes

- Consider a set of functional dependencies  $F$  and let  $\alpha \rightarrow \beta$  be in  $F$ 
  - Attribute  $A$  in  $\alpha$  is extraneous if  $F$  logically implies  $(F - \{\alpha \rightarrow \beta\}) \cup \{(\alpha - A) \rightarrow \beta\}$
  - Attribute  $A$  in  $\beta$  is extraneous if the set  $(F - \{\alpha \rightarrow \beta\}) \cup \{\alpha \rightarrow (\beta - A)\}$  logically implies  $F$
- For a relational schema  $R$  and functional dependencies  $F$  and  $\alpha \rightarrow \beta$  in  $F$ ,  
Let  $A$  be an attribute in  $\alpha \rightarrow \beta$ 
  - If  $A$  is in  $\beta$  : To check if  $A$  is extraneous, consider the set of functional dependencies:  
 $F^* = (F - \{\alpha \rightarrow \beta\}) \cup \{\alpha \rightarrow (\beta - A)\}$  and check if  $\alpha \rightarrow \beta$  can be inferred from  $F^*$  (By computing  $\alpha^+$  under  $F^*$ )
  - If  $A$  is in  $\alpha$  : To check if  $A$  is extraneous, consider  $\gamma = \alpha - \{A\}$  and check if  $\gamma \rightarrow \beta$  can be inferred under  $F$  (By computing  $\gamma^+$ )

# Canonical Cover

- **Canonical Cover** of a set of functional dependencies  $F$  is given by  $F_c$  such that  $F$  logically implies  $F_c$  and vice-versa, such that the following conditions hold:
  - No functional dependencies in  $F_c$  contains extraneous attributes
  - Each left side of a functional dependency in  $F_c$  is unique(There are no two dependencies  $\alpha_1 \rightarrow \beta_1$  and  $\alpha_2 \rightarrow \beta_2$  in  $F_c$  such that  $\alpha_1 = \alpha_2$ )

$F_c = F$

**repeat**

Use the union rule to replace any dependencies in  $F_c$  of the form

$\alpha_1 \rightarrow \beta_1$  and  $\alpha_1 \rightarrow \beta_2$  with  $\alpha_1 \rightarrow \beta_1 \beta_2$ .

Find a functional dependency  $\alpha \rightarrow \beta$  in  $F_c$  with an extraneous attribute either in  $\alpha$  or in  $\beta$ .

/\* Note: the test for extraneous attributes is done using  $F_c$ , not  $F$  \*/

If an extraneous attribute is found, delete it from  $\alpha \rightarrow \beta$  in  $F_c$ .

**until** ( $F_c$  does not change)

# Canonical Cover

- **Canonical Cover** of a set of functional dependencies  $F$  is given by  $F_c$  such that  $F$  logically implies  $F_c$  and vice-versa, such that the following conditions hold:
  - No functional dependencies in  $F_c$  contains extraneous attributes
  - Each left side of a functional dependency in  $F_c$  is unique  
(There are no two dependencies  $\alpha_1 \rightarrow \beta_1$  and  $\alpha_2 \rightarrow \beta_2$  in  $F_c$  such that  $\alpha_1 = \alpha_2$ )
- **$F$  and  $F_c$**  have same closures.
- Testing if  $F$  is satisfied is equivalent to testing if  $F_c$  is satisfied
- It is cheaper to test  $F_c$  than  $F$  itself

# Canonical Cover : Example

- Let  $r(A, B, C)$  be a relational schema with the following functional dependencies:

$A \rightarrow BC$

$B \rightarrow C$

$A \rightarrow B$

$AB \rightarrow C$

- Step 0:  $F_c = \{ A \rightarrow BC, B \rightarrow C, A \rightarrow B, AB \rightarrow C \}$
- Step 1 : Combine 1st and 3rd FD and obtain  $F_c = \{ A \rightarrow BC, B \rightarrow C, AB \rightarrow C \}$
- Step 2:  $A$  is extraneous in  $AB \rightarrow C$ 
  - because  $F_c$  implies  $\{ B \rightarrow C, A \rightarrow BC \} \cup \{ B \rightarrow C \}$ 
    - $F_c = \{ A \rightarrow BC, B \rightarrow C \}$
- Step 3 :  $C$  is extraneous in  $A \rightarrow BC$ 
  - Because  $\{ A \rightarrow B, B \rightarrow C \}$  logically implies  $A \rightarrow BC$ 
    - $F_c = \{ A \rightarrow B, B \rightarrow C \}$



# Dependency Preservation

- Let  $F$  be a set of functional dependencies on  $R$  and let  $R_1 R_2 \dots R_n$  be a decomposition of  $R$
- Restriction of  $F$  to  $R_i$  is the set  $F_i$  is the set of all functional dependencies that includes only the attributes of  $R_i$
- Example: If  $r(A,B,C)$  is a schema with  $F = \{ A \rightarrow B, B \rightarrow C, A \rightarrow C \}$  and the decomposition is  $R_1 = (A,B)$  and  $R_2 = (A,C)$   
Then  $F_1 = \{ A \rightarrow B \}$  and  $F_2 = \{ A \rightarrow C \}$
- Decomposition of  $R$  to  $R_1 R_2 \dots R_n$  is said to be dependency preserving if:

$$F^+ = (F_1 \cup F_2 \cup \dots \cup F_m)^+$$

```
compute  $F^+$ ;  
for each schema  $R_i$  in  $D$  do  
  begin  
     $F_i :=$  the restriction of  $F^+$  to  $R_i$ ;  
  end  
 $F' := \emptyset$   
for each restriction  $F_i$  do  
  begin  
     $F' = F' \cup F_i$   
  end  
compute  $F'^+$ ;  
if  $(F'^+ = F^+)$  then return (true)  
  else return (false);
```

# Decomposition Algorithms : BCNF

- A relation schema  $R$  is in BCNF with respect to a set  $F$  of functional dependencies if, for all functional dependencies in  $F^+$  of the form  $\alpha \rightarrow \beta$ , where  $\alpha \subseteq R$  and  $\beta \subseteq R$  : at least one of the following holds:
  - $\alpha \rightarrow \beta$  is a trivial functional dependency (that is,  $\beta \subseteq \alpha$  )
  - $\alpha$  is a superkey for schema  $R$
- Testing for BCNF
  - Goal is to see if there exists a non-trivial  $\alpha \rightarrow \beta$  in  $F^+$  where  $\alpha$  is not a superkey
    - For every non-trivial  $\alpha \rightarrow \beta$  in  $F^+$  Compute  $\alpha^+$  and see if it contains all the attributes of  $R$
  - There are Other clever ways to do it that avoids computing  $F^+$

# Decomposition Algorithms : BCNF

- A relation schema  $R$  is in BCNF with respect to a set  $F$  of functional dependencies if, for all functional dependencies in  $F^+$  of the form  $\alpha \rightarrow \beta$ , where  $\alpha \subseteq R$  and  $\beta \subseteq R$  : at least one of the following holds:
  - $\alpha \rightarrow \beta$  is a trivial functional dependency (that is,  $\beta \subseteq \alpha$  )
  - $\alpha$  is a superkey for schema  $R$
- BCNF Decomposition Algorithm

```
result := {R};
done := false;
compute  $F^+$ ;
while (not done) do
    if (there is a schema  $R_i$  in result that is not in BCNF)
        then begin
            let  $\alpha \rightarrow \beta$  be a nontrivial functional dependency that holds
            on  $R_i$  such that  $\alpha \rightarrow R_i$  is not in  $F^+$ , and  $\alpha \cap \beta = \emptyset$ ;
            result := (result -  $R_i$ )  $\cup$  ( $R_i - \beta$ )  $\cup$  ( $\alpha, \beta$ );
        end
    else done := true;
```

# Decomposition Algorithms : BCNF

- Example:  
class ( course\_id, title, dept\_name, credits, sec\_id, semester, year, building, room\_number, capacity, time\_slot\_id)
- course\_id → title, dept\_name, credits
- building, room\_number → capacity
- course\_id, sec\_id, semester, year → building, room\_number, time\_slot\_id
- Candidate Key = { course\_id, sec\_id, semester, year }
- course\_id → title, dept\_name, credits (is non-trivial FD and course\_id is not superkey)
  - course ( course\_id , title, dept\_name, credits)
  - class1 (course\_id, sec\_id, semester, year, building, room\_number, capacity, time\_slot\_id)
- building, room\_number → capacity (is non-trivial FD and course\_id is not superkey)
  - course ( course\_id , title, dept\_name, credits)
  - class11 (building, Room\_number, capacity)
  - class12 ( course\_id, sec\_id, year, time\_slot\_id)
- All 3 are in BCNF ( This is lossless and dependency preserving decomposition )
- Algorithm ensures losslessness but dependency preservation might be violated

# Decomposition Algorithms : 3NF

- A relation schema  $r(R)$  is in Third Normal Form (3NF) with respect to a set  $F$  of functional dependencies if, for all functional dependencies in  $F^+$  of the form  $\alpha \rightarrow \beta$ , where  $\alpha \subseteq R$  and  $\beta \subseteq R$  : at least one of the following holds:
  - $\alpha \rightarrow \beta$  is a trivial functional dependency (that is,  $\beta \subseteq \alpha$ ).
  - $\alpha$  is a superkey for schema  $R$ .
  - Each attribute  $A$  in  $\beta - \alpha$  is contained in some candidate key of  $R$

```
let  $F_c$  be a canonical cover for  $F$ ;  
 $i := 0$ ;  
for each functional dependency  $\alpha \rightarrow \beta$  in  $F_c$   
   $i := i + 1$ ;  
   $R_i := \alpha \beta$ ;  
if none of the schemas  $R_j, j = 1, 2, \dots, i$  contains a candidate key for  $R$   
  then  
     $i := i + 1$ ;  
     $R_i :=$  any candidate key for  $R$ ;  
/* Optionally, remove redundant relations */  
repeat  
  if any schema  $R_j$  is contained in another schema  $R_k$   
  then  
    /* Delete  $R_j$  */  
     $R_j := R_i$ ;  
     $i := i - 1$ ;  
until no more  $R_j$ s can be deleted  
return  $(R_1, R_2, \dots, R_i)$ 
```

# Decomposition Algorithms : 3NF

- Example:

dept\_advisors (s\_ID, i\_ID, dept\_name)

- $i\_ID \rightarrow dept\_name$

- $s\_ID, dept\_name \rightarrow i\_ID$

- $F = F_c$  (no extraneous attributes)

- $R_1(i\_ID, dept\_name)$

- $R_2(s\_ID, dept\_name, i\_ID)$

- Delete  $R_1$  and retain only  $R_2$

- Algorithm results in Lossless and Dependency preserving 3NF decomposition

let  $F_c$  be a canonical cover for  $F$ ;

$i := 0$ ;

**for each** functional dependency  $\alpha \rightarrow \beta$  in  $F_c$

$i := i + 1$ ;

$R_i := \alpha \beta$ ;

**if** none of the schemas  $R_j$ ,  $j = 1, 2, \dots, i$  contains a candidate key for  $R$

**then**

$i := i + 1$ ;

$R_i :=$  any candidate key for  $R$ ;

    /\* Optionally, remove redundant relations \*/

**repeat**

**if** any schema  $R_j$  is contained in another schema  $R_k$

**then**

            /\* Delete  $R_j$  \*/

$R_j := R_i$ ;

$i := i - 1$ ;

**until** no more  $R_j$ s can be deleted

**return** ( $R_1, R_2, \dots, R_i$ )

# Comparing BCNF and 3NF

- It is **always possible to obtain 3NF** without sacrificing dependency preservation and losslessness
  - But might lead to having many nulls and repetition of information
- Goal:
  - BCNF + Lossless decomposition + Dependency Preservation
  - All 3 are not possible simultaneously
  - Choose BCNF or Dependency Preservation + 3NF
- SQL does not allow specification of FDs (except primary key and super keys using unique )
  - Can be done using assertions

# Multivalued dependencies

- In some cases BCNF does not reduce repetitions
- Suppose one instructor can be associated with multiple departments. Moreover Instructors can have several addresses (office, current address, permanent address.. )
  - inst ( ID, dept\_name, name, street, city )
  - FD : ID  $\rightarrow$  name ( but ID is not the key for inst )
  - $r_1$  ( ID, dept\_name, street, city) and  $r_2$  ( ID, name )
- Decomposition is in BCNF but there still can be repetition in  $r_1$ 
  - $r_{11}$  ( ID, dept\_name ) and  $r_{12}$  ( ID, street, city )
- How to identify this in the absence of additional constraints ?



# Multivalued dependencies

- A relation schema  $r(R)$  be a relational schema and let  $\alpha \subseteq R$  and  $\beta \subseteq R$ . The multivalued dependency  $\alpha \twoheadrightarrow \beta$  holds if for every legal instance  $r$  of  $r(R)$  and tuples  $t_1$  and  $t_2$  of  $r$  such that  $t_1[\alpha] = t_2[\alpha]$  then there exists  $t_3$  and  $t_4$  such that

- $t_1[\alpha] = t_2[\alpha] = t_3[\alpha] = t_4[\alpha]$
- $t_3[\beta] = t_1[\beta]$
- $t_3[R - \beta] = t_2[R - \beta]$
- $t_4[\beta] = t_2[\beta]$
- $t_4[R - \beta] = t_1[R - \beta]$

	$\alpha$	$\beta$	$R - \alpha - \beta$
$t_1$	$a_1 \dots a_i$	$a_{i+1} \dots a_j$	$a_{j+1} \dots a_n$
$t_2$	$a_1 \dots a_i$	$b_{i+1} \dots b_j$	$b_{j+1} \dots b_n$
$t_3$	$a_1 \dots a_i$	$a_{i+1} \dots a_j$	$b_{j+1} \dots b_n$
$t_4$	$a_1 \dots a_i$	$b_{i+1} \dots b_j$	$a_{j+1} \dots a_n$

- Multivalued dependencies are part of more general **tuple generating dependencies**
- Functional dependencies are part of more general **equality generating dependencies**

# Multivalued dependencies : Closure

- If  $D$  is a set of FDs + MVDs then let  $D^+$  denote the closure of  $D$  that contains all the dependencies that are logically implied by  $D$
- Computed using axioms. Sample axioms:
  - If  $\alpha \twoheadrightarrow \beta$  then  $\alpha \twoheadrightarrow \beta$
  - If  $\alpha \twoheadrightarrow \beta$  then  $\alpha \twoheadrightarrow R - \alpha - \beta$

# Multivalued dependencies : Fourth Normal Form

- MVD  $\alpha \twoheadrightarrow \beta$  over  $r(R)$  is trivial if  $\beta \subseteq \alpha$  or  $\alpha \cup \beta = R$
- A relational schema  $r(R)$  is in Fourth Normal Form (4NF) with respect to a set  $D$  of FD + MVD if for all MVDs  $\alpha \twoheadrightarrow \beta$  in  $D^+$  where  $\alpha \subseteq R$  and  $\beta \subseteq R$  one of the following holds:
  - $\alpha \twoheadrightarrow \beta$  is a trivial MVD
  - $\alpha$  is a superkey of  $R$
- A database design is in 4NF if every relational schema is in 4NF
- Note that 4NF implies BCNF

# Multivalued dependencies : 4NF decomposition

- Preserves Losslessness
- But does not preserve dependencies

```
result := {R};  
done := false;  
compute  $D^+$ ; Given schema  $R_i$ , let  $D_i$  denote the restriction of  $D^+$  to  $R_i$   
while (not done) do  
    if (there is a schema  $R_i$  in result that is not in 4NF w.r.t.  $D_i$ )  
        then begin  
            let  $\alpha \twoheadrightarrow \beta$  be a nontrivial multivalued dependency that holds  
            on  $R_i$  such that  $\alpha \rightarrow R_i$  is not in  $D_i$ , and  $\alpha \cap \beta = \emptyset$ ;  
            result := (result -  $R_i$ )  $\cup$  ( $R_i - \beta$ )  $\cup$  ( $\alpha, \beta$ );  
        end  
    else done := true;
```

# Higher Normal Forms

- Join dependencies generalize multivalued dependencies
  - Leads to Project-Join Normal Form (5NF)
- Domain Key Normal Form
- Disadvantages of higher normal forms:
  - Hard to reason
  - No sound and complete axiom set

# Database Design and Normalization

- Generated while converting an E-R diagram into a relational schema
  - **Good E-R diagram** does not need too much normalization when translated to relational schemas
  - **Relationship sets involving more than two entities** can result in non-normalized schema
  - **Universal-relation approach** : Design one big relational schema that contains everything and then normalize
- Naming Relationship and Attributes
  - **Unique Role assumption** : Every name of attribute has a unique meaning
  - Convention to designate primary key attributes at the beginning
  - **Prefix helps in disambiguation** : student\_ID, instructor\_ID

# Database Design and Normalization

- Denormalization for Performance
  - Occasionally designers choose to retain redundancy
  - Improves performance  
(Normalized databases need join operations to evaluate queries)
  - Alternative is to use normalized database and store denormalized database as materialized views

# Database Design and Normalization

- Some aspects not considered in normalization process
- Example:
  - Store number of instructors in each department in various years  
`(dept_name, year, num_inst)`  
with `dept_name, year → num_inst` as FD
  - Alternative is to have one table for each year  
`inst_count_2020 ( dept_name num_inst) inst_count_2021 (dept_name, num_inst)`  
With `dept_name → num_inst` as FD in each table
  - Another Alternative  
`dept_year ( dept_name, num_inst_2020, num_inst_2021)`  
With `dept_name → num_inst_2020, num_inst_2021`  
Close to spreadsheet representation
- Last 2 alternatives are bad design choices though they are in BCNF



# Modelling Temporal Data

- Temporal Data is associated with a time interval where the data is valid (called snapshot)
- Functional dependency should take temporal aspect into account
- Example : (ID, street, city, from, to)
- Temporal functional dependency :  $X \xrightarrow{T} Y$ 
  - Makes the design and functional dependence analysis complicated
  - First design without temporal data (by considering a snapshot) then decide what temporal constraints are needed
- What should be the end time if it is currently ongoing?
  - Null ? (Problem with primary key)
  - Something far in future
- Relationship between time dependent and time independent data
  - Advisor relationship and department budget (though depends on current budget)
  - Solution: Not to add time in main relation, but maintain history separately

Reference:

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Database System Concepts by Silberschatz, Korth and Sudarshan  
(6th edition)  
Chapter 8