# User Guide for SuiteSparse:GraphBLAS

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#### Abstract

SuiteSparse:GraphBLAS is a full implementation of the Graph-BLAS standard, which defines a set of sparse matrix operations on an extended algebra of semirings using an almost unlimited variety of operators and types. When applied to sparse adjacency matrices, these algebraic operations are equivalent to computations on graphs. GraphBLAS provides a powerful and expressive framework for creating high-performance graph algorithms based on the elegant mathematics of sparse matrix operations on a semiring.

When compared with MATLAB R2021a, some methods in Graph-BLAS are up to a million times faster than MATLAB, even when using the same syntax. Typical speedups are in the range 2x to 30x, with a few 1000x speedups. The statement C(M)=A when using MATLAB sparse matrices takes  $O(e^2)$  time where e is the number of entries in C. GraphBLAS can perform the same computation with the exact same syntax, but in  $O(e\log e)$  time (or O(e) in some cases), and in practice that means GraphBLAS can compute C(M)=A for a large problem in under a second, while MATLAB takes about 4 to 5 days.

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### 1 Introduction

The GraphBLAS standard defines sparse matrix and vector operations on an extended algebra of semirings. The operations are useful for creating a wide range of graph algorithms.

For example, consider the matrix-matrix multiplication,  $\mathbf{C} = \mathbf{AB}$ . Suppose  $\mathbf{A}$  and  $\mathbf{B}$  are sparse n-by-n Boolean adjacency matrices of two undirected graphs. If the matrix multiplication is redefined to use logical AND instead of scalar multiply, and if it uses the logical OR instead of add, then the matrix  $\mathbf{C}$  is the sparse Boolean adjacency matrix of a graph that has an edge (i,j) if node i in  $\mathbf{A}$  and node j in  $\mathbf{B}$  share any neighbor in common. The OR-AND pair forms an algebraic semiring, and many graph operations like this one can be succinctly represented by matrix operations with different semirings and different numerical types. GraphBLAS provides a wide range of built-in types and operators, and allows the user application to create new types and operators without needing to recompile the GraphBLAS library.

For more details on SuiteSparse:GraphBLAS, and its use in LAGraph, see [Dav19, Dav18, DAK19, ACD+20, MDK+19].

A full and precise definition of the GraphBLAS specification is provided in *The GraphBLAS C API Specification* by Aydın Buluç, Timothy Mattson, Scott McMillan, José Moreira, Carl Yang, and Benjamin Brock [BMM<sup>+</sup>17a, BMM<sup>+</sup>17b], based on *GraphBLAS Mathematics* by Jeremy Kepner [Kep17]. The GraphBLAS C API Specification is available at <a href="http://graphblas.org">http://graphblas.org</a>. This version of SuiteSparse:GraphBLAS conforms to most of Version 2.0.0 (Sept 29, 2021 (draft)) of *The GraphBLAS C API specification*, except for GrB\_wait and GrB\_Info.

In this User Guide, aspects of the GraphBLAS specification that would be true for any GraphBLAS implementation are simply called "GraphBLAS." Details unique to this particular implementation are referred to as Suite-Sparse:GraphBLAS.

All functions, objects, and macros with a name of the form GxB\_\* are SuiteSparse-specific extensions to the spec.

**SPEC:** Non-obvious deviations or additions to the GraphBLAS C API Specification are highlighted in a box like this one, except for GxB\* methods. They are not highlighted since their name makes it clear that they are extensions to the GraphBLAS C API.

#### 1.1 Release Notes

- Version 6.0.0 (future)
  - this release will contain only a few changes that cause a break with backward compatibility.
  - It will be fully compliant with the v2.0 C API Specification. Two changes from the v2.0 C API Spec are not backward compatible (GrB\_\*wait, and GrB\_Info). GxB\_init has also changed.
    - \* GrB\_wait (object, mode): was GrB\_wait (&object).
    - \* GrB\_Info: changed enum values
    - \* GxB\_init (mode, malloc, calloc, realloc, free, is\_thread\_safe): the last parameter, is\_thread\_safe, will be deleted. The malloc, calloc, realloc, and free functions must be thread-safe.
- Version 5.2.0 (FIXME)
  - Added for the v2.0 C API Specification: only features that are backward compatible with SuiteSparse:GraphBLAS v5.x have been added to v5.2.0:
    - \* GrB\_Scalar: replaces GxB\_Scalar, GxB\_Scalar\_\* functions renamed GrB
    - \* GrB\_IndexUnaryOp\verb: new, free, fprint, wait
    - \* GrB\_select\verb: selection via GrB\_IndexUnaryOp
    - \* GrB\_apply: with GrB\_IndexUnaryOp
    - \* GrB\_reduce: reduce matrix or vector to GrB\_Scalar
    - \* GrB\_assign, GrB\_subassion: with GrB\_Scalar input
    - \* GrB\_\*\_extractElement\_Scalar: get GrB\_Scalar from a matrix or vector
    - \* GrB\*build: when dup is NULL, duplicates result in an error.
    - \* GrB import/export: import/export from/to user-provided arrays
    - \* Grb\_EMPTY\_OBJECT, Grb\_NOT\_IMPLEMENTED: error codes added
    - \* GrB\_\*\_setElement\_Scalar: set an entry in a matrix or vector, from a GrB\_Scalar
    - \* GrB\_Matrix\_diag: same as GxB\_Matrix\_diag (C, v, k, NULL)
    - \* GrB\_\*\_serialize/deserialize: with compression

- GxB\_IGNORE\_DUP: special operator for build methods only; if dup is this operator, then duplicates are ignored (not an error)
- GxB\_IndexUnaryOp\_new: create a named index-unary operator
- GxB\_BinaryOp\_new: create a named binary operator
- GxB\_UnaryOp\_new: create a named unary operator
- GxB\_Type\_new: to create a named type
- GxB\_Type\_name: to query the name of a type
- added GxB\_\*type\_name methods to query the name of a type as a string.
- GxB methods that query an object return a GrB\_type such as GxB\_Matrix\_type are declared historical; will be kept but not recommended (use GxB\_\*type\_name methods).
- GxB\_Matrix\_serialize/deserialize: with compression; optional descriptor.
- added LZ4/LZ4HC: compression library, http://www.lz4.org (BSD 2), v1.9.3, Copyright (c) 2011-2016, Yann Collet.
- Version 5.1.7 (Aug 23, 2021)
  - (30) bug fix: GrB\_apply, when done in-place and matrix starts non-iso and becomes iso, gave the wrong iso result. Caught by Fabian Murariu.
- Version 5.1.6 (Aug 16, 2021)
  - one-line change to C=A\*B: faster symbolic analysis when a vector
     C(:,j) is dense (for CSC) or C(i,:) for CSR.
- Version 5.1.5 (July 15, 2021)
  - submission to ACM Transactions on Mathematical Software as a Collected Algorithm of the ACM.
- Version 5.1.4 (July 6, 2021)
  - faster Octave interface. Octave v7 or later is required.
  - (30) bug fix: 1-based printing not enabled for pending tuples. Caught by Will Kimmerer, while working on the Julia interface.

- Version 5.1.3 (July 3, 2021)
  - added GxB\_Matrix\_iso and GxB\_Vector\_iso: to query if a matrix or vector is held as iso-valued
  - (29) bug fix: Matrix\_pack\_\*R into a matrix previously held by column, or Matrix\_pack\*C into a matrix by row, would flip the dimensions. Caught by Erik Welch, Anaconda.
  - (28) bug fix: kron(A,B) with iso input matrices A and B fixed. Caught by Michel Pelletier, Graphegon.
  - (27) bug fix: v5.1.0 had a wrong version of a file; posted by mistake. Caught by Michel Pelletier, Graphegon.
- Version 5.1.2 (June 30, 2021)
  - iso matrices added: these are matrices and vectors whose values in the sparsity pattern are all the same. This is an internal change to the opaque data structures of the GrB\_Matrix and GrB\_Vector with very little change to the API.
  - added GxB\_Matrix\_build\_Scalar and GxB\_Vector\_build\_Scalar, which always build iso matrices and vectors.
  - import/export methods can now import/export iso matrices and vectors.
  - added GrB.argmin/argmax to Octave/MATLAB interface
  - added GxB\_\*\_pack/unpack methods as alternatives to import/export.
  - added GxB\_PRINT\_1BASED to the global settings.
  - added GxB\_\*\_memoryUsage
  - port to Octave: gbmake and gbtest work in Octave7 to build and test the @GrB interface to GraphBLAS. Octave 7.0.0 is required.
- Version 5.0.6 (May 24, 2021)
  - BFS and triangle counting demos removed from GraphBLAS/Demo: see LAGraph for these algorithms. Eventually, all of Graph-BLAS/Demo will be deleted, once LAGraph includes all the methods included there.
- Version 5.0.5 (May 17, 2021)

- (26) performance bug fix: reduce-to-vector where A is hypersparse CSR with a transposed descriptor (or CSC with no transpose), and some cases for GrB\_mxm/mxv/vxm when computing C=A\*B with A hypersparse CSC and B bitmap/full (or A bitmap/full and B hypersparse CSR), the wrong internal method was being selected via the auto-selection strategy, resulting in a significant slowdown in some cases.
- Version 5.0.4 (May 13, 2021)
  - @GrB Octave/MATLAB interface: changed license to GNU General Public License v3.0 or later.
- Version 5.0.3 (May 12, 2021)
  - (25) bug fix: disabling ANY\_PAIR semirings by editting Source/GB\_control.h would cause a segfault if those disabled semirings were used.
  - demos are no longer built by default
  - (24) bug fix: new functions in v5.0.2 not declared as extern in GraphBLAS.h.
  - GrB\_Matrix\_reduce\_BinaryOp reinstated from v4.0.3; same limit on built-in ops that correspond to known monoids.
- Version 5.0.2 (May 5, 2021)
  - (23) bug fix: GrB\_Matrix\_apply\_BinaryOp1st and 2nd were using the wrong descriptors for GrB\_INPO and GrB\_INP1. Caught by Erik Welch, Anaconda.
  - memory pool added for faster allocation/free of small blocks
  - QGrB interface ported to MATLAB R2021a.
  - ${\tt GxB\_PRINTF}$  and  ${\tt GxB\_FLUSH}$  global options added.
  - GxB\_Matrix\_diag: construct a diagonal matrix from a vector
  - GxB\_Vector\_diag: extract a diagonal from a matrix
  - concat/split: methods to concatenate and split matrices.
  - import/export: size of arrays now in bytes, not entries. This change is required for better internal memory management, and it is not backward compatible with the GxB\*import/export functions in v4.0. A new parameter, is\_uniform, has been added to

- all import/export methods, which indicates that the matrix values are all the same.
- (22) bug fix: SIMD vectorization was missing reduction(+,task\_cnvals) in GB\_dense\_subassign\_06d\_template.c. Caught by Jeff Huang, Texas A&M, with his software package for race-condition detection.
- GrB\_Matrix\_reduce\_BinaryOp: removed. Use a monoid instead,
   with GrB\_reduce or GrB\_Matrix\_reduce\_Monoid.
- Version 4.0.3 (Jan 19, 2021)
  - faster min/max monoids
  - G=GrB(G) converts G from v3 object to v4
- Version 4.0.2 (Jan 13, 2021)
  - ability to load \*.mat files saved with the v3 GrB
- Version 4.0.1 (Jan 4, 2021)
  - significant performance improvements: compared with v3.3.3, up to 5x faster in breadth-first-search (using LAGraph\_bfs\_parent2), and 2x faster in Betweenness-Centrality (using LAGraph\_bc\_batch5).
  - GrB\_wait(void), with no inputs: removed
  - GrB\_wait(&object): polymorphic function added
  - GrB\_\*\_nvals: no longer guarantees completion; use GrB\_wait(&object)
    or non-polymorphic GrB\_\*\_wait (&object) instead
  - GrB\_error: now has two parameters: a string (char \*\*) and an object.
  - GrB\_Matrix\_reduce\_BinaryOp limited to built-in operators that correspond to known monoids.
  - GrB\_\*\_extractTuples: may return indices out of order
  - removed internal features: GBI iterator, slice and hyperslice matrices
  - bitmap/full matrices and vectors added
  - positional operators and semirings: GxB\_FIRSTI\_INT32 and related ops

- jumbled matrices: sort left pending, like zombies and pending tuples
- GxB\_get/set: added GxB\_SPARSITY\_\* (hyper, sparse, bitmap, or full) and GxB\_BITMAP\_SWITCH.
- GxB\_HYPER: enum renamed to GxB\_HYPER\_SWITCH
- GxB\*import/export: API modified
- GxB\_SelectOp: nrows and ncols removed from function signature.
- OpenMP tasking removed from mergesort and replaced with parallel for loops. Just as fast on Linux/Mac; now the performance ports to Windows.
- GxB\_BURBLE added as a supported feature. This was an undocumented feature of prior versions.
- bug fix: A({lo,hi})=scalar A(lo:hi)=scalar was OK
- Version 3.3.3 (July 14, 2020). Bug fix: w<m>=A\*u with mask non-empty and u empty.
- Version 3.3.2 (July 3, 2020). Minor changes to build system.
- Version 3.3.1 (June 30, 2020). Bug fix to GrB\_assign and GxB\_subassign when the assignment is simple (C=A) but with typecasting.
- Version 3.3.0 (June 26, 2020). Compliant with V1.3 of the C API (except that the polymorphic GrB\_wait(&object) doesn't appear yet; it will appear in V4.0).

Added complex types (GxB\_FC32 and GxB\_FC64), many unary operators, binary operators, monoids, and semirings. Added bitwise operators, and their monoids and semirings. Added the predefined monoids and semirings from the v1.3 spec. @GrB interface: added complex matrices and operators, and changed behavior of integer operations to more closely match the behavior on built-in integer matrices. The rules for typecasting large floating point values to integers has changed. The specific object-based GrB\_Matrix\_wait, GrB\_Vector\_wait, etc, functions have been added. The no-argument GrB\_wait() is deprecated. Added GrB\_getVersion, GrB\_Matrix\_resize, GrB\_Vector\_resize, GrB\_kronecker, GrB\_\*\_wait, scalar binding with binary operators for GrB\_apply, GrB\_Matrix\_removeElement, and GrB\_Vector\_removeElement.

- Version 3.2.0 (Feb 20, 2020). Faster GrB\_mxm, GrB\_mxv, and GrB\_vxm, and faster operations on dense matrices/vectors. Removed compiletime user objects (GxB\_\*\_define), since these were not compatible with the faster matrix operations. Added the ANY and PAIR operators. Added the predefined descriptors, GrB\_DESC\_\*. Added the structural mask option. Changed default chunk size to 65,536. @GrB interface modified: GrB.init is now optional.
- Version 3.1.2 (Dec, 2019). Changes to allow SuiteSparse:GraphBLAS to be compiled with the Microsoft Visual Studio compiler. This compiler does not support the \_Generic keyword, so the polymorphic functions are not available. Use the equivalent non-polymorphic functions instead, when compiling GraphBLAS with MS Visual Studio. In addition, variable-length arrays are not supported, so user-defined types are limited to 128 bytes in size. These changes have no effect if you have an ANSI C11 compliant compiler.

**QGrB** interface modified: **GrB**.init is now required.

- Version 3.1.0 (Oct 1, 2019). @GrB interface added. See the GraphBLAS/GraphBLAS folder for details and documentation, and Section 3.1.
- Version 3.0 (July 26, 2019), with OpenMP parallelism.

The version number is increased to 3.0, since this version is not backward compatible with V2.x. The GxB\_select operation changes; the Thunk parameter was formerly a const void \* pointer, and is now a GxB\_Scalar. A new parameter is added to GxB\_SelectOp\_new, to define the expected type of Thunk. A new parameter is added to GxB\_init, to specify whether or not the user-provided memory management functions are thread safe.

The remaining changes add new features, and are upward compatible with V2.x. The major change is the addition of OpenMP parallelism. This addition has no effect on the API, except that round-off errors can differ with the number of threads used, for floating-point types. GxB\_set can optionally define the number of threads to use (the default is omp\_get\_max\_threads). The number of threads can also defined globally, and/or in the GrB\_Descriptor. The RDIV and

RMINUS operators are added, which are defined as f(x,y) = y/x and f(x,y) = y-x, respectively. Additional options are added to GxB\_get.

- Version 2.3.3 (May 2019): Collected Algorithm of the ACM. No changes from V2.3.2 other than the documentation.
- Version 2.3 (Feb 2019) improves the performance of many GraphBLAS operations, including an early-exit for monoids. These changes have a significant impact on breadth-first-search (a performance bug was also fixed in the two BFS Demo codes). The matrix and vector import/export functions were added (Section 6.11), in support of the new LAGraph project (https://github.com/GraphBLAS/LAGraph, see also Section 13.1). LAGraph includes a push-pull BFS in GraphBLAS that is faster than two versions in the Demo folder. GxB\_init was added to allow the memory manager functions (malloc, etc) to be specified.
- Version 2.2 (Nov 2018) adds user-defined objects at compile-time, via user \*.m4 files placed in GraphBLAS/User, which use the GxB\_\*\_define macros (NOTE: feature removed in v3.2). The default matrix format is now GxB\_BY\_ROW. Also added are the GxB\_\*print methods for printing the contents of each GraphBLAS object (Section 11). PageRank demos have been added to the Demos folder.
- Version 2.1 (Oct 2018) was a major update with support for new matrix formats (by row or column, and hypersparse matrices), and colon notation (I=begin:end or I=begin:inc:end). Some graph algorithms are more naturally expressed with matrices stored by row, and this version includes the new GxB\_BY\_ROW format. The default format in Version 2.1 and prior versions is by column. New extensions to Graph-BLAS in this version include GxB\_get, GxB\_set, and GxB\_AxB\_METHOD, GxB\_RANGE, GxB\_STRIDE, and GxB\_BACKWARDS, and their related definitions, described in Sections 6.13, 8, and 9.
- Version 2.0 (March 2018) addressed changes in the GraphBLAS C API Specification and added GxB\_kron and GxB\_resize.
- Version 1.1 (Dec 2017) primarily improved the performance.
- Version 1.0 was released on Nov 25, 2017.

### 1.1.1 Regarding historical and deprecated functions and symbols

When a GxB\* function or symbol is added to the C API Specification with a GrB\* name, the new GrB\* name should be used instead, if possible. However, the old GxB\* name will be kept as long as possible for historical reasons. Historical functions and symbols will not always be documented here in the SuiteSparse:GraphBLAS User Guide, but they will be kept in GraphbBLAS.h and kept in good working order in the library. Historical functions and symbols would only be removed in the very unlikely case that they cause a serious conflict with future methods.

The only methods that have been fully deprecated and removed are the older versions of <code>GrB\_wait</code> and <code>GrB\_error</code> methods, which are incompatible with the latest versions.

# 2 Basic Concepts

Since the *GraphBLAS C API Specification* provides a precise definition of GraphBLAS, not every detail of every function is provided here. For example, some error codes returned by GraphBLAS are self-explanatory, but since a specification must precisely define all possible error codes a function can return, these are listed in detail in the *GraphBLAS C API Specification*. However, including them here is not essential and the additional information on the page might detract from a clearer view of the essential features of the GraphBLAS functions.

This User Guide also assumes the reader is familiar with Octave/MATLAB. MATLAB supports only the conventional plus-times semiring on sparse double and complex matrices, but a MATLAB-like notation easily extends to the arbitrary semirings used in GraphBLAS. The matrix multiplication in the example in the Introduction can be written in MATLAB notation as C=A\*B, if the Boolean OR-AND semiring is understood. Relying on a MATLAB-like notation allows the description in this User Guide to be expressive, easy to understand, and terse at the same time. The GraphBLAS C API Specification also makes use of some MATLAB-like language, such as the colon notation.

MATLAB notation will always appear here in fixed-width font, such as C=A\*B(:,j). In standard mathematical notation it would be written as the matrix-vector multiplication  $C=Ab_j$  where  $b_j$  is the jth column of the matrix B. The GraphBLAS standard is a C API and SuiteSparse:GraphBLAS is written in C, and so a great deal of C syntax appears here as well, also in fixed-width font. This User Guide alternates between all three styles as needed.

## 2.1 Graphs and sparse matrices

Graphs can be huge, with many nodes and edges. A dense adjacency matrix  $\mathbf{A}$  for a graph of n nodes takes  $O(n^2)$  memory, which is impossible if n is, say, a million. Let  $|\mathbf{A}|$  denote the number of entries in a matrix. Most graphs arising in practice are sparse, however, with only  $|\mathbf{A}| = O(n)$  edges, where  $|\mathbf{A}|$  denotes the number of edges in the graph, or the number of explicit entries present in the data structure for the matrix  $\mathbf{A}$ . Sparse graphs with millions of nodes and edges can easily be created by representing them as sparse matrices, where only explicit values need to be stored. Some graphs

are hypersparse, with  $|\mathbf{A}| \ll n$ . SuiteSparse:GraphBLAS supports two kinds of sparse matrix formats: a regular sparse format, taking  $O(n + |\mathbf{A}|)$  space, and a hypersparse format taking only  $O(|\mathbf{A}|)$  space. As a result, creating a sparse matrix of size n-by-n where  $n = 2^{60}$  (about  $10^{18}$ ) can be done on quite easily on a commodity laptop, limited only by  $|\mathbf{A}|$ .

A sparse matrix data structure only stores a subset of the possible  $n^2$  entries, and it assumes the values of entries not stored have some implicit value. In conventional linear algebra, this implicit value is zero, but it differs with different semirings. Explicit values are called *entries* and they appear in the data structure. The *pattern* of a matrix defines where its explicit entries appear. It will be referenced in one of two equivalent ways. It can be viewed as a set of indices (i, j), where (i, j) is in the pattern of a matrix **A** if  $\mathbf{A}(i, j)$  is an explicit value. It can also be viewed as a Boolean matrix **S** where  $\mathbf{S}(i, j)$  is true if (i, j) is an explicit entry and false otherwise. In MATLAB notation,  $\mathbf{S=spones}(\mathbf{A})$  or  $\mathbf{S=(A^{\sim}=0)}$ , if the implicit value is zero. The (i, j) pairs, and their values, can also be extracted from the matrix via the MATLAB expression  $[\mathbf{I}, \mathbf{J}, \mathbf{X}] = \mathbf{find}(\mathbf{A})$ , where the kth tuple  $(\mathbf{I}(\mathbf{k}), \mathbf{J}(\mathbf{k}), \mathbf{X}(\mathbf{k}))$  represents the explicit entry  $\mathbf{A}(\mathbf{I}(\mathbf{k}), \mathbf{J}(\mathbf{k}))$ , with numerical value  $\mathbf{X}(\mathbf{k})$  equal to  $a_{ij}$ , with row index  $i=\mathbf{I}(\mathbf{k})$  and column index  $j=\mathbf{J}(\mathbf{k})$ .

The entries in the pattern of **A** can take on any value, including the implicit value, whatever it happens to be. This differs slightly from MAT-LAB, which always drops all explicit zeros from its sparse matrices. This is a minor difference but it cannot be done in GraphBLAS. For example, in the max-plus tropical algebra, the implicit value is negative infinity, and zero has a different meaning. Here, the MATLAB notation used will assume that no explicit entries are ever dropped because their explicit value happens to match the implicit value.

Graph Algorithms in the Language on Linear Algebra, Kepner and Gilbert, eds., provides a framework for understanding how graph algorithms can be expressed as matrix computations [KG11]. For additional background on sparse matrix algorithms, see also [Dav06] and [DRSL16].

## 2.2 Overview of GraphBLAS methods and operations

GraphBLAS provides a collection of *methods* to create, query, and free its of objects: sparse matrices, sparse vectors, scalars, types, operators, monoids, semirings, and a descriptor object used for parameter settings. Details are given in Section 6. Once these objects are created they can be used in

mathematical operations (not to be confused with the how the term operator is used in GraphBLAS). A short summary of these operations and their nearest Octave/MATLAB analog is given in the table below.

operation	approximate Octave/MATLAB analog
matrix multiplication	C=A*B
element-wise operations	C=A+B and C=A.*B
reduction to a vector or scalar	s=sum(A)
apply unary operator	C=-A
transpose	C=A'
submatrix extraction	C=A(I,J)
submatrix assignment	C(I,J)=A
select	C=tril(A)

GraphBLAS can do far more than what Octave/MATLAB can do in these rough analogs, but the list provides a first step in describing what GraphBLAS can do. Details of each GraphBLAS operation are given in Section 10. With this brief overview, the full scope of GraphBLAS extensions of these operations can now be described.

GraphBLAS has 13 built-in scalar types: Boolean, single and double precision floating-point (real and complex), and 8, 16, 32, and 64-bit signed and unsigned integers. In addition, user-defined scalar types can be created from nearly any C typedef, as long as the entire type fits in a fixed-size contiguous block of memory (of arbitrary size). All of these types can be used to create GraphBLAS sparse matrices, vectors, or scalars.

The scalar addition of conventional matrix multiplication is replaced with a monoid. A monoid is an associative and commutative binary operator z=f(x,y) where all three domains are the same (the types of x, y, and z), and where the operator has an identity value id such that f(x,id)=f(id,x)=x. Performing matrix multiplication with a semiring uses a monoid in place of the "add" operator, scalar addition being just one of many possible monoids. The identity value of addition is zero, since x + 0 = 0 + x = x. Graph-BLAS includes many built-in operators suitable for use as a monoid: min (with an identity value of positive infinity), max (whose identity is negative infinity), add (identity is zero), multiply (with an identity of one), four logical operators: AND, OR, exclusive-OR, and Boolean equality (XNOR), four bitwise operators (AND, OR, XOR, and XNOR), and the ANY operator. User-created monoids can be defined with any associative and commutative operator that has an identity value.

Finally, a semiring can use any built-in or user-defined binary operator z=f(x,y) as its "multiply" operator, as long as the type of its output, z matches the type of the semiring's monoid. The user application can create any semiring based on any types, monoids, and multiply operators, as long these few rules are followed.

Just considering built-in types and operators, GraphBLAS can perform C=A\*B in thousands of unique semirings. With typecasting, any of these semirings can be applied to matrices C, A, and B of 13 predefined types, in any combination. This results in millions of possible kinds of sparse matrix multiplication supported by GraphBLAS, and this is counting just built-in types and operators. By contrast, MATLAB provides just two semirings for its sparse matrix multiplication C=A\*B: plus-times-double and plus-times-complex, not counting the typecasting that MATLAB does when multiplying a real matrix times a complex matrix.

A monoid can also be used in a reduction operation, like s=sum(A) in MATLAB. MATLAB provides the plus, times, min, and max reductions of a real or complex sparse matrix as s=sum(A), s=prod(A), s=min(A), and s=max(A), respectively. In GraphBLAS, any monoid can be used (min, max, plus, times, AND, OR, exclusive-OR, equality, bitwise operators, or any user-defined monoid on any user-defined type).

Element-wise operations are also expanded from what can be done in MATLAB. Consider matrix addition, C=A+B in MATLAB. The pattern of the result is the set union of the pattern of A and B. In GraphBLAS, any binary operator can be used in this set-union "addition." The operator is applied to entries in the intersection. Entries in A but not B, or visa-versa, are copied directly into C, without any application of the binary operator. The accumulator operation for  $\mathbf{Z} = \mathbf{C} \odot \mathbf{T}$  described in Section 2.3 is one example of this set-union application of an arbitrary binary operator.

Consider element-wise multiplication, C=A.\*B in MATLAB. The operator (multiply in this case) is applied to entries in the set intersection, and the pattern of C just this set intersection. Entries in A but not B, or visa-versa, do not appear in C. In GraphBLAS, any binary operator can be used in this manner, not just scalar multiplication. The difference between element-wise "add" and "multiply" is not the operators, but whether or not the pattern of the result is the set union or the set intersection. In both cases, the operator is only applied to the set intersection.

Finally, GraphBLAS includes a *non-blocking* mode where operations can be left pending, and saved for later. This is very useful for submatrix as-

signment (C(I,J)=A where I and J are integer vectors), or scalar assignment (C(i,j)=x where i and j are scalar integers). Because of how MATLAB stores its matrices, adding and deleting individual entries is very costly. For example, this is very slow in MATLAB, taking  $O(nz^2)$  time:

```
A = sparse (m,n);  % an empty sparse matrix
for k = 1:nz
    compute a value x, row index i, and column index j
    A (i,j) = x;
end
```

The above code is very easy read and simple to write, but exceedingly slow. In MATLAB, the method below is preferred and is far faster, taking at most  $O(|\mathbf{A}| \log |\mathbf{A}| + n)$  time. It can easily be a million times faster than the method above. Unfortunately the second method below is a little harder to read and a little less natural to write:

GraphBLAS can do both methods. SuiteSparse:GraphBLAS stores its matrices in a format that allows for pending computations, which are done later in bulk, and as a result it can do both methods above equally as fast as the MATLAB sparse function, allowing the user to write simpler code.

#### 2.3 The accumulator and the mask

Most GraphBLAS operations can be modified via transposing input matrices, using an accumulator operator, applying a mask or its complement, and by clearing all entries the matrix C after using it in the accumulator operator but before the final results are written back into it. All of these steps are optional, and are controlled by a descriptor object that holds parameter settings (see Section 6.13) that control the following options:

- the input matrices A and/or B can be transposed first.
- an accumulator operator can be used, like the plus in the statement C=C+A\*B. The accumulator operator can be any binary operator, and an element-wise "add" (set union) is performed using the operator.
- an optional mask can be used to selectively write the results to the output. The mask is a sparse Boolean matrix Mask whose size is the same size as the result. If Mask(i,j) is true, then the corresponding entry in the output can be modified by the computation. If Mask(i,j) is false, then the corresponding in the output is protected and cannot be modified by the computation. The Mask matrix acts exactly like logical matrix indexing in MATLAB, with one minor difference: in GraphBLAS notation, the mask operation is  $C\langle M \rangle = Z$ , where the mask M appears only on the left-hand side. In MATLAB, it would appear on both sides as C(Mask) = Z(Mask). If no mask is provided, the Mask matrix is implicitly all true. This is indicated by passing the value GrB\_NULL in place of the Mask argument in GraphBLAS operations.

This process can be described in mathematical notation as:

```
\mathbf{A} = \mathbf{A}^{\mathsf{T}}, if requested via descriptor (first input option)

\mathbf{B} = \mathbf{B}^{\mathsf{T}}, if requested via descriptor (second input option)
```

T is computed according to the specific operation

 $\mathbf{C}\langle \mathbf{M} \rangle = \mathbf{C} \odot \mathbf{T}$ , accumulating and writing the results back via the mask

The application of the mask and the accumulator operator is written as  $\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{C}\odot\mathbf{T}$  where  $\mathbf{Z} = \mathbf{C}\odot\mathbf{T}$  denotes the application of the accumulator operator, and  $\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{Z}$  denotes the mask operator via the Boolean matrix  $\mathbf{M}$ . The Accumulator Phase,  $\mathbf{Z} = \mathbf{C}\odot\mathbf{T}$ , is performed as follows:

```
 \begin{aligned} \textbf{Accumulator Phase} \colon & \text{compute } \mathbf{Z} = \mathbf{C} \odot \mathbf{T} ; \\ & \text{if accum is NULL} \\ & \mathbf{Z} = \mathbf{T} \\ & \text{else} \\ & \mathbf{Z} = \mathbf{C} \odot \mathbf{T} \end{aligned}
```

The accumulator operator is  $\odot$  in GraphBLAS notation, or accum in the code. The pattern of  $\mathbb{C} \odot \mathbb{T}$  is the set union of the patterns of  $\mathbb{C}$  and  $\mathbb{T}$ , and the operator is applied only on the set intersection of  $\mathbb{C}$  and  $\mathbb{T}$ . Entries in neither the pattern of  $\mathbb{C}$  nor  $\mathbb{T}$  do not appear in the pattern of  $\mathbb{Z}$ . That is:

```
for all entries (i, j) in \mathbf{C} \cap \mathbf{T} (that is, entries in both \mathbf{C} and \mathbf{T})
z_{ij} = c_{ij} \odot t_{ij}
for all entries (i, j) in \mathbf{C} \setminus \mathbf{T} (that is, entries in \mathbf{C} but not \mathbf{T})
z_{ij} = c_{ij}
for all entries (i, j) in \mathbf{T} \setminus \mathbf{C} (that is, entries in \mathbf{T} but not \mathbf{C})
z_{ij} = t_{ij}
```

The Accumulator Phase is followed by the Mask/Replace Phase,  $\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{Z}$  as controlled by the  $\mathtt{GrB\_REPLACE}$  and  $\mathtt{GrB\_COMP}$  descriptor options:

```
\begin{aligned} \mathbf{Mask/Replace\ Phase:\ compute\ C\langle M\rangle = Z:} \\ & \text{if\ } (\mathtt{GrB\_REPLACE}) \ \text{delete\ all\ entries\ in\ C} \\ & \text{if\ } \mathtt{Mask\ is\ NULL} \\ & \text{if\ } (\mathtt{GrB\_COMP}) \\ & \mathbf{C} \ \text{is\ not\ modified} \\ & \text{else} \\ & \mathbf{C} = \mathbf{Z} \\ & \text{else} \\ & \text{if\ } (\mathtt{GrB\_COMP}) \\ & \mathbf{C\langle \neg M\rangle = Z} \\ & \text{else} \\ & \mathbf{C\langle M\rangle = Z} \end{aligned}
```

Both phases of the accum/mask process are illustrated in MATLAB notation in Figure 1. A GraphBLAS operation starts with its primary computation, producing a result T; for matrix multiply, T=A\*B, or if A is transposed first, T=A'\*B, for example. Applying the accumulator, mask (or its complement) to obtain the final result matrix C can be expressed in the MATLAB accum\_mask function shown in the figure. This function is an exact, fully functional, and nearly-complete description of the GraphBLAS accumulator/mask operation. The only aspects it does not consider are typecasting (see Section 2.4), and the value of the implicit identity (for those, see another version in the Test folder).

One aspect of GraphBLAS cannot be as easily expressed in a MATLAB sparse matrix: namely, what is the implicit value of entries not in the pattern? To accommodate this difference in the accum\_mask MATLAB function, each sparse matrix A is represented with its values A.matrix and its pattern, A.pattern. The latter could be expressed as the sparse matrix A.pattern=spones(A) or A.pattern=(A~=0) in MATLAB, if the implicit

```
function C = accum_mask (C, Mask, accum, T, C_replace, Mask_complement)
[m n] = size (C.matrix) ;
Z.matrix = zeros (m, n) ;
Z.pattern = false (m, n) ;
if (isempty (accum))
   Z = T;
               % no accum operator
  % Z = accum (C,T), like Z=C+T but with an binary operator, accum
  p = C.pattern & T.pattern; Z.matrix (p) = accum (C.matrix (p), T.matrix (p));
  p = C.pattern & ~T.pattern; Z.matrix (p) = C.matrix (p);
  p = ~C.pattern & T.pattern ; Z.matrix (p) = T.matrix (p) ;
  Z.pattern = C.pattern | T.pattern ;
end
% = 1000 apply the mask to the values and pattern
C.matrix = mask (C.matrix, Mask, Z.matrix, C_replace, Mask_complement);
C.pattern = mask (C.pattern, Mask, Z.pattern, C_replace, Mask_complement) ;
end
function C = mask (C, Mask, Z, C_replace, Mask_complement)
% replace C if requested
if (C_replace)
   C(:,:) = 0;
if (isempty (Mask))
                                % if empty, Mask is implicit ones(m,n)
  % implicitly, Mask = ones (size (C))
   if (~Mask_complement)
     C = Z;
                                % this is the default
   else
                                % Z need never have been computed
     C = C;
   end
else
   % apply the mask
   if (~Mask_complement)
     C (Mask) = Z (Mask);
   else
     C (^{\sim}Mask) = Z (^{\sim}Mask);
   end
end
end
```

Figure 1: Applying the mask and accumulator,  $\mathbf{C}\langle \mathbf{M} \rangle = \mathbf{C} \odot \mathbf{T}$ 

value is zero. With different semirings, entries not in the pattern can be 1, +Inf, -Inf, or whatever is the identity value of the monoid. As a result, Figure 1 performs its computations on two MATLAB matrices: the values in A.matrix and the pattern in the logical matrix A.pattern. Implicit values are untouched.

The final computation in Figure 1 with a complemented Mask is easily expressed in MATLAB as C(~Mask)=Z(~Mask) but this is costly if Mask is very sparse (the typical case). It can be computed much faster in MATLAB without complementing the sparse Mask via:

```
R = Z; R (Mask) = C (Mask); C = R;
```

A set of MATLAB functions that precisely compute the  $\mathbf{C}\langle\mathbf{M}\rangle=\mathbf{C}\odot\mathbf{T}$  operation according to the full GraphBLAS specification is provided in Suite-Sparse:GraphBLAS as  $\mathtt{GB\_spec\_accum.m}$ , which computes  $\mathbf{Z}=\mathbf{C}\odot\mathbf{T}$ , and  $\mathtt{GB\_spec\_mask.m}$ , which computes  $\mathbf{C}\langle\mathbf{M}\rangle=\mathbf{Z}$ . SuiteSparse:GraphBLAS includes a complete list of  $\mathtt{GB\_spec\_*}$  functions that illustrate every GraphBLAS operation.

The methods in Figure 1 rely heavily on MATLAB's logical matrix indexing. For those unfamiliar with logical indexing in MATLAB, here is short summary. Logical matrix indexing in MATLAB is written as A(Mask) where A is any matrix and Mask is a logical matrix the same size as A. The expression x=A(Mask) produces a column vector x consisting of the entries of A where Mask is true. On the left-hand side, logical submatrix assignment A(Mask)=x does the opposite, copying the components of the vector x into the places in A where Mask is true. For example, to negate all values greater than 10 using logical indexing in MATLAB:

```
>> A = magic (4)
A =
    16
                    3
                          13
     5
           11
                   10
                           8
            7
                    6
                          12
      4
           14
                   15
>> A (A>10) =
                  A (A>10)
A =
   -16
             2
                    3
                         -13
                   10
     5
          -11
                           8
     9
            7
                    6
                         -12
          -14
                 -15
                           1
```

In MATLAB, logical indexing with a sparse matrix A and sparse logical matrix Mask is a built-in method. The Mask operator in GraphBLAS works identically as sparse logical indexing in MATLAB, but is typically far faster in SuiteSparse:GraphBLAS than the same operation using MATLAB sparse matrices.

### 2.4 Typecasting

If an operator z=f(x) or z=f(x,y) is used with inputs that do not match its inputs x or y, or if its result z does not match the type of the matrix it is being stored into, then the values are typecasted. Typecasting in Graph-BLAS extends beyond just operators. Almost all GraphBLAS methods and operations are able to typecast their results, as needed.

If one type can be typecasted into the other, they are said to be *compatible*. All built-in types are compatible with each other. GraphBLAS cannot typecast user-defined types thus any user-defined type is only compatible with itself. When GraphBLAS requires inputs of a specific type, or when one type cannot be typecast to another, the GraphBLAS function returns an error code, Grb\_DOMAIN\_MISMATCH (refer to Section 5.6 for a complete list of error codes). Typecasting can only be done between built-in types, and it follows the rules of the ANSI C language (not MATLAB) wherever the rules of ANSI C are well-defined.

However, unlike MATLAB, the ANSI C11 language specification states that the results of typecasting a float or double to an integer type is not always defined. In SuiteSparse:GraphBLAS, whenever C leaves the result undefined the rules used in MATLAB are followed. In particular +Inf converts to the largest integer value, -Inf converts to the smallest (zero for unsigned integers), and NaN converts to zero. Positive values outside the range of the integer are converted to the largest positive integer, and negative values less than the most negative integer are converted to that most negative integer. Other than these special cases, SuiteSparse:GraphBLAS trusts the C compiler for the rest of its typecasting.

Typecasting to bool is fully defined in the C language specification, even for NaN. The result is false if the value compares equal to zero, and true otherwise. Thus NaN converts to true. This is unlike MATLAB, which does not allow a typecast of a NaN to the MATLAB logical type.

**SPEC:** the GraphBLAS API C Specification states that typecasting follows the rules of ANSI C. Yet C leaves some typecasting undefined. All typecasting between built-in types in SuiteSparse:GraphBLAS is precisely defined, as an extension to the spec.

## 2.5 Notation and list of GraphBLAS operations

As a summary of what GraphBLAS can do, the following table lists all GraphBLAS operations. Upper case letters denote a matrix, lower case letters are vectors, and  ${\bf AB}$  denote the multiplication of two matrices over a semiring.

GrB_mxm	matrix-matrix multiply	$\mathbf{C}\langle\mathbf{M} angle=\mathbf{C}\odot\mathbf{AB}$
${\tt GrB\_vxm}$	vector-matrix multiply	$\mathbf{w}^T \langle \mathbf{m}^T  angle = \mathbf{w}^T \odot \mathbf{u}^T \mathbf{A}$
<pre>GrB_mxv</pre>	matrix-vector multiply	$\mathbf{w}\langle\mathbf{m}\rangle=\mathbf{w}\odot\mathbf{A}\mathbf{u}$
GrB_eWiseMult	element-wise,	$\mathbf{C}\langle \mathbf{M} \rangle = \mathbf{C} \odot (\mathbf{A} \otimes \mathbf{B})$
	set intersection	$\mathbf{w}\langle\mathbf{m}\rangle=\mathbf{w}\odot(\mathbf{u}\otimes\mathbf{v})$
GrB_eWiseAdd	element-wise,	$\mathbf{C}\langle \mathbf{M} \rangle = \mathbf{C} \odot (\mathbf{A} \oplus \mathbf{B})$
	set union	$\mathbf{w}\langle\mathbf{m} angle=\mathbf{w}\odot(\mathbf{u}\oplus\mathbf{v})$
GrB_extract	extract submatrix	$\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{C}\odot\mathbf{A}(\mathbf{I},\mathbf{J})$
		$\mathbf{w}\langle\mathbf{m} angle=\mathbf{w}\odot\mathbf{u}(\mathbf{i})$
GxB_subassign	assign submatrix	$\mathbf{C}(\mathbf{I},\mathbf{J})\langle\mathbf{M} angle = \mathbf{C}(\mathbf{I},\mathbf{J})\odot\mathbf{A}$
	(with submask for $\mathbf{C}(\mathbf{I}, \mathbf{J})$ )	$\mathbf{w}(\mathbf{i})\langle\mathbf{m}\rangle=\mathbf{w}(\mathbf{i})\odot\mathbf{u}$
GrB_assign	assign submatrix	$\mathbf{C}\langle\mathbf{M} angle(\mathbf{I},\mathbf{J})=\mathbf{C}(\mathbf{I},\mathbf{J})\odot\mathbf{A}$
	(with mask for $\mathbf{C}$ )	$\mathbf{w}\langle\mathbf{m} angle(\mathbf{i})=\mathbf{w}(\mathbf{i})\odot\mathbf{u}$
<pre>GrB_apply</pre>	apply unary operator	$\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{C}\odot f(\mathbf{A})$
		$\mathbf{w}\langle\mathbf{m} angle=\mathbf{w}\odot f(\mathbf{u})$
	apply binary operator	$\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{C}\odot f(\mathbf{A}, y)$
		$\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{C}\odot f(x,\mathbf{A})$
		$\mathbf{w}\langle\mathbf{m}\rangle = \mathbf{w}\odot f(\mathbf{u}, y)$
		$\mathbf{w}\langle\mathbf{m}\rangle = \mathbf{w}\odot f(x, \mathbf{u})$
	apply index-unary op	$\mathbf{C}\langle\mathbf{M} angle=\mathbf{C}\odot f(\mathbf{A}),\mathbf{i},\mathbf{j},\mathbf{s}$
		$\mathbf{w}\langle\mathbf{m}\rangle = \mathbf{w}\odot f(\mathbf{u}), \mathbf{i}, 0, \mathbf{s}$
<pre>GrB_select</pre>	apply select operator	$\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{C}\odot f(\mathbf{A},k)$
		$\mathbf{w}\langle\mathbf{m}\rangle = \mathbf{w}\odot f(\mathbf{u},k)$
<pre>GrB_reduce</pre>	reduce to vector	$\mathbf{w}\langle\mathbf{m}\rangle = \mathbf{w}\odot[\oplus_{j}\mathbf{A}(:,j)]$
	reduce to scalar	$s = s \odot [\oplus_{ij} \mathbf{A}(i,j)]$
GrB_transpose	transpose	$\mathbf{C}\langle\mathbf{M} angle=\mathbf{C}\odot\mathbf{A}^T$
GrB_kronecker	Kronecker product	$\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{C}\odot \mathrm{kron}(\mathbf{A},\mathbf{B})$

Each operation takes an optional  $GrB_Descriptor$  argument that modifies the operation. The input matrices A and B can be optionally transposed, the mask M can be complemented, and C can be cleared of its entries after it is used in  $Z = C \odot T$  but before the  $C\langle M \rangle = Z$  assignment. Vectors are

never transposed via the descriptor.

Let  $\mathbf{A} \oplus \mathbf{B}$  denote the element-wise operator that produces a set union pattern (like A+B in MATLAB). Any binary operator can be used this way in GraphBLAS, not just plus. Let  $\mathbf{A} \otimes \mathbf{B}$  denote the element-wise operator that produces a set intersection pattern (like A.\*B in MATLAB); any binary operator can be used this way, not just times.

Reduction of a matrix **A** to a vector reduces the *i*th row of **A** to a scalar  $w_i$ . This is like w=sum(A') since by default, MATLAB reduces down the columns, not across the rows.

# 3 Interfaces to Octave, MATLAB, Python, Julia, Java

The Octave/MATLAB interface to SuiteSparse:GraphBLAS is included with this distribution, described in Section 3.1. It is fully polished, and fully tested, but does have some limitations that will be addressed in future releases. A beta version of a Python interface is now available, as is a Julia interface. These are not part of the SuiteSparse:GraphBLAS distribution. See the links below (see Sections 3.2 and 3.3).

## 3.1 Octave/MATLAB Interface

An easy-to-use Octave/MATLAB interface for SuiteSparse:GraphBLAS is available; see the documentation in the GraphBLAS/GraphBLAS folder for details. Start with the README.md file in that directory. An easy-to-read output of the MATLAB demos can be found in GraphBLAS/GraphBLAS/demo/html.

The Octave/MATLAB interface adds the @GrB class, which is an opaque Octave/MATLAB object that contains a GraphBLAS matrix, either double or single precision (real or complex), boolean, or any of the built-in integer types. Octave/MATLAB sparse and full matrices can be arbitrarily mixed with GraphBLAS matrices. The following overloaded operators and methods all work as you would expect for any matrix. The matrix multiplication A\*B uses the conventional PLUS\_TIMES semiring.

A+B A-B A\*B A.\*B A./B A.\B A.\b A.\b C=A(I,J)   
-A +A 
$$^{\sim}A$$
 A' A.' A&B A|B b\A C(I,J)=A   
A^=B A>B A=B A<=B A>=B A

For a list of overloaded operations and static methods, type methods GrB in Octave/MATLAB, or help GrB for more details.

**Limitations:** Some features for Octave/MATLAB sparse matrices are not yet available for GraphBLAS matrices. Some of these may be added in future releases.

 GrB matrices with dimension larger than 2<sup>53</sup> do not display properly in the whos command. The size is displayed correctly with disp or display.

- Non-blocking mode is not exploited; this would require a Octave/MATLAB mexFunction to modify its inputs, which is technically possible but not permitted by the Octave/MATLAB API. This can have significant impact on performance, if an m-file makes many repeated tiny changes to a matrix. This can be done in the C API but not Octave/MATLAB.
- Linear indexing: A(:) for a 2D matrix, and I=find(A).
- Singleton expansion.
- Dynamically growing arrays, where C(i)=x can increase the size of C.
- Saturating element-wise binary and unary operators for integers. For C=A+B with MATLAB uint8 matrices, results saturate if they exceed 255. This is not compatible with a monoid for C=A\*B, and thus MATLAB does not support matrix-matrix multiplication with uint8 matrices. In GraphBLAS, uint8 addition acts in a modulo fashion.
- Solvers, so that x=A\b could return a GF(2) solution, for example.
- Sparse matrices with dimension higher than 2.

### 3.2 Python Interface

See Michel Pelletier's Python interface at https://github.com/michelp/pygraphblas; it also appears at https://anaconda.org/conda-forge/pygraphblas.

See Jim Kitchen and Erik Welch's (both from Anaconda, Inc.) Python interface at https://github.com/metagraph-dev/grblas. See also https://anaconda.org/conda-forge/graphblas.

Both of them allow for pending work to be left pending in a GrB\_Matrix.

### 3.3 Julia Interface

The Julia interface is at https://github.com/JuliaSparse/SuiteSparseGraphBLAS.jl, developed by Will Kimmerer, Abhinav Mehndiratta, and Viral Shah. Unlike the Octave/MATLAB interface (and like the Python interfaces) the Julia interface can keep pending work (zombies, pending tuples, jumbled state) in a GrB\_Matrix. This makes Python and Julia the best high-level interfaces for SuiteSparse:GraphBLAS. MATLAB is not as well suited, since it does not allow inputs to a function or mexFunction to be modified, so any pending work must be finished before a matrix can be used as input.

### 3.4 Java Interface

Fabian Murariu is working on a Java interface. See https://github.com/fabianmurariu/graphblas-java-native.

# 4 Performance of MATLAB versus Graph-BLAS

MATLAB R2021a includes v3.3 of SuiteSparse:GraphBLAS as a built-in library, but uses it only for C=A\*B when both A and B are sparse. In prior versions of MATLAB, C=A\*B relied on the SFMULT and SSMULT packages in SuiteSparse, which are single-threaded (also written by this author). The GraphBLAS GrB\_mxm is up to 30x faster on a 20-core Intel Xeon, compared with C=A\*B in MATLAB R2020b and earlier. With MATLAB R2021a and later, the performance of C=A\*B when using MATLAB sparse matrices is identical to the performance for GraphBLAS matrices, since the same code is being used by both (GrB\_mxm).

Other methods in GraphBLAS are also faster, some extremely so. In particular, the statement C(M)=A (where M is a logical matrix) takes under a second for a large sparse problem when using GraphBLAS via its @GrB interface. By stark contrast, MATLAB would take about 4 or 5 days, a speedup of about 500,000x. For a smaller problem, GraphBLAS takes 0.4 seconds while MATLAB takes 28 hours (a speedup of about 250,000x). Both cases use the same statement with the same syntax (C(M)=A) and compute exactly the same result. Below are the results for n-by-n matrices in GraphBLAS v5.0.6 and MATLAB R2020a, on a Dell XPS13 laptop (16GB RAM, Intel(R) Core(TM) i7-8565U CPU @ 1.80GHz with 4 hardware cores). GraphBLAS is using 4 threads.

n	nnz(C)	nnz(M)	GraphBLAS (sec)	MATLAB (sec)	speedup
2,048	20,432	2,048	0.005	0.024	4.7
4,096	40,908	4,096	0.003	0.115	39
8,192	81,876	8,191	0.009	0.594	68
16,384	163,789	16,384	0.009	2.53	273
32,768	327,633	32,767	0.014	12.4	864
$65,\!536$	$655,\!309$	65,536	0.025	65.9	2,617
131,072	1,310,677	131,070	0.055	276.2	4,986
262,144	2,621,396	262,142	0.071	1,077	15,172
524,288	5,242,830	524,288	0.114	5,855	51,274
1,048,576	$10,\!485,\!713$	1,048,576	0.197	27,196	137,776
2,097,152	20,971,475	2,097,152	0.406	100,799	248,200
4,194,304	41,942,995	4,194,304	0.855	4 to 5 days?	500,000?

The assignment C(I,J)=A in MATLAB, when using @GrB objects, is up to 1000x faster than the same statement with the same syntax, when using MATLAB sparse matrices instead. Matrix concatenation C = [A B] is about 17 times faster in GraphBLAS, on a 20-core Intel Xeon. For more details, see the GraphBLAS/GraphBLAS/demo folder and its contents.

# 5 GraphBLAS Context and Sequence

A user application that directly relies on GraphBLAS must include the GraphBLAS.h header file:

```
#include "GraphBLAS.h"
```

The GraphBLAS.h file defines functions, types, and macros prefixed with GrB\_ and GxB\_ that may be used in user applications. The prefix GrB\_ denote items that appear in the official *GraphBLAS C API Specification*. The prefix GxB\_ refers to SuiteSparse-specific extensions to the GraphBLAS API.

The GraphBLAS.h file includes all the definitions required to use Graph-BLAS, including the following macros that can assist a user application in compiling and using GraphBLAS.

There are two version numbers associated with SuiteSparse:GraphBLAS: the version of the *GraphBLAS C API Specification* it conforms to, and the version of the implementation itself. These can be used in the following manner in a user application:

```
#if GxB_SPEC_VERSION >= GxB_VERSION (2,0,3)
... use features in GraphBLAS specification 2.0.3 ...
#else
... only use features in early specifications
#endif

#if GxB_IMPLEMENTATION >= GxB_VERSION (5,2,0)
... use features from version 5.2.0 (or later)
of a specific GraphBLAS implementation
#endif
```

SuiteSparse:GraphBLAS also defines the following strings with #define. Refer to the GraphBLAS.h file for details.

Macro	purpose
GxB_IMPLEMENTATION_ABOUT	this particular implementation, copyright, and URL
GxB_IMPLEMENTATION_DATE	the date of this implementation
GxB_SPEC_ABOUT	the GraphBLAS specification for this implementation
GxB_SPEC_DATE	the date of the GraphBLAS specification
GxB_IMPLEMENTATION_LICENSE	the license for this particular implementation

Finally, SuiteSparse:GraphBLAS gives itself a unique name of the form GxB\_SUITESPARSE\_GRAPHBLAS that the user application can use in #ifdef

tests. This is helpful in case a particular implementation provides non-standard features that extend the GraphBLAS specification, such as additional predefined built-in operators, or if a GraphBLAS implementation does not yet fully implement all of the GraphBLAS specification. The Suite-Sparse:GraphBLAS name is provided in its GraphBLAS.h file as:

```
#define GxB_SUITESPARSE_GRAPHBLAS
```

For example, SuiteSparse:GraphBLAS predefines additional built-in operators not in the specification. If the user application wishes to use these in any GraphBLAS implementation, an #ifdef can control when they are used. Refer to the examples in the GraphBLAS/Demo folder.

As another example, the GraphBLAS API states that an implementation need not define the order in which <code>GrB\_Matrix\_build</code> assembles duplicate tuples in its <code>[I,J,X]</code> input arrays. As a result, no particular ordering should be relied upon in general. However, SuiteSparse:GraphBLAS does guarantee an ordering, and this guarantee will be kept in future versions of SuiteSparse:GraphBLAS as well. Since not all implementations will ensure a particular ordering, the following can be used to exploit the ordering returned by SuiteSparse:GraphBLAS.

```
#ifdef GxB_SUITESPARSE_GRAPHBLAS
// duplicates in I, J, X assembled in a specific order;
// results are well-defined even if op is not associative.
GrB_Matrix_build (C, I, J, X, nvals, op);
#else
// duplicates in I, J, X assembled in no particular order;
// results are undefined if op is not associative.
GrB_Matrix_build (C, I, J, X, nvals, op);
#endif
```

The remainder of this section describes GraphBLAS functions that create, modify, and destroy the GraphBLAS context, or provide utility methods for dealing with errors:

GraphBLAS function	purpose	Section
GrB_init	start up GraphBLAS	5.2
<pre>GrB_getVersion</pre>	C API supported by the library	5.3
GxB_init	start up GraphBLAS with different malloc	5.4
<pre>GrB_Info</pre>	status code returned by GraphBLAS functions	5.5
GrB_error	get more details on the last error	5.6
GrB_finalize	finish GraphBLAS	5.7

# 5.1 GrB\_Index: the GraphBLAS integer

Matrix and vector dimensions and indexing rely on a specific integer, GrB\_Index, which is defined in GraphBLAS.h as

```
typedef uint64_t GrB_Index ;
```

Row and column indices of an nrows-by-ncols matrix range from zero to the nrows-1 for the rows, and zero to ncols-1 for the columns. Indices are zero-based, like C, and not one-based, like Octave/MATLAB. In Suite-Sparse:GraphBLAS, the largest size permitted for any integer of GrB\_Index is  $2^{60}$  (defined by GxB\_INDEX\_MAX). The largest GrB\_Matrix that SuiteSparse:GraphBLAS can construct is thus  $2^{60}$ -by- $2^{60}$ . An n-by-n matrix A that size can easily be constructed in practice with  $O(|\mathbf{A}|)$  memory requirements, where  $|\mathbf{A}|$  denotes the number of entries that explicitly appear in the pattern of  $\mathbf{A}$ . The time and memory required to construct a matrix that large does not depend on n, since SuiteSparse:GraphBLAS can represent  $\mathbf{A}$  in hypersparse form (see Section 8.3). The largest GrB\_Vector that can be constructed is  $2^{60}$ -by-1.

# 5.2 GrB\_init: initialize GraphBLAS

GrB\_init must be called before any other GraphBLAS operation. It defines the mode that GraphBLAS will use: blocking or non-blocking. With blocking mode, all operations finish before returning to the user application. With non-blocking mode, operations can be left pending, and are computed only when needed. Non-blocking mode can be much faster than blocking mode, by many orders of magnitude in extreme cases. Blocking

mode should be used only when debugging a user application. The mode cannot be changed once it is set by GrB\_init.

GraphBLAS objects are opaque. This allows GraphBLAS to postpone operations and then do them later in a more efficient manner by rearranging them and grouping them together. In non-blocking mode, the computations required to construct an opaque GraphBLAS object might not be finished when the GraphBLAS method or operation returns to the user. However, user-provided arrays are not opaque, and GraphBLAS methods and operations that read them (such as GrB\_Matrix\_build) or write to them (such as GrB\_Matrix\_extractTuples) always finish reading them, or creating them, when the method or operation returns to the user application.

All methods and operations that extract values from a GraphBLAS object and return them into non-opaque user arrays always ensure that the user-visible arrays are fully populated when they return: GrB\_\*\_reduce (to scalar), GrB\_\*\_nvals, GrB\_\*\_extractElement, and GrB\_\*\_extractTuples. These functions do *not* guarantee that the opaque objects they depend on are finalized. To do that, use GrB\_wait instead.

SuiteSparse:GraphBLAS is multithreaded internally, via OpenMP, and it is also safe to use in a multithreaded user application. See Section 14 for details. User threads must not operate on the same matrices at the same time, with one exception. Multiple user threads can use the same matrices or vectors as read-only inputs to GraphBLAS operations or methods, but only if they have no pending operations (use GrB\_wait first). User threads cannot simultaneously modify a matrix or vector via any GraphBLAS operation or method.

It is safe to use the internal parallelism in SuiteSparse:GraphBLAS on matrices, vectors, and scalars that are not yet completed. The library handles this on its own. The <code>GrB\_wait</code> function is only needed when a user application makes multiple calls to GraphBLAS in parallel, from multiple user threads.

With multiple user threads, exactly one user thread must call <code>GrB\_init</code> before any user thread may call any <code>GrB\_\*</code> or <code>GxB\_\*</code> function. When the user application is finished, exactly one user thread must call <code>GrB\_finalize</code>, after which no user thread may call any <code>GrB\_\*</code> or <code>GxB\_\*</code> function. The mode of a GraphBLAS session can be queried with <code>GxB\_get</code>; see Section 8 for details.

## 5.3 GrB\_getVersion: determine the C API Version

GraphBLAS defines two compile-time constants that define the version of the C API Specification that is implemented by the library: GRB\_VERSION and GRB\_SUBVERSION. If the user program was compiled with one version of the library but linked with a different one later on, the compile-time version check with GRB\_VERSION would be stale. GrB\_getVersion thus provides a runtime access of the version of the C API Specification supported by the library.

**SPEC:** This version of SuiteSparse:GraphBLAS supports the majority of 2.0.0 (Sept 29, 2021 (draft)) of the C API Specification, with the exception of h GrB\_wait, and GrB\_info.

#### 5.4 GxB\_init: initialize with alternate malloc

GxB\_init is identical to GrB\_init, except that it also redefines the memory management functions that SuiteSparse:GraphBLAS will use. Giving the user application control over this is particularly important when using the GxB\_\*import, GxB\_\*export, GxB\_\*pack, GxB\_\*unpack, and GxB\_\*serialize functions described in Section 6.11, since they require the user application and GraphBLAS to use the same memory manager.

user\_calloc\_function and user\_realloc\_function are optional, and may be NULL. If NULL, then the user\_malloc\_function is relied on instead, for all memory allocations.

These functions can only be set once, when GraphBLAS starts. Either GrB\_init or GxB\_init must be called before any other GraphBLAS operation, but not both. The functions passed to GxB\_init must be thread-safe.

The following usage is identical to GrB\_init(mode):

scalable\_free, true) ;

```
GxB_init (mode, malloc, calloc, realloc, free, true);

To use the scalable Intel TBB memory manager:

#include "tbb/scalable_allocator.h"
#include "GraphBLAS.h"
```

The last parameter (ignored) is ignored in this version (v5.2) and will be removed in v6.0.

GxB\_init (mode, scalable\_malloc, scalable\_calloc, scalable\_realloc,

## 5.5 GrB\_Info: status code returned by GraphBLAS

Each GraphBLAS method and operation returns its status to the caller as its return value, an enumerated type (an enum) called GrB\_Info. The first two values in the following table denote a successful status, the rest are error codes. SuiteSparse:GraphBLAS v5.x and earlier use the enum values in the v5 column, since the C API Specification did not define them. The values are now defined in the v2.0 C API Specification, and will appear in SuiteSparse:GraphBLAS v6.0.0 with the values in the v6 column.

Error	v5	v6	description
GrB_SUCCESS	0	0	the method or operation was successful
GrB_NO_VALUE	1	1	the method was successful, but the entry
			does not appear in the matrix or vector.
GrB_UNINITIALIZED_OBJECT	2	-1	object has not been initialized
GrB_NULL_POINTER	4	-2	input pointer is NULL
GrB_INVALID_VALUE	5	-3	generic error code; some value is bad
<pre>GrB_INVALID_INDEX</pre>	6	-4	a row or column index is out of bounds
GrB_DOMAIN_MISMATCH	7	-5	object domains are not compatible
GrB_DIMENSION_MISMATCH	8	-6	matrix dimensions do not match
GrB_OUTPUT_NOT_EMPTY	9	-7	output matrix already has values in it
GrB_NOT_IMPLEMENTED	-8	-8	not implemented in SS:GrB
GrB_PANIC	13	-101	unrecoverable error
GrB_OUT_OF_MEMORY	10	-102	out of memory
<pre>GrB_INSUFFICIENT_SPACE</pre>	11	-103	output array not large enough
GrB_INVALID_OBJECT	3	-104	object is corrupted
<pre>GrB_INDEX_OUT_OF_BOUNDS</pre>	12	-105	a row or column index is out of bounds
GrB_EMPTY_OBJECT	-106	-106	a input scalar has no entry

Not all GraphBLAS methods or operations can return all status codes. In the discussions of each method and operation in this User Guide, most of the obvious error code returns are not discussed. For example, if a required input is a NULL pointer, then GrB\_NULL\_POINTER is returned. Only error codes specific to the method or that require elaboration are discussed here. For a full list of the status codes that each GraphBLAS function can return, refer to *The GraphBLAS C API Specification* [BMM+17b].

## 5.6 GrB\_error: get more details on the last error

Each GraphBLAS method and operation returns a GrB\_Info error code. The GrB\_error function returns additional information on the error for a particular object in a null-terminated string. The string returned by GrB\_error is never a NULL string, but it may have length zero (with the first entry being the '\0' string-termination value). The string must not be freed or modified.

```
info = GrB_some_method_here (C, ...);
if (! (info == GrB_SUCCESS || info == GrB_NO_VALUE))
{
    char *err;
    GrB_error (&err, C);
    printf ("info: %d error: %s\n", info, err);
}
```

If C has no error status, or if the error is not recorded in the string, an empty non-null string is returned. In particular, out-of-memory conditions result in an empty string fro GrB\_error.

SuiteSparse:GraphBLAS reports many helpful details via GrB\_error. For example, if a row or column index is out of bounds, the report will state what those bounds are. If a matrix dimension is incorrect, the mismatching dimensions will be provided. GrB\_BinaryOp\_new, GrB\_UnaryOp\_new, and GrB\_IndexUnaryOp\_new record the name the function passed to them, and GrB\_Type\_new records the name of its type parameter, and these are printed if the user-defined types and operators are used incorrectly. Refer to the output of the example programs in the Demo and Test folder, which intentionally generate errors to illustrate the use of GrB\_error.

The only functions in GraphBLAS that return an error string are functions that have a single input/output argument C, as a GrB\_Matrix, GrB\_Vector, GrB\_Scalar, or GrB\_Descriptor. Methods that create these objects (such as GrB\_Matrix\_new) return a NULL object on failure, so these methods cannot also return an error string in C.

Any subsequent GraphBLAS method that modifies the object C clears the error string.

Note that GrB\_NO\_VALUE is an not error, but an informational status. GrB\_\*\_extractElment(&x,A,i,j), which does x=A(i,j), returns this value to indicate that A(i,j) is not present in the matrix. That method does not have an input/output object so it cannot return an error string.

The GrB\_error function is a polymorphic function for the following variants:

```
GrB_Info GrB_Type_error
                                (const char **err, const GrB_Type type) ;
                                (const char **err, const GrB_UnaryOp op) ;
GrB_Info GrB_UnaryOp_error
GrB_Info GrB_BinaryOp_error
                                (const char **err, const GrB_BinaryOp op) ;
GrB_Info GrB_IndexUnaryOp_error (const char **err, const GrB_IndexUnaryOp op) ;
GrB_Info GrB_Monoid_error
                                (const char **err, const GrB_Monoid monoid) ;
GrB_Info GrB_Semiring_error
                                (const char **err, const GrB_Semiring semiring)
                                (const char **err, const GrB_Scalar s) ;
GrB_Info GrB_Scalar_error
GrB_Info GrB_Vector_error
                                (const char **err, const GrB_Vector v) ;
GrB_Info GrB_Matrix_error
                                (const char **err, const GrB_Vector A) ;
GrB_Info GrB_Descriptor_error
                                (const char **err, const GrB_Descriptor d) ;
```

Currently, only GrB\_Matrix\_error, GrB\_Vector\_error, GrB\_Scalar\_error, and GrB\_Descriptor\_error are able to return non-empty error strings. The latter can return an error string only from GrB\_Descriptor\_set and GxB\_set(d,...).

The only GraphBLAS methods (Section 6) that return an error string are \*setElement, \*removeElement, GxB\_Matrix\_Option\_set(A,...), GxB\_Vector\_Option\_set(v,...), GrB\_Descriptor\_set, and GxB\_Desc\_set(d,...). All GraphBLAS operations discussed in Section 10 can return an error string in their input/output object, except for GrB\_reduce when reducing to a scalar.

# 5.7 GrB\_finalize: finish GraphBLAS

```
GrB_Info GrB_finalize ( );  // finish GraphBLAS
```

GrB\_finalize must be called as the last GraphBLAS operation, even after all calls to GrB\_free. All GraphBLAS objects created by the user application should be freed first, before calling GrB\_finalize since GrB\_finalize will not free those objects. In non-blocking mode, GraphBLAS may leave some computations as pending. These computations can be safely abandoned if the user application frees all GraphBLAS objects it has created and then calls GrB\_finalize. When the user application is finished, exactly one user thread must call GrB\_finalize.

# 6 GraphBLAS Objects and their Methods

GraphBLAS defines ten different objects to represent matrices, vectors, scalars, data types, operators (binary, unary, and index-unary), monoids, semirings, and a *descriptor* object used to specify optional parameters that modify the behavior of a GraphBLAS operation.

The GraphBLAS API makes a distinction between *methods* and *operations*. A method is a function that works on a GraphBLAS object, creating it, destroying it, or querying its contents. An operation (not to be confused with an operator) acts on matrices and/or vectors in a semiring.

a scalar data type
a unary operator $z = f(x)$ , where z and x are scalars
a binary operator $z = f(x, y)$ , where z, x, and y are scalars
a index-unary operator
an associative and commutative binary operator
and its identity value
a monoid that defines the "plus" and a binary operator
that defines the "multiply" for an algebraic semiring
a 2D sparse matrix of any type
a 1D sparse column vector of any type
a scalar of any type
a collection of parameters that modify an operation

Each of these objects is implemented in C as an opaque handle, which is a pointer to a data structure held by GraphBLAS. User applications may not examine the content of the object directly; instead, they can pass the handle back to GraphBLAS which will do the work. Assigning one handle to another is valid but it does not make a copy of the underlying object.

# 6.1 The GraphBLAS type: GrB\_Type

A GraphBLAS GrB\_Type defines the type of scalar values that a matrix or vector contains, and the type of scalar operands for a unary or binary operator. There are 13 built-in types, and a user application can define any types of its own as well. The built-in types correspond to built-in types in C (#include <stdbool.h> and #include <stdint.h>) as listed in the following table.

GraphBLAS	C type	description	range
type			
GrB_BOOL	bool	Boolean	true $(1)$ , false $(0)$
GrB_INT8	int8_t	8-bit signed integer	-128 to 127
GrB_INT16	int16_t	16-bit integer	$-2^{15}$ to $2^{15}-1$
GrB_INT32	int32_t	32-bit integer	$-2^{31}$ to $2^{31}-1$
GrB_INT64	int64_t	64-bit integer	$-2^{63}$ to $2^{63}-1$
GrB_UINT8	uint8_t	8-bit unsigned integer	0 to 255
GrB_UINT16	uint16_t	16-bit unsigned integer	0 to $2^{16} - 1$
GrB_UINT32	uint32_t	32-bit unsigned integer	0 to $2^{32} - 1$
GrB_UINT64	uint64_t	64-bit unsigned integer	0 to $2^{64} - 1$
GrB_FP32	float	32-bit IEEE 754	-Inf to +Inf
GrB_FP64	double	64-bit IEEE $754$	-Inf to +Inf
GxB_FC32	float complex	32-bit complex	-Inf to +Inf
GxB_FC64	double complex	64-bit complex	-Inf to +Inf

The ANSI C11 definitions of float complex and double complex are not always available. The GraphBLAS.h header defines them as GxB\_FC32\_t and GxB\_FC64\_t, respectively.

The user application can also define new types based on any typedef in the C language whose values are held in a contiguous region of memory. For example, a user-defined GrB\_Type could be created to hold any C struct whose content is self-contained. A C struct containing pointers might be problematic because GraphBLAS would not know to dereference the pointers to traverse the entire "scalar" entry, but this can be done if the objects referenced by these pointers are not moved. A user-defined complex type with real and imaginary types can be defined, or even a "scalar" type containing a fixed-sized dense matrix (see Section 6.1.1). The possibilities are endless. GraphBLAS can create and operate on sparse matrices and vectors in any of these types, including any user-defined ones. For user-defined types, GraphBLAS simply moves the data around itself (via memcpy), and then passes the

values back to user-defined functions when it needs to do any computations on the type. The next sections describe the methods for the GrB\_Type object:

```
GrB_Type_new create a user-defined type
GxB_Type_new create a user-defined type, with name and definition
GrB_Type_wait wait for a user-defined type
GxB_Type_size return the size of a type
GxB_Type_name return the name of a type
GxB_Type_from_name return the type from its name
GrB_Type_free free a user-defined type
```

#### 6.1.1 GrB\_Type\_new: create a user-defined type

GrB\_Type\_new creates a new user-defined type. The type is a handle, or a pointer to an opaque object. The handle itself must not be NULL on input, but the content of the handle can be undefined. On output, the handle contains a pointer to a newly created type. The ctype is the type in C that will be used to construct the new GraphBLAS type. It can be either a built-in C type, or defined by a typedef. The second parameter should be passed as sizeof(ctype). The only requirement on the C type is that sizeof(ctype) is valid in C, and that the type reside in a contiguous block of memory so that it can be moved with memcpy. For example, to create a user-defined type called Complex for double-precision complex values using the ANSI C11 double complex type, the following can be used. A complete example can be found in the usercomplex.c and usercomplex.h files in the Demo folder.

```
#include <math.h>
#include <complex.h>
GrB_Type Complex ;
GrB_Type_new (&Complex, sizeof (double complex)) ;
```

To demonstrate the flexibility of the GrB\_Type, consider a "scalar" consisting of 4-by-4 floating-point matrix and a string. This type might be useful

for the 4-by-4 translation/rotation/scaling matrices that arise in computer graphics, along with a string containing a description or even a regular expression that can be parsed and executed in a user-defined operator. All that is required is a fixed-size type, where sizeof(ctype) is a constant.

```
typedef struct
{
    float stuff [4][4];
    char whatstuff [64];
}
wildtype;
GrB_Type WildType;
GrB_Type_new (&WildType, sizeof (wildtype));
```

With this type a sparse matrix can be created in which each entry consists of a 4-by-4 dense matrix stuff and a 64-character string whatstuff. GraphBLAS treats this 4-by-4 as a "scalar." Any GraphBLAS method or operation that simply moves data can be used with this type without any further information from the user application. For example, entries of this type can be assigned to and extracted from a matrix or vector, and matrices containing this type can be transposed. A working example (wildtype.c in the Demo folder) creates matrices and multiplies them with a user-defined semiring with this type.

Performing arithmetic on matrices and vectors with user-defined types requires operators to be defined. Refer to Section 13.7 for more details on these example user-defined types.

# 6.1.2 GxB\_Type\_new: create a user-defined type (with name and definition)

GxB\_Type\_new creates a type with a name and definition that are known to GraphBLAS, as strings. The type\_name is any valid string (max length of 128 characters, including the required null-terminating character) that may

appear as the name of a C type created by a C typedef statement. It must not contain any white-space characters. Example, creating a type of size 16\*4+1=65 bytes, with a 4-by-4 dense float array and a 32-bit integer:

The type\_name and type\_defn are both null-terminated strings. Currently, type\_defn is unused, but it will be required for best performance when a JIT is implemented in SuiteSparse:GraphBLAS (both on the CPU and GPU). User defined types created by GrB\_Type\_new will not work with a JIT.

At most GxB\_MAX\_NAME\_LEN characters are accessed in type\_name; characters beyond that limit are silently ignored.

#### **6.1.3** GrB\_Type\_wait: wait for a type

After creating a user-defined type, a GraphBLAS library may choose to exploit non-blocking mode to delay its creation. GrB\_Type\_wait(&type) ensures the type is completed. SuiteSparse:GraphBLAS currently does nothing for GrB\_Type\_wait(&type), except to ensure that type is valid. The function will change in SS:GrB v6.0.0 (for the v2.0 C API Specification).

### 6.1.4 GxB\_Type\_size: return the size of a type

This function acts just like sizeof(type) in the C language. For example GxB\_Type\_size (&s, GrB\_INT32) sets s to 4, the same as sizeof(int32\_t).

#### 6.1.5 GxB\_Type\_name: return the name of a type

Returns the name of a type, as a string. For built-in type, the name is the same as the C type. For example, <code>GxB\_Type\_name(type\_name,GrB\_FP32)</code> returns the name as "float". The following table lists the names of the 13 built-in types.

GraphBLAS type
GrB_BOOL
GrB_INT8
GrB_INT16
GrB_INT32
GrB_INT64
GrB_UINT8
GrB_UINT16
GrB_UINT32
GrB_UINT64
GrB_FP32
GrB_FP64
GxB_FC32
GxB_FC64

#### 6.1.6 GxB\_Type\_from\_name: return the type from its name

Returns the built-in type from the corresponding name of the type. For example, GxB\_Type\_from\_name (&type, "bool") returns GrB\_B00L. If the name is from a user-defined type, the type is returned as NULL. This is not an error condition. The user application must itself do this translation since GraphBLAS does not keep a registry of all user-defined types.

With this function, a user application can manage the translation for both built-in types and its own user-defined types, as in the following example.

```
typedef struct { double x ; char stuff [16] ; } myfirsttype ;
typedef struct { float z [4][4] ; int color ; } myquaternion ;
GrB_Type MyType1, MyQType ;
GxB_Type_new (&MyType1, sizeof (myfirsttype), "myfirsttype",
    "typedef struct { double x ; char stuff [16] ; } myfirsttype ;") ;
GxB_Type_new (&MyQType, sizeof (myquaternion), "myquaternion",
    "typedef struct { float z [4][4] ; int color ; } myquaternion ;") ;
GrB_Matrix A ;
// ... create a matrix A of some built-in or user-defined type
// later on, to query the type of A:
size_t typesize ;
GxB_Type_size (&typesize, type) ;
                                        // works for any type
GrB_Type atype ;
char atype_name [GxB_MAX_NAME_LEN] ;
GxB_Matrix_type_name (atype_name, A) ;
GxB_Type_from_name (&atype, atype_name);
if (atype == NULL)
    // This is not yet an error. It means that A has a user-defined type.
    if ((strcmp (atype_name, "myfirsttype")) == 0) atype = MyType1;
    else if ((strcmp (atype_name, "myquaternion")) == 0) atype = MyQType ;
    else { ... this is now an error ... the type of A is unknown. }
}
```

#### 6.1.7 GrB\_Type\_free: free a user-defined type

```
GrB_Info GrB_free // free a user-defined type
(
GrB_Type *type // handle of user-defined type to free
);
```

GrB\_Type\_free frees a user-defined type. Either usage:

```
GrB_Type_free (&type) ;
GrB_free (&type) ;
```

frees the user-defined type and sets type to NULL. It safely does nothing if passed a NULL handle, or if type == NULL on input.

It is safe to attempt to free a built-in type. SuiteSparse:GraphBLAS silently ignores the request and returns GrB\_SUCCESS. A user-defined type should not be freed until all operations using the type are completed. Suite-Sparse:GraphBLAS attempts to detect this condition but it must query a freed object in its attempt. This is hazardous and not recommended. Operations on such objects whose type has been freed leads to undefined behavior.

It is safe to first free a type, and then a matrix of that type, but after the type is freed the matrix can no longer be used. The only safe thing that can be done with such a matrix is to free it.

The function signature of GrB\_Type\_free uses the generic name GrB\_free, which can free any GraphBLAS object. See Section 6.14 details. GraphBLAS includes many such generic functions. When describing a specific variation, a function is described with its specific name in this User Guide (such as GrB\_Type\_free). When discussing features applicable to all specific forms, the generic name is used instead (such as GrB\_free).

# **6.2** GraphBLAS unary operators: GrB\_UnaryOp, z = f(x)

A unary operator is a scalar function of the form z = f(x). The domain (type) of z and x need not be the same.

In the notation in the tables below, T is any of the 13 built-in types and is a place-holder for BOOL, INT8, UINT8, ... FP32, FP64, FC32, or FC64. For example,  $GrB_AINV_INT32$  is a unary operator that computes z=-x for two values x and z of type  $GrB_INT32$ .

The notation R refers to any real type (all but FC32 and FC64), I refers to any integer type (INT\* and UINT\*), F refers to any real or complex floating point type (FP32, FP64, FC32, or FC64), and Z refers to any complex floating point type (FC32 or FC64).

The logical negation operator  $\mathtt{GrB\_LNOT}$  only works on Boolean types. The  $\mathtt{GxB\_LNOT}\_R$  functions operate on inputs of type R, implicitly typecasting their input to Boolean and returning result of type R, with a value 1 for true and 0 for false. The operators  $\mathtt{GxB\_LNOT\_BOOL}$  and  $\mathtt{GrB\_LNOT}$  are identical.

Unary operators for all types			
GraphBLAS name	types (domains)	z = f(x)	description
$\mathtt{GxB\_ONE\_}T$	$T \to T$	z = 1	one
${\tt GrB\_IDENTITY\_}T$	$T \to T$	z = x	identity
${\tt GrB\_AINV\_}T$	$T \to T$	z = -x	additive inverse
${\tt GrB\_MINV\_}T$	$T \to T$	z = 1/x	multiplicative inverse

Unary operators for real and integer types			
GraphBLAS name	types (domains)	z = f(x)	description
${\tt GrB\_ABS\_}T$	$R \to R$	z =  x	absolute value
GrB_LNOT	$\mathtt{bool} \to \mathtt{bool}$	$z = \neg x$	logical negation
$\mathtt{GxB\_LNOT}\_R$	$R \to R$	$z = \neg(x \neq 0)$	logical negation
${\tt GrB\_BNOT\_}I$	I  o I	$z = \neg x$	bitwise negation

Positional unary operators for any type (including user-defined)				
GraphBLAS name	types (domains)	$z = f(a_{ij})$	description	
$\texttt{GxB\_POSITIONI}\_T$	$\rightarrow T$	z = i	row index (0-based)	
${\tt GxB\_POSITIONI1\_}T$	$\rightarrow T$	z = i + 1	row index (1-based)	
${\tt GxB\_POSITIONJ\_}T$	$\rightarrow T$	z = j	column index (0-based)	
${\tt GxB\_POSITIONJ1\_}T$	$\rightarrow T$	z = j + 1	column index (1-based)	

Unary operators for floating-point types (real and complex)				
GraphBLAS name	types (domains)	z = f(x)	description	
$\mathtt{GxB\_SQRT\_}F$	$F \to F$	$z = \sqrt{(x)}$	square root	
${\tt GxB\_LOG\_}F$	$F \to F$	$z = \log_e(x)$	natural logarithm	
${\tt GxB\_EXP\_} F$	$F \to F$	$z = e^x$	natural exponent	
${\tt GxB\_LOG10\_}F$	$F \to F$	$z = \log_{10}(x)$	base-10 logarithm	
${\tt GxB\_LOG2\_}F$	$F \to F$	$z = \log_2(x)$	base-2 logarithm	
$\mathtt{GxB}\_\mathtt{EXP2}\_F$	$F \to F$	$z = 2^x$	base-2 exponent	
${\tt GxB\_EXPM1\_}F$	$F \to F$	$z = e^x - 1$	natural exponent - 1	
${\tt GxB\_LOG1P\_}F$	$F \to F$	$z = \log(x+1)$	natural log of $x+1$	
$\mathtt{GxB\_SIN\_}F$	$F \to F$	$z = \sin(x)$	sine	
$\mathtt{GxB\_COS\_}F$	$F \to F$	$z = \cos(x)$	cosine	
$\mathtt{GxB\_TAN}\_F$	$F \to F$	$z = \tan(x)$	tangent	
${\tt GxB\_ASIN\_}F$	$F \to F$	$z = \sin^{-1}(x)$	inverse sine	
$\mathtt{GxB\_ACOS\_}F$	$F \to F$	$z = \cos^{-1}(x)$	inverse cosine	
$\mathtt{GxB\_ATAN\_}F$	$F \to F$	$z = \tan^{-1}(x)$	inverse tangent	
$\mathtt{GxB\_SINH\_}F$	$F \to F$	$z = \sinh(x)$	hyperbolic sine	
$\mathtt{GxB\_COSH\_}F$	$F \to F$	$z = \cosh(x)$	hyperbolic cosine	
$GxB_TANH_F$	$F \to F$	$z = \tanh(x)$	hyperbolic tangent	
$\mathtt{GxB\_ASINH\_}F$	$F \to F$	$z = \sinh^{-1}(x)$	inverse hyperbolic sine	
$\mathtt{GxB\_ACOSH\_}F$	$F \to F$	$z = \cosh^{-1}(x)$	inverse hyperbolic cosine	
$\mathtt{GxB\_ATANH\_}F$	$F \to F$	$z = \tanh^{-1}(x)$	inverse hyperbolic tangent	
${\tt GxB\_SIGNUM\_}F$	$F \to F$	$z = \operatorname{sgn}(x)$	sign, or signum function	
$\mathtt{GxB\_CEIL\_}F$	$F \to F$	$z = \lceil x \rceil$	ceiling function	
${\tt GxB\_FLOOR\_}F$	$F \to F$	$z = \lfloor x \rfloor$	floor function	
$\mathtt{GxB}_\mathtt{ROUND}_\mathtt{F}$	$F \to F$	z = round(x)	round to nearest	
$\texttt{GxB\_TRUNC\_}F$	$F \to F$	$z = \operatorname{trunc}(x)$	round towards zero	
${\tt GxB\_LGAMMA\_}F$	$F \to F$	$z = \log( \Gamma(x) )$	log of gamma function	
${\tt GxB\_TGAMMA\_}F$	$F \to F$	$z = \Gamma(x)$	gamma function	
$\mathtt{GxB\_ERF}\_F$	$F \to F$	$z = \operatorname{erf}(x)$	error function	
$\mathtt{GxB\_ERFC\_}F$	$F \to F$	$z = \operatorname{erfc}(x)$	complimentary error function	
$\mathtt{GxB\_FREXPX\_}F$	$F \to F$	z = frexpx(x)	normalized fraction	
$\mathtt{GxB\_FREXPE\_}F$	$F \to F$	z = frexpe(x)	normalized exponent	
$\mathtt{GxB\_ISINF}\_F$	$F  o  exttt{bool}$	z = isinf(x)	true if $\pm \infty$	
${\tt GxB\_ISNAN\_}F$	$F  o  exttt{bool}$	$z = i\operatorname{snan}(x)$	true if NaN	
$\texttt{GxB\_ISFINITE\_}F$	$F  o  exttt{bool}$	z = isfinite(x)	true if finite	

1	Unary operators for	r complex type	S
GraphBLAS name	types (domains)	z = f(x)	description
$\mathtt{GxB\_CONJ\_}Z$	Z  o Z	$z = \overline{x}$	complex conjugate
${ t GxB\_ABS\_}Z$	$Z \to F$	z =  x	absolute value
${\tt GxB\_CREAL\_}Z$	$Z \to F$	z = real(x)	real part
${\tt GxB\_CIMAG\_\it Z}$	$Z \to F$	z = imag(x)	imaginary part
${\tt GxB\_CARG\_}{Z}$	$Z \to F$	$z = \operatorname{carg}(x)$	angle

A positional unary operator return the row or column index of an entry. For a matrix  $z = f(a_{ij})$  returns z = i or z = j, or +1 for 1-based indices. The latter is useful in the Octave/MATLAB interface, where row and column indices are 1-based. When applied to a vector, j is always zero, and i is the index in the vector. Positional unary operators come in two types: INT32 and INT64, which is the type of the output, z. The functions are agnostic to the type of their inputs; they only depend on the position of the entries, not their values. User-defined positional operators cannot be defined by  $GrB_UnaryOp_new$ .

GxB\_FREXPX GxB\_FREXPE return the mantissa and exponent, respectively, from the ANSI C11 frexp function. The exponent is returned as a floating-point value, not an integer.

The operators <code>GxB\_EXPM1\_FC\*</code> and <code>GxB\_LOG1P\_FC\*</code> for complex types are currently not accurate. They will be revised in a future version.

The functions casin, casinf, casinh, and casinhf provided by Microsoft Visual Studio for computing  $\sin^{-1}(x)$  and  $\sinh^{-1}(x)$  when x is complex do not compute the correct result. Thus, the unary operators  $GxB_ASIN_FC32$ ,  $GxB_ASIN_FC64$   $GxB_ASINH_FC32$ , and  $GxB_ASINH_FC64$  do not work properly if the MS Visual Studio compiler is used. These functions work properly if the gcc, icc, or clang compilers are used on Linux or MacOS.

Integer division by zero normally terminates an application, but this is avoided in SuiteSparse:GraphBLAS. For details, see the binary  $\mathtt{GrB\_DIV\_}T$  operators.

**SPEC:** The definition of integer division by zero is an extension to the spec.

The next sections define the following methods for the GrB\_UnaryOp object:

GraphBLAS function	purpose	Section
GrB_UnaryOp_new	create a user-defined unary operator	6.2.1
<pre>GxB_UnaryOp_new</pre>	create a named user-defined unary operator	6.2.2
<pre>GrB_UnaryOp_wait</pre>	wait for a user-defined unary operator	6.2.3
<pre>GxB_UnaryOp_ztype_name</pre>	return the name of the type of the output z for $z = f(x)$	6.2.4
${\tt GxB\_UnaryOp\_xtype\_name}$	return the name of the type of the input $x$ for $z = f(x)$	6.2.5
<pre>GrB_UnaryOp_free</pre>	free a user-defined unary operator	6.2.6

#### 6.2.1 GrB\_UnaryOp\_new: create a user-defined unary operator

GrB\_UnaryOp\_new creates a new unary operator. The new operator is returned in the unaryop handle, which must not be NULL on input. On output, its contents contains a pointer to the new unary operator.

The two types xtype and ztype are the GraphBLAS types of the input x and output z of the user-defined function z = f(x). These types may be built-in types or user-defined types, in any combination. The two types need not be the same, but they must be previously defined before passing them to GrB\_UnaryOp\_new.

The function argument to GrB\_UnaryOp\_new is a pointer to a user-defined function with the following signature:

```
void (*f) (void *z, const void *x);
```

When the function f is called, the arguments z and x are passed as (void \*) pointers, but they will be pointers to values of the correct type, defined by ztype and xtype, respectively, when the operator was created.

**NOTE:** The pointers may not be unique. That is, the user function may be called with multiple pointers that point to the same space, such as when z=f(z,y) is to be computed by a binary operator, or z=f(z) for a unary operator. Any parameters passed to the user-callable function may be aliased to each other.

# **6.2.2** GxB\_UnaryOp\_new: create a named user-defined unary operator

Creates a named GrB\_UnaryOp. Only the first 127 characters of unop\_name are used. The unop\_defn is a string containing the entire function itself. For example:

Currently, only the unop\_name is used, but future versions will rely on the unop\_defn when employing a JIT for better performance.

#### 6.2.3 GrB\_UnaryOp\_wait: wait for a unary operator

After creating a user-defined unary operator, a GraphBLAS library may choose to exploit non-blocking mode to delay its creation. GrB\_UnaryOp\_wait(&unaryop) ensures the op is completed. SuiteSparse:GraphBLAS currently does nothing for GrB\_UnaryOp\_wait(&unaryop), except to ensure that the unaryop is valid. The function will change in SS:GrB v6.0.0 (for the v2.0 C API Specification).

#### 6.2.4 GxB\_UnaryOp\_ztype\_name: return the name of the type of z

GxB\_UnaryOp\_ztype\_name returns the name of the ztype of the unary operator, which is the type of z in the function z = f(x).

#### 6.2.5 GxB\_UnaryOp\_xtype\_name: return the name of the type of x

GxB\_UnaryOp\_xtype\_name returns the name of the xtype of the unary operator, which is the type of x in the function z = f(x).

#### 6.2.6 GrB\_UnaryOp\_free: free a user-defined unary operator

 ${\tt GrB\_Unary0p\_free}\ {\tt frees}\ {\tt a}\ {\tt user-defined}\ {\tt unary}\ {\tt operator}.\ {\tt Either}\ {\tt usage:}$ 

```
GrB_UnaryOp_free (&unaryop) ;
GrB_free (&unaryop) ;
```

frees the unaryop and sets unaryop to NULL. It safely does nothing if passed a NULL handle, or if unaryop == NULL on input. It does nothing at all if passed a built-in unary operator.

# **6.3** GraphBLAS binary operators: GrB\_BinaryOp, z = f(x,y)

A binary operator is a scalar function of the form z = f(x, y). The types of z, x, and y need not be the same. The built-in binary operators are listed in the tables below. The notation T refers to any of the 13 built-in types, but two of those types are SuiteSparse extensions (GxB\_FC32 and GxB\_FC64). For those types, the operator name always starts with GxB, not GrB).

The six  $GxB_IS*$  comparators and the  $GxB_*$  logical operators all return a result one for true and zero for false, in the same domain T or R as their inputs. These six comparators are useful as "multiply" operators for creating semirings with non-Boolean monoids.

Binary operators for all 13 types				
GraphBLAS name	types (domains)	z = f(x, y)	description	
${\tt GrB\_FIRST\_}T$	$T \times T \to T$	z = x	first argument	
${\tt GrB\_SECOND\_}T$	$T \times T \to T$	z = y	second argument	
$\mathtt{GxB\_ANY\_}T$	$T \times T \to T$	z = x  or  y	pick $x$ or $y$ arbitrarily	
${\tt GxB\_PAIR\_}T$	$T\times T\to T$	z = 1	one	
${ t GrB\_PLUS\_T}$	$T\times T\to T$	z = x + y	addition	
${\tt GrB\_MINUS\_}T$	$T\times T\to T$	z = x - y	subtraction	
${\tt GxB\_RMINUS\_}T$	$T\times T\to T$	z = y - x	reverse subtraction	
${\tt GrB\_TIMES\_}T$	$T\times T\to T$	z = xy	multiplication	
${\tt GrB\_DIV\_}T$	$T\times T\to T$	z = x/y	division	
${\tt GxB\_RDIV\_}T$	$T\times T\to T$	z = y/x	reverse division	
$\mathtt{GxB\_POW\_}T$	$T\times T\to T$	$z = x^y$	power	
${\tt GxB\_ISEQ\_T}$	$T \times T \to T$	z = (x == y)	equal	
$\mathtt{GxB\_ISNE\_}T$	$T \times T \to T$	$z = (x \neq y)$	not equal	

The  $GxB_POW_*$  operators for real types do not return a complex result, and thus  $z = f(x, y) = x^y$  is undefined if x is negative and y is not an integer. To compute a complex result, use  $GxB_POW_FC32$  or  $GxB_POW_FC64$ .

Operators that require the domain to be ordered (MIN, MAX, less-than, greater-than, and so on) are not defined for complex types. These are listed in the following table:

Binary operators for all non-complex types				
GraphBLAS name	types (domains)	z = f(x, y)	description	
${\tt GrB\_MIN\_}R$	$R \times R \to R$	$z = \min(x, y)$	minimum	
${\tt GrB\_MAX\_}R$	$R \times R \to R$	$z = \max(x, y)$	maximum	
$\texttt{GxB\_ISGT\_}R$	$R \times R \to R$	z = (x > y)	greater than	
${\tt GxB\_ISLT\_}R$	$R \times R \to R$	z = (x < y)	less than	
${\tt GxB\_ISGE\_}R$	$R \times R \to R$	$z = (x \ge y)$	greater than or equal	
${\tt GxB\_ISLE\_}R$	$R \times R \to R$	$z = (x \le y)$	less than or equal	
GxB_LOR_R	$R \times R \to R$	$z = (x \neq 0) \lor (y \neq 0)$	logical OR	
${\tt GxB\_LAND\_}R$	$R \times R \to R$	$z = (x \neq 0) \land (y \neq 0)$	logical AND	
${\tt GxB\_LXOR\_}R$	$R \times R \to R$	$z = (x \neq 0) \veebar (y \neq 0)$	logical XOR	

Another set of six kinds of built-in comparators have the form  $T \times T \to bool$ . Note that when T is bool, the six operators give the same results as the six  $GxB_IS*_B00L$  operators in the table above. These six comparators are useful as "multiply" operators for creating semirings with Boolean monoids.

Binary comparators for all 13 types			
GraphBLAS name types (domains) $z = f(x, y)$ description			
${\tt GrB\_EQ\_}T$	$T \times T  o \mathtt{bool}$	z = (x == y)	equal
${\tt GrB\_NE\_}T$	$T\times T\to \texttt{bool}$	$z = (x \neq y)$	not equal

Binary comparators for non-complex types			
GraphBLAS name	types (domains)	z = f(x, y)	description
${\tt GrB\_GT\_}R$	$R \times R  o  exttt{bool}$	z = (x > y)	greater than
$\mathtt{GrB\_LT}\_R$	$R\times R\to \texttt{bool}$	z = (x < y)	less than
${\tt GrB\_GE\_}R$	$R\times R\to \texttt{bool}$	$z = (x \ge y)$	greater than or equal
${\tt GrB\_LE\_}R$	$R\times R\to \texttt{bool}$	$z = (x \le y)$	less than or equal

GraphBLAS has four built-in binary operators that operate purely in the Boolean domain. The first three are identical to the <code>GxB\_L\*\_BOOL</code> operators described above, just with a shorter name. The <code>GrB\_LXNOR</code> operator is the same as <code>GrB\_EQ\_BOOL</code>.

Binary operators for the boolean type only				
GraphBLAS name types (domains) $z = f(x, y)$ description				
GrB_LOR	$\texttt{bool} \times \texttt{bool} \to \texttt{bool}$	$z = x \vee y$	logical OR	
GrB_LAND	$\texttt{bool} \times \texttt{bool} \to \texttt{bool}$	$z = x \wedge y$	logical AND	
GrB_LXOR	$\mathtt{bool} \times \mathtt{bool}  o \mathtt{bool}$	$z = x \veebar y$	logical XOR	
GrB_LXNOR	$\texttt{bool} \times \texttt{bool} \to \texttt{bool}$	$z = \neg(x \veebar y)$	logical XNOR	

The following operators are defined for real floating-point types only (GrB\_FP32 and GrB\_FP64). They are identical to the ANSI C11 functions of the same name. The last one in the table constructs the corresponding complex type.

J P				
Binary operators for the real floating-point types only				
GraphBLAS name	AS name types (domains) $z = f(x, y)$ description		description	
$\mathtt{GxB\_ATAN2\_}F$	$F \times F \to F$	$z = \tan^{-1}(y/x)$	4-quadrant arc tangent	
${\tt GxB\_HYPOT\_}F$	$F\times F\to F$	$z = \sqrt{x^2 + y^2}$	hypotenuse	
${\tt GxB\_FMOD\_}F$	$F\times F\to F$	·	ANSI C11 fmod	
${\tt GxB\_REMAINDER\_}F$	$F\times F\to F$		ANSI C11 remainder	
$\mathtt{GxB\_LDEXP\_}F$	$F\times F\to F$		ANSI C11 ldexp	
${\tt GxB\_COPYSIGN\_}F$	$F\times F\to F$		ANSI C11 copysign	
$\mathtt{GxB\_CMPLX\_}F$	$F \times F \to Z$	$z = x + y \times i$	complex from real & imag	

Eight bitwise operators are predefined for signed and unsigned integers.

Binary operators for signed and unsigned integers				
GraphBLAS name types (domains) $z = f(x, y)$ description			description	
${\tt GrB\_BOR\_\it I}$	$I \times I \to I$	z=x y	bitwise logical OR	
${\tt GrB\_BAND\_}I$	$I \times I \to I$	z=x&y	bitwise logical AND	
${\tt GrB\_BXOR\_}I$	$I \times I \to I$	z=x^y	bitwise logical XOR	
${\tt GrB\_BXNOR\_}I$	$I\times I\to I$	z=~(x^y)	bitwise logical XNOR	
GxB_BGET_I	$I \times I \to I$		get bit y of x	
$\texttt{GxB\_BSET}\_I$	$I \times I \to I$		set bit y of x	
$\mathtt{GxB\_BCLR}\_I$	$I \times I \to I$		clear bit y of x	
${\tt GxB\_BSHIFT\_}I$	$I{ imes}{ exttt{int8}}{ ightarrow}\ I$		bit shift	

There are two sets of built-in comparators in SuiteSparse:GraphBLAS, but they are not redundant. They are identical except for the type (domain) of their output, z. The  $\mathtt{GrB\_EQ\_}T$  and related operators compare their inputs of type T and produce a Boolean result of true or false. The  $\mathtt{GxB\_ISEQ\_}T$  and related operators compute the same thing and produce a result with same type T as their input operands, returning one for true or zero for false. The  $\mathtt{IS*}$  comparators are useful when combining comparators with other non-Boolean operators. For example, a PLUS-ISEQ semiring counts how many terms are true. With this semiring, matrix multiplication  $\mathbf{C} = \mathbf{AB}$  for two weighted undirected graphs  $\mathbf{A}$  and  $\mathbf{B}$  computes  $c_{ij}$  as the number of edges node i and j have in common that have identical edge weights. Since the output type of the "multiplier" operator in a semiring must match the type of its monoid, the Boolean  $\mathtt{EQ}$  cannot be combined with a non-Boolean PLUS

monoid to perform this operation.

Likewise, SuiteSparse:GraphBLAS has two sets of logical OR, AND, and XOR operators. Without the  $\_T$  suffix, the three operators  $\texttt{GrB\_LOR}$ ,  $\texttt{GrB\_LAND}$ , and  $\texttt{GrB\_LXOR}$  operate purely in the Boolean domain, where all input and output types are  $\texttt{GrB\_BOOL}$ . The second set  $(\texttt{GxB\_LOR\_}T \texttt{GxB\_LAND\_}T$  and  $\texttt{GxB\_LXOR\_}T$ ) provides Boolean operators to all 11 real domains, implicitly typecasting their inputs from type T to Boolean and returning a value of type T that is 1 for true or zero for false. The set of  $\texttt{GxB\_L*\_}T$  operators are useful since they can be combined with non-Boolean monoids in a semiring.

Floating-point operations follow the IEEE 754 standard. Thus, computing x/0 for a floating-point x results in +Inf if x is positive, -Inf if x is negative, and NaN if x is zero. The application is not terminated. However, integer division by zero normally terminates an application. Suite-Sparse:GraphBLAS avoids this by adopting the same rules as MATLAB, which are analogous to how the IEEE standard handles floating-point division by zero. For integers, when x is positive, x/0 is the largest positive integer, for negative x it is the minimum integer, and 0/0 results in zero. For example, for an integer x of type  $GrB_INT32$ , 1/0 is  $2^{31} - 1$  and (-1)/0 is  $-2^{31}$ . Refer to Section 6.1 for a list of integer ranges.

Eight positional operators are predefined. They differ when used in a semiring and when used in GrB\_eWise\* and GrB\_apply. Positional operators cannot be used in GrB\_build, nor can they be used as the accum operator for any operation.

The positional binary operators do not depend on the type or numerical value of their inputs, just their position in a matrix or vector. For a vector, j is always 0, and i is the index into the vector. There are two types T available: INT32 and INT64, which is the type of the output z. User-defined positional operators cannot be defined by  $GrB_BinaryOp_new$ .

Positional binary operators for any type (including user-defined)					
wh	when used as a multiplicative operator in a semiring				
GraphBLAS name	types (domains)	$z = f(a_{ik}, b_{kj})$	description		
$\texttt{GxB\_FIRSTI\_}T$	$\rightarrow T$	z = i	row index of $a_{ik}$ (0-based)		
$\texttt{GxB\_FIRSTI1\_}T$	$\rightarrow T$	z = i + 1	row index of $a_{ik}$ (1-based)		
$\mathtt{GxB\_FIRSTJ\_}T$	$\rightarrow T$	z = k	column index of $a_{ik}$ (0-based)		
$\mathtt{GxB\_FIRSTJ1\_}T$	$\rightarrow T$	z = k + 1	column index of $a_{ik}$ (1-based)		
$\mathtt{GxB\_SECONDI\_}T$	$\rightarrow T$	z = k	row index of $b_{kj}$ (0-based)		
${\tt GxB\_SECONDI1\_}T$	$\rightarrow T$	z = k + 1	row index of $b_{kj}$ (1-based)		
$\mathtt{GxB\_SECONDJ\_}T$	$\rightarrow T$	z = j	column index of $b_{kj}$ (0-based)		
${\tt GxB\_SECONDJ1\_}T$	$\rightarrow T$	z = j + 1	column index of $b_{kj}$ (1-based)		

Positional binary operators for any type (including user-defined)					
	when used in all other methods				
GraphBLAS name	types (domains)	$z = f(a_{ij}, b_{ij})$	description		
$\mathtt{GxB\_FIRSTI\_}T$	$\rightarrow T$	z = i	row index of $a_{ij}$ (0-based)		
$\mathtt{GxB\_FIRSTI1\_}T$	$\rightarrow T$	z = i + 1	row index of $a_{ij}$ (1-based)		
${\tt GxB\_FIRSTJ\_}T$	$\rightarrow T$	z = j	column index of $a_{ij}$ (0-based)		
${\tt GxB\_FIRSTJ1\_}T$	$\rightarrow T$	z = j + 1	column index of $a_{ij}$ (1-based)		
$\mathtt{GxB\_SECONDI\_}T$	$\rightarrow T$	z = i	row index of $b_{ij}$ (0-based)		
$\mathtt{GxB\_SECONDI1\_}T$	$\rightarrow T$	z = i + 1	row index of $b_{ij}$ (1-based)		
$\mathtt{GxB\_SECONDJ\_}T$	$\rightarrow T$	z = j	column index of $b_{ij}$ (0-based)		
${\tt GxB\_SECONDJ1\_}T$	$\rightarrow T$	z = j + 1	column index of $b_{ij}$ (1-based)		

Finally, one special binary operator can only be used as input to GrB\_Matrix\_build or GrB\_Vector\_build: the GxB\_IGNORE\_DUP operator. If dup is NULL, any duplicates in the GrB\*build methods result in an error. If dup is the special binary operator GxB\_IGNORE\_DUP, then any duplicates are ignored. If duplicates appear, the last one in the list of tuples is taken and the prior ones ignored. This is not an error.

The next sections define the following methods for the GrB\_BinaryOp object:

GraphBLAS function	purpose	Section
GrB_BinaryOp_new	create a user-defined binary operator	6.3.1
<pre>GxB_BinaryOp_new</pre>	create a named user-defined binary operator	6.3.2
<pre>GrB_BinaryOp_wait</pre>	wait for a user-defined binary operator	6.3.3
<pre>GxB_BinaryOp_ztype_name</pre>	return the type of the output z for $z = f(x, y)$	6.3.4
<pre>GxB_BinaryOp_xtype_name</pre>	return the type of the input $x$ for $z = f(x, y)$	6.3.5
<pre>GxB_BinaryOp_ytype_name</pre>	return the type of the input y for $z = f(x, y)$	6.3.6
GrB_BinaryOp_free	free a user-defined binary operator	6.3.7

#### 6.3.1 GrB\_BinaryOp\_new: create a user-defined binary operator

GrB\_BinaryOp\_new creates a new binary operator. The new operator is returned in the binaryop handle, which must not be NULL on input. On output, its contents contains a pointer to the new binary operator.

The three types xtype, ytype, and ztype are the GraphBLAS types of the inputs x and y, and output z of the user-defined function z = f(x, y). These types may be built-in types or user-defined types, in any combination. The three types need not be the same, but they must be previously defined before passing them to  $GrB_BinaryOp_new$ .

The final argument to GrB\_BinaryOp\_new is a pointer to a user-defined function with the following signature:

```
void (*f) (void *z, const void *x, const void *y) ;
```

When the function f is called, the arguments z, x, and y are passed as (void \*) pointers, but they will be pointers to values of the correct type, defined by ztype, xtype, and ytype, respectively, when the operator was created.

**NOTE:** SuiteSparse:GraphBLAS may call the function with the pointers z and x equal to one another, in which case z=f(z,y) should be computed. Future versions may use additional pointer aliasing.

# **6.3.2** GxB\_BinaryOp\_new: create a named user-defined binary operator

```
GrB_Info GxB_BinaryOp_new
   GrB_BinaryOp *op,
                                   // handle for the new binary operator
   GxB_binary_function function,
                                   // pointer to the binary function
   GrB_Type ztype,
                                   // type of output z
   GrB_Type xtype,
                                   // type of input x
   GrB_Type ytype,
                                   // type of input y
   const char *binop_name,
                                  // name of the user function
   const char *binop_defn
                                   // definition of the user function
);
```

Creates a named GrB\_BinaryOp. Only the first 127 characters of binop\_name are used. The binop\_defn is a string containing the entire function itself. For example:

Currently, only the binop\_name is used, but future versions will rely on the binop\_defn when employing a JIT for better performance.

#### 6.3.3 GrB\_BinaryOp\_wait: wait for a binary operator

After creating a user-defined binary operator, a GraphBLAS library may choose to exploit non-blocking mode to delay its creation. GrB\_BinaryOp\_wait(&binaryop) ensures the binaryop is completed. SuiteSparse:GraphBLAS currently does nothing for GrB\_BinaryOp\_wait(&binaryop), except to ensure that the binaryop is valid. The function will change in SS:GrB v6.0.0 (for the v2.0 C API Specification).

### 6.3.4 GxB\_BinaryOp\_ztype\_name: return the name of the type of z

GxB\_BinaryOp\_ztype\_name returns name of the ztype of the binary operator, which is the type of z in the function z = f(x, y).

#### 6.3.5 GxB\_BinaryOp\_xtype\_name: return the name of the type of x

GxB\_BinaryOp\_xtype\_name returns name of the xtype of the binary operator, which is the type of x in the function z = f(x, y).

#### 6.3.6 GxB\_BinaryOp\_ytype\_name: return the name of the type of y

GxB\_BinaryOp\_ytype\_name returns name of the ytype of the binary operator, which is the type of y in the function z = f(x, y).

#### 6.3.7 GrB\_BinaryOp\_free: free a user-defined binary operator

GrB\_BinaryOp\_free frees a user-defined binary operator. Either usage:

```
GrB_BinaryOp_free (&op) ;
GrB_free (&op) ;
```

frees the op and sets op to NULL. It safely does nothing if passed a NULL handle, or if op == NULL on input. It does nothing at all if passed a built-in binary operator.

#### 6.3.8 ANY and PAIR operators

The PAIR operator is simple to describe: just f(x,y) = 1. It is called the PAIR operator since it returns 1 in a semiring when a pair of entries  $a_{ik}$  and  $b_{kj}$  is found in the matrix multiply. This operator is simple yet very useful. It allows purely symbolic computations to be performed on matrices of any type, without having to typecast them to Boolean with all values being true. Typecasting need not be performed on the inputs to the PAIR operator, and the PAIR operator does not have to access the values of the matrix, so it is a very fast operator to use.

The ANY operator is very unusual, but very powerful. It is the function  $f_{any}(x,y) = x$ , or y, where GraphBLAS has to freedom to select either x, or y, at its own discretion. Do not confuse the ANY operator with the any

function in Octave/MATLAB, which computes a reduction using the logical OR operator.

The ANY function is associative and commutative, and can thus serve as an operator for a monoid. The selection of x are y is not randomized. Instead, SuiteSparse:GraphBLAS uses this freedom to compute as fast a result as possible. When used in a dot product,

$$c_{ij} = \sum_{k} a_{ik} b_{kj}$$

for example, the computation can terminate as soon as any matching pair of entries is found. When used in a parallel saxpy-style computation, the ANY operator allows for a relaxed form of synchronization to be used, resulting in a fast benign race condition.

Because of this benign race condition, the result of the ANY monoid can be non-deterministic, unless it is coupled with the PAIR multiplicative operator. In this case, the ANY\_PAIR semiring will return a deterministic result, since  $f_{any}(1,1)$  is always 1.

When paired with a different operator, the results are non-deterministic. This gives a powerful method when computing results for which any value selected by the ANY operator is valid. One such example is the breadth-first-search tree. Suppose node j is at level v, and there are multiple nodes i at level v-1 for which the edge (i,j) exists in the graph. Any of these nodes i can serve as a valid parent in the BFS tree. Using the ANY operator, GraphBLAS can quickly compute a valid BFS tree; if it used again on the same inputs, it might return a different, yet still valid, BFS tree, due to the non-deterministic nature of intra-thread synchronization.

# 6.4 GraphBLAS IndexUnaryOp operators: GrB\_IndexUnaryOp

An index-unary operator is a scalar function of the form  $z = f(a_{ij}, i, j, \text{thunk})$  that is applied to the entries  $a_{ij}$  of an m-by-n matrix. It can be used in  $\texttt{GrB\_apply}$  (Section 10.11) or in  $\texttt{GrB\_select}$  (Section 10.12) to select entries from a matrix or vector.

The signature of the index-unary function f is as follows:

```
void f
                       // output value z, of type ztype
   void *z,
                       // input value x of type xtype; value of v(i) or A(i,j)
   const void *x,
// draft v2.0 spec:
                              // [i 0] for v(i) or [i j] of A(i,j)
// const GrB_Index *indices,
// GrB_Index n,
                               // 1 for vector index, 2 for matrix indices
// a better approach:
   int64_t i,
                      // row index of A(i,j)
                       // column index of A(i,j), or zero for v(i)
   int64_t j,
                      // input scalar thunk
   const void *thunk
);
```

The following built-in operators are available. Operators that do not depend on the value of A(i,j) can be used on any matrix or vector, including those of user-defined type. In the table, k refers to the thunk scalar, whose type matches the suffix of the operator. The VALUENE and VALUENE operators are defined for any built-in type. The other VALUE operators are defined only for real (not complex) built-in types. Any index computations are done in int64\_t arithmetic; the result is typecasted to int32\_t for the \*INDEX\_INT32 operators.

GraphBLAS name	Octave/MATLAB	description
	analog	
GrB_ROWINDEX_INT32	z=i+k	row index of A(i,j), as int32
GrB_ROWINDEX_INT64	z=i+k	row index of A(i,j), as int64
<pre>GrB_COLINDEX_INT32</pre>	z=j+k	column index of A(i,j), as int32
GrB_COLINDEX_INT64	z=j+k	column index of A(i,j), as int64
<pre>GrB_DIAGINDEX_INT32</pre>	z=j-(i+k)	column diagonal index of A(i,j), as int32
GrB_DIAGINDEX_INT64	z=j-(i+k)	column diagonal index of A(i,j), as int64
GrB_TRIL_INT64	z=(j<=(i+k))	true for entries on or below the kth diagonal
GrB_TRIU_INT64	z=(j>=(i+k))	true for entries on or above the kth diagonal
GrB_DIAG_INT64	z=(j==(i+k))	true for entries on the kth diagonal
GrB_OFFDIAG_INT64	z=(j!=(i+k))	true for entries not on the kth diagonal
GrB_COLLE_INT64	$z=(j \le k)$	true for entries in columns 0 to k
GrB_COLGT_INT64	z=(j>k)	true for entries in columns $k+1$ and above
GrB_ROWLE_INT64	$z=(i \le k)$	true for entries in rows 0 to k
GrB_ROWGT_INT64	z=(i>k)	true for entries in rows $k+1$ and above
GrB_VALUENE_T	z=(aij!=k)	true if A(i,j) is not equal to k
GrB_VALUEEQ_T	z=(aij==k)	true if A(i,j) is equal to k
GrB_VALUEGT_T	z=(aij>k)	true if A(i,j) is greater than k
GrB_VALUEGE_T	z=(aij>=k)	true if A(i,j) is greater than or equal to k
GrB_VALUELT_T	z=(aij <k)< td=""><td>true if A(i,j) is less than k</td></k)<>	true if A(i,j) is less than k
GrB_VALUELE_T	z=(aij<=k)	true if A(i,j) is less than or equal to k

The following methods operate on the GrB\_IndexUnaryOp object:

GraphBLAS function	purpose	Section
GrB_IndexUnaryOp_new	create a user-defined index-unary operator	6.4.1
<pre>GxB_IndexUnaryOp_new</pre>	create a named user-defined index-unary operator	6.4.2
<pre>GrB_IndexUnaryOp_wait</pre>	wait for a user-defined index-unary operator	6.4.3
<pre>GrB_IndexUnaryOp_ztype_name</pre>	return the type of the output $z$	6.4.4
<pre>GrB_IndexUnaryOp_xtype_name</pre>	return the type of the input $x$	6.4.5
<pre>GrB_IndexUnaryOp_ttype_name</pre>	return the type of the input thunk	6.4.6
<pre>GrB_IndexUnaryOp_free</pre>	free a user-defined select operator	6.4.7

# $\textbf{6.4.1} \quad \mathsf{GrB\_IndexUnaryOp\_new: \ create \ a \ user-defined \ index-unary \ operator }$

```
GrB_Info GrB_IndexUnaryOp_new // create a new user-defined IndexUnary op

(
GrB_IndexUnaryOp *op, // handle for the new IndexUnary operator 
void *function, // pointer to IndexUnary function 
GrB_Type ztype, // type of output z
```

```
GrB_Type xtype, // type of input x (the A(i,j) entry)
GrB_Type ttype // type of input thunk
);
```

GrB\_IndexUnaryOp\_new creates a new index-unary operator. The new operator is returned in the op handle, which must not be NULL on input. On output, its contents contains a pointer to the new index-unary operator.

The function argument to  $GrB_IndexUnaryOp_new$  is a pointer to a user-defined function whose signature is given at the beginning of Section 6.4. Given the properties of an entry  $a_{ij}$  in a matrix, the function should return z as true if the entry should be kept in the output of  $GrB_select$ , or false if it should not appear in the output. If the return value is not  $GrB_BOOL$ , it is typecasted to  $GrB_BOOL$  by  $GrB_select$ .

The type xtype is the GraphBLAS type of the input x of the user-defined function z = f(x, i, j, thunk), which is used for the entry A(i,j) of a matrix or v(i) of a vector. The type may be built-in or user-defined.

The type ttype is the GraphBLAS type of the input thunk of the user-defined function z = f(x, i, j, thunk). The type may be built-in or user-defined.

# **6.4.2** GxB\_IndexUnaryOp\_new: create a named user-defined index-unary operator

```
GrB_Info GxB_IndexUnaryOp_new // create a named user-created IndexUnaryOp
    GrB_IndexUnaryOp *op,
                                    // handle for the new IndexUnary operator
    GxB_index_unary_function function,
                                          // pointer to index_unary function
    GrB_Type ztype,
                                    // type of output z
    GrB_Type xtype,
                                    // type of input x
    GrB_Type ttype,
                                   // type of input thunk
    const char *idxop_name,
                                    // name of the user function
    const char *idxop_defn
                                    // definition of the user function
);
```

Creates a named GrB\_IndexUnaryOp. Only the first 127 characters of idxop\_name are used. The ixdop\_defn is a string containing the entire function itself. Currently, only the idxop\_name is used, but future versions will rely on the idxop\_defn when employing a JIT for better performance.

### 6.4.3 GrB\_IndexUnaryOp\_wait: wait for an index-unary operator

After creating a user-defined select operator, a GraphBLAS library may choose to exploit non-blocking mode to delay its creation. GrB\_IndexUnaryOp\_wait(&op) ensures the op is completed. SuiteSparse:GraphBLAS currently does nothing for GrB\_IndexUnaryOp\_wait(&op), except to ensure that the op is valid. The function will change in SS:GrB v6.0.0 (for the v2.0 C API Specification).

# 6.4.4 GxB\_IndexUnaryOp\_ztype\_name: return the name of the type of

GrB\_IndexUnaryOp\_ztype\_name returns the ztype of the index-unary operator, which is the type of z in the function z = f(x, i, j, thunk).

# **6.4.5** GxB\_IndexUnaryOp\_xtype\_name: return the name of the type of x

GrB\_IndexUnaryOp\_xtype\_name returns the xtype of the index-unary operator, which is the type of x in the function z = f(x, i, j, thunk). This input is used for the entry A(i,j) of a matrix or v(i) of a vector.

# 6.4.6 GxB\_IndexUnaryOp\_ttype\_name: return the name of the type of thunk

GrB\_IndexUnaryOp\_ttype\_name returns the ttype of the index-unary operator, which is the type of the thunk in the function z = f(x, i, j, thunk).

# 6.4.7 GrB\_IndexUnaryOp\_free: free a user-defined index-unary operator

GrB\_IndexUnaryOp\_free frees a user-defined index-unary operator. Either usage:

```
GrB_IndexUnaryOp_free (&op) ;
GrB_free (&op) ;
```

frees the op and sets op to NULL. It safely does nothing if passed a NULL handle, or if op == NULL on input. It does nothing at all if passed a built-in index-unary operator.

# 6.5 GraphBLAS monoids: GrB\_Monoid

A monoid is defined on a single domain (that is, a single type), T. It consists of an associative binary operator z = f(x, y) whose three operands x, y, and z are all in this same domain T (that is  $T \times T \to T$ ). The associative operator must also have an identity element, or "zero" in this domain, such that f(x,0) = f(0,x) = x. Recall that an associative operator f(x,y) is one for which the condition f(a, f(b,c)) = f(f(a,b),c) always holds. That is, operator can be applied in any order and the results remain the same.

Predefined binary operators that can be used to form monoids are listed in the table below. Most of these are the binary operators of predefined monoids, except that the bitwise monoids are predefined only for the unsigned integer types, not the signed integers.

Recall that T denotes any built-in type (including boolean, integer, floating point real, and complex), R denotes any non-complex type, and I denotes any integer type.

GraphBLAS	types (domains)	expression	identity	terminal
operator		z = f(x, y)		
${\tt GrB\_PLUS\_}T$	$T \times T \to T$	z = x + y	0	none
${\tt GrB\_TIMES\_}T$	$T \times T \to T$	z = xy	1	0 or none (see note)
${\tt GxB\_ANY\_}T$	$T \times T \to T$	z = x  or  y	any	any
${\tt GrB\_MIN\_}R$	$R \times R \to R$	$z = \min(x, y)$	$+\infty$	$-\infty$
${\tt GrB\_MAX\_}R$	$R \times R \to R$	$z = \max(x, y)$	$-\infty$	$+\infty$
GrB_LOR	$\texttt{bool} \times \texttt{bool} \to \texttt{bool}$	$z = x \vee y$	false	true
GrB_LAND	$\mathtt{bool} \times \mathtt{bool} \to \mathtt{bool}$	$z = x \wedge y$	true	false
GrB_LXOR	$\mathtt{bool} \times \mathtt{bool} \to \mathtt{bool}$	$z = x \veebar y$	false	none
GrB_LXNOR	$\mathtt{bool} \times \mathtt{bool} \to \mathtt{bool}$	z = (x == y)	true	none
${\tt GrB\_BOR\_}I$	$I \times I \to I$	z=x y	all bits zero	all bits one
${\tt GrB\_BAND\_}I$	$I \times I \to I$	z=x&y	all bits one	all bits zero
${\tt GrB\_BXOR\_}I$	$I \times I \to I$	z=x^y	all bits zero	none
${\tt GrB\_BXNOR\_}I$	$I \times I \to I$	$z=(x^y)$	all bits one	none

The above table lists the GraphBLAS operator, its type, expression, identity value, and terminal value (if any). For these built-in operators, the terminal values are the annihilators of the function, which is the value z so that z = f(z, y) regardless of the value of y. For example  $\min(-\infty, y) = -\infty$  for any y. For integer domains,  $+\infty$  and  $-\infty$  are the largest and smallest integer in their range. With unsigned integers, the smallest value is zero, and thus  $\mathtt{GrB\_MIN\_UINT8}$  has an identity of 255 and a terminal value of 0.

When computing with a monoid, the computation can terminate early if the terminal value arises. No further work is needed since the result will not change. This value is called the terminal value instead of the annihilator, since a user-defined operator can be created with a terminal value that is not an annihilator. See Section 6.5.3 for an example.

The GxB\_ANY\_\* monoid can terminate as soon as it finds any value at all. NOTE: The GrB\_TIMES\_FP\* operators do not have a terminal value of zero, since they comply with the IEEE 754 standard, and O\*NaN is not zero, but NaN. Technically, their terminal value is NaN, but this value is rare in practice and thus the terminal condition is not worth checking.

The C API Specification includes 44 predefined monoids, with the naming convention GrB\_op\_MONOID\_type. Forty monoids are available for the four operators MIN, MAX, PLUS, and TIMES, each with the 10 non-boolean real types. Four boolean monoids are predefined: GrB\_LOR\_MONOID\_BOOL, GrB\_LAND\_MONOID\_BOOL, GrB\_LXOR\_MONOID\_BOOL, and GrB\_LXNOR\_MONOID\_BOOL.

These all appear in SuiteSparse:GraphBLAS, which adds 33 additional predefined GxB\* monoids, with the naming convention GxB\_op\_type\_MONOID. The ANY operator can be used for all 13 types (including complex). The PLUS and TIMES operators are provided for both complex types, for 4 additional complex monoids. Sixteen monoids are predefined for four bitwise operators (BOR, BAND, BXOR, and BNXOR), each with four unsigned integer types (UINT8, UINT16, UINT32, and UINT64).

The next sections define the following methods for the GrB\_Monoid object:

GraphBLAS function	purpose	Section
GrB_Monoid_new	create a user-defined monoid	6.5.1
<pre>GrB_Monoid_wait</pre>	wait for a user-defined monoid	6.5.2
<pre>GxB_Monoid_terminal_new</pre>	create a monoid that has a terminal value	6.5.3
<pre>GxB_Monoid_operator</pre>	return the monoid operator	6.5.4
<pre>GxB_Monoid_identity</pre>	return the monoid identity value	6.5.5
<pre>GxB_Monoid_terminal</pre>	return the monoid terminal value (if any)	6.5.6
GrB_Monoid_free	free a monoid	6.5.7

#### 6.5.1 GrB Monoid new: create a monoid

GrB\_Monoid\_new creates a monoid. The operator, op, must be an associative binary operator, either built-in or user-defined.

In the definition above, <type> is a place-holder for the specific type of the monoid. For built-in types, it is the C type corresponding to the built-in type (see Section 6.1), such as bool, int32\_t, float, or double. In this case, identity is a scalar value of the particular type, not a pointer. For user-defined types, <type> is void \*, and thus identity is a not a scalar itself but a void \* pointer to a memory location containing the identity value of the user-defined operator, op.

If op is a built-in operator with a known identity value, then the identity parameter is ignored, and its known identity value is used instead.

#### 6.5.2 GrB\_Monoid\_wait: wait for a monoid

After creating a user-defined monoid, a GraphBLAS library may choose to exploit non-blocking mode to delay its creation. GrB\_Monoid\_wait(&monoid) ensures the monoid is completed. SuiteSparse:GraphBLAS currently does nothing for GrB\_Monoid\_wait(&monoid), except to ensure that the monoid is valid. The function will change in SS:GrB v6.0.0 (for the v2.0 C API Specification).

#### 6.5.3 GxB\_Monoid\_terminal\_new: create a monoid with terminal

GxB\_Monoid\_terminal\_new is identical to GrB\_Monoid\_new, except that it allows for the specification of a *terminal value*. The <type> of the terminal value is the same as the identity parameter; see Section 6.5.1 for details.

The terminal value of a monoid is the value z for which z = f(z, y) for any y, where z = f(x, y) is the binary operator of the monoid. This is also called the *annihilator*, but the term *terminal value* is used here. This is because all annihilators are terminal values, but a terminal value need not be an annihilator, as described in the MIN example below.

If the terminal value is encountered during computation, the rest of the computations can be skipped. This can greatly improve the performance of  $GrB\_reduce$ , and matrix multiply in specific cases (when a dot product method is used). For example, using  $GrB\_reduce$  to compute the sum of all entries in a  $GrB\_FP32$  matrix with e entries takes O(e) time, since a monoid based on  $GrB\_PLUS\_FP32$  has no terminal value. By contrast, a reduction using  $GrB\_LOR$  on a  $GrB\_BOOL$  matrix can take as little as O(1) time, if a true value is found in the matrix very early.

Monoids based on the built-in GrB\_MIN\_\* and GrB\_MAX\_\* operators (for any type), the boolean GrB\_LOR, and the boolean GrB\_LAND operators all have terminal values. For example, the identity value of GrB\_LOR is false, and its terminal value is true. When computing a reduction of a set of boolean values to a single value, once a true is seen, the computation can exit early since the result is now known.

If op is a built-in operator with known identity and terminal values, then the identity and terminal parameters are ignored, and its known identity and terminal values are used instead.

There may be cases in which the user application needs to use a non-standard terminal value for a built-in operator. For example, suppose the matrix has type GrB\_FP32, but all values in the matrix are known to be non-negative. The annihilator value of MIN is -INFINITY, but this will never be seen. However, the computation could could terminate when finding the

value zero. This is an example of using a terminal value that is not actually an annihilator, but it functions like one since the monoid will operate strictly on non-negative values. In this case, a monoid created with <code>GrB\_MIN\_FP32</code> will not terminate early. To create a monoid that can terminate early, create a user-defined operator that computes the same thing as <code>GrB\_MIN\_FP32</code>, and then create a monoid based on this user-defined operator with a terminal value of zero and an identity of <code>+INFINITY</code>.

#### 6.5.4 GxB\_Monoid\_operator: return the monoid operator

GxB\_Monoid\_operator returns the binary operator of the monoid.

### **6.5.5** GxB\_Monoid\_identity: return the monoid identity

GxB\_Monoid\_identity returns the identity value of the monoid. The void \* pointer, identity, must be non-NULL and must point to a memory space of size at least equal to the size of the type of the monoid. The type size can be obtained via GxB\_Monoid\_operator to return the monoid additive operator, then GxB\_BinaryOp\_ztype to obtain the ztype, followed by GxB\_Type\_size to get its size.

#### 6.5.6 GxB Monoid terminal: return the monoid terminal value

```
GrB_Info GxB_Monoid_terminal  // return the monoid terminal (

bool *has_terminal,  // true if the monoid has a terminal value void *terminal,  // returns the terminal of the monoid GrB_Monoid monoid  // monoid to query

);
```

GxB\_Monoid\_terminal returns the terminal value of the monoid (if any). The void \* pointer, terminal, must be non-NULL and must point to a memory space of size at least equal to the size of the type of the monoid. The type size can be obtained via GxB\_Monoid\_operator to return the monoid additive operator, then GxB\_BinaryOp\_ztype to obtain the ztype, followed by GxB\_Type\_size to get its size.

If the monoid has a terminal value, then has\_terminal is true, and its value is returned in the terminal parameter. If it has no terminal value, then has\_terminal is false, and the terminal parameter is not modified.

#### 6.5.7 GrB Monoid free: free a monoid

GrB\_Monoid\_frees frees a monoid. Either usage:

```
GrB_Monoid_free (&monoid) ;
GrB_free (&monoid) ;
```

frees the monoid and sets monoid to NULL. It safely does nothing if passed a NULL handle, or if monoid == NULL on input. It does nothing at all if passed a built-in monoid.

# 6.6 GraphBLAS semirings: GrB\_Semiring

A semiring defines all the operators required to define the multiplication of two sparse matrices in GraphBLAS,  $\mathbf{C} = \mathbf{AB}$ . The "add" operator is a commutative and associative monoid, and the binary "multiply" operator defines a function z = fmult(x,y) where the type of z matches the exactly with the monoid type. SuiteSparse:GraphBLAS includes 1,473 predefined built-in semirings. The next sections define the following methods for the GrB\_Semiring object:

GraphBLAS function	purpose	Section
GrB_Semiring_new	create a user-defined semiring	6.6.1
<pre>GrB_Semiring_wait</pre>	wait for a user-defined semiring	6.6.2
${\tt GxB\_Semiring\_add}$	return the additive monoid of a semiring	6.6.3
GxB_Semiring_multiply	return the binary operator of a semiring	6.6.4
<pre>GrB_Semiring_free</pre>	free a semiring	6.6.5

### 6.6.1 GrB\_Semiring\_new: create a semiring

GrB\_Semiring\_new creates a new semiring, with add being the additive monoid and multiply being the binary "multiply" operator. In addition to the standard error cases, the function returns GrB\_DOMAIN\_MISMATCH if the output (ztype) domain of multiply does not match the domain of the add monoid. Using built-in types and operators, 2,438 semirings can be built. This count excludes redundant Boolean operators (for example GrB\_TIMES\_BOOL and GrB\_LAND are different operators but they are redundant since they always return the same result).

The v1.3 C API Specification for GraphBLAS includes 124 predefined semirings, with names of the form  $GrB_add_mult_SEMIRING_type$ , where add is the operator of the additive monoid, mult is the multiply operator, and type is the type of the input x to the multiply operator, f(x,y). The name of the domain for the additive monoid does not appear in the name, since it always matches the type of the output of the mult operator.

Twelve kinds of GrB\* semirings are available for all 10 real, non-boolean types: PLUS\_TIMES, PLUS\_MIN, MIN\_PLUS, MIN\_TIMES, MIN\_FIRST, MIN\_SECOND, MIN\_MAX, MAX\_PLUS, MAX\_TIMES, MAX\_FIRST, MAX\_SECOND, and MAX\_MIN. Four semirings are for boolean types only: LOR\_LAND, LAND\_LOR, LXOR\_LAND, and LXNOR\_LOR.

SuiteSparse:GraphBLAS pre-defines 1,553 semirings from built-in types and operators, listed below. The naming convention is <code>GxB\_add\_mult\_type</code>. The 124 <code>GrB\*</code> semirings are a subset of the list below, included with two names: <code>GrB\*</code> and <code>GxB\*</code>. If the <code>GrB\*</code> name is provided, its use is preferred, for portability to other <code>GraphBLAS</code> implementations.

- 1000 semirings with a multiplier  $T \times T \to T$  where T is any of the 10 non-Boolean, real types, from the complete cross product of:
  - 5 monoids (MIN, MAX, PLUS, TIMES, ANY)
  - 20 multiply operators (FIRST, SECOND, PAIR, MIN, MAX, PLUS, MINUS, RMINUS, TIMES, DIV, RDIV, ISEQ, ISNE, ISGT, ISLT, ISGE, ISLE, LOR, LAND, LXOR).
  - 10 non-Boolean types, T
- 300 semirings with a comparator  $T \times T \to \texttt{bool}$ , where T is non-Boolean and real, from the complete cross product of:
  - 5 Boolean monoids (LAND, LOR, LXOR, EQ, ANY)
  - 6 multiply operators (EQ, NE, GT, LT, GE, LE)
  - 10 non-Boolean types, T
- 55 semirings with purely Boolean types, bool × bool → bool, from the complete cross product of:
  - 5 Boolean monoids (LAND, LOR, LXOR, EQ, ANY)
  - 11 multiply operators (FIRST, SECOND, PAIR, LOR, LAND, LXOR, EQ, GT, LT, GE, LE)
- 54 complex semirings,  $Z \times Z \to Z$  where Z is GxB\_FC32 (single precision complex) or GxB\_FC64 (double precision complex):
  - 3 complex monoids (PLUS, TIMES, ANY)

- 9 complex multiply operators (FIRST, SECOND, PAIR, PLUS, MINUS, TIMES, DIV, RDIV, RMINUS)
- -2 complex types, Z
- 64 bitwise semirings,  $U \times U \to U$  where U is an unsigned integer.
  - 4 bitwise monoids (BOR, BAND, BXOR, BXNOR)
  - 4 bitwise multiply operators (the same list)
  - 4 unsigned integer types
- 80 positional semirings,  $X \times X \to T$  where T is INT32 or INT64:
  - 5 monoids (MIN, MAX, PLUS, TIMES, ANY)
  - 8 positional operators (FIRSTI, FIRSTI1, FIRSTJ1, FIRSTJ1, SECONDI, SECONDI1, SECONDJ, SECONDJ1)
  - 2 integer types (INT32, INT64)

### **6.6.2** GrB\_Semiring\_wait: wait for a semiring

After creating a user-defined semiring, a GraphBLAS library may choose to exploit non-blocking mode to delay its creation. GrB\_Semiring\_wait(&semiring) ensures the semiring is completed. SuiteSparse:GraphBLAS currently does nothing for GrB\_Semiring\_wait(&semiring), except to ensure that the semiring is valid. The function will change in SS:GrB v6.0.0 (for the v2.0 C API Specification).

# 6.6.3 GxB\_Semiring\_add: return the additive monoid of a semiring

GxB\_Semiring\_add returns the additive monoid of a semiring.

# 6.6.4 GxB\_Semiring\_multiply: return multiply operator of a semiring

GxB\_Semiring\_multiply returns the binary multiplicative operator of a semiring.

## 6.6.5 GrB\_Semiring\_free: free a semiring

GrB\_Semiring\_free frees a semiring. Either usage:

```
GrB_Semiring_free (&semiring) ;
GrB_free (&semiring) ;
```

frees the semiring and sets semiring to NULL. It safely does nothing if passed a NULL handle, or if semiring == NULL on input. It does nothing at all if passed a built-in semiring.

# 6.7 GraphBLAS scalars: GrB\_Scalar

This section describes a set of methods that create, modify, query, and destroy a GraphBLAS scalar, GrB\_Scalar:

GraphBLAS function	purpose	Section
GrB_Scalar_new	create a scalar	6.7.1
<pre>GrB_Scalar_wait</pre>	wait for a scalar	6.7.2
<pre>GrB_Scalar_dup</pre>	copy a scalar	6.7.3
<pre>GrB_Scalar_clear</pre>	clear a scalar of its entry	6.7.4
<pre>GrB_Scalar_nvals</pre>	return number of entries in a scalar	6.7.5
<pre>GxB_Scalar_type_name</pre>	return name of the type of a scalar	6.7.6
<pre>GrB_Scalar_setElement</pre>	set the single entry of a scalar	6.7.7
<pre>GrB_Scalar_extractElement</pre>	get the single entry from a scalar	6.7.8
<pre>GxB_Scalar_memoryUsage</pre>	memory used by a scalar	6.7.9
GrB_Scalar_free	free a scalar	6.7.10

#### 6.7.1 GrB\_Scalar\_new: create a scalar

```
GrB_Info GrB_Scalar_new  // create a new GrB_Scalar with no entry
(
    GrB_Scalar *s,  // handle of GrB_Scalar to create
    GrB_Type type  // type of GrB_Scalar to create
);
```

GrB\_Scalar\_new creates a new scalar with no entry in it, of the given type. This is analogous to Octave/MATLAB statement s = sparse(0), except that GraphBLAS can create scalars any type. The pattern of the new scalar is empty.

#### 6.7.2 GrB\_Scalar\_wait: wait for a scalar

In non-blocking mode, the computations for a GrB\_Scalar may be delayed. In this case, the scalar is not yet safe to use by multiple independent user threads. A user application may force completion of a scalar s via GrB\_Scalar\_wait(&s). After this call, different user threads may safely call GraphBLAS operations that use the scalar s as an input parameter. The function will change in SS:GrB v6.0.0 (for the v2.0 C API Specification).

### 6.7.3 GrB\_Scalar\_dup: copy a scalar

GrB\_Scalar\_dup makes a deep copy of a scalar. In GraphBLAS, it is possible, and valid, to write the following:

Then s and t can be used interchangeably. However, only a pointer reference is made, and modifying one of them modifies both, and freeing one of them leaves the other as a dangling handle that should not be used. If two different scalars are needed, then this should be used instead:

Then **s** and **t** are two different scalars that currently have the same value, but they do not depend on each other. Modifying one has no effect on the other.

#### 6.7.4 GrB\_Scalar\_clear: clear a scalar of its entry

GrB\_Scalar\_clear clears the entry from a scalar. The pattern of s is empty, just as if it were created fresh with GrB\_Scalar\_new. Analogous with s = sparse (0) in Octave/MATLAB. The type of s does not change. Any pending updates to the scalar are discarded.

#### 6.7.5 GrB\_Scalar\_nyals: return the number of entries in a scalar

GrB\_Scalar\_nvals returns the number of entries in a scalar, which is either 0 or 1. Roughly analogous to nvals = nnz(s) in Octave/MATLAB, except that the implicit value in GraphBLAS need not be zero and nnz (short for "number of nonzeros") in MATLAB is better described as "number of entries" in GraphBLAS.

# 6.7.6 GxB\_Scalar\_type\_name: return name of the type of a scalar

GxB\_Scalar\_type\_name returns the name of the type of a scalar. Analogous to type = class (s) in MATLAB.

#### 6.7.7 GrB\_Scalar\_setElement: set the single entry of a scalar

GrB\_Scalar\_setElement sets the single entry in a scalar, like s = sparse(x) in MATLAB notation. For further details of this function, see GrB\_Matrix\_setElement in Section 6.9.11. If an error occurs, GrB\_error(&err,s) returns details about the error.

### 6.7.8 GrB\_Scalar\_extractElement: get the single entry from a scalar

GrB\_Scalar\_extractElement extracts the single entry from a sparse scalar, like x = full(s) in MATLAB. Further details of this method are discussed in Section 6.9.12, which discusses GrB\_Matrix\_extractElement. NOTE: if no entry is present in the scalar s, then x is not modified, and the return value of GrB\_Scalar\_extractElement is GrB\_NO\_VALUE.

#### 6.7.9 GxB\_Scalar\_memoryUsage: memory used by a scalar

```
GrB_Info GxB_Scalar_memoryUsage // return # of bytes used for a scalar
(
    size_t *size, // # of bytes used by the scalar s
    const GrB_Scalar s // GrB_Scalar to query
);
```

Returns the memory space required for a scalar, in bytes.

#### 6.7.10 GrB\_Scalar\_free: free a scalar

GrB\_Scalar\_free frees a scalar. Either usage:

```
GrB_Scalar_free (&s) ;
GrB_free (&s) ;
```

frees the scalar s and sets s to NULL. It safely does nothing if passed a NULL handle, or if s == NULL on input. Any pending updates to the scalar are abandoned.

# 6.8 GraphBLAS vectors: GrB\_Vector

This section describes a set of methods that create, modify, query, and destroy a GraphBLAS sparse vector,  $\mathtt{GrB\_Vector}$ :

GraphBLAS function	purpose	Section
GrB_Vector_new	create a vector	6.8.1
<pre>GrB_Vector_wait</pre>	wait for a vector	6.8.2
<pre>GrB_Vector_dup</pre>	copy a vector	6.8.3
<pre>GrB_Vector_clear</pre>	clear a vector of all entries	6.8.4
<pre>GrB_Vector_size</pre>	size of a vector	6.8.5
<pre>GrB_Vector_nvals</pre>	number of entries in a vector	6.8.6
<pre>GxB_Vector_type_name</pre>	name of the type of a vector	6.8.7
GrB_Vector_build	build a vector from tuples	6.8.8
<pre>GxB_Vector_build_Scalar</pre>	build a vector from tuples	6.8.9
<pre>GrB_Vector_setElement</pre>	add an entry to a vector	6.8.10
<pre>GrB_Vector_extractElement</pre>	get an entry from a vector	6.8.11
<pre>GrB_Vector_removeElement</pre>	remove an entry from a vector	6.8.12
<pre>GrB_Vector_extractTuples</pre>	get all entries from a vector	6.8.13
<pre>GrB_Vector_resize</pre>	resize a vector	6.8.14
<pre>GxB_Vector_diag</pre>	extract a diagonal from a matrix	6.8.15
<pre>GxB_Vector_iso</pre>	query iso status	6.8.16
<pre>GxB_Vector_memoryUsage</pre>	memory used by a vector	6.8.17
<pre>GrB_Vector_serializeSize</pre>	return size of serialized vector	6.10.1
<pre>GrB_Vector_serialize</pre>	serialize a vector	6.10.2
<pre>GxB_Vector_serialize</pre>	serialize a vector	6.10.3
<pre>GrB_Vector_deserialize</pre>	deserialize a vector	6.10.4
<pre>GxB_Vector_deserialize</pre>	deserialize a vector	6.10.5
<pre>GrB_Vector_free</pre>	free a vector	6.8.18
GxB_Vector_import_CSC, GxB_Vector_pack_CSC	import in CSC format	6.11.1, 6.11.2
<pre>GxB_Vector_export_CSC, GxB_Vector_unpack_CSC</pre>	export in CSC format	6.11.3,6.11.4
GxB_Vector_import_Bitmap, GxB_Vector_pack_Bitmap	import in bitmap format	6.11.5, 6.11.6
<pre>GxB_Vector_export_Bitmap, GxB_Vector_unpack_Bitmap</pre>	export in bitmap format	6.11.7,  6.11.8
GxB_Vector_import_Full, GxB_Vector_pack_Full	import in full format	6.11.9, 6.11.10
<pre>GxB_Vector_export_Full, GxB_Vector_unpack_Full</pre>	export in full format	6.11.11, 6.11.12

Refer to Section 6.11 for a discussion the import/export/pack/unpack methods.

## 6.8.1 GrB\_Vector\_new: create a vector

```
GrB_Info GrB_Vector_new // create a new vector with no entries

(
GrB_Vector *v, // handle of vector to create
GrB_Type type, // type of vector to create
GrB_Index n // vector dimension is n-by-1

);
```

GrB\_Vector\_new creates a new n-by-1 sparse vector with no entries in it, of the given type. This is analogous to Octave/MATLAB statement v = sparse (n,1), except that GraphBLAS can create sparse vectors any type. The pattern of the new vector is empty.

#### 6.8.2 GrB\_Vector\_wait: wait for a vector

In non-blocking mode, the computations for a GrB\_Vector may be delayed. In this case, the vector is not yet safe to use by multiple independent user threads. A user application may force completion of a vector w via GrB\_Vector\_wait(&w). After this call, different user threads may safely call GraphBLAS operations that use the vector w as an input parameter. The function will change in SS:GrB v6.0.0 (for the v2.0 C API Specification).

## 6.8.3 GrB\_Vector\_dup: copy a vector

GrB\_Vector\_dup makes a deep copy of a sparse vector. In GraphBLAS, it is possible, and valid, to write the following:

Then w and u can be used interchangeably. However, only a pointer reference is made, and modifying one of them modifies both, and freeing one of them leaves the other as a dangling handle that should not be used. If two different vectors are needed, then this should be used instead:

```
GrB_Vector u, w ;
GrB_Vector_new (&u, GrB_FP64, n) ;
GrB_Vector_dup (&w, u) ;  // like w = u, but making a deep copy
```

Then w and u are two different vectors that currently have the same set of values, but they do not depend on each other. Modifying one has no effect on the other.

#### 6.8.4 GrB\_Vector\_clear: clear a vector of all entries

GrB\_Vector\_clear clears all entries from a vector. All values v(i) are now equal to the implicit value, depending on what semiring ring is used to perform computations on the vector. The pattern of v is empty, just as if it were created fresh with GrB\_Vector\_new. Analogous with v (:) = sparse(0) in MATLAB. The type and dimension of v do not change. Any pending updates to the vector are discarded.

#### 6.8.5 GrB\_Vector\_size: return the size of a vector

 $GrB_Vector\_size$  returns the size of a vector (the number of rows). Analogous to n = length(v) or n = size(v, 1) in MATLAB.

#### 6.8.6 GrB Vector nyals: return the number of entries in a vector

GrB\_Vector\_nvals returns the number of entries in a vector. Roughly analogous to nvals = nnz(v) in MATLAB, except that the implicit value in GraphBLAS need not be zero and nnz (short for "number of nonzeros") in MATLAB is better described as "number of entries" in GraphBLAS.

## 6.8.7 GxB\_Vector\_type\_name: return name of the type of a vector

GxB\_Vector\_type\_name returns the name of the type of a vector. Analogous to type = class (v) in MATLAB.

## 6.8.8 GrB\_Vector\_build: build a vector from a set of tuples

GrB\_Vector\_build constructs a sparse vector w from a set of tuples, I and X, each of length nvals. The vector w must have already been initialized with GrB\_Vector\_new, and it must have no entries in it before calling GrB\_Vector\_build.

This function is just like GrB\_Matrix\_build (see Section 6.9.9), except that it builds a sparse vector instead of a sparse matrix. For a description

of what GrB\_Vector\_build does, refer to GrB\_Matrix\_build. For a vector, the list of column indices J in GrB\_Matrix\_build is implicitly a vector of length nvals all equal to zero. Otherwise the methods are identical.

If dup is NULL, any duplicates result in an error. If dup is the special binary operator GxB\_IGNORE\_DUP, then any duplicates are ignored. If duplicates appear, the last one in the list of tuples is taken and the prior ones ignored. This is not an error.

**SPEC:** Results are defined even if dup is non-associative.

#### 6.8.9 GxB\_Vector\_build\_Scalar: build a vector from a set of tuples

GxB\_Vector\_build\_Scalar constructs a sparse vector w from a set of tuples defined by the index array I of length nvals, and a scalar. The scalar is the value of all of the tuples. Unlike GrB\_Vector\_build, there is no dup operator to handle duplicate entries. Instead, any duplicates are silently ignored (if the number of duplicates is desired, simply compare the input nvals with the value returned by GrB\_Vector\_nvals after the vector is constructed). All entries in the sparsity pattern of w are identical, and equal to the input scalar value.

#### 6.8.10 GrB\_Vector\_setElement: add an entry to a vector

GrB\_Vector\_setElement sets a single entry in a vector, w(i) = x. The operation is exactly like setting a single entry in an n-by-1 matrix, A(i,0) = x, where the column index for a vector is implicitly j=0. For further details of

this function, see GrB\_Matrix\_setElement in Section 6.9.11. If an error occurs, GrB\_error(&err,w) returns details about the error.

### 6.8.11 GrB\_Vector\_extractElement: get an entry from a vector

TODO: add GrB\_Scalar

GrB\_Vector\_extractElement extracts a single entry from a vector,  $\mathbf{x} = \mathbf{v}(\mathbf{i})$ . The method is identical to extracting a single entry  $\mathbf{x} = \mathbf{A}(\mathbf{i}, \mathbf{0})$  from an n-by-1 matrix, so further details of this method are discussed in Section 6.9.12, which discusses GrB\_Matrix\_extractElement. In this case, the column index is implicitly j=0. If  $\mathbf{v}(\mathbf{i})$  is not present, then  $\mathbf{x}$  is not modified, and GrB\_Vector\_extractElement returns GrB\_NO\_VALUE.

#### 6.8.12 GrB\_Vector\_removeElement: remove an entry from a vector

GrB\_Vector\_removeElement removes a single entry w(i) from a vector. If no entry is present at w(i), then the vector is not modified. If an error occurs, GrB\_error(&err,w) returns details about the error.

#### 6.8.13 GrB\_Vector\_extractTuples: get all entries from a vector

GrB\_Vector\_extractTuples extracts all tuples from a sparse vector, analogous to [I,~,X] = find(v) in Octave/MATLAB. This function is identical to its GrB\_Matrix\_extractTuples counterpart, except that the array

of column indices J does not appear in this function. Refer to Section 6.9.14 where further details of this function are described.

#### 6.8.14 GrB Vector resize: resize a vector

```
GrB_Info GrB_Vector_resize // change the size of a vector

(
GrB_Vector u, // vector to modify
GrB_Index nrows_new // new number of rows in vector
);
```

GrB\_Vector\_resize changes the size of a vector. If the dimension decreases, entries that fall outside the resized vector are deleted.

#### 6.8.15 GxB\_Vector\_diag: extract a diagonal from a matrix

GxB\_Vector\_diag extracts a vector v from an input matrix A, which may be rectangular. If k = 0, the main diagonal of A is extracted; k > 0 denotes diagonals above the main diagonal of A, and k < 0 denotes diagonals below the main diagonal of A. Let A have dimension m-by-n. If k is in the range 0 to n - 1, then v has length  $\min(m, n - k)$ . If k is negative and in the range -1 to -m + 1, then v has length  $\min(m + k, n)$ . If k is outside these ranges, v has length 0 (this is not an error). This function computes the same thing as the Octave/MATLAB statement v=diag(A,k) when A is a matrix, except that GxB\_Vector\_diag can also do typecasting.

The vector  $\mathbf{v}$  must already exist on input, and  $\mathtt{GrB\_Vector\_size}$  (&len,  $\mathbf{v}$ ) must return  $\mathtt{len} = 0$  if  $\mathbf{k} \geq n$  or  $\mathbf{k} \leq -m$ ,  $\mathtt{len} = \min(m, n-k)$  if  $\mathbf{k}$  is in the range 0 to n-1, and  $\mathtt{len} = \min(m+k,n)$  if  $\mathbf{k}$  is in the range -1 to -m+1. Any existing entries in  $\mathbf{v}$  are discarded. The type of  $\mathbf{v}$  is preserved, so that if the type of  $\mathbf{A}$  and  $\mathbf{v}$  differ, the entries are typecasted into the type of  $\mathbf{v}$ . Any settings made to  $\mathbf{v}$  by  $\mathtt{GxB\_Vector\_Option\_set}$  (bitmap switch and sparsity control) are unchanged.

## 6.8.16 GxB\_Vector\_iso: query iso status of a vector

Returns the true if the vector is iso-valued, false otherwise.

## 6.8.17 GxB\_Vector\_memoryUsage: memory used by a vector

Returns the memory space required for a vector, in bytes.

#### 6.8.18 GrB\_Vector\_free: free a vector

GrB\_Vector\_free frees a vector. Either usage:

```
GrB_Vector_free (&v) ;
GrB_free (&v) ;
```

frees the vector v and sets v to NULL. It safely does nothing if passed a NULL handle, or if v == NULL on input. Any pending updates to the vector are abandoned.

# $\mathbf{6.9}\quad \mathbf{GraphBLAS}\ \mathbf{matrices:}\ \mathsf{GrB}\_\mathsf{Matrix}$

This section describes a set of methods that create, modify, query, and destroy a GraphBLAS sparse matrix, <code>GrB\_Matrix</code>:

GraphBLAS function	purpose	Section
GrB_Matrix_new	create a matrix	6.9.1
GrB_Matrix_wait	wait for a matrix	6.9.2
GrB_Matrix_dup	copy a matrix	6.9.3
GrB_Matrix_clear	clear a matrix of all entries	6.9.4
GrB_Matrix_nrows	number of rows of a matrix	6.9.5
<pre>GrB_Matrix_ncols</pre>	number of columns of a matrix	6.9.6
<pre>GrB_Matrix_nvals</pre>	number of entries in a matrix	6.9.7
<pre>GxB_Matrix_type_name</pre>	type of a matrix	6.9.8
<pre>GrB_Matrix_build</pre>	build a matrix from tuples	6.9.9
<pre>GxB_Matrix_build_Scalar</pre>	build a matrix from tuples	6.9.10
<pre>GrB_Matrix_setElement</pre>	add an entry to a matrix	6.9.11
<pre>GrB_Matrix_extractElement</pre>	get an entry from a matrix	6.9.12
<pre>GrB_Matrix_removeElement</pre>	remove an entry from a matrix	6.9.13
<pre>GrB_Matrix_extractTuples</pre>	get all entries from a matrix	6.9.14
GrB_Matrix_resize	resize a matrix	6.9.15
GxB_Matrix_concat	concatenate matrices	6.9.16
GxB_Matrix_split	split a matrix into matrices	6.9.17
GrB_Matrix_diag	diagonal matrix from vector	6.9.18
GxB_Matrix_diag	diagonal matrix from vector	6.9.19
GxB_Matrix_iso	query iso status	6.9.20
GxB_Matrix_memoryUsage	memory used by a matrix	6.9.21
GrB_Matrix_serializeSize	return size of serialized matrix	6.10.6
GrB_Matrix_serialize	serialize a matrix	6.10.7
GxB_Matrix_serialize	serialize a matrix	6.10.8
<pre>GrB_Matrix_deserialize</pre>	deserialize a matrix	6.10.9
<pre>GxB_Matrix_deserialize</pre>	deserialize a matrix	6.10.10
GrB_Matrix_free	free a matrix	6.9.22
GxB_Matrix_import_CSR, GxB_Matrix_pack_CSR	import CSR	6.11.13, 6.11.14
<pre>GxB_Matrix_export_CSR, GxB_Matrix_unpack_CSR</pre>	export CSR	6.11.15, 6.11.16
<pre>GxB_Matrix_import_CSC, GxB_Matrix_pack_CSC</pre>	import CSC	6.11.17, 6.11.18
<pre>GxB_Matrix_export_CSC, GxB_Matrix_unpack_CSC</pre>	export CSC	6.11.19, 6.11.20
GxB_Matrix_import_HyperCSR, GxB_Matrix_pack_HyperCSR	import HyperCSR	6.11.21, 6.11.22
<pre>GxB_Matrix_export_HyperCSR, GxB_Matrix_unpack_HyperCSR</pre>	export HyperCSR	6.11.23,  6.11.24
GxB_Matrix_import_HyperCSC, GxB_Matrix_pack_HyperCSC	import HyperCSC	6.11.25, 6.11.26
<pre>GxB_Matrix_export_HyperCSC, GxB_Matrix_unpack_HyperCSC</pre>	export HyperCSC	6.11.27,  6.11.28
GxB_Matrix_import_BitmapR, GxB_Matrix_pack_BitmapR	import BitmapR	6.11.29, 6.11.30
GxB_Matrix_export_BitmapR, GxB_Matrix_unpack_BitmapR	export BitmapR	6.11.31, 6.11.32
GxB_Matrix_import_BitmapC, GxB_Matrix_pack_BitmapC	import BitmapC	6.11.33, 6.11.34
GxB_Matrix_export_BitmapC, GxB_Matrix_unpack_BitmapC	export BitmapC	6.11.35, 6.11.36
GxB_Matrix_import_FullR, GxB_Matrix_pack_FullR	import FullR	6.11.37, 6.11.38
GxB_Matrix_export_FullR, GxB_Matrix_unpack_FullR	export FullR	6.11.39, 6.11.40
GxB_Matrix_import_FullC, GxB_Matrix_pack_FullC	import FullC	6.11.41, 6.11.42
GxB_Matrix_export_FullC, GxB_Matrix_unpack_FullC	export FullC	6.11.43, 6.11.44
GrB_Matrix_import	import in various formats	6.12.1
GrB_Matrix_export	export in various formats	6.12.2
GrB_Matrix_exportSize	array sizes required for export	6.12.3
GrB_Matrix_exportHint	hint best export format	6.12.4
1	r	

Refer to Section 6.11 for a discussion of import/export/pack/unpack.

#### 6.9.1 GrB\_Matrix\_new: create a matrix

```
GrB_Info GrB_Matrix_new // create a new matrix with no entries

(
GrB_Matrix *A, // handle of matrix to create
GrB_Type type, // type of matrix to create
GrB_Index nrows, // matrix dimension is nrows-by-ncols
GrB_Index ncols
);
```

GrB\_Matrix\_new creates a new nrows-by-ncols sparse matrix with no entries in it, of the given type. This is analogous to the MATLAB statement A = sparse (nrows, ncols), except that GraphBLAS can create sparse matrices of any type.

#### 6.9.2 GrB\_Matrix\_wait: wait for a matrix

In non-blocking mode, the computations for a GrB\_Matrix may be delayed. In this case, the matrix is not yet safe to use by multiple independent user threads. A user application may force completion of a matrix C via GrB\_Matrix\_wait(&C). After this call, different user threads may safely call GraphBLAS operations that use the matrix C as an input parameter. The function will change in SS:GrB v6.0.0 (for the v2.0 C API Specification).

### 6.9.3 GrB\_Matrix\_dup: copy a matrix

```
GrB_Info GrB_Matrix_dup // make an exact copy of a matrix

(
GrB_Matrix *C, // handle of output matrix to create const GrB_Matrix A // input matrix to copy
);
```

GrB\_Matrix\_dup makes a deep copy of a sparse matrix. In GraphBLAS, it is possible, and valid, to write the following:

Then C and A can be used interchangeably. However, only a pointer reference is made, and modifying one of them modifies both, and freeing one of them leaves the other as a dangling handle that should not be used. If two different matrices are needed, then this should be used instead:

```
GrB_Matrix A, C ;
GrB_Matrix_new (&A, GrB_FP64, n) ;
GrB_Matrix_dup (&C, A) ;  // like C = A, but making a deep copy
```

Then C and A are two different matrices that currently have the same set of values, but they do not depend on each other. Modifying one has no effect on the other.

#### 6.9.4 GrB\_Matrix\_clear: clear a matrix of all entries

GrB\_Matrix\_clear clears all entries from a matrix. All values A(i,j) are now equal to the implicit value, depending on what semiring ring is used to perform computations on the matrix. The pattern of A is empty, just as if it were created fresh with GrB\_Matrix\_new. Analogous with A (:,:) = 0 in MATLAB. The type and dimensions of A do not change. Any pending updates to the matrix are discarded.

## 6.9.5 GrB\_Matrix\_nrows: return the number of rows of a matrix

GrB\_Matrix\_nrows returns the number of rows of a matrix (nrows=size(A,1) in MATLAB).

#### 6.9.6 GrB\_Matrix\_ncols: return the number of columns of a matrix

GrB\_Matrix\_ncols returns the number of columns of a matrix (ncols=size(A,2) in MATLAB).

#### 6.9.7 GrB\_Matrix\_nvals: return the number of entries in a matrix

GrB\_Matrix\_nvals returns the number of entries in a matrix. Roughly analogous to nvals = nnz(A) in MATLAB, except that the implicit value in GraphBLAS need not be zero and nnz (short for "number of nonzeros") in MATLAB is better described as "number of entries" in GraphBLAS.

### 6.9.8 GxB\_Matrix\_type\_name: return name of the type of a matrix

GxB\_Matrix\_type\_name returns the name of the type of a matrix, like type=class(A) in MATLAB.

#### 6.9.9 GrB\_Matrix\_build: build a matrix from a set of tuples

GrB\_Matrix\_build constructs a sparse matrix C from a set of tuples, I, J, and X, each of length nvals. The matrix C must have already been initialized with GrB\_Matrix\_new, and it must have no entries in it before calling GrB\_Matrix\_build. Thus the dimensions and type of C are not changed by this function, but are inherited from the prior call to GrB\_Matrix\_new or GrB\_matrix\_dup.

An error is returned (GrB\_INDEX\_OUT\_OF\_BOUNDS) if any row index in I is greater than or equal to the number of rows of C, or if any column index in J is greater than or equal to the number of columns of C

Any duplicate entries with identical indices are assembled using the binary dup operator provided on input. All three types (x, y, z for z=dup(x,y)) must be identical. The types of dup, C and X must all be compatible. See Section 2.4 regarding typecasting and compatibility. The values in X are typecasted, if needed, into the type of dup. Duplicates are then assembled into a matrix T of the same type as dup, using  $T(i,j) = dup \ (T \ (i,j), X \ (k))$ . After T is constructed, it is typecasted into the result C. That is, typecasting does not occur at the same time as the assembly of duplicates.

If dup is NULL, any duplicates result in an error. If dup is the special binary operator GxB\_IGNORE\_DUP, then any duplicates are ignored. If duplicates appear, the last one in the list of tuples is taken and the prior ones ignored. This is not an error.

**SPEC:** As an extension to the spec, results are defined even if dup is non-associative.

The GraphBLAS API requires dup to be associative so that entries can be assembled in any order, and states that the result is undefined if dup is not associative. However, SuiteSparse:GraphBLAS guarantees a well-defined order of assembly. Entries in the tuples [I,J,X] are first sorted in increasing order of row and column index, with ties broken by the position of the tuple in the [I,J,X] list. If duplicates appear, they are assembled in the order they appear in the [I,J,X] input. That is, if the same indices i and j appear in positions k1, k2, k3, and k4 in [I,J,X], where k1 < k2 < k3 < k4, then the following operations will occur in order:

```
T (i,j) = X (k1) ;

T (i,j) = dup (T (i,j), X (k2)) ;

T (i,j) = dup (T (i,j), X (k3)) ;

T (i,j) = dup (T (i,j), X (k4)) ;
```

This is a well-defined order but the user should not depend upon it when using other GraphBLAS implementations since the GraphBLAS API does not require this ordering.

However, SuiteSparse:GraphBLAS guarantees this ordering, even when it compute the result in parallel. With this well-defined order, several operators become very useful. In particular, the SECOND operator results in the last tuple overwriting the earlier ones. The FIRST operator means the value of the first tuple is used and the others are discarded.

The acronym dup is used here for the name of binary function used for assembling duplicates, but this should not be confused with the \_dup suffix in the name of the function GrB\_Matrix\_dup. The latter function does not apply any operator at all, nor any typecasting, but simply makes a pure deep copy of a matrix.

The parameter X is a pointer to any C equivalent built-in type, or a void \* pointer. The GrB\_Matrix\_build function uses the \_Generic feature of ANSI C11 to detect the type of pointer passed as the parameter X. If X is

a pointer to a built-in type, then the function can do the right typecasting. If X is a void \* pointer, then it can only assume X to be a pointer to a user-defined type that is the same user-defined type of C and dup. This function has no way of checking this condition that the void \* X pointer points to an array of the correct user-defined type, so behavior is undefined if the user breaks this condition.

The GrB\_Matrix\_build method is analogous to C = sparse (I,J,X) in MATLAB, with several important extensions that go beyond that which MATLAB can do. In particular, the MATLAB sparse function only provides one option for assembling duplicates (summation), and it can only build double, double complex, and logical sparse matrices.

#### 6.9.10 GxB\_Matrix\_build\_Scalar: build a matrix from a set of tuples

GxB\_Matrix\_build\_Scalar constructs a sparse matrix C from a set of tuples defined the index arrays I and J of length nvals, and a scalar. The scalar is the value of all of the tuples. Unlike GrB\_Matrix\_build, there is no dup operator to handle duplicate entries. Instead, any duplicates are silently ignored (if the number of duplicates is desired, simply compare the input nvals with the value returned by GrB\_Vector\_nvals after the matrix is constructed). All entries in the sparsity pattern of C are identical, and equal to the input scalar value.

## 6.9.11 GrB\_Matrix\_setElement: add an entry to a matrix

);

 $GrB_Matrix_setElement$  sets a single entry in a matrix, C(i,j)=x. If the entry is already present in the pattern of C, it is overwritten with the new value. If the entry is not present, it is added to C. In either case, no entry is ever deleted by this function. Passing in a value of x=0 simply creates an explicit entry at position (i,j) whose value is zero, even if the implicit value is assumed to be zero.

An error is returned (GrB\_INVALID\_INDEX) if the row index i is greater than or equal to the number of rows of C, or if the column index j is greater than or equal to the number of columns of C. Note that this error code differs from the same kind of condition in GrB\_Matrix\_build, which returns GrB\_INDEX\_OUT\_OF\_BOUNDS. This is because GrB\_INVALID\_INDEX is an API error, and is caught immediately even in non-blocking mode, whereas GrB\_INDEX\_OUT\_OF\_BOUNDS is an execution error whose detection may wait until the computation completes sometime later.

The scalar x is typecasted into the type of C. Any value can be passed to this function and its type will be detected, via the \_Generic feature of ANSI C11. For a user-defined type, x is a void \* pointer that points to a memory space holding a single entry of this user-defined type. This user-defined type must exactly match the user-defined type of C since no typecasting is done between user-defined types.

Performance considerations: SuiteSparse:GraphBLAS exploits the non-blocking mode to greatly improve the performance of this method. Refer to the example shown in Section 2.2. If the entry exists in the pattern already, it is updated right away and the work is not left pending. Otherwise, it is placed in a list of pending updates, and the later on the updates are done all at once, using the same algorithm used for GrB\_Matrix\_build. In other words, setElement in SuiteSparse:GraphBLAS builds its own internal list of tuples [I,J,X], and then calls GrB\_Matrix\_build whenever the matrix is needed in another computation, or whenever GrB\_Matrix\_wait is called.

As a result, if calls to setElement are mixed with calls to most other methods and operations (even extractElement) then the pending updates are assembled right away, which will be slow. Performance will be good if many setElement updates are left pending, and performance will be poor if the updates are assembled frequently.

A few methods and operations can be intermixed with setElement, in

particular, some forms of the GrB\_assign and GxB\_subassign operations are compatible with the pending updates from setElement. Section 10.10 gives more details on which GxB\_subassign and GrB\_assign operations can be interleaved with calls to setElement without forcing updates to be assembled. Other methods that do not access the existing entries may also be done without forcing the updates to be assembled, namely GrB\_Matrix\_clear (which erases all pending updates), GrB\_Matrix\_free, GrB\_Matrix\_ncols, GrB\_Matrix\_nrows, GxB\_Matrix\_type, and of course GrB\_Matrix\_setElement itself. All other methods and operations cause the updates to be assembled. Future versions of SuiteSparse:GraphBLAS may extend this list.

See Section 13.3 for an example of how to use GrB\_Matrix\_setElement. If an error occurs, GrB\_error(&err,C) returns details about the error.

#### 6.9.12 GrB\_Matrix\_extractElement: get an entry from a matrix

TODO: add GrB\_Scalar

 $GrB\_Matrix\_extractElement$  extracts a single entry from a matrix x=A(i,j). An error is returned  $(GrB\_INVALID\_INDEX)$  if the row index i is greater than or equal to the number of rows of C, or if column index j is greater than or equal to the number of columns of C.

**NOTE:** if no entry is present at A(i,j), then x is not modified, and the return value of GrB\_Matrix\_extractElement is GrB\_NO\_VALUE.

If the entry is not present then GraphBLAS does not know its value, since its value depends on the implicit value, which is the identity value of the additive monoid of the semiring. It is not a characteristic of the matrix itself, but of the semiring it is used in. A matrix can be used in any compatible semiring, and even a mixture of semirings, so the implicit value can change as the semiring changes.

As a result, if the entry is present, x=A(i,j) is performed and the scalar x is returned with this value. The method returns  $GrB\_SUCCESS$ . If the entry

is not present, x is not modified, and  $GrB_NO_VALUE$  is returned to the caller. What this means is up to the caller.

The function knows the type of the pointer x, so it can do typecasting as needed, from the type of A into the type of x. User-defined types cannot be typecasted, so if A has a user-defined type then x must be a void \* pointer that points to a memory space the same size as a single scalar of the type of A

Currently, this method causes all pending updates from GrB\_setElement, GrB\_assign, or GxB\_subassign to be assembled, so its use can have performance implications. Calls to this function should not be arbitrarily intermixed with calls to these other two functions. Everything will work correctly and results will be predictable, it will just be slow.

## 6.9.13 GrB\_Matrix\_removeElement: remove an entry from a matrix

GrB\_Matrix\_removeElement removes a single entry A(i,j) from a matrix. If no entry is present at A(i,j), then the matrix is not modified. If an error occurs, GrB\_error(&err,A) returns details about the error.

#### 6.9.14 GrB\_Matrix\_extractTuples: get all entries from a matrix

GrB\_Matrix\_extractTuples extracts all the entries from the matrix A, returning them as a list of tuples, analogous to [I,J,X]=find(A) in MAT-LAB. Entries in the tuples [I,J,X] are unique. No pair of row and column indices (i,j) appears more than once.

The GraphBLAS API states the tuples can be returned in any order. If GrB\_wait(&A) is called first, then SuiteSparse:GraphBLAS chooses to always return them in sorted order, depending on whether the matrix is stored by row or by column. Otherwise, the indices can be returned in any order.

The number of tuples in the matrix A is given by GrB\_Matrix\_nvals(&anvals,A). If anvals is larger than the size of the arrays (nvals in the parameter list), an error GrB\_INSUFFICIENT\_SIZE is returned, and no tuples are extracted. If nvals is larger than anvals, then only the first anvals entries in the arrays I J, and X are modified, containing all the tuples of A, and the rest of I J, and X are left unchanged. On output, nvals contains the number of tuples extracted.

**SPEC:** As an extension to the spec, the arrays I, J, and/or X may be passed in as NULL pointers. In this case, GrB\_Matrix\_extractTuples does not return a component specified as NULL. This is not an error condition.

#### 6.9.15 GrB\_Matrix\_resize: resize a matrix

GrB\_Matrix\_resize changes the size of a matrix. If the dimensions decrease, entries that fall outside the resized matrix are deleted.

#### 6.9.16 GxB\_Matrix\_concat: concatenate matrices

GxB\_Matrix\_concat concatenates an array of matrices (Tiles) into a single GrB\_Matrix C.

Tiles is an m-by-n dense array of matrices held in row-major format, where Tiles [i\*n+j] is the (i, j)th tile, and where m > 0 and n > 0 must hold. Let  $A_{i,j}$  denote the (i, j)th tile. The matrix C is constructed by concatenating these tiles together, as:

$$C = \begin{bmatrix} A_{0,0} & A_{0,1} & A_{0,2} & \cdots & A_{0,n-1} \\ A_{1,0} & A_{1,1} & A_{1,2} & \cdots & A_{1,n-1} \\ \vdots & \vdots & & & & \\ A_{m-1,0} & A_{m-1,1} & A_{m-1,2} & \cdots & A_{m-1,n-1} \end{bmatrix}$$

On input, the matrix C must already exist. Any existing entries in C are discarded. C must have dimensions **nrows** by **ncols** where **nrows** is the sum of the number of rows in the matrices  $A_{i,0}$  for all i, and **ncols** is the sum of the number of columns in the matrices  $A_{0,j}$  for all j. All matrices in any given tile row i must have the same number of rows (that is, and all matrices in any given tile column j must have the same number of columns).

The type of C is unchanged, and all matrices  $A_{i,j}$  are typecasted into the type of C. Any settings made to C by  $GxB_Matrix_Option_set$  (format by row or by column, bitmap switch, hyper switch, and sparsity control) are unchanged.

#### 6.9.17 GxB\_Matrix\_split: split a matrix

GxB\_Matrix\_split does the opposite of GxB\_Matrix\_concat. It splits a single input matrix A into a 2D array of tiles. On input, the Tiles array must be a non-NULL pointer to a previously allocated array of size at least m\*n where both m and n must be greater than zero. The Tiles\_nrows array has size m, and Tiles\_ncols has size m. The (i,j)th tile has dimension Tiles\_nrows[i]-by-Tiles\_ncols[j]. The sum of Tiles\_nrows [0:m-1] must equal the number of rows of A, and the sum of Tiles\_ncols [0:n-1]

must equal the number of columns of A. The type of each tile is the same as the type of A; no typecasting is done.

## 6.9.18 GrB\_Matrix\_diag: construct a diagonal matrix

GrB\_Matrix\_diag constructs a matrix from a vector. Let n be the length of the v vector, from GrB\_Vector\_size (&n, v). If k = 0, then C is an n-by-n diagonal matrix with the entries from v along the main diagonal of C, with C(i,i)=v(i). If k is nonzero, C is square with dimension n+|k|. If k is positive, it denotes diagonals above the main diagonal, with C(i,i+k)=v(i). If k is negative, it denotes diagonals below the main diagonal of C, with C(i-k,i)=v(i). This behavior is identical to the MATLAB statement C=diag(v,k), where v is a vector, except that C=diag(v,k) and C=diag(v,k) is a vector, except that C=diag(v,k) is a vector of C=diag(v,k).

C must already exist on input, of the correct size. Any existing entries in C are discarded. The type of C is preserved, so that if the type of C and v differ, the entries are typecasted into the type of C. Any settings made to C by GxB\_Matrix\_Option\_set (format by row or by column, bitmap switch, hyper switch, and sparsity control) are unchanged.

## 6.9.19 GxB\_Matrix\_diag: construct a diagonal matrix

Identical to GrB\_Matrix\_diag, except for the extra parameter: a descriptor to provide control over the number of threads used.

#### 6.9.20 GxB\_Matrix\_iso: query iso status of a matrix

Returns the true if the matrix is iso-valued, false otherwise.

## 6.9.21 GxB\_Matrix\_memoryUsage: memory used by a matrix

```
GrB_Info GxB_Matrix_memoryUsage // return # of bytes used for a matrix
(
    size_t *size, // # of bytes used by the matrix A
    const GrB_Matrix A // matrix to query
);
```

Returns the memory space required for a matrix, in bytes.

### 6.9.22 GrB\_Matrix\_free: free a matrix

```
GrB_Info GrB_free // free a matrix
(
GrB_Matrix *A // handle of matrix to free
);
```

GrB\_Matrix\_free frees a matrix. Either usage:

```
GrB_Matrix_free (&A) ;
GrB_free (&A) ;
```

frees the matrix A and sets A to NULL. It safely does nothing if passed a NULL handle, or if A == NULL on input. Any pending updates to the matrix are abandoned.

# 6.10 Serialize/deserialize methods

Serialization takes an opaque GraphBLAS object (a vector or matrix) and encodes it in a single non-opaque array of bytes, the blob. The blob can only be deserialized by the same library that created it (SuiteSparse:GraphBLAS in this case). The array of bytes can be written to a file, sent to another process over an MPI channel, or operated on in any other way that moves the bytes around. The contents of the array cannot be interpretted except by deserialization back into a vector or matrix.

There are two forms of serialization: GrB\*serialize and GxB\*serialize. For the GrB form, the blob must first be allocated by the user application, and it must be large enough to hold the matrix or vector.

By default, LZ4 compression is used for serialization, but other options can be selected via the descriptor: GxB\_set (desc, GxB\_COMPRESSION, method), where method is an integer selected from the following options:

method	description
GxB_COMPRESSION_NONE	no compression
GxB_COMPRESSION_DEFAULT	LZ4
GxB_COMPRESSION_LZ4	LZ4
GxB_COMPRESSION_LZ4HC	LZ4HC, with default level 9

The LZ4HC method can be modified by adding a level of zero to 9, with 9 being the default. Higher levels lead to a more compact blob, at the cost of extra computational time. This level is simply added to the method, so to compress a vector with LZ4HC with level 6, use:

```
GxB_set (desc, GxB_COMPRESSION, GxB_COMPRESSION_LZ4HC + 6) ;
```

Descrialization of untrusted data is a common security problem; see <a href="https://cwe.mitre.org/data/definitions/502.html">https://cwe.mitre.org/data/definitions/502.html</a>. The descrialization methods do a few basic checks so that no out-of-bounds access occurs during descrialization, but the output matrix or vector itself may still be corrupted. If the data is untrusted, use this to check the matrix or vector after descrializing it. For a vector:

```
info = GxB_Vector_fprint (w, "w deserialized", GrB_SILENT, NULL) ;
if (info != GrB_SUCCESS) GrB_free (&w) ;
```

For a matrix:

```
info = GxB_Matrix_fprint (A, "A deserialized", GrB_SILENT, NULL) ;
if (info != GrB_SUCCESS) GrB_free (&A) ;
```

The following methods are described in this Section:

GraphBLAS function	purpose	Section
GrB_Vector_serializeSize	return size of serialized vector	6.10.1
<pre>GrB_Vector_serialize</pre>	serialize a vector	6.10.2
<pre>GxB_Vector_serialize</pre>	serialize a vector	6.10.3
<pre>GrB_Vector_deserialize</pre>	deserialize a vector	6.10.4
<pre>GxB_Vector_deserialize</pre>	deserialize a vector	6.10.5
GrB_Matrix_serializeSize	return size of serialized matrix	6.10.6
<pre>GrB_Matrix_serialize</pre>	serialize a matrix	6.10.7
<pre>GxB_Matrix_serialize</pre>	serialize a matrix	6.10.8
<pre>GrB_Matrix_deserialize</pre>	deserialize a matrix	6.10.9
<pre>GxB_Matrix_deserialize</pre>	deserialize a matrix	6.10.10
GrB_deserialize_type_name	return the name of type of the blob	6.10.11

## 6.10.1 GrB\_Vector\_serializeSize: return size of serialized vector

GrB\_Vector\_serializeSize returns an upper bound on the size of the blob needed to serialize a GrB\_Vector using GrB\_Vector\_serialize. After the vector is serialized, the actual size used is returned, and the blob may be realloc'd to that size if desired. This method is not required for GxB\_Vector\_serialize.

#### 6.10.2 GrB\_Vector\_serialize: serialize a vector

GrB\_Vector\_serialize serializes a vector into a single array of bytes (the blob), which must be already allocated by the user application. On input, &blob\_size is the size of the allocated blob in bytes. On output, it is reduced to the numbed of bytes actually used to serialize the vector. After calling GrB\_Vector\_serialize, the blob may be realloc'd to this revised size if desired (this is optional). LZ4 compression is used to construct a compact blob.

#### 6.10.3 GxB\_Vector\_serialize: serialize a vector

GxB\_Vector\_serialize is identical to GrB\_Vector\_serialize, except that it does not require a pre-allocated blob. Instead, it malloc's the blob internally, and fills it with the serialized vector.

#### 6.10.4 GrB\_Vector\_deservalize: deservalize a vector

```
GrB_Info GrB_Vector_deserialize
                                     // deserialize blob into a GrB_Vector
    // output:
    GrB_Vector *w,
                        // output vector created from the blob
    // input:
                        \ensuremath{//} type of the vector w. Required if the blob holds a
    GrB_Type type,
                         // vector of user-defined type. May be NULL if blob
                         // holds a built-in type; otherwise must match the
                        // type of w.
                             // the blob
    const void *blob,
                             // size of the blob
    GrB_Index blob_size
);
```

This method creates a vector w by descrializing the contents of the blob, constructed by either GrB\_Vector\_serialize or GxB\_Vector\_serialize.

#### 6.10.5 GxB\_Vector\_deserialize: deserialize a vector

```
GrB_Info GxB_Vector_deserialize
                                    // deserialize blob into a GrB_Vector
    // output:
    GrB_Vector *w,
                        // output vector created from the blob
    // input:
   GrB_Type type,
                        // type of the vector w. Required if the blob holds a
                        // vector of user-defined type. May be NULL if blob
                        // holds a built-in type; otherwise must match the
                        // type of w.
    const void *blob,
                            // the blob
    GrB_Index blob_size,
                            // size of the blob
                                   // to control # of threads used
    const GrB_Descriptor desc
);
```

Identical to GrB\_Vector\_deserialize, except that the descriptor appears as the last parameter to control the number of threads used.

#### 6.10.6 GrB Matrix serializeSize: return size of serialized matrix

```
GrB_Info GrB_Matrix_serializeSize  // estimate the size of a blob

(
    // output:
    GrB_Index *blob_size_handle,  // upper bound on the required size of the  // blob on output.

    // input:
    GrB_Matrix A  // matrix to serialize

);
```

GrB\_Matrix\_serializeSize returns an upper bound on the size of the blob needed to serialize a GrB\_Matrix with GrB\_Matrix\_serialize. After the matrix is serialized, the actual size used is returned, and the blob may be realloc'd to that size if desired.

This method is not required for GxB\_Matrix\_serialize.

#### 6.10.7 GrB\_Matrix\_serialize: serialize a matrix

GrB\_Matrix\_serialize serializes a matrix into a single array of bytes (the blob), which must be already allocated by the user application. On input, &blob\_size is the size of the allocated blob in bytes. On output, it is reduced to the numbed of bytes actually used to serialize the matrix. After calling GrB\_Matrix\_serialize, the blob may be realloc'd to this revised size if desired (this is optional). LZ4 compression is used to construct a compact blob.

#### 6.10.8 GxB\_Matrix\_serialize: serialize a matrix

GxB\_Matrix\_serialize is identical to GrB\_Matrix\_serialize, except that it does not require a pre-allocated blob. Instead, it malloc's the blob internally, and fills it with the serialized vector.

By default, LZ4 compression is used, but other options can be selected via the descriptor. Refer to Section 6.10.3 for details.

#### 6.10.9 GrB\_Matrix\_deservative: deservative a matrix

```
GrB_Info GrB_Matrix_deserialize
                                    // deserialize blob into a GrB_Matrix
    // output:
    GrB_Matrix *C,
                        // output matrix created from the blob
    // input:
    GrB_Type type,
                        // type of the matrix C. Required if the blob holds a
                        // matrix of user-defined type. May be NULL if blob
                        // holds a built-in type; otherwise must match the
                        // type of C.
                            // the blob
    const void *blob,
                            // size of the blob
    GrB_Index blob_size
);
```

This method creates a matrix A by descrializing the contents of the blob, constructed by either GrB\_Matrix\_serialize or GxB\_Matrix\_serialize.

#### 6.10.10 GxB\_Matrix\_deservalize: deservalize a matrix

```
GrB_Info GxB_Matrix_deserialize
                                    // deserialize blob into a GrB_Matrix
    // output:
    GrB_Matrix *C,
                       // output matrix created from the blob
    // input:
    GrB_Type type,
                       // type of the matrix C. Required if the blob holds a
                        // matrix of user-defined type. May be NULL if blob
                        // holds a built-in type; otherwise must match the
                        // type of C.
                            // the blob
    const void *blob,
                           // size of the blob
    GrB_Index blob_size,
    const GrB_Descriptor desc
                                   // to control # of threads used
);
```

Identical to GrB\_Matrix\_deserialize, except that the descriptor appears as the last parameter to control the number of threads used.

## 6.10.11 GxB\_deserialize\_type\_name: name of the type of a blob

GrB\_deserialize\_type\_name returns the name of type of the matrix or vector serialized into the blob. This method works for any blob, from GrB\_Vector\_serialize, GxB\_Vector\_serialize, GrB\_Matrix\_serialize, or GxB\_Matrix\_serialize.

# 6.11 GraphBLAS import/export/pack/unpack: using move semantics

The import/export/pack/unpack functions allow the user application to create a GrB\_Matrix or GrB\_Vector object, and to extract its contents, faster and with less memory overhead than the GrB\_\*\_build and GrB\_\*\_extractTuples functions.

The GrB\_Matrix\_import and GrB\_Matrix\_export are not described in this section. Refer to Section 6.12 instead.

The semantics of the GxB import/export/pack/unpack are the same as the *move constructor* in C++. On import/pack, the user provides a set of arrays that have been previously allocated via the ANSI C malloc, calloc, or realloc functions (by default), or by the corresponding functions passed to GxB\_init. The arrays define the content of the matrix or vector. Unlike GrB\_\*\_build, the GraphBLAS library then takes ownership of the user's input arrays and may either:

- 1. incorporate them into its internal data structure for the new GrB\_Matrix or GrB\_Vector, potentially creating the GrB\_Matrix or GrB\_Vector in constant time with no memory copying performed, or
- 2. if the library does not support the import format directly, then it may convert the input to its internal format, and then free the user's input arrays.
- 3. A GraphBLAS implementation may also choose to use a mix of the two strategies.

SuiteSparse:GraphBLAS takes the first approach, and so the import/pack functions always take O(1) time, and require O(1) memory space to be allocated.

Regardless of the method chosen, as listed above, the input arrays are no longer owned by the user application. If A is a GrB\_Matrix created by an import/pack, the user input arrays are freed no later than GrB\_free(&A), and may be freed earlier, at the discretion of the GraphBLAS library. The data structure of the GrB\_Matrix and GrB\_Vector remain opaque.

The GxB export/unpack of a GrB\_Matrix or GrB\_Vector is symmetric with the import/pack operation. The export/unpack changes the ownership of the arrays, which are returned to the user and which contain the matrix

or vector in the requested format. Ownership of these arrays is given to the user application, which is then responsible for freeing them via the ANSI C free function (by default), or by the free\_function that was passed in to GxB\_init. Alternatively, these arrays can be re-imported/re-packed into a GrB\_Matrix or GrB\_Vector, at which point they again become the responsibility of GraphBLAS.

For an export/unpack, if the output format matches the current internal format of the matrix or vector then these arrays are returned to the user application in O(1) time and with no memory copying performed. Otherwise, the  ${\tt GrB\_Matrix}$  or  ${\tt GrB\_Vector}$  is first converted into the requested format, and then exported.

The import and pack methods are nearly identical. They differ only in how the matrix or vector is created. For the import, the matrix/vector argument is passed in as &A, and the object is created, just like GrB\_Matrix\_new, and its contents are then populated with the new content. For the pack methods, the A matrix/vector must already exist on input, and its contents are populated with the new content, just like GrB\_Matrix\_build.

The export and unpack methods are also nearly identical. They differ only in how the matrix or vector is handled on return. For the export methods, &A is passed in, and the object is destroyed after its contents are exported, and A is set to NULL to indicate this. For the unpack methods, A is passed in, and the matrix/vector still exists on return, just with no entries. Its type and dimensions are preserved.

Exporting/unpacking a matrix or vector forces completion of any pending operations on the matrix, with one exception. SuiteSparse:GraphBLAS supports three kinds of pending operations: zombies (pending deletions), pending tuples (pending insertions), and a lazy sort. In the latter, if the matrix or vector is left in a jumbled state, indices in any row or column may appear out of order. If unjumbled, the indices always appear in ascending order.

The vector import/export/pack/unpack methods use a three formats for a GrB\_Vector. Eight different formats are provided for the import/export/pack/unpack of a GrB\_Matrix. For each format, the numerical value array (Ax or vx) has a C type corresponding to one of the 13 built-in types in GraphBLAS (bool, int\*\_t, uint\*\_t, float, double float complex, double complex), or that corresponds with the user-defined type. No typecasting is done.

If iso is true, then all entries present in the matrix or vector have the same value, and the Ax array (for matrices) or vx array (for vectors) only

need to be large enough to hold a single value.

The export/unpack of a GrB\_Vector in CSC format may return the indices in a jumbled state, in any order.

For a GrB\_Matrix in CSR or HyperCSR format, if the matrix is returned as jumbled, the column indices in any given row may appear out of order. For CSC or HyperCSC formats, if the matrix is returned as jumbled, the row indices in any given column may appear out of order.

On import/pack, if the user-provided arrays contain jumbled row or column vectors, then the input flag jumbled must be passed in as true. On export, if \*jumbled is NULL, this indicates to the export method that the user expects the exported matrix or vector to be returned in an ordered, unjumbled state. If \*jumbled is provided, then it is return as true if the indices may appear out of order, or false if they are known to be in ascending order.

Matrices and vectors in bitmap or full format are never jumbled.

If data is imported/packed using the  $GxB*\_import_*$  or  $GxB*\_pack_*$ , the default is to trust the input data so that the import/pack can be done in O(1) time. However, if the data comes from an untrusted source, additional checks should be made during the import. This is indicated with a descriptor setting, and then passing the descriptor to the GxB import/pack methods:

```
GxB_set (desc, GxB_IMPORT, GxB_SECURE_IMPORT) ;
```

The table below lists the methods presented in this section.

method	purpose	Section
GxB_Vector_import_CSC	import a vector in CSC format	6.11.1
<pre>GxB_Vector_pack_CSC</pre>	pack a vector in CSC format	6.11.2
<pre>GxB_Vector_export_CSC</pre>	export a vector in CSC format	6.11.3
<pre>GxB_Vector_unpack_CSC</pre>	unpack a vector in CSC format	6.11.4
GxB_Vector_import_Bitmap	import a vector in bitmap format	6.11.5
<pre>GxB_Vector_pack_Bitmap</pre>	pack a vector in bitmap format	6.11.6
<pre>GxB_Vector_export_Bitmap</pre>	export a vector in bitmap format	6.11.7
<pre>GxB_Vector_unpack_Bitmap</pre>	unpack a vector in bitmap format	6.11.8
GxB_Vector_import_Full	import a vector in full format	6.11.9
<pre>GxB_Vector_pack_Full</pre>	pack a vector in full format	6.11.10
<pre>GxB_Vector_export_Full</pre>	export a vector in full format	6.11.11
<pre>GxB_Vector_unpack_Full</pre>	unpack a vector in full format	6.11.12
GxB_Matrix_import_CSR	import a matrix in CSR form	6.11.13
<pre>GxB_Matrix_pack_CSR</pre>	pack a matrix in CSR form	6.11.14
<pre>GxB_Matrix_export_CSR</pre>	export a matrix in CSR form	6.11.15
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<pre>GxB_Matrix_pack_CSC</pre>	pack a matrix in CSC form	6.11.18
${ t GxB\_Matrix\_export\_CSC}$	export a matrix in CSC form	6.11.19
<pre>GxB_Matrix_unpack_CSC</pre>	unpack a matrix in CSC form	6.11.20
GxB_Matrix_import_HyperCSR	import a matrix in HyperCSR form	6.11.21
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${ t GxB\_Matrix\_pack\_HyperCSC}$	pack a matrix in HyperCSC form	6.11.26
${ t GxB\_Matrix\_export\_HyperCSC}$	export a matrix in HyperCSC form	6.11.27
<pre>GxB_Matrix_unpack_HyperCSC</pre>	unpack a matrix in HyperCSC form	6.11.28
<pre>GxB_Matrix_import_BitmapR</pre>	import a matrix in BitmapR form	6.11.29
${\tt GxB\_Matrix\_pack\_BitmapR}$	pack a matrix in BitmapR form	6.11.30
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<pre>GxB_Matrix_unpack_BitmapC</pre>	unpack a matrix in BitmapC form	6.11.36
<pre>GxB_Matrix_import_FullR</pre>	import a matrix in FullR form	6.11.37
${\tt GxB\_Matrix\_pack\_FullR}$	pack a matrix in FullR form	6.11.38
${\tt GxB\_Matrix\_export\_FullR}$	export a matrix in FullR form	6.11.39
GxB_Matrix_unpack_FullR	unpack a matrix in FullR form	6.11.40
<pre>GxB_Matrix_import_FullC</pre>	import a matrix in FullC form	6.11.41
<pre>GxB_Matrix_pack_FullC</pre>	pack a matrix in FullC form	6.11.42
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GxB_Matrix_unpack_FullC	unpack a matrix in FullC form	6.11.44

## 6.11.1 GxB\_Vector\_import\_CSC import a vector in CSC form

```
GrB_Info GxB_Vector_import_CSC // import a vector in CSC format
    GrB_Vector *v,
                       // handle of vector to create
    GrB_Type type,
                       // type of vector to create
    GrB_Index n,
                       // vector length
   GrB_Index **vi,
                       // indices, vi_size >= nvals(v)*sizeof(int64_t)
    void **vx,
                       // values, vx_size >= nvals(v)*(type size)
                       // or vx_size >= (type size), if iso is true
    GrB_Index vi_size, // size of vi in bytes
    GrB_Index vx_size, // size of vx in bytes
                       // if true, v is iso
    bool iso.
                       // # of entries in vector
    GrB_Index nvals,
   bool jumbled,
                       // if true, indices may be unsorted
    const GrB_Descriptor desc
);
```

GxB\_Vector\_import\_CSC is analogous to GxB\_Matrix\_import\_CSC. Refer to the description of GxB\_Matrix\_import\_CSC for details (Section 6.11.17).

If successful, v is created as a n-by-1 GrB\_Vector. Its entries are the row indices given by vi, with the corresponding values in vx. The two pointers vi and vx are returned as NULL, which denotes that they are no longer owned by the user application. They have instead been moved into the new GrB\_Vector v. If jumbled is true, the row indices in vi must appear in sorted order. No duplicates can appear. These conditions are not checked, so results are undefined if they are not met exactly. The user application can check the resulting vector v with GxB\_print, if desired, which will determine if these conditions hold.

If not successful, v is returned as NULL and vi and vx are not modified.

## 6.11.2 GxB\_Vector\_pack\_CSC pack a vector in CSC form

```
GrB_Info GxB_Vector_pack_CSC // pack a vector in CSC format
                       // vector to create (type and length unchanged)
   GrB_Vector v,
   GrB_Index **vi,
                      // indices, vi_size >= nvals(v) * sizeof(int64_t)
   void **vx,
                       // values, vx_size >= nvals(v) * (type size)
                       // or vx_size >= (type size), if iso is true
   GrB_Index vi_size, // size of vi in bytes
   GrB_Index vx_size, // size of vx in bytes
   bool iso,
                      // if true, v is iso
   GrB_Index nvals, // # of entries in vector
   bool jumbled, // if true, indices may be unsorted
   const GrB_Descriptor desc
);
```

Identical to GxB\_Vector\_import\_CSC, except that the vector must exist on input with the right type and length. No typecasting is done.

## 6.11.3 GxB\_Vector\_export\_CSC: export a vector in CSC form

```
GrB_Info GxB_Vector_export_CSC // export and free a CSC vector
   GrB_Vector *v,
                       // handle of vector to export and free
   GrB_Type *type,
                       // type of vector exported
   GrB_Index *n,
                       // length of the vector
   GrB_Index **vi,
                       // indices, vi_size >= nvals(v)*sizeof(int64_t)
   void **vx,
                       // values, vx_size >= nvals(v)*(type size)
                       // or vx_size >= (type size), if iso is true
   GrB_Index *vi_size, // size of vi in bytes
   GrB_Index *vx_size, // size of vx in bytes
                // if true, v is iso
   bool *iso.
                       // # of entries in vector
   GrB_Index *nvals,
   bool *jumbled,
                       // if true, indices may be unsorted
   const GrB_Descriptor desc
);
```

GxB\_Vector\_export\_CSC is analogous to GxB\_Matrix\_export\_CSC. Refer to the description of GxB\_Matrix\_export\_CSC for details (Section 6.11.19).

Exporting a vector forces completion of any pending operations on the vector, except that indices may be exported out of order (jumbled is true if they may be out of order, false if sorted in ascending order). If jumbled is NULL on input, then the indices are always returned in sorted order.

If successful, v is returned as NULL, and its contents are returned to the user, with its type, dimension n, and number of entries nvals. A list of row indices of entries that were in v is returned in vi, and the corresponding numerical values are returned in vx. If nvals is zero, the vi and vx arrays are returned as NULL; this is not an error condition.

If not successful, v is unmodified and vi and vx are not modified.

# 6.11.4 GxB\_Vector\_unpack\_CSC: unpack a vector in CSC form

Identical to GxB\_Vector\_export\_CSC, except that the vector still exists on output with the type and length unchanged, but with no entries.

#### 6.11.5 GxB\_Vector\_import\_Bitmap import a vector in bitmap form

```
GrB_Info GxB_Vector_import_Bitmap // import a bitmap vector
   GrB_Vector *v,
                       // handle of vector to create
   GrB_Type type,
                       // type of vector to create
   GrB_Index n,
                       // vector length
   int8_t **vb,
                       // bitmap, vb_size >= n
   void **vx,
                       // values, vx_size >= n * (type size)
                       // or vx_size >= (type size), if iso is true
   GrB_Index vb_size, // size of vb in bytes
   GrB_Index vx_size, // size of vx in bytes
                       // if true, v is iso
   bool iso.
                       // # of entries in bitmap
   GrB_Index nvals,
   const GrB_Descriptor desc
) ;
```

GxB\_Vector\_import\_Bitmap is analogous to GxB\_Matrix\_import\_BitmapC. Refer to the description of GxB\_Matrix\_import\_BitmapC for details (Section 6.11.33).

If successful, v is created as a n-by-1 GrB\_Vector. Its entries are determined by vb, where vb[i]=1 denotes that the entry v(i) is present with value given by vx[i], and vb[i]=0 denotes that the entry v(i) is not present (vx[i]) is ignored in this case).

The two pointers vb and vx are returned as NULL, which denotes that they are no longer owned by the user application. They have instead been moved into the new GrB\_Vector v.

The vb array must not hold any values other than 0 and 1. The value nvals must exactly match the number of 1s in the vb array. These conditions are not checked, so results are undefined if they are not met exactly. The user application can check the resulting vector v with GxB\_print, if desired, which will determine if these conditions hold.

If not successful, v is returned as NULL and vb and vx are not modified.

## 6.11.6 GxB\_Vector\_pack\_Bitmap pack a vector in bitmap form

Identical to GxB\_Vector\_import\_Bitmap, except that the vector must exist on input with the right type and length. No typecasting is done.

#### 6.11.7 GxB\_Vector\_export\_Bitmap: export a vector in bitmap form

```
GrB_Info GxB_Vector_export_Bitmap
                                   // export and free a bitmap vector
   GrB_Vector *v,
                       // handle of vector to export and free
                       // type of vector exported
   GrB_Type *type,
   GrB_Index *n,
                       // length of the vector
   int8_t **vb,
                       // bitmap, vb_size >= n
   void **vx,
                       // values, vx_size >= n * (type size)
                       // or vx_size >= (type size), if iso is true
   GrB_Index *vb_size, // size of vb in bytes
   GrB_Index *vx_size, // size of vx in bytes
   bool *iso, // if true, v is iso
   GrB_Index *nvals, // # of entries in bitmap
   const GrB_Descriptor desc
) ;
```

GxB\_Vector\_export\_Bitmap is analogous to GxB\_Matrix\_export\_BitmapC. Refer to the description of GxB\_Matrix\_export\_BitmapC for details (Section 6.11.35).

Exporting a vector forces completion of any pending operations on the vector.

If successful, v is returned as NULL, and its contents are returned to the user, with its type, dimension n, and number of entries nvals. The entries that were in v are returned in vb, where vb[i]=1 means v(i) is present with value vx[i], and vb[i]=0 means v(i) is not present (vx[i] is undefined in this case). The corresponding numerical values are returned in vx.

If not successful, v is unmodified and vb and vx are not modified.

## 6.11.8 GxB\_Vector\_unpack\_Bitmap: unpack a vector in bitmap form

Identical to GxB\_Vector\_export\_Bitmap, except that the vector still exists on output with the type and length unchanged, but with no entries.

#### 6.11.9 GxB\_Vector\_import\_Full import a vector in full form

GxB\_Vector\_import\_Full is analogous to GxB\_Matrix\_import\_FullC. Refer to the description of GxB\_Matrix\_import\_BitmapC for details (Section 6.11.41).

If successful, v is created as a n-by-1 GrB\_Vector. All entries are present, and the value of v(i) is given by vx[i].

The pointer vx is returned as NULL, which denotes that it is no longer owned by the user application. It has instead been moved into the new GrB\_Vector v.

If not successful, v is returned as NULL and vx is not modified.

## 6.11.10 GxB\_Vector\_pack\_Full pack a vector in full form

Identical to GxB\_Vector\_import\_Full, except that the vector must exist on input with the right type and length. No typecasting is done.

#### 6.11.11 GxB\_Vector\_export\_Full: export a vector in full form

```
GrB_Info GxB_Vector_export_Full
                                  // export and free a full vector
    GrB_Vector *v,
                       // handle of vector to export and free
    GrB_Type *type,
                       // type of vector exported
    GrB_Index *n,
                       // length of the vector
    void **vx.
                       // values, vx_size >= nvals(v) * (type size)
                       // or vx_size >= (type size), if iso is true
   GrB_Index *vx_size, // size of vx in bytes
    bool *iso,
                       // if true, v is iso
    const GrB_Descriptor desc
) ;
```

GxB\_Vector\_export\_Full is analogous to GxB\_Matrix\_export\_FullC. Refer to the description of GxB\_Matrix\_export\_FullC for details (Section 6.11.43).

Exporting a vector forces completion of any pending operations on the vector. All entries in v must be present. In other words, prior to the export, GrB\_Vector\_nvals for a vector of length n must report that the vector contains n entries; GrB\_INVALID\_VALUE is returned if this condition does not hold.

If successful, v is returned as NULL, and its contents are returned to the user, with its type and dimension n. The entries that were in v are returned in the array vx, vb, where vb[i]=1 means v(i) is present with value where the value of v(i) is vx[i].

If not successful, v is unmodified and vx is not modified.

# 6.11.12 GxB\_Vector\_unpack\_Full: unpack a vector in full form

Identical to GxB\_Vector\_export\_Full, except that the vector still exists on output with the type and length unchanged, but with no entries.

## 6.11.13 GxB\_Matrix\_import\_CSR: import a CSR matrix

```
GrB_Info GxB_Matrix_import_CSR
                                    // import a CSR matrix
    GrB_Matrix *A,
                       // handle of matrix to create
    GrB_Type type,
                       // type of matrix to create
    GrB_Index nrows,
                       // number of rows of the matrix
   GrB_Index ncols,
                       // number of columns of the matrix
    GrB_Index **Ap,
                       // row "pointers", Ap_size >= (nrows+1)*sizeof(int64_t)
                       // column indices, Aj_size >= nvals(A)*sizeof(int64_t)
    GrB_Index **Aj,
    void **Ax,
                        // values, Ax_size >= nvals(A) * (type size)
                       // or Ax_size >= (type size), if iso is true
    GrB_Index Ap_size, // size of Ap in bytes
    GrB_Index Aj_size, // size of Aj in bytes
    GrB_Index Ax_size, // size of Ax in bytes
    bool iso,
                       // if true, A is iso
                       // if true, indices in each row may be unsorted
    bool jumbled,
    const GrB_Descriptor desc
);
```

GxB\_Matrix\_import\_CSR imports a matrix from 3 user arrays in CSR format. In the resulting GrB\_Matrix A, the CSR format is a sparse matrix with a format (GxB\_FORMAT) of GxB\_BY\_ROW.

The first four arguments of GxB\_Matrix\_import\_CSR are the same as all four arguments of GrB\_Matrix\_new, because this function is similar. It creates a new GrB\_Matrix A, with the given type and dimensions. The GrB\_Matrix A does not exist on input.

Unlike GrB\_Matrix\_new, this function also populates the new matrix A with the three arrays Ap, Aj and Ax, provided by the user, all of which must have been created with the ANSI C malloc, calloc, or realloc functions (by default), or by the corresponding malloc\_function, calloc\_function, or realloc\_function provided to GxB\_init. These arrays define the pattern and values of the new matrix A:

- GrB\_Index Ap [nrows+1]; The Ap array is the row "pointer" array. It does not actual contain pointers. More precisely, it is an integer array that defines where the column indices and values appear in Aj and Ax, for each row. The number of entries in row i is given by the expression Ap [i+1] Ap [i].
- GrB\_Index Aj [nvals]; The Aj array defines the column indices of entries in each row.

• ctype Ax [nvals]; The Ax array defines the values of entries in each row. It is passed in as a (void \*) pointer, but it must point to an array of size nvals values, each of size sizeof(ctype), where ctype is the exact type in C that corresponds to the GrB\_Type type parameter. That is, if type is GrB\_INT32, then ctype is int32\_t. User types may be used, just the same as built-in types.

The content of the three arrays Ap Aj, and Ax is very specific. This content is not checked, since this function takes only O(1) time. Results are undefined if the following specification is not followed exactly.

The column indices of entries in the ith row of the matrix are held in Aj [Ap [i] ... Ap[i+1]], and the corresponding values are held in the same positions in Ax. Column indices must be in the range 0 to ncols-1. If jumbled is false, column indices must appear in ascending order within each row. If jumbled is true, column indices may appear in any order within each row. No duplicate column indices may appear in any row. Ap [0] must equal zero, and Ap [nrows] must equal nvals. The Ap array must be of size nrows+1 (or larger), and the Aj and Ax arrays must have size at least nvals.

If nvals is zero, then the content of the Aj and Ax arrays is not accessed and they may be NULL on input (if not NULL, they are still freed and returned as NULL, if the method is successful).

An example of the CSR format is shown below. Consider the following matrix with 10 nonzero entries, and suppose the zeros are not stored.

$$A = \begin{bmatrix} 4.5 & 0 & 3.2 & 0 \\ 3.1 & 2.9 & 0 & 0.9 \\ 0 & 1.7 & 3.0 & 0 \\ 3.5 & 0.4 & 0 & 1.0 \end{bmatrix}$$
 (1)

The Ap array has length 5, since the matrix is 4-by-4. The first entry must always zero, and Ap [5] = 10 is the number of entries. The content of the arrays is shown below:

Spaces have been added to the Ap array, just for illustration. Row zero is in Aj [0..1] (column indices) and Ax [0..1] (values), starting at Ap [0] = 0 and ending at Ap [0+1]-1 = 1. The list of column indices of

row one is at Aj [2..4] and row two is in Aj [5..6]. The last row (three) appears Aj [7..9], because Ap [3] = 7 and Ap [4]-1 = 10-1 = 9. The corresponding numerical values appear in the same positions in Ax.

To iterate over the rows and entries of this matrix, the following code can be used (assuming it has type GrB\_FP64):

On successful creation of A, the three pointers Ap, Aj, and Ax are set to NULL on output. This denotes to the user application that it is no longer responsible for freeing these arrays. Internally, GraphBLAS has moved these arrays into its internal data structure. They will eventually be freed no later than when the user does GrB\_free(&A), but they may be freed or resized later, if the matrix changes. If an export is performed, the freeing of these three arrays again becomes the responsibility of the user application.

The  $GxB\_Matrix\_import\_CSR$  function will rarely fail, since it allocates just O(1) space. If it does fail, it returns  $GrB\_OUT\_OF\_MEMORY$ , and it leaves the three user arrays unmodified. They are still owned by the user application, which is eventually responsible for freeing them with free(Ap), etc.

# 6.11.14 GxB\_Matrix\_pack\_CSR: pack a CSR matrix

```
GrB_Info GxB_Matrix_pack_CSR
                                  // pack a CSR matrix
    GrB_Matrix A,
                       // matrix to create (type, nrows, ncols unchanged)
    GrB_Index **Ap,
                       // row "pointers", Ap_size >= (nrows+1)* sizeof(int64_t)
    GrB_Index **Aj,
                       // column indices, Aj_size >= nvals(A) * sizeof(int64_t)
   void **Ax,
                        // values, Ax_size >= nvals(A) * (type size)
                       // or Ax_size >= (type size), if iso is true
    GrB_Index Ap_size, // size of Ap in bytes
    GrB_Index Aj_size, // size of Aj in bytes
    GrB_Index Ax_size, // size of Ax in bytes
    bool iso,
                       // if true, A is iso
                       // if true, indices in each row may be unsorted
    bool jumbled,
    const GrB_Descriptor desc
);
```

Identical to GxB\_Matrix\_import\_CSR, except that the matrix must exist on input with the right type and dimensions. No typecasting is done.

# 6.11.15 GxB\_Matrix\_export\_CSR: export a CSR matrix

```
GrB_Info GxB_Matrix_export_CSR // export and free a CSR matrix
    GrB_Matrix *A,
                        // handle of matrix to export and free
    GrB_Type *type,
                        // type of matrix exported
    GrB_Index *nrows,
                        // number of rows of the matrix
   GrB_Index *ncols,
                        // number of columns of the matrix
    GrB_Index **Ap,
                        // row "pointers", Ap_size >= (nrows+1)*sizeof(int64_t)
                        // column indices, Aj_size >= nvals(A)*sizeof(int64_t)
    GrB_Index **Aj,
    void **Ax,
                        // values, Ax_size >= nvals(A)*(type size)
                        // or Ax_size >= (type size), if iso is true
    GrB_Index *Ap_size, // size of Ap in bytes
    GrB_Index *Aj_size, // size of Aj in bytes
    GrB_Index *Ax_size, // size of Ax in bytes
    bool *iso,
                        // if true, A is iso
                       // if true, indices in each row may be unsorted
    bool *jumbled,
    const GrB_Descriptor desc
);
```

GxB\_Matrix\_export\_CSR exports a matrix in CSR form.

If successful, the GrB\_Matrix A is freed, and A is returned as NULL. Its type is returned in the type parameter, its dimensions in nrows and ncols, and the CSR format is in the three arrays Ap, Aj, and Ax. If the matrix has no entries, the Aj and Ax arrays may be returned as NULL; this is not an error, and GxB\_Matrix\_import\_CSR also allows these two arrays to be NULL on input when the matrix has no entries. After a successful export, the user application is responsible for freeing these three arrays via free (or the free function passed to GxB\_init). The CSR format is described in Section 6.11.13.

If jumbled is returned as false, column indices will appear in ascending order within each row. If jumbled is returned as true, column indices may appear in any order within each row. If jumbled is passed in as NULL, then column indices will be returned in ascending order in each row. No duplicate column indices will appear in any row. Ap [0] is zero, and Ap [nrows] is equal to the number of entries in the matrix (nvals). The Ap array will be of size nrows+1 (or larger), and the Aj and Ax arrays will have size at least nvals.

This method takes O(1) time if the matrix is already in CSR format internally. Otherwise, the matrix is converted to CSR format and then exported.

# 6.11.16 GxB\_Matrix\_unpack\_CSR: unpack a CSR matrix

```
GrB_Info GxB_Matrix_unpack_CSR // unpack a CSR matrix
                       // matrix to unpack (type, nrows, ncols unchanged)
   GrB_Matrix A,
GrB_Index **Ap,
   GrB_Matrix A,
                       // row "pointers"
    GrB_Index **Aj,
                       // column indices
   void **Ax,
                        // values
    GrB_Index *Ap_size, // size of Ap in bytes
    GrB_Index *Aj_size, // size of Aj in bytes
   GrB_Index *Ax_size, // size of Ax in bytes
                      // if true, A is iso
    bool *iso,
   bool *jumbled,
                     // if true, indices in each row may be unsorted
    const GrB_Descriptor desc
);
```

Identical to GxB\_Matrix\_export\_CSR, except that the matrix still exists on output with the type and dimensions unchanged, but with no entries.

#### 6.11.17 GxB\_Matrix\_import\_CSC: import a CSC matrix

```
GrB_Info GxB_Matrix_import_CSC
                                   // import a CSC matrix
   GrB_Matrix *A,
                       // handle of matrix to create
                       // type of matrix to create
   GrB_Type type,
   GrB_Index nrows,
                       // number of rows of the matrix
   GrB_Index ncols,
                       // number of columns of the matrix
   GrB_Index **Ap,
                       // col "pointers", Ap_size >= (ncols+1)*sizeof(int64_t)
                       // row indices, Ai_size >= nvals(A)*sizeof(int64_t)
   GrB_Index **Ai,
   void **Ax,
                       // values, Ax_size >= nvals(A) * (type size)
                       // or Ax_size >= (type size), if iso is true
   GrB_Index Ap_size, // size of Ap in bytes
   GrB_Index Ai_size, // size of Ai in bytes
   GrB_Index Ax_size, // size of Ax in bytes
   bool iso,
                       // if true, A is iso
                       // if true, indices in each column may be unsorted
   bool jumbled,
    const GrB_Descriptor desc
);
```

GxB\_Matrix\_import\_CSC imports a matrix from 3 user arrays in CSC format. The GrB\_Matrix A is created in the CSC format, which is a GxB\_FORMAT of GxB\_BY\_COL. The arguments are identical to GxB\_Matrix\_import\_CSR, except for how the 3 user arrays are interpreted. The column "pointer" array has size ncols+1. The row indices of the columns are in Ai, and must appear in ascending order in each column. The corresponding numerical values are held in Ax. The row indices of column j are held in Ai [Ap [j]...Ap [j+1]-1], and the corresponding numerical values are in the same locations in Ax.

The same matrix from Equation 1 in the last section (repeated here):

$$A = \begin{bmatrix} 4.5 & 0 & 3.2 & 0 \\ 3.1 & 2.9 & 0 & 0.9 \\ 0 & 1.7 & 3.0 & 0 \\ 3.5 & 0.4 & 0 & 1.0 \end{bmatrix}$$
 (2)

is held in CSC form as follows:

That is, the row indices of column 1 (the second column) are in Ai [3..5], and the values in the same place in Ax, since Ap [1] = 3 and Ap [2]-1 = 5.

To iterate over the columns and entries of this matrix, the following code can be used (assuming it has type GrB\_FP64):

The method is identical to GxB\_Matrix\_import\_CSR; just the format is transposed.

If Ap [ncols] is zero, then the content of the Ai and Ax arrays is not accessed and they may be NULL on input (if not NULL, they are still freed and returned as NULL, if the method is successful).

# 6.11.18 GxB\_Matrix\_pack\_CSC: pack a CSC matrix

```
GrB_Info GxB_Matrix_pack_CSC
                                 // pack a CSC matrix
    GrB_Matrix A,
                       // matrix to create (type, nrows, ncols unchanged)
    GrB_Index **Ap,
                       // col "pointers", Ap_size >= (ncols+1)*sizeof(int64_t)
    GrB_Index **Ai,
                       // row indices, Ai_size >= nvals(A)*sizeof(int64_t)
    void **Ax,
                       // values, Ax_size >= nvals(A) * (type size)
                       // or Ax_size >= (type size), if iso is true
    GrB_Index Ap_size, // size of Ap in bytes
    GrB_Index Ai_size, // size of Ai in bytes
    GrB_Index Ax_size, // size of Ax in bytes
    bool iso,
                       // if true, A is iso
                       // if true, indices in each column may be unsorted
    bool jumbled,
    const GrB_Descriptor desc
);
```

Identical to GxB\_Matrix\_import\_CSC, except that the matrix must exist on input with the right type and dimensions. No typecasting is done.

#### 6.11.19 GxB\_Matrix\_export\_CSC: export a CSC matrix

```
GrB_Info GxB_Matrix_export_CSC // export and free a CSC matrix
    GrB_Matrix *A,
                       // handle of matrix to export and free
   GrB_Type *type,
                       // type of matrix exported
    GrB_Index *nrows,
                       // number of rows of the matrix
    GrB_Index *ncols,
                       // number of columns of the matrix
    GrB_Index **Ap,
                       // column "pointers", Ap_size >= ncols+1
                       // row indices, Ai_size >= nvals(A)*sizeof(int64_t)
    GrB_Index **Ai,
    void **Ax,
                        // values, Ax_size >= nvals(A)*(type size)
                       // or Ax_size >= (type size), if iso is true
    GrB_Index *Ap_size, // size of Ap in bytes
    GrB_Index *Ai_size, // size of Ai in bytes
   GrB_Index *Ax_size, // size of Ax in bytes
    bool *iso,
                       // if true, A is iso
                       // if true, indices in each column may be unsorted
    bool *jumbled,
    const GrB_Descriptor desc
);
```

GxB\_Matrix\_export\_CSC exports a matrix in CSC form.

If successful, the GrB\_Matrix A is freed, and A is returned as NULL. Its type is returned in the type parameter, its dimensions in nrows and ncols, and the CSC format is in the three arrays Ap, Ai, and Ax. If the matrix has no entries, Ai and Ax arrays are returned as NULL; this is not an error, and GxB\_Matrix\_import\_CSC also allows these two arrays to be NULL on input when the matrix has no entries. After a successful export, the user application is responsible for freeing these three arrays via free (or the free function passed to GxB\_init). The CSC format is described in Section 6.11.17.

This method takes O(1) time if the matrix is already in CSC format internally. Otherwise, the matrix is converted to CSC format and then exported.

# 6.11.20 GxB\_Matrix\_unpack\_CSC: unpack a CSC matrix

```
GrB_Info GxB_Matrix_unpack_CSC // unpack a CSC matrix
   GrB_Index **Ap,
                       // matrix to unpack (type, nrows, ncols unchanged)
                      // column "pointers"
   GrB_Index **Ai,
                       // row indices
   void **Ax,
                       // values
   GrB_Index *Ap_size, // size of Ap in bytes
   GrB_Index *Ai_size, // size of Ai in bytes
   GrB_Index *Ax_size, // size of Ax in bytes
   bool *iso,
                     // if true, A is iso
   bool *jumbled,
                    // if true, indices in each column may be unsorted
   const GrB_Descriptor desc
);
```

Identical to GxB\_Matrix\_unpack\_CSC, except that the matrix still exists on output with the type and dimensions unchanged, but with no entries.

#### 6.11.21 GxB\_Matrix\_import\_HyperCSR: import a HyperCSR matrix

```
GrB_Info GxB_Matrix_import_HyperCSR
                                        // import a hypersparse CSR matrix
   GrB_Matrix *A,
                       // handle of matrix to create
   GrB_Type type,
                       // type of matrix to create
   GrB_Index nrows,
                       // number of rows of the matrix
   GrB_Index ncols,
                       // number of columns of the matrix
   GrB_Index **Ap,
                       // row "pointers", Ap_size >= (nvec+1)*sizeof(int64_t)
                       // row indices, Ah_size >= (nvec)*sizeof(int64_t)
   GrB_Index **Ah,
   GrB_Index **Aj,
                       // column indices, Aj_size >= nvals(A)*sizeof(int64_t)
   void **Ax,
                       // values, Ax_size >= nvals(A) * (type size)
                       // or Ax_size >= (type size), if iso is true
   GrB_Index Ap_size, // size of Ap in bytes
   GrB_Index Ah_size, // size of Ah in bytes
   GrB_Index Aj_size, // size of Aj in bytes
   GrB_Index Ax_size, // size of Ax in bytes
   bool iso,
                       // if true, A is iso
   GrB_Index nvec,
                       // number of rows that appear in Ah
   bool jumbled,
                    // if true, indices in each row may be unsorted
    const GrB_Descriptor desc
);
```

GxB\_Matrix\_import\_HyperCSR imports a matrix in hypersparse CSR format. The hypersparse HyperCSR format is identical to the CSR format, except that the Ap array itself becomes sparse, if the matrix has rows that are completely empty. An array Ah of size nvec provides a list of rows that appear in the data structure. For example, consider Equation 3, which is a sparser version of the matrix in Equation 1. Row 2 and column 1 of this matrix are all zero.

$$A = \begin{bmatrix} 4.5 & 0 & 3.2 & 0 \\ 3.1 & 0 & 0 & 0.9 \\ 0 & 0 & 0 & 0 \\ 3.5 & 0 & 0 & 1.0 \end{bmatrix}$$
 (3)

The conventional CSR format would appear as follows. Since the third row (row 2) is all zero, accessing Ai [Ap [2] ... Ap [3]-1] gives an empty set ([2..1]), and the number of entries in this row is Ap [i+1] - Ap [i] = Ap [3] - Ap [2] = 0.

```
int64_t Ap [] = { 0, 2, 2, 4, 5 };
int64_t Aj [] = { 0, 2, 0, 3, 0 3 }
double Ax [] = { 4.5, 3.2, 3.1, 0.9, 3.5, 1.0 };
```

A hypersparse CSR format for this same matrix would discard these duplicate integers in Ap. Doing so requires another array, Ah, that keeps track of the rows that appear in the data structure.

Note that the Aj and Ax arrays are the same in the CSR and HyperCSR formats. The row indices in Ah must appear in ascending order, and no duplicates can appear. To iterate over this data structure (assuming it has type GrB\_FP64):

This is more complex than the CSR format, but it requires at most O(e) space, where A is m-by-n with  $e = \mathtt{nvals}$  entries. The CSR format requires O(m+e) space. If e << m, then the size m+1 of Ap can dominate the memory required. In the hypersparse form, Ap takes on size  $\mathtt{nvec+1}$ , and Ah has size  $\mathtt{nvec}$ , where  $\mathtt{nvec}$  is the number of rows that appear in the data structure. The CSR format can be viewed as a dense array (of size  $\mathtt{nrows}$ ) of sparse row vectors. By contrast, the hypersparse CSR format is a sparse array (of size  $\mathtt{nvec}$ ) of sparse row vectors.

The import takes O(1) time. If successful, the four arrays Ah, Ap, Aj, and Ax are returned as NULL, and the hypersparse  $GrB_Matrix$  A is created.

If the matrix has no entries, then the content of the Aj and Ax arrays is not accessed and they may be NULL on input (if not NULL, they are still freed and returned as NULL, if the method is successful).

# 6.11.22 GxB\_Matrix\_pack\_HyperCSR: pack a HyperCSR matrix

```
GrB_Info GxB_Matrix_pack_HyperCSR
                                      // pack a hypersparse CSR matrix
   GrB_Matrix A,
                       // matrix to create (type, nrows, ncols unchanged)
   GrB_Index **Ap,
                      // row "pointers", Ap_size >= (nvec+1)*sizeof(int64_t)
   GrB_Index **Ah,
                      // row indices, Ah_size >= nvec*sizeof(int64_t)
   GrB_Index **Aj,
                       // column indices, Aj_size >= nvals(A)*sizeof(int64_t)
   void **Ax,
                       // values, Ax_size >= nvals(A) * (type size)
                       // or Ax_size >= (type size), if iso is true
   GrB_Index Ap_size, // size of Ap in bytes
   GrB_Index Ah_size, // size of Ah in bytes
   GrB_Index Aj_size, // size of Aj in bytes
   GrB_Index Ax_size, // size of Ax in bytes
   bool iso,
                       // if true, A is iso
   GrB_Index nvec,
                      // number of rows that appear in Ah
   bool jumbled, // if true, indices in each row may be unsorted
   const GrB_Descriptor desc
);
```

Identical to GxB\_Matrix\_import\_HyperCSR, except that the matrix must exist on input with the right type and dimensions. No typecasting is done.

# 6.11.23 GxB\_Matrix\_export\_HyperCSR: export a HyperCSR matrix

```
GrB_Info GxB_Matrix_export_HyperCSR // export and free a hypersparse CSR matrix
    GrB_Matrix *A,
                        // handle of matrix to export and free
    GrB_Type *type,
                       // type of matrix exported
    GrB_Index *nrows,
                       // number of rows of the matrix
                       // number of columns of the matrix
    GrB_Index *ncols,
    GrB_Index **Ap,
                       // row "pointers", Ap_size >= (nvec+1)*sizeof(int64_t)
                       // row indices, Ah_size >= nvec*sizeof(int64_t)
    GrB_Index **Ah,
    GrB_Index **Aj,
                       // column indices, Aj_size >= nvals(A)*sizeof(int64_t)
    void **Ax,
                       // values, Ax_size >= nvals(A)*(type size)
                       // or Ax_size >= (type size), if iso is true
    GrB_Index *Ap_size, // size of Ap in bytes
   GrB_Index *Ah_size, // size of Ah in bytes
    GrB_Index *Aj_size, // size of Aj in bytes
    GrB_Index *Ax_size, // size of Ax in bytes
                       // if true, A is iso
    bool *iso,
    GrB_Index *nvec,
                       // number of rows that appear in Ah
    bool *jumbled,
                       // if true, indices in each row may be unsorted
    const GrB_Descriptor desc
);
```

GxB\_Matrix\_export\_HyperCSR exports a matrix in HyperCSR format.

If successful, the GrB\_Matrix A is freed, and A is returned as NULL. Its type is returned in the type parameter, its dimensions in nrows and ncols. and the number of non-empty rows in nvec. The hypersparse CSR format is in the four arrays Ah, Ap, Aj, and Ax. If the matrix has no entries, the Aj and Ax arrays are returned as NULL; this is not an error. After a successful export, the user application is responsible for freeing these three arrays via free (or the free function passed to GxB\_init). The hypersparse CSR format is described in Section 6.11.21.

This method takes O(1) time if the matrix is already in HyperCSR format internally. Otherwise, the matrix is converted to HyperCSR format and then exported.

# 6.11.24 GxB\_Matrix\_unpack\_HyperCSR: unpack a HyperCSR matrix

```
GrB_Info GxB_Matrix_unpack_HyperCSR // unpack a hypersparse CSR matrix
                       // matrix to unpack (type, nrows, ncols unchanged)
    GrB_Matrix A,
   GrB_Index **Ap,
                       // row "pointers"
    GrB_Index **Ah,
                       // row indices
   GrB_Index **Aj,
                       // column indices
    void **Ax,
                       // values
    GrB_Index *Ap_size, // size of Ap in bytes
    GrB_Index *Ah_size, // size of Ah in bytes
    GrB_Index *Aj_size, // size of Aj in bytes
    GrB_Index *Ax_size, // size of Ax in bytes
                       // if true, A is iso
    bool *iso,
    GrB_Index *nvec,
                       // number of rows that appear in Ah
   bool *jumbled,
                       // if true, indices in each row may be unsorted
    const GrB_Descriptor desc
);
```

Identical to GxB\_Matrix\_export\_HyperCSR, except that the matrix still exists on output with the type and dimensions unchanged, but with no entries.

# 6.11.25 GxB\_Matrix\_import\_HyperCSC: import a HyperCSC matrix

```
GrB_Info GxB_Matrix_import_HyperCSC
                                        // import a hypersparse CSC matrix
   GrB_Matrix *A,
                       // handle of matrix to create
   GrB_Type type,
                       // type of matrix to create
   GrB_Index nrows,
                       // number of rows of the matrix
                       // number of columns of the matrix
   GrB_Index ncols,
   GrB_Index **Ap,
                       // col "pointers", Ap_size >= (nvec+1)*sizeof(int64_t)
   GrB_Index **Ah,
                       // column indices, Ah_size >= nvec*sizeof(int64_t)
   GrB_Index **Ai,
                       // row indices, Ai_size >= nvals(A)*sizeof(int64_t)
   void **Ax,
                       // values, Ax_size >= nvals(A)*(type size)
                       // or Ax_size >= (type size), if iso is true
   GrB_Index Ap_size, // size of Ap in bytes
   GrB_Index Ah_size, // size of Ah in bytes
   GrB_Index Ai_size, // size of Ai in bytes
   GrB_Index Ax_size, // size of Ax in bytes
   bool iso,
                       // if true, A is iso
   GrB_Index nvec,
                       // number of columns that appear in Ah
   bool jumbled,
                    // if true, indices in each column may be unsorted
    const GrB_Descriptor desc
);
```

GxB\_Matrix\_import\_HyperCSC imports a matrix in hypersparse CSC format. It is identical to GxB\_Matrix\_import\_HyperCSR, except the data structure defined by the four arrays Ah, Ap, Ai, and Ax holds the matrix as a sparse array of nvec sparse column vectors. The following code iterates over the matrix, assuming it has type GrB\_FP64:

# 6.11.26 GxB\_Matrix\_pack\_HyperCSC: pack a HyperCSC matrix

```
GrB_Info GxB_Matrix_pack_HyperCSR
                                      // pack a hypersparse CSR matrix
   GrB_Matrix A,
                       // matrix to create (type, nrows, ncols unchanged)
   GrB_Index **Ap,
                      // row "pointers", Ap_size >= (nvec+1)*sizeof(int64_t)
   GrB_Index **Ah,
                      // row indices, Ah_size >= nvec*sizeof(int64_t)
   GrB_Index **Aj,
                       // column indices, Aj_size >= nvals(A)*sizeof(int64_t)
   void **Ax,
                       // values, Ax_size >= nvals(A) * (type size)
                       // or Ax_size >= (type size), if iso is true
   GrB_Index Ap_size, // size of Ap in bytes
   GrB_Index Ah_size, // size of Ah in bytes
   GrB_Index Aj_size, // size of Aj in bytes
   GrB_Index Ax_size, // size of Ax in bytes
   bool iso,
                       // if true, A is iso
   GrB_Index nvec,
                      // number of rows that appear in Ah
   bool jumbled, // if true, indices in each row may be unsorted
   const GrB_Descriptor desc
);
```

Identical to GxB\_Matrix\_import\_HyperCSC, except that the matrix must exist on input with the right type and dimensions. No typecasting is done.

# 6.11.27 GxB\_Matrix\_export\_HyperCSC: export a HyperCSC matrix

```
GrB_Info GxB_Matrix_export_HyperCSC // export and free a hypersparse CSC matrix
    GrB_Matrix *A,
                        // handle of matrix to export and free
    GrB_Type *type,
                       // type of matrix exported
    GrB_Index *nrows,
                       // number of rows of the matrix
    GrB_Index *ncols,
                       // number of columns of the matrix
    GrB_Index **Ap,
                       // column "pointers", Ap_size >= nvec+1
                       // column indices, Ah_size >= nvec
    GrB_Index **Ah,
    GrB_Index **Ai,
                       // row indices, Ai_size >= nvals(A)
    void **Ax,
                       // values, Ax_size >= nvals(A)*(type size)
                       // or Ax_size >= (type size), if iso is true
   GrB_Index *Ap_size, // size of Ap
    GrB_Index *Ah_size, // size of Ah
    GrB_Index *Ai_size, // size of Ai
    GrB_Index *Ax_size, // size of Ax
                       // if true, A is iso
    bool *iso,
    GrB_Index *nvec,
                       // number of columns that appear in Ah
    bool *jumbled,
                       // if true, indices in each column may be unsorted
    const GrB_Descriptor desc
);
```

GxB\_Matrix\_export\_HyperCSC exports a matrix in HyperCSC form.

If successful, the GrB\_Matrix A is freed, and A is returned as NULL. Its type is returned in the type parameter, its dimensions in nrows and ncols, and the number of non-empty rows in nvec. The hypersparse CSC format is in the four arrays Ah, Ap, Ai, and Ax. If the matrix has no entries, the Ai and Ax arrays are returned as NULL; this is not an error. After a successful export, the user application is responsible for freeing these three arrays via free (or the free function passed to GxB\_init). The hypersparse CSC format is described in Section 6.11.25.

This method takes O(1) time if the matrix is already in HyperCSC format internally. Otherwise, the matrix is converted to HyperCSC format and then exported.

# 6.11.28 GxB\_Matrix\_unpack\_HyperCSC: unpack a HyperCSC matrix

```
GrB_Info GxB_Matrix_unpack_HyperCSR // unpack a hypersparse CSR matrix
                       // matrix to unpack (type, nrows, ncols unchanged)
    GrB_Matrix A,
   GrB_Index **Ap,
                       // row "pointers"
    GrB_Index **Ah,
                       // row indices
   GrB_Index **Aj,
                       // column indices
   void **Ax,
                       // values
   GrB_Index *Ap_size, // size of Ap in bytes
    GrB_Index *Ah_size, // size of Ah in bytes
   GrB_Index *Aj_size, // size of Aj in bytes
    GrB_Index *Ax_size, // size of Ax in bytes
                       // if true, A is iso
    bool *iso,
    GrB_Index *nvec,
                       // number of rows that appear in Ah
                       // if true, indices in each row may be unsorted
    bool *jumbled,
    const GrB_Descriptor desc
);
```

Identical to GxB\_Matrix\_export\_HyperCSC, except that the matrix still exists on output with the type and dimensions unchanged, but with no entries.

#### 6.11.29 GxB\_Matrix\_import\_BitmapR: import a BitmapR matrix

```
GrB_Info GxB_Matrix_import_BitmapR // import a bitmap matrix, held by row
    GrB_Matrix *A,
                        // handle of matrix to create
    GrB_Type type,
                       // type of matrix to create
    GrB_Index nrows,
                       // number of rows of the matrix
    GrB_Index ncols,
                       // number of columns of the matrix
    int8_t **Ab,
                       // bitmap, Ab_size >= nrows*ncols
    void **Ax,
                        // values, Ax_size >= nrows*ncols * (type size)
                       // or Ax_size >= (type size), if iso is true
   GrB_Index Ab_size, // size of Ab in bytes
    GrB_Index Ax_size, // size of Ax in bytes
                       // if true, A is iso
   bool iso,
    GrB_Index nvals,
                       // # of entries in bitmap
    const GrB_Descriptor desc
);
```

GxB\_Matrix\_import\_BitmapR imports a matrix from 2 user arrays in BitmapR format.

The first four arguments of GxB\_Matrix\_import\_BitmapR are the same as all four arguments of GrB\_Matrix\_new, because this function is similar. It creates a new GrB\_Matrix A, with the given type and dimensions. The GrB\_Matrix A does not exist on input.

The GrB\_Matrix A is created from the arrays Ab and Ax, each of which are size nrows\*ncols. Both arrays must have been created with the ANSI C malloc, calloc, or realloc functions (by default), or by the corresponding malloc\_function, calloc\_function, or realloc\_function provided to GxB\_init. These arrays define the pattern and values of the new matrix A:

- int8\_t Ab [nrows\*ncols]; The Ab array defines which entries of A are present. If Ab[i\*ncols+j]=1, then the entry A(i,j) is present, with value Ax[i\*ncols+j]. If Ab[i\*ncols+j]=0, then the entry A(i,j) is not present. The Ab array must contain only 0s and 1s. The nvals input must exactly match the number of 1s in the Ab array.
- ctype Ax [nrows\*ncols]; The Ax array defines the values of entries in the matrix. It is passed in as a (void \*) pointer, but it must point to an array of size nrows\*ncols values, each of size sizeof(ctype), where ctype is the exact type in C that corresponds to the GrB\_Type type parameter. That is, if type is GrB\_INT32, then ctype is int32\_t. User

types may be used, just the same as built-in types. If Ab[p] is zero, the value of Ax[p] is ignored.

To iterate over the rows and entries of this matrix, the following code can be used (assuming it has type GrB\_FP64):

On successful creation of A, the two pointers Ab, Ax, are set to NULL on output. This denotes to the user application that it is no longer responsible for freeing these arrays. Internally, GraphBLAS has moved these arrays into its internal data structure. They will eventually be freed no later than when the user does GrB\_free(&A), but they may be freed or resized later, if the matrix changes. If an export is performed, the freeing of these three arrays again becomes the responsibility of the user application.

The  $GxB\_Matrix\_import\_BitmapR$  function will rarely fail, since it allocates just O(1) space. If it does fail, it returns  $GrB\_OUT\_OF\_MEMORY$ , and it leaves the two user arrays unmodified. They are still owned by the user application, which is eventually responsible for freeing them with free(Ab), etc.

# 6.11.30 GxB\_Matrix\_pack\_BitmapR: pack a BitmapR matrix

Identical to GxB\_Matrix\_import\_BitmapR, except that the matrix must exist on input with the right type and dimensions. No typecasting is done.

#### 6.11.31 GxB\_Matrix\_export\_BitmapR: export a BitmapR matrix

```
GrB_Info GxB_Matrix_export_BitmapR // export and free a bitmap matrix, by row
    GrB_Matrix *A,
                       // handle of matrix to export and free
    GrB_Type *type,
                       // type of matrix exported
   GrB_Index *nrows,
                       // number of rows of the matrix
    GrB_Index *ncols,
                       // number of columns of the matrix
    int8_t **Ab,
                       // bitmap, Ab_size >= nrows*ncols
    void **Ax,
                       // values, Ax_size >= nrows*ncols * (type size)
                       // or Ax_size >= (type size), if iso is true
    GrB_Index *Ab_size, // size of Ab in bytes
    GrB_Index *Ax_size, // size of Ax in bytes
                       // if true, A is iso
    bool *iso,
                       // # of entries in bitmap
    GrB_Index *nvals,
    const GrB_Descriptor desc
);
```

GxB\_Matrix\_export\_BitmapR exports a matrix in BitmapR form.

If successful, the GrB\_Matrix A is freed, and A is returned as NULL. Its type is returned in the type parameter, its dimensions in nrows and ncols, and the number of entries in nvals.

The BitmapR format is in the two arrays Ab, and Ax. After a successful export, the user application is responsible for freeing these three arrays via free (or the free function passed to GxB\_init). The BitmapR format is described in Section 6.11.29. If Ab[p] is zero, the value of Ax[p] is undefined.

This method takes O(1) time if the matrix is already in BitmapR format internally. Otherwise, the matrix is converted to BitmapR format and then exported.

# 6.11.32 GxB\_Matrix\_unpack\_BitmapR: unpack a BitmapR matrix

Identical to GxB\_Matrix\_export\_BitmapR, except that the matrix still exists on output with the type and dimensions unchanged, but with no entries.

# 6.11.33 GxB\_Matrix\_import\_BitmapC: import a BitmapC matrix

```
GrB_Info GxB_Matrix_import_BitmapC // import a bitmap matrix, held by column
   GrB_Matrix *A,
                       // handle of matrix to create
   GrB_Type type,
                       // type of matrix to create
   GrB_Index nrows,
                       // number of rows of the matrix
   GrB_Index ncols,
                      // number of columns of the matrix
   int8_t **Ab,
                       // bitmap, Ab_size >= nrows*ncols
   void **Ax,
                       // values, Ax_size >= nrows*ncols * (type size)
                       // or Ax_size >= (type size), if iso is true
   GrB_Index Ab_size, // size of Ab in bytes
   GrB_Index Ax_size, // size of Ax in bytes
                       // if true, A is iso
   bool iso,
   GrB_Index nvals, // # of entries in bitmap
   const GrB_Descriptor desc
) ;
```

GxB\_Matrix\_import\_BitmapC imports a matrix from 2 user arrays in BitmapC format. It is identical to GxB\_Matrix\_import\_BitmapR, except that the entry A(i,j) is held in Ab[i+j\*nrows] and Ax[i+j\*nrows], in column-major format.

# 6.11.34 GxB\_Matrix\_pack\_BitmapC: pack a BitmapC matrix

Identical to GxB\_Matrix\_import\_BitmapC, except that the matrix must exist on input with the right type and dimensions. No typecasting is done.

# 6.11.35 GxB\_Matrix\_export\_BitmapC: export a BitmapC matrix

```
GrB_Info GxB_Matrix_export_BitmapC // export and free a bitmap matrix, by col
                       // handle of matrix to export and free
   GrB_Matrix *A,
   GrB_Type *type,
                       // type of matrix exported
   GrB_Index *nrows,
                       // number of rows of the matrix
   GrB_Index *ncols, // number of columns of the matrix
   int8_t **Ab,
                       // bitmap, Ab_size >= nrows*ncols
   void **Ax,
                       // values, Ax_size >= nrows*ncols * (type size)
                       // or Ax_size >= (type size), if iso is true
   GrB_Index *Ab_size, // size of Ab in bytes
   GrB_Index *Ax_size, // size of Ax in bytes
                       // if true, A is iso
   bool *iso,
                     // # of entries in bitmap
   GrB_Index *nvals,
   const GrB_Descriptor desc
) ;
```

GxB\_Matrix\_export\_BitmapC exports a matrix in BitmapC form. It is identical to GxB\_Matrix\_export\_BitmapR, except that the entry A(i,j) is held in Ab[i+j\*nrows] and Ax[i+j\*nrows], in column-major format.

# 6.11.36 GxB\_Matrix\_unpack\_BitmapC: unpack a BitmapC matrix

Identical to GxB\_Matrix\_export\_BitmapC, except that the matrix still exists on output with the type and dimensions unchanged, but with no entries.

#### 6.11.37 GxB\_Matrix\_import\_FullR: import a FullR matrix

```
GrB_Info GxB_Matrix_import_FullR // import a full matrix, held by row
   GrB_Matrix *A,
                       // handle of matrix to create
                       // type of matrix to create
   GrB_Type type,
   GrB_Index nrows,
                       // number of rows of the matrix
   GrB_Index ncols,
                       // number of columns of the matrix
   void **Ax,
                       // values, Ax_size >= nrows*ncols * (type size)
                       // or Ax_size >= (type size), if iso is true
   GrB_Index Ax_size, // size of Ax in bytes
   bool iso,
                       // if true, A is iso
   const GrB_Descriptor desc
);
```

GxB\_Matrix\_import\_FullR imports a matrix from a user arrays in FullR format. The FullR format is identical to BitmapR, except that all entries are present. The value of A(i, j) is Ax[i\*ncols+j].

To iterate over the rows and entries of this matrix, the following code can be used (assuming it has type  $GrB_FP64$ ). Note that if A is both full and iso, it takes O(1) memory, regardless of nrows and ncols.

# 6.11.38 GxB\_Matrix\_pack\_FullR: pack a FullR matrix

Identical to GxB\_Matrix\_import\_FullR, except that the matrix must exist on input with the right type and dimensions. No typecasting is done.

# 6.11.39 GxB\_Matrix\_export\_FullR: export a FullR matrix

```
GrB_Info GxB_Matrix_export_FullR // export and free a full matrix, by row
   GrB_Matrix *A,
                       // handle of matrix to export and free
   GrB_Type *type,
                       // type of matrix exported
   GrB_Index *nrows,
                       // number of rows of the matrix
   GrB_Index *ncols,
                       // number of columns of the matrix
                       // values, Ax_size >= nrows*ncols * (type size)
   void **Ax,
                       // or Ax_size >= (type size), if iso is true
   GrB_Index *Ax_size, // size of Ax in bytes
                       // if true, A is iso
   bool *iso,
   const GrB_Descriptor desc
) ;
```

GxB\_Matrix\_export\_FullR exports a matrix in FullR form. It is identical to GxB\_Matrix\_export\_BitmapR, except that all entries must be present.

That is, prior to export, GrB\_Matrix\_nvals (&nvals, A) must return nvals equal to nrows\*ncols. Otherwise, if the A is exported with GxB\_Matrix\_export\_FullR, an error is returned (GrB\_INVALID\_VALUE) and the matrix is not exported.

# 6.11.40 GxB\_Matrix\_unpack\_FullR: unpack a FullR matrix

Identical to GxB\_Matrix\_export\_FullR, except that the matrix still exists on output with the type and dimensions unchanged, but with no entries.

#### 6.11.41 GxB\_Matrix\_import\_FullC: import a FullC matrix

```
GrB_Info GxB_Matrix_import_FullC // import a full matrix, held by column
   GrB_Matrix *A,
                       // handle of matrix to create
                       // type of matrix to create
   GrB_Type type,
   GrB_Index nrows,
                       // number of rows of the matrix
   GrB_Index ncols,
                       // number of columns of the matrix
   void **Ax,
                       // values, Ax_size >= nrows*ncols * (type size)
                       // or Ax_size >= (type size), if iso is true
   GrB_Index Ax_size, // size of Ax in bytes
                       // if true, A is iso
   bool iso,
   const GrB_Descriptor desc
);
```

GxB\_Matrix\_import\_FullC imports a matrix from a user arrays in FullC format. The FullC format is identical to BitmapC, except that all entries are present. The value of A(i, j) is Ax[i+j\*nrows].

To iterate over the rows and entries of this matrix, the following code can be used (assuming it has type  $GrB_FP64$ ). Note that if A is both full and iso, it takes O(1) memory, regardless of nrows and ncols.

# 6.11.42 GxB\_Matrix\_pack\_FullC: pack a FullC matrix

Identical to GxB\_Matrix\_import\_FullC, except that the matrix must exist on input with the right type and dimensions. No typecasting is done.

# 6.11.43 GxB\_Matrix\_export\_FullC: export a FullC matrix

```
GrB_Info GxB_Matrix_export_FullC // export and free a full matrix, by column
   GrB_Matrix *A,
                       // handle of matrix to export and free
   GrB_Type *type,
                       // type of matrix exported
   GrB_Index *nrows,
                       // number of rows of the matrix
   GrB_Index *ncols,
                       // number of columns of the matrix
                       // values, Ax_size >= nrows*ncols * (type size)
   void **Ax,
                       // or Ax_size >= (type size), if iso is true
   GrB_Index *Ax_size, // size of Ax in bytes
                       // if true, A is iso
   bool *iso,
   const GrB_Descriptor desc
) ;
```

GxB\_Matrix\_export\_FullC exports a matrix in FullC form. It is identical to GxB\_Matrix\_export\_BitmapC, except that all entries must be present.

That is, prior to export, GrB\_Matrix\_nvals (&nvals, A) must return nvals equal to nrows\*ncols. Otherwise, if the A is exported with GxB\_Matrix\_export\_FullC, an error is returned (GrB\_INVALID\_VALUE) and the matrix is not exported.

# 6.11.44 GxB\_Matrix\_unpack\_FullC: unpack a FullC matrix

Identical to GxB\_Matrix\_export\_FullC, except that the matrix still exists on output with the type and dimensions unchanged, but with no entries.

# 6.12 GraphBLAS import/export: using copy semantics

The v2.0 C API includes import/export methods for matrices (not vectors) using a different strategy as compared to the GxB import/export/pack/unpack methods. The GxB methods are based on move semantics, in which ownership of arrays is passed between SuiteSparse:GraphBLAS and the user application. This allows import/export/pack/unpack to work in O(1) time, and require no additional memory, but it requires that GraphBLAS and the user application agree on which memory manager to use. This is done via GxB\_init. This allows GraphBLAS to malloc an array that can be later freed by the user applicatin, and visa versa.

The GrB import/export methods take a different approach. The data is always copied in and out between the opaque GraphBLAS matrix and the user arrays. This takes  $\Omega(e)$  time, if the matrix has e entries, and requires more memory. It has the advantage that it does not require GraphBLAS and the user application to agree on what memory manager to use, since no ownership of allocated arrays is changed.

The format for GrB\_Matrix\_import and GrB\_Matrix\_export is controlled by the following enum:

#### 6.12.1 GrB\_Matrix\_import: import a matrix

```
GrB_Info GrB_Matrix_import // import a matrix
    GrB_Matrix *A,
                            // handle of matrix to create
    GrB_Type type,
                            // type of matrix to create
   GrB_Index nrows,
                            // number of rows of the matrix
    GrB_Index ncols,
                            // number of columns of the matrix
    const GrB_Index *Ap,
                            // pointers for CSR, CSC, column indices for COO
    const GrB_Index *Ai,
                            // row indices for CSR, CSC
    const void *Ax,
                            // values
                            // number of entries in Ap (not # of bytes)
    GrB_Index Ap_len,
    GrB_Index Ai_len,
                            // number of entries in Ai (not # of bytes)
    GrB_Index Ax_len,
                            // number of entries in Ax (not # of bytes)
    GrB Format format
                            // import format
);
```

The GrB\_Matrix\_import method copies from user-provided arrays into an opaque GrB\_Matrix and GrB\_Matrix\_export copies data out, from an opaque GrB\_Matrix into user-provided arrays. Unlike the GxB import/export methods, memory is not handed off between the user application and Graph-BLAS. The three arrays Ap, Ai. and Ax are not modified, and are still owned by the user application when the method finishes.

The matrix can be imported in one of five different formats:

- GrB\_CSR\_FORMAT: Compressed-row format. Ap is an array of size nrows+1. The arrays Ai and Ax are of size nvals = Ap [nrows], and Ap[0] must be zero. The column indices of entries in the ith row appear in Ai[Ap[i]...Ap[i+1]-1], and the values of those entries appear in the same locations in Ax. The column indices need not be in any particular order.
- GrB\_CSC\_FORMAT: Compressed-column format. Ap is an array of size ncols+1. The arrays Ai and Ax are of size nvals = Ap [ncols], and Ap[0] must be zero. The row indices of entries in the jth column appear in Ai[Ap[j]...Ap[j+1]-1], and the values of those entries appear in the same locations in Ax. The row indices need not be in any particular order.
- GrB\_COO\_FORMAT: Coordinate format. This is the same format as GrB\_Matrix\_build. The three arrays Ap, Ai, and Ax have the same size. The kth tuple has row index Ai[k], column index Ap[k], and value Ax[k]. The tuples can appear any order, but no duplicates are permitted.

- Grb\_Dense\_row\_format: Dense matrix format, held by row. Only the Ax array is used, of size nrows\*ncols. It holds the matrix in dense format, in row major order.
- GrB\_DENSE\_COL\_FORMAT: Dense matrix format, held by column. Only the Ax array is used, of size nrows\*ncols. It holds the matrix in dense format, in colum major order.

#### **6.12.2** GrB\_Matrix\_export: export a matrix

GrB\_Matrix\_export copies the contents of a matrix into three user-provided arrays, using any one of the five different formats described in Section 6.12.1. The size of the arrays must be at least as large as the lengths returned by GrB\_Matrix\_exportSize. The matrix A is not modified.

The GrB\_DENSE\_ROW\_FORMAT and GrB\_DENSE\_COL\_FORMAT formats can only be used if all entries are present in the matrix. That is, GrB\_Matrix\_nvals (&nvals,A) must return nvals equal to nrows\*ncols.

#### 6.12.3 GrB\_Matrix\_exportSize: determine size of export

Returns the required sizes of the arrays Ap, Ai, and Ax for exporting a matrix using GrB\_Matrix\_export.

#### 6.12.4 GrB\_Matrix\_exportHint: determine best export format

```
GrB_Info GrB_Matrix_exportHint // suggest the best export format
(
    GrB_Format *format, // export format
    GrB_Matrix A // matrix to export
);
```

This method suggests the most efficient format for the export of a given matrix. For SuiteSparse:GraphBLAS, the hint depends on the current format of the GrB\_Matrix:

- GxB\_SPARSE, GxB\_BY\_ROW: export as GrB\_CSR\_FORMAT
- GxB\_SPARSE, GxB\_BY\_COL: export as GrB\_CSC\_FORMAT
- GxB\_HYPERSPARSE: export as GrB\_COO\_FORMAT
- GxB\_BITMAP, GxB\_BY\_ROW: export as GrB\_CSR\_FORMAT
- GxB\_BITMAP, GxB\_BY\_COL: export as GrB\_CSC\_FORMAT
- GxB\_FULL, GxB\_BY\_ROW: export as GrB\_DENSE\_ROW\_FORMAT
- GxB\_FULL, GxB\_BY\_COL: export as GrB\_DENSE\_COL\_FORMAT

## 6.13 GraphBLAS descriptors: GrB\_Descriptor

A GraphBLAS descriptor modifies the behavior of a GraphBLAS operation. If the descriptor is GrB\_NULL, defaults are used.

The access to these parameters and their values is governed by two enum types, GrB\_Desc\_Field and GrB\_Desc\_Value:

```
#define GxB_NTHREADS 5 // for both GrB_Desc_field and GxB_Option_field
#define GxB_CHUNK 7
typedef enum
    GrB_OUTP = 0, // descriptor for output of a method
    GrB\_MASK = 1,
                  // descriptor for the mask input of a method
    GrB_INPO = 2, // descriptor for the first input of a method
    GrB_INP1 = 3, // descriptor for the second input of a method
    GxB_DESCRIPTOR_NTHREADS = GxB_NTHREADS,
                                             // number of threads to use
   GxB_DESCRIPTOR_CHUNK = GxB_CHUNK,
                                       // chunk size for small problems
    GxB_AxB_METHOD = 1000, // descriptor for selecting C=A*B algorithm
    GxB_SORT = 35  // control sort in GrB_mxm
    GxB_COMPRESSION = 36, // select compression for serialize
    GxB_IMPORT = 37,
                           // secure vs fast import
GrB_Desc_Field ;
typedef enum
    // for all GrB_Descriptor fields:
    GxB_DEFAULT = 0,
                       // default behavior of the method
    // for GrB_OUTP only:
                      // clear the output before assigning new values to it
    GrB_REPLACE = 1,
    // for GrB_MASK only:
    GrB\_COMP = 2,
                       // use the complement of the mask
    GrB_STRUCTURE = 4, // use the structure of the mask
    // for GrB_INPO and GrB_INP1 only:
    GrB_TRAN = 3,
                       // use the transpose of the input
    // for GxB_AxB_METHOD only:
    GxB_AxB_GUSTAVSON = 1001, // gather-scatter saxpy method
    GxB_AxB_DOT
                     = 1003, // dot product
    GxB_AxB_HASH
                     = 1004, // hash-based saxpy method
                     = 1005
                               // saxpy method (any kind)
    GxB_AxB_SAXPY
    // for GxB_IMPORT only:
    GxB_SECURE_IMPORT = 502
                              // import/pack trust their input data
GrB_Desc_Value ;
```

• GrB\_OUTP is a parameter that modifies the output of a GraphBLAS operation. In the default case, the output is not cleared, and  $\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{Z} = \mathbf{C}\odot\mathbf{T}$  is computed as-is, where  $\mathbf{T}$  is the results of the particular GraphBLAS operation.

In the non-default case,  $\mathbf{Z} = \mathbf{C} \odot \mathbf{T}$  is first computed, using the results of  $\mathbf{T}$  and the accumulator  $\odot$ . After this is done, if the  $\mathtt{GrB\_OUTP}$  descriptor field is set to  $\mathtt{GrB\_REPLACE}$ , then the output is cleared of its entries. Next, the assignment  $\mathbf{C}\langle \mathbf{M} \rangle = \mathbf{Z}$  is performed.

• GrB\_MASK is a parameter that modifies the Mask, even if the mask is not present.

If this parameter is set to its default value, and if the mask is not present (Mask==NULL) then implicitly Mask(i,j)=1 for all i and j. If the mask is present then Mask(i,j)=1 means that C(i,j) is to be modified by the  $C\langle M \rangle = Z$  update. Otherwise, if Mask(i,j)=0, then C(i,j) is not modified, even if Z(i,j) is an entry with a different value; that value is simply discarded.

If the GrB\_MASK parameter is set to GrB\_COMP, then the use of the mask is complemented. In this case, if the mask is not present (Mask==NULL) then implicitly Mask(i,j)=0 for all i and j. This means that none of C is modified and the entire computation of Z might as well have been skipped. That is, a complemented empty mask means no modifications are made to the output object at all, except perhaps to clear it in accordance with the GrB\_OUTP descriptor. With a complemented mask, if the mask is present then Mask(i,j)=0 means that C(i,j) is to be modified by the  $C\langle M \rangle = Z$  update. Otherwise, if Mask(i,j)=1, then C(i,j) is not modified, even if Z(i,j) is an entry with a different value; that value is simply discarded.

If the GrB\_MASK parameter is set to GrB\_STRUCTURE, then the values of the mask are ignored, and just the pattern of the entries is used. Any entry M(i,j) in the pattern is treated as if it were true.

The GrB\_COMP and GrB\_STRUCTURE settings can be combined, either by setting the mask option twice (once with each value), or by setting the mask option to GrB\_COMP+GrB\_STRUCTURE (the latter is an extension to the spec).

Using a parameter to complement the Mask is very useful because constructing the actual complement of a very sparse mask is impossible since it has too many entries. If the number of places in C that should be modified is very small, then use a sparse mask without complementing it. If the number of places in C that should be protected from modification is very small, then use a sparse mask to indicate those places, and use a descriptor GrB\_MASK that complements the use of the mask.

• GrB\_INPO and GrB\_INP1 modify the use of the first and second input matrices A and B of the GraphBLAS operation.

If the GrB\_INPO is set to GrB\_TRAN, then A is transposed before using it in the operation. Likewise, if GrB\_INP1 is set to GrB\_TRAN, then the second input, typically called B, is transposed.

Vectors and scalars are never transposed via the descriptor. If a method's first parameter is a matrix and the second a vector or scalar, then GrB\_INPO modifies the matrix parameter and GrB\_INP1 is ignored. If a method's first parameter is a vector or scalar and the second a matrix, then GrB\_INP1 modifies the matrix parameter and GrB\_INPO is ignored.

To clarify this in each function, the inputs are labeled as first input: and second input: in the function signatures.

- GxB\_AxB\_METHOD suggests the method that should be used to compute C=A\*B. All the methods compute the same result, except they may have different floating-point roundoff errors. This descriptor should be considered as a hint; SuiteSparse:GraphBLAS is free to ignore it.
  - GxB\_DEFAULT means that a method is selected automatically.
  - GxB\_AxB\_SAXPY: select any saxpy-based method: GxB\_AxB\_GUSTAVSON, and/or GxB\_AxB\_HASH, or any mix of the two, in contrast to the dot-product method.
  - GxB\_AxB\_GUSTAVSON: an extended version of Gustavson's method [Gus78], which is a very good general-purpose method, but sometimes the workspace can be too large. Assuming all matrices are stored by column, it computes C(:,j)=A\*B(:,j) with a sequence of saxpy operations (C(:,j)+=A(:,k)\*B(k:,j) for each nonzero

- B(k,j)). In the coarse Gustavson method, each internal thread requires workspace of size m, to the number of rows of C, which is not suitable if the matrices are extremely sparse or if there are many threads. For the fine Gustavson method, threads can share workspace and update it via atomic operations. If all matrices are stored by row, then it computes C(i,:)=A(i,:)\*B in a sequence of sparse saxpy operations, and using workspace of size n per thread, or group of threads, corresponding to the number of columns of C.
- GxB\_AxB\_HASH: a hash-based method, based on [NMAB18]. It is very efficient for hypersparse matrices, matrix-vector-multiply, and when |B| is small. SuiteSparse:GraphBLAS includes a coarse hash method, in which each thread has its own hash workspace, and a fine hash method, in which groups of threads share a single hash workspace, as concurrent data structure, using atomics.
- GxB\_AxB\_DOT: computes C(i,j)=A(i,:)\*B(j,:)', for each entry C(i,j). If the mask is present and not complemented, only entries for which M(i,j)=1 are computed. This is a very specialized method that works well only if the mask is present, very sparse, and not complemented, when C is small, or when C is bitmap or full. For example, it works very well when A and B are tall and thin, and C<M>=A\*B' or C=A\*B' are computed. These expressions assume all matrices are in CSR format. If in CSC format, then the dot-product method used for A'\*B. The method is impossibly slow if C is large and the mask is not present, since it takes Ω(mn) time if C is m-by-n in that case. It does not use any workspace at all. Since it uses no workspace, it can work very well for extremely sparse or hypersparse matrices, when the mask is present and not complemented.
- GxB\_NTHREADS controls how many threads a method uses. By default (if set to zero, or GxB\_DEFAULT), all available threads are used. The maximum available threads is controlled by the global setting, which is omp\_get\_max\_threads ( ) by default. If set to some positive integer nthreads less than this maximum, at most nthreads threads will be used. See Section 8.1 for details.
- GxB\_CHUNK is a double value that controls how many threads a method

uses for small problems. See Section 8.1 for details.

- GxB\_SORT provides a hint to GrB\_mxm, GrB\_mxv, GrB\_vxm, and GrB\_reduce (to vector). These methods can leave the output matrix or vector in a jumbled state, where the final sort is left as pending work. This is typically fastest, since some algorithms can tolerate jumbled matrices on input, and sometimes the sort can be skipped entirely. However, if the matrix or vector will be immediately exported in unjumbled form, or provided as input to a method that requires it to not be jumbled, then sorting it during the matrix multiplication is faster. By default, these methods leave the result in jumbled form (a lazy sort), if GxB\_SORT is set to zero (GxB\_DEFAULT). A nonzero value will inform the matrix multiplication to sort its result, instead.
- GxB\_COMPRESSION selects the compression method for serialization. The default is LZ4. See Section 6.10 for other options.
- GxB\_IMPORT informs the GxB import and pack methods that they can trust their input data, or not. The default is to trust the input, for faster import. If the data is being imported from an untrusted source, then additional checks should be made, and the following descriptor setting should be used:

```
GxB_set (desc, GxB_IMPORT, GxB_SECURE_IMPORT) ;
```

The next sections describe the methods for a GrB\_Descriptor:

GraphBLAS function	purpose	Section
GrB_Descriptor_new	create a descriptor	6.13.1
<pre>GrB_Descriptor_wait</pre>	wait for a descriptor	6.13.2
<pre>GrB_Descriptor_set</pre>	set a parameter in a descriptor	6.13.3
<pre>GxB_Desc_set</pre>	set a parameter in a descriptor	6.13.4
GxB_Desc_get	get a parameter from a descriptor	6.13.5
<pre>GrB_Descriptor_free</pre>	free a descriptor	6.13.6

#### 6.13.1 GrB\_Descriptor\_new: create a new descriptor

```
GrB_Info GrB_Descriptor_new // create a new descriptor
(
    GrB_Descriptor *descriptor // handle of descriptor to create
);
```

GrB\_Descriptor\_new creates a new descriptor, with all fields set to their defaults (output is not replaced, the mask is not complemented, the mask is valued not structural, neither input matrix is transposed, the method used in C=A\*B is selected automatically, and GrB\_mxm leaves the final sort as pending work).

#### 6.13.2 GrB\_Descriptor\_wait: wait for a descriptor

After creating a user-defined descriptor, a GraphBLAS library may choose to exploit non-blocking mode to delay its creation. GrB\_Descriptor\_wait(&d) ensures the descriptor d is completed. SuiteSparse:GraphBLAS currently does nothing for GrB\_Descriptor\_wait(&d), except to ensure that d is valid. The function will change in SS:GrB v6.0.0 (for the v2.0 C API Specification).

#### 6.13.3 GrB\_Descriptor\_set: set a parameter in a descriptor

GrB\_Descriptor\_set sets a descriptor field (GrB\_OUTP, GrB\_MASK, GrB\_INPO, GrB\_INP1, or GxB\_AxB\_METHOD) to a particular value. Use GxB\_Dec\_set to set the value of GxB\_NTHREADS, GxB\_CHUNK, and GxB\_SORT. If an error occurs, GrB\_error(&err,desc) returns details about the error.

Descriptor field	Default	Non-default
GrB_OUTP	GxB_DEFAULT: The output matrix is not cleared. The operation computes $\mathbf{C}\langle\mathbf{M}\rangle=\mathbf{C}\odot\mathbf{T}.$	GrB_REPLACE: After computing $\mathbf{Z} = \mathbf{C} \odot \mathbf{T}$ , the output $\mathbf{C}$ is cleared of all entries. Then $\mathbf{C}\langle \mathbf{M} \rangle = \mathbf{Z}$ is performed.
GrB_MASK	GxB_DEFAULT: The Mask is not complemented. Mask(i,j)=1 means the value $C_{ij}$ can be modified by the operation, while Mask(i,j)=0 means the value $C_{ij}$ shall not be modified by the operation.	GrB_COMP: The Mask is complemented. Mask(i,j)=0 means the value $C_{ij}$ can be modified by the operation, while Mask(i,j)=1 means the value $C_{ij}$ shall not be modified by the operation. GrB_STRUCTURE: The values of the Mask are ignored. If Mask(i,j) is an entry in the Mask matrix, it is treated as if Mask(i,j)=1. The two options GrB_COMP and GrB_STRUCTURE can be combined, with two subsequent calls, or with a single call with the setting GrB_COMP+GrB_STRUCTURE.
GrB_INPO	GxB_DEFAULT: The first input is not transposed prior to using it in the operation.	GrB_TRAN: The first input is transposed prior to using it in the operation. Only matrices are transposed, never vectors.
GrB_INP1	GxB_DEFAULT: The second input is not transposed prior to using it in the operation.	GrB_TRAN: The second input is transposed prior to using it in the operation. Only matrices are transposed, never vectors.
GrB_AxB_METHOD	GxB_DEFAULT: The method for C=A*B is selected automatically.	GxB_AxB_method: The selected method is used to compute C=A*B.

#### 6.13.4 GxB\_Desc\_set: set a parameter in a descriptor

GxB\_Desc\_set is like GrB\_Descriptor\_set, except that the type of the third parameter can vary with the field. This function can modify all descriptor settings, including those that do not have the type GrB\_Desc\_Value. See also GxB\_set described in Section 8. If an error occurs, GrB\_error(&err,desc) returns details about the error.

#### 6.13.5 GxB\_Desc\_get: get a parameter from a descriptor

GxB\_Desc\_get returns the value of a single field in a descriptor. The type of the third parameter is a pointer to a variable type, whose type depends on the field. See also GxB\_get described in Section 8.

#### 6.13.6 GrB\_Descriptor\_free: free a descriptor

GrB\_Descriptor\_free frees a descriptor. Either usage:

```
GrB_Descriptor_free (&descriptor) ;
GrB_free (&descriptor) ;
```

frees the descriptor and sets descriptor to NULL. It safely does nothing if passed a NULL handle, or if descriptor == NULL on input.

## 6.13.7 GrB\_DESC\_\*: predefined descriptors

Version 1.3 of the GraphBLAS C API Specification adds predefined descriptors, and these have been added as of v3.2.0 of SuiteSparse:GraphBLAS. They are listed in the table below. A dash in the table indicates the default. These descriptors may not be modified or freed. Attempts to modify them result in an error (Grb\_INVALID\_VALUE); attempts to free them are silently ignored.

Descriptor	OUTP	MASK	MASK	INPO	INP1
		structural	complement		
GrB_NULL	=	=	-	-	-
GrB_DESC_T1	-	-	-	-	GrB_TRAN
GrB_DESC_TO	-	-	-	GrB_TRAN	-
GrB_DESC_TOT1	-	-	-	GrB_TRAN	GrB_TRAN
GrB_DESC_C	-	-	GrB_COMP	_	_
GrB_DESC_CT1	-	-	GrB_COMP	-	GrB_TRAN
GrB_DESC_CTO	-	-	GrB_COMP	GrB_TRAN	-
GrB_DESC_CTOT1	-	-	GrB_COMP	GrB_TRAN	GrB_TRAN
GrB_DESC_S	-	GrB_STRUCTURE	_	-	-
GrB_DESC_ST1	-	GrB_STRUCTURE	_	_	GrB_TRAN
GrB_DESC_STO	-	GrB_STRUCTURE	_	GrB_TRAN	_
GrB_DESC_STOT1	-	GrB_STRUCTURE	-	GrB_TRAN	GrB_TRAN
GrB_DESC_SC	-	GrB_STRUCTURE	GrB_COMP	-	-
GrB_DESC_SCT1	-	GrB_STRUCTURE	GrB_COMP	_	GrB_TRAN
GrB_DESC_SCTO	-	GrB_STRUCTURE	GrB_COMP	GrB_TRAN	_
GrB_DESC_SCTOT1	-	GrB_STRUCTURE	GrB_COMP	GrB_TRAN	GrB_TRAN
GrB_DESC_R	GrB_REPLACE	-	_	-	-
GrB_DESC_RT1	GrB_REPLACE	-	_	-	GrB_TRAN
GrB_DESC_RTO	GrB_REPLACE	-	_	GrB_TRAN	_
GrB_DESC_RTOT1	GrB_REPLACE	-	_	GrB_TRAN	GrB_TRAN
GrB_DESC_RC	GrB_REPLACE	-	GrB_COMP	-	-
GrB_DESC_RCT1	GrB_REPLACE	-	GrB_COMP	-	GrB_TRAN
GrB_DESC_RCTO	GrB_REPLACE	-	GrB_COMP	GrB_TRAN	_
GrB_DESC_RCTOT1	GrB_REPLACE	-	GrB_COMP	GrB_TRAN	GrB_TRAN
GrB_DESC_RS	GrB_REPLACE	GrB_STRUCTURE	_	-	-
GrB_DESC_RST1	GrB_REPLACE	GrB_STRUCTURE	_	-	GrB_TRAN
GrB_DESC_RSTO	GrB_REPLACE	GrB_STRUCTURE	_	GrB_TRAN	_
GrB_DESC_RSTOT1	GrB_REPLACE	GrB_STRUCTURE	_	GrB_TRAN	GrB_TRAN
GrB_DESC_RSC	GrB_REPLACE	GrB_STRUCTURE	GrB_COMP	-	-
GrB_DESC_RSCT1	GrB_REPLACE	GrB_STRUCTURE	GrB_COMP	_	GrB_TRAN
GrB_DESC_RSCTO	GrB_REPLACE	GrB_STRUCTURE	GrB_COMP	GrB_TRAN	-
GrB_DESC_RSCTOT1	GrB_REPLACE	GrB_STRUCTURE	GrB_COMP	GrB_TRAN	GrB_TRAN

## 6.14 GrB\_free: free any GraphBLAS object

Each of the ten objects has GrB\_\*\_new and GrB\_\*\_free methods that are specific to each object. They can also be accessed by a generic function, GrB\_free, that works for all ten objects. If G is any of the ten objects, the statement

```
GrB_free (&G) ;
```

frees the object and sets the variable G to NULL. It is safe to pass in a NULL handle, or to free an object twice:

```
GrB_free (NULL); // SuiteSparse:GraphBLAS safely does nothing GrB_free (&G); // the object G is freed and G set to NULL GrB_free (&G); // SuiteSparse:GraphBLAS safely does nothing
```

However, the following sequence of operations is not safe. The first two are valid but the last statement will lead to undefined behavior.

```
H = G;  // valid; creates a 2nd handle of the same object GrB\_free (&G);  // valid; G is freed and set to NULL; H now undefined GrB\_some\_method (H);  // not valid; H is undefined
```

Some objects are predefined, such as the built-in types. If a user application attempts to free a built-in object, SuiteSparse:GraphBLAS will safely do nothing. The GrB\_free function in SuiteSparse:GraphBLAS always returns GrB\_SUCCESS.

# 7 The mask, accumulator, and replace option

After a GraphBLAS operation computes a result  $\mathbf{T}$ , (for example,  $\mathbf{T} = \mathbf{AB}$  for  $\mathtt{GrB\_mxm}$ ), the results are assigned to an output matrix  $\mathbf{C}$  via the mask/accumulator phase, written as  $\mathbf{C}\langle\mathbf{M}\rangle=\mathbf{C}\odot\mathbf{T}$ . This phase is affected by the  $\mathtt{GrB\_REPLACE}$  option in the descriptor, the presence of an optional binary accumulator operator  $(\odot)$ , the presence of the optional mask matrix  $\mathbf{M}$ , and the status of the mask descriptor. The interplay of these options is summarized in Table 1.

The mask  $\mathbf{M}$  may be present, or not. It may be structural or valued, and it may be complemented, or not. These options may be combined, for a total of 8 cases, although the structural/valued option as no effect if  $\mathbf{M}$  is not present. If  $\mathbf{M}$  is not present and not complemented, then  $m_{ij}$  is implicitly true. If not present yet complemented, then all  $m_{ij}$  entries are implicitly zero; in this case,  $\mathbf{T}$  need not be computed at all. Either  $\mathbf{C}$  is not modified, or all its entries are cleared if the replace option is enabled. If  $\mathbf{M}$  is present, and the structural option is used, then  $m_{ij}$  is treated as true if it is an entry in the matrix (its value is ignored). Otherwise, the value of  $m_{ij}$  is used. In both cases, entries not present are implicitly zero. These values are negated if the mask is complemented. All of these various cases are combined to give a single effective value of the mask at position ij.

The combination of all these options are presented in the Table 1. The first column is the  $GrB_REPLACE$  option. The second column lists whether or not the accumulator operator is present. The third column lists whether or not  $c_{ij}$  exists on input to the mask/accumulator phase (a dash means that it does not exist). The fourth column lists whether or not the entry  $t_{ij}$  is present in the result matrix  $\mathbf{T}$ . The mask column is the final effective value of  $m_{ij}$ , after accounting for the presence of  $\mathbf{M}$  and the mask options. Finally, the last column states the result of the mask/accum step; if no action is listed in this column, then  $c_{ij}$  is not modified.

Several important observations can be made from this table. First, if no mask is present (and the mask-complement descriptor option is not used), then only the first half of the table is used. In this case, the <code>Grb\_REPLACE</code> option has no effect. The entire matrix  ${\bf C}$  is modified.

Consider the cases when  $c_{ij}$  is present but  $t_{ij}$  is not, and there is no mask or the effective value of the mask is true for this ij position. With no accumulator operator,  $c_{ij}$  is deleted. If the accumulator operator is present and the replace option is not used,  $c_{ij}$  remains unchanged.

repl	accum	$\mathbf{C}$	$\mathbf{T}$	mask	action taken by $\mathbf{C}\langle \mathbf{M} \rangle = \mathbf{C} \odot \mathbf{T}$
-	-	$c_{ij}$	$t_{ij}$	1	$c_{ij} = t_{ij}$ , update
-	-	-	$t_{ij}$	1	$c_{ij} = t_{ij}$ , insert
-	-	$c_{ij}$	-	1	delete $c_{ij}$ because $t_{ij}$ not present
-	-	-	-	1	
-	-	$c_{ij}$	$t_{ij}$	0	
-	-	-	$t_{ij}$	0	
-	-	$c_{ij}$	-	0	
-	-	-	-	0	
yes	-	$c_{ij}$	$t_{ij}$	1	$c_{ij} = t_{ij}$ , update
yes	-	-	$t_{ij}$	1	$c_{ij} = t_{ij}$ , insert
yes	-	$c_{ij}$	-	1	delete $c_{ij}$ because $t_{ij}$ not present
yes	-	-	-	1	
yes	-	$c_{ij}$	$t_{ij}$	0	delete $c_{ij}$ (because of $GrB_REPLACE$ )
yes	-	-	$t_{ij}$	0	
yes	-	$c_{ij}$	-	0	delete $c_{ij}$ (because of $GrB_REPLACE$ )
yes	-	-	-	0	
-	yes	$c_{ij}$	$t_{ij}$	1	$c_{ij} = c_{ij} \odot t_{ij}$ , apply accumulator
-	yes	-	$t_{ij}$	1	$c_{ij} = t_{ij}$ , insert
-	yes	$c_{ij}$	-	1	
-	yes	-	-	1	
-	yes	$c_{ij}$	$t_{ij}$	0	
-	yes	-	$t_{ij}$	0	
-	yes	$c_{ij}$	-	0	
_	yes	-	-	0	
yes	yes	$c_{ij}$	$t_{ij}$	1	$c_{ij} = c_{ij} \odot t_{ij}$ , apply accumulator
yes	yes	-	$t_{ij}$	1	$c_{ij} = t_{ij}$ , insert
yes	yes	$c_{ij}$	-	1	
yes	yes	-	-	1	
yes	yes	$c_{ij}$	$t_{ij}$	0	delete $c_{ij}$ (because of GrB_REPLACE)
yes	yes	-	$t_{ij}$	0	
yes	yes	$c_{ij}$	-	0	delete $c_{ij}$ (because of $GrB_REPLACE$ )
yes	yes	-	-	0	

Table 1: Results of the mask/accumulator phase

# 8 SuiteSparse:GraphBLAS Options

SuiteSparse:GraphBLAS includes two type-generic methods, GxB\_set and GxB\_get, that set and query various options and parameters settings, including a generic way to set values in the GrB\_Descriptor object. Using these methods, the user application can provide hints to SuiteSparse:GraphBLAS on how it should store and operate on its matrices. These hints have no effect on the results of any GraphBLAS operation (except perhaps floating-point roundoff differences), but they can have a great impact on the amount of time or memory taken.

• GxB\_set (field, value) sets global options.

field	value	description
GxB_HYPER_SWITCH	double	hypersparsity control (0 to 1)
GxB_BITMAP_SWITCH	double [8]	bitmap control
GxB_FORMAT	int	<pre>GxB_BY_ROW or GxB_BY_COL</pre>
${\tt GxB\_GLOBAL\_NTHREADS}$	int	number of threads to use
GxB_NTHREADS	int	number of threads to use
$\texttt{GxB\_GLOBAL\_CHUNK}$	double	chunk size
GxB_CHUNK	double	chunk size
GxB_BURBLE	int	diagnostic output
GxB_PRINTF	see below	diagnostic output
GxB_FLUSH	see below	diagnostic output
GxB_MEMORY_POOL	int64_t [64]	memory pool control
GxB_PRINT_1BASED	int	for printing matrices/vectors

• GxB\_set (GrB\_Matrix A, field, value) provides hints to SuiteSparse: GraphBLAS on how to store a particular matrix.

field	value	description
GxB_HYPER_SWITCH	double	hypersparsity control (0 to 1)
GxB_BITMAP_SWITCH	double	bitmap control (0 to 1)
GxB_FORMAT	int	<pre>GxB_BY_ROW or GxB_BY_COL</pre>
GxB_SPARSITY_CONTROL	int	0 to 15

• GxB\_set (GrB\_Vector v, field, value) provides hints to SuiteSparse: GraphBLAS on how to store a particular vector.

field	value	description
GxB_BITMAP_SWITCH	double	bitmap control (0 to 1)
GxB_SPARSITY_CONTROL	int	0 to 15

• GxB\_set (GrB\_Descriptor desc, field, value) sets the value of a field in a GrB\_Descriptor.

field	value	description
GrB_OUTP	GrB_Desc_Value	replace option
GrB_MASK	<pre>GrB_Desc_Value</pre>	mask option
GrB_INPO	<pre>GrB_Desc_Value</pre>	transpose input 0
GrB_INP1	<pre>GrB_Desc_Value</pre>	transpose input 1
GxB_DESCRIPTOR_NTHREADS	int	number of threads to use
GxB_NTHREADS	int	number of threads to use
GxB_DESCRIPTOR_CHUNK	double	chunk size
GxB_CHUNK	double	chunk size
GxB_AxB_METHOD	int	method for matrix multiply
GxB_SORT	int	lazy vs aggressive sort
GxB_COMPRESSION	int	compression for serialization
GxB_IMPORT	<pre>GrB_Desc_Value</pre>	trust data on import/pack

 ${\tt GxB\_get}$  queries a  ${\tt GrB\_Descriptor},$  a  ${\tt GrB\_Matrix},$  a  ${\tt GrB\_Vector},$  or the global options.

• GxB\_get (field, &value) retrieves the value of a global option.

field	value	description
GxB_HYPER_SWITCH	double	hypersparsity control (0 to 1)
GxB_BITMAP_SWITCH	double [8]	bitmap control
GxB_FORMAT	int	<pre>GxB_BY_ROW or GxB_BY_COL</pre>
$\texttt{GxB\_GLOBAL\_NTHREADS}$	int	number of threads to use
GxB_NTHREADS	int	number of threads to use
$\texttt{GxB\_GLOBAL\_CHUNK}$	double	chunk size
GxB_CHUNK	double	chunk size
GxB_BURBLE	int	diagnostic output
GxB_PRINTF	see below	diagnostic output
GxB_FLUSH	see below	diagnostic output
GxB_MEMORY_POOL	int64_t [64]	memory pool control
GxB_PRINT_1BASED	int	for printing matrices/vectors
GxB_MODE	int	blocking/non-blocking
GxB_LIBRARY_NAME	char *	name of library
GxB_LIBRARY_VERSION	int [3]	library version
GxB_LIBRARY_DATE	char *	release date
GxB_LIBRARY_ABOUT	char *	about the library
GxB_LIBRARY_LICENSE	char *	license
GxB_LIBRARY_COMPILE_DATE	char *	date of compilation
GxB_LIBRARY_COMPILE_TIME	char *	time of compilation
GxB_LIBRARY_URL	char *	url of library
GxB_API_VERSION	int [3]	C API version
GxB_API_DATE	char *	C API date
GxB_API_ABOUT	char *	about the C API
GxB_API_URL	char *	http://graphblas.org

• GxB\_get (GrB\_Matrix A, field, &value) retrieves the current value of an option from a particular matrix A.

field	value	description
GxB_HYPER_SWITCH	double	hypersparsity control (0 to 1)
GxB_BITMAP_SWITCH	double	bitmap control (0 to 1)
GxB_FORMAT	int	<pre>GxB_BY_ROW or GxB_BY_COL</pre>
GxB_SPARSITY_CONTROL	int	0 to 15
GxB_SPARSITY_STATUS	int	1, 2, 4, or 8

• GxB\_get (GrB\_Vector A, field, &value) retrieves the current value of an option from a particular vector v.

field	value	description
GxB_BITMAP_SWITCH	double	bitmap control (0 to 1)
GxB_FORMAT	int	<pre>GxB_BY_ROW or GxB_BY_COL</pre>
GxB_SPARSITY_CONTROL	int	0 to 15
GxB_SPARSITY_STATUS	int	1, 2, 4, or 8

• GxB\_get (GrB\_Descriptor desc, field, &value) retrieves the value of a field in a descriptor.

field	value	description
GrB_OUTP	<pre>GrB_Desc_Value</pre>	replace option
GrB_MASK	<pre>GrB_Desc_Value</pre>	mask option
GrB_INPO	<pre>GrB_Desc_Value</pre>	transpose input 0
GrB_INP1	<pre>GrB_Desc_Value</pre>	transpose input 1
GxB_DESCRIPTOR_NTHREADS	int	number of threads to use
GxB_NTHREADS	int	number of threads to use
GxB_DESCRIPTOR_CHUNK	double	chunk size
GxB_CHUNK	double	chunk size
GxB_AxB_METHOD	int	method for matrix multiply
GxB_SORT	int	lazy vs aggressive sort
GxB_COMPRESSION	int	compression for serialization
GxB_IMPORT	<pre>GrB_Desc_Value</pre>	trust data on import/pack

# 8.1 OpenMP parallelism

SuiteSparse:GraphBLAS is a parallel library, based on OpenMP. By default, all GraphBLAS operations will use up to the maximum number of threads specified by the omp\_get\_max\_threads OpenMP function. For small problems, GraphBLAS may choose to use fewer threads, using two parameters: the maximum number of threads to use (which may differ from the omp\_get\_max\_threads value), and a parameter called the chunk. Suppose

work is a measure of the work an operation needs to perform (say the number of entries in the two input matrices for GrB\_eWiseAdd). No more than floor(work/chunk) threads will be used (or one thread if the ratio is less than 1).

The default **chunk** value is 65,536, but this may change in future versions, or it may be modified when GraphBLAS is installed on a particular machine. Both parameters can be set in two ways:

• Globally: If the following methods are used, then all subsequent Graph-BLAS operations will use these settings. Note the typecast, (double) chunk. This is necessary if a literal constant such as 20000 is passed as this argument. The type of the constant must be double.

```
int nthreads_max = 40 ;
GxB_set (GxB_NTHREADS, nthreads_max) ;
GxB_set (GxB_CHUNK, (double) 20000) ;
```

Per operation: Most GraphBLAS operations take a GrB\_Descriptor input, and this can be modified to set the number of threads and chunk size for the operation that uses this descriptor. Note that chunk is a double.

```
GrB_Descriptor desc ;
GrB_Descriptor_new (&desc)
int nthreads_max = 40 ;
GxB_set (desc, GxB_NTHREADS, nthreads_max) ;
double chunk = 20000 ;
GxB_set (desc, GxB_CHUNK, chunk) ;
```

The smaller of nthreads\_max and floor(work/chunk) is used for any given GraphBLAS operation, except that a single thread is used if this value is zero or less.

If either parameter is set to GxB\_DEFAULT, then default values are used. The default for nthreads\_max is the return value from omp\_get\_max\_threads, and the default chunk size is currently 65,536.

If a descriptor value for either parameter is left at its default, or set to GxB\_DEFAULT, then the global setting is used. This global setting may have been modified from its default, and this modified value will be used.

For example, suppose omp\_get\_max\_threads reports 8 threads. If GxB\_set (GxB\_NTHREADS, 4) is used, then the global setting is four threads, not eight. If a descriptor is used but its GxB\_NTHREADS is not set, or set to GxB\_DEFAULT, then any operation that uses this descriptor will use 4 threads.

## 8.2 Storing a matrix by row or by column

The GraphBLAS GrB\_Matrix is entirely opaque to the user application, and the GraphBLAS API does not specify how the matrix should be stored. However, choices made in how the matrix is represented in a particular implementation, such as SuiteSparse:GraphBLAS, can have a large impact on performance.

Many graph algorithms are just as fast in any format, but some algorithms are much faster in one format or the other. For example, suppose the user application stores a directed graph as a matrix A, with the edge (i, j) represented as the value A(i,j), and the application makes many accesses to the ith row of the matrix, with  $GrB_Col_extract(w, ..., A, GrB_ALL, ..., i, desc)$  with the transposed descriptor ( $GrB_INPO$  set to  $GrB_TRAN$ ). If the matrix is stored by column this can be extremely slow, just like the expression w=A(i,:) in MATLAB, where i is a scalar. Since this is a typical usecase in graph algorithms, the default format in SuiteSparse:GraphBLAS is to store its matrices by row, in Compressed Sparse Row format (CSR).

MATLAB stores its sparse matrices by column, in "non-hypersparse" format, in what is called the Compressed Sparse Column format, or CSC for short. An  $\mathfrak{m}$ -by- $\mathfrak{m}$  matrix in MATLAB is represented as a set of  $\mathfrak{n}$  column vectors, each with a sorted list of row indices and values of the nonzero entries in that column. As a result,  $\mathfrak{w}=A(:,j)$  is very fast in MATLAB, since the result is already held in the data structure a single list, the jth column vector. However,  $\mathfrak{w}=A(i,:)$  is very slow in MATLAB, since every column in the matrix has to be searched to see if it contains row i. In MATLAB, if many such accesses are made, it is much better to transpose the matrix (say AT=A') and then use  $\mathfrak{w}=AT(:,i)$  instead. This can have a dramatic impact on the performance of MATLAB.

Likewise, if u is a very sparse column vector and A is stored by column, then  $w=u^*A$  (via  $GrB_vxm$ ) is slower than w=A\*u (via  $GrB_mxv$ ). The opposite is true if the matrix is stored by row.

SuiteSparse:GraphBLAS stores its sparse matrices by row, by default. In Versions 2.1 and earlier, the matrices were stored by column, by default.

However, it can also be instructed to store any selected matrices, or all matrices, by column instead (just like MATLAB), so that w=A(:,j) (via GrB\_Col\_extract) is very fast. The change in data format has no effect on the result, just the time and memory usage. To use a column-oriented format by default, the following can be done in a user application that tends to access its matrices by column.

```
GrB_init (...);
// just after GrB_init: do the following:
#ifdef GxB_SUITESPARSE_GRAPHBLAS
GxB_set (GxB_FORMAT, GxB_BY_COL);
#endif
```

If this is done, and no other <code>GxB\_set</code> calls are made with <code>GxB\_FORMAT</code>, all matrices will be stored by column. Alternatively, SuiteSparse:GraphBLAS can be compiled with <code>-DBYCOL</code>, which changes the default format to <code>GxB\_BY\_COL</code>, with no calls to any <code>GxB\_\*</code> function. The default format is now <code>GxB\_BY\_ROW</code>.

## 8.3 Hypersparse matrices

MATLAB can store an m-by-n matrix with a very large value of m, since a CSC data structure takes  $O(n + |\mathbf{A}|)$  memory, independent of m, where  $|\mathbf{A}|$  is the number of nonzeros in the matrix. It cannot store a matrix with a huge n, and this structure is also inefficient when  $|\mathbf{A}|$  is much smaller than n. In contrast, SuiteSparse:GraphBLAS can store its matrices in hypersparse format, taking only  $O(|\mathbf{A}|)$  memory, independent of how it is stored (by row or by column) and independent of both m and n [BG08, BG12].

In both the CSR and CSC formats, the matrix is held as a set of sparse vectors. In non-hypersparse format, the set of sparse vectors is itself dense; all vectors are present, even if they are empty. For example, an m-by-n matrix in non-hypersparse CSC format contains n sparse vectors. Each column vector takes at least one integer to represent, even for a column with no entries. This allows for quick lookup for a particular vector, but the memory required is  $O(n+|\mathbf{A}|)$ . With a hypersparse CSC format, the set of vectors itself is sparse, and columns with no entries take no memory at all. The drawback of the hypersparse format is that finding an arbitrary column vector  $\mathbf{j}$ , such as for the computation  $C=A(:,\mathbf{j})$ , takes  $O(\log k)$  time if there  $k \leq n$  vectors in the data structure. One advantage of the hypersparse structure is the memory required for an m-by-n hypersparse CSC matrix is only  $O(|\mathbf{A}|)$ , independent

of m and n. Algorithms that must visit all non-empty columns of a matrix are much faster when working with hypersparse matrices, since empty columns can be skipped.

The hyper\_switch parameter controls the hypersparsity of the internal data structure for a matrix. The parameter is typically in the range 0 to 1. The default is hyper\_switch = GxB\_HYPER\_DEFAULT, which is an extern const double value, currently set to 0.0625, or 1/16. This default ratio may change in the future.

The hyper\_switch determines how the matrix is converted between the hypersparse and non-hypersparse formats. Let n be the number of columns of a CSC matrix, or the number of rows of a CSR matrix. The matrix can have at most n non-empty vectors.

Let k be the actual number of non-empty vectors. That is, for the CSC format,  $k \leq n$  is the number of columns that have at least one entry. Let h be the value of hyper\_switch.

If a matrix is currently hypersparse, it can be converted to non-hypersparse if the either condition  $n \leq 1$  or k > 2nh holds, or both. Otherwise, it stays hypersparse. Note that if  $n \leq 1$  the matrix is always stored as non-hypersparse.

If currently non-hypersparse, it can be converted to hypersparse if both conditions n > 1 and  $k \le nh$  hold. Otherwise, it stays non-hypersparse. Note that if  $n \le 1$  the matrix always remains non-hypersparse.

The default value of hyper\_switch is assigned at startup by GrB\_init, and can then be modified globally with GxB\_set. All new matrices are created with the same hyper\_switch, determined by the global value. Once a particular matrix A has been constructed, its hypersparsity ratio can be modified from the default with:

```
double hyper_switch = 0.2 ;
GxB_set (A, GxB_HYPER_SWITCH, hyper_switch) ;
```

To force a matrix to always be non-hypersparse, use hyper\_switch equal to GxB\_NEVER\_HYPER. To force a matrix to always stay hypersparse, set hyper\_switch to GxB\_ALWAYS\_HYPER.

A GrB\_Matrix can thus be held in one of four formats: any combination of hyper/non-hyper and CSR/CSC. All GrB\_Vector objects are always stored in non-hypersparse CSC format.

A new matrix created via  $\mathtt{GrB\_Matrix\_new}$  starts with k=0 and is created in hypersparse form by default unless  $n\leq 1$  or if h<0, where h is the

global hyper\_switch value. The matrix is created in either GxB\_BY\_ROW or GxB\_BY\_COL format, as determined by the last call to GxB\_set(GxB\_FORMAT,...) or GrB\_init.

A new matrix C created via GrB\_dup (&C,A) inherits the CSR/CSC format, hypersparsity format, and hyper\_switch from A.

## 8.4 Bitmap matrices

By default, SuiteSparse:GraphBLAS switches between all four formats (hypersparse, sparse, bitmap, and full) automatically. Let  $d = |\mathbf{A}|/mn$  for an m-by-n matrix  $\mathbf{A}$  with  $|\mathbf{A}|$  entries. If the matrix is currently in sparse or hypersparse format, and is modified so that d exceeds a given threshold, it is converted into bitmap format. The default threshold is controlled by the GxB\_BITMAP\_SWITCH setting, which can be set globally, or for a particular matrix or vector.

The default value of the switch to bitmap format depends on  $\min(m, n)$ , for a matrix of size m-by-n. For the global setting, the bitmap switch is a double array of size  $\texttt{GxB\_NBITMAP\_SWITCH}$ . The defaults are given below:

parameter	default	matrix sizes
bitmap_switch [0]	] 0.04	$\min(m, n) = 1$ (and all vectors)
bitmap_switch [1]	0.05	$\min(m,n) = 2$
bitmap_switch [2]	0.06	$\min(m,n) = 3 \text{ to } 4$
bitmap_switch [3]	0.08	$\min(m,n) = 5 \text{ to } 8$
bitmap_switch [4]	0.10	$\min(m,n) = 9 \text{ to } 16$
bitmap_switch [5]	] 0.20	$\min(m,n) = 17 \text{ to } 32$
bitmap_switch [6]	] 0.30	$\min(m,n) = 33 \text{ to } 64$
bitmap_switch [7]	0.40	$\min(m,n) > 64$

That is, by default a GrB\_Vector is held in bitmap format if its density exceeds 4%. To change the global settings, do the following:

```
double bswitch [GxB_NBITMAP_SWITCH] = { 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8 }; GxB_set (GxB_BITMAP_SWITCH, bswitch);
```

If the matrix is currently in bitmap format, it is converted to full if all entries are present, or to sparse/hypersparse if d drops below b/2, if its bitmap switch is b. A matrix or vector with d between b/2 and b remains in its current format.

## 8.5 Parameter types

The GxB\_Option\_Field enumerated type gives the type of the field parameter for the second argument of GxB\_set and GxB\_get, for setting global options or matrix options.

```
typedef enum
   // for matrix/vector get/set and global get/set:
   GxB_HYPER_SWITCH = 0,
                        // defines switch to hypersparse (double value)
   GxB_BITMAP_SWITCH = 34, // defines switch to hypersparse (double value)
   GxB_FORMAT = 1,
                     // defines CSR/CSC format: GxB_BY_ROW or GxB_BY_COL
   GxB_SPARSITY_CONTROL = 32, // control the sparsity of a matrix or vector
   // for global get/set only:
   GxB_GLOBAL_NTHREADS = GxB_NTHREADS, // max number of threads to use
   GxB_GLOBAL_CHUNK = GxB_CHUNK, // chunk size for small problems
   GxB\_BURBLE = 99,
                                    // diagnositic output
                                // printf function for diagnostic output
   GxB_PRINTF = 101,
   GxB_FLUSH = 102,
                                // flush function for diagnostic output
   GxB_MEMORY_POOL = 103, // memory pool control
   GxB_PRINT_1BASED = 104, // print matrices as 0-based or 1-based
   // for matrix/vector get only:
   GxB_SPARSITY_STATUS = 33, // query the sparsity of a matrix or vector
   // for global get only:
   GxB\_MODE = 2,
                     // mode passed to GrB_init (blocking or non-blocking)
   GxB_LIBRARY_COMPILE_TIME = 15, // time library was compiled (char *)
   GxB\_API\_VERSION = 16,
                           // API version (3 int's)
                             GxB\_API\_DATE = 17,
   GxB\_API\_ABOUT = 18,
   GxB\_API\_URL = 19,
                               // URL for the API (char *)
GxB_Option_Field ;
```

The GxB\_FORMAT field can be by row or by column, set to a value with the type GxB\_Format\_Value:

The default format is given by the predefined value GxB\_FORMAT\_DEFAULT, which is equal to GxB\_BY\_ROW if default compile-time options are used. To change the default at compile time to GxB\_BY\_COL, compile the SuiteSparse: GraphBLAS library with -DBYCOL. This changes GxB\_FORMAT\_DEFAULT to GxB\_BY\_COL. The default hypersparsity ratio is 0.0625 (1/16), but this value may change in the future.

Setting the GxB\_HYPER\_SWITCH field to GxB\_ALWAYS\_HYPER ensures a matrix always stays hypersparse. If set to GxB\_NEVER\_HYPER, it always stays non-hypersparse. At startup, GrB\_init defines the following initial settings:

```
GxB_set (GxB_HYPER_SWITCH, GxB_HYPER_DEFAULT) ;
GxB_set (GxB_FORMAT, GxB_FORMAT_DEFAULT) ;
```

That is, by default, all new matrices are held by column in CSR format, unless -DBYCOL is used at compile time, in which case the default is to store all new matrices by row in CSC format. If a matrix has fewer than n/16 columns, it can be converted to hypersparse format. If it has more than n/8 columns, it can be converted to non-hypersparse format. These options can be changed for all future matrices with  $GxB\_set$ . For example, to change all future matrices to be in non-hypersparse CSC when created, use:

```
GxB_set (GxB_HYPER_SWITCH, GxB_NEVER_HYPER) ;
GxB_set (GxB_FORMAT, GxB_BY_COL) ;
Then if a particular matrix needs a different format, then (as an example):
GxB_set (A, GxB_HYPER_SWITCH, 0.1) ;
GxB_set (A, GxB_FORMAT, GxB_BY_ROW) ;
```

This changes the matrix A so that it is stored by row, and it is converted from non-hypersparse to hypersparse format if it has fewer than 10% non-empty columns. If it is hypersparse, it is a candidate for conversion to non-hypersparse if has 20% or more non-empty columns. If it has between 10% and 20% non-empty columns, it remains in its current format. MATLAB only supports a non-hypersparse CSC format. The format in SuiteSparse:GraphBLAS that is equivalent to the MATLAB format is:

```
GrB_init (...);
GxB_set (GxB_HYPER_SWITCH, GxB_NEVER_HYPER);
GxB_set (GxB_FORMAT, GxB_BY_COL);
// no subsequent use of GxB_HYPER_SWITCH or GxB_FORMAT
```

The GxB\_HYPER\_SWITCH and GxB\_FORMAT options should be considered as suggestions from the user application as to how SuiteSparse:GraphBLAS can obtain the best performance for a particular application. SuiteSparse:GraphBLAS is free to ignore any of these suggestions, both now and in the future, and the available options and formats may be augmented in the future. Any prior options no longer needed in future versions of SuiteSparse:GraphBLAS will be silently ignored, so the use these options is safe for future updates.

The sparsity status of a matrix can be queried with the following, which returns a value of GxB\_HYPERSPARSE GxB\_SPARSE GxB\_BITMAP or GxB\_FULL.

```
int sparsity ;
GxB_get (A, GxB_SPARSITY_STATUS, &sparsity) ;
```

The sparsity format of a matrix can be controlled with GxB\_set, which can be any mix (a sum or bitwise or) of GxB\_HYPERSPARSE GxB\_SPARSE GxB\_BITMAP, and GxB\_FULL. By default, a matrix or vector can be held in any format, with the default setting GxB\_AUTO\_SPARSITY, which is equal to GxB\_HYPERSPARSE + GxB\_SPARSE + GxB\_BITMAP + GxB\_FULL. To enable a matrix to take on just GxB\_SPARSE or GxB\_FULL formats, but not GxB\_HYPERSPARSE or GxB\_BITMAP, for example, use the following:

```
GxB_set (A, GxB_SPARSITY_CONTROL, GxB_SPARSE + GxB_FULL) ;
```

In this case, SuiteSparse:GraphBLAS will hold the matrix in sparse format (CSC or CSC, depending on its GxB\_FORMAT), unless all entries are present, in which case it will be converted to full format.

Only the least 4 bits of the sparsity control are considered, so the formats can be bitwise negated. For example, to allow for any format except full:

```
GxB_set (A, GxB_SPARSITY_CONTROL, ~GxB_FULL) ;
```

## 8.6 GxB\_BURBLE, GxB\_PRINTF, GxB\_FLUSH: diagnostics

GxB\_set (GxB\_BURBLE, ...) controls the burble setting. It can also be controlled via GrB.burble(b) in the Octave/MATLAB interface.

```
GxB_set (GxB_BURBLE, true) ; // enable burble
GxB_set (GxB_BURBLE, false) ; // disable burble
```

If enabled, SuiteSparse:GraphBLAS reports which internal kernels it uses, and how much time is spent. If you see the word generic, it means that SuiteSparse:GraphBLAS was unable to use is faster kernels in Source/Generated2, but used a generic kernel that relies on function pointers. This is done for user-defined types and operators, and when typecasting is performed, and it is typically slower than the kernels in Source/Generated2.

If you see a lot of wait statements, it may mean that a lot of time is spent finishing a matrix or vector. This may be the result of an inefficient use of the setElement and assign methods. If this occurs you might try changing the sparsity format of a vector or matrix to GxB\_BITMAP, assuming there's enough space for it.

GxB\_set (GxB\_PRINTF, printf) allows the user application to change the function used to print diagnostic output. This also controls the output of the GxB\_\*print functions. By default this parameter is NULL, in which case the ANSI C11 printf function is used. The parameter is a function pointer with the same signature as the ANSI C11 printf function. The Octave/MATLAB interface to GraphBLAS uses the following so that GraphBLAS can print to the Octave/MATLAB Command Window:

```
GxB_set (GxB_PRINTF, mexPrintf)
```

After each call to the printf function, an optional flush function is called, which is NULL by default. If NULL, the function is not used. This can be changed with GxB\_set (GxB\_FLUSH, flush). The flush function takes no arguments, and returns an int which is 0 if successful, or any nonzero value on failure (the same output as the ANSI C11 fflush function, except that flush has no inputs).

# 8.7 Other global options

GxB\_MODE can only be queried by GxB\_get; it cannot be modified by GxB\_set. The mode is the value passed to GrB\_init (blocking or non-blocking).

All threads in the same user application share the same global options, including hypersparsity, bitmap options, and CSR/CSC format determined by GxB\_set, and the blocking mode determined by GrB\_init. Specific format

and hypersparsity parameters of each matrix are specific to that matrix and can be independently changed.

The GxB\_LIBRARY\_\* options can be used with GxB\_get to query the current implementation. For all of these, GxB\_get returns a string (char \*), except for GxB\_LIBRARY\_VERSION, which takes as input an int array of size three. The GxB\_API\_\* options can be used with GxB\_get to query the current GraphBLAS C API Specification. For all of these, GxB\_get returns a string (char \*), except for GxB\_API\_VERSION, which takes as input an int array of size three.

## 8.8 GxB\_Global\_Option\_set: set a global option

This usage of GxB\_set sets the value of a global option. The field parameter can be GxB\_HYPER\_SWITCH, GxB\_BITMAP\_SWITCH, GxB\_FORMAT, GxB\_NTHREADS, GxB\_CHUNK, GxB\_BURBLE, GxB\_PRINTF, GxB\_FLUSH, GxB\_MEMORY\_POOL, or GxB\_PRINT\_1BASED.

For example, the following usage sets the global hypersparsity ratio to 0.2, the format of future matrices to GxB\_BY\_COL, the maximum number of threads to 4, the chunk size to 10000, and enables the burble. No existing matrices are changed.

```
GxB_set (GxB_HYPER_SWITCH, 0.2) ;
GxB_set (GxB_FORMAT, GxB_BY_COL) ;
GxB_set (GxB_NTHREADS, 4) ;
GxB_set (GxB_CHUNK, (double) 10000) ;
GxB_set (GxB_BURBLE, true) ;
GxB_set (GxB_PRINTF, mexPrintf) ;
```

The memory pool parameter sets an upper bound on the number of freed blocks of memory that SuiteSparse:GraphBLAS keeps in its internal memory pool for future allocations.  $free_pool_limit$  is an  $int64_t$  array of size 64, and  $free_pool_limit$  [k] is the upper bound on the number of blocks of size  $2^k$  that are kept in the pool. Passing in a NULL pointer sets the defaults. Passing in an array of size 64 whose entries are all zero disables the memory pool entirely.

## 8.9 GxB\_Matrix\_Option\_set: set a matrix option

This usage of GxB\_set sets the value of a matrix option, for a particular matrix. The field parameter can be GxB\_HYPER\_SWITCH, GxB\_BITMAP\_SWITCH, GxB\_SPARSITY\_CONTROL, or GxB\_FORMAT.

For example, the following usage sets the hypersparsity ratio to 0.2, and the format of GxB\_BY\_COL, for a particular matrix A, and sets the sparsity control to GxB\_SPARSE+GxB\_FULL (allowing the matrix to be held in CSC or FullC formats, but not BitmapC or HyperCSC). SuiteSparse:GraphBLAS currently applies these changes immediately, but since they are simply hints, future versions of SuiteSparse:GraphBLAS may delay the change in format if it can obtain better performance.

If the setting is just GxB\_FULL and some entries are missing, then the matrix is held in bitmap format.

```
GxB_set (A, GxB_HYPER_SWITCH, 0.2);
GxB_set (A, GxB_FORMAT, GxB_BY_COL);
GxB_set (A, GxB_SPARSITY_CONTROL, GxB_SPARSE + GxB_FULL);
```

For performance, the matrix option should be set as soon as it is created with GrB\_Matrix\_new, so the internal transformation takes less time.

If an error occurs, GrB\_error(&err,A) returns details about the error.

## 8.10 GxB\_Desc\_set: set a GrB\_Descriptor value

This usage is similar to GrB\_Descriptor\_set, just with a name that is consistent with the other usages of this generic function. Unlike GrB\_Descriptor\_set,

the field may also be GxB\_NTHREADS, GxB\_CHUNK, GxB\_SORT, GxB\_COMPRESSION, or GxB\_IMPORT. Refer to Sections 6.13.3 and 6.13.4 for details. If an error occurs, GrB\_error(&err,desc) returns details about the error.

## 8.11 GxB\_Global\_Option\_get: retrieve a global option

This usage of GxB\_get retrieves the value of a global option. The field parameter can be one of the following:

GxB_HYPER_SWITCH	sparse/hyper setting	
GxB_BITMAP_SWITCH	bitmap/sparse setting	
GxB_FORMAT	by row/col setting	
GxB_MODE	blocking / non-blocking	
GxB_NTHREADS	default number of threads	
GxB_CHUNK	default chunk size	
GxB_BURBLE	burble setting	
GxB_PRINTF	printf function	
GxB_FLUSH	flush function	
GxB_MEMORY_POOL	memory pool control	
GxB_PRINT_1BASED	for printing matrices/vectors	
GxB_LIBRARY_NAME	the string "SuiteSparse:GraphBLAS"	
GxB_LIBRARY_VERSION	int array of size 3	
GxB_LIBRARY_DATE	date of release	
GxB_LIBRARY_ABOUT	author, copyright	
GxB_LIBRARY_LICENSE	license for the library	
<pre>GxB_LIBRARY_COMPILE_DATE</pre>	date of compilation	
<pre>GxB_LIBRARY_COMPILE_TIME</pre>	time of compilation	
GxB_LIBRARY_URL	URL of the library	
GxB_API_VERSION	GraphBLAS C API Specification Version	
GxB_API_DATE	date of the C API Spec.	
GxB_API_ABOUT	about of the C API Spec.	
GxB_API_URL	URL of the spec	

For example:

```
double h ;
GxB_get (GxB_HYPER_SWITCH, &h) ;
```

```
printf ("hyper_switch = %g for all new matrices\n", h) ;
double b [GxB_BITMAP_SWITCH] ;
GxB_get (GxB_BITMAP_SWITCH, b) ;
for (int k = 0 ; k < GxB_NBITMAP_SWITCH ; k++)</pre>
   printf ("bitmap_switch [%d] = %g ", k, b [k]);
   if (k == 0)
        printf ("for vectors and matrices with 1 row or column\n") ;
    else if (k == GxB_NBITMAP_SWITCH - 1)
        printf ("for matrices with min dimension > d\n", 1 << (k-1));
    }
    else
    {
        printf ("for matrices with min dimension %d to %d\n",
            (1 << (k-1)) + 1, 1 << k);
}
GxB_Format_Value s ;
GxB_get (GxB_FORMAT, &s) ;
if (s == GxB_BY_COL) printf ("all new matrices are stored by column\n");
else printf ("all new matrices are stored by row\n");
GrB_mode mode ;
GxB_get (GxB_MODE, &mode) ;
if (mode == GrB_BLOCKING) printf ("GrB_init(GrB_BLOCKING) was called.\n") ;
else printf ("GrB_init(GrB_NONBLOCKING) was called.\n") ;
int nthreads_max ;
GxB_get (GxB_NTHREADS, &nthreads_max) ;
printf ("max # of threads to use: %d\n", nthreads_max);
double chunk ;
GxB_get (GxB_CHUNK, &chunk) ;
printf ("chunk size: %g\n", chunk);
int64_t free_pool_limit [64] ;
GxB_get (GxB_MEMORY_POOL, free_pool_limit) ;
for (int k = 0; k < 64; k++)
    printf ("pool %d: limit %ld\n", free_pool_limit [k]);
```

```
char *name ;
int ver [3] ;
GxB_get (GxB_LIBRARY_NAME, &name) ;
GxB_get (GxB_LIBRARY_VERSION, ver) ;
printf ("Library %s, version %d.%d\n", name, ver [0], ver [1], ver [2]) ;
```

## 8.12 GxB\_Matrix\_Option\_get: retrieve a matrix option

```
GrB_Info GxB_get  // gets the current option of a matrix

(
GrB_Matrix A,  // matrix to query
GxB_Option_Field field,  // option to query
...  // return value of the matrix option
);
```

This usage of GxB\_get retrieves the value of a matrix option. The field parameter can be GxB\_HYPER\_SWITCH, GxB\_BITMAP\_SWITCH, GxB\_SPARSITY\_CONTROL, GxB\_SPARSITY\_STATUS, or GxB\_FORMAT. For example:

```
double h, b ;
int sparsity, scontrol ;
GxB_get (A, GxB_SPARSITY_STATUS, &sparsity) ;
GxB_get (A, GxB_HYPER_SWITCH, &h) ;
printf ("matrix A has hyper_switch = %g\n", h);
GxB_get (A, GxB_BITMAP_SWITCH, &b) ;
printf ("matrix A has bitmap_switch = %g\n", b) ;
switch (sparsity)
   case GxB_HYPERSPARSE: printf ("matrix A is hypersparse\n") ; break ;
   case GxB_SPARSE:
                     printf ("matrix A is sparse\n"
                                                          ) ; break ;
   case GxB_BITMAP:
                         printf ("matrix A is bitmap\n"
                                                            ); break;
   case GxB_FULL:
                         printf ("matrix A is full\n"
                                                            ); break;
}
GxB_Format_Value s ;
GxB_get (A, GxB_FORMAT, &s) ;
printf ("matrix A is stored by %s\n", (s == GxB_BY_COL) ? "col" : "row") ;
GxB_get (A, GxB_SPARSITY_CONTROL, &scontrol) ;
if (scontrol & GxB_HYPERSPARSE) printf ("A may become hypersparse\n") ;
if (scontrol & GxB_SPARSE
                             ) printf ("A may become sparse\n");
if (scontrol & GxB_BITMAP
                             ) printf ("A may become bitmap\n");
if (scontrol & GxB_FULL
                             ) printf ("A may become full\n");
```

## **8.13** GxB\_Desc\_get: retrieve a GrB\_Descriptor value

This usage is the same as GxB\_Desc\_get. The field parameter can be GrB\_OUTP, GrB\_MASK, GrB\_INPO, GrB\_INP1, GxB\_AxB\_METHOD, GxB\_NTHREADS, GxB\_CHUNK, GxB\_SORT, GxB\_COMPRESSION, or GxB\_IMPORT. Refer to Section 6.13.5 for details.

## 8.14 Summary of usage of GxB\_set and GxB\_get

The different usages of GxB\_set and GxB\_get are summarized below. To set/get the global options:

```
GxB_set (GxB_HYPER_SWITCH, double h) ;
GxB_set (GxB_HYPER_SWITCH, GxB_ALWAYS_HYPER) ;
GxB_set (GxB_HYPER_SWITCH, GxB_NEVER_HYPER) ;
GxB_get (GxB_HYPER_SWITCH, double *h) ;
double b [GxB_NBITMAP_SWITCH] ;
GxB_set (GxB_BITMAP_SWITCH, b) ;
GxB_set (GxB_BITMAP_SWITCH, NULL) ;
                                        // set defaults
GxB_get (GxB_BITMAP_SWITCH, b) ;
GxB_set (GxB_FORMAT, GxB_BY_ROW) ;
GxB_set (GxB_FORMAT, GxB_BY_COL) ;
GxB_get (GxB_FORMAT, GxB_Format_Value *s) ;
GxB_set (GxB_NTHREADS, int nthreads_max) ;
GxB_get (GxB_NTHREADS, int *nthreads_max) ;
GxB_set (GxB_CHUNK, double chunk) ;
GxB_get (GxB_CHUNK, double *chunk) ;
GxB_set (GxB_BURBLE, bool burble) ;
GxB_get (GxB_BURBLE, bool *burble) ;
GxB_set (GxB_PRINTF, void *printf_function) ;
GxB_get (GxB_PRINTF, void **printf_function) ;
GxB_set (GxB_FLUSH, void *flush_function) ;
GxB_get (GxB_FLUSH, void **flush_function) ;
int64_t free_pool_limit [64] ;
GxB_set (GxB_MEMORY_POOL, free_pool_limit) ;
GxB_set (GxB_MEMORY_POOL, NULL) ; // set defaults
GxB_get (GxB_MEMORY_POOL, free_pool_limit) ;
```

```
GxB_set (GxB_PRINT_1BASED, bool onebased) ;
   GxB_get (GxB_PRINT_1BASED, bool *onebased) ;
To get global options that can be queried but not modified:
    GxB_get (GxB_MODE,
                                       GrB_Mode *mode) ;
    GxB_get (GxB_LIBRARY_NAME,
                                       char **);
   GxB_get (GxB_LIBRARY_VERSION,
                                       int *);
   GxB_get (GxB_LIBRARY_DATE,
                                       char **);
                                       char **);
   GxB_get (GxB_LIBRARY_ABOUT,
   GxB_get (GxB_LIBRARY_LICENSE,
                                       char **);
   GxB_get (GxB_LIBRARY_COMPILE_DATE, char **) ;
    GxB_get (GxB_LIBRARY_COMPILE_TIME, char **);
   GxB_get (GxB_LIBRARY_URL,
                                       char **);
   GxB_get (GxB_API_VERSION,
                                      int *);
   GxB_get (GxB_API_DATE,
                                      char **) ;
    GxB_get (GxB_API_ABOUT,
                                      char **);
    GxB_get (GxB_API_URL,
                                       char **);
To set/get a matrix option or status
    GxB_set (GrB_Matrix A, GxB_HYPER_SWITCH, double h) ;
    GxB_set (GrB_Matrix A, GxB_HYPER_SWITCH, GxB_ALWAYS_HYPER) ;
   GxB_set (GrB_Matrix A, GxB_HYPER_SWITCH, GxB_NEVER_HYPER) ;
   GxB_get (GrB_Matrix A, GxB_HYPER_SWITCH, double *h) ;
   GxB_set (GrB_Matrix A, GxB_BITMAP_SWITCH, double b) ;
   GxB_get (GrB_Matrix A, GxB_BITMAP_SWITCH, double *b) ;
   GxB_set (GrB_Matrix A, GxB_FORMAT, GxB_BY_ROW) ;
    GxB_set (GrB_Matrix A, GxB_FORMAT, GxB_BY_COL) ;
   GxB_get (GrB_Matrix A, GxB_FORMAT, GxB_Format_Value *s) ;
   GxB_set (GrB_Matrix A, GxB_SPARSITY_CONTROL, GxB_AUTO_SPARSITY) ;
   GxB_set (GrB_Matrix A, GxB_SPARSITY_CONTROL, scontrol) ;
    GxB_get (GrB_Matrix A, GxB_SPARSITY_CONTROL, int *scontrol) ;
    GxB_get (GrB_Matrix A, GxB_SPARSITY_STATUS, int *sparsity) ;
To set/get a vector option or status:
    GxB_set (GrB_Vector v, GxB_BITMAP_SWITCH, double b) ;
   GxB_get (GrB_Vector v, GxB_BITMAP_SWITCH, double *b) ;
   GxB_set (GrB_Vector v, GxB_FORMAT, GxB_BY_ROW) ;
   GxB_set (GrB_Vector v, GxB_FORMAT, GxB_BY_COL) ;
   GxB_get (GrB_Vector v, GxB_FORMAT, GxB_Format_Value *s) ;
   GxB_set (GrB_Vector v, GxB_SPARSITY_CONTROL, GxB_AUTO_SPARSITY) ;
   GxB_set (GrB_Vector v, GxB_SPARSITY_CONTROL, scontrol) ;
    GxB_get (GrB_Vector v, GxB_SPARSITY_CONTROL, int *scontrol) ;
    GxB_get (GrB_Vector v, GxB_SPARSITY_STATUS, int *sparsity) ;
```

#### To set/get a descriptor field:

```
GxB_set (GrB_Descriptor d, GrB_OUTP, GxB_DEFAULT) ;
GxB_set (GrB_Descriptor d, GrB_OUTP, GrB_REPLACE) ;
GxB_get (GrB_Descriptor d, GrB_OUTP, GrB_Desc_Value *v) ;
GxB_set (GrB_Descriptor d, GrB_MASK, GxB_DEFAULT) ;
GxB_set (GrB_Descriptor d, GrB_MASK, GrB_COMP) ;
GxB_set (GrB_Descriptor d, GrB_MASK, GrB_STRUCTURE) ;
GxB_set (GrB_Descriptor d, GrB_MASK, GrB_COMP+GrB_STRUCTURE) ;
GxB_get (GrB_Descriptor d, GrB_MASK, GrB_Desc_Value *v) ;
GxB_set (GrB_Descriptor d, GrB_INPO, GxB_DEFAULT) ;
GxB_set (GrB_Descriptor d, GrB_INPO, GrB_TRAN) ;
GxB_get (GrB_Descriptor d, GrB_INPO, GrB_Desc_Value *v) ;
GxB_set (GrB_Descriptor d, GrB_INP1, GxB_DEFAULT) ;
GxB_set (GrB_Descriptor d, GrB_INP1, GrB_TRAN) ;
GxB_get (GrB_Descriptor d, GrB_INP1, GrB_Desc_Value *v) ;
GxB_set (GrB_Descriptor d, GxB_AxB_METHOD, GxB_DEFAULT) ;
GxB_set (GrB_Descriptor d, GxB_AxB_METHOD, GxB_AxB_GUSTAVSON) ;
GxB_set (GrB_Descriptor d, GxB_AxB_METHOD, GxB_AxB_HASH) ;
GxB_set (GrB_Descriptor d, GxB_AxB_METHOD, GxB_AxB_SAXPY) ;
GxB_set (GrB_Descriptor d, GxB_AxB_METHOD, GxB_AxB_DOT) ;
GxB_get (GrB_Descriptor d, GrB_AxB_METHOD, GrB_Desc_Value *v) ;
GxB_set (GrB_Descriptor d, GxB_NTHREADS, int nthreads) ;
GxB_get (GrB_Descriptor d, GxB_NTHREADS, int *nthreads) ;
GxB_set (GrB_Descriptor d, GxB_CHUNK, double chunk) ;
GxB_get (GrB_Descriptor d, GxB_CHUNK, double *chunk) ;
GxB_set (GrB_Descriptor d, GxB_SORT, sort) ;
GxB_get (GrB_Descriptor d, GxB_SORT, int *sort) ;
GxB_set (GrB_Descriptor d, GxB_COMPRESSION, GxB_FAST_IMPORT) ;
GxB_set (GrB_Descriptor d, GxB_COMPRESSION, GxB_SECURE_IMPORT) ;
GxB_get (GrB_Descriptor d, GxB_COMPRESSION, GrB_Desc_Value *method) ;
GxB_set (GrB_Descriptor d, GxB_IMPORT, int method) ;
GxB_get (GrB_Descriptor d, GxB_IMPORT, int *method) ;
```

# 9 SuiteSparse:GraphBLAS Colon and Index Notation

Octave/MATLAB uses a colon notation to index into matrices, such as C=A(2:4,3:8), which extracts C as 3-by-6 submatrix from A, from rows 2 through 4 and columns 3 to 8 of the matrix A. A single colon is used to denote all rows, C=A(:,9), or all columns, C=A(12,:), which refers to the 9th column and 12th row of A, respectively. An arbitrary integer list can be given as well, such as the Octave/MATLAB statements:

```
I = [2 1 4];

J = [3 5];

C = A (I,J);
```

which creates the 3-by-2 matrix C as follows:

$$C = \left[ \begin{array}{cc} a_{2,3} & a_{2,5} \\ a_{1,3} & a_{1,5} \\ a_{4,3} & a_{4,5} \end{array} \right]$$

The GraphBLAS API can do the equivalent of C=A(I,J), C=A(:,J), C=A(I,:), and C=A(:,:), by passing a parameter const GrB\_Index \*I as either an array of size ni, or as the special value GrB\_ALL, which corresponds to the stand-alone colon C=A(:,J), and the same can be done for J.. To compute C=A(2:4,3:8) in GraphBLAS requires the user application to create two explicit integer arrays I and J of size 3 and 5, respectively, and then fill them with the explicit values [2,3,4] and [3,4,5,6,7,8]. This works well if the lists are small, or if the matrix has more entries than rows or columns.

However, particularly with hypersparse matrices, the size of the explicit arrays I and J can vastly exceed the number of entries in the matrix. When using its hypersparse format, SuiteSparse:GraphBLAS allows the user application to create a GrB\_Matrix with dimensions up to  $2^{60}$ , with no memory constraints. The only constraint on memory usage in a hypersparse matrix is the number of entries in the matrix.

For example, creating a n-by-n matrix A of type GrB\_FP64 with  $n=2^{60}$  and one million entries is trivial to do in Version 2.1 (and later) of Suite-Sparse:GraphBLAS, taking at most 24MB of space. SuiteSparse:GraphBLAS

Version 2.1 (or later) could do this on an old smartphone. However, using just the pure GraphBLAS API, constructing C=A(0:(n/2),0:(n/2)) in SuiteSparse Version 2.0 would require the creation of an integer array I of size 2<sup>59</sup>, containing the sequence 0, 1, 2, 3, ...., requiring about 4 ExaBytes of memory (4 million terabytes). This is roughly 1000 times larger than the memory size of the world's largest computer in 2018.

SuiteSparse:GraphBLAS Version 2.1 and later extends the GraphBLAS API with a full implementation of the MATLAB colon notation for integers, I=begin:inc:end. This extension allows the construction of the matrix C=A(0:(n/2),0:(n/2)) in this example, with dimension 2<sup>59</sup>, probably taking just milliseconds on an old smartphone.

The GrB\_extract, GrB\_assign, and GxB\_subassign operations (described in the Section 10) each have parameters that define a list of integer indices, using two parameters:

```
const GrB\_Index *I ; // an array, or a special value GrB\_ALL GrB\_Index ni ; // the size of I, or a special value
```

These two parameters define five kinds of index lists, which can be used to specify either an explicit or implicit list of row indices and/or column indices. The length of the list of indices is denoted |I|. This discussion applies equally to the row indices I and the column indices J. The five kinds are listed below.

- 1. An explicit list of indices, such as I = [2 1 4 7 2] in MATLAB notation, is handled by passing in I as a pointer to an array of size 5, and passing ni=5 as the size of the list. The length of the explicit list is ni=|I|. Duplicates may appear, except that for some uses of GrB\_assign and GxB\_subassign, duplicates lead to undefined behavior according to the GraphBLAS C API Specification. Suite-Sparse:GraphBLAS specifies how duplicates are handled in all cases, as an addition to the specification. See Section 10.9 for details.
- 2. To specify all rows of a matrix, use I = GrB\_ALL. The parameter ni is ignored. This is equivalent to C=A(:,J) in MATLAB. In GraphBLAS, this is the sequence 0:(m-1) if A has m rows, with length |I|=m. If J is used the columns of an m-by-n matrix, then J=GrB\_ALL refers to all columns, and is the sequence 0:(n-1), of length |J|=n.

3. To specify a contiguous range of indices, such as I=10:20 in MATLAB, the array I has size 2, and ni is passed to SuiteSparse:GraphBLAS as the special value ni = GxB\_RANGE. The beginning index is I[GxB\_BEGIN] and the ending index is I[GxB\_END]. Both values must be non-negative since GrB\_Index is an unsigned integer (uint64\_t). The value of I[GxB\_INC] is ignored.

Let  $b = I[GxB\_BEGIN]$ , let  $e = I[GxB\_END]$ , The sequence has length zero if b > e; otherwise the length is |I| = (e - b) + 1.

4. To specify a strided range of indices with a non-negative stride, such as I=3:2:10, the array I has size 3, and ni has the special value GxB\_STRIDE. This is the sequence 3, 5, 7, 9, of length 4. Note that 10 does not appear in the list. The end point need not appear if the increment goes past it.

The GxB\_STRIDE sequence is the same as the List generated by the following for loop:

```
int64_t k = 0 ;
GrB_Index *List = (a pointer to an array of large enough size)
for (int64_t i = I [GxB_BEGIN] ; i <= I [GxB_END] ; i += I [GxB_INC])
{
    // i is the kth entry in the sequence
    List [k++] = i ;
}</pre>
```

Then passing the explicit array List and its length ni=k has the same effect as passing in the array I of size 3, with ni=GxB\_STRIDE. The latter is simply much faster to produce, and much more efficient for SuiteSparse:GraphBLAS to process.

Let  $b = I[GxB\_BEGIN]$ , let  $e = I[GxB\_END]$ , and let  $\Delta = I[GxB\_INC]$ . The sequence has length zero if b > e or  $\Delta = 0$ . Otherwise, the length of the sequence is

$$|I| = \left| \frac{e - b}{\Lambda} \right| + 1$$

5. In MATLAB notation, if the stride is negative, the sequence is decreasing. For example, 10:-2:1 is the sequence 10, 8, 6, 4, 2, in that order. In SuiteSparse:GraphBLAS, use ni = GxB\_BACKWARDS, with an array I of size 3. The following example specifies defines the equivalent of the MATLAB expression 10:-2:1 in SuiteSparse:GraphBLAS:

```
// to specify I = 10:-2:1
GrB_Index I [3], ni = GxB_BACKWARDS ;
I [GxB_BEGIN ] = 10 ;  // the start of the sequence
I [GxB_INC ] = 2 ;  // the magnitude of the increment
I [GxB_END ] = 1 ;  // the end of the sequence
```

The value -2 cannot be assigned to the GrB\_Index array I, since that is an unsigned type. The signed increment is represented instead with the special value ni = GxB\_BACKWARDS. The GxB\_BACKWARDS sequence is the same as generated by the following for loop:

```
int64_t k = 0 ;
GrB_Index *List = (a pointer to an array of large enough size)
for (int64_t i = I [GxB_BEGIN] ; i >= I [GxB_END] ; i -= I [GxB_INC])
{
    // i is the kth entry in the sequence
    List [k++] = i ;
}
```

Let  $b = I[GxB\_BEGIN]$ , let  $e = I[GxB\_END]$ , and let  $\Delta = I[GxB\_INC]$  (note that  $\Delta$  is not negative). The sequence has length zero if b < e or  $\Delta = 0$ . Otherwise, the length of the sequence is

$$|I| = \left\lfloor \frac{b - e}{\Delta} \right\rfloor + 1$$

Since GrB\_Index is an unsigned integer, all three values I[GxB\_BEGIN], I[GxB\_INC], and I[GxB\_END] must be non-negative.

Just as in MATLAB, it is valid to specify an empty sequence of length zero. For example, I = 5:3 has length zero in MATLAB and the same is true for a GxB\_RANGE sequence in SuiteSparse:GraphBLAS, with I [GxB\_BEGIN] = 5 and I [GxB\_END] = 3. This has the same effect as array I with ni=0.

# 10 GraphBLAS Operations

The next sections define each of the GraphBLAS operations, also listed in the table below.

GrB_mxm	matrix-matrix multiply	$\mathbf{C}\langle\mathbf{M} angle=\mathbf{C}\odot\mathbf{AB}$
${\tt GrB\_vxm}$	vector-matrix multiply	$\mathbf{w}^{T} \langle \mathbf{m}^{T} \rangle = \mathbf{w}^{T} \odot \mathbf{u}^{T} \mathbf{A}$
GrB_mxv	matrix-vector multiply	$\mathbf{w}\langle\mathbf{m}\rangle=\mathbf{w}\odot\mathbf{A}\mathbf{u}$
GrB_eWiseMult	element-wise,	$\mathbf{C}\langle \mathbf{M}  angle = \mathbf{C} \odot (\mathbf{A} \otimes \mathbf{B})$
	set intersection	$\mathbf{w}\langle\mathbf{m}\rangle=\mathbf{w}\odot(\mathbf{u}\otimes\mathbf{v})$
GrB_eWiseAdd	element-wise,	$\mathbf{C}\langle\mathbf{M}\rangle=\mathbf{C}\odot(\mathbf{A}\oplus\mathbf{B})$
	set union	$\mathbf{w}\langle\mathbf{m} angle=\mathbf{w}\odot(\mathbf{u}\oplus\mathbf{v})$
<pre>GrB_extract</pre>	extract submatrix	$\mathbf{C}\langle\mathbf{M} angle=\mathbf{C}\odot\mathbf{A}(\mathbf{I},\mathbf{J})$
		$\mathbf{w}\langle\mathbf{m} angle=\mathbf{w}\odot\mathbf{u}(\mathbf{i})$
GxB_subassign	assign submatrix,	$\mathbf{C}(\mathbf{I},\mathbf{J})\langle\mathbf{M} angle = \mathbf{C}(\mathbf{I},\mathbf{J})\odot\mathbf{A}$
	with submask for $\mathbf{C}(\mathbf{I}, \mathbf{J})$	$\mathbf{w}(\mathbf{i})\langle\mathbf{m} angle=\mathbf{w}(\mathbf{i})\odot\mathbf{u}$
${\tt GrB\_assign}$	assign submatrix	$\mathbf{C}\langle\mathbf{M} angle(\mathbf{I},\mathbf{J})=\mathbf{C}(\mathbf{I},\mathbf{J})\odot\mathbf{A}$
	with submask for $\mathbf{C}$	$\mathbf{w}\langle\mathbf{m} angle(\mathbf{i})=\mathbf{w}(\mathbf{i})\odot\mathbf{u}$
<pre>GrB_apply</pre>	apply unary operator	$\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{C}\odot f(\mathbf{A})$
		$\mathbf{w}\langle\mathbf{m} angle=\mathbf{w}\odot f(\mathbf{u})$
	apply binary operator	$\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{C}\odot f(x,\mathbf{A})$
		$\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{C}\odot f(\mathbf{A},y)$
		$\mathbf{w}\langle\mathbf{m}\rangle = \mathbf{w}\odot f(x, \mathbf{x})$
		$\mathbf{w}\langle\mathbf{m}\rangle = \mathbf{w}\odot f(\mathbf{u}, y)$
<pre>GrB_select</pre>	apply select operator	$\mathbf{C}\langle\mathbf{M} angle=\mathbf{C}\odot f(\mathbf{A},\mathbf{i},\mathbf{j},\mathbf{k})$
		$\mathbf{w}\langle\mathbf{m}\rangle = \mathbf{w}\odot f(\mathbf{u}, \mathbf{i}, \mathbf{j}, \mathbf{k})$
<pre>GrB_reduce</pre>	reduce to vector	$\mathbf{w}\langle\mathbf{m} angle=\mathbf{w}\odot[\oplus_{j}\mathbf{A}(:,j)]$
	reduce to scalar	$s = s \odot [\oplus_{ij} \mathbf{A}(I,J)]$
GrB_transpose	transpose	$\mathbf{C}\langle\mathbf{M} angle=\mathbf{C}\odot\mathbf{A}^T$
GrB_kronecker	Kronecker product	$\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{C}\odot \mathrm{kron}(\mathbf{A},\mathbf{B})$

If an error occurs, GrB\_error(&err,C) or GrB\_error(&err,w) returns details about the error, for operations that return a modified matrix C or vector w. The only operation that cannot return an error string is reduction to a scalar with GrB\_reduce.

# 10.1 GrB\_mxm: matrix-matrix multiply

```
GrB_Info GrB_mxm
                                     // C<Mask> = accum (C, A*B)
                                    // input/output matrix for results
    GrB_Matrix C,
   const GrB_Matrix Mask,
                                    // optional mask for C, unused if NULL
    const GrB_BinaryOp accum,
                                    // optional accum for Z=accum(C,T)
    const GrB_Semiring semiring,
                                    // defines '+' and '*' for A*B
    const GrB_Matrix A,
                                    // first input: matrix A
    const GrB_Matrix B,
                                    // second input: matrix B
    const GrB_Descriptor desc
                                    // descriptor for C, Mask, A, and B
);
```

GrB\_mxm multiplies two sparse matrices A and B using the semiring. The input matrices A and B may be transposed according to the descriptor, desc (which may be NULL) and then typecasted to match the multiply operator of the semiring. Next, T=A\*B is computed on the semiring, precisely defined in the GB\_spec\_mxm.m script in GraphBLAS/Test. The actual algorithm exploits sparsity and does not take  $O(n^3)$  time, but it computes the following:

```
[m s] = size (A.matrix) ;
[s n] = size (B.matrix) ;
T.matrix = zeros (m, n, multiply.ztype) ;
T.pattern = zeros (m, n, 'logical') ;
T.matrix (:,:) = identity ;
                                        % the identity of the semiring's monoid
T.class = multiply.ztype ;
                                        % the ztype of the semiring's multiply op
A = cast (A.matrix, multiply.xtype);
                                       % the xtype of the semiring's multiply op
                                        % the ytype of the semiring's multiply op
B = cast (B.matrix, multiply.ytype);
for j = 1:n
   for i = 1:m
       for k = 1:s
            % T (i,j) += A (i,k) * B (k,j), using the semiring
            if (A.pattern (i,k) && B.pattern (k,j))
                z = multiply (A (i,k), B (k,j));
                T.matrix (i,j) = add (T.matrix (i,j), z);
                T.pattern(i,j) = true;
            end
        end
    end
end
```

Finally, T is typecasted into the type of C, and the results are written back into C via the accum and Mask,  $C\langle M \rangle = C \odot T$ . The latter step is reflected in the MATLAB function  $GB_spec_accum_mask.m$ , discussed in Section 2.3.

Performance considerations: Suppose all matrices are in GxB\_BY\_COL format, and B is extremely sparse but A is not as sparse. Then computing C=A\*B is very fast, and much faster than when A is extremely sparse. For example, if A is square and B is a column vector that is all nonzero except for one entry B(j,0)=1, then C=A\*B is the same as extracting column A(:,j). This is very fast if A is stored by column but slow if A is stored by row. If A is a sparse row with a single entry A(0,i)=1, then C=A\*B is the same as extracting row B(i,:). This is fast if B is stored by row but slow if B is stored by column.

If the user application needs to repeatedly extract rows and columns from a matrix, whether by matrix multiplication or by <code>GrB\_extract</code>, then keep two copies: one stored by row, and other by column, and use the copy that results in the fastest computation.

By default, GrB\_mxm, GrB\_mxv, GrB\_vxm, and GrB\_reduce (to vector) can return their result in a jumbled state, with the sort left pending. It can sometimes be faster for these methods to do the sort as they compute their result. Use the GxB\_SORT descriptor setting to select this option. Refer to Section 6.13 for details.

# 10.2 GrB\_vxm: vector-matrix multiply

```
GrB_Info GrB_vxm
                                    // w'<mask> = accum (w, u'*A)
    GrB_Vector w,
                                    // input/output vector for results
    const GrB_Vector mask,
                                    // optional mask for w, unused if NULL
    const GrB_BinaryOp accum,
                                    // optional accum for z=accum(w.t)
                                    // defines '+' and '*' for u'*A
    const GrB_Semiring semiring,
    const GrB_Vector u,
                                    // first input: vector u
    const GrB_Matrix A,
                                    // second input: matrix A
    const GrB_Descriptor desc
                                    // descriptor for w, mask, and A
);
```

GrB\_vxm multiplies a row vector u' times a matrix A. The matrix A may be first transposed according to desc (as the second input, GrB\_INP1); the column vector u is never transposed via the descriptor. The inputs u and A are typecasted to match the xtype and ytype inputs, respectively, of the multiply operator of the semiring. Next, an intermediate column vector t=A'\*u is computed on the semiring using the same method as GrB\_mxm. Finally, the column vector t is typecasted from the ztype of the multiply operator of the semiring into the type of w, and the results are written back into w using the optional accumulator accum and mask.

The last step is  $\mathbf{w}\langle \mathbf{m} \rangle = \mathbf{w} \odot \mathbf{t}$ , as described in Section 2.3, except that all the terms are column vectors instead of matrices.

Performance considerations: If the GxB\_FORMAT of A is GxB\_BY\_ROW, and the default descriptor is used (A is not transposed), then GrB\_vxm is faster than than GrB\_mxv with its default descriptor, when the vector u is very sparse. However, if the GxB\_FORMAT of A is GxB\_BY\_COL, then GrB\_mxv with its default descriptor is faster than GrB\_vxm with its default descriptor, when the vector u is very sparse. Using the non-default GrB\_TRAN descriptor for A makes the GrB\_vxm operation equivalent to GrB\_mxv with its default descriptor (with the operands reversed in the multiplier, as well). The reverse is true as well; GrB\_mxv with GrB\_TRAN is the same as GrB\_vxm with a default descriptor.

# 10.3 GrB\_mxv: matrix-vector multiply

```
GrB_Info GrB_mxv
                                    // w<mask> = accum (w, A*u)
    GrB_Vector w,
                                    // input/output vector for results
    const GrB_Vector mask,
                                    // optional mask for w, unused if NULL
    const GrB_BinaryOp accum,
                                    // optional accum for z=accum(w,t)
                                    // defines '+' and '*' for A*B
    const GrB_Semiring semiring,
    const GrB_Matrix A,
                                    // first input: matrix A
    const GrB_Vector u,
                                    // second input: vector u
    const GrB_Descriptor desc
                                    // descriptor for w, mask, and A
);
```

GrB\_mxv multiplies a matrix A times a column vector u. The matrix A may be first transposed according to desc (as the first input); the column vector u is never transposed via the descriptor. The inputs A and u are typecasted to match the xtype and ytype inputs, respectively, of the multiply operator of the semiring. Next, an intermediate column vector t=A\*u is computed on the semiring using the same method as GrB\_mxm. Finally, the column vector t is typecasted from the ztype of the multiply operator of the semiring into the type of w, and the results are written back into w using the optional accumulator accum and mask.

The last step is  $\mathbf{w}\langle \mathbf{m} \rangle = \mathbf{w} \odot \mathbf{t}$ , as described in Section 2.3, except that all the terms are column vectors instead of matrices.

Performance considerations: Refer to the discussion of GrB\_vxm. In SuiteSparse:GraphBLAS, GrB\_mxv is very efficient when u is sparse or dense, when the default descriptor is used, and when the matrix is GxB\_BY\_COL. When u is very sparse and GrB\_INPO is set to its non-default GrB\_TRAN, then this method is not efficient if the matrix is in GxB\_BY\_COL format. If an application needs to perform A'\*u repeatedly where u is very sparse, then use the GxB\_BY\_ROW format for A instead.

# 10.4 GrB\_eWiseMult: element-wise operations, set intersection

Element-wise "multiplication" is shorthand for applying a binary operator element-wise on two matrices or vectors A and B, for all entries that appear in the set intersection of the patterns of A and B. This is like A.\*B for two sparse matrices in MATLAB, except that in GraphBLAS any binary operator can be used, not just multiplication.

The pattern of the result of the element-wise "multiplication" is exactly this set intersection. Entries in A but not B, or visa versa, do not appear in the result.

Let  $\otimes$  denote the binary operator to be used. The computation  $\mathbf{T} = \mathbf{A} \otimes \mathbf{B}$  is given below. Entries not in the intersection of  $\mathbf{A}$  and  $\mathbf{B}$  do not appear in the pattern of  $\mathbf{T}$ . That is:

for all entries 
$$(i, j)$$
 in  $\mathbf{A} \cap \mathbf{B}$   
 $t_{ij} = a_{ij} \otimes b_{ij}$ 

Depending on what kind of operator is used and what the implicit value is assumed to be, this can give the Hadamard product. This is the case for A.\*B in MATLAB since the implicit value is zero. However, computing a Hadamard product is not necessarily the goal of the eWiseMult operation. It simply applies any binary operator, built-in or user-defined, to the set intersection of A and B, and discards any entry outside this intersection. Its usefulness in a user's application does not depend upon it computing a Hadamard product in all cases. The operator need not be associative, commutative, nor have any particular property except for type compatibility with A and B, and the output matrix C.

The generic name for this operation is **GrB\_eWiseMult**, which can be used for both matrices and vectors.

# 10.4.1 GrB\_eWiseMult\_Vector: element-wise vector multiply

```
GrB_Info GrB_eWiseMult
                                    // w<mask> = accum (w, u.*v)
    GrB_Vector w,
                                    // input/output vector for results
    const GrB_Vector mask,
                                    // optional mask for w, unused if NULL
    const GrB_BinaryOp accum,
                                    // optional accum for z=accum(w,t)
    const <operator> multiply,
                                    // defines '.*' for t=u.*v
    const GrB_Vector u,
                                    // first input: vector u
    const GrB_Vector v,
                                    // second input: vector v
    const GrB_Descriptor desc
                                    // descriptor for w and mask
);
```

GrB\_Vector\_eWiseMult computes the element-wise "multiplication" of two vectors  $\mathbf{u}$  and  $\mathbf{v}$ , element-wise using any binary operator (not just times). The vectors are not transposed via the descriptor. The vectors  $\mathbf{u}$  and  $\mathbf{v}$  are first typecasted into the first and second inputs of the multiply operator. Next, a column vector  $\mathbf{t}$  is computed, denoted  $\mathbf{t} = \mathbf{u} \otimes \mathbf{v}$ . The pattern of  $\mathbf{t}$  is the set intersection of  $\mathbf{u}$  and  $\mathbf{v}$ . The result  $\mathbf{t}$  has the type of the output ztype of the multiply operator.

The operator is typically a GrB\_BinaryOp, but the method is typegeneric for this parameter. If given a monoid (GrB\_Monoid), the additive operator of the monoid is used as the multiply binary operator. If given a semiring (GrB\_Semiring), the multiply operator of the semiring is used as the multiply binary operator.

The next and final step is  $\mathbf{w}\langle\mathbf{m}\rangle = \mathbf{w}\odot\mathbf{t}$ , as described in Section 2.3, except that all the terms are column vectors instead of matrices. Note for all GraphBLAS operations, including this one, the accumulator  $\mathbf{w}\odot\mathbf{t}$  is always applied in a set union manner, even though  $\mathbf{t} = \mathbf{u}\otimes\mathbf{v}$  for this operation is applied in a set intersection manner.

# 10.4.2 GrB\_eWiseMult\_Matrix: element-wise matrix multiply

```
GrB_Info GrB_eWiseMult
                                    // C<Mask> = accum (C, A.*B)
    GrB_Matrix C,
                                    // input/output matrix for results
    const GrB_Matrix Mask,
                                    // optional mask for C, unused if NULL
    const GrB_BinaryOp accum,
                                    // optional accum for Z=accum(C,T)
    const <operator> multiply,
                                    // defines '.*' for T=A.*B
    const GrB_Matrix A,
                                    // first input: matrix A
    const GrB_Matrix B,
                                    // second input: matrix B
    const GrB_Descriptor desc
                                    // descriptor for C, Mask, A, and B
);
```

GrB\_Matrix\_eWiseMult computes the element-wise "multiplication" of two matrices A and B, element-wise using any binary operator (not just times). The input matrices may be transposed first, according to the descriptor desc. They are then typecasted into the first and second inputs of the multiply operator. Next, a matrix T is computed, denoted  $\mathbf{T} = \mathbf{A} \otimes \mathbf{B}$ . The pattern of T is the set intersection of A and B. The result T has the type of the output ztype of the multiply operator.

The multiply operator is typically a GrB\_BinaryOp, but the method is type-generic for this parameter. If given a monoid (GrB\_Monoid), the additive operator of the monoid is used as the multiply binary operator. If given a semiring (GrB\_Semiring), the multiply operator of the semiring is used as the multiply binary operator.

The operation can be expressed in MATLAB notation as:

```
[nrows, ncols] = size (A.matrix);
T.matrix = zeros (nrows, ncols, multiply.ztype);
T.class = multiply.ztype;
p = A.pattern & B.pattern;
A = cast (A.matrix (p), multiply.xtype);
B = cast (B.matrix (p), multiply.ytype);
T.matrix (p) = multiply (A, B);
T.pattern = p;
```

The final step is  $\mathbf{C}\langle \mathbf{M} \rangle = \mathbf{C} \odot \mathbf{T}$ , as described in Section 2.3. Note for all GraphBLAS operations, including this one, the accumulator  $\mathbf{C} \odot \mathbf{T}$  is always applied in a set union manner, even though  $\mathbf{T} = \mathbf{A} \otimes \mathbf{B}$  for this operation is applied in a set intersection manner.

# 10.5 GrB\_eWiseAdd: element-wise operations, set union

Element-wise "addition" is shorthand for applying a binary operator element-wise on two matrices or vectors A and B, for all entries that appear in the set intersection of the patterns of A and B. This is like A+B for two sparse matrices in MATLAB, except that in GraphBLAS any binary operator can be used, not just addition. The pattern of the result of the element-wise "addition" is the set union of the pattern of A and B. Entries in neither in A nor in B do not appear in the result.

Let  $\oplus$  denote the binary operator to be used. The computation  $\mathbf{T} = \mathbf{A} \oplus \mathbf{B}$  is exactly the same as the computation with accumulator operator as described in Section 2.3. It acts like a sparse matrix addition, except that any operator can be used. The pattern of  $\mathbf{A} \oplus \mathbf{B}$  is the set union of the patterns of  $\mathbf{A}$  and  $\mathbf{B}$ , and the operator is applied only on the set intersection of  $\mathbf{A}$  and  $\mathbf{B}$ . Entries not in either the pattern of  $\mathbf{A}$  or  $\mathbf{B}$  do not appear in the pattern of  $\mathbf{T}$ . That is:

```
for all entries (i, j) in \mathbf{A} \cap \mathbf{B}

t_{ij} = a_{ij} \oplus b_{ij}

for all entries (i, j) in \mathbf{A} \setminus \mathbf{B}

t_{ij} = a_{ij}

for all entries (i, j) in \mathbf{B} \setminus \mathbf{A}

t_{ij} = b_{ij}
```

The only difference between element-wise "multiplication" ( $\mathbf{T} = \mathbf{A} \otimes \mathbf{B}$ ) and "addition" ( $\mathbf{T} = \mathbf{A} \oplus \mathbf{B}$ ) is the pattern of the result, and what happens to entries outside the intersection. With  $\otimes$  the pattern of  $\mathbf{T}$  is the intersection; with  $\oplus$  it is the set union. Entries outside the set intersection are dropped for  $\otimes$ , and kept for  $\oplus$ ; in both cases the operator is only applied to those (and only those) entries in the intersection. Any binary operator can be used interchangeably for either operation.

Element-wise operations do not operate on the implicit values, even implicitly, since the operations make no assumption about the semiring. As a result, the results can be different from MATLAB, which can always assume the implicit value is zero. For example, C=A-B is the conventional matrix subtraction in MATLAB. Computing A-B in GraphBLAS with eWiseAdd will apply the MINUS operator to the intersection, entries in A but not B will be unchanged and appear in C, and entries in neither A nor B do not appear in C. For these cases, the results matches the MATLAB C=A-B. Entries in B but not A do appear in C but they are not negated; they cannot be subtracted

from an implicit value in A. This is by design. If conventional matrix subtraction of two sparse matrices is required, and the implicit value is known to be zero, use GrB\_apply to negate the values in B, and then use eWiseAdd with the PLUS operator, to compute A+(-B).

The generic name for this operation is GrB\_eWiseAdd, which can be used for both matrices and vectors.

There is another minor difference in two variants of the element-wise functions. If given a semiring, the eWiseAdd functions use the binary operator of the semiring's monoid, while the eWiseMult functions use the multiplicative operator of the semiring.

#### 10.5.1 GrB\_eWiseAdd\_Vector: element-wise vector addition

```
// w<mask> = accum (w, u+v)
GrB_Info GrB_eWiseAdd
    GrB_Vector w,
                                    // input/output vector for results
    const GrB_Vector mask,
                                    // optional mask for w, unused if NULL
                                    // optional accum for z=accum(w.t)
    const GrB_BinaryOp accum,
    const <operator> add,
                                    // defines '+' for t=u+v
    const GrB_Vector u,
                                    // first input: vector u
    const GrB_Vector v,
                                    // second input: vector v
    const GrB_Descriptor desc
                                    // descriptor for w and mask
);
```

GrB\_Vector\_eWiseAdd computes the element-wise "addition" of two vectors u and v, element-wise using any binary operator (not just plus). The vectors are not transposed via the descriptor. Entries in the intersection of u and v are first typecasted into the first and second inputs of the add operator. Next, a column vector t is computed, denoted  $\mathbf{t} = \mathbf{u} \oplus \mathbf{v}$ . The pattern of t is the set union of u and v. The result t has the type of the output ztype of the add operator.

The add operator is typically a GrB\_BinaryOp, but the method is typegeneric for this parameter. If given a monoid (GrB\_Monoid), the additive operator of the monoid is used as the add binary operator. If given a semiring (GrB\_Semiring), the additive operator of the monoid of the semiring is used as the add binary operator.

The final step is  $\mathbf{w}\langle \mathbf{m} \rangle = \mathbf{w} \odot \mathbf{t}$ , as described in Section 2.3, except that all the terms are column vectors instead of matrices.

#### 10.5.2 GrB eWiseAdd Matrix: element-wise matrix addition

```
GrB_Info GrB_eWiseAdd
                                    // C<Mask> = accum (C, A+B)
    GrB_Matrix C,
                                    // input/output matrix for results
    const GrB_Matrix Mask,
                                    // optional mask for C, unused if NULL
    const GrB_BinaryOp accum,
                                    // optional accum for Z=accum(C,T)
                                    // defines '+' for T=A+B
    const <operator> add,
    const GrB_Matrix A,
                                    // first input: matrix A
    const GrB_Matrix B,
                                    // second input: matrix B
    const GrB_Descriptor desc
                                    // descriptor for C, Mask, A, and B
);
```

GrB\_Matrix\_eWiseAdd computes the element-wise "addition" of two matrices A and B, element-wise using any binary operator (not just plus). The input matrices may be transposed first, according to the descriptor desc. Entries in the intersection then typecasted into the first and second inputs of the add operator. Next, a matrix T is computed, denoted  $\mathbf{T} = \mathbf{A} \oplus \mathbf{B}$ . The pattern of T is the set union of A and B. The result T has the type of the output ztype of the add operator.

The add operator is typically a GrB\_BinaryOp, but the method is type-generic for this parameter. If given a monoid (GrB\_Monoid), the additive operator of the monoid is used as the add binary operator. If given a semiring (GrB\_Semiring), the additive operator of the monoid of the semiring is used as the add binary operator.

The operation can be expressed in MATLAB notation as:

```
[nrows, ncols] = size (A.matrix);
T.matrix = zeros (nrows, ncols, add.ztype);
p = A.pattern & B.pattern;
A = GB_mex_cast (A.matrix (p), add.xtype);
B = GB_mex_cast (B.matrix (p), add.ytype);
T.matrix (p) = add (A, B);
p = A.pattern & ~B.pattern; T.matrix (p) = cast (A.matrix (p), add.ztype);
p = ~A.pattern & B.pattern; T.matrix (p) = cast (B.matrix (p), add.ztype);
T.pattern = A.pattern | B.pattern;
T.class = add.ztype;
```

Except for when typecasting is performed, this is identical to how the accum operator is applied in Figure 1.

The final step is  $C(M) = C \odot T$ , as described in Section 2.3.

# 10.6 GrB extract: submatrix extraction

The GrB\_extract function is a generic name for three specific functions: GrB\_Vector\_extract, GrB\_Col\_extract, and GrB\_Matrix\_extract. The generic name appears in the function signature, but the specific function name is used when describing what each variation does.

# 10.6.1 GrB\_Vector\_extract: extract subvector from vector

```
GrB_Info GrB_extract
                                    // w<mask> = accum (w, u(I))
                                    // input/output vector for results
   GrB_Vector w,
    const GrB_Vector mask,
                                    // optional mask for w, unused if NULL
                                    // optional accum for z=accum(w,t)
    const GrB_BinaryOp accum,
    const GrB_Vector u,
                                    // first input: vector u
    const GrB_Index *I,
                                    // row indices
    const GrB_Index ni,
                                    // number of row indices
    const GrB_Descriptor desc
                                    // descriptor for w and mask
) ;
```

GrB\_Vector\_extract extracts a subvector from another vector, identical to  $\mathbf{t} = \mathbf{u}$  (I) in MATLAB where I is an integer vector of row indices. Refer to GrB\_Matrix\_extract for further details; vector extraction is the same as matrix extraction with n-by-1 matrices. See Section 9 for a description of I and ni. The final step is  $\mathbf{w}\langle\mathbf{m}\rangle = \mathbf{w}\odot\mathbf{t}$ , as described in Section 2.3, except that all the terms are column vectors instead of matrices.

#### 10.6.2 GrB\_Matrix\_extract: extract submatrix from matrix

```
GrB_Info GrB_extract
                                    // C<Mask> = accum (C, A(I,J))
    GrB_Matrix C,
                                    // input/output matrix for results
    const GrB_Matrix Mask,
                                    // optional mask for C, unused if NULL
    const GrB_BinaryOp accum,
                                    // optional accum for Z=accum(C,T)
    const GrB_Matrix A,
                                    // first input: matrix A
    const GrB_Index *I,
                                    // row indices
    const GrB_Index ni,
                                    // number of row indices
                                    // column indices
    const GrB_Index *J,
    const GrB_Index nj,
                                    // number of column indices
    const GrB_Descriptor desc
                                    // descriptor for C, Mask, and A
);
```

GrB\_Matrix\_extract extracts a submatrix from another matrix, identical to T = A(I,J) in MATLAB where I and J are integer vectors of row and column indices, respectively, except that indices are zero-based in Graph-BLAS and one-based in MATLAB. The input matrix A may be transposed first, via the descriptor. The type of T and A are the same. The size of C is |I|-by-|J|. Entries outside A(I,J) are not accessed and do not take part in the computation. More precisely, assuming the matrix A is not transposed, the matrix T is defined as follows:

If duplicate indices are present in I or J, the above method defines the result in T. Duplicates result in the same values of A being copied into different places in T. See Section 9 for a description of the row indices I and ni, and the column indices J and nj. The final step is  $\mathbf{C}\langle\mathbf{M}\rangle=\mathbf{C}\odot\mathbf{T}$ , as described in Section 2.3.

**Performance considerations:** If A is not transposed via input descriptor: if |I| is small, then it is fastest if A is GxB\_BY\_ROW; if |J| is small, then it is fastest if A is GxB\_BY\_COL. The opposite is true if A is transposed.

### 10.6.3 GrB\_Col\_extract: extract column vector from matrix

```
GrB_Info GrB_extract
                                    // w<mask> = accum (w, A(I,j))
    GrB_Vector w,
                                    // input/output matrix for results
   const GrB_Vector mask,
                                    // optional mask for w, unused if NULL
    const GrB_BinaryOp accum,
                                    // optional accum for z=accum(w,t)
    const GrB_Matrix A,
                                    // first input: matrix A
    const GrB_Index *I,
                                    // row indices
    const GrB_Index ni,
                                    // number of row indices
    const GrB_Index j,
                                    // column index
    const GrB_Descriptor desc
                                    // descriptor for w, mask, and A
);
```

GrB\_Col\_extract extracts a subvector from a matrix, identical to t = A (I,j) in MATLAB where I is an integer vector of row indices and where j is a single column index. The input matrix A may be transposed first, via the descriptor, which results in the extraction of a single row j from the matrix A, the result of which is a column vector w. The type of t and A are the same. The size of w is |I|-by-1.

See Section 9 for a description of the row indices I and ni. The final step is  $\mathbf{w}\langle\mathbf{m}\rangle = \mathbf{w}\odot\mathbf{t}$ , as described in Section 2.3, except that all the terms are column vectors instead of matrices.

**Performance considerations:** If A is not transposed: it is fastest if the format of A is GxB\_BY\_COL. The opposite is true if A is transposed.

# 10.7 GxB\_subassign: submatrix assignment

### TODO: add GrB\_scalar

The methods described in this section are all variations of the form C(I,J)=A, which modifies a submatrix of the matrix C. All methods can be used in their generic form with the single name  $GxB\_subassign$ . This is reflected in the prototypes. However, to avoid confusion between the different kinds of assignment, the name of the specific function is used when describing each variation. If the discussion applies to all variations, the simple name  $GxB\_subassign$  is used.

See Section 9 for a description of the row indices I and ni, and the column indices J and nj.

GxB\_subassign is very similar to GrB\_assign, described in Section 10.8. The two operations are compared and contrasted in Section 10.10. For a discussion of how duplicate indices are handled in I and J, see Section 10.9.

# 10.7.1 GxB\_Vector\_subassign: assign to a subvector

```
GrB_Info GxB_subassign
                                     // w(I) < mask > = accum (w(I),u)
                                     // input/output matrix for results
    GrB_Vector w,
    const GrB_Vector mask,
                                     // optional mask for w(I), unused if NULL
    const GrB_BinaryOp accum,
                                     // optional accum for z=accum(w(I),t)
    const GrB_Vector u,
                                     // first input: vector u
    const GrB_Index *I,
                                     // row indices
    const GrB_Index ni,
                                     // number of row indices
    const GrB_Descriptor desc
                                     // descriptor for w(I) and mask
);
```

GxB\_Vector\_subassign operates on a subvector w(I) of w, modifying it with the vector u. The method is identical to GxB\_Matrix\_subassign described in Section 10.7.2, where all matrices have a single column each. The mask has the same size as w(I) and u. The only other difference is that the input u in this method is not transposed via the GrB\_INPO descriptor.

# 10.7.2 GxB\_Matrix\_subassign: assign to a submatrix

```
GrB_Info GxB_subassign
                                    // C(I,J) < Mask > = accum (C(I,J),A)
    GrB_Matrix C,
                                    // input/output matrix for results
    const GrB_Matrix Mask,
                                    // optional mask for C(I,J), unused if NULL
    const GrB_BinaryOp accum,
                                    // optional accum for Z=accum(C(I,J),T)
                                    // first input: matrix A
    const GrB_Matrix A,
    const GrB_Index *I,
                                    // row indices
                                    // number of row indices
    const GrB_Index ni,
    const GrB_Index *J,
                                    // column indices
    const GrB_Index nj,
                                    // number of column indices
    const GrB_Descriptor desc
                                    // descriptor for C(I,J), Mask, and A
);
```

GxB\_Matrix\_subassign operates only on a submatrix S of C, modifying it with the matrix A. For this operation, the result is not the entire matrix C, but a submatrix S=C(I,J) of C. The steps taken are as follows, except that A may be optionally transposed via the GrB\_INPO descriptor option.

Step	GraphBLAS	description
	notation	
1	S = C(I, J)	extract the $C(I, J)$ submatrix
2	$\mathbf{S}\langle\mathbf{M} angle=\mathbf{S}\odot\mathbf{A}$	apply the accumulator/mask to the submatrix ${f S}$
3	$\mathbf{C}(\mathbf{I},\mathbf{J}) = \mathbf{S}$	put the submatrix $S$ back into $C(I, J)$

The accumulator/mask step in Step 2 is the same as for all other Graph-BLAS operations, described in Section 2.3, except that for  $GxB\_subassign$ , it is applied to just the submatrix S = C(I, J), and thus the Mask has the same size as A, S, and C(I, J).

The GxB\_subassign operation is the reverse of matrix extraction:

- For submatrix extraction, GrB\_Matrix\_extract, the submatrix A(I,J) appears on the right-hand side of the assignment, C=A(I,J), and entries outside of the submatrix are not accessed and do not take part in the computation.
- For submatrix assignment, GxB\_Matrix\_subassign, the submatrix C(I, J) appears on the left-hand-side of the assignment, C(I, J)=A, and entries outside of the submatrix are not accessed and do not take part in the computation.

In both methods, the accumulator and mask modify the submatrix of the assignment; they simply differ on which side of the assignment the submatrix resides on. In both cases, if the Mask matrix is present it is the same size as the submatrix:

- For submatrix extraction,  $\mathbf{C}\langle\mathbf{M}\rangle=\mathbf{C}\odot\mathbf{A}(\mathbf{I},\mathbf{J})$  is computed, where the submatrix is on the right. The mask  $\mathbf{M}$  has the same size as the submatrix  $\mathbf{A}(\mathbf{I},\mathbf{J})$ .
- For submatrix assignment,  $\mathbf{C}(\mathbf{I}, \mathbf{J})\langle \mathbf{M} \rangle = \mathbf{C}(\mathbf{I}, \mathbf{J}) \odot \mathbf{A}$  is computed, where the submatrix is on the left. The mask  $\mathbf{M}$  has the same size as the submatrix  $\mathbf{C}(\mathbf{I}, \mathbf{J})$ .

In Step 1, the submatrix S is first computed by the GrB\_Matrix\_extract operation, S=C(I,J).

Step 2 accumulates the results  $S\langle M \rangle = S \odot T$ , exactly as described in Section 2.3, but operating on the submatrix S, not C, using the optional Mask and accum operator. The matrix T is simply T = A, or  $T = A^T$  if A is transposed via the desc descriptor,  $GrB_INPO$ . The  $GrB_REPLACE$  option in the descriptor clears S after computing C = T or  $C = C \odot T$ , not all of C since this operation can only modify the specified submatrix of C.

Finally, Step 3 writes the result (which is the modified submatrix S and not all of C) back into the C matrix that contains it, via the assignment C(I,J)=S, using the reverse operation from the method described for matrix extraction:

**Performance considerations:** If A is not transposed: if |I| is small, then it is fastest if the format of C is GxB\_BY\_ROW; if |J| is small, then it is fastest if the format of C is GxB\_BY\_COL. The opposite is true if A is transposed.

# 10.7.3 GxB\_Col\_subassign: assign to a sub-column of a matrix

```
GrB_Info GxB_subassign
                                    // C(I,j) < mask > = accum (C(I,j),u)
    GrB_Matrix C,
                                    // input/output matrix for results
                                    // optional mask for C(I,j), unused if NULL
    const GrB_Vector mask,
    const GrB_BinaryOp accum,
                                    // optional accum for z=accum(C(I,j),t)
   const GrB_Vector u,
                                    // input vector
    const GrB_Index *I,
                                    // row indices
                                    // number of row indices
    const GrB_Index ni,
    const GrB_Index j,
                                    // column index
    const GrB_Descriptor desc
                                    // descriptor for C(I,j) and mask
);
```

GxB\_Col\_subassign modifies a single sub-column of a matrix C. It is the same as GxB\_Matrix\_subassign where the index vector J[0]=j is a single column index (and thus nj=1), and where all matrices in GxB\_Matrix\_subassign (except C) consist of a single column. The mask vector has the same size as u and the sub-column C(I,j). The input descriptor GrB\_INPO is ignored; the input vector u is not transposed. Refer to GxB\_Matrix\_subassign for further details.

Performance considerations: GxB\_Col\_subassign is much faster than GxB\_Row\_subassign if the format of C is GxB\_BY\_COL. GxB\_Row\_subassign is much faster than GxB\_Col\_subassign if the format of C is GxB\_BY\_ROW.

### 10.7.4 GxB\_Row\_subassign: assign to a sub-row of a matrix

```
GrB_Info GxB_subassign
                                       // C(i,J) < mask' > = accum (C(i,J),u')
                                    // input/output matrix for results
    GrB_Matrix C,
    const GrB_Vector mask,
                                    // optional mask for C(i,J), unused if NULL
    const GrB_BinaryOp accum,
                                    // optional accum for z=accum(C(i,J),t)
    const GrB_Vector u,
                                    // input vector
    const GrB_Index i,
                                    // row index
    const GrB_Index *J,
                                    // column indices
                                    // number of column indices
    const GrB_Index nj,
    const GrB_Descriptor desc
                                    // descriptor for C(i,J) and mask
);
```

GxB\_Row\_subassign modifies a single sub-row of a matrix C. It is the same as GxB\_Matrix\_subassign where the index vector I[0]=i is a single

row index (and thus ni=1), and where all matrices in GxB\_Matrix\_subassign (except C) consist of a single row. The mask vector has the same size as u and the sub-column C(I,j). The input descriptor GrB\_INPO is ignored; the input vector u is not transposed. Refer to GxB\_Matrix\_subassign for further details.

Performance considerations: GxB\_Col\_subassign is much faster than GxB\_Row\_subassign if the format of C is GxB\_BY\_COL. GxB\_Row\_subassign is much faster than GxB\_Col\_subassign if the format of C is GxB\_BY\_ROW.

# 10.7.5 GxB\_Vector\_subassign\_<type>: assign a scalar to a subvector

```
GrB_Info GxB_subassign
                                       // w(I) < mask > = accum (w(I),x)
   GrB_Vector w,
                                    // input/output vector for results
                                    // optional mask for w(I), unused if NULL
    const GrB_Vector mask,
    const GrB_BinaryOp accum,
                                    // optional accum for z=accum(w(I),x)
    const <type> x,
                                    // scalar to assign to w(I)
    const GrB_Index *I,
                                    // row indices
    const GrB_Index ni,
                                    // number of row indices
    const GrB_Descriptor desc
                                    // descriptor for w(I) and mask
);
```

GxB\_Vector\_subassign\_<type> assigns a single scalar to an entire subvector of the vector w. The operation is exactly like setting a single entry in an n-by-1 matrix, A(I,0) = x, where the column index for a vector is implicitly j=0. For further details of this function, see GxB\_Matrix\_subassign\_<type> in Section 10.7.6.

# 10.7.6 GxB\_Matrix\_subassign\_<type>: assign a scalar to a submatrix

```
GrB_Info GxB_subassign
                                        // C(I,J) < Mask > = accum (C(I,J),x)
    GrB_Matrix C,
                                     // input/output matrix for results
    const GrB_Matrix Mask,
                                     // optional mask for C(I,J), unused if NULL
    const GrB_BinaryOp accum,
                                     // optional accum for Z=accum(C(I,J),x)
                                     // scalar to assign to C(I,J)
    const <type> x,
    const GrB_Index *I,
                                     // row indices
    const GrB_Index ni,
                                     // number of row indices
    const GrB_Index *J,
                                     // column indices
    const GrB_Index nj,
                                     // number of column indices
    const GrB_Descriptor desc
                                     // descriptor for C(I,J) and Mask
);
```

GxB\_Matrix\_subassign\_<type> assigns a single scalar to an entire submatrix of C, like the scalar expansion C(I,J)=x in MATLAB. The scalar x is implicitly expanded into a matrix A of size ni by nj, and then the matrix A is assigned to C(I,J) using the same method as in GxB\_Matrix\_subassign. Refer to that function in Section 10.7.2 for further details. For the accumulation step, the scalar x is typecasted directly into the type of C when the accum operator is not applied to it, or into the ytype of the accum operator, if accum is not NULL, for entries that are already present in C.

The <type> x notation is otherwise the same as GrB\_Matrix\_setElement (see Section 6.9.11). Any value can be passed to this function and its type will be detected, via the \_Generic feature of ANSI C11. For a user-defined type, x is a void \* pointer that points to a memory space holding a single entry of a scalar that has exactly the same user-defined type as the matrix C. This user-defined type must exactly match the user-defined type of C since no typecasting is done between user-defined types.

If a void \* pointer is passed in and the type of the underlying scalar does not exactly match the user-defined type of C, then results are undefined. No error status will be returned since GraphBLAS has no way of catching this error.

**Performance considerations:** If A is not transposed: if |I| is small, then it is fastest if the format of C is GxB\_BY\_ROW; if |J| is small, then it is fastest if the format of C is GxB\_BY\_COL. The opposite is true if A is transposed.

# 10.8 GrB\_assign: submatrix assignment

### TODO: add GrB\_scalar

The methods described in this section are all variations of the form C(I,J)=A, which modifies a submatrix of the matrix C. All methods can be used in their generic form with the single name  $GrB_assign$ . These methods are very similar to their  $GxB_subassign$  counterparts in Section 10.7. They differ primarily in the size of the Mask, and how the  $GrB_REPLACE$  option works. Section 10.10 compares  $GxB_subassign$  and  $GrB_assign$ .

See Section 9 for a description of I, ni, J, and nj.

# 10.8.1 GrB\_Vector\_assign: assign to a subvector

```
GrB_Info GrB_assign
                                    // w<mask>(I) = accum (w(I),u)
    GrB_Vector w,
                                    // input/output matrix for results
    const GrB_Vector mask,
                                    // optional mask for w, unused if NULL
    const GrB_BinaryOp accum,
                                    // optional accum for z=accum(w(I),t)
    const GrB_Vector u,
                                    // first input: vector u
    const GrB_Index *I,
                                    // row indices
    const GrB_Index ni,
                                    // number of row indices
    const GrB_Descriptor desc
                                    // descriptor for w and mask
);
```

GrB\_Vector\_assign operates on a subvector w(I) of w, modifying it with the vector u. The mask vector has the same size as w. The method is identical to GrB\_Matrix\_assign described in Section 10.8.2, where all matrices have a single column each. The only other difference is that the input u in this method is not transposed via the GrB\_INPO descriptor.

# 10.8.2 GrB\_Matrix\_assign: assign to a submatrix

```
GrB_Info GrB_assign
                                    // C<Mask>(I,J) = accum (C(I,J),A)
    GrB_Matrix C,
                                    // input/output matrix for results
                                    // optional mask for C, unused if NULL
    const GrB_Matrix Mask,
    const GrB_BinaryOp accum,
                                    // optional accum for Z=accum(C(I,J),T)
                                    // first input: matrix A
    const GrB_Matrix A,
    const GrB_Index *I,
                                    // row indices
                                    // number of row indices
    const GrB_Index ni,
    const GrB_Index *J,
                                    // column indices
    const GrB_Index nj,
                                    // number of column indices
    const GrB_Descriptor desc
                                    // descriptor for C, Mask, and A
);
```

GrB\_Matrix\_assign operates on a submatrix S of C, modifying it with the matrix A. It may also modify all of C, depending on the input descriptor desc and the Mask.

Step	GraphBLAS	description
	notation	
1	$\mathbf{S} = \mathbf{C}(\mathbf{I}, \mathbf{J})$	extract $\mathbf{C}(\mathbf{I}, \mathbf{J})$ submatrix
2	$\mathbf{S} = \mathbf{S} \odot \mathbf{A}$	apply the accumulator (but not the mask) to ${\bf S}$
3	$\mathbf{Z} = \mathbf{C}$	make a copy of C
4	$\mathbf{Z}(\mathbf{I}, \mathbf{J}) = \mathbf{S}$	put the submatrix into $\mathbf{Z}(\mathbf{I}, \mathbf{J})$
5	$\mathbf{C}\langle\mathbf{M} angle=\mathbf{Z}$	apply the mask/replace phase to all of ${\bf C}$

In contrast to GxB\_subassign, the Mask has the same as C.

Step 1 extracts the submatrix and then Step 2 applies the accumulator (or S = A if accum is NULL). The Mask is not yet applied.

Step 3 makes a copy of the C matrix, and then Step 4 writes the submatrix S into Z. This is the same as Step 3 of GxB\_subassign, except that it operates on a temporary matrix Z.

Finally, Step 5 writes  $\mathbf{Z}$  back into  $\mathbf{C}$  via the Mask, using the Mask/Replace Phase described in Section 2.3. If  $\mathtt{GrB\_REPLACE}$  is enabled, then all of  $\mathbf{C}$  is cleared prior to writing  $\mathbf{Z}$  via the mask. As a result, the  $\mathtt{GrB\_REPLACE}$  option can delete entries outside the  $\mathbf{C}(\mathbf{I},\mathbf{J})$  submatrix.

**Performance considerations:** If A is not transposed: if |I| is small, then it is fastest if the format of C is GxB\_BY\_ROW; if |J| is small, then it is fastest if the format of C is GxB\_BY\_COL. The opposite is true if A is transposed.

# 10.8.3 GrB\_Col\_assign: assign to a sub-column of a matrix

```
// C<mask>(I,j) = accum (C(I,j),u)
GrB_Info GrB_assign
   GrB_Matrix C,
                                    // input/output matrix for results
                                    // optional mask for C(:,j), unused if NULL
   const GrB_Vector mask,
    const GrB_BinaryOp accum,
                                    // optional accum for z=accum(C(I,j),t)
    const GrB_Vector u,
                                    // input vector
    const GrB_Index *I,
                                    // row indices
                                    // number of row indices
    const GrB_Index ni,
    const GrB_Index j,
                                    // column index
    const GrB_Descriptor desc
                                    // descriptor for C(:,j) and mask
);
```

GrB\_Col\_assign modifies a single sub-column of a matrix C. It is the same as GrB\_Matrix\_assign where the index vector J[0]=j is a single column index, and where all matrices in GrB\_Matrix\_assign (except C) consist of a single column.

Unlike GrB\_Matrix\_assign, the mask is a vector with the same size as a single column of C.

The input descriptor GrB\_INPO is ignored; the input vector **u** is not transposed. Refer to GrB\_Matrix\_assign for further details.

Performance considerations: GrB\_Col\_assign is much faster than GrB\_Row\_assign if the format of C is GxB\_BY\_COL. GrB\_Row\_assign is much faster than GrB\_Col\_assign if the format of C is GxB\_BY\_ROW.

# 10.8.4 GrB\_Row\_assign: assign to a sub-row of a matrix

```
GrB_Info GrB_assign
                                    // C < mask' > (i,J) = accum (C(i,J),u')
   GrB_Matrix C,
                                    // input/output matrix for results
                                    // optional mask for C(i,:), unused if NULL
   const GrB_Vector mask,
    const GrB_BinaryOp accum,
                                    // optional accum for z=accum(C(i,J),t)
    const GrB_Vector u,
                                    // input vector
    const GrB_Index i,
                                    // row index
                                    // column indices
    const GrB_Index *J,
    const GrB_Index nj,
                                    // number of column indices
    const GrB_Descriptor desc
                                    // descriptor for C(i,:) and mask
);
```

GrB\_Row\_assign modifies a single sub-row of a matrix C. It is the same as GrB\_Matrix\_assign where the index vector I[0]=i is a single row index, and where all matrices in GrB\_Matrix\_assign (except C) consist of a single row.

Unlike GrB\_Matrix\_assign, the mask is a vector with the same size as a single row of C.

The input descriptor GrB\_INPO is ignored; the input vector **u** is not transposed. Refer to GrB\_Matrix\_assign for further details.

Performance considerations: GrB\_Col\_assign is much faster than GrB\_Row\_assign if the format of C is GxB\_BY\_COL. GrB\_Row\_assign is much faster than GrB\_Col\_assign if the format of C is GxB\_BY\_ROW.

# 10.8.5 GrB\_Vector\_assign\_<type>: assign a scalar to a subvector

```
GrB_Info GrB_assign
                                    // w<mask>(I) = accum (w(I),x)
    GrB_Vector w,
                                    // input/output vector for results
    const GrB_Vector mask,
                                    // optional mask for w, unused if NULL
    const GrB_BinaryOp accum,
                                    // optional accum for z=accum(w(I),x)
    const <type> x,
                                    // scalar to assign to w(I)
    const GrB_Index *I,
                                    // row indices
                                    // number of row indices
    const GrB_Index ni,
    const GrB_Descriptor desc
                                    // descriptor for w and mask
);
```

GrB\_Vector\_assign\_<type> assigns a single scalar to an entire subvector of the vector w. The operation is exactly like setting a single entry in an n-by-1 matrix, A(I,0) = x, where the column index for a vector is implicitly j=0. The mask vector has the same size as w. For further details of this function, see GrB\_Matrix\_assign\_<type> in the next section.

Following the C API Specification, results are well-defined if I contains duplicate indices. Duplicate indices are simply ignored. See Section 10.9 for more details.

# 10.8.6 GrB\_Matrix\_assign\_<type>: assign a scalar to a submatrix

```
// C<Mask>(I,J) = accum (C(I,J),x)
GrB_Info GrB_assign
    GrB_Matrix C,
                                    // input/output matrix for results
    const GrB_Matrix Mask,
                                    // optional mask for C, unused if NULL
    const GrB_BinaryOp accum,
                                    // optional accum for Z=accum(C(I,J),x)
    const <type> x,
                                    // scalar to assign to C(I,J)
                                    // row indices
    const GrB_Index *I,
    const GrB_Index ni,
                                    // number of row indices
    const GrB_Index *J,
                                    // column indices
    const GrB_Index nj,
                                    // number of column indices
    const GrB_Descriptor desc
                                    // descriptor for C and Mask
);
```

GrB\_Matrix\_assign\_<type> assigns a single scalar to an entire submatrix of C, like the scalar expansion C(I,J)=x in MATLAB. The scalar x is implicitly expanded into a matrix A of size ni by nj, and then the matrix A is assigned to C(I,J) using the same method as in GrB\_Matrix\_assign. Refer to that function in Section 10.8.2 for further details.

The Mask has the same size as C.

For the accumulation step, the scalar  $\mathbf{x}$  is typecasted directly into the type of C when the accum operator is not applied to it, or into the ytype of the accum operator, if accum is not NULL, for entries that are already present in C.

The <type> x notation is otherwise the same as GrB\_Matrix\_setElement (see Section 6.9.11). Any value can be passed to this function and its type will be detected, via the \_Generic feature of ANSI C11. For a user-defined type, x is a void \* pointer that points to a memory space holding a single entry of a scalar that has exactly the same user-defined type as the matrix C. This user-defined type must exactly match the user-defined type of C since no typecasting is done between user-defined types.

If a void \* pointer is passed in and the type of the underlying scalar does not exactly match the user-defined type of C, then results are undefined. No error status will be returned since GraphBLAS has no way of catching this error.

Following the C API Specification, results are well-defined if I or J contain duplicate indices. Duplicate indices are simply ignored. See Section 10.9 for more details.

**Performance considerations:** If A is not transposed: if |I| is small, then it is fastest if the format of C is GxB\_BY\_ROW; if |J| is small, then it is fastest if the format of C is GxB\_BY\_COL. The opposite is true if A is transposed.

# 10.9 Duplicate indices in GrB\_assign and GxB\_subassign

According to the GraphBLAS C API Specification if the index vectors I or J contain duplicate indices, the results are undefined for GrB\_Matrix\_assign, GrB\_Matrix\_assign, GrB\_Col\_assign, and GrB\_Row\_assign. Only the scalar assignment operations (GrB\_Matrix\_assign\_TYPE and GrB\_Matrix\_assign\_TYPE) are well-defined when duplicates appear in I and J. In those two functions, duplicate indices are ignored.

As an extension to the specification, SuiteSparse:GraphBLAS provides a definition of how duplicate indices are handled in all cases. If I has duplicate indices, they are ignored and the last unique entry in the list is used. When no mask and no accumulator is present, the results are identical to how MAT-LAB handles duplicate indices in the built-in expression C(I,J)=A. Details of how this is done is shown below.

```
function C = subassign (C, I, J, A)
% submatrix assignment with pre-sort of I and J; and remove duplicates
% delete duplicates from I, keeping the last one seen
[I2 I2k] = sort(I);
Idupl = [(I2 (1:end-1) == I2 (2:end)), false];
I2 = I2 ( (Idupl) ;
I2k = I2k (~Idupl);
assert (isequal (I2, unique (I)))
% delete duplicates from J, keeping the last one seen
[J2 \ J2k] = sort(J);
Jdupl = [(J2 (1:end-1) == J2 (2:end)), false];
J2 = J2 (~Jdupl);
J2k = J2k (~Jdupl);
assert (isequal (J2, unique (J)))
% do the submatrix assignment, with no duplicates in I2 or J2
C (I2,J2) = A (I2k,J2k);
```

If a mask is present, then it is replaced with M = M (I2k, J2k) for  $GxB\_subassign$ , or with M = M (I2, J2) for  $GrB\_assign$ . If an accumulator operator is present, it is applied after the duplicates are removed, as (for example):

```
C(I2,J2) = C(I2,J2) + A(I2k,J2k);
```

These definitions allow the Octave/MATLAB interface to GraphBLAS to return the same results for C(I, J)=A for a GrB object as they do for built-in Octave/MATLAB matrices. They also allow the assignment to be done in parallel.

Results are always well-defined in SuiteSparse:GraphBLAS, but they might not be what you expect. For example, suppose the MIN operator is being used the following assignment to the vector  $\mathbf{x}$ , and suppose I contains the entries [0 0]. Suppose  $\mathbf{x}$  is initially empty, of length 1, and suppose  $\mathbf{y}$  is a vector of length 2 with the values [5 7].

```
#include "GraphBLAS.h"
#include <stdio.h>
int main (void)
{
   GrB_init (GrB_NONBLOCKING) ;
   GrB_Vector x, y ;
   GrB_Vector_new (&x, GrB_INT32, 1) ;
   GrB_Vector_new (&y, GrB_INT32, 2);
   GrB_Index I [2] = \{0, 0\};
   GrB_Vector_setElement (y, 5, 0);
   GrB_Vector_setElement (y, 7, 1) ;
   GrB_Vector_wait (&y) ;
    GxB_print(x, 3);
    GxB_print (y, 3);
    GrB_assign (x, NULL, GrB_MIN_INT32, y, I, 2, NULL) ;
    GrB_Vector_wait (&y) ;
   GxB_print (x, 3);
    GrB_finalize ( );
}
```

You might (wrongly) expect the result to be the vector  $\mathbf{x}(0)=5$ , since two entries seem to be assigned, and the min operator might be expected to take the minimum of the two. This is not how SuiteSparse:GraphBLAS handles duplicates.

Instead, the first duplicate index of I is discarded (I [0] = 0, and y(0)=5). and only the second entry is used (I [1] = 0, and y(1)=7). The output of the above program is:

```
1x1 GraphBLAS int32_t vector, sparse by col:
```

```
x, no entries

2x1 GraphBLAS int32_t vector, sparse by col:
y, 2 entries

(0,0) 5
(1,0) 7

1x1 GraphBLAS int32_t vector, sparse by col:
x, 1 entry

(0,0) 7
```

You see that the result is x(0)=7, since the y(0)=5 entry has been ignored because of the duplicate indices in I.

**SPEC:** Providing a well-defined behavior for duplicate indices with matrix and vector assignment is an extension to the spec. The spec only defines the behavior when assigning a scalar into a matrix or vector, and states that duplicate indices otherwise lead to undefined results.

# 10.10 Comparing GrB\_assign and GxB\_subassign

The GxB\_subassign and GrB\_assign operations are very similar, but they differ in two ways:

- 1. The Mask has a different size: The mask in GxB\_subassign has the same dimensions as w(I) for vectors and C(I,J) for matrices. In GrB\_assign, the mask is the same size as w or C, respectively (except for the row/col variants). The two masks are related. If M is the mask for GrB\_assign, then M(I,J) is the mask for GxB\_subassign. If there is no mask, or if I and J are both GrB\_ALL, the two masks are the same. For GrB\_Row\_assign and GrB\_Col\_assign, the mask vector is the same size as a row or column of C, respectively. For the corresponding GxB\_Row\_subassign and GxB\_Col\_subassign operations, the mask is the same size as the sub-row C(i,J) or subcolumn C(I,j), respectively.
- 2. GrB\_REPLACE is different: They differ in how C is affected in areas outside the C(I,J) submatrix. In GxB\_subassign, the C(I,J) submatrix is the only part of C that can be modified, and no part of C outside the submatrix is ever modified. In GrB\_assign, it is possible to delete entries in C outside the submatrix, but only in one specific manner. Suppose the mask M is present (or, suppose it is not present but GrB\_COMP is true). After (optionally) complementing the mask, the value of M(i,j) can be 0 for some entry outside the C(I,J) submatrix. If the GrB\_REPLACE descriptor is true, GrB\_assign deletes this entry.

GxB\_subassign and GrB\_assign are identical if GrB\_REPLACE is set to its default value of false, and if the masks happen to be the same. The two masks can be the same in two cases: either the Mask input is NULL (and it is not complemented via GrB\_COMP), or I and J are both GrB\_ALL. If all these conditions hold, the two algorithms are identical and have the same performance. Otherwise, GxB\_subassign is much faster than GrB\_assign when the latter must examine the entire matrix C to delete entries (when GrB\_REPLACE is true), and if it must deal with a much larger Mask matrix. However, both methods have specific uses.

Consider using C(I,J)+=F for many submatrices F (for example, when assembling a finite-element matrix). If the Mask is meant as a specification for which entries of C should appear in the final result, then use GrB\_assign.

If instead the Mask is meant to control which entries of the submatrix C(I,J) are modified by the finite-element F, then use GxB\_subassign. This is particularly useful is the Mask is a template that follows along with the finite-element F, independent of where it is applied to C. Using GrB\_assign would be very difficult in this case since a new Mask, the same size as C, would need to be constructed for each finite-element F.

In GraphBLAS notation, the two methods can be described as follows:

matrix and vector subassign	$\mathbf{C}(\mathbf{I},\mathbf{J})\langle\mathbf{M} angle=\mathbf{C}(\mathbf{I},\mathbf{J})\odot\mathbf{A}$
matrix and vector assign	$\mathbf{C}\langle\mathbf{M} angle(\mathbf{I},\mathbf{J})=\mathbf{C}(\mathbf{I},\mathbf{J})\odot\mathbf{A}$

This notation does not include the details of the GrB\_COMP and GrB\_REPLACE descriptors, but it does illustrate the difference in the Mask. In the subassign, Mask is the same size as C(I,J) and A. If I[0]=i and J[0]=j, Then Mask(0,0) controls how C(i,j) is modified by the subassign, from the value A(0,0). In the assign, Mask is the same size as C, and Mask(i,j) controls how C(i,j) is modified.

The GxB\_subassign and GrB\_assign functions have the same signatures; they differ only in how they consider the Mask and the GrB\_REPLACE descriptor

Details of each step of the two operations are listed below:

Step	<pre>GrB_Matrix_assign</pre>	GxB_Matrix_subassign
1	$\mathbf{S} = \mathbf{C}(\mathbf{I}, \mathbf{J})$	$\mathbf{S} = \mathbf{C}(\mathbf{I}, \mathbf{J})$
2	$\mathbf{S} = \mathbf{S} \odot \mathbf{A}$	$\mathbf{S}\langle \mathbf{M} \rangle = \mathbf{S} \odot \mathbf{A}$
3	$\mathbf{Z} = \mathbf{C}$	$\mathbf{C}(\mathbf{I},\mathbf{J})=\mathbf{S}$
4	$\mathbf{Z}(\mathbf{I},\mathbf{J}) = \mathbf{S}$	
5	$\mathbf{C}\langle\mathbf{M} angle=\mathbf{Z}$	

Step 1 is the same. In the Accumulator Phase (Step 2), the expression  $S \odot A$ , described in Section 2.3, is the same in both operations. The result is simply **A** if accum is NULL. It only applies to the submatrix **S**, not the whole matrix. The result  $S \odot A$  is used differently in the Mask/Replace phase.

The Mask/Replace Phase, described in Section 2.3 is different:

• For Grb\_assign (Step 5), the mask is applied to all of C. The mask has the same size as C. Just prior to making the assignment via the mask, the Grb\_REPLACE option can be used to clear all of C first. This is the only way in which entries in C that are outside the C(I, J) submatrix can be modified by this operation.

• For GxB\_subassign (Step 2b), the mask is applied to just S. The mask has the same size as C(I, J), S, and A. Just prior to making the assignment via the mask, the GrB\_REPLACE option can be used to clear S first. No entries in C that are outside the C(I, J) can be modified by this operation. Thus, GrB\_REPLACE has no effect on entries in C outside the C(I, J) submatrix.

The differences between  $GrB_assign$  and  $GxB_subassign$  can be seen in Tables 2 and 3. The first table considers the case when the entry  $c_{ij}$  is in the C(I, J) submatrix, and it describes what is computed for both  $GrB_assign$  and  $GxB_subassign$ . They perform the exact same computation; the only difference is how the value of the mask is specified. Compare Table 2 with Table 1 in Section 7.

The first column of Table 2 is yes if  $GrB_REPLACE$  is enabled, and a dash otherwise. The second column is yes if an accumulator operator is given, and a dash otherwise. The third column is  $c_{ij}$  if the entry is present in  $\mathbf{C}$ , and a dash otherwise. The fourth column is  $a_{i'j'}$  if the corresponding entry is present in  $\mathbf{A}$ , where  $i = \mathbf{I}(i')$  and  $j = \mathbf{J}(i')$ .

The mask column is 1 if the effective value of the mask mask allows C to be modified, and 0 otherwise. This is  $m_{ij}$  for  $\mathtt{GrB\_assign}$ , and  $m_{i'j'}$  for  $\mathtt{GrB\_subassign}$ , to reflect the difference in the mask, but this difference is not reflected in the table. The value 1 or 0 is the value of the entry in the mask after it is optionally complemented via the  $\mathtt{GrB\_COMP}$  option.

Finally, the last column is the action taken in this case. It is left blank if no action is taken, in which case  $c_{ij}$  is not modified if present, or not inserted into  $\mathbf{C}$  if not present.

repl	accum	$\mathbf{C}$	$\mathbf{A}$	$\max k$	action taken by GrB_assign and GxB_subassign
-	-	$c_{ij}$	$a_{i'j'}$	1	$c_{ij} = a_{i'j'}$ , update
-	-	-	$a_{i'j'}$	1	$c_{ij} = a_{i'j'}$ , insert
-	-	$c_{ij}$	-	1	delete $c_{ij}$ because $a_{i'j'}$ not present
-	-	-	-	1	
-	-	$c_{ij}$	$a_{i'j'}$	0	
-	-	-	$a_{i'j'}$	0	
-	-	$c_{ij}$	-	0	
_	-	-	-	0	
yes	-	$c_{ij}$	$a_{i'j'}$	1	$c_{ij} = a_{i'j'}$ , update
yes	-	-	$a_{i'j'}$	1	$c_{ij} = a_{i'j'}$ , insert
yes	-	$c_{ij}$	-	1	delete $c_{ij}$ because $a_{i'j'}$ not present
yes	-	-	-	1	
yes	-	$c_{ij}$	$a_{i'j'}$	0	delete $c_{ij}$ (because of GrB_REPLACE)
yes	-	-	$a_{i'j'}$	0	
yes	-	$c_{ij}$	-	0	delete $c_{ij}$ (because of $GrB_REPLACE$ )
yes	-	-	-	0	
-	yes	$c_{ij}$	$a_{i'j'}$	1	$c_{ij} = c_{ij} \odot a_{i'j'}$ , apply accumulator
-	yes	-	$a_{i'j'}$	1	$c_{ij} = a_{i'j'}$ , insert
-	yes	$c_{ij}$	-	1	
-	yes	-	-	1	
-	yes	$c_{ij}$	$a_{i'j'}$	0	
-	yes	-	$a_{i'j'}$	0	
-	yes	$c_{ij}$	-	0	
_	yes	-	-	0	
yes	yes	$c_{ij}$	$a_{i'j'}$	1	$c_{ij} = c_{ij} \odot a_{i'j'}$ , apply accumulator
yes	yes	-	$a_{i'j'}$	1	$c_{ij} = a_{i'j'}$ , insert
yes	yes	$c_{ij}$	-	1	
yes	yes	-	-	1	
yes	yes	$c_{ij}$	$a_{i'j'}$	0	delete $c_{ij}$ (because of $GrB_REPLACE$ )
yes	yes	-	$a_{i'j'}$	0	
yes	yes	$c_{ij}$	-	0	delete $c_{ij}$ (because of GrB_REPLACE)
yes	yes	-	-	0	

Table 2: Results of assign and subassign for entries in the  $\mathbf{C}(\mathbf{I},\mathbf{J})$  submatrix

repl	accum	$\mathbf{C}$	$\mathbf{C} = \mathbf{Z}$	mask	action taken by GrB_assign
-	-	$c_{ij}$	$c_{ij}$	1	
-	-	-	-	1	
-	-	$c_{ij}$	$c_{ij}$	0	
-	-	-	-	0	
yes	-	$c_{ij}$	$c_{ij}$	1	
yes	-	-	-	1	
yes	-	$c_{ij}$	$c_{ij}$	0	delete $c_{ij}$ (because of $GrB_REPLACE$ )
yes	-	-	-	0	
-	yes	$c_{ij}$	$c_{ij}$	1	
-	yes	-	-	1	
-	yes	$c_{ij}$	$c_{ij}$	0	
-	yes	-	-	0	
yes	yes	$c_{ij}$	$c_{ij}$	1	
yes	yes	-	-	1	
yes	yes	$c_{ij}$	$c_{ij}$	0	delete $c_{ij}$ (because of $GrB_REPLACE$ )
yes	yes	-	-	0	

Table 3: Results of assign for entries outside the C(I, J) submatrix. Subassign has no effect on these entries.

Table 3 illustrates how  $GrB_assign$  and  $GxB_subassign$  differ for entries outside the submatrix.  $GxB_subassign$  never modifies any entry outside the C(I, J) submatrix, but  $GrB_assign$  can modify them in two cases listed in Table 3. When the  $GrB_REPLACE$  option is selected, and when the Mask(i,j) for an entry  $c_{ij}$  is false (or if the Mask(i,j) is true and  $GrB_COMP$  is enabled via the descriptor), then the entry is deleted by  $GrB_assign$ .

The fourth column of Table 3 differs from Table 2, since entries in **A** never affect these entries. Instead, for all index pairs outside the  $I \times J$  submatrix, **C** and **Z** are identical (see Step 3 above). As a result, each section of the table includes just two cases: either  $c_{ij}$  is present, or not. This in contrast to Table 2, where each section must consider four different cases.

The GrB\_Row\_assign and GrB\_Col\_assign operations are slightly different. They only affect a single row or column of C. For GrB\_Row\_assign, Table 3 only applies to entries in the single row C(i, J) that are outside the list of indices, J. For GrB\_Col\_assign, Table 3 only applies to entries in the single column C(I,j) that are outside the list of indices, I.

#### 10.10.1 Example

The difference between GxB\_subassign and GrB\_assign is illustrated in the following example. Consider the 2-by-2 matrix C where all entries are present.

$$\mathbf{C} = \left[ \begin{array}{cc} 11 & 12 \\ 21 & 22 \end{array} \right]$$

Suppose GrB\_REPLACE is true, and GrB\_COMP is false. Let the Mask be:

$$\mathbf{M} = \left[ \begin{array}{cc} 1 & 1 \\ 0 & 1 \end{array} \right].$$

Let  $\mathbf{A} = 100$ , and let the index sets be  $\mathbf{I} = 0$  and  $\mathbf{J} = 1$ . Consider the computation  $\mathbf{C}\langle\mathbf{M}\rangle(0,1) = \mathbf{C}(0,1) + \mathbf{A}$ , using the GrB\_assign operation. The result is:

$$\mathbf{C} = \left[ \begin{array}{cc} 11 & 112 \\ - & 22 \end{array} \right].$$

The (0,1) entry is updated and the (1,0) entry is deleted because its Mask is zero. The other two entries are not modified since  $\mathbf{Z} = \mathbf{C}$  outside the submatrix, and those two values are written back into  $\mathbf{C}$  because their Mask values are 1. The (1,0) entry is deleted because the entry  $\mathbf{Z}(1,0) = 21$  is prevented from being written back into  $\mathbf{C}$  since Mask(1,0)=0.

Now consider the analogous GxB\_subassign operation. The Mask has the same size as A, namely:

$$\mathbf{M} = [1].$$

After computing  $C(0,1)\langle \mathbf{M} \rangle = C(0,1) + \mathbf{A}$ , the result is

$$\mathbf{C} = \left[ \begin{array}{cc} 11 & 112 \\ 21 & 22 \end{array} \right].$$

Only the  $\mathbf{C}(\mathbf{I}, \mathbf{J})$  submatrix, the single entry  $\mathbf{C}(0, 1)$ , is modified by  $\mathtt{GxB\_subassign}$ . The entry  $\mathbf{C}(1, 0) = 21$  is unaffected by  $\mathtt{GxB\_subassign}$ , but it is deleted by  $\mathtt{GrB\_assign}$ .

#### 10.10.2 Performance of GxB\_subassign, GrB\_assign and GrB\_\*\_setElement

When SuiteSparse:GraphBLAS uses non-blocking mode, the modifications to a matrix by GxB\_subassign, GrB\_assign, and GrB\_\*\_setElement can postponed, and computed all at once later on. This has a huge impact on performance.

A sequence of assignments is fast if their completion can be postponed for as long as possible, or if they do not modify the pattern at all. Modifying the pattern can be costly, but it is fast if non-blocking mode can be fully exploited.

Consider a sequence of t submatrix assignments C(I,J)=C(I,J)+A to an n-by-n matrix C where each submatrix A has size a-by-a with s entries, and where C starts with s entries. Assume the matrices are all stored in non-hypersparse form, by row  $(GxB\_BY\_ROW)$ .

If blocking mode is enabled, or if the sequence requires the matrix to be completed after each assignment, each of the t assignments takes  $O(a + s \log n)$  time to process the A matrix and then  $O(n + c + s \log s)$  time to complete C. The latter step uses  $GrB_*build$  to build an update matrix and then merge it with C. This step does not occur if the sequence of assignments does not add new entries to the pattern of C, however. Assuming in the worst case that the pattern does change, the total time is  $O(t [a + s \log n + n + c + s \log s])$ .

If the sequence can be computed with all updates postponed until the end of the sequence, then the total time is no worse than  $O(a+s\log n)$  to process each A matrix, for t assignments, and then a single build at the end, taking  $O(n+c+st\log st)$  time. The total time is  $O(t\left[a+s\log n\right]+(n+c+st\log st))$ . If no new entries appear in C the time drops to  $O(t\left[a+s\log n\right])$ , and in this case, the time for both methods is the same; both are equally efficient.

A few simplifying assumptions are useful to compare these times. Consider a graph of n nodes with O(n) edges, and with a constant bound on the degree of each node. The asymptotic bounds assume a worst-case scenario where C has a least some dense rows (thus the  $\log n$  terms). If these are not present, if both t and c are O(n), and if a and s are constants, then the total time with blocking mode becomes  $O(n^2)$ , assuming the pattern of C changes at each assignment. This very high for a sparse graph problem. In contrast, the non-blocking time becomes  $O(n \log n)$  under these same assumptions, which is asymptotically much faster.

The difference in practice can be very dramatic, since n can be many millions for sparse graphs with n nodes and O(n), which can be handled on a commodity laptop.

The following guidelines should be considered when using GxB\_subassign, GrB\_assign and GrB\_\*\_setElement.

- 1. A sequence of assignments that does not modify the pattern at all is fast, taking as little as  $\Omega(1)$  time per entry modified. The worst case time complexity is  $O(\log n)$  per entry, assuming they all modify a dense row of C with n entries, which can occur in practice. It is more common, however, that most rows of C have a constant number of entries, independent of n. No work is ever left pending when the pattern of C does not change.
- 2. A sequence of assignments that modifies the entries that already exist in the pattern of a matrix, or adds new entries to the pattern (using the same accum operator), but does not delete any entries, is fast. The matrix is not completed until the end of the sequence.
- 3. Similarly, a sequence that modifies existing entries, or deletes them, but does not add new ones, is also fast. This sequence can also repeatedly delete pre-existing entries and then reinstate them and still be fast. The matrix is not completed until the end of the sequence.
- 4. A sequence that mixes assignments of types (2) and (3) above can be costly, since the matrix may need to be completed after each assignment. The time complexity can become quadratic in the worst case.
- 5. However, any single assignment takes no more than  $O(a+s\log n+n+c+s\log s)$  time, even including the time for a matrix completion, where C is n-by-n with c entries and A is a-by-a with s entries. This time is essentially linear in the size of the matrix C, if A is relatively small and sparse compared with C. In this case, n+c are the two dominant terms.
- 6. In general, GxB\_subassign is faster than GrB\_assign. If GrB\_REPLACE is used with GrB\_assign, the entire matrix C must be traversed. This is much slower than GxB\_subassign, which only needs to examine the C(I,J) submatrix. Furthermore, GrB\_assign must deal with a much larger Mask matrix, whereas GxB\_subassign has a smaller mask. Since

its mask is smaller, GxB\_subassign takes less time than GrB\_assign to access the mask.

Submatrix assignment in SuiteSparse:GraphBLAS is extremely efficient, even without considering the advantages of non-blocking mode discussed in Section 10.10. It can be up to 1000x faster than MATLAB R2019b, or even higher depending on the kind of matrix assignment. MATLAB logical indexing (the mask of GraphBLAS) is extremely faster with GraphBLAS as compared in MATLAB R2019b; differences of up to 250,000x have been observed (0.4 seconds in GraphBLAS versus 28 hours in MATLAB).

All of the 28 variants (each with their own source code) are either asymptotically optimal, or to within a log factor of being asymptotically optimal. The methods are also fully parallel. For hypersparse matrices, the term n in the expressions in the above discussion is dropped, and is replaced with  $h \log h$ , at the worst case, where h << n is the number of non-empty columns of a hypersparse matrix stored by column, or the number of non-empty rows of a hypersparse matrix stored by row. In many methods, n is replaced with h, not  $h \log h$ .

# 10.11 GrB\_apply: apply a unary, binary, or index-unary operator

TODO: add GrB\_IndexUnaryOp

GrB\_apply is the generic name for 62 specific functions. GrB\_Vector\_apply and GrB\_Matrix\_apply apply a unary operator to the entries of a matrix.  $GrB_*apply_Binary0p1st*$  applies a binary operator where a single scalar is provided as the x input to the binary operator.  $GrB_*apply_Binary0p2nd*$  applies a binary operator where a single scalar is provided as the y input to the binary operator. The generic name appears in the function prototypes, but the specific function name is used when describing each variation. When discussing features that apply to all versions, the simple name  $GrB_apply$  is used.

#### 10.11.1 GrB\_Vector\_apply: apply a unary operator to a vector

GrB\_Vector\_apply applies a unary operator to the entries of a vector, analogous to  $\mathbf{t} = op(\mathbf{u})$  in MATLAB except the operator op is only applied to entries in the pattern of  $\mathbf{u}$ . Implicit values outside the pattern of  $\mathbf{u}$  are not affected. The entries in  $\mathbf{u}$  are typecasted into the xtype of the unary operator. The vector  $\mathbf{t}$  has the same type as the ztype of the unary operator. The final step is  $\mathbf{w}\langle\mathbf{m}\rangle = \mathbf{w}\odot\mathbf{t}$ , as described in Section 2.3, except that all the terms are column vectors instead of matrices.

#### 10.11.2 GrB\_Matrix\_apply: apply a unary operator to a matrix

GrB\_Matrix\_apply applies a unary operator to the entries of a matrix, analogous to T = op(A) in MATLAB except the operator op is only applied to entries in the pattern of A. Implicit values outside the pattern of A are not affected. The input matrix A may be transposed first. The entries in A are typecasted into the xtype of the unary operator. The matrix T has the same type as the ztype of the unary operator. The final step is  $C\langle M \rangle = C \odot T$ , as described in Section 2.3.

The built-in  $GrB_IDENTITY_T$  operators (one for each built-in type T) are very useful when combined with this function, enabling it to compute  $C\langle M \rangle = C \odot A$ . This makes  $GrB_apply$  a direct interface to the accumulator/mask function for both matrices and vectors. The  $GrB_IDENTITY_T$  operators also provide the fastest stand-alone typecasting methods in Suite-Sparse:GraphBLAS, with all  $13 \times 13 = 169$  methods appearing as individual functions, to typecast between any of the 13 built-in types.

To compute  $\mathbf{C}\langle \mathbf{M} \rangle = \mathbf{A}$  or  $\mathbf{C}\langle \mathbf{M} \rangle = \mathbf{C} \odot \mathbf{A}$  for user-defined types, the user application would need to define an identity operator for the type. Since GraphBLAS cannot detect that it is an identity operator, it must call the operator to make the full copy T=A and apply the operator to each entry of the matrix or vector.

The other GraphBLAS operation that provides a direct interface to the accumulator/mask function is GrB\_transpose, which does not require an operator to perform this task. As a result, GrB\_transpose can be used as an efficient and direct interface to the accumulator/mask function for both built-in and user-defined types. However, it is only available for matrices, not vectors.

## 10.11.3 GrB\_Vector\_apply\_BinaryOp1st: apply a binary operator to a vector; 1st scalar binding

```
GrB_Info GrB_apply
                                    // w<mask> = accum (w, op(x,u))
    GrB_Vector w,
                                    // input/output vector for results
    const GrB_Vector mask,
                                    // optional mask for w, unused if NULL
    const GrB_BinaryOp accum,
                                    // optional accum for z=accum(w,t)
    const GrB_BinaryOp op,
                                    // operator to apply to the entries
    <type> x,
                                    // first input: scalar x
    const GrB_Vector u,
                                    // second input: vector u
    const GrB_Descriptor desc
                                    // descriptor for w and mask
);
```

GrB\_Vector\_apply\_BinaryOp1st\_<type> applies a binary operator z = f(x,y) to a vector, where a scalar x is bound to the first input of the operator. It is otherwise identical to GrB\_Vector\_apply. With no suffix, GxB\_Vector\_apply\_BinaryOp1st takes as input a GrB\_Scalar.

# 10.11.4 GrB\_Vector\_apply\_BinaryOp2nd: apply a binary operator to a vector; 2nd scalar binding

```
// w<mask> = accum (w, op(u,y))
GrB_Info GrB_apply
                                    // input/output vector for results
    GrB_Vector w,
    const GrB_Vector mask,
                                    // optional mask for w, unused if NULL
    const GrB_BinaryOp accum,
                                    // optional accum for z=accum(w,t)
    const GrB_BinaryOp op,
                                    // operator to apply to the entries
    const GrB_Vector u,
                                    // first input: vector u
    <type> y,
                                    // second input: scalar y
    const GrB_Descriptor desc
                                    // descriptor for w and mask
);
```

GrB\_Vector\_apply\_BinaryOp2nd\_<type> applies a binary operator z = f(x,y) to a vector, where a scalar y is bound to the second input of the operator. It is otherwise identical to GrB\_Vector\_apply. With no suffix, GxB\_Vector\_apply\_BinaryOp2nd takes as input a GrB\_Scalar.

## 10.11.5 GrB\_Matrix\_apply\_BinaryOp1st: apply a binary operator to a matrix; 1st scalar binding

```
GrB_Info GrB_apply
                                     // C < M > = accum(C, op(x, A))
   GrB_Matrix C,
                                     // input/output matrix for results
                                     // optional mask for C, unused if NULL
    const GrB_Matrix Mask,
    const GrB_BinaryOp accum,
                                     // optional accum for Z=accum(C,T)
    const GrB_BinaryOp op,
                                     // operator to apply to the entries
    <type> x,
                                     // first input: scalar x
    const GrB_Matrix A,
                                    // second input: matrix A
    const GrB_Descriptor desc
                                    // descriptor for C, mask, and A
);
```

 ${\tt GrB\_Matrix\_apply\_Binary0p1st\_<type>}$  applies a binary operator z=f(x,y) to a matrix, where a scalar x is bound to the first input of the operator. It is otherwise identical to  ${\tt GrB\_Matrix\_apply}$ . With no suffix,  ${\tt GxB\_Matrix\_apply\_Binary0p1st}$  takes as input a  ${\tt GrB\_Scalar}$ . To transpose the input matrix, use the  ${\tt GrB\_INP1}$  descriptor setting.

## 10.11.6 GrB\_Matrix\_apply\_BinaryOp2nd: apply a binary operator to a matrix; 2nd scalar binding

```
// C<M>=accum(C,op(A,y))
GrB_Info GrB_apply
(
    GrB_Matrix C,
                                    // input/output matrix for results
                                    // optional mask for C, unused if NULL
    const GrB_Matrix Mask,
    const GrB_BinaryOp accum,
                                    // optional accum for Z=accum(C,T)
    const GrB_BinaryOp op,
                                    // operator to apply to the entries
    const GrB_Matrix A,
                                    // first input: matrix A
    <type> y,
                                    // second input: scalar y
    const GrB_Descriptor desc
                                    // descriptor for C, mask, and A
);
```

GrB\_Matrix\_apply\_BinaryOp2nd\_<type> applies a binary operator z = f(x,y) to a matrix, where a scalar x is bound to the second input of the operator. It is otherwise identical to GrB\_Matrix\_apply. With no suffix, GxB\_Matrix\_apply\_BinaryOp2nd takes as input a GrB\_Scalar. To transpose the input matrix, use the GrB\_INPO descriptor setting.

# 10.12 GxB\_select: apply a select operator (FIXME: GrB-select)

TODO: change to GrB\_select with GrB\_IndexUnaryOp. Remove all mention of GxB\_SelectOp.

The GxB\_select function is the generic name for two specific functions: GxB\_Vector\_select and GxB\_Matrix\_select. The generic name appears in the function prototypes, but the specific function name is used when describing each variation. When discussing features that apply to both versions, the simple name GxB\_select is used.

#### 10.12.1 GxB\_Vector\_select: apply a select operator to a vector

```
GrB_Info GxB_select
                                    // w<mask> = accum (w, op(u,k))
   GrB_Vector w,
                                    // input/output vector for results
    const GrB_Vector mask,
                                    // optional mask for w, unused if NULL
    const GrB_BinaryOp accum,
                                    // optional accum for z=accum(w,t)
    const GxB_SelectOp op,
                                    // operator to apply to the entries
    const GrB_Vector u,
                                    // first input: vector u
    const GxB_Scalar Thunk,
                                    // optional input for the select operator
    const GrB_Descriptor desc
                                    // descriptor for w and mask
);
```

GxB\_Vector\_select applies a select operator to the entries of a vector, analogous to t = u.\*op(u) in MATLAB except the operator op is only applied to entries in the pattern of u. Implicit values outside the pattern of u are not affected. If the operator is not type-generic, the entries in u are type-casted into the xtype of the select operator. The vector t has the same type and size as u. The final step is  $\mathbf{w}\langle \mathbf{m}\rangle = \mathbf{w}\odot \mathbf{t}$ , as described in Section 2.3, except that all the terms are column vectors instead of matrices.

This operation operates on vectors just as if they were m-by-1 matrices, except that GraphBLAS never transposes a vector via the descriptor. The op is passed n=1 as the number of columns. Refer to the next section on GxB\_Matrix\_select for more details.

#### 10.12.2 GxB\_Matrix\_select: apply a select operator to a matrix

```
// C<Mask> = accum (C, op(A,k)) or op(A',k)
GrB_Info GxB_select
    GrB_Matrix C,
                                    // input/output matrix for results
                                    // optional mask for C, unused if NULL
    const GrB_Matrix Mask,
    const GrB_BinaryOp accum,
                                    // optional accum for Z=accum(C,T)
                                    // operator to apply to the entries
    const GxB_SelectOp op,
    const GrB_Matrix A,
                                    // first input: matrix A
    const GxB_Scalar Thunk,
                                    // optional input for the select operator
    const GrB_Descriptor desc
                                    // descriptor for C, mask, and A
);
```

GxB\_Matrix\_select applies a select operator to the entries of a matrix, analogous to T = A .\* op(A) in MATLAB except the operator op is only applied to entries in the pattern of A. Implicit values outside the pattern of A are not affected. The input matrix A may be transposed first. If the operator is not type-generic, the entries in A are typecasted into the xtype of the select operator. The final step is  $C\langle M \rangle = C \odot T$ , as described in Section 2.3.

The matrix T has the same size and type as A (or the transpose of A if the input is transposed via the descriptor). The entries of T are a subset of those of A. Each entry A(i,j) of A is passed to the op, as  $z = f(i,j,m,n,a_{ij},\text{thunk})$ , where A is m-by-n. If A is transposed first then the operator is applied to entries in the transposed matrix, A'. If z is returned as true, then the entry is copied into T, unchanged. If it returns false, the entry does not appear in T.

If Thunk is not NULL, it must be a valid GrB\_Scalar. If it has no entry, it is treated as if it had a single entry equal to zero, for built-in types (not user-defined types).

For user-defined select operators, the entry is passed to the user-defined select operator, with no typecasting. Its type must be identical to ttype of the select operator.

For the  $GxB_TRIL$ ,  $GxB_TRIU$ ,  $GxB_DIAG$ , and  $GxB_DFFDIAG$ , the Thunk parameter may be NULL, or it may be present but contain no entry. In this case, these operators use the value of k=0, the main diagonal. If present, the Thunk can be any built-in type. The value of this entry is typecasted:  $k = (int64_t)$  Thunk. The value k=0 specifies the main diagonal of the matrix, k=1 is the +1 diagonal (the entries just above the main diagonal), k=-1 is the -1 diagonal, and so on.

For the GxB\_\*ZERO select operators, Thunk is ignored, and may be NULL.

For built-in types, with the GxB\_\*THUNK operators, the value of Thunk is typecasted to the same type as the A matrix. For user-defined types, Thunk is passed to the select operator without typecasting.

The action of GxB\_select with the built-in select operators is described in the table below. The MATLAB analogs are precise for tril and triu, but shorthand for the other operations. The MATLAB diag function returns a column with the diagonal, if A is a matrix, whereas the matrix T in GxB\_select always has the same size as A (or its transpose if the GrB\_INPO is set to GrB\_TRAN). In the MATLAB analog column, diag is as if it operates like GxB\_select, where T is a matrix.

The following operators may be used on matrices with a user-defined type: GxB\_TRIL, GxB\_TRIU, GxB\_DIAG, GxB\_OFFIAG, GxB\_NONZERO, GxB\_EQ\_ZERO, GxB\_NE\_THUNK, and GxB\_EQ\_THUNK.

The comparators <code>GxB\_GT\_\*GxB\_GE\_\*GxB\_LT\_\*</code>, and <code>GxB\_LE\_\*</code> only work for built-in types. All other built-in select operators can be used for any type, both built-in and any user-defined type.

**NOTE:** For floating-point values, compares with NaN always return false. The built-in select operators should not be used with a scalar **thunk** that is equal to NaN. For this case, create a user-defined select operator that performs the test with the ANSI C **isnan** function instead.

GraphBLAS	Octave/MATLAB	
name	analog	
GxB_TRIL	T=tril(A,k)	Entries in T are the entries on and below the kth diagonal of A.
GxB_TRIU	T=triu(A,k)	Entries in T are the entries on and above the kth diagonal of A.
GxB_DIAG	T=diag(A,k)	Entries in T are the entries on the kth diagonal of A.
GxB_OFFDIAG	T=A-diag(A,k)	Entries in T are all entries not on the kth diagonal of A.
GxB_NONZERO	$T=A(A^{\sim}=0)$	Entries in T are all entries in A that have nonzero value.
GxB_EQ_ZERO	T=A(A==0)	Entries in T are all entries in A that are equal to zero.
GxB_GT_ZERO	T=A(A>0)	Entries in T are all entries in A that are greater than zero.
GxB_GE_ZERO	$T=A(A \le 0)$	Entries in T are all entries in A that are greater than or equal to zero.
GxB_LT_ZERO	T=A(A<0)	Entries in T are all entries in A that are less than zero.
GxB_LE_ZERO	$T=A(A \le 0)$	Entries in T are all entries in A that are less than or equal to zero.
GxB_NE_THUNK	$T=A(A^{\sim}=k)$	Entries in T are all entries in A that are not equal to k.
GxB_EQ_THUNK	T=A(A==k)	Entries in T are all entries in A that are equal to k.
GxB_GT_THUNK	T=A(A>k)	Entries in T are all entries in A that are greater than k.
GxB_GE_THUNK	T=A(A>=k)	Entries in T are all entries in A that are greater than or equal to k.
GxB_LT_THUNK	T=A(A <k)< td=""><td>Entries in T are all entries in A that are less than k.</td></k)<>	Entries in T are all entries in A that are less than k.
GxB_LE_THUNK	$T=A(A\leq k)$	Entries in $T$ are all entries in $A$ that are less than or equal to $k$ .

#### 10.13 GrB reduce: reduce to a vector or scalar

#### TODO: add GrB\_Scalar

The generic function name **GrB\_reduce** may be used for all specific functions discussed in this section. When the details of a specific function are discussed, the specific name is used for clarity.

#### 10.13.1 GrB\_Matrix\_reduce\_Monoid reduce a matrix to a vector

GrB\_Matrix\_reduce\_Monoid reduces a matrix to a column vector using a monoid, roughly analogous to t = sum (A') in MATLAB, in the default case, where t is a column vector. By default, the method reduces across the rows to obtain a column vector; use GrB\_TRAN to reduce down the columns.

The input matrix A may be transposed first. Its entries are then typecast into the type of the reduce operator or monoid. The reduction is applied to all entries in A (i,:) to produce the scalar t (i). This is done without the use of the identity value of the monoid. If the ith row A (i,:) has no entries, then (i) is not an entry in t and its value is implicit. If A (i,:) has a single entry, then that is the result t (i) and reduce is not applied at all for the ith row. Otherwise, multiple entries in row A (i,:) are reduced via the reduce operator or monoid to obtain a single scalar, the result t (i).

The final step is  $\mathbf{w}\langle \mathbf{m} \rangle = \mathbf{w} \odot \mathbf{t}$ , as described in Section 2.3, except that all the terms are column vectors instead of matrices.

GrB\_reduce can also be passed a GrB\_BinaryOp in place of the monoid parameter, but the binary operator must correspond to a known built-in monoid. This provides a limited implementation of GrB\_Matrix\_reduce\_BinaryOp.

#### 10.13.2 GrB\_Vector\_reduce\_<type>: reduce a vector to a scalar

GrB\_Vector\_reduce\_<type> reduces a vector to a scalar, analogous to t = sum (u) in MATLAB, except that in GraphBLAS any commutative and associative monoid can be used in the reduction.

If the vector u has no entries, that identity value of the monoid is copied into the scalar t. Otherwise, all of the entries in the vector are reduced to a single scalar using the monoid.

The scalar type is any of the built-in types, or a user-defined type. In the function signature it is a C type: bool, int8\_t, ... float, double, or void \* for a user-defined type. The user-defined type must be identical to the type of the vector u. This cannot be checked by GraphBLAS and thus results are undefined if the types are not the same.

The descriptor is unused, but it appears in case it is needed in future versions of the GraphBLAS API. This function has no mask so its accumulator/mask step differs from the other GraphBLAS operations. It does not use the methods described in Section 2.3, but uses the following method instead.

If accum is NULL, then the scalar t is typecast into the type of c, and c = t is the final result. Otherwise, the scalar t is typecast into the ytype of the accum operator, and the value of c (on input) is typecast into the xtype of the accum operator. Next, the scalar z = accum (c,t) is computed, of the ztype of the accum operator. Finally, z is typecast into the final result, c.

Since this operation does not have a GraphBLAS input/output object, it cannot return an error string for GrB\_error.

#### 10.13.3 GrB\_Matrix\_reduce\_<type>: reduce a matrix to a scalar

GrB\_Matrix\_reduce\_<type> reduces a matrix A to a scalar, roughly analogous to t = sum (A (:)) in MATLAB. This function is identical to reducing a vector to a scalar, since the positions of the entries in a matrix or vector have no effect on the result. Refer to the reduction to scalar described in the previous Section 10.13.2.

Since this operation does not have a GraphBLAS input/output object, it cannot return an error string for GrB\_error.

## 10.14 GrB\_transpose: transpose a matrix

GrB\_transpose transposes a matrix A, just like the array transpose T = A.' in MATLAB. The internal result matrix T = A' (or merely T = A if A is transposed via the descriptor) has the same type as A. The final step is  $\mathbf{C}\langle\mathbf{M}\rangle=\mathbf{C}\odot\mathbf{T}$ , as described in Section 2.3, which typecasts T as needed and applies the mask and accumulator.

To be consistent with the rest of the GraphBLAS API regarding the descriptor, the input matrix A may be transposed first. It may seem counterintuitive, but this has the effect of not doing any transpose at all. As a result,  $GrB\_transpose$  is useful for more than just transposing a matrix. It can be used as a direct interface to the accumulator/mask operation,  $C\langle M\rangle = C\odot A$ . This step also does any typecasting needed, so  $GrB\_transpose$  can be used to typecast a matrix A into another matrix C. To do this, simply use NULL for the Mask and accum, and provide a non-default descriptor desc that sets the transpose option:

```
// C = typecasted copy of A
GrB_Descriptor_set (desc, GrB_INPO, GrB_TRAN) ;
GrB_transpose (C, NULL, NULL, A, desc) ;
```

If the types of C and A match, then the above two lines of code are the same as GrB\_Matrix\_dup (&C, A), except that for GrB\_transpose the matrix C must already exist and be the right size. If C does not exist, the work of GrB\_Matrix\_dup can be replicated with this:

```
// C = create an exact copy of A, just like GrB_Matrix_dup
GrB_Matrix C;
GrB_Type type;
GrB_Index nrows, ncols;
GrB_Descriptor desc;
GxB_Matrix_type (&type, A);
GrB_Matrix_nrows (&nrows, A);
```

```
GrB_Matrix_ncols (&ncols, A) ;
GrB_Matrix_new (&C, type, nrows, ncols) ;
GrB_Descriptor_new (&desc) ;
GrB_Descriptor_set (desc, GrB_INPO, GrB_TRAN) ;
GrB_transpose (C, NULL, NULL, A, desc) ;
```

Since the input matrix A is transposed by the descriptor, SuiteSparse:Graph-BLAS does the right thing and does not transpose the matrix at all. Since T = A is not typecasted, SuiteSparse:GraphBLAS can construct T internally in O(1) time and using no memory at all. This makes  $Grb_transpose$  a fast and direct interface to the accumulator/mask function in GraphBLAS.

This example is of course overkill, since the work can all be done by a single call to the <code>GrB\_Matrix\_dup</code> function. However, the <code>GrB\_Matrix\_dup</code> function can only create <code>C</code> as an exact copy of <code>A</code>, whereas variants of the code above can do many more things with these two matrices. For example, the <code>type</code> in the example can be replaced with any other type, perhaps selected from another matrix or from an operator.

Consider the following code excerpt, which uses  $GrB\_transpose$  to remove all diagonal entries from a square matrix. It first creates a diagonal Mask, which is complemented so that  $\mathbf{C}\langle \neg \mathbf{M}\rangle = \mathbf{A}$  does not modify the diagonal of  $\mathbf{C}$ . The REPLACE ensures that  $\mathbf{C}$  is cleared first, and then  $\mathbf{C}\langle \neg \mathbf{M}\rangle = \mathbf{A}$  modifies all entries in  $\mathbf{C}$  where the mask  $\mathbf{M}$  is false. These correspond to all the off-diagonal entries. The descriptor ensures that  $\mathbf{A}$  is not transposed at all. The Mask can have any pattern, of course, and wherever it is set true, the corresponding entries in  $\mathbf{A}$  are deleted from the copy  $\mathbf{C}$ .

```
// remove all diagonal entries from the matrix A
// Mask = speye (n);
GrB_Matrix_new (&Mask, GrB_BOOL, n, n);
for (int64_t i = 0; i < n; i++)
{
        GrB_Matrix_setElement (Mask, (bool) true, i, i);
}
// C<~Mask> = A, clearing C first. No transpose.
GrB_Descriptor_new (&desc);
GrB_Descriptor_set (desc, GrB_INPO, GrB_TRAN);
GrB_Descriptor_set (desc, GrB_MASK, GrB_COMP);
GrB_Descriptor_set (desc, GrB_OUTP, GrB_REPLACE);
GrB_transpose (A, Mask, NULL, A, desc);
```

## 10.15 GrB\_kronecker: Kronecker product

```
GrB_Info GrB_kronecker
                                    // C<Mask> = accum (C, kron(A,B))
    GrB_Matrix C,
                                    // input/output matrix for results
                                    // optional mask for C, unused if NULL
   const GrB_Matrix Mask,
   const GrB_BinaryOp accum,
                                    // optional accum for Z=accum(C,T)
                                    // defines '*' for T=kron(A,B)
    const <operator> op,
    const GrB_Matrix A,
                                    // first input: matrix A
    const GrB_Matrix B,
                                    // second input: matrix B
    const GrB_Descriptor desc
                                    // descriptor for C, Mask, A, and B
);
```

 ${\tt GrB\_kronecker}\ computes\ the\ Kronecker\ product,\ {\bf C}\langle {\bf M}\rangle={\bf C}\odot kron({\bf A},{\bf B})$  where

$$\operatorname{kron}(\mathbf{A}, \mathbf{B}) = \begin{bmatrix} a_{00} \otimes \mathbf{B} & \dots & a_{0,n-1} \otimes \mathbf{B} \\ \vdots & \ddots & \vdots \\ a_{m-1,0} \otimes \mathbf{B} & \dots & a_{m-1,n-1} \otimes \mathbf{B} \end{bmatrix}$$

The  $\otimes$  operator is defined by the op parameter. It is applied in an element-wise fashion (like  $GrB_eWiseMult$ ), where the pattern of the submatrix  $a_{ij}\otimes B$  is the same as the pattern of B if  $a_{ij}$  is an entry in the matrix A, or empty otherwise. The input matrices A and B can be of any dimension, and both matrices may be transposed first via the descriptor, desc. Entries in A and B are typecast into the input types of the op. The matrix T=kron(A,B) has the same type as the ztype of the binary operator, op. The final step is  $C\langle M \rangle = C \odot T$ , as described in Section 2.3.

The operator op may be a GrB\_BinaryOp, a GrB\_Monoid, or a GrB\_Semiring. In the latter case, the multiplicative operator of the semiring is used.

## 11 Printing GraphBLAS objects

The ten different objects handled by SuiteSparse:GraphBLAS are all opaque, although nearly all of their contents can be extracted via methods such as GrB\_Matrix\_extractTuples, GrB\_Matrix\_extractElement, GxB\_Matrix\_type, and so on. The GraphBLAS C API has no mechanism for printing all the contents of GraphBLAS objects, but this is helpful for debugging. Ten type-specific methods and two type-generic methods are provided:

<pre>GxB_Type_fprint</pre>	print and check a GrB_Type
<pre>GxB_UnaryOp_fprint</pre>	print and check a GrB_UnaryOp
<pre>GxB_BinaryOp_fprint</pre>	print and check a GrB_BinaryOp
<pre>GxB_SelectOp_fprint</pre>	print and check a GxB_SelectOp
<pre>GxB_Monoid_fprint</pre>	print and check a GrB_Monoid
<pre>GxB_Semiring_fprint</pre>	print and check a GrB_Semiring
<pre>GxB_Descriptor_fprint</pre>	print and check a GrB_Descriptor
<pre>GxB_Matrix_fprint</pre>	print and check a GrB_Matrix
<pre>GxB_Vector_fprint</pre>	print and check a GrB_Vector
<pre>GxB_Scalar_fprint</pre>	print and check a GrB_Scalar
GxB_fprint	print/check any object to a file
GxB_print	print/check any object to stdout

These methods do not modify the status of any object, and thus they cannot return an error string for use by GrB\_error.

If a matrix or vector has not been completed, the pending computations are guaranteed to *not* be performed. The reason is simple. It is possible for a bug in the user application (such as accessing memory outside the bounds of an array) to mangle the internal content of a GraphBLAS object, and the GxB\_\*print methods can be helpful tools to track down this bug. If GxB\_\*print attempted to complete any computations prior to printing or checking the contents of the matrix or vector, then further errors could occur, including a segfault.

By contrast, GraphBLAS methods and operations that return values into user-provided arrays or variables might finish pending operations before the return these values, and this would change their state. Since they do not change the state of any object, the <code>GxB\_\*print</code> methods provide a useful alternative for debugging, and for a quick understanding of what GraphBLAS is computing while developing a user application.

Each of the methods has a parameter of type GxB\_Print\_Level that specifies the amount to print:

The ten type-specific functions include an additional argument, the name string. The name is printed at the beginning of the display (assuming the print level is not GxB\_SILENT) so that the object can be more easily identified in the output. For the type-generic methods GxB\_fprint and GxB\_print, the name string is the variable name of the object itself.

If the file f is NULL, stdout is used. If name is NULL, it is treated as the empty string. These are not error conditions.

The methods check their input objects carefully and extensively, even when pr is equal to GxB\_SILENT. The following error codes can be returned:

- Grb\_Success: object is valid
- Grb\_Uninitialized\_Object: object is not initialized
- Grb\_INVALID\_OBJECT: object is not valid
- Grb\_NULL\_POINTER: object is a NULL pointer
- Grb\_INVALID\_VALUE: fprintf returned an I/O error.

The content of any GraphBLAS object is opaque, and subject to change. As a result, the exact content and format of what is printed is implementation-dependent, and will change from version to version of SuiteSparse:GraphBLAS. Do not attempt to rely on the exact content or format by trying to parse the resulting output via another program. The intent of these functions is to produce a report of an object for visual inspection. If the user application needs to extract content from a GraphBLAS matrix or vector, use GrB\_\*cextractTuples or the import/export methods instead.

GraphBLAS matrices and vectors are zero-based, where indices of an n-by-n matrix are in the range 0 to n-1. However, Octave, MATLAB, and Julia prefer to print their matrices and vectors as one-based. To enable 1-based printing, use  $GxB\_set$  ( $GxB\_PRINT\_1BASED$ , true). Printing is done as zero-based by default.

## 11.1 GxB\_fprint: Print a GraphBLAS object to a file

The GxB\_fprint function prints the contents of any of the ten Graph-BLAS objects to the file f. If f is NULL, the results are printed to stdout. For example, to print the entire contents of a matrix A to the file f, use GxB\_fprint (A, GxB\_COMPLETE, f).

## 11.2 GxB\_print: Print a GraphBLAS object to stdout

GxB\_print is the same as GxB\_fprint, except that it prints the contents of the object to stdout instead of a file f. For example, to print the entire contents of a matrix A, use GxB\_print (A, GxB\_COMPLETE).

## 11.3 GxB\_Type\_fprint: Print a GrB\_Type

For example, GxB\_Type\_fprint (GrB\_BOOL, "boolean type", GxB\_COMPLETE, f) prints the contents of the GrB\_BOOL object to the file f.

## 11.4 GxB\_UnaryOp\_fprint: Print a GrB\_UnaryOp

For example, GxB\_UnaryOp\_fprint (GrB\_LNOT, "not", GxB\_COMPLETE, f) prints the GrB\_LNOT unary operator to the file f.

## 11.5 GxB\_BinaryOp\_fprint: Print a GrB\_BinaryOp

For example, GxB\_BinaryOp\_fprint (GrB\_PLUS\_FP64, "plus", GxB\_COMPLETE, f) prints the GrB\_PLUS\_FP64 binary operator to the file f.

## 11.6 GxB\_IndexUnaryOp\_fprint: Print a GrB\_IndexUnaryOp

For example, GrB\_IndexUnaryOp\_fprint (GrB\_TRIL\_INT64, "tril", GxB\_COMPLETE, f) prints the GrB\_TRIL\_INT64 index-unary operator to the file f.

## 11.7 GxB\_Monoid\_fprint: Print a GrB\_Monoid

For example, GxB\_Monoid\_fprint (GxB\_PLUS\_FP64\_MONOID, "plus monoid", GxB\_COMPLETE, f) prints the predefined GxB\_PLUS\_FP64\_MONOID (based on the binary operator GrB\_PLUS\_FP64) to the file f.

## 11.8 GxB\_Semiring\_fprint: Print a GrB\_Semiring

For example, GxB\_Semiring\_fprint (GxB\_PLUS\_TIMES\_FP64, "standard", GxB\_COMPLETE, f) prints the predefined GxB\_PLUS\_TIMES\_FP64 semiring to the file f.

## 11.9 GxB\_Descriptor\_fprint: Print a GrB\_Descriptor

For example, GxB\_Descriptor\_fprint (d, "descriptor", GxB\_COMPLETE, f) prints the descriptor d to the file f.

## 11.10 GxB\_Matrix\_fprint: Print a GrB\_Matrix

For example, GxB\_Matrix\_fprint (A, "my matrix", GxB\_SHORT, f) prints about 30 entries from the matrix A to the file f.

## 11.11 GxB\_Vector\_fprint: Print a GrB\_Vector

For example, GxB\_Vector\_fprint (v, "my vector", GxB\_SHORT, f) prints about 30 entries from the vector v to the file f.

## 11.12 GxB\_Scalar\_fprint: Print a GrB\_Scalar

For example, GxB\_Scalar\_fprint (s, "my scalar", GxB\_SHORT, f) prints a short description of the scalar s to the file f.

## 11.13 Performance and portability considerations

Even when the print level is GxB\_SILENT, these methods extensively check the contents of the objects passed to them, which can take some time. They should be considered debugging tools only, not for final use in production.

The return value of the <code>GxB\_\*print</code> methods can be relied upon, but the output to the file (or <code>stdout</code>) can change from version to version. If these methods are eventually added to the GraphBLAS C API Specification, a conforming implementation might never print anything at all, regardless of the <code>pr</code> value. This may be essential if the GraphBLAS library is installed in a dedicated device, with no file output, for example.

Some implementations may wish to print nothing at all if the matrix is not yet completed, or just an indication that the matrix has pending operations and cannot be printed, when non-blocking mode is employed. In this case, use <code>GrB\_Matrix\_wait</code>, <code>GrB\_Vector\_wait</code>, or <code>GxB\_Scalar\_wait</code> to finish all pending computations first. If a matrix or vector has pending operations, SuiteSparse:GraphBLAS prints a list of the *pending tuples*, which are the entries not yet inserted into the primary data structure. It can also print out entries that remain in the data structure but are awaiting deletion; these are called *zombies* in the output report.

Most of the rest of the report is self-explanatory.

## 12 Iso-Valued Matrices and Vectors

The GraphBLAS C API states that the entries in all  $GrB_Matrix$  and  $GrB_Vector$  objects have a numerical value, with either a built-in or user-defined type. Representing an unweighted graph requires a value to be placed on each edge, typically  $a_{ij} = 1$ . Adding a structure-only data type would not mix well with the rest of GraphBLAS, where all operators, monoids, and semirings need to operate on a value, of some data type. And yet unweighted graphs are very important in graph algorithms.

The solution is simple, and exploiting it in SuiteSparse:GraphBLAS requires nearly no extensions to the GraphBLAS C API. SuiteSparse:GraphBLAS can often detect when the user application is creating a matrix or vector where all entries in the sparsity pattern take on the same numerical value.

For example,  $\mathbf{C}\langle\mathbf{C}\rangle=1$ , when the mask is structural, sets all entries in  $\mathbf{C}$  to the value 1. SuiteSparse:GraphBLAS detects this, and performs this assignment in O(1) time. It stores a single copy of this "iso-value" and sets an internal flag in the opaque data structure for  $\mathbf{C}$ , which states that all entries in the pattern of  $\mathbf{C}$  are equal to 1. This saves both time and memory and allows for the efficient representation of sparse adjacency matrices of unweighted graphs, yet does not change the C API. To the user application, it still appears that  $\mathbf{C}$  has  $\mathtt{nvals}(\mathbf{C})$  entries, all equal to 1.

Creating and operating on iso-valued matrices (or just iso matrices for short) is significantly faster than creating matrices with different data values. A matrix that is iso requires only O(1) space for its numerical values. The sparse and hypersparse formats require an additional O(n+e) or O(e) integer space to hold the pattern of an n-by-n matrix C, respectively, and a matrix C in bitmap format requires  $O(n^2)$  space for the bitmap. A full matrix requires no integer storage, so a matrix that is both iso and full requires only O(1) space, regardless of its dimension.

The sections below a describe the methods that can be used to create iso matrices and vectors. Let a, b, and c denote the iso values of A, B, and C, respectively.

# 12.1 Using iso matrices and vectors in a graph algorithm

There are two primary useful ways to use iso-valued matrices and vectors: (1) as iso sparse/hypersparse adjacency matrices for unweighted graphs, and

(2) as iso full matrices or vectors used with operations that do not need to access all of the content of the iso full matrix or vector.

In the first use case, simply create a  $GrB_Matrix$  with values that are all the same (those in the sparsity pattern). The  $GxB_Matrix_build_Scalar$  method can be used for this, since it guarantees that the time and work spent on the numerical part of the array is only O(1). The method still must spend O(e) or  $O(e \log e)$  time on the integer arrays that represent the sparsity pattern, but the reduction in time and work on the numerical part of the matrix will improve performance.

The use of GxB\_Matrix\_build\_Scalar is optional. Matrices can also be constructed with GrB\* methods. In particular, GrB\_Matrix\_build\_\* can be used. It first builds a non-iso matrix and then checks if all of the values are the same, after assembling any duplicate entries. This does not save time or memory for the construction of the matrix itself, but it will lead to savings in time and memory later on, when the matrix is used.

To ensure a matrix C is iso-valued, simply use  $GrB_assign$  to compute C<C,struct>=1, or assign whatever value of scalar you wish. It is essential to use a structural mask. Otherwise, it is not clear that all entries in C will be assigned the same value. The following code takes O(1) time, and it resets the size of the numerical part of the C matrix to be O(1) in size:

The Octave/MATLAB analog of the code above is C=spones(C).

The second case for where iso matrices and vectors are useful is to use them with operations that do not necessarily access all of their content. Suppose you have a matrix A of arbitrarily large dimension (say n-by-n where n=2^60, of type GrB\_FP64. A matrix this large can be represented by Suite-Sparse:GraphBLAS, but only in a hypersparse form.

Now, suppose you wish to compute the maximum value in each row, reducing the matrix to a vector. This can be done with GrB\_reduce:

```
GrB_Vector_new (&v, GrB_FP64, n) ;
GrB_reduce (v, NULL, GrB_MAX_MONOID_FP64, A, NULL) ;
```

It can also be done with  $\mathtt{GrB\_mxv}$ , by creating an iso full vector  $\mathbf{x}$ . The creation of  $\mathbf{x}$  takes O(1) time and memory, and the  $\mathtt{GrB\_mxv}$  computation takes O(e) time (with modest assumptions; if  $\mathbf{A}$  needs to be transposed the time would be  $O(e \log e)$ ).

```
GrB_Vector_new (&v, GrB_FP64, n);
GrB_Vector_new (&x, GrB_FP64, n);
GrB_assign (x, NULL, NULL, 1, GrB_ALL, n, NULL);
GrB_mxv (v, NULL, NULL, GrB_MAX_FIRST_SEMIRING_FP64, A, x, NULL);
```

The above computations are identical in SuiteSparse:GraphBLAS. Internally, GrB\_reduce creates x and calls GrB\_mxv. Using GrB\_mxm directly gives the user application additional flexibility in creating new computations that exploit the multiplicative operator in the semiring. GrB\_reduce always uses the FIRST operator in its semiring, but any other binary operator can be used instead when using GrB\_mxv.

Below is a method for computing the argmax of each row of a square matrix A of dimension n and type GrB\_FP64. The vector x contains the maximum value in each row, and the vector p contains the zero-based column index of the maximum value in each row. If there are duplicate maximum values in each row, any one of them is selected arbitrarily using the ANY monoid. To select the minimum column index of the duplicate maximum values, use the GxB\_MIN\_SECONDI\_INT64 semiring instead (this will be slightly slower than the ANY monoid if there are many duplicates).

To compute the argmax of each column, use the GrB\_DESC\_TO descriptor in GrB\_mxv, and compute G=A\*D instead of G=D\*A with GrB\_mxm. See the GrB.argmin and GrB.argmax functions in the Octave/MATLAB interface for details.

```
GrB_Vector_new (&x, GrB_FP64, n) ;
GrB_Vector_new (&y, GrB_FP64, n) ;
GrB_Vector_new (&p, GrB_INT64, n) ;
// y (:) = 1, an iso full vector
GrB_assign (y, NULL, NULL, 1, GrB_ALL, n, NULL) ;
// x = max (A) where x(i) = max (A (i,:))
GrB_mxv (x, NULL, NULL, GrB_MAX_FIRST_SEMIRING_FP64, A, y, NULL) ;
// D = diag (x)
GrB_Matrix_new (&D, GrB_FP64, n, n);
GrB_Matrix_diag (D, x, 0);
// G = D*A using the ANY_EQ semiring
GrB_Matrix_new (&G, GrB_BOOL, n, n) ;
GrB_mxm (G, NULL, NULL, GxB_ANY_EQ_FP64, D, A, NULL) ;
// drop explicit zeros from G
GxB_select (G, NULL, NULL, GxB_NONZERO, G, NULL, NULL) ;
// find the position of any max entry in each row: p = G*y,
// so that p(i) = j if x(i) = A(i,j) = max (A(i,:))
GrB_mxv (p, NULL, NULL, GxB_ANY_SECONDI_INT64, G, y, NULL) ;
```

No part of the above code takes  $\Omega(n)$  time or memory. The data type of the iso full vector  $\mathbf{y}$  can be anything, and its iso value can be anything. It is operated on by the FIRST operator in the first  $\mathtt{GrB\_mxv}$ , and the SECONDI positional operator in the second  $\mathtt{GrB\_mxv}$ , and both operators are oblivious to the content and even the type of  $\mathbf{y}$ . The semirings simply note that  $\mathbf{y}$  is a full vector and compute their result according, by accessing the matrices only (A and G, respectively).

For floating-point values, NaN values are ignored, and treated as if they were not present in the input matrix, unless all entries in a given row are equal to NaN. In that case, if all entries in A(i,:) are equal to NaN, then x(i) is NaN and the entry p(i) is not present.

## 12.2 Iso matrices from matrix multiplication

Consider  $GrB_mxm$ ,  $GrB_mxv$ , and  $GrB_vxm$ , and let C=A\*B, where no mask is present, or C<M>=A\*B where C is initially empty. If C is not initially empty, then these rules apply to a temporary matrix T<M>=A\*B, which is initially empty and is then assigned to C via C<M>=T.

The iso property of C is determined with the following rules, where the first rule that fits defines the property and value of C.

- If the semiring includes a positional multiplicative operator (GxB\_FIRSTI, GrB\_SECONDI, and related operators), then C is never iso.
- Define an iso-monoid as a built-in monoid with the property that reducing a set of n>1 identical values x returns the same value x. These are the MIN MAX LOR LAND BOR BAND and ANY monoids. All other monoids are not iso monoids: PLUS, TIMES, LXNOR, EQ, BXOR, BXNOR, and all user-defined monoids. Currently, there is no mechanism for telling SuiteSparse:GraphBLAS that a user-defined monoid is an iso-monoid.
- If the multiplicative op is PAIR, and the monoid is an iso-monoid, or the EQ or TIMES monoids, then C is iso with a value of 1.
- If both B and the monoid are iso, and the multiplicative op is SECOND or ANY, then C is iso with a value of b.
- If both A and the monoid are iso, and the multiplicative op is FIRST or ANY, then C is iso with a value of a.

- If A, B, and the monoid are all iso, then C is iso, with a value c = f(a, b), where f is any multiplicative op (including user-defined, which assumes that a user-defined f has no side effects).
- If A and B are both iso and full (all entries present, regardless of the format of the matrices), then C is iso and full. Its iso value is computed in  $O(\log(n))$  time, via a reduction of n copies of the value t = f(a, b) to a scalar. The storage required to represent C is just O(1), regardless of its dimension. Technically, the PLUS monoid could be computed as c = nt in O(1) time, but the log-time reduction works for any monoid, including user-defined ones.
- Otherwise, C is not iso.

#### 12.3 Iso matrices from eWiseMult and kronecker

Consider  $GrB_eWiseMult$ . Let C=A.\*B, or C<M>=A.\*B with any mask and where C is initially empty, where .\* denotes a binary operator f(x,y) applied with eWiseMult. These rules also apply to  $GrB_kronecker$ .

- If the operator is positional (GxB\_FIRSTI and related) then C is not iso.
- If the op is PAIR, then C is iso with c=1.
- If B is iso and the op is SECOND or ANY, then C is iso with c = b.
- If A is iso and the op is FIRST or ANY, then C is iso with c=a.
- If both A and B are iso, then C is iso with c = f(a, b).
- Otherwise, C is not iso.

#### 12.4 Iso matrices from eWiseAdd

Consider  $GrB_eWiseAdd$ , and also the accumulator phase of C<M>=T when an accumulator operator is present. Let C=A+B, or C<M>=A+B with any mask and where C is initially empty.

• If both A and B are full (all entries present), then the rules for eWiseMult in Section 12.3 are used instead.

- If the operator is positional (GxB\_FIRSTI and related) then C is not iso.
- If a and b differ (when typecasted to the type of C), then C is not iso.
- If c = f(a, b) = a = b holds, then C is iso, where f(a, b) is the operator.
- Otherwise, C is not iso.

## 12.5 Reducing iso matrices to a scalar or vector

If A is iso with e entries, reducing it to a scalar takes  $O(\log(e))$  time, regardless of the monoid used to reduce the matrix to a scalar. Reducing A to a vector c is the same as the matrix-vector multiply c=A\*x or c=A\*x, depending on the descriptor, where x is an iso full vector (refer to Section 12.2).

## 12.6 Iso matrices from apply

Let C=f(A) denote the application of a unary operator f, and let C=f(A,s) and C=f(s,A) denote the application of a binary operator with s a scalar.

- If the operator is positional (GxB\_POSITION\*, GxB\_FIRSTI, and related) then C is not iso.
- If the operator is ONE or PAIR, then C iso with c=1.
- If the operator is FIRST or ANY with C=f(s,A), then C iso with c=s.
- If the operator is SECOND or ANY with C=f(A,s), then C iso with c=s.
- If A is iso then C is iso, with the following value of c:
  - If the op is IDENTITY, then c = a.
  - If the op is unary with C=f(A), then c=f(a).
  - If the op is binary with C=f(s,A), then c = f(s,a).
  - If the op is binary with C=f(A,s), then c = f(a,s).
- Otherwise, C is not iso.

#### 12.7 Iso matrices from select

Let C=select(A) denote the application of a select operator.

- If A is iso, then C is iso with c = a.
- If the select operator is EQ\_ZERO then C is always iso, with c=0.
- If the select operator is NONZERO and A is boolean, then C is iso with c=1.
- If the selector operator is EQ\_THUNK then C is always iso, with c=t where t is the value of the thunk scalar.
- Otherwise, C is not iso.

#### 12.8 Iso matrices from assign and subassign

These rules are somewhat complex. Consider the assignment C<M>(I,J)=... with  $GrB_assign$ . Internally, this assignment is converted into C(I,J)<M(I,J)>=... and then  $GxB_subassign$  is used. Thus, all of the rules below assume the form C(I,J)<M>=... where M has the same size as the submatrix C(I,J).

#### 12.8.1 Assignment with no accumulator operator

If no accumulator operator is present, the following rules are used.

- For matrix assignment, A must be iso. For scalar assignment, the single scalar is implicitly expanded into an iso matrix A of the right size. If these rules do not hold, C is not iso.
- If A is not iso, or if C is not iso on input, then C is not iso on output.
- If C is iso or empty on input, and A is iso (or scalar assignment is begin performed) and the iso values c and a (or the scalar s) match, then the following forms of assignment result in an iso matrix C on output:
  - -C(I,J) = scalar
  - C(I,J) < M > = scalar
  - C(I,J) < !M > = scalar

```
- C(I,J)<M,replace> = scalar
- C(I,J)<!M,replace> = scalar
- C(I,J) = A
- C(I,J)<M> = A
- C(I,J)<!M> = A
- C(I,J)<!M> = A
- C(I,J)<M,replace> = A
- C(I,J)<!M,replace> = A
```

- For these forms of assignment, C is always iso on output, regardless of its iso property on input:
  - C = scalar
  - C<M, struct>=scalar; C empty on input.
  - C<C,struct>=scalar
- For these forms of assignment, C is always iso on output if A is iso:
  - C = A
  - C<M,str> = A; C empty on input.

#### 12.8.2 Assignment with an accumulator operator

If an accumulator operator is present, the following rules are used. Positional operators (GxB\_FIRSTI and related) cannot be used as accumulator operators, so these rules do not consider that case.

- For matrix assignment, A must be iso. For scalar assignment, the single scalar is implicitly expanded into an iso matrix A of the right size. If these rules do not hold, C is not iso.
- For these forms of assignment C is iso if C is empty on input, or if c = c + a for the where a is the iso value of A or the value of the scalar for scalar assignment.
  - C(I,J) += scalar
  - C(I,J) < M > += scalar

```
- C(I,J)<!M> += scalar
- C(I,J)<M,replace> += scalar
- C(I,J)<!M,replace> += scalar
- C(I,J)<M,replace> += A
- C(I,J)<!M,replace> += A
- C(I,J) += A
- C(I,J) += A
- C(I,J)<M> += A
- C(I,J)<!M> += A
- C(I,J)<!M> += A
```

#### 12.9 Iso matrices from build methods

GxB\_Matrix\_build\_Scalar and GxB\_Vector\_build\_Scalar always construct an iso matrix/vector.

GrB\_Matrix\_build and GrB\_Vector\_build can also construct iso matrices and vectors. A non-iso matrix/vector is constructed first, and then the entries are checked to see if they are all equal. The resulting iso-valued matrix/vector will be efficient to use and will use less memory than a non-iso matrix/vector. However, constructing an iso matrix/vector with GrB\_Matrix\_build and GrB\_Vector\_build will take more time and memory than constructing the matrix/vector with GxB\_Matrix\_build\_Scalar or GxB\_Vector\_build\_Scalar.

#### 12.10 Iso matrices from other methods

- For GrB\_Matrix\_dup and GrB\_Vector\_dup, the output matrix/vector has the same iso property as the input matrix/vector.
- GrB\_\*\_setElement\_\* preserves the iso property of the matrix/vector it modifies, if the input scalar is equal to the iso value of the matrix/vector. If the matrix or vector has no entries, the first call to setElement makes it iso. This allows a sequence of setElement calls with the same scalar value to create an entire iso matrix or vector, if starting from an empty matrix or vector.
- GxB\_Matrix\_concat constructs an iso matrix as its result if all input tiles are either empty or iso.

- GxB\_Matrix\_split constructs its output tiles as iso if its input matrix is iso.
- GxB\_Matrix\_diag and GrB\_Matrix\_diag construct an iso matrix if its input vector is iso.
- GxB\_Vector\_diag constructs an iso vector if its input matrix is iso.
- GrB\_\*extract constructs an iso matrix/vector if its input matrix/vector is iso.
- GrB\_transpose constructs an iso matrix if its input is iso.
- The GxB\_import/export/pack/unpack methods preserve the iso property of their matrices/vectors.

## 12.11 Iso matrices not exploited

There are many cases where an matrix may have the iso property but it is not detected by SuiteSparse:GraphBLAS. For example, if A is non-iso, C=A(I,J) from GrB\_extract may be iso, if all entries in the extracted submatrix have the same value. Future versions of SuiteSparse:GraphBLAS may extend the rules described in this section to detect these cases.

## 13 Examples

**NOTE:** The programs in the Demo folder are not always the fastest methods. They are simple methods for illustration only, not performance. Do not benchmark them. Refer to the latest (draft) LAGraph package for the fastest methods. Be sure to use the right combination of package versions between LAGraph and SuiteSparse:GraphBLAS. Contact the author (davis@tamu.edu) if you have any questions about how to properly benchmark LAGraph + SuiteSparse:GraphBLAS.

Several examples of how to use GraphBLAS are listed below. They all appear in the Demo folder of SuiteSparse:GraphBLAS.

- 1. finding a maximal independent set
- 2. creating a random matrix
- 3. creating a finite-element matrix
- 4. reading a matrix from a file
- 5. complex numbers as a user-defined type
- 6. PageRank
- 7. matrix import/export

Additional examples appear in the newly created LAGraph project, currently in progress.

## 13.1 LAGraph

The LAGraph project is a community-wide effort to create graph algorithms based on GraphBLAS (any implementation of the API, not just SuiteSparse: GraphBLAS). Some of the algorithms and utilities in LAGraph are listed in the table below. Many additional algorithms are planned. Refer to <a href="https://github.com/GraphBLAS/LAGraph">https://github.com/GraphBLAS/LAGraph</a> for a current list of algorithms. All functions in the Demo/ folder in SuiteSparse:GraphBLAS will eventually be translated into algorithms or utilities for LAGraph, and then removed from GraphBLAS/Demo.

To use LAGraph with SuiteSparse:GraphBLAS, place the two folders LAGraph and GraphBLAS in the same parent directory. This allows the cmake script in LAGraph to find the copy of GraphBLAS. Alternatively, the GraphBLAS source could be placed anywhere, as long as sudo make install is performed.

### 13.2 Maximal independent set

The maximal independent set problem is to find a set of nodes S such that no two nodes in S are adjacent to each other (an independent set), and all nodes not in S are adjacent to at least one node in S (and thus S is maximal since it cannot be augmented by any node while remaining an independent set). The mis function in the Demo folder solves this problem using Luby's method [Lub86]. The key operations in the method are replicated on the next page.

The gist of the algorithm is this. In each phase, all candidate nodes are given a random score. If a node has a score higher than all its neighbors, then it is added to the independent set. All new nodes added to the set cause their neighbors to be removed from the set of candidates. The process must be repeated for multiple phases until no new nodes can be added. This is because in one phase, a node  $\mathbf i$  might not be added because one of its neighbors  $\mathbf j$  has a higher score, yet that neighbor  $\mathbf j$  might not be added because one of its neighbors  $\mathbf k$  is added to the independent set instead. The node  $\mathbf j$  is no longer a candidate and can never be added to the independent set, but node  $\mathbf i$  could be added to S in a subsequent phase.

The initialization step, before the while loop, computes the degree of each node with a PLUS reduction. The set of candidates is Boolean vector, the ith component is true if node i is a candidate. A node with no neighbors causes the algorithm to stall, so these nodes are not candidates. Instead, they are immediately added to the independent set, represented by another Boolean vector iset. Both steps are done with an assign, using the degree as a mask, except the assignment to iset uses the complement of the mask, via the sr\_desc descriptor. Finally, the GrB\_Vector\_nvals statement counts how many candidates remain.

Each phase of Luby's algorithm consists of 11 calls to GraphBLAS operations, all of which are either parallel, or take O(1) time. Not all of them are described here since they are commented in the code itself. The two matrix-vector multiplications are the important parts and also take the most

time. They also make interesting use of semirings and masks. The first one computes the largest score of all the neighbors of each node in the candidate set:

```
// compute the max probability of all neighbors
GrB_vxm (neighbor_max, candidates, NULL, maxFirst, prob, A, r_desc);
```

A is a symmetric Boolean matrix and prob is a sparse real vector (of type FP32), where prob(i) is nonzero only if node i is a candidate. The prob vector is computed from a random vector computed by a utility function prand\_xget, in the Demo folder. It uses two calls to GrB\_apply to construct n random numbers in parallel, using a repeatable pseudo-random number generator.

The maxFirst semiring uses z=FIRST(x,y) as the multiplier operator. The column A(:,j) is the adjacency of node j, and the dot product prob'\*A(:,j) applies the FIRST operator on all entries that appear in the intersection of prob and A(:,j), where z=FIRST(prob(i),A(i,j)) which is just prob(i) if A(i,j) is present. If A(i,j) not an explicit entry in the matrix, then this term is not computed and does not take part in the reduction by the MAX monoid.

Thus, each term z=FIRST(prob(i),A(i,j)) is the score, prob(i), of all neighbors i of node j that have a score. Node i does not have a score if it is not also a candidate and so this is skipped. These terms are then "summed" up by taking the maximum score, using MAX as the additive monoid.

Finally, the results of this matrix-vector multiply are written to the result, neighbor\_max. The r\_desc descriptor has the REPLACE option enabled. Since neighbor\_max does not also take part in the computation prob'\*A, it is simply cleared first. Next, is it modified only in those positions i where candidates(i) is true, using candidates as a mask. This sets the neighbor\_max only for candidate nodes, and leaves the other components of neighbor\_max as zero (implicit values not in the pattern of the vector).

All of the above work is done in a single matrix-vector multiply, with an elegant use of the maxFirst semiring coupled with a mask. The matrix-vector multiplication is described above as if it uses dot products of rows of A with the column vector prob, but SuiteSparse:GraphBLAS does not compute it that way. Sparse dot products are much slower the optimal method for multiplying a sparse matrix times a sparse vector. The result is the same, however.

The second matrix-vector multiplication is more straight-forward. Once the set of new members in the independent is found, it is used to remove all neighbors of those new members from the set of candidates.

```
// compute the degree of each node
GrB_reduce (degrees, NULL, NULL, GrB_PLUS_FP64, A, NULL) ;
// singletons are not candidates; they are added to iset first instead
// candidates[degree != 0] = 1
GrB_assign (candidates, degrees, NULL, true, GrB_ALL, n, NULL);
// add all singletons to iset
// iset[degree == 0] = 1
GrB_assign (iset, degrees, NULL, true, GrB_ALL, n, sr_desc) ;
// Iterate while there are candidates to check.
GrB_Index nvals ;
GrB_Vector_nvals (&nvals, candidates);
while (nvals > 0)
    // sparsify the random number seeds (just keep it for each candidate)
    GrB_assign (Seed, candidates, NULL, Seed, GrB_ALL, n, r_desc) ;
    // compute a random probability scaled by inverse of degree
    prand_xget (X, Seed) ; // two calls to GrB_apply
    GrB_eWiseMult (prob, candidates, NULL, set_random, degrees, X, r_desc) ;
    // compute the max probability of all neighbors
    GrB_vxm (neighbor_max, candidates, NULL, maxFirst, prob, A, r_desc) ;
    // select node if its probability is > than all its active neighbors
    GrB_eWiseAdd (new_members, NULL, NULL, GrB_GT_FP64, prob, neighbor_max,0);
    // add new members to independent set.
    GrB_eWiseAdd (iset, NULL, NULL, GrB_LOR, iset, new_members, NULL) ;
    // remove new members from set of candidates c = c & !new
    GrB_apply (candidates, new_members, NULL, GrB_IDENTITY_BOOL,
        candidates, sr_desc) ;
    GrB_Vector_nvals (&nvals, candidates);
    if (nvals == 0) { break ; }
                                                 // early exit condition
    // Neighbors of new members can also be removed from candidates
    GrB_vxm (new_neighbors, candidates, NULL, Boolean,
        new_members, A, NULL);
    GrB_apply (candidates, new_neighbors, NULL, GrB_IDENTITY_BOOL,
        candidates, sr_desc) ;
    GrB_Vector_nvals (&nvals, candidates) ;
```

### 13.3 Creating a random matrix

The random\_matrix function in the Demo folder generates a random matrix with a specified dimension and number of entries, either symmetric or unsymmetric, and with or without self-edges (diagonal entries in the matrix). It relies on simple\_rand\* functions in the Demo folder to provide a portable random number generator that creates the same sequence on any computer and operating system.

random\_matrix can use one of two methods: GrB\_Matrix\_setElement and GrB\_Matrix\_build. The former method is very simple to use:

```
GrB_Matrix_new (&A, GrB_FP64, nrows, ncols) ;
for (int64_t k = 0 ; k < ntuples ; k++)
{
    GrB_Index i = simple_rand_i () % nrows ;
    GrB_Index j = simple_rand_i () % ncols ;
    if (no_self_edges && (i == j)) continue ;
    double x = simple_rand_x () ;
    // A (i,j) = x
    GrB_Matrix_setElement (A, x, i, j) ;
    if (make_symmetric)
    {
        // A (j,i) = x
        GrB_Matrix_setElement (A, x, j, i) ;
    }
}</pre>
```

The above code can generate a million-by-million sparse double matrix with 200 million entries in 66 seconds (6 seconds of which is the time to generate the random i, j, and x), including the time to finish all pending computations. The user application does not need to create a list of all the tuples, nor does it need to know how many entries will appear in the matrix. It just starts from an empty matrix and adds them one at a time in arbitrary order. GraphBLAS handles the rest. This method is not feasible in MATLAB.

The next method uses GrB\_Matrix\_build. It is more complex to use than setElement since it requires the user application to allocate and fill the tuple lists, and it requires knowledge of how many entries will appear in the matrix, or at least a good upper bound, before the matrix is constructed. It is slightly faster, creating the same matrix in 60 seconds, 51 seconds of which is spent in GrB\_Matrix\_build.

```
GrB_Index *I, *J;
double *X ;
int64_t s = ((make_symmetric) ? 2 : 1) * nedges + 1 ;
I = malloc (s * sizeof (GrB_Index)) ;
J = malloc (s * sizeof (GrB_Index));
X = malloc (s * sizeof (double
                                 ));
if (I == NULL || J == NULL || X == NULL)
{
    // out of memory
    if (I != NULL) free (I);
    if (J != NULL) free (J);
    if (X != NULL) free (X);
   return (GrB_OUT_OF_MEMORY) ;
int64_t ntuples = 0 ;
for (int64_t k = 0 ; k < nedges ; k++)
    GrB_Index i = simple_rand_i ( ) % nrows ;
    GrB_Index j = simple_rand_i ( ) % ncols ;
    if (no_self_edges && (i == j)) continue ;
    double x = simple_rand_x ( );
    // A (i,j) = x
    I [ntuples] = i ;
    J [ntuples] = j ;
    X [ntuples] = x ;
   ntuples++;
    if (make_symmetric)
        // A (j,i) = x
        I [ntuples] = j ;
        J [ntuples] = i ;
        X [ntuples] = x ;
        ntuples++;
    }
GrB_Matrix_build (A, I, J, X, ntuples, GrB_SECOND_FP64) ;
```

The equivalent sprandsym function in MATLAB takes 150 seconds, but sprandsym uses a much higher-quality random number generator to create the tuples [I,J,X]. Considering just the time for sparse(I,J,X,n,n) in sprandsym (equivalent to GrB\_Matrix\_build), the time is 70 seconds. That is, each of these three methods, setElement and build in Suite-Sparse:GraphBLAS, and sparse in MATLAB, are equally fast.

### 13.4 Creating a finite-element matrix

Suppose a finite-element matrix is being constructed, with k=40,000 finite-element matrices, each of size 8-by-8. The following operations (in pseudo-MATLAB notation) are very efficient in SuiteSparse:GraphBLAS.

```
A = sparse (m,n) ; % create an empty n-by-n sparse GraphBLAS matrix
for i = 1:k
    construct a 8-by-8 sparse or dense finite-element F
    I and J define where the matrix F is to be added:
    I = a list of 8 row indices
    J = a list of 8 column indices
    % using GrB_assign, with the 'plus' accum operator:
    A (I,J) = A (I,J) + F
end
```

If this were done in MATLAB or in GraphBLAS with blocking mode enabled, the computations would be extremely slow. A far better approach is to construct a list of tuples [I,J,X] and to use sparse(I,J,X,n,n). This is identical to creating the same list of tuples in GraphBLAS and using the GrB\_Matrix\_build, which is equally fast.

In SuiteSparse:GraphBLAS, the performance of both methods is essentially identical, and roughly as fast as sparse in MATLAB. Inside SuiteSparse:GraphBLAS, GrB\_assign is doing the same thing. When performing A(I,J)=A(I,J)+F, if it finds that it cannot quickly insert an update into the A matrix, it creates a list of pending tuples to be assembled later on. When the matrix is ready for use in a subsequent GraphBLAS operation (one that normally cannot use a matrix with pending computations), the tuples are assembled all at once via GrB\_Matrix\_build.

GraphBLAS operations on other matrices have no effect on when the pending updates of a matrix are completed. Thus, any GraphBLAS method or operation can be used to construct the F matrix in the example above, without affecting when the pending updates to A are completed.

The MATLAB wathen.m script is part of Higham's gallery of matrices [Hig02]. It creates a finite-element matrix with random coefficients for a 2D mesh of size nx-by-ny, a matrix formulation by Wathen [Wat87]. The pattern of the matrix is fixed; just the values are randomized. The GraphBLAS equivalent can use either GrB\_Matrix\_build, or GrB\_assign. Both methods have good performance. The GrB\_Matrix\_build version below is about 15% to 20% faster than the MATLAB wathen.m function, regardless of the problem size. It uses the identical algorithm as wathen.m.

```
int64_t ntriplets = nx*ny*64 ;
I = malloc (ntriplets * sizeof (int64_t));
J = malloc (ntriplets * sizeof (int64_t));
X = malloc (ntriplets * sizeof (double )) ;
if (I == NULL || J == NULL || X == NULL)
    FREE_ALL ;
    return (GrB_OUT_OF_MEMORY) ;
ntriplets = 0 ;
for (int j = 1; j \le ny; j++)
   for (int i = 1; i <= nx; i++)
        nn [0] = 3*j*nx + 2*i + 2*j + 1;
        nn [1] = nn [0] - 1;
        nn [2] = nn [1] - 1;
        nn [3] = (3*j-1)*nx + 2*j + i - 1;
        nn [4] = 3*(j-1)*nx + 2*i + 2*j - 3;
        nn [5] = nn [4] + 1;
        nn [6] = nn [5] + 1;
        nn [7] = nn [3] + 1;
        for (int krow = 0; krow < 8; krow++) nn [krow]--;
        for (int krow = 0; krow < 8; krow++)
            for (int kcol = 0; kcol < 8; kcol++)
            {
               I [ntriplets] = nn [krow] ;
                J [ntriplets] = nn [kcol] ;
               X [ntriplets] = em (krow,kcol) ;
               ntriplets++ ;
            }
        }
   }
// A = sparse (I,J,X,n,n);
GrB_Matrix_build (A, I, J, X, ntriplets, GrB_PLUS_FP64) ;
```

The GrB\_assign version has the advantage of not requiring the user application to construct the tuple list, and is almost as fast as using GrB\_Matrix\_build. The code is more elegant than either the MATLAB wathen.m function or its GraphBLAS equivalent above. Its performance is comparable with the other two methods, but slightly slower, being about 5% slower than the MATLAB wathen, and 20% slower than the GraphBLAS method above.

```
GrB_Matrix_new (&F, GrB_FP64, 8, 8);
for (int j = 1; j \le ny; j++)
    for (int i = 1; i \le nx; i++)
        nn [0] = 3*j*nx + 2*i + 2*j + 1;
        nn [1] = nn [0] - 1;
        nn [2] = nn [1] - 1;
        nn [3] = (3*j-1)*nx + 2*j + i - 1;
        nn [4] = 3*(j-1)*nx + 2*i + 2*j - 3;
       nn [5] = nn [4] + 1;
       nn [6] = nn [5] + 1;
       nn [7] = nn [3] + 1;
        for (int krow = 0; krow < 8; krow++) nn [krow]--;
        for (int krow = 0 ; krow < 8 ; krow++)</pre>
        {
            for (int kcol = 0; kcol < 8; kcol++)
                // F (krow,kcol) = em (krow, kcol)
               GrB_Matrix_setElement (F, em (krow,kcol), krow, kcol) ;
        }
        // A (nn,nn) += F
        GrB_assign (A, NULL, GrB_PLUS_FP64, F, nn, 8, nn, 8, NULL);
    }
}
```

Since there is no Mask, and since GrB\_REPLACE is not used, the call to GrB\_assign in the example above is identical to GxB\_subassign. Either one can be used, and their performance would be identical.

Refer to the wathen.c function in the Demo folder, which uses GraphBLAS to implement the two methods above, and two additional ones.

### 13.5 Reading a matrix from a file

See also LAGraph\_mmread and LAGraph\_mmwrite, which can read and write any matrix in Matrix Market format, and LAGraph\_binread and LAGraph\_binwrite, which read/write a matrix from a binary file. The binary file I/O functions are much faster than the read\_matrix function described here, and also much faster than LAGraph\_mmread and LAGraph\_mmwrite.

The read\_matrix function in the Demo reads in a triplet matrix from a file, one line per entry, and then uses GrB\_Matrix\_build to create the matrix. It creates a second copy with GrB\_Matrix\_setElement, just to test that method and compare the run times. Section 13.3 has already compared build versus setElement.

The function can return the matrix as-is, which may be rectangular or unsymmetric. If an input parameter is set to make the matrix symmetric, read\_matrix computes A=(A+A')/2 if A is square (turning all directed edges into undirected ones). If A is rectangular, it creates a bipartite graph, which is the same as the augmented matrix,  $A = [0 \ A \ ; A' \ 0]$ . If C is an n-by-n matrix, then C=(C+C')/2 can be computed as follows in GraphBLAS, (the scale2 function divides an entry by 2):

```
GrB_Descriptor_new (&dt2) ;
GrB_Descriptor_set (dt2, GrB_INP1, GrB_TRAN) ;
GrB_Matrix_new (&A, GrB_FP64, n, n) ;
GrB_eWiseAdd (A, NULL, NULL, GrB_PLUS_FP64, C, C, dt2) ; // A=C+C'
GrB_free (&C) ;
GrB_Matrix_new (&C, GrB_FP64, n, n) ;
GrB_UnaryOp_new (&scale2_op, scale2, GrB_FP64, GrB_FP64) ;
GrB_apply (C, NULL, NULL, scale2_op, A, NULL) ; // C=A/2
GrB_free (&A) ;
GrB_free (&scale2_op) ;
```

This is of course not nearly as elegant as A=(A+A')/2 in MATLAB, but with minor changes it can work on any type and use any built-in operators instead of PLUS, or it can use any user-defined operators and types. The above code in SuiteSparse:GraphBLAS takes 0.60 seconds for the Freescale2 matrix, slightly slower than MATLAB (0.55 seconds).

Constructing the augmented system is more complicated using the Graph-BLAS C API Specification since it does not yet have a simple way of specifying a range of row and column indices, as in A(10:20,30:50) in MATLAB (GxB\_RANGE is a SuiteSparse:GraphBLAS extension that is not in the Speci-

fication). Using the C API in the Specification, the application must instead build a list of indices first, I=[10, 11 ... 20].

Thus, to compute the MATLAB equivalent of  $A = [0 \ A \ ; \ A' \ 0]$ , index lists I and J must first be constructed:

```
int64_t n = nrows + ncols ;
I = malloc (nrows * sizeof (int64_t)) ;
J = malloc (ncols * sizeof (int64_t)) ;
// I = 0:nrows-1
// J = nrows:n-1
if (I == NULL || J == NULL)
{
    if (I != NULL) free (I) ;
    if (J != NULL) free (J) ;
    return (GrB_OUT_OF_MEMORY) ;
}
for (int64_t k = 0 ; k < nrows ; k++) I [k] = k ;
for (int64_t k = 0 ; k < ncols ; k++) J [k] = k + nrows ;</pre>
```

Once the index lists are generated, however, the resulting GraphBLAS operations are fairly straightforward, computing A=[0 C; C'0].

```
GrB_Descriptor_new (&dt1) ;
GrB_Descriptor_set (dt1, GrB_INPO, GrB_TRAN) ;
GrB_Matrix_new (&A, GrB_FP64, n, n) ;
// A (nrows:n-1, 0:nrows-1) = C'
GrB_assign (A, NULL, NULL, C, J, ncols, I, nrows, dt1) ;
// A (0:nrows-1, nrows:n-1) = C
GrB_assign (A, NULL, NULL, C, I, nrows, J, ncols, NULL) ;
```

This takes 1.38 seconds for the Freescale2 matrix, almost as fast as A=[sparse(m,m) C; C' sparse(n,n)] in MATLAB (1.25 seconds).

Both calls to GrB\_assign use no accumulator, so the second one causes the partial matrix A=[0 0; C'0] to be built first, followed by the final build of A=[0 C; C'0]. A better method, but not an obvious one, is to use the GrB\_FIRST\_FP64 accumulator for both assignments. An accumulator enables SuiteSparse:GraphBLAS to determine that that entries created by the first assignment cannot be deleted by the second, and thus it need not force completion of the pending updates prior to the second assignment.

SuiteSparse:GraphBLAS also adds a  $GxB_RANGE$  mechanism that mimics the MATLAB colon notation. This speeds up the method and simplifies the code the user needs to write to compute  $A=[0\ C\ ;\ C'\ 0]$ :

Any operator will suffice because it is not actually applied. An operator is only applied to the set intersection, and the two assignments do not overlap. If an accum operator is used, only the final matrix is built, and the time in GraphBLAS drops slightly to 1.25 seconds. This is a very small improvement because in this particular case, SuiteSparse:GraphBLAS is able to detect that no sorting is required for the first build, and the second one is a simple concatenation. In general, however, allowing GraphBLAS to postpone pending updates can lead to significant reductions in run time.

### 13.6 PageRank

The Demo folder contains three methods for computing the PageRank of the nodes of a graph. One uses floating-point arithmetic (GrB\_FP64) and two user-defined unary operators (dpagerank.c). The second (ipagerank.c) is very similar, relying on integer arithmetic instead (GrB\_UINT64). Neither method include a stopping condition. They simply compute a fixed number of iterations. The third example is more extensive (dpagerank2.c), and serves as an example of the power and flexibility of user-defined types, operators, monoids, and semirings. It creates a semiring for the entire PageRank computation. It terminates if the 2-norm of the change in the rank vector r is below a threshold.

LAGraph contains a much faster pagerank method, as LAGraph\_VertexCentrality\_PageRankGAP, which can be used for the GAP benchmarks of Scott Beamer. Do **not** benchmark the pagerank methods in the GraphBLAS/Demo folder.

### 13.7 User-defined types and operators

The Demo folder contains two working examples of user-defined types, first discussed in Section 6.1.1: double complex, and a user-defined typedef called wildtype with a struct containing a string and a 4-by-4 float matrix.

**Double Complex:** Prior to v3.3, GraphBLAS did not have a native complex type. It now appears as the GxB\_FC64 predefined type, but a complex type can also easily added as a user-defined type. The Complex\_init function in the usercomplex.c file in the Demo folder creates the Complex type based on the ANSI C11 double complex type. It creates a full suite of operators that correspond to every built-in GraphBLAS operator, both binary and unary. In addition, it creates the operators listed in the following table, where D is double and C is Complex.

name	types	Octave/MATLAB equivalent	description
Complex_complex	$D \times D \to C$	z=complex(x,y)	complex from real and imag.
Complex_conj	$C \to C$	z=conj(x)	complex conjugate
Complex_real	$C \to D$	z=real(x)	real part
Complex_imag	$C \to D$	z=imag(x)	imaginary part
Complex_angle	$C \to D$	z=angle(x)	phase angle
Complex_complex_real	$D \to C$	z=complex(x,0)	real to complex real
Complex_complex_imag	$D \to C$	z=complex(0,x)	real to complex imag.

The Complex\_init function creates two monoids (Complex\_add\_monoid and Complex\_times\_monoid) and a semiring Complex\_plus\_times that corresponds to the conventional linear algebra for complex matrices. The include file usercomplex.h in the Demo folder is available so that this user-defined Complex type can easily be imported into any other user application. When the user application is done, the Complex\_finalize function frees the Complex type and its operators, monoids, and semiring. NOTE: the Complex type is not supported in this Demo in Microsoft Visual Studio.

Struct-based: In addition, the wildtype.c program creates a user-defined typedef of a struct containing a dense 4-by-4 float matrix, and a 64-character string. It constructs an additive monoid that adds two 4-by-4 dense matrices, and a multiplier operator that multiplies two 4-by-4 matrices. Each of these 4-by-4 matrices is treated by GraphBLAS as a "scalar" value, and they can be manipulated in the same way any other GraphBLAS type can be manipulated. The purpose of this type is illustrate the endless possibilities of user-defined types and their use in GraphBLAS.

## 13.8 User applications using OpenMP or other threading models

An example demo program (openmp\_demo) is included that illustrates how a multi-threaded user application can use GraphBLAS.

The results from the openmp\_demo program may appear out of order. This is by design, simply to show that the user application is running in parallel. The output of each thread should be the same. In particular, each thread generates an intentional error, and later on prints it with GrB\_error. It will print its own error, not an error from another thread. When all the threads finish, the leader thread prints out each matrix generated by each thread.

GraphBLAS can also be combined with user applications that rely on MPI, the Intel TBB threading library, POSIX pthreads, Microsoft Windows threads, or any other threading library. In all cases, GraphBLAS will be thread safe.

## 14 Compiling and Installing SuiteSparse:GraphBLAS

#### 14.1 On Linux and Mac

GraphBLAS makes extensive use of features in the ANSI C11 standard, and thus a C compiler supporting this version of the C standard is required to use all features of GraphBLAS. On the Mac (OS X), clang 8.0.0 in Xcode version 8.2.1 is sufficient, although earlier versions of Xcode may work as well. For the GNU gcc compiler, version 4.9 or later is required. For the Intel icc compiler, version 18.0 or later is required. Version 2.8.12 or later of cmake is required; version 3.0.0 is preferred.

If you are using a pre-C11 ANSI C compiler, or Microsoft Visual Studio, then the \_Generic keyword is not available. SuiteSparse:GraphBLAS will still compile, but you will not have access to polymorphic functions such as GrB\_assign. You will need to use the non-polymorphic functions instead.

**NOTE:** icc is generally an excellent compiler, but it will generate slower code than gcc for SuiteSparse:GraphBLAS. This is because of how the two compilers treat #pragma omp atomic. The use of gcc is strongly recommended. Atomics are far slower in icc as compared to gcc. Intel has a new icx compiler that has much better performance for atomics, but I have not tested it with GraphBLAS.

To compile SuiteSparse:GraphBLAS, simply type make in the main Graph-BLAS folder, which compiles the library. This will be a single-threaded compilation, which will take a long time. To compile in parallel (40 threads for example), use:

make JOBS=40

To use a non-default compiler with 4 threads:

make CC=icc CXX=icc JOBS=4

After compiling the library, you can compile the demos with make all and then make run.

If cmake or make fail, it might be that your default compiler does not support ANSI C11. Try another compiler. For example, try one of these options. Go into the build directory and type one of these:

```
CC=gcc cmake ..

CC=gcc-6 cmake ..

CC=xlc cmake ..

CC=icc cmake ..
```

You can also do the following in the top-level GraphBLAS folder instead:

```
CC=gcc make
CC=gcc-6 cmake
CC=xlc cmake
CC=icc cmake
```

For faster compilation, you can specify a parallel make. For example, to use 32 parallel jobs and the gcc compiler, do the following:

```
JOBS=32 CC=gcc make
```

If you do not have cmake, refer to Section 14.8.

#### 14.2 More details on the Mac

SuiteSparse:GraphBLAS requires OpenMP for its internal parallelism, but OpenMP is not on the Mac by default.

If you have the Intel compiler and OpenMP library, then use the following in the top-level GraphBLAS folder. OpenMP will be found automatically:

```
make CC=icc CXX=icc
```

The following instructions work on MacOS Big Sur (v11.3), using cmake 3.13 or later:

First install Xcode (see https://developer.apple.com/xcode, and then install the command line tools for Xcode:

```
cd /Applications/Utilities
xcode-select install
```

Next, install brew, at https://brew.sh.

If not used for the MATLAB mexFunction interface, Clang now works with libomp and the GraphBLAS/CMakeLists.txt. To use the MATLAB mexFunction, however, you must use gcc-11.

With MacOSx Big Sur install gcc-11, cmake, and OpenMP, and then compile GraphBLAS. cmake 3.13 or later is required. For the MATLAB mexFunctions, you must use gcc-11; the libomp from brew will allow you to compile the mexFunctions but they will not work properly.

```
brew install cmake
brew install libomp
brew install gcc
cd GraphBLAS/GraphBLAS
make CC=gcc-11 CXX=g++-11 JOBS=8
```

The above instructions assume MATLAB R2021a, using libgraphblas\_renamed.dylib, since that version of MATLAB includes its own copy of SuiteSparse:GraphBLAS (libmwgraphblas.dylib) but at version v3.3.3, not the latest version.

Next, compile the MATLAB mexFunctions. I had to edit this file first:

/Users/davis/Library/Application Support/MathWorks/MATLAB/R2021a/mex\_C\_maci64.xml

where you would replace davis with your MacOS user name. Change lines 4 and 18, where both cases of MACOSX\_DEPLOYMENT\_TARGET=10.14 must become MACOSX\_DEPLOYMENT\_TARGET=11.3. Otherwise, MATLAB complains that the libgraphblas\_renamed.dylib was built for 11.3 but linked for 10.14.

Next, type the following in the MATLAB Command Window:

```
cd GraphBLAS/GraphBLAS/@GrB/private
gbmake
```

Then add the paths to your startup.m file (usually in ~/Documents/MATLAB/startup.m). For example, my path is:

```
addpath ('/Users/davis/GraphBLAS/GraphBLAS');
addpath ('/Users/davis/GraphBLAS/GraphBLAS/build');
```

Finally, you can run the tests to see if your installation works:

```
cd ../../test
gbtest
```

#### 14.3 On Microsoft Windows

SuiteSparse:GraphBLAS is now ported to Microsoft Visual Studio. However, that compiler is not ANSI C11 compliant. As a result, GraphBLAS on Windows will have a few minor limitations.

- The MS Visual Studio compiler does not support the \_Generic keyword, required for the polymorphic GraphBLAS functions. So for example, you will need to use GrB\_Matrix\_free instead of just GrB\_free.
- Variable-length arrays are not supported, so user-defined types are limited to 128 bytes in size. This can be changed by editing GB\_VLA\_MAXSIZE in Source/GB\_compiler.h, and recompiling SuiteSparse:GraphBLAS.

If you use a recent gcc or icc compiler on Windows other than the Microsoft Compiler (cl), these limitations can be avoided.

The following instructions apply to Windows 10, CMake 3.16, and Visual Studio 2019, but may work for earlier versions.

- 1. Install CMake 3.16 or later, if not already installed. See https://cmake.org/ for details.
- 2. Install Microsoft Visual Studio, if not already installed. See <a href="https://visualstudio.microsoft.com/">https://visualstudio.microsoft.com/</a> for details. Version 2019 is preferred, but earlier versions may also work.
- 3. Open a terminal window and type this in the SuiteSparse/GraphBLAS/build folder:

cmake ..

- 4. The cmake command generates many files in SuiteSparse/GraphBLAS/build, and the file graphblas.sln in particular. Open the generated graphblas.sln file in Visual Studio.
- 5. Optionally: right-click graphblas in the left panel (Solution Explorer) and select properties; then navigate to Configuration Properties, C/C++, General and change the parameter Multiprocessor Compilation to Yes (/MP). Click OK. This will significantly speed up the compilation of GraphBLAS.

- 6. Select the Build menu item at the top of the window and select Build Solution. This should create a folder called Release and place the compiled graphblas.dll, graphblas.lib, and graphblas.exp files there. Please be patient; some files may take a while to compile and sometimes may appear to be stalled. Just wait.
- 7. Add the GraphBLAS/build/Release folder to the Windows System path:
  - Open the Start Menu and type Control Panel.
  - Select the Control Panel app.
  - When the app opens, select System and Security.
  - Under System and Security, select System.
  - From the top left side of the System window, select Advanced System Settings. You may have to authenticate at this step.
  - The Systems Properties window should appear with the Advanced tab selected; select Environment Variables.
  - The Environment Variables window displays 2 sections, one for User variables and the other for System variables. Under the Systems variable section, scroll to and select Path, then select Edit. A editor window appears allowing to add, modify, delete or re-order the parts of the Path.
  - Add the full path of the GraphBLAS\build\Release folder (typically starting with C:\Users\you\..., where you is your Windows username) to the Path.
  - If the above steps do not work, you can instead copy the graphblas.\* files from GraphBLAS\build\Release into any existing folder listed in your Path.
- 8. The GraphBLAS/Include/GraphBLAS.h file must be included in user applications via #include "GraphBLAS.h". This is already done for you in the Octave/MATLAB interface discussed in the next section.

# 14.4 Compiling the Octave/MATLAB interface (for Octave, and for MATLAB R2020a and earlier)

First, compile the SuiteSparse:GraphBLAS dynamic library (libgraphblas.so for Linux, libgraphblas.dylib for Mac, or graphblas.dll for Windows), as described in the prior two subsections. Next:

1. In the Octave/MATLAB command window:

cd GraphBLAS/GraphBLAS/@GrB/private
gbmake

- 2. Follow the remaining instructions in the GraphBLAS/GraphBLAS/README.md file, to revise your Octave/MATLAB path and startup.m file.
- 3. As a quick test, try the command GrB(1), which creates and displays a 1-by-1 GraphBLAS matrix. For a longer test, do the following:

cd GraphBLAS/GraphBLAS/test
gbtest

4. In Windows, if the tests fail with an error stating that the mex file is invalid because the module could not be found, it means that MAT-LAB could not find the compiled graphblas.lib, \*.dll or \*.exp files in the build/Release folder. This can happen if your Windows System path is not set properly, or if Windows is not recognizing the GraphBLAS/build/Release folder (see Section 14.3) Or, you might have permission to change your Windows System path. In this case, do the following in the MATLAB Command Window:

cd GraphBLAS/build/Release
GrB(1)

After this step, the GraphBLAS library will be loaded into MATLAB. You may need to add the above lines in your Documents/MATLAB/startup.m file, so that they are done each time MATLAB starts. You will also need to do this after clear all or clear mex, since those MATLAB commands remove all loaded libraries from MATLAB.

You might also get an error "the specified procedure cannot be found." This can occur if you have upgraded your GraphBLAS library from a prior version, and some of the compiled files @GrB/private/\*.mex\* are stale. Try the command gbmake all in the MATLAB Command Window, which forces all of the MATLAB interface to be recompiled. Or, try deleting all @GrB/private/\*.mex\* files and running gbmake again.

5. On Windows, the casin, casinf, casinh, and casinhf functions provided by Microsoft do not return the correct imaginary part. As a result, GxB\_ASIN\_FC32, GxB\_ASIN\_FC64 GxB\_ASINH\_FC32, and GxB\_ASINH\_FC64 do not work properly on Windows. This affects the GrB/asin, GrB/acsc, GrB/asinh, and GrB/acsch, functions in the MATLAB interface. See the MATLAB tests bypassed in gbtest76.m for details, in the GraphBLAS/GraphBLAS/test folder.

# 14.5 Compiling the Octave/MATLAB interface (for MATLAB R2021a and later)

MATLAB R2021a includes its own copy of SuiteSparse:GraphBLAS v3.3.3, as the file libmwgraphblas.so, which is used for the built-in C=A\*B when both A and B are sparse (see the Release Notes of MATLAB R2021a, which discusses the performance gained in MATLAB by using GraphBLAS).

That's great news for the impact of GraphBLAS on MATLAB itself, and the domain of high performance computing in general, but it causes a linking problem when using this MATLAB interface for GraphBLAS. The two use different versions of the same library, and a segfault arises if the MATLAB interface for v5.x tries to link with the older GraphBLAS v3.3.3 library. Likewise, the built-in C=A\*B causes a segfault if it tries to use the newer GraphBLAS v4.x or v5.x libraries.

To resolve this issue, a second GraphBLAS library must be compiled, libgraphblas\_renamed, where the internal symbols are all renamed so they do not conflict with the libmwgraphblas library. Then both libraries can co-exist in the same instance of MATLAB.

To do this, go to the GraphBLAS/GraphBLAS folder, containing the MAT-LAB interface. That folder contains a CMakeLists.txt file to compile the libgraphblas\_renamed library. See the instructions for how to compile the C library libgraphblas, and repeat them but using the folder

SuiteSparse/GraphBLAS/GraphBLAS/build instead of SuiteSparse/GraphBLAS/build.

This will compile the renamed SuiteSparse:GraphBLAS dynamic library (libgraphblas\_renamed.so for Linux, libgraphblas\_renamed.dylib for Mac, or graphblas\_renamed.dll for Windows). These can be placed in the same system-wide location as the standard libgraphblas libraries, such as /usr/local/lib for Linux. The two pairs of libraries share the identical GraphBLAS.h include file.

Next, compile the MATLAB interface as described in Section 14.4. For any instructions in that Section that refer to the GraphBLAS/build folder (Linux and Mac) or GraphBLAS/build/Release (Windows), use GraphBLAS/GraphBLAS/build (Linux and Mac) or GraphBLAS/GraphBLAS/build/Release (Windows) instead.

The resulting functions for your **@GrB** object will now work just fine; no other changes are needed. You can even use the GraphBLAS mexFunctions compiled in MATLAB R2021a in earlier versions of MATLAB (such as R2020a).

### 14.6 Default matrix format

By default, SuiteSparse:GraphBLAS stores its matrices by row, using the GxB\_BY\_ROW format. You can change the default at compile time to GxB\_BY\_COL using cmake -DBYCOL=1. For example:

```
cmake -DBYCOL=1 ..
```

The user application can also use GxB\_get and GxB\_set to set and query the global option (see also Sections 8.10 and 8.11):

```
GxB_Format_Value s ;
GxB_get (GxB_FORMAT, &s) ;
if (s == GxB_BY_COL) printf ("all new matrices are stored by column\n") ;
else printf ("all new matrices are stored by row\n") ;
```

## 14.7 Setting the C flags and using CMake

The above options can also be combined. For example, to use the gcc compiler, to change the default format GxB\_FORMAT\_DEFAULT to GxB\_BY\_COL, use the following cmake command while in the GraphBLAS/build directory:

```
CC=gcc cmake -DBYCOL=1 ...
```

Then do make in the build directory. If this still fails, see the CMakeLists.txt file. You can edit that file to pass compiler-specific options to your compiler. Locate this section in the CMakeLists.txt file. Use the set command in cmake, as in the example below, to set the compiler flags you need.

```
# check which compiler is being used. If you need to make
# compiler-specific modifications, here is the place to do it.
if ("${CMAKE_C_COMPILER_ID}" STREQUAL "GNU")
    # cmake 2.8 workaround: gcc needs to be told to do ANSI C11.
    # cmake 3.0 doesn't have this problem.
    set ( CMAKE_C_FLAGS "${CMAKE_C_FLAGS} -std=c11 -lm " )
    ...
elseif ("${CMAKE_C_COMPILER_ID}" STREQUAL "Intel")
    ...
elseif ("${CMAKE_C_COMPILER_ID}" STREQUAL "Clang")
    ...
elseif ("${CMAKE_C_COMPILER_ID}" STREQUAL "MSVC")
    ...
endif ( )
```

To compile SuiteSparse:GraphBLAS without running the demos, use make library in the top-level directory, or make in the build directory.

Several compile-time options can be selected by editing the Source/GB.h file, but these are meant only for code development of SuiteSparse:GraphBLAS itself, not for end-users of SuiteSparse:GraphBLAS.

## 14.8 Using a plain makefile

The GraphBLAS/alternative directory contains a simple Makefile that can be used to compile SuiteSparse:GraphBLAS. This is a useful option if you do not have the required version of cmake. This Makefile can even compile the entire library with a C++ compiler, which cannot be done with CMake.

This alternative Makefile does not build the libgraphblas\_renamed.so library required for MATLAB R2021a (see Section 14.5). This can be done by revising the Makefile, however: add the -DGBRENAME=1 flag, and change the library name from libgraphblas to libgraphbas\_renamed.

### 14.9 Running the Demos

After make in the top-level directory to compile the library, type make run to run the demos. You can also run the demos after compiling:

```
cd Demo
```

The ./demo command is a script that runs the demos with various input matrices in the Demo/Matrix folder. The output of the demos will be compared with expected output files in Demo/Output.

DO NOT publish benchmarks of these demos, and do not link against the demo library in any user application. These codes are sometimes slow, and are meant as simple illustrations only, not for performance. The fastest methods are in LAGraph, not in SuiteSparse/GraphBLAS/Demo. Benchmark LAGraph instead. Eventually, all GraphBLAS/Demos methods will be removed, and LAGraph will serve all uses: for illustration, benchmarking, and production uses.

## 14.10 Installing SuiteSparse:GraphBLAS

To install the library (typically in /usr/local/lib and /usr/local/include for Linux systems), go to the top-level GraphBLAS folder and type:

sudo make install

## 14.11 Linking issues

My linux distro (Ubuntu 18.04) includes a copy of libgraphblas.so.1, which as SuiteSparse:GraphBLAS v1.1.2. After installing SuiteSparse:GraphBLAS in /usr/local/lib (with sudo make install), compiling a simple standalone program links against libgraphblas.so.1 instead of the latest version, while at the same time accessing the v5.0 include file as /usr/local/include/GraphBLAS.h. This command fails:

```
gcc prog.c -lgraphblas
```

Revising my LD\_LIBRARY\_PATH to put /usr/local/lib first in the library director order didn't help. If you encounter this problem, try one of the following options (all four work for me, and link against the proper version, /usr/local/lib/libgraphblas.so.5.2.0):

```
gcc prog.c -1:libgraphblas.so.5
gcc prog.c -1:libgraphblas.so.5.2.0
gcc prog.c /usr/local/lib/libgraphblas.so
gcc prog.c -Wl,-v -L/usr/local/lib -lgraphblas

This prog.c test program is a trivial one, which works in v1.0 and later:
#include <GraphBLAS.h>
int main (void)
{
    GrB_init (GrB_NONBLOCKING) ;
    GrB_finalize ( ) ;
```

Compile the program above, then use this command to ensure libgraphblas.so.5 appears:

ldd a.out

}

## 14.12 Running the tests

To run a short test, type make run at the top-level GraphBLAS folder. This will run all the demos in GraphBLAS/Demos. MATLAB is not required.

To perform the extensive tests in the Test folder, and the statement coverage tests in Tcov, MATLAB R2017A is required. See the README.txt files in those two folders for instructions on how to run the tests. The tests in the Test folder have been ported to MATLAB on Linux, MacOS, and Windows. The Tcov tests do not work on Windows. The MATLAB interface test (gbtest) works on all platforms; see the GraphBLAS/GraphBLAS folder for more details.

### 14.13 Cleaning up

To remove all compiled files, type make distclean in the top-level Graph-BLAS folder.

## 15 About NUMA systems

I have tested this package extensively on multicore single-socket systems, but have not yet optimized it for multi-socket systems with a NUMA architecture. That will be done in a future release. If you publish benchmarks with this package, please state the SuiteSparse:GraphBLAS version, and a caveat if appropriate. If you see significant performance issues when going from a single-socket to multi-socket system, I would like to hear from you so I can look into it.

## 16 Acknowledgments

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faster than the fastest graph databases (https://youtu.be/9h3Qco\_x0QE https://redislabs.com/blog/new-redisgraph-1-0-achieves-600x-faster-performance-graph-databases/). SuiteSparse:GraphBLAS was developed with support from NVIDIA, Intel, MIT Lincoln Lab, Redis Labs, IBM, the National Science Foundation

### 17 Additional Resources

(1514406, 1835499), and Julia Computing.

See http://graphblas.org for the GraphBLAS community page. See https://github.com/GraphBLAS/GraphBLAS-Pointers for an up-to-date list of additional resources on GraphBLAS, maintained by Gábor Szárnyas.

## References

- [ACD+20] Mohsen Aznaveh, Jinhao Chen, Timothy A. Davis, Bálint Hegyi, Scott P. Kolodziej, Timothy G. Mattson, and Gábor Szárnyas. Parallel GraphBLAS with OpenMP. In *CSC20*, *SIAM Workshop on Combinatorial Scientific Computing*. SIAM, 2020. (accepted) https://www.siam.org/conferences/cm/conference/csc20.
- [BG08] A. Buluç and J. Gilbert. On the representation and multiplication of hypersparse matrices. In *IPDPS'80: 2008 IEEE Intl. Symp. on Parallel and Distributed Processing*, pages 1–11, April 2008. https://dx.doi.org/10.1109/IPDPS.2008.4536313.
- [BG12] A. Buluç and J. Gilbert. Parallel sparse matrix-matrix multiplication and indexing: Implementation and experiments. SIAM Journal on Scientific Computing, 34(4):C170–C191, 2012. https://dx.doi.org/10.1137/110848244.
- [BMM+17a] A. Buluç, T. Mattson, S. McMillan, J. Moreira, and C. Yang. Design of the GraphBLAS API for C. In 2017 IEEE International Parallel and Distributed Processing Symposium Workshops (IPDPSW), pages 643–652, May 2017. https://dx.doi.org/10.1109/IPDPSW.2017.117.
- [BMM<sup>+</sup>17b] A. Buluç, T. Mattson, S. McMillan, J. Moreira, and C. Yang. The GraphBLAS C API specification. Technical report, 2017. http://graphblas.org/.
- [DAK19] T. A. Davis, M. Aznaveh, and S. Kolodziej. Write quick, run fast: Sparse deep neural network in 20 minutes of development time via SuiteSparse:GraphBLAS. In *IEEE HPEC'19*. IEEE, 2019. Grand Challenge Champion, for high performance. See <a href="http://www.ieee-hpec.org/">http://www.ieee-hpec.org/</a>.
- [Dav06] T. A. Davis. Direct Methods for Sparse Linear Systems. SIAM, Philadelphia, PA, 2006.

Provides a basic overview of many sparse matrix algorithms and a simple sparse matrix data structure. A series of 42 lectures are available on YouTube; see the link at http://faculty.cse.tamu.edu/davis/publications.html For the book, see https://dx.doi.org/10.1137/1.9780898718881

[Dav18] T. A. Davis. Graph algorithms via SuiteSparse:GraphBLAS: triangle counting and K-truss. In *IEEE HPEC'18*. IEEE, 2018. Grand Challenge Innovation Award. See http://www.ieee-hpec.org/.

- [Dav19] Timothy A. Davis. Algorithm 1000: SuiteSparse:GraphBLAS: Graph algorithms in the language of sparse linear algebra. *ACM Trans. Math. Softw.*, 45(4), December 2019.
- [DRSL16] T. A. Davis, S. Rajamanickam, and W. M. Sid-Lakhdar. A survey of direct methods for sparse linear systems. Acta Numerica, 25:383–566, 2016.

Abstract: Wilkinson defined a sparse matrix as one with enough zeros that it pays to take advantage of them. This informal yet practical definition captures the essence of the goal of direct methods for solving sparse matrix problems. They exploit the sparsity of a matrix to solve problems economically: much faster and using far less memory than if all the entries of a matrix were stored and took part in explicit computations. These methods form the backbone of a wide range of problems in computational science. A glimpse of the breadth of applications relying on sparse solvers can be seen in the origins of matrices in published matrix benchmark collections (Duff and Reid 1979a, Duff, Grimes and Lewis 1989a, Davis and Hu 2011). The goal of this survey article is to impart a working knowledge of the underlying theory and practice of sparse direct methods for solving linear systems and least-squares problems, and to provide an overview of the algorithms, data structures, and software available to solve these problems, so that the reader can both understand the methods and know how best to use them. DOI: https://dx.doi.org/10.1017/S0962492916000076

- [Gus78] F. G. Gustavson. Two fast algorithms for sparse matrices: Multiplication and permuted transposition. ACM Transactions on Mathematical Software, 4(3):250–269, 1978. https://dx.doi.org/10.1145/355791.355796.
- [Hig02] N. Higham. Accuracy and Stability of Numerical Algorithms. SIAM, 2nd edition, 2002. https://dx.doi.org/10.1137/1.9780898718027.
- [Kep17] J. Kepner. GraphBLAS mathematics. Technical report, 2017. http://www.mit.edu/~kepner/GraphBLAS/GraphBLAS-Math-release.pdf.
- [KG11] J. Kepner and J. Gilbert. Graph Algorithms in the Language of Linear Algebra. SIAM, Philadelphia, PA, 2011.

From the preface: Graphs are among the most important abstract data types in computer science, and the algorithms that

operate on them are critical to modern life. Graphs have been shown to be powerful tools for modeling complex problems because of their simplicity and generality. Graph algorithms are one of the pillars of mathematics, informing research in such diverse areas as combinatorial optimization, complexity theory, and topology. Algorithms on graphs are applied in many ways in today's worldfrom Web rankings to metabolic networks, from finite element meshes to semantic graphs. The current exponential growth in graph data has forced a shift to parallel computing for executing graph algorithms. Implementing parallel graph algorithms and achieving good parallel performance have proven difficult. This book addresses these challenges by exploiting the well-known duality between a canonical representation of graphs as abstract collections of vertices and edges and a sparse adjacency matrix representation. This linear algebraic approach is widely accessible to scientists and engineers who may not be formally trained in computer science. The authors show how to leverage existing parallel matrix computation techniques and the large amount of software infrastructure that exists for these computations to implement efficient and scalable parallel graph algorithms. The benefits of this approach are reduced algorithmic complexity, ease of implementation, and improved performance. DOI: https://dx.doi.org/10.1137/1.9780898719918

- [Lub86] M. Luby. A simple parallel algorithm for the maximal independent set problem. SIAM J. Comput., 15(4), 1986. https://dx.doi.org/10.1137/0215074.
- [MDK<sup>+</sup>19] T. Mattson, T. A. Davis, M. Kumar, A. Buluç, S. McMillan, J. Moreira, and C. Yang. LAGraph: a community effort to collect graph algorithms built on top of the GraphBLAS. In *GrAPL'19: Workshop on Graphs, Architectures, Programming, and Learning*. IEEE, May 2019. https://hpc.pnl.gov/grapl/previous/2019, part of IPDPS'19, at http://www.ipdps.org/jpdps2019.
- [NMAB18] Yusuke Nagasaka, Satoshi Matsuoka, Ariful Azad, and Aydın Buluç. High-performance sparse matrix-matrix products on intel knl and multicore architectures. In *Proceedings of the 47th International Conference on Parallel Processing Companion*, ICPP 18, New York, NY, USA, 2018. Association for Computing Machinery.
- [Wat87] A. J. Wathen. Realistic eigenvalue bounds for the Galerkin mass matrix. *IMA J. Numer. Anal.*, 7:449–457, 1987. https://dx.doi.org/10.1093/imanum/7.4.449.