DAA Tutorial - 04

PAGE NO.
DATE. / /

Satyam Kumar, Sec - D, Roll no:- 34.

Tutorial - 4

1. $T(n) = 3T(n/2) + n^2$

$\Rightarrow c = \log_2 3 = 1.58$

$\Rightarrow n^c = n^{1.58}$

$\Rightarrow n^2 > n^{1.58}$

$\Rightarrow f(n) > n^c$

$\Rightarrow T(n) = \theta(n^2)$.

2. $T(n) = 4T(n/2) + n^2$

$\Rightarrow c = \log_2 4 = 2$

$\Rightarrow n^c = n^2$

$\Rightarrow n^c = f(n)$

$\Rightarrow T(n) = \theta(n^2 \log n)$.

3. $T(n) = T(n/2) + 2^n$

$\Rightarrow c = \log_2 1 = 0$

$\Rightarrow n^c = n^0 = 1$.

$f(n) = 2^n$

$\Rightarrow f(n) > n^c$

$\Rightarrow T(n) = \theta(2^n)$.

4. $T(n) = 2^n T(n/2) + n^n$

$\Rightarrow c = \log_2 2^n$

$\Rightarrow n^c = n^{2^n}$

$\because f(n) = n^n$

$\Rightarrow n^c > f(n)$

$\Rightarrow T(n) = \theta(n^{2^n})$.

PAGE NO.
DATE. / /

5. $T(n) = 16T(n/4) + n$

$\Rightarrow c = \log_4 16 = 2$

$n^c = n^2$

$f(n) = n$

$\Rightarrow \because n^2 > n$

$\Rightarrow n^c > f(n)$

$T(n) = \theta(n^2)$.

6. $T(n) = 2T(n/2) + n\log n$

$\Rightarrow c = \log_2 2 = 1$

$n^c = n$

$f(n) = n\log n$

$\because n\log n > n$

$\Rightarrow f(n) > n^c$

$\Rightarrow T(n) = \theta(n\log n)$

7. $T(n) = 2T(n/2) + \frac{n}{\log n}$

$c = \log_2 2 = 1$

$\Rightarrow n^c = n$

$f(n) = \frac{n}{\log n}$

$\because \frac{n}{\log n} < n$

$\Rightarrow f(n) < n^c$

$\Rightarrow T(n) = \theta(n)$.

$100^2 =$ $10^2 = 10^{\log 10}$ $10! =$

PAGE NO.
DATE. / /

8. $T(n) = 2T(n/4) + n^{0.51}$

$c = \log_4 2 = 0.5$

$n^c = n^{0.5}$

$\because n^{0.5} < n^{0.51}$

$\Rightarrow n^c < f(n)$

$\Rightarrow T(n) = \theta(n^{0.51})$.

9. $T(n) = 0.5\,T(n/2) + 1/n$

~~$c = \log$~~

$\because a = 0.5$

$\Rightarrow a < 1$, for applying master's theorem, condition should be $a \geq 1$.

$\Rightarrow$ Here, Master theorem does not apply.

10. $T(n) = 16\,T(n/4) + n!$

$c = \log_4 16 = 2$

$n^c = n^2$

$\because n^2 < n!$

$\Rightarrow n^c < f(n)$

$\Rightarrow T(n) = \theta(n!)$

11. $T(n) = 4\,T(n/2) + \log n$

$\Rightarrow c = \log_2 4 = 2$

$n^c = n^2$

$\Rightarrow n^2 > \log n$

$n^c > f(n)$

$\Rightarrow T(n) = \theta(n^2)$.

PAGE NO.
DATE. / /

12. $T(n) = \sqrt{n}\, T(n/2) + \log n$

$c = \log_2 \sqrt{n}$

$\Rightarrow c = \log_2 n^{1/2}$

$c = \frac{1}{2} \log_2 n$

$\Rightarrow n^c = n^{(\log_2 n)/2}$

$\because n^{(\log_2 n/2)} > \log n$

$\Rightarrow n^c > f(n)$

$\Rightarrow T(n) = \theta\left(n^{(\log_2 n/2)}\right)$

13. $T(n) = 3T(n/2) + n$

$\Rightarrow c = \log_2 3 = 1.584.$

$n^{1.584} > n$

$\Rightarrow n^c > f(n)$

$\Rightarrow T(n) = \theta(n^{1.584})$

14. $T(n) = 3T(n/3) + \text{sqrt}(n).$

$\Rightarrow c = \log_3 3 = 1$

$n^c = n, \quad f(n) = \sqrt{n}$

$\Rightarrow \because n > \sqrt{n}$

$\Rightarrow n^c > f(n)$

$\Rightarrow T(n) = \theta(n).$

15. $T(n) = 4T(n/2) + cn$

$\Rightarrow k = \log_2 4 = 2$

$\Rightarrow n^k = n^2$

$\because n^2 > cn$ [$\because$ for any constant c]

$\Rightarrow T(n) = \theta(n^2).$

PAGE NO.
DATE. / /

16. $T(n) = 3T(n/4) + n\log n$

$\Rightarrow a = 3, \quad b = 4$

$c = \log_4 3 = 0.792$

$\Rightarrow n^c = n^{0.792}$

$\because n^{0.792} < n\log n$

$\Rightarrow T(n) = \theta(n\log n)$

17. $T(n) = 3T(n/3) + n/2$

$\Rightarrow a = 3, \quad b = 3$

$c = \log_3 3 = 1$

$n^c = n$

$\because n > \frac{n}{2}$

$\Rightarrow n^c > f(n)$

$\Rightarrow T(n) = \theta(n).$

18. $T(n) = 6T(n/3) + n^2\log n$

$a = 6, \quad b = 3$

$\Rightarrow c = \log_3 6 = 1.6309$

$\Rightarrow n^c = n^{1.6309}.$

$\because n^{1.6309} < n^2\log n$

$\Rightarrow T(n) = \theta(n^2\log n).$

19. $T(n) = 4T(n/2) + n/\log n$

$a = 4, \quad b = 2 \Rightarrow c = \log_2 4 = 2$

$\Rightarrow n^c = n^2.$

$\because n^2 > n/\log n$

$\Rightarrow T(n) = \theta(n^2).$

PAGE NO.
DATE. / /

20. $T(n) = 64\,T(n/8) - n^2 \log n$

$a = 64, \quad b = 8$

$c = \log_8 64 = 2$

$\Rightarrow n^c = n^2$, but

$f(n) = -n^2 \log n$

$\therefore f(n) < 0 \Rightarrow$ master's theorem can't be applied.

21. $T(n) = 7T\left(\frac{n}{3}\right) + n^2$

$a = 7, \quad b = 3$

$\Rightarrow c = \log_3 7 = 1.771$

$\Rightarrow n^c = n^{1.771}$

$\therefore n^2 > n^{1.771}$

$\Rightarrow f(n) > n^c$

$\Rightarrow T(n) = \Theta(f(n))$

$= \Theta(n^2)$.

22. $T(n) = T(n/2) + n(2 - \cos n)$

$a = 1, \quad b = 2$

$c = \log_2 1 = 0$

$\Rightarrow n^c = n^0 = 1$.

Now, $f(n) = n(2 - \cos n)$

$\because (2 - \cos n) \in [-1, 1]$

$\Rightarrow$ highest value of $n(2 - \cos n)$ can be,

$n(2 - (-1)) = 3n$.

$\Rightarrow 3n > n^1$

$\Rightarrow f(n) > 1$

$\Rightarrow T(n) = \Theta(3n) = \Theta(n)$.