User Guide for SuiteSparse:GraphBLAS

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VERSION 3.0.1, July 19, 2019 (full test coverage) (DRAFT)

Abstract

SuiteSparse:GraphBLAS is a full implementation of the Graph-BLAS standard, which defines a set of sparse matrix operations on an extended algebra of semirings using an almost unlimited variety of operators and types. When applied to sparse adjacency matrices, these algebraic operations are equivalent to computations on graphs. GraphBLAS provides a powerful and expressive framework for creating graph algorithms based on the elegant mathematics of sparse matrix operations on a semiring.

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1 Introduction

The GraphBLAS standard defines sparse matrix and vector operations on an extended algebra of semirings. The operations are useful for creating a wide range of graph algorithms.

For example, consider the matrix-matrix multiplication, $\mathbf{C} = \mathbf{AB}$. Suppose $\mathbf{A}$ and $\mathbf{B}$ are sparse n-by-n Boolean adjacency matrices of two undirected graphs. If the matrix multiplication is redefined to use logical AND instead of scalar multiply, and if it uses the logical OR instead of add, then the matrix $\mathbf{C}$ is the sparse Boolean adjacency matrix of a graph that has an edge (i, j) if node i in $\mathbf{A}$ and node j in $\mathbf{B}$ share any neighbor in common. The OR-AND pair forms an algebraic semiring, and many graph operations like this one can be succinctly represented by matrix operations with different semirings and different numerical types. GraphBLAS provides a wide range of built-in types and operators, and allows the user application to create new types and operators without needing to recompile the GraphBLAS library.

See [Dav19] for a journal article on SuiteSparse:GraphBLAS, and [Dav18] for a more recent conference paper. A full and precise definition of the GraphBLAS specification is provided in *The GraphBLAS C API Specification* by Aydın Buluç, Timothy Mattson, Scott McMillan, José Moreira, and Carl Yang [BMM+17a, BMM+17b], based on *GraphBLAS Mathematics* by Jeremy Kepner [Kep17]. The GraphBLAS C API Specification is available at http://graphblas.org. This version of SuiteSparse:GraphBLAS fully conforms to Version 1.2.0 (May 18, 2018) of that specification. In this User Guide, aspects of the GraphBLAS specification that would be true for any GraphBLAS implementation are simply called "GraphBLAS." Details unique to this particular implementation are referred to as SuiteSparse:GraphBLAS.

SPEC: See the tag **SPEC:** for SuiteSparse extensions to the spec. They are also placed in text boxes like this one. All functions, objects, and macros with a name of the form `GxB_*` are extensions to the spec.

1.1 Release Notes:

- Version 3.0 (2019): The version number is increased to 3.0, since this version is not backward compatible with V2.x. The `GxB_select` operation changes; the `Thunk` parameter was formerly a `const void *` pointer, and is now a `GrB_Vector` of length 1. A new parameter is added to `GxB_init`, to specify whether or not the user-provided memory management functions are thread safe.

  The remaining changes add new features, and are upward compatible with V2.x. The major change is the addition of OpenMP parallelism. This addition has no effect on the API, except that round-off errors can differ with the number of threads used, for floating-point types. `GxB_set` can optionally define the number of threads to use (the default is `omp_get_max_threads`). The number of threads can also defined globally, and/or in the `GrB_Descriptor`. The RDIV and RMINUS operators are added, which are defined as $f(x, y) = y/x$ and $f(x, y) = y - x$, respectively. Additional options are added to `GxB_get`.

- Version 2.3.3 (May 2019): Collected Algorithm of the ACM. No changes from V2.3.2 other than the documentation.

- Version 2.3 (Feb 2019) improves the performance of many GraphBLAS operations, including an early-exit for monoids. These changes have a significant impact on breadth-first-search (a performance bug was also fixed in the two BFS `Demo` codes). The matrix and vector import/export functions were added (Section 4.9), in support of the new LAGraph project (https://github.com/GraphBLAS/LAGraph, see also Section 10.1). LAGraph includes a push-pull BFS in GraphBLAS that is faster than two versions in the `Demo` folder. `GxB_init` was added to allow the memory manager functions (`malloc`, etc) to be specified.

- Version 2.2 (Nov 2018) adds user-defined objects at compile-time, via user `*.m4` files placed in `GraphBLAS/User`, which use the `GxB_*_define` macros described in Section 9. The default matrix format is now `GxB_BY_ROW`. Also added are the `GxB_*print` methods for printing the contents of each GraphBLAS object (Section 8). PageRank demos have been added to the `Demos` folder. Prior versions required GraphBLAS to be compiled with OpenMP, for it to be thread-safe. It can now be compiled with POSIX pthreads. The `cmake` script automatically detects

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if OpenMP and/or POSIX pthreads are available. Demos have been added to show how GraphBLAS can be called from a multi-threaded user application.

- Version 2.1 (Oct 2018) was a major update with support for new matrix formats (by row or column, and hypersparse matrices), and MATLAB-like colon notation (`I=begin:end` or `I=begin:inc:end`). Some graph algorithms are more naturally expressed with matrices stored by row, and this version includes the new `GxB_BY_ROW` format. The default format in Version 2.1 and prior versions is by column. New extensions to GraphBLAS in this version include `GxB_get`, `GxB_set`, and `GxB_AxB_METHOD`, `GxB_RANGE`, `GxB_STRIDE`, and `GxB_BACKWARDS`, and their related definitions, described in Sections 4.10, 5, and 6.

- Version 2.0 (March 2018) addressed changes in the GraphBLAS C API Specification and added `GxB_kron` and `GxB_resize`.

- Version 1.1 (Dec 2017) primarily improved the performance.

- Version 1.0 was released on Nov 25, 2017.

2 Basic Concepts

Since the *GraphBLAS C API Specification* provides a precise definition of GraphBLAS, not every detail of every function is provided here. For example, some error codes returned by GraphBLAS are self-explanatory, but since a specification must precisely define all possible error codes a function can return, these are listed in detail in the *GraphBLAS C API Specification*. However, including them here is not essential and the additional information on the page might detract from a clearer view of the essential features of the GraphBLAS functions.

This User Guide also assumes the reader is familiar with the MATLAB language, created by Cleve Moler. MATLAB supports only the conventional plus-times semiring on sparse double and complex matrices, but a MATLAB-like notation easily extends to the arbitrary semirings used in GraphBLAS. The matrix multiplication in the example in the Introduction can be written in MATLAB notation as `C=A*B`, if the Boolean `OR-AND` semiring is understood. Relying on a MATLAB-like notation allows the description in this User Guide to be expressive, easy to understand, and terse at the same time. *The GraphBLAS C API Specification* also makes use of some MATLAB-like language, such as the colon notation.

MATLAB notation will always appear here in fixed-width font, such as `C=A*B(:,j)`. In standard mathematical notation it would be written as the matrix-vector multiplication $\mathbf{C} = \mathbf{A}\mathbf{b}_j$ where $\mathbf{b}_j$ is the jth column of the matrix $\mathbf{B}$. The GraphBLAS standard is a C API and SuiteSparse:GraphBLAS is written in C, and so a great deal of C syntax appears here as well, also in fixed-width font. This User Guide alternates between all three styles as needed.

2.1 Graphs and sparse matrices

Graphs can be huge, with many nodes and edges. A dense adjacency matrix $\mathbf{A}$ for a graph of n nodes takes $O(n^2)$ memory, which is impossible if n is, say, a million. Most graphs arising in practice are sparse, however, with only $|\mathbf{A}| = O(n)$ edges, where $|\mathbf{A}|$ denotes the number of edges in the graph, or the number of explicit entries present in the data structure for the matrix $\mathbf{A}$. Sparse graphs with millions of nodes and edges can easily be created by representing them as sparse matrices, where only explicit values need to be stored. Some graphs are *hypersparse*, with $|\mathbf{A}| << n$. SuiteSparse:GraphBLAS sup-

ports two kinds of sparse matrix formats: a regular sparse format, taking $O(n + |\mathbf{A}|)$ space, and a hypersparse format taking only $O(|\mathbf{A}|)$ space. As a result, creating a sparse matrix of size n-by-n where $n = 2^{60}$ (about 10^{18}) can be done on quite easily on a commodity laptop, limited only by $|\mathbf{A}|$.

A sparse matrix data structure only stores a subset of the possible n^2 entries, and it assumes the values of entries not stored have some implicit value. In conventional linear algebra, this implicit value is zero, but it differs with different semirings. Explicit values are called *entries* and they appear in the data structure. The *pattern* of a matrix defines where its explicit entries appear. It will be referenced in one of two equivalent ways. It can be viewed as a set of indices (i, j), where (i, j) is in the pattern of a matrix $\mathbf{A}$ if $\mathbf{A}(i, j)$ is an explicit value. It can also be viewed as a Boolean matrix $\mathbf{S}$ where $\mathbf{S}(i, j)$ is true if (i, j) is an explicit entry and false otherwise. In MATLAB notation, S=spones(A) or S=(A~=0), if the implicit value is zero. The (i,j) pairs, and their values, can also be extracted from the matrix via the MATLAB expression [I,J,X]=find(A), where the kth tuple (I(k),J(k),X(k)) represents the explicit entry A(I(k),J(k)), with numerical value X(k) equal to a_{ij}, with row index i=I(k) and column index j=J(k).

The entries in the pattern of $\mathbf{A}$ can take on any value, including the implicit value, whatever it happens to be. This differs slightly from MATLAB, which always drops all explicit zeros from its sparse matrices. This is a minor difference but it cannot be done in GraphBLAS. For example, in the max-plus tropical algebra, the implicit value is negative infinity, and zero has a different meaning. Here, the MATLAB notation used will assume that no explicit entries are ever dropped because their explicit value happens to match the implicit value.

Graph Algorithms in the Language on Linear Algebra, Kepner and Gilbert, eds., provides a framework for understanding how graph algorithms can be expressed as matrix computations [KG11]. For additional background on sparse matrix algorithms, see also [Dav06] and [DRSL16].

2.2   Overview of GraphBLAS methods and operations

GraphBLAS provides a collection of *methods* to create, query, and free its of objects: sparse matrices, sparse vectors, types, operators, monoids, semirings, and a descriptor object used for parameter settings. Details are given in Section 4. Once these objects are created they can be used in mathematical *operations* (not to be confused with the how the term *operator* is

used in GraphBLAS). A short summary of these operations and their nearest MATLAB analog is given in the table below.

operation	approximate MATLAB analog
matrix multiplication	`C=A*B`
element-wise operations	`C=A+B` and `C=A.*B`
reduction to a vector or scalar	`s=sum(A)`
apply unary operator	`C=-A`
transpose	`C=A'`
submatrix extraction	`C=A(I,J)`
submatrix assignment	`C(I,J)=A`

GraphBLAS can do far more than what MATLAB can do in these rough analogs, but the list provides a first step in describing what GraphBLAS can do. Details of each GraphBLAS operation are given in Section 7. With this brief overview, the full scope of GraphBLAS extensions of these operations can now be described.

GraphBLAS has 11 built-in scalar types: Boolean, single and double precision floating-point, and 8, 16, 32, and 64-bit signed and unsigned integers. In addition, user-defined scalar types can be created from nearly any C `typedef`, as long as the entire type fits in a fixed-size contiguous block of memory (of arbitrary size). All of these types can be used to create Graph-BLAS sparse matrices or vectors.

The scalar addition of conventional matrix multiplication is replaced with a *monoid*. A monoid is an associative and commutative binary operator `z=f(x,y)` where all three domains are the same (the types of `x`, `y`, and `z`), and where the operator has an identity value `id` such that `f(x,id)=f(id,x)=x`. Performing matrix multiplication with a semiring uses a monoid in place of the "add" operator, scalar addition being just one of many possible monoids. The identity value of addition is zero, since $x + 0 = 0 + x = x$. GraphBLAS includes eight built-in operators suitable for use as a monoid: min (with an identity value of positive infinity), max (whose identity is negative infinity), add (identity is zero) multiply (with an identity of one), and four logical operators: AND, OR, exclusive-OR, and Boolean equality. User-created monoids can be defined with any associative and commutative operator that has an identity value.

Finally, a semiring can use any built-in or user-defined binary operator `z=f(x,y)` as its "multiply" operator, as long as the type of its output, `z` matches the type of the semiring's monoid. The user application can create

any semiring based on any types, monoids, and multiply operators, as long these few rules are followed.

Just considering built-in types and operators, GraphBLAS can perform C=A*B in 1040 unique semirings. With typecasting, any of these 1040 semirings can be applied to matrices C, A, and B of any of the 11 types, in any combination. This gives $1040 \times 11^3 = 1,384,240$ possible kinds of sparse matrix multiplication supported by GraphBLAS, and this is counting just built-in types and operators. By contrast, MATLAB provides just two semirings for its sparse matrix multiplication C=A*B: plus-times-double and plus-times-complex, not counting the typecasting that MATLAB does when multiplying a real matrix times a complex matrix. All of the 1.4 million forms of matrix multiplication methods in SuiteSparse:GraphBLAS are typically just as fast as computing C=A*B in MATLAB using its own native sparse matrix multiplication methods, and often faster when parallelism can be effectively used.

A monoid can also be used in a reduction operation, like s=sum(A) in MATLAB. MATLAB provides the plus, times, min, and max reductions of a real or complex sparse matrix as s=sum(A), s=prod(A), s=min(A), and s=max(A), respectively. In GraphBLAS, any monoid can be used (min, max, plus, times, AND, OR, exclusive-OR, equality, or any user-defined monoid, on any user-defined type).

Element-wise operations are also expanded from what can be done in MATLAB. Consider matrix addition, C=A+B in MATLAB. The pattern of the result is the set union of the pattern of A and B. In GraphBLAS, any binary operator can be used in this set-union "addition." The operator is applied to entries in the intersection. Entries in A but not B, or visa-versa, are copied directly into C, without any application of the binary operator. The accumulator operation for $\mathbf{Z} = \mathbf{C} \odot \mathbf{T}$ described in Section 2.3 is one example of this set-union application of an arbitrary binary operator.

Consider element-wise multiplication, C=A.*B in MATLAB. The operator (multiply in this case) is applied to entries in the set intersection, and the pattern of C just this set intersection. Entries in A but not B, or visa-versa, do not appear in C. In GraphBLAS, any binary operator can be used in this manner, not just scalar multiplication. The difference between element-wise "add" and "multiply" is not the operators, but whether or not the pattern of the result is the set union or the set intersection. In both cases, the operator is only applied to the set intersection.

Finally, GraphBLAS includes a *non-blocking* mode where operations can

be left pending, and saved for later. This is very useful for submatrix assignment (`C(I,J)=A` where `I` and `J` are integer vectors), or scalar assignment (`C(i,j)=x` where `i` and `j` are scalar integers). Because of how MATLAB stores its matrices, adding and deleting individual entries is very costly. For example, this is very slow in MATLAB, taking $O(nz^2)$ time:

```
A = sparse (m,n) ;   % an empty sparse matrix
for k = 1:nz
    compute a value x, row index i, and column index j
    A (i,j) = x ;
end
```

The above code is very easy read and simple to write, but exceedingly slow. In MATLAB, the method below is preferred and is far faster, taking only $O(|\mathbf{A}|)$ time. It can easily be a million times faster than the method above. Unfortunately the second method below is a little harder to read and a little less natural to write:

```
I = zeros (nz,1) ;
J = zeros (nz,1) ;
X = zeros (nz,1) ;
for k = 1:nz
    compute a value x, row index i, and column index j
    I (k) = i ;
    J (k) = j ;
    X (k) = x ;
end
A = sparse (I,J,X,m,n) ;
```

GraphBLAS can do both methods. SuiteSparse:GraphBLAS stores its matrices in a format that allows for pending computations, which are done later in bulk, and as a result it can do both methods above equally as fast as the MATLAB `sparse` function, allowing the user to write simpler code.

2.3   The accumulator and the mask

Most GraphBLAS operations can be modified via transposing input matrices, using an accumulator operator, applying a mask or its complement, and by clear all entries the matrix `C` after using it in the accumulator operator but before the final results are written back into it. All of these steps are optional, and are controlled by a descriptor object that holds parameter settings (see Section 4.10) that control the following options:

- the input matrices `A` and/or `B` can be transposed first.

- an accumulator operator can be used, like the plus in the statement `C=C+A*B`. The accumulator operator can be any binary operator, and an element-wise "add" (set union) is performed using the operator.

- an optional *mask* can be used to selectively write the results to the output. The mask is a sparse Boolean matrix `Mask` whose size is the same size as the result. If `Mask(i,j)` is true, then the corresponding entry in the output can be modified by the computation. If `Mask(i,j)` is false, then the corresponding in the output is protected and cannot be modified by the computation. The `Mask` matrix acts exactly like logical matrix indexing in MATLAB, with one minor difference: in GraphBLAS notation, the mask operation is $\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{Z}$, where the mask $\mathbf{M}$ appears only on the left-hand side. In MATLAB, it would appear on both sides as `C(Mask)=Z(Mask)`. If no mask is provided, the `Mask` matrix is implicitly all true. This is indicated by passing the value `GrB_NULL` in place of the `Mask` argument in GraphBLAS operations.

This process can be described in mathematical notation as:

$\mathbf{A} = \mathbf{A}^{\mathsf{T}}$, if requested via descriptor (first input option)
$\mathbf{B} = \mathbf{B}^{\mathsf{T}}$, if requested via descriptor (second input option)
$\mathbf{T}$ is computed according to the specific operation
$\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{C} \odot \mathbf{T}$, accumulating and writing the results back via the mask

The application of the mask and the accumulator operator is written as $\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{C} \odot \mathbf{T}$ where $\mathbf{Z} = \mathbf{C} \odot \mathbf{T}$ denotes the application of the accumulator operator, and $\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{Z}$ denotes the mask operator via the Boolean matrix $\mathbf{M}$. The Accumulator Phase, $\mathbf{Z} = \mathbf{C} \odot \mathbf{T}$, is performed as follows:

Accumulator Phase: compute $\mathbf{Z} = \mathbf{C} \odot \mathbf{T}$:
    if `accum` is NULL
        $\mathbf{Z} = \mathbf{T}$
    else
        $\mathbf{Z} = \mathbf{C} \odot \mathbf{T}$

The accumulator operator is $\odot$ in GraphBLAS notation, or `accum` in the code. The pattern of $\mathbf{C} \odot \mathbf{T}$ is the set union of the patterns of $\mathbf{C}$ and $\mathbf{T}$, and the operator is applied only on the set intersection of $\mathbf{C}$ and $\mathbf{T}$. Entries in neither the pattern of $\mathbf{C}$ nor $\mathbf{T}$ do not appear in the pattern of $\mathbf{Z}$. That is:

for all entries (i, j) in $\mathbf{C} \cap \mathbf{T}$ (that is, entries in both $\mathbf{C}$ and $\mathbf{T}$)
$$z_{ij} = c_{ij} \odot t_{ij}$$
for all entries (i, j) in $\mathbf{C} \setminus \mathbf{T}$ (that is, entries in $\mathbf{C}$ but not $\mathbf{T}$)
$$z_{ij} = c_{ij}$$
for all entries (i, j) in $\mathbf{T} \setminus \mathbf{C}$ (that is, entries in $\mathbf{T}$ but not $\mathbf{C}$)
$$z_{ij} = t_{ij}$$

The Accumulator Phase is followed by the Mask/Replace Phase, $\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{Z}$ as controlled by the `GrB_REPLACE` and `GrB_SCMP` descriptor options:

Mask/Replace Phase: compute $\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{Z}$:
    if (`GrB_REPLACE`) delete all entries in $\mathbf{C}$
    if `Mask` is `NULL`
        if (`GrB_SCMP`)
            $\mathbf{C}$ is not modified
        else
            $\mathbf{C} = \mathbf{Z}$
    else
        if (`GrB_SCMP`)
            $\mathbf{C}\langle\neg\mathbf{M}\rangle = \mathbf{Z}$
        else
            $\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{Z}$

Both phases of the accum/mask process are illustrated in MATLAB notation in Figure 1. A GraphBLAS operation starts with its primary computation, producing a result `T`; for matrix multiply, `T=A*B`, or if `A` is transposed first, `T=A'*B`, for example. Applying the accumulator, mask (or its complement) to obtain the final result matrix `C` can be expressed in the MATLAB `accum_mask` function shown in the figure. This function is an exact, fully functional, and nearly-complete description of the GraphBLAS accumulator/mask operation. The only aspects it does not consider are typecasting (see Section 2.4), and the value of the implicit identity (for those, see another version in the `Test` folder).

One aspect of GraphBLAS cannot be as easily expressed in a MATLAB sparse matrix: namely, what is the implicit value of entries not in the pattern? To accommodate this difference in the `accum_mask` MATLAB function, each sparse matrix `A` is represented with its values `A.matrix` and its pattern, `A.pattern`. The latter could be expressed as the sparse matrix `A.pattern=spones(A)` or `A.pattern=(A~=0)` in MATLAB, if the implicit

```
function C = accum_mask (C, Mask, accum, T, C_replace, Mask_complement)
[m n] = size (C.matrix) ;
Z.matrix  = zeros (m, n) ;
Z.pattern = false (m, n) ;

if (isempty (accum))
   Z = T ;      % no accum operator
else
   % Z = accum (C,T), like Z=C+T but with an binary operator, accum
   p =  C.pattern &  T.pattern ; Z.matrix (p) = accum (C.matrix (p), T.matrix (p));
   p =  C.pattern & ~T.pattern ; Z.matrix (p) = C.matrix (p) ;
   p = ~C.pattern &  T.pattern ; Z.matrix (p) = T.matrix (p) ;
   Z.pattern = C.pattern | T.pattern ;
end

% apply the mask to the values and pattern
C.matrix  = mask (C.matrix,  Mask, Z.matrix,  C_replace, Mask_complement) ;
C.pattern = mask (C.pattern, Mask, Z.pattern, C_replace, Mask_complement) ;
end

function C = mask (C, Mask, Z, C_replace, Mask_complement)
% replace C if requested
if (C_replace)
   C (:,:) = 0 ;
end
if (isempty (Mask))              % if empty, Mask is implicit ones(m,n)
   % implicitly, Mask = ones (size (C))
   if (~Mask_complement)
      C = Z ;                    % this is the default
   else
      C = C ;                    % Z need never have been computed
   end
else
   % apply the mask
   if (~Mask_complement)
      C (Mask) = Z (Mask) ;
   else
      C (~Mask) = Z (~Mask) ;
   end
end
end
```

Figure 1: Applying the mask and accumulator, $\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{C} \odot \mathbf{T}$

value is zero. With different semirings, entries not in the pattern can be `1`, `+Inf`, `-Inf`, or whatever is the identity value of the monoid. As a result, Figure 1 performs its computations on two MATLAB matrices: the values in `A.matrix` and the pattern in the logical matrix `A.pattern`. Implicit values are untouched.

The final computation in Figure 1 with a complemented `Mask` is easily expressed in MATLAB as `C(~Mask)=Z(~Mask)` but this is costly if `Mask` is very sparse (the typical case). It can be computed much faster in MATLAB without complementing the sparse `Mask` via:

```
R = Z ; R (Mask) = C (Mask) ; C = R ;
```

A set of MATLAB functions that precisely compute the $\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{C} \odot \mathbf{T}$ operation according to the full GraphBLAS specification is provided in Suite-Sparse:GraphBLAS as `GB_spec_accum.m`, which computes $\mathbf{Z} = \mathbf{C} \odot \mathbf{T}$, and `GB_spec_mask.m`, which computes $\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{Z}$. SuiteSparse:GraphBLAS includes a complete list of `GB_spec_*` functions that illustrate every Graph-BLAS operation; these are discussed in in Section 7.1.

The methods in Figure 1 rely heavily on MATLAB's logical matrix indexing. For those unfamiliar with logical indexing in MATLAB, here is short summary. Logical matrix indexing in MATLAB is written as `A(Mask)` where `A` is any matrix and `Mask` is a logical matrix the same size as `A`. The expression `x=A(Mask)` produces a column vector `x` consisting of the entries of `A` where `Mask` is true. On the left-hand side, logical submatrix assignment `A(Mask)=x` does the opposite, copying the components of the vector `x` into the places in `A` where `Mask` is true. For example, to negate all values greater than 10 using logical indexing in MATLAB:

```
>> A = magic (4)
A =
    16     2     3    13
     5    11    10     8
     9     7     6    12
     4    14    15     1
>> A (A>10) = - A (A>10)
A =
   -16     2     3   -13
     5   -11    10     8
     9     7     6   -12
     4   -14   -15     1
```

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In MATLAB, logical indexing with a sparse matrix `A` and sparse logical matrix `Mask` is very efficient since MATLAB supports sparse logical matrices. The Mask operator in GraphBLAS works identically as sparse logical indexing in MATLAB, and is equally as fast (or faster) in SuiteSparse:GraphBLAS.

2.4 Typecasting

If an operator `z=f(x)` or `z=f(x,y)` is used with inputs that do not match its inputs `x` or `y`, or if its result `z` does not match the type of the matrix it is being stored into, then the values are typecasted. Typecasting in Graph-BLAS extends beyond just operators. Almost all GraphBLAS methods and operations are able to typecast their results, as needed.

If one type can be typecasted into the other, they are said to be *compatible*. All built-in types are compatible with each other. GraphBLAS cannot typecast user-defined types thus any user-defined type is only compatible with itself. When GraphBLAS requires inputs of a specific type, or when one type cannot be typecast to another, the GraphBLAS function returns an error code, `GrB_DOMAIN_MISMATCH` (refer to Section 3.5 for a complete list of error codes). Typecasting can only be done between built-in types, and it follows the rules of the ANSI C language (not MATLAB) wherever the rules of ANSI C are well-defined. In particular, a large integer outside the range of a smaller one is wrapped, modulo style. This differs from MATLAB.

However, unlike MATLAB, the C language specification states that the results of typecasting a `float` or `double` to an integer type is not always defined. In SuiteSparse:GraphBLAS, whenever C leaves the result undefined the rules used in MATLAB are followed. In particular `+Inf` converts to the largest integer value, `-Inf` converts to the smallest (zero for unsigned integers), and `NaN` converts to zero. Other than these special cases, Suite-Sparse:GraphBLAS trusts the C compiler for the rest of its typecasting.

Typecasting to `bool` is fully defined in the C language specification, even for `NaN`. The result is `false` if the value compares equal to zero, and true otherwise. Thus `NaN` converts to `true`.

> **SPEC:** the GraphBLAS API states that typecasting follows the rules of ANSI C. Yet C leaves some typecasting undefined. SuiteSparse:GraphBLAS provides a precise definition for all typecasting as an extension to the spec.

2.5 Notation and list of GraphBLAS operations

As a summary of what GraphBLAS can do, the following table lists all Graph-BLAS operations (where `GxB_*` are in SuiteSparse:GraphBLAS only). Upper case letters denote a matrix, lower case letters are vectors, and $\mathbf{AB}$ denote the multiplication of two matrices over a semiring.

`GrB_mxm`	matrix-matrix multiply	$\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{C} \odot \mathbf{AB}$
`GrB_vxm`	vector-matrix multiply	$\mathbf{w}^\mathsf{T}\langle\mathbf{m}^\mathsf{T}\rangle = \mathbf{w}^\mathsf{T} \odot \mathbf{u}^\mathsf{T}\mathbf{A}$
`GrB_mxv`	matrix-vector multiply	$\mathbf{w}\langle\mathbf{m}\rangle = \mathbf{w} \odot \mathbf{Au}$
`GrB_eWiseMult`	element-wise,	$\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{C} \odot (\mathbf{A} \otimes \mathbf{B})$
	set union	$\mathbf{w}\langle\mathbf{m}\rangle = \mathbf{w} \odot (\mathbf{u} \otimes \mathbf{v})$
`GrB_eWiseAdd`	element-wise,	$\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{C} \odot (\mathbf{A} \oplus \mathbf{B})$
	set intersection	$\mathbf{w}\langle\mathbf{m}\rangle = \mathbf{w} \odot (\mathbf{u} \oplus \mathbf{v})$
`GrB_extract`	extract submatrix	$\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{C} \odot \mathbf{A}(\mathbf{I},\mathbf{J})$
		$\mathbf{w}\langle\mathbf{m}\rangle = \mathbf{w} \odot \mathbf{u}(\mathbf{i})$
`GxB_subassign`	assign submatrix	$\mathbf{C}(\mathbf{I},\mathbf{J})\langle\mathbf{M}\rangle = \mathbf{C}(\mathbf{I},\mathbf{J}) \odot \mathbf{A}$
	(with submask for $\mathbf{C}(\mathbf{I},\mathbf{J})$)	$\mathbf{w}(\mathbf{i})\langle\mathbf{m}\rangle = \mathbf{w}(\mathbf{i}) \odot \mathbf{u}$
`GrB_assign`	assign submatrix	$\mathbf{C}\langle\mathbf{M}\rangle(\mathbf{I},\mathbf{J}) = \mathbf{C}(\mathbf{I},\mathbf{J}) \odot \mathbf{A}$
	(with mask for $\mathbf{C}$)	$\mathbf{w}\langle\mathbf{m}\rangle(\mathbf{i}) = \mathbf{w}(\mathbf{i}) \odot \mathbf{u}$
`GrB_apply`	apply unary operator	$\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{C}\odot f(\mathbf{A})$
		$\mathbf{w}\langle\mathbf{m}\rangle = \mathbf{w}\odot f(\mathbf{u})$
`GxB_select`	apply select operator	$\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{C}\odot f(\mathbf{A},\mathbf{k})$
		$\mathbf{w}\langle\mathbf{m}\rangle = \mathbf{w}\odot f(\mathbf{u},\mathbf{k})$
`GrB_reduce`	reduce to vector	$\mathbf{w}\langle\mathbf{m}\rangle = \mathbf{w}\odot[\oplus_j \mathbf{A}(:,j)]$
	reduce to scalar	$s = s \odot [\oplus_{ij}\mathbf{A}(I,J)]$
`GrB_transpose`	transpose	$\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{C} \odot \mathbf{A}^\mathsf{T}$
`GxB_kron`	Kronecker product	$\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{C} \odot \mathrm{kron}(\mathbf{A},\mathbf{B})$

Each operation takes an optional `GrB_Descriptor` argument that modifies the operation. The input matrices $\mathbf{A}$ and $\mathbf{B}$ can be optionally transposed, the mask $\mathbf{M}$ can be complemented, and $\mathbf{C}$ can be cleared of its entries after it is used in $\mathbf{Z} = \mathbf{C} \odot \mathbf{T}$ but before the $\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{Z}$ assignment. Vectors are never transposed via the descriptor.

Let $\mathbf{A} \oplus \mathbf{B}$ denote the element-wise operator that produces a set union pattern (like `A+B` in MATLAB). Any binary operator can be used this way in GraphBLAS, not just plus. Let $\mathbf{A} \otimes \mathbf{B}$ denote the element-wise operator that produces a set intersection pattern (like `A.*B` in MATLAB); any binary operator can be used this way, not just times.

Reduction of a matrix $\mathbf{A}$ to a vector reduces the ith row of $\mathbf{A}$ to a scalar w_i. This is like `w=sum(A')` since by default, MATLAB reduces down the columns, not across the rows.

3 GraphBLAS Context and Sequence

A user application that directly relies on GraphBLAS must include the `GraphBLAS.h` header file:

```
#include "GraphBLAS.h"
```

The `GraphBLAS.h` file defines functions, types, and macros prefixed with `GrB_` and `GxB_` that may be used in user applications. The prefix `GrB_` denote items that appear in the official *GraphBLAS C API Specification*. The prefix `GxB_` refers to SuiteSparse-specific extensions to the GraphBLAS API. Both may be used in user applications but be aware that items with prefixes `GxB_` will not appear in other implementations of the GraphBLAS standard.

> **SPEC:** The following macros are extensions to the spec.

The `GraphBLAS.h` file includes all the definitions required to use Graph-BLAS, including the following macros that can assist a user application in compiling and using GraphBLAS.

There are two version numbers associated with SuiteSparse:GraphBLAS: the version of the *GraphBLAS C API Specification* it conforms to, and the version of the implementation itself. These can be used in the following manner in a user application:

```
#if GxB_SPEC_VERSION >= GxB_VERSION (2,0,3)
... use features in GraphBLAS specification 2.0.3 ...
#else
... only use features in early specifications
#endif

#if GxB_IMPLEMENTATION > GxB_VERSION (1,4,0)
... use features from version 1.4.0 of a specific GraphBLAS implementation
#endif
```

SuiteSparse:GraphBLAS also defines the following strings with `#define`. Refer to the `GraphBLAS.h` file for details.

Macro	purpose
GxB_IMPLEMENTATION_ABOUT	this particular implementation, copyright, and URL
GxB_IMPLEMENTATION_DATE	the date of this implementation
GxB_SPEC_ABOUT	the GraphBLAS specification for this implementation
GxB_SPEC_DATE	the date of the GraphBLAS specification
GxB_IMPLEMENTATION_LICENSE	the license for this particular implementation

Finally, SuiteSparse:GraphBLAS gives itself a unique name of the form `GxB_SUITESPARSE_GRAPHBLAS` that the user application can use in `#ifdef` tests. This is helpful in case a particular implementation provides non-standard features that extend the GraphBLAS specification, such as additional predefined built-in operators, or if a GraphBLAS implementation does not yet fully implement all of the GraphBLAS specification. The SuiteSparse:GraphBLAS name is provided in its `GraphBLAS.h` file as:

```
#define GxB_SUITESPARSE_GRAPHBLAS
```

For example, SuiteSparse:GraphBLAS predefines additional built-in operators not in the specification. If the user application wishes to use these in any GraphBLAS implementation, an `#ifdef` can control when they are used. Refer to the examples in the `GraphBLAS/Demo` folder.

As another example, the GraphBLAS API states that an implementation need not define the order in which `GrB_Matrix_build` assembles duplicate tuples in its `[I,J,X]` input arrays. As a result, no particular ordering should be relied upon in general. However, SuiteSparse:GraphBLAS does guarantee an ordering, and this guarantee will be kept in future versions of SuiteSparse:GraphBLAS as well. Since not all implementations will ensure a particular ordering, the following can be used to exploit the ordering returned by SuiteSparse:GraphBLAS.

```
#ifdef GxB_SUITESPARSE_GRAPHBLAS
// duplicates in I, J, X assembled in a specific order;
// results are well-defined even if op is not associative.
GrB_Matrix_build (C, I, J, X, nvals, op) ;
#else
// duplicates in I, J, X assembled in no particular order;
// results are undefined if op is not associative.
GrB_Matrix_build (C, I, J, X, nvals, op) ;
#endif
```

The remainder of this section describes GraphBLAS functions that create, modify, and destroy the GraphBLAS context, or provide utility methods for dealing with errors:

GraphBLAS function	purpose	Section
GrB_init	start up GraphBLAS	3.1
GxB_init	start up GraphBLAS with different `malloc`	3.2
GrB_wait	force completion of pending operations	3.3
GrB_Info	status code returned by GraphBLAS functions	3.4
GrB_error	get more details on the last error	3.5
GrB_finalize	finish GraphBLAS	3.6

3.1 GrB_init: initialize GraphBLAS

```
typedef enum
{
    GrB_NONBLOCKING = 0,    // methods may return with pending computations
    GrB_BLOCKING = 1        // no computations are ever left pending
}
GrB_Mode ;
```

```
GrB_Info GrB_init          // start up GraphBLAS
(
    GrB_Mode mode          // blocking or non-blocking mode
) ;
```

`GrB_init` must be called before any other GraphBLAS operation. It defines the mode that GraphBLAS will use: blocking or non-blocking. With blocking mode, all operations finish before returning to the user application. With non-blocking mode, operations can be left pending, and are computed only when needed. Non-blocking mode can be much faster than blocking mode, by many orders of magnitude in extreme cases. Blocking mode should be used only when debugging a user application. The mode cannot be changed once it is set by `GrB_init`.

GraphBLAS objects are opaque to the user application. This allows GraphBLAS to postpone operations and then do them later in a more efficient manner by rearranging them and grouping them together. In non-blocking mode, the computations required to construct an opaque GraphBLAS object might not be finished when the GraphBLAS method or operation returns to the user. However, user-provided arrays are not opaque, and GraphBLAS methods and operations that read them (such as `GrB_Matrix_build`) or write to them (such as `GrB_Matrix_extractTuples`) always finish reading them, or creating them, when the method or operation returns to the user application.

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In addition, all methods and operations that extract values from a Graph-BLAS object and return them into non-opaque user arrays always ensure that the computations for that object are completed when the method returns, namely: `GrB_*_nvals`, `GrB_*_extractElement`, `GrB_*_extractTuples`, and `GrB_*_reduce` (to scalar). These methods only ensure that the computations for a single object are completed. Use `GrB_wait` to ensure that all computations are completed (see Section 3.3).

SuiteSparse:GraphBLAS is not yet multithreaded, but it is safe to use in a multithreaded user application. SuiteSparse:GraphBLAS must be compiled for each specific user threading model; see Section 11 for details.

User threads must not operate on the same matrices at the same time, with one exception. Multiple threads can use the same matrices or vectors as read-only inputs to GraphBLAS operations or methods, but only if they have no pending operations (use `GrB_Matrix_nvals` or `GrB_wait` first). User threads cannot simultaneously modify a matrix or vector via any GraphBLAS operation or method.

With multiple user threads, exactly one user thread must call `GrB_init` before any user thread may call any `GrB_*` or `GxB_*` function. When the user application is finished, exactly one user thread must call `GrB_finalize`, after which no user thread may call any `GrB_*` or `GxB_*` function.

You can query the mode of a GraphBLAS session with the following (see Section 5), which returns the `mode` passed to `GrB_init`:

```
GrB_mode mode ;
GxB_get (GxB_MODE, &mode) ;
```

3.2 GxB_init: **initialize with alternate malloc**

```
GrB_Info GxB_init           // start up GraphBLAS and also define malloc, etc
(
    GrB_Mode mode,          // blocking or non-blocking mode

    // pointers to memory management functions.
    void * (* user_malloc_function  ) (size_t),
    void * (* user_calloc_function  ) (size_t, size_t),
    void * (* user_realloc_function ) (void *, size_t),
    void   (* user_free_function    ) (void *),
    bool user_malloc_is_thread_safe
) ;
```

`GxB_init` is identical to `GrB_init`, except that it also redefines the memory management functions that SuiteSparse:GraphBLAS will use. Giving the user application control over this is particularly important when using the `GxB_*import` and `GxB_*export` functions described in Section 4.9, since they require the user application and GraphBLAS to use the same memory manager.

These functions can only be set once, when GraphBLAS starts. Either `GrB_init` or `GxB_init` must be called before any other GraphBLAS operation, but not both. The last argument to `GxB_init` informs GraphBLAS as to whether or not the functions are thread-safe. The ANSI C and TBB functions are thread-safe, but the MATLAB `mxMalloc` and related functions are not thread-safe. If not thread-safe, GraphBLAS calls the functions from inside an OpenMP critical section.

The following usage is identical to `GrB_init(mode)`:

```
GxB_init (mode, malloc, calloc, realloc, free, true) ;
```

SuiteSparse:GraphBLAS can be compiled as normal (outside of MATLAB) and then linked into a MATLAB `mexFunction`. However, a `mexFunction` should use the MATLAB memory managers. To do this, use the following instead of `GrB_init(mode)` in a MATLAB `mexFunction`.

```
#include "mex.h"
#include "GraphBLAS.h"
...
GxB_init (mode, mxMalloc, mxCalloc, mxRealloc, mxFree, false) ;
```

As another example, the scalable TBB memory manager can be used:

```
#include "tbb/scalable_allocator.h"
#include "GraphBLAS.h"
...
GxB_init (mode, scalable_malloc, scalable_calloc, scalable_realloc,
    scalable_free, true) ;
```

SPEC: `GxB_init` is an extension to the spec.

3.3 GrB_wait: wait for pending operations to finish

```
GrB_Info GrB_wait ( ) ;     // finish all pending computations
```

GrB_wait forces all pending operations to complete. Blocking mode acts as if GrB_wait is called whenever a GraphBLAS method or operation returns to the user application.

Unless specific rules are followed, non-blocking mode can be unpredictable if user-defined functions have side effects or if they rely on global variables not under the control of GraphBLAS. Suppose the user application creates a user-defined operator that accesses a global variable. That operator is then used in a GraphBLAS operation, which is left pending. If the user application then changes the global variable before pending operations complete, the pending operations will be eventually computed with this different value.

Worse yet, a user-defined operator might be freed before it is needed to finish a pending operation. This causes undefined behavior.

For best results with GraphBLAS, user-defined functions should not have side effects, nor should they access global variables outside the control of GraphBLAS. This allows the non-blocking mode to be used at its fullest level of performance. However, both of these features can safely be used in user-defined functions if the following specific rules are followed.

- User-defined functions may be called in any order when used in a GraphBLAS operation. This order may change in non-obvious ways, even in the same GraphBLAS operation. For example, SuiteSparse:-GraphBLAS relies on multiple algorithms for matrix multiplication, and selects between them automatically. The methods will call user-defined multiply and add operators in the semiring, in different order. The user application should not rely on any particular order used in a specific implementation of GraphBLAS.

- User-defined functions are permitted to access global variables. However, if they do so, the global variables they rely on should not be changed if any GraphBLAS methods or operations are still pending, assuming GraphBLAS is executing in non-blocking mode (see Section 3.1). To ensure this, the user application must call GrB_wait before changing any global variables relied upon by user-defined functions. Alternatively, computations can be forced to complete on selected matrices and vectors via GrB_*_nvals, GrB_*_extractElement,

`GrB_*_extractTuples`, and `GrB_*_reduce` (to scalar) applied to selected matrices and vectors. The `GrB_*_nvals` function is particularly well-suited for this purpose since it is otherwise an extremely lightweight computation in SuiteSparse:GraphBLAS.

- If any GraphBLAS methods or operations are still pending, freeing user-defined types, operators, monoids, semirings, vectors, matrices, or descriptors leads to undefined behavior. A user application must call `GrB_wait` before freeing any user-defined object, if a pending operation relies on it, or by selective completion via, say, `GrB_*_nvals`. Alternatively, if the user application is about to terminate GraphBLAS (see `GrB_finalize` below), then all GraphBLAS objects may be freed in any order, without calling `GrB_wait`. Pending computations will simply be abandoned.

`GrB_wait` ensures that all computations are completed for all objects. For specific objects, `GrB_*_nvals`, `GrB_*_extractElement`, `GrB_*_extractTuples`, and `GrB_*_reduce` (to scalar) ensure that the pending operations are completed just for the matrix or vector they operate on. No other GraphBLAS method or operation guarantees the completion of pending computations, even though they may happen to do so in any particular implementation. In the current version, SuiteSparse:GraphBLAS exploits the non-blocking mode in the `GrB_*_setElement` methods and the `GrB_assign` and `GxB_subassign` operations. Future versions of SuiteSparse:GraphBLAS may extend this to other methods and operations. Refer to the example at the end of Section 2.2.

If multiple user threads have created matrices or vectors, and those have pending operations, then a single call by one thread to `GrB_wait` causes all pending operations left by all threads to be completed. If other user threads are working on any of those matrices, this would result in a race condition. Therefore, `GrB_wait` should be called only when no other user thread is operating on any other matrix. Functions that cause a specific matrix to be finalized (`GrB_*_nvals`, `GrB_*_extractElement`, `GrB_*_extractTuples`, and `GrB_*_reduce` (to scalar)) can be safely called by multiple user threads on different matrices.

3.4 GrB_Info: status code returned by GraphBLAS

Each GraphBLAS method and operation returns its status to the caller as its return value, an enumerated type (an `enum`) called `GrB_Info`. The first two values in the following table denote a successful status, the rest are error codes.

`GrB_SUCCESS`	0	the method or operation was successful
`GrB_NO_VALUE`	1	`A(i,j)` requested but not there. Its value is implicit.
`GrB_UNINITIALIZED_OBJECT`	2	object has not been initialized
`GrB_INVALID_OBJECT`	3	object is corrupted
`GrB_NULL_POINTER`	4	input pointer is `NULL`
`GrB_INVALID_VALUE`	5	generic error code; some value is bad
`GrB_INVALID_INDEX`	6	a row or column index is out of bounds; for indices passed as scalars, not in a list.
`GrB_DOMAIN_MISMATCH`	7	object domains are not compatible
`GrB_DIMENSION_MISMATCH`	8	matrix dimensions do not match
`GrB_OUTPUT_NOT_EMPTY`	9	output matrix already has values in it
`GrB_OUT_OF_MEMORY`	10	out of memory
`GrB_INSUFFICIENT_SPACE`	11	output array not large enough
`GrB_INDEX_OUT_OF_BOUNDS`	12	a row or column index is out of bounds; for indices in a list of indices.
`GrB_PANIC`	13	unrecoverable error.

Not all GraphBLAS methods or operations can return all status codes. Any GraphBLAS method or operation can return an out-of-memory condition, `GrB_OUT_OF_MEMORY`, or a panic, `GrB_PANIC`. These two errors, and the `GrB_INDEX_OUT_OF_BOUNDS` error, are called *execution errors*. The other errors are called *API* errors. An API error is detecting immediately, regardless of the blocking mode. The detection of an execution error may be deferred until the pending operations complete.

In the discussions of each method and operation in this User Guide, most of the obvious error code returns are not discussed. For example, if a required input is a `NULL` pointer, then `GrB_NULL_POINTER` is returned. Only error codes specific to the method or that require elaboration are discussed here. For a full list of the status codes that each GraphBLAS function can return, refer to *The GraphBLAS C API Specification* [BMM+17b].

3.5 GrB_error: get more details on the last error

```
const char *GrB_error ( ) ;      // return a string describing the last error
```

Each GraphBLAS method and operation returns a `GrB_Info` error code. The `GrB_error` function returns additional information on the error in a thread-safe null-terminated string. The string returned by `GrB_error` is allocated in thread local storage and must not be freed or modified. Each user thread has its own error status. The simplest way to use it is just to print it out, such as:

```
info = GrB_some_method_here (...) ;
if (! (info == GrB_SUCCESS || info == GrB_NO_VALUE))
{
    printf ("info: %d error: %s\n", info, GrB_error ( )) ;
}
```

SuiteSparse:GraphBLAS reports many helpful details via `GrB_error`. For example, if a row or column index is out of bounds, the report will state what those bounds are. If a matrix dimension is incorrect, the mismatching dimensions will be provided. `GrB_BinaryOp_new`, `GrB_UnaryOp_new`, and `GxB_SelectOp_new` record the name the function passed to them, and `GrB_Type_new` records the name of its type parameter, and these are printed if the user-defined types and operators are used incorrectly. Refer to the output of the example programs in the `Demo` folder, which intentionally generate errors to illustrate the use of `GrB_error`.

Successful GraphBLAS methods do not modify the last error message recorded. If a GraphBLAS method fails and then subsequent GraphBLAS method succeeds, the error message is not modified from the last failure. Only a subsequent failure will cause `GrB_error` to return a different error message.

Note that `GrB_NO_VALUE` is an not error, but an informational status. `GrB_*_extractElment(&x,A,i,j)`, which does `x=A(i,j)`, returns this value to indicate that `A(i,j)` is not present in the matrix.

In SuiteSparse:GraphBLAS, some failures cannot be safely recorded for `GrB_error` to print. These include `GrB_PANIC` and errors in `GrB_init` and `GxB_init`.

3.6 GrB_finalize: **finish GraphBLAS**

```
GrB_Info GrB_finalize ( ) ;     // finish GraphBLAS
```

GrB_finalize must be called as the last GraphBLAS operation, even after all calls to GrB_free. All GraphBLAS objects created by the user application should be freed first, before calling GrB_finalize since GrB_finalize will not free those objects. In non-blocking mode, GraphBLAS may leave some computations as pending. These computations can be safely abandoned if the user application frees all GraphBLAS objects it has created and then calls GrB_finalize. There is no need to call GrB_wait in this case. When the user application is finished, exactly one user thread must call GrB_finalize.

4   GraphBLAS Objects and their Methods

GraphBLAS defines eight different objects to represent matrices and vectors, their scalar data type (or domain), binary and unary operators on scalar types, monoids, semirings, and a *descriptor* object used to specify optional parameters that modify the behavior of a GraphBLAS operation. Suite-Sparse:GraphBLAS adds an operator for selecting entries from a matrix or vector.

The GraphBLAS API makes a distinction between *methods* and *operations*. A method is a function that works on a GraphBLAS object, creating it, destroying it, or querying its contents. An operation (not to be confused with an operator) acts on matrices and/or vectors in a semiring.

`GrB_Type`	a scalar data type
`GrB_UnaryOp`	a unary operator $z = f(x)$, where z and x are scalars
`GrB_BinaryOp`	a binary operator $z = f(x, y)$, where z, x, and y are scalars
`GxB_SelectOp`	a select operator
`GrB_Monoid`	an associative and commutative binary operator and its identity value
`GrB_Semiring`	a monoid that defines the "plus" and a binary operator that defines the "multiply" for an algebraic semiring
`GrB_Matrix`	a 2D sparse matrix of any type
`GrB_Vector`	a 1D sparse column vector of any type
`GrB_Descriptor`	a collection of parameters that modify an operation

Each of these objects is implemented in C as an opaque handle, which is a pointer to a data structure held by GraphBLAS. User applications may not examine the content of the object directly; instead, they can pass the handle back to GraphBLAS which will do the work. Assigning one handle to another is valid but it does not make a copy of the underlying object.

GraphBLAS provides 11 built-in types and 157 built-in operators; Suite-Sparse:GraphBLAS adds 121 additional built-in operators. With these, 44 unique monoids and 1040 unique semirings can be constructed.

SPEC: SuiteSparse:GraphBLAS predefines all unique monoids and semirings that can be constructed from built-in types and operators, as an extension to the spec. They appear in `GraphBLAS.h`. The `GxB_SelectOp` object is an extension to GraphBLAS.

4.1 The GraphBLAS type: GrB_Type

A GraphBLAS `GrB_Type` defines the type of scalar values that a matrix or vector contains, and the type of scalar operands for a unary or binary operator. There are eleven built-in types, and a user application can define any types of its own as well. The built-in types correspond to built-in types in C (`#include <stdbool.h>` and `#include <stdint.h>`), and the classes in MATLAB, as listed in the following table.

GraphBLAS type	C type	MATLAB class	description	range
GrB_BOOL	bool	logical	Boolean	true (1), false (0)
GrB_INT8	int8_t	int8	8-bit signed integer	-128 to 127
GrB_UINT8	uint8_t	uint8	8-bit unsigned integer	0 to 255
GrB_INT16	int16_t	int16	16-bit integer	-2^{15} to $2^{15}-1$
GrB_UINT16	uint16_t	uint16	16-bit unsigned integer	0 to $2^{16}-1$
GrB_INT32	int32_t	int32	32-bit integer	-2^{31} to $2^{31}-1$
GrB_UINT32	uint32_t	uint32	32-bit unsigned integer	0 to $2^{32}-1$
GrB_INT64	int64_t	int64	64-bit integer	-2^{63} to $2^{63}-1$
GrB_UINT64	uint64_t	uint64	64-bit unsigned integer	0 to $2^{64}-1$
GrB_FP32	float	single	32-bit IEEE 754	-Inf to +Inf
GrB_FP64	double	double	64-bit IEEE 754	-Inf to +Inf

The user application can also define new types based on any `typedef` in the C language whose values are held in a contiguous region of memory. For example, a user-defined `GrB_Type` could be created to hold any C `struct` whose content is self-contained. A C `struct` containing pointers might be problematic because GraphBLAS would not know to dereference the pointers to traverse the entire "scalar" entry, but this can be done if the objects referenced by these pointers are not moved. A user-defined complex type with real and imaginary types can be defined, or even a "scalar" type containing a fixed-sized dense matrix (see Section 4.1.1). The possibilities are endless. GraphBLAS can create and operate on sparse matrices and vectors in any of these types, including any user-defined ones. For user-defined types, GraphBLAS simply moves the data around itself (via `memcpy`), and then passes the values back to user-defined functions when it needs to do any computations on the type. The next sections describe the methods for the `GrB_Type` object:

GrB_Type_new	create a user-defined type
GxB_Type_size	return the size of a type
GrB_Type_free	free a user-defined type

4.1.1 GrB_Type_new: create a user-defined type

```
GrB_Info GrB_Type_new           // create a new GraphBLAS type
(
    GrB_Type *type,             // handle of user type to create
    size_t sizeof_ctype         // size = sizeof (ctype) of the C type
) ;
```

GrB_Type_new creates a new user-defined type. The type is a handle, or a pointer to an opaque object. The handle itself must not be NULL on input, but the content of the handle can be undefined. On output, the handle contains a pointer to a newly created type. The ctype is the type in C that will be used to construct the new GraphBLAS type. It can be either a built-in C type, or defined by a typedef. The second parameter should be passed as sizeof(ctype). The only requirement on the C type is that sizeof(ctype) is valid in C, and that the type reside in a contiguous block of memory so that it can be moved with memcpy. For example, to create a user-defined type called Complex for double-precision complex values using the ANSI C11 double complex type, the following can be used. A complete example can be found in the usercomplex.c and usercomplex.h files in the Demo folder.

```
#include <math.h>
#include <complex.h>
GrB_Type Complex ;
GrB_Type_new (&Complex, sizeof (double complex)) ;
```

To demonstrate the flexibility of the GrB_Type, consider a "scalar" consisting of 4-by-4 floating-point matrix and a string. This type might be useful for the 4-by-4 translation/rotation/scaling matrices that arise in computer graphics, along with a string containing a description or even a regular expression that can be parsed and executed in a user-defined operator. All that is required is a fixed-size type, where sizeof(ctype) is a constant.

```
typedef struct
{
    float stuff [4][4] ;
    char whatstuff [64] ;
}
wildtype ;
GrB_Type WildType ;
GrB_Type_new (&WildType, sizeof (wildtype)) ;
```

With this type a sparse matrix can be created in which each entry consists of a 4-by-4 dense matrix `stuff` and a 64-character string `whatstuff`. GraphBLAS treats this 4-by-4 as a "scalar." Any GraphBLAS method or operation that simply moves data can be used with this type without any further information from the user application. For example, entries of this type can be assigned to and extracted from a matrix or vector, and matrices containing this type can be transposed. A working example (`wildtype.c` in the `Demo` folder) creates matrices and multiplies them with a user-defined semiring with this type.

Performing arithmetic on matrices and vectors with user-defined types requires operators to be defined. For example, the user application can define its own type for complex numbers, but then transposing the matrix with GraphBLAS will not compute the complex conjugate transpose. This corresponds to the array transpose in MATLAB (`C=A.'`) instead of the complex conjugate transpose (`C=A'`). To compute the complex conjugate transpose, the application would need to create a user-defined unary operator to conjugate a user-defined complex scalar, and then apply it to the matrix before or after the transpose, via `GrB_apply`. An extensive set of complex operators are provided in the `usercomplex.c` example in the `Demo` folder, along with an include file, `usercomplex.h`, that is suitable for inclusion in any user application. Thus, while GraphBLAS does not include any complex types or operators, SuiteSparse:GraphBLAS provides them in two simple "user" files in the `Demo` folder. Refer to Section 10.9 for more details on these two example user-defined types.

4.1.2   GxB_Type_size: **return the size of a type**

```
GrB_Info GxB_Type_size          // determine the size of the type
(
    size_t *size,               // the sizeof the type
    GrB_Type type               // type to determine the sizeof
) ;
```

This function acts just like `sizeof(type)` in the C language. For example `GxB_Type_size (&s, GrB_INT32)` sets `s` to 4, the same as `sizeof(int32_t)`.

SPEC: `GxB_Type_size` is an extension to the spec.

4.1.3 GrB_Type_free: **free a user-defined type**

```
GrB_Info GrB_free                    // free a user-defined type
(
    GrB_Type *type                   // handle of user-defined type to free
) ;
```

`GrB_Type_free` frees a user-defined type. Either usage:

```
GrB_Type_free (&type) ;
GrB_free (&type) ;
```

frees the user-defined `type` and sets `type` to `NULL`. It safely does nothing if passed a `NULL` handle, or if `type == NULL` on input.

It is safe to attempt to free a built-in type. SuiteSparse:GraphBLAS silently ignores the request and returns `GrB_SUCCESS`. A user-defined type should not be freed until all operations using the type are completed. Suite-Sparse:GraphBLAS attempts to detect this condition but it must query a freed object in its attempt. This is hazardous and not recommended. Operations on such objects whose type has been freed leads to undefined behavior.

It is safe to first free a type, and then a matrix of that type, but after the type is freed the matrix can no longer be used. The only safe thing that can be done with such a matrix is to free it.

Note the function signature of `GrB_Type_free`, above. It is illustrated with the generic name, `GrB_free`. Any GraphBLAS object can be freed with the single function, `GrB_free`. Refer to Section 4.11 for more details.

GraphBLAS includes many such generic functions. When describing a specific variation, a function is described with its specific name in this User Guide (such as `GrB_Type_free`). When discussing features applicable to all specific forms, the generic name is used instead (such as `GrB_free`).

4.2 GraphBLAS unary operators: GrB_UnaryOp, $z = f(x)$

A unary operator is a scalar function of the form $z = f(x)$. The domain (type) of z and x need not be the same.

There are six kinds of built-in unary operators: one, identity, additive inverse, absolute value, multiplicative inverse, and logical negation. In the notation in the table below, T is any of the 11 built-in types and is a place-holder for BOOL, INT8, UINT8, ... FP32, or FP64. For example, GrB_AINV_INT32 is a unary operator that computes z=-x for two values x and z of type GrB_INT32.

The logical negation operator GrB_LNOT only works on Boolean types. The GxB_LNOT_T functions operate on inputs of type T, implicitly typecasting their input to Boolean and returning result of type T, with a value 1 for true and 0 for false. The operators GxB_LNOT_BOOL and GrB_LNOT are identical. Considering all combinations, there are thus 67 built-in unary operators ((6 kinds of operators) × (11 types), and GrB_LNOT).

GraphBLAS name	types (domains)	expression $z = f(x)$	description
GxB_ONE_T	$T \to T$	$z = 1$	one
GrB_IDENTITY_T	$T \to T$	$z = x$	identity
GrB_AINV_T	$T \to T$	$z = -x$	additive inverse
GxB_ABS_T	$T \to T$	$z = |x|$	absolute value
GrB_MINV_T	$T \to T$	$z = 1/x$	multiplicative inverse
GxB_LNOT_T	$T \to T$	$z = \neg(x \neq 0)$	logical negation
GrB_LNOT	bool $\to$ bool	$z = \neg x$	logical negation

> **SPEC:** GxB_ONE_T, GxB_ABS_T and GxB_LNOT_T are extensions to the spec.

Integer division by zero normally terminates an application, but this is avoided in SuiteSparse:GraphBLAS. For details, see the binary GrB_DIV_T operators.

> **SPEC:** The definition of integer division by zero is an extension to the spec.

The next sections define the following methods for the GrB_UnaryOp object:

`GrB_UnaryOp_new`	create a user-defined unary operator
`GxB_UnaryOp_ztype`	return the type of the output z for $z = f(x)$
`GxB_UnaryOp_xtype`	return the type of the input x for $z = f(x)$
`GrB_UnaryOp_free`	free a user-defined unary operator

4.2.1 GrB_UnaryOp_new: **create a user-defined unary operator**

```
GrB_Info GrB_UnaryOp_new             // create a new user-defined unary operator
(
    GrB_UnaryOp *unaryop,            // handle for the new unary operator
    void *function,                  // pointer to the unary function
    GrB_Type ztype,                  // type of output z
    GrB_Type xtype                   // type of input x
) ;
```

`GrB_UnaryOp_new` creates a new unary operator. The new operator is returned in the `unaryop` handle, which must not be `NULL` on input. On output, its contents contains a pointer to the new unary operator.

The two types `xtype` and `ztype` are the GraphBLAS types of the input x and output z of the user-defined function $z = f(x)$. These types may be built-in types or user-defined types, in any combination. The two types need not be the same, but they must be previously defined before passing them to `GrB_UnaryOp_new`.

The `function` argument to `GrB_UnaryOp_new` is a pointer to a user-defined function with the following signature:

```
void (*f) (void *z, const void *x) ;
```

When the function `f` is called, the arguments `z` and `x` are passed as (`void *`) pointers, but they will be pointers to values of the correct type, defined by `ztype` and `xtype`, respectively, when the operator was created. **NOTE:** Currently, the pointers will be unique. That is, the user function is not called with pointers that point to the same space. Future versions of SuiteSparse:GraphBLAS may call the function with the pointers `z` and `x` equal to one another, in which case `z=f(z)` should be computed.

4.2.2 GxB_UnaryOp_ztype: return the type of z

```
GrB_Info GxB_UnaryOp_ztype          // return the type of z
(
    GrB_Type *ztype,                // return type of output z
    GrB_UnaryOp unaryop             // unary operator
) ;
```

GxB_UnaryOp_ztype returns the ztype of the unary operator, which is the type of z in the function $z = f(x)$.

SPEC: GxB_UnaryOp_ztype is an extension to the spec.

4.2.3 GxB_UnaryOp_xtype: return the type of x

```
GrB_Info GxB_UnaryOp_xtype          // return the type of x
(
    GrB_Type *xtype,                // return type of input x
    GrB_UnaryOp unaryop             // unary operator
) ;
```

GxB_UnaryOp_xtype returns the xtype of the unary operator, which is the type of x in the function $z = f(x)$.

SPEC: GxB_UnaryOp_xtype is an extension to the spec.

4.2.4 GrB_UnaryOp_free: free a user-defined unary operator

```
GrB_Info GrB_free                   // free a user-created unary operator
(
    GrB_UnaryOp *unaryop            // handle of unary operator to free
) ;
```

GrB_UnaryOp_free frees a user-defined unary operator. Either usage:

```
GrB_UnaryOp_free (&unaryop) ;
GrB_free (&unaryop) ;
```

frees the unaryop and sets unaryop to NULL. It safely does nothing if passed a NULL handle, or if unaryop == NULL on input. It does nothing at all if passed a built-in unary operator.

4.3 GraphBLAS binary operators: GrB_BinaryOp, $z = f(x, y)$

A binary operator is a scalar function of the form $z = f(x, y)$. The types of z, x, and y need not be the same.

SuiteSparse:GraphBLAS has 19 kinds of built-in binary operators of the form $T \times T \to T$ that work on all 11 of the built-in types, T, for a total of 209 binary operators of this form. These are listed in the table below. For each of these operators, all domains (types) of the three operands are the same. The six comparison operators and three logical operators all return a result one for true and zero for false, in the same domain T as their inputs. These six comparison operators are useful as "multiply" operators for creating semirings with non-Boolean monoids.

GraphBLAS name	types (domains)	expression $z = f(x, y)$	description
GrB_FIRST_T	$T \times T \to T$	$z = x$	first argument
GrB_SECOND_T	$T \times T \to T$	$z = y$	second argument
GrB_MIN_T	$T \times T \to T$	$z = \min(x, y)$	minimum
GrB_MAX_T	$T \times T \to T$	$z = \max(x, y)$	maximum
GrB_PLUS_T	$T \times T \to T$	$z = x + y$	addition
GrB_MINUS_T	$T \times T \to T$	$z = x - y$	subtraction
GxB_RMINUS_T	$T \times T \to T$	$z = y - x$	reverse subtraction
GrB_TIMES_T	$T \times T \to T$	$z = xy$	multiplication
GrB_DIV_T	$T \times T \to T$	$z = x/y$	division
GxB_RDIV_T	$T \times T \to T$	$z = y/x$	reverse division
GxB_ISEQ_T	$T \times T \to T$	$z = (x == y)$	equal
GxB_ISNE_T	$T \times T \to T$	$z = (x \neq y)$	not equal
GxB_ISGT_T	$T \times T \to T$	$z = (x > y)$	greater than
GxB_ISLT_T	$T \times T \to T$	$z = (x < y)$	less than
GxB_ISGE_T	$T \times T \to T$	$z = (x \geq y)$	greater than or equal
GxB_ISLE_T	$T \times T \to T$	$z = (x \leq y)$	less than or equal
GxB_LOR_T	$T \times T \to T$	$z = (x \neq 0) \vee (y \neq 0)$	logical OR
GxB_LAND_T	$T \times T \to T$	$z = (x \neq 0) \wedge (y \neq 0)$	logical AND
GxB_LXOR_T	$T \times T \to T$	$z = (x \neq 0) \veebar (y \neq 0)$	logical XOR

> **SPEC:** The GxB_IS*_T GxB_RMINUS_T, and GxB_RDIV_T operators, and the Boolean GxB_L*_T are extensions to the spec.

Another set of six kinds of built-in comparison operators have the form $T \times T \to$ bool. They are defined for all eleven built-in types, for a total of 66

binary operators. Note that when T is `bool`, the six operators give the same results as the six `GxB_IS*_BOOL` operators in the table above. These six comparison operators are useful as "multiply" operators for creating semirings with Boolean monoids.

GraphBLAS name	types (domains)	expression $z = f(x, y)$	description
`GrB_EQ_`T	$T \times T \rightarrow$`bool`	$z = (x == y)$	equal
`GrB_NE_`T	$T \times T \rightarrow$`bool`	$z = (x \neq y)$	not equal
`GrB_GT_`T	$T \times T \rightarrow$`bool`	$z = (x > y)$	greater than
`GrB_LT_`T	$T \times T \rightarrow$`bool`	$z = (x < y)$	less than
`GrB_GE_`T	$T \times T \rightarrow$`bool`	$z = (x \geq y)$	greater than or equal
`GrB_LE_`T	$T \times T \rightarrow$`bool`	$z = (x \leq y)$	less than or equal

Finally, GraphBLAS has three built-in binary operators that operate purely in the Boolean domain. These three are identical to the `GxB_L*_BOOL` operators described above, just with a shorter name.

GraphBLAS name	types (domains)	expression $z = f(x, y)$	description
`GrB_LOR`	`bool` $\times$ `bool` $\rightarrow$ `bool`	$z = x \vee y$	logical OR
`GrB_LAND`	`bool` $\times$ `bool` $\rightarrow$ `bool`	$z = x \wedge y$	logical AND
`GrB_LXOR`	`bool` $\times$ `bool` $\rightarrow$ `bool`	$z = x \veebar y$	logical XOR

This gives a total of 278 built-in binary operators listed in the tables above: 209 of the form $T \times T \rightarrow T$, 66 of the form $T \times T \rightarrow$ `bool`, and three purely Boolean. There are 260 unique operators since 18 of the 28 Boolean operators are redundant.

There are two sets of built-in comparison operators in SuiteSparse:GraphBLAS, but they are not redundant. They are identical except for the type (domain) of their output, z. The `GrB_EQ_`T and related operators compare their inputs of type T and produce a Boolean result of true or false. The `GxB_ISEQ_`T and related operators do the same comparison and produce a result with same type T as their input operands, returning one for true or zero for false. The `IS*` comparison operators are useful when combining comparisons with other non-Boolean operators. For example, a `PLUS-ISEQ` semiring counts how many terms of the comparison are true. With this semiring, matrix multiplication $\mathbf{C} = \mathbf{AB}$ for two weighted undirected graphs $\mathbf{A}$ and $\mathbf{B}$ computes c_{ij} as the number of edges node i and j have in common that have identical edge weights. Since the output type of the "multiplier" operator

in a semiring must match the type of its monoid, the Boolean `EQ` cannot be combined with a non-Boolean `PLUS` monoid to perform this operation.

Likewise, SuiteSparse:GraphBLAS has two sets of logical OR, AND, and XOR operators. Without the $\_T$ suffix, the three operators `GrB_LOR`, `GrB_LAND`, and `GrB_LXOR` operate purely in the Boolean domain, where all input and output types are `GrB_BOOL`. The second set (`GxB_LOR`$\_T$ `GxB_LAND`$\_T$ and `GxB_LXOR`$\_T$) provides Boolean operators to all 11 domains, implicitly typecasting their inputs from type T to Boolean and returning a value of type T that is 1 for true or zero for false. The set of `GxB_L*_`T operators are useful since they can be combined with non-Boolean monoids in a semiring.

SPEC: The definition of integer division by zero is an extension to the spec.

Floating-point operations follow the IEEE 754 standard. Thus, computing $x/0$ for a floating-point x results in `+Inf` if x is positive, `-Inf` if x is negative, and `NaN` if x is zero. The application is not terminated. However, integer division by zero normally terminates an application. SuiteSparse:GraphBLAS avoids this by adopting the same rules as MATLAB, which are analogous to how the IEEE standard handles floating-point division by zero. For integers, when x is positive, $x/0$ is the largest positive integer, for negative x it is the minimum integer, and $0/0$ results in zero. For example, for an integer x of type `GrB_INT32`, $1/0$ is $2^{31} - 1$ and $(-1)/0$ is -2^{31}. Refer to Section 4.1 for a list of integer ranges.

The next sections define the following methods for the `GrB_BinaryOp` object:

`GrB_BinaryOp_new`	create a user-defined binary operator
`GxB_BinaryOp_ztype`	return the type of the output z for $z = f(x, y)$
`GxB_BinaryOp_xtype`	return the type of the input x for $z = f(x, y)$
`GxB_BinaryOp_ytype`	return the type of the input y for $z = f(x, y)$
`GrB_BinaryOp_free`	free a user-defined binary operator

4.3.1 GrB_BinaryOp_new: **create a user-defined binary operator**

```
GrB_Info GrB_BinaryOp_new
(
    GrB_BinaryOp *binaryop,         // handle for the new binary operator
    void *function,                 // pointer to the binary function
    GrB_Type ztype,                 // type of output z
    GrB_Type xtype,                 // type of input x
    GrB_Type ytype                  // type of input y
) ;
```

`GrB_BinaryOp_new` creates a new binary operator. The new operator is returned in the `binaryop` handle, which must not be `NULL` on input. On output, its contents contains a pointer to the new binary operator.

The three types `xtype`, `ytype`, and `ztype` are the GraphBLAS types of the inputs x and y, and output z of the user-defined function $z = f(x, y)$. These types may be built-in types or user-defined types, in any combination. The three types need not be the same, but they must be previously defined before passing them to `GrB_BinaryOp_new`.

The final argument to `GrB_BinaryOp_new` is a pointer to a user-defined function with the following signature:

```
void (*f) (void *z, const void *x, const void *y) ;
```

When the function `f` is called, the arguments `z`, `x`, and `y` are passed as `(void *)` pointers, but they will be pointers to values of the correct type, defined by `ztype`, `xtype`, and `ytype`, respectively, when the operator was created. **NOTE:** SuiteSparse:GraphBLAS may call the function with the pointers `z` and `x` equal to one another, in which case `z=f(z,y)` should be computed. Future versions may use additional pointer aliasing.

4.3.2 GxB_BinaryOp_ztype: **return the type of** z

```
GrB_Info GxB_BinaryOp_ztype         // return the type of z
(
    GrB_Type *ztype,                // return type of output z
    GrB_BinaryOp binaryop           // binary operator to query
) ;
```

`GxB_BinaryOp_ztype` returns the `ztype` of the binary operator, which is the type of z in the function $z = f(x, y)$.

4.3.3 GxB_BinaryOp_xtype: **return the type of** x

```
GrB_Info GxB_BinaryOp_xtype         // return the type of x
(
    GrB_Type *xtype,                // return type of input x
    GrB_BinaryOp binaryop           // binary operator to query
) ;
```

GxB_BinaryOp_xtype returns the `xtype` of the binary operator, which is the type of x in the function $z = f(x, y)$.

4.3.4 GxB_BinaryOp_ytype: **return the type of** y

```
GrB_Info GxB_BinaryOp_ytype         // return the type of y
(
    GrB_Type *ytype,                // return type of input y
    GrB_BinaryOp binaryop           // binary operator to query
) ;
```

GxB_BinaryOp_ytype returns the `ytype` of the binary operator, which is the type of y in the function $z = f(x, y)$.

4.3.5 GrB_BinaryOp_free: **free a user-defined binary operator**

```
GrB_Info GrB_free                   // free a user-created binary operator
(
    GrB_BinaryOp *binaryop          // handle of binary operator to free
) ;
```

GrB_BinaryOp_free frees a user-defined binary operator. Either usage:

```
GrB_BinaryOp_free (&op) ;
GrB_free (&op) ;
```

frees the `op` and sets `op` to `NULL`. It safely does nothing if passed a `NULL` handle, or if `op == NULL` on input. It does nothing at all if passed a built-in binary operator.

4.4 SuiteSparse:GraphBLAS select operators: GxB_SelectOp

A select operator is a scalar function of the form $z = f(i, j, m, n, a_{ij}, \text{thunk})$ that is applied to the entries a_{ij} of an m-by-n matrix. The domain (type) of z is always boolean. The domain (type) of a_{ij} can be any built-in or user-defined type, or it can be GrB_NULL if the operator is type-generic.

The GxB_SelectOp operator is used by GxB_select (see Section 7.12) to select entries from a matrix. Each entry A(i,j) is evaluated with the operator, which returns true if the entry is to be kept in the output, or false if it is not to appear in the output. The signature of the select function f is as follows:

```
bool f                      // returns true if A(i,j) is kept
(
    const GrB_Index i,      // row index of A(i,j)
    const GrB_Index j,      // column index of A(i,j)
    const GrB_Index nrows,  // number of rows of A
    const GrB_Index ncols,  // number of columns of A
    const void *x,          // value of A(i,j), or NULL if f is type-generic
    const void *thunk       // user-defined auxiliary data
) ;
```

Operators can be used on any type, including user-defined types, except that the comparisons GT, GE, LT, and LE can only be used with built-in types. User-defined select operators can also be created.

GraphBLAS name	MATLAB analog	description
GxB_TRIL	C=tril(A,k)	true for A(i,j) if (j-i) <= k
GxB_TRIU	C=triu(A,k)	true for A(i,j) if (j-i) >= k
GxB_DIAG	C=diag(A,k)	true for A(i,j) if (j-i) == k
GxB_OFFDIAG	C=A-diag(A,k)	true for A(i,j) if (j-i) != k
GxB_NONZERO	C=A(A~=0)	true if A(i,j) is nonzero
GxB_EQ_ZERO	C=A(A==0)	true if A(i,j) is zero
GxB_GT_ZERO	C=A(A>0)	true if A(i,j) is greater than zero
GxB_GE_ZERO	C=A(A>=0)	true if A(i,j) is greater than or equal to zero
GxB_LT_ZERO	C=A(A<0)	true if A(i,j) is less than zero
GxB_LE_ZERO	C=A(A<=0)	true if A(i,j) is less than or equal to zero
GxB_NE_THUNK	C=A(A~=k)	true if A(i,j) is not equal to k
GxB_EQ_THUNK	C=A(A==k)	true if A(i,j) is equal to k
GxB_GT_THUNK	C=A(A>k)	true if A(i,j) is greater than k
GxB_GE_THUNK	C=A(A>=k)	true if A(i,j) is greater than or equal to k
GxB_LT_THUNK	C=A(A<k)	true if A(i,j) is less than k
GxB_LE_THUNK	C=A(A<=k)	true if A(i,j) is less than or equal to k

> **SPEC:** `GxB_SelectOp` and the table above are extensions to the spec.

The following methods operate on the `GxB_SelectOp` object:

`GxB_SelectOp_new`	create a user-defined select operator
`GxB_SelectOp_xtype`	return the type of the input x
`GxB_SelectOp_free`	free a user-defined select operator

4.4.1 GxB_SelectOp_new: **create a user-defined select operator**

```
GrB_Info GxB_SelectOp_new        // create a new user-defined select operator
(
    GxB_SelectOp *selectop,      // handle for the new select operator
    void *function,              // pointer to the select function
    GrB_Type xtype,              // type of input x, or NULL if type-generic
    GrB_Type ttype               // type of input thunk, or NULL if type-generic
) ;
```

`GxB_SelectOp_new` creates a new select operator. The new operator is returned in the `selectop` handle, which must not be `NULL` on input. On output, its contents contains a pointer to the new select operator.

The `function` argument to `GxB_SelectOp_new` is a pointer to a user-defined function whose signature is given at the beginning of Section 4.4. Given the properties of an entry a_{ij} in an m-by-n matrix, the `function` should return `true` if the entry should be kept in the output of `GxB_select`, or `false` if it should not appear in the output.

The type `xtype` is the GraphBLAS type of the input x of the user-defined function $z = f(i, j, m, n, x, \text{thunk})$. The type may be built-in or user-defined, or it may even be `GrB_NULL`. If the `xtype` is `GrB_NULL`, then the `selectop` is type-generic.

The type `ttype` is the GraphBLAS type of the input *thunk* of the user-defined function $z = f(i, j, m, n, x, \text{thunk})$. The type may be built-in or user-defined, or it may even be `GrB_NULL`. If the `ttype` is `GrB_NULL`, then the `selectop` is type-generic.

For example, to delete all entries in a matrix equal to the additive identity of a monoid, use the `GxB_NE_THUNK` select operator, and pass in the identity value as the `Thunk` parameter to `GxB_select`.

4.4.2 GxB_SelectOp_xtype: **return the type of** x

```
GrB_Info GxB_SelectOp_xtype      // return the type of x
(
    GrB_Type *xtype,             // return type of input x
    GxB_SelectOp selectop        // select operator
) ;
```

GxB_SelectOp_xtype returns the xtype of the select operator, which is the type of x in the function $z = f(i, j, m, n, x, \text{thunk})$. If the select operator is type-generic, xtype is returned as GrB_NULL. This is not an error condition, but simply indicates that the GxB_SelectOp is type-generic.

4.4.3 GxB_SelectOp_ttype: **return the type of the** *thunk*

```
GrB_Info GxB_SelectOp_ttype      // return the type of thunk
(
    GrB_Type *ttype,             // return type of input thunk
    GxB_SelectOp selectop        // select operator
) ;
```

GxB_SelectOp_ttype returns the ttype of the select operator, which is the type of *thunk* in the function $z = f(i, j, m, n, x, \text{thunk})$. If the select operator does not use this parameter, ttype is returned as GrB_NULL. This is not an error condition, but simply indicates that the GxB_SelectOp is type-generic.

4.4.4 GxB_SelectOp_free: **free a user-defined select operator**

```
GrB_Info GxB_free                   // free a user-created select operator
(
    GxB_SelectOp *selectop      // handle of select operator to free
) ;
```

GxB_SelectOp_free frees a user-defined select operator. Either usage:

```
GxB_SelectOp_free (&selectop) ;
GrB_free (&selectop) ;
```

frees the selectop and sets selectop to NULL. It safely does nothing if passed a NULL handle, or if selectop == NULL on input. It does nothing at all if passed a built-in select operator.

4.5 GraphBLAS monoids: GrB_Monoid

A *monoid* is defined on a single domain (that is, a single type), T. It consists of an associative binary operator $z = f(x, y)$ whose three operands x, y, and z are all in this same domain T (that is $T \times T \to T$). The associative operator must also have an identity element, or "zero" in this domain, such that $f(x, 0) = f(0, x) = 0$. Recall that an associative operator $f(x, y)$ is one for which the condition $f(a, f(b, c)) = f(f(a, b), c)$ always holds. That is, operator can be applied in any order and the results remain the same.

Four kinds of built-in operators (MIN, MAX, PLUS, TIMES) can be used to form monoids for each of the ten non-Boolean built-in types, and 12 can be used for Boolean monoids, all of which are listed in the table below. This is a total of 52 valid monoids that can be constructed from built-in types and operators, although 8 of the 12 Boolean monoids are redundant (the four remaining being OR, AND, XOR, and EQ). There are thus a total of 44 unique monoids that can be constructed using built-in binary operators. Since the built-in monoids are also commutative, all of them can be used to create a semiring. Recall that a commutative operator $f(x, y)$ is one for which the condition $f(a, b) = f(b, a)$ always holds. That is, the two operands can be swapped and the results remain the same. One of the components of a semiring is a commutative monoid.

GraphBLAS operator	types (domains)	expression $z = f(x, y)$	identity	terminal
GrB_MIN_T	$T \times T \to T$	$z = \min(x, y)$	$+\infty$	$-\infty$
GrB_MAX_T	$T \times T \to T$	$z = \max(x, y)$	$-\infty$	$+\infty$
GrB_PLUS_T	$T \times T \to T$	$z = x + y$	0	none
GrB_TIMES_T	$T \times T \to T$	$z = xy$	1	0 (not fp)
GrB_LOR	bool $\times$ bool $\to$ bool	$z = x \lor y$	false	true
GrB_LAND	bool $\times$ bool $\to$ bool	$z = x \land y$	true	false
GrB_LXOR	bool $\times$ bool $\to$ bool	$z = x \veebar y$	false	none
GrB_EQ_BOOL	bool $\times$ bool $\to$ bool	$z = (x == y)$	true	none

The above table lists the GraphBLAS operator, its type, expression, identity value, and *terminal* value (if any). For these built-in operators, the terminal values are the *annihilators* of the function, which is the value z so that $z = f(z, y)$ regardless of the value of y. For example $\min(-\infty, y) = -\infty$ for any y. For integer domains, $+\infty$ and $-\infty$ are the largest and smallest integer in their range. With unsigned integers, the smallest value is zero, and thus GrB_MIN_UINT8 has an identity of 255 and a terminal value of 0.

When computing with a monoid, the computation can terminate early if the terminal value arises. No further work is needed since the result will not change. This value is called the terminal value instead of the annihilator, since a user-defined operator can be created with a terminal value that is not an annihilator. See Section 4.5.2 for an example.

The `GrB_TIMES_FP*` operators do not have a terminal value of zero, since they comply with the IEEE 754 standard, and `0*NaN` is not zero, but `NaN`. Technically, their terminal value is `NaN`, but this value is rare in practice and thus the terminal condition is not worth checking.

SuiteSparse:GraphBLAS predefines each of the 44 unique monoids that can be constructed with built-in types and operators, with the naming convention `GxB_op_type_MONOID`. For the first 40, `op` is `MIN`, `MAX`, `PLUS`, or `TIMES`, and `type` is all but `BOOL`. The four Booolean monoids are `GxB_LOR_BOOL_MONOID`, `GxB_LAND_BOOL_MONOID`, `GxB_LXOR_BOOL_MONOID`, and `GxB_EQ_BOOL_MONOID`.

The next sections define the following methods for the `GrB_Monoid` object:

`GrB_Monoid_new`	create a monoid
`GxB_Monoid_terminal_new`	create a monoid that has a terminal value
`GxB_Monoid_operator`	return the monoid operator
`GxB_Monoid_identity`	return the monoid identity value
`GxB_Monoid_terminal`	return the monoid terminal value (if any)
`GrB_Monoid_free`	free a monoid

SPEC: The predefined monoids are an extension to the spec.

4.5.1 GrB_Monoid_new: **create a monoid**

```
GrB_Info GrB_Monoid_new             // create a monoid
(
    GrB_Monoid *monoid,             // handle of monoid to create
    GrB_BinaryOp op,                // binary operator of the monoid
    <type> identity                 // identity value of the monoid
) ;
```

`GrB_Monoid_new` creates a monoid. The operator, `op`, must be an associative binary operator, either built-in or user-defined.

In the definition above, `<type>` is a place-holder for the specific type of the monoid. For built-in types, it is the C type corresponding to the built-in type (see Section 4.1), such as `bool`, `int32_t`, `float`, or `double`. In this case,

`identity` is a scalar value of the particular type, not a pointer. For user-defined types, `<type>` is `void *`, and thus `identity` is a not a scalar itself but a `void *` pointer to a memory location containing the identity value of the user-defined operator, `op`.

If `op` is a built-in operator with a known identity value, then the `identity` parameter is ignored, and its known identity value is used instead.

4.5.2   GxB_Monoid_terminal_new: **create a monoid with terminal**

```
GrB_Info GxB_Monoid_terminal_new    // create a monoid that has a terminal value
(
    GrB_Monoid *monoid,             // handle of monoid to create
    GrB_BinaryOp op,                // binary operator of the monoid
    <type> identity,                // identity value of the monoid
    <type> terminal                 // terminal value of the monoid
) ;
```

`GxB_Monoid_terminal_new` is identical to `GrB_Monoid_new`, except that it allows for the specification of a *terminal value*. The `<type>` of the terminal value is the same as the `identity` parameter; see Section 4.5.1 for details.

The terminal value of a monoid is the value z for which $z = f(z, y)$ for any y, where $z = f(x, y)$ is the binary operator of the monoid. This is also called the *annihilator*, but the term *terminal value* is used here. This is because all annihilators are terminal values, but a terminal value need not be an annihilator, as described in the `MIN` example below.

If the terminal value is encountered during computation, the rest of the computations can be skipped. This can greatly improve the performance of `GrB_reduce`, and matrix multiply in specific cases (when a dot product method is used). For example, using `GrB_reduce` to compute the sum of all entries in a `GrB_FP32` matrix with e entries takes $O(e)$ time, since a monoid based on `GrB_PLUS_FP32` has no terminal value. By contrast, a reduction using `GrB_LOR` on a `GrB_BOOL` matrix can take as little as $O(1)$ time, if a `true` value is found in the matrix very early.

Monoids based on the built-in `GrB_MIN_*` and `GrB_MAX_*` operators (for any type), the boolean `GrB_LOR`, and the boolean `GrB_LAND` operators all have terminal values. For example, the identity value of `GrB_LOR` is `false`, and its terminal value is `true`. When computing a reduction of a set of boolean values to a single value, once a `true` is seen, the computation can exit early since the result is now known.

If `op` is a built-in operator with known identity and terminal values, then the `identity` and `terminal` parameters are ignored, and its known identity and terminal values are used instead.

There may be cases in which the user application needs to use a non-standard terminal value for a built-in operator. For example, suppose the matrix has type `GrB_FP32`, but all values in the matrix are known to be non-negative. The annihilator value of `MIN` is `-INFINITY`, but this will never be seen. However, the computation could could terminate when finding the value zero. This is an example of using a terminal value that is not actually an annihilator, but it functions like one since the monoid will operate stricly on non-negative values. In this case, a monoid created with `GrB_MIN_FP32` will not terminate early. To create a monoid that can terminate early, create a user-defined operator that computes the same thing as `GrB_MIN_FP32`, and then create a monoid based on this user-defined operator with a terminal value of zero and an identity of `+INFINITY`.

SPEC: `GxB_Monoid_terminal_new` is an extension to the spec.

4.5.3 GxB_Monoid_operator: **return the monoid operator**

```
GrB_Info GxB_Monoid_operator       // return the monoid operator
(
    GrB_BinaryOp *op,              // returns the binary op of the monoid
    GrB_Monoid monoid              // monoid to query
) ;
```

`GxB_Monoid_operator` returns the binary operator of the monoid.

SPEC: `GxB_Monoid_operator` is an extension to the spec.

4.5.4 GxB_Monoid_identity: **return the monoid identity**

```
GrB_Info GxB_Monoid_identity       // return the monoid identity
(
    void *identity,                // returns the identity of the monoid
    GrB_Monoid monoid              // monoid to query
) ;
```

`GxB_Monoid_identity` returns the identity value of the monoid. The `void *` pointer, `identity`, must be non-`NULL` and must point to a memory

space of size at least equal to the size of the type of the `monoid`. The type size can be obtained via `GxB_Monoid_operator` to return the monoid additive operator, then `GxB_BinaryOp_ztype` to obtain the `ztype`, followed by `GxB_Type_size` to get its size.

> **SPEC:** `GxB_Monoid_identity` is an extension to the spec.

4.5.5 GxB_Monoid_terminal: **return the monoid terminal value**

```
GrB_Info GxB_Monoid_terminal        // return the monoid terminal
(
    bool *has_terminal,             // true if the monoid has a terminal value
    void *terminal,                 // returns the terminal of the monoid
    GrB_Monoid monoid               // monoid to query
) ;
```

`GxB_Monoid_terminal` returns the terminal value of the monoid (if any). The `void *` pointer, `terminal`, must be non-NULL and must point to a memory space of size at least equal to the size of the type of the `monoid`. The type size can be obtained via `GxB_Monoid_operator` to return the monoid additive operator, then `GxB_BinaryOp_ztype` to obtain the `ztype`, followed by `GxB_Type_size` to get its size.

If the monoid has a terminal value, then `has_terminal` is `true`, and its value is returned in the `terminal` parameter. If it has no terminal value, then `has_terminal` is `false`, and the `terminal` parameter is not modified.

> **SPEC:** `GxB_Monoid_terminal` is an extension to the spec.

4.5.6 GrB_Monoid_free: **free a monoid**

```
GrB_Info GrB_free                   // free a user-created monoid
(
    GrB_Monoid *monoid              // handle of monoid to free
) ;
```

`GrB_Monoid_frees` frees a monoid. Either usage:

```
GrB_Monoid_free (&monoid) ;
GrB_free (&monoid) ;
```

frees the `monoid` and sets `monoid` to NULL. It safely does nothing if passed a NULL handle, or if `monoid` == NULL on input. It does nothing at all if passed a built-in monoid.

4.6   GraphBLAS semirings: GrB_Semiring

A *semiring* defines all the operators required to define the multiplication of two sparse matrices in GraphBLAS, $\mathbf{C} = \mathbf{AB}$. The "add" operator is a commutative and associative monoid, and the binary "multiply" operator defines a function $z = fmult(x, y)$ where the type of z matches the exactly with the monoid type. SuiteSparse:GraphBLAS includes 1040 predefined built-in semirings, which are all those that can be constructed from built-in types and operators. The next sections define the following methods for the `GrB_Semiring` object:

`GrB_Semiring_new`	create a semiring
`GxB_Semiring_add`	return the additive monoid of a semiring
`GxB_Semiring_multiply`	return the binary operator of a semiring
`GrB_Semiring_free`	free a semiring

4.6.1   GrB_Semiring_new: **create a semiring**

```
GrB_Info GrB_Semiring_new            // create a semiring
(
    GrB_Semiring *semiring,          // handle of semiring to create
    GrB_Monoid add,                  // add monoid of the semiring
    GrB_BinaryOp multiply            // multiply operator of the semiring
) ;
```

`GrB_Semiring_new` creates a new semiring, with `add` being the additive monoid and `multiply` being the binary "multiply" operator. In addition to the standard error cases, the function returns `GrB_DOMAIN_MISMATCH` if the output (`ztype`) domain of `multiply` does not match the domain of the `add` monoid. Using built-in types and operators, 1040 unique semirings can be built. This count excludes redundant Boolean operators (for example `GrB_TIMES_BOOL` and `GrB_LAND` are different operators but they are redundant since they always return the same result).

- 760 semirings with a multiplier $T \times T \rightarrow T$ where T is non-Boolean, from the complete cross product of:

    - 4 add monoids (`MIN`, `MAX`, `PLUS`, `TIMES`)
    - 19 multiply operators (`FIRST`, `SECOND`, `MIN`, `MAX`, `PLUS`, `MINUS`, `RMINUS`, `TIMES`, `DIV`, `RDIV`, `ISEQ`, `ISNE`, `ISGT`, `ISLT`, `ISGE`, `ISLE`, `LOR`, `LAND`, `LXOR`).

- 10 non-Boolean types, T

- 240 semirings with a comparison operator $T \times T \to$ `bool`, where T is non-Boolean, from the complete cross product of:

  - 4 Boolean add monoids (`LAND`, `LOR`, `LXOR`, `EQ`)
  - 6 multiply operators (`EQ`, `NE`, `GT`, `LT`, `GE`, `LE`)
  - 10 non-Boolean types, T

- 40 semirings with purely Boolean types, `bool` $\times$ `bool` $\to$ `bool`, from the complete cross product of:

  - 4 Boolean add monoids (`LAND`, `LOR`, `LXOR`, `EQ`)
  - 10 multiply operators (`FIRST`, `SECOND`, `LOR`, `LAND`, `LXOR`, `EQ`, `GT`, `LT`, `GE`, `LE`)

SuiteSparse:GraphBLAS pre-defines all 1040 unique semirings that can be constructed from built-in types and operators, as an extension to the spec. The naming convention is `GxB_add_mult_type`, where `add` is the operator of the addititive monoid, `mult` is the multiply operator, and `type` is the type of inputs to the multiply operator. The name of the domain for the addititve monoid does not appear, since it always matches the type of the output of the `mult` operator. For example `GxB_LAND_EQ_FP32` uses the `GxB_LAND_BOOL_MONOID` as its additive monoid, and the `GrB_EQ_FP32` as the binary multiplicative operator.

> **SPEC:** Predefined semirings are an extension to the spec.

4.6.2 GxB_Semiring_add: **return the additive monoid of a semiring**

```
GrB_Info GxB_Semiring_add           // return the add monoid of a semiring
(
    GrB_Monoid *add,                // returns add monoid of the semiring
    GrB_Semiring semiring           // semiring to query
) ;
```

GxB_Semiring_add returns the additive monoid of a semiring.

SPEC: GxB_Semiring_add is an extension to the spec.

4.6.3 GxB_Semiring_multiply: **return multiply operator of a semiring**

```
GrB_Info GxB_Semiring_multiply      // return multiply operator of a semiring
(
    GrB_BinaryOp *multiply,         // returns multiply operator of the semiring
    GrB_Semiring semiring           // semiring to query
) ;
```

GxB_Semiring_multiply returns the binary multiplicative operator of a semiring.

SPEC: GxB_Semiring_multiply is an extension to the spec.

4.6.4 GrB_Semiring_free: **free a semiring**

```
GrB_Info GrB_free                   // free a user-created semiring
(
    GrB_Semiring *semiring          // handle of semiring to free
) ;
```

GrB_Semiring_free frees a semiring. Either usage:

```
GrB_Semiring_free (&semiring) ;
GrB_free (&semiring) ;
```

frees the semiring and sets semiring to NULL. It safely does nothing if passed a NULL handle, or if semiring == NULL on input. It does nothing at all if passed a built-in semiring.

4.7 GraphBLAS vectors: GrB_Vector

Many of the methods for GraphBLAS vectors require a row index or a size. Many methods for matrices require both a row and column index, or a row and column dimension. These are all integers of a specific type, `GrB_Index`, which is defined in `GraphBLAS.h` as

```
typedef uint64_t GrB_Index ;
```

Row and column indices of an `nrows`-by-`ncols` matrix range from zero to the `nrows-1` for the rows, and zero to `ncols-1` for the columns. Indices are zero-based, like C, and not one-based, like MATLAB. In Suite-Sparse:GraphBLAS, the largest size permitted for any integer of `GrB_Index` is 2^{60}. The largest `GrB_Matrix` that SuiteSparse:GraphBLAS can construct is thus 2^{60}-by-2^{60}. An n-by-n matrix A that size can easily be constructed in practice with $O(|\mathbf{A}|)$ memory requirements, where $|\mathbf{A}|$ denotes the number of entries that explicitly appear in the pattern of $\mathbf{A}$. The time and memory required to construct a matrix that large does not depend on n, since Suite-Sparse:GraphBLAS can represent $\mathbf{A}$ in hypersparse form (see Section 5.3). The largest `GrB_Vector` that can be constructed is 2^{60}-by-1.

This section describes a set of methods that create, modify, query, and destroy a GraphBLAS sparse vector, `GrB_Vector`:

GrB_Vector_new	create a vector
GrB_Vector_dup	copy a vector
GrB_Vector_clear	clear a vector of all entries
GrB_Vector_size	return the size of a vector
GrB_Vector_nvals	return the number of entries in a vector
GxB_Vector_type	return the type of a vector
GrB_Vector_build	build a vector from a set of tuples
GrB_Vector_setElement	add a single entry to a vector
GrB_Vector_extractElement	get a single entry from a vector
GrB_Vector_extractTuples	get all entries from a vector
GxB_Vector_resize	resize a vector
GrB_Vector_free	free a vector
GxB_Vector_import	import a vector (see Section 4.9)
GxB_Vector_export	export a vector (see Section 4.9)

4.7.1 GrB_Vector_new: **create a vector**

```
GrB_Info GrB_Vector_new     // create a new vector with no entries
(
    GrB_Vector *v,          // handle of vector to create
    GrB_Type type,          // type of vector to create
    GrB_Index n             // vector dimension is n-by-1
) ;
```

`GrB_Vector_new` creates a new n-by-1 sparse vector with no entries in it, of the given type. This is analogous to MATLAB statement `v = sparse (n,1)`, except that GraphBLAS can create sparse vectors any type. The pattern of the new vector is empty.

4.7.2 GrB_Vector_dup: **copy a vector**

```
GrB_Info GrB_Vector_dup     // make an exact copy of a vector
(
    GrB_Vector *w,          // handle of output vector to create
    const GrB_Vector u      // input vector to copy
) ;
```

`GrB_Vector_dup` makes a deep copy of a sparse vector, like `w=u` in MATLAB. In GraphBLAS, it is possible, and valid, to write the following:

```
GrB_Vector u, w ;
GrB_Vector_new (&u, GrB_FP64, n) ;
w = u ;                          // w is a shallow copy of u
```

Then `w` and `u` can be used interchangeably. However, only a pointer reference is made, and modifying one of them modifies both, and freeing one of them leaves the other as a dangling handle that should not be used. If two different vectors are needed, then this should be used instead:

```
GrB_Vector u, w ;
GrB_Vector_new (&u, GrB_FP64, n) ;
GrB_Vector_dup (&w, u) ;       // like w = u, but making a deep copy
```

Then `w` and `u` are two different vectors that currently have the same set of values, but they do not depend on each other. Modifying one has no effect on the other.

4.7.3 GrB_Vector_clear: **clear a vector of all entries**

```
GrB_Info GrB_Vector_clear    // clear a vector of all entries;
(                            // type and dimension remain unchanged.
    GrB_Vector v             // vector to clear
) ;
```

`GrB_Vector_clear` clears all entries from a vector. All values `v(i)` are now equal to the implicit value, depending on what semiring ring is used to perform computations on the vector. The pattern of `v` is empty, just as if it were created fresh with `GrB_Vector_new`. Analogous with `v (:) = 0` in MATLAB. The type and dimension of v do not change. In SuiteSparse:GraphBLAS, any pending updates to the vector are discarded.

4.7.4 GrB_Vector_size: **return the size of a vector**

```
GrB_Info GrB_Vector_size     // get the dimension of a vector
(
    GrB_Index *n,            // vector dimension is n-by-1
    const GrB_Vector v       // vector to query
) ;
```

`GrB_Vector_size` returns the size of a vector (the number of rows). Analogous to `n = length(v)` or `n = size(v,1)` in MATLAB.

4.7.5 GrB_Vector_nvals: **return the number of entries in a vector**

```
GrB_Info GrB_Vector_nvals   // get the number of entries in a vector
(
    GrB_Index *nvals,       // vector has nvals entries
    const GrB_Vector v      // vector to query
) ;
```

`GrB_Vector_nvals` returns the number of entries in a vector. Roughly analogous to `nvals = nnz(v)` in MATLAB, except that the implicit value in GraphBLAS need not be zero and `nnz` (short for "number of nonzeros") in MATLAB is better described as "number of entries" in GraphBLAS.

Forced completion: All computations for the vector v are guaranteed to be finished when `GrB_Vector_nvals` method returns. That is, it acts like an object-specific `GrB_wait` for just this particular vector v, which is a side-effect useful in its own right. For example, suppose the computations required for v rely upon a user-defined operator that accesses a user-controlled global variable outside the scope or control of GraphBLAS. If the user-application needs to modify or free the variable, `GrB_Vector_nvals` can be used to force all pending operations for this vector v to complete. The user application can then safely modify the global variable. A call to `GrB_Vector_nvals(&nvals,v)` only ensures that the computations require to compute v are finished; other pending computations for other objects may remain. To ensure that all pending computations are complete for all GraphBLAS objects, use `GrB_wait` instead.

4.7.6 GxB_Vector_type: **return the type of a vector**

```
GrB_Info GxB_Vector_type    // get the type of a vector
(
    GrB_Type *type,         // returns the type of the vector
    const GrB_Vector v      // vector to query
) ;
```

`GxB_Vector_type` returns the type of a vector. Analogous to `type = class (v)` in MATLAB.

SPEC: `GxB_Vector_type` is an extension to the spec.

4.7.7 GrB_Vector_build: **build a vector from a set of tuples**

```
GrB_Info GrB_Vector_build            // build a vector from (I,X) tuples
(
    GrB_Vector w,                    // vector to build
    const GrB_Index *I,              // array of row indices of tuples
    const <type> *X,                 // array of values of tuples
    GrB_Index nvals,                 // number of tuples
    const GrB_BinaryOp dup           // binary function to assemble duplicates
) ;
```

GrB_Vector_build constructs a sparse vector w from a set of tuples, I and X, each of length nvals. The vector w must have already been initialized with GrB_Vector_new, and it must have no entries in it before calling GrB_Vector_build.

This function is just like GrB_Matrix_build (see Section 4.8.8), except that it builds a sparse vector instead of a sparse matrix. For a description of what GrB_Vector_build does, refer to GrB_Matrix_build. For a vector, the list of column indices J in GrB_Matrix_build is implicitly a vector of length nvals all equal to zero. Otherwise the methods are identical.

> **SPEC:** As an extension to the spec, results are defined even if dup is non-associative.

4.7.8 GrB_Vector_setElement: **add a single entry to a vector**

```
GrB_Info GrB_Vector_setElement       // w(i) = x
(
    GrB_Vector w,                    // vector to modify
    <type> x,                        // scalar to assign to w(i)
    GrB_Index i                      // row index
) ;
```

GrB_Vector_setElement sets a single entry in a vector, w(i) = x. The operation is exactly like setting a single entry in an n-by-1 matrix, A(i,0) = x, where the column index for a vector is implicitly j=0. For further details of this function, see GrB_Matrix_setElement in Section 4.8.9.

4.7.9   GrB_Vector_extractElement: **get a single entry from a vector**

```
GrB_Info GrB_Vector_extractElement  // x = v(i)
(
    <type> *x,                      // scalar extracted
    const GrB_Vector v,             // vector to extract an entry from
    GrB_Index i                     // row index
) ;
```

GrB_Vector_extractElement extracts a single entry from a vector, x = v(i). The method is identical to extracting a single entry x = A(i,0) from an n-by-1 matrix, so further details of this method are discussed in Section 4.8.10, which discusses GrB_Matrix_extractElement. In this case, the column index is implicitly j=0. **NOTE:** if no entry is present at v(i), then x is not modified, and the return value of GrB_Vector_extractElement is GrB_NO_VALUE.

Forced completion: All computations for the vector v are guaranteed to be finished when the method returns.

4.7.10   GrB_Vector_extractTuples: **get all entries from a vector**

```
GrB_Info GrB_Vector_extractTuples              // [I,~,X] = find (v)
(
    GrB_Index *I,             // array for returning row indices of tuples
    <type> *X,                // array for returning values of tuples
    GrB_Index *nvals,         // I, X size on input; # tuples on output
    const GrB_Vector v        // vector to extract tuples from
) ;
```

GrB_Vector_extractTuples extracts all tuples from a sparse vector, analogous to [I,~,X] = find(v) in MATLAB. This function is identical to its GrB_Matrix_extractTuples counterpart, except that the array of column indices J does not appear in this function. Refer to Section 4.8.11 where further details of this function are described.

Forced completion: All computations for the vector v are guaranteed to be finished when the method returns.

4.7.11   GxB_Vector_resize: **resize a vector**

```
GrB_Info GxB_Vector_resize      // change the size of a vector
(
    GrB_Vector u,               // vector to modify
    GrB_Index nrows_new         // new number of rows in vector
) ;
```

`GxB_Vector_resize` changes the size of a vector. If the dimension decreases, entries that fall outside the resized vector are deleted.

4.7.12   GrB_Vector_free: **free a vector**

```
GrB_Info GrB_free               // free a vector
(
    GrB_Vector *v               // handle of vector to free
) ;
```

`GrB_Vector_free` frees a vector. Either usage:

```
GrB_Vector_free (&v) ;
GrB_free (&v) ;
```

frees the vector `v` and sets `v` to `NULL`. It safely does nothing if passed a `NULL` handle, or if `v == NULL` on input. In SuiteSparse:GraphBLAS, any pending updates to the vector are abandoned.

4.8 GraphBLAS matrices: GrB_Matrix

This section describes a set of methods that create, modify, query, and destroy a GraphBLAS sparse matrix, `GrB_Matrix`:

GrB_Matrix_new	create a matrix
GrB_Matrix_dup	copy a matrix
GrB_Matrix_clear	clear a matrix of all entries
GrB_Matrix_nrows	return the number of rows of a matrix
GrB_Matrix_ncols	return the number of columns of a matrix
GrB_Matrix_nvals	return the number of entries in a matrix
GxB_Matrix_type	return the type of a matrix
GrB_Matrix_build	build a matrix from a set of tuples
GrB_Matrix_setElement	add a single entry to a matrix
GrB_Matrix_extractElement	get a single entry from a matrix
GrB_Matrix_extractTuples	get all entries from a matrix
GxB_Matrix_resize	resize a matrix
GrB_Matrix_free	free a matrix
GxB_Matrix_import_CSR	import a matrix in CSR form (see Section 4.9)
GxB_Matrix_import_CSC	import a matrix in CSC form (see Section 4.9)
GxB_Matrix_import_HyperCSR	import a matrix in HyperCSR form (see Section 4.9)
GxB_Matrix_import_HyperCSC	import a matrix in HyperCSC form (see Section 4.9)
GxB_Matrix_export_CSR	export a matrix in CSR form (see Section 4.9)
GxB_Matrix_export_CSC	export a matrix in CSC form (see Section 4.9)
GxB_Matrix_export_HyperCSR	export a matrix in HyperCSR form (see Section 4.9)
GxB_Matrix_export_HyperCSC	export a matrix in HyperCSC form (see Section 4.9)

4.8.1 GrB_Matrix_new: create a matrix

```
GrB_Info GrB_Matrix_new      // create a new matrix with no entries
(
    GrB_Matrix *A,           // handle of matrix to create
    GrB_Type type,           // type of matrix to create
    GrB_Index nrows,         // matrix dimension is nrows-by-ncols
    GrB_Index ncols
) ;
```

`GrB_Matrix_new` creates a new `nrows`-by-`ncols` sparse matrix with no entries in it, of the given type. This is analogous to the MATLAB statement `A = sparse (nrows, ncols)`, except that GraphBLAS can create sparse matrices of any type.

4.8.2 GrB_Matrix_dup: **copy a matrix**

```
GrB_Info GrB_Matrix_dup      // make an exact copy of a matrix
(
    GrB_Matrix *C,           // handle of output matrix to create
    const GrB_Matrix A       // input matrix to copy
) ;
```

`GrB_Matrix_dup` makes a deep copy of a sparse matrix, like `C=A` in MAT-LAB. In GraphBLAS, it is possible, and valid, to write the following:

```
GrB_Matrix A, C ;
GrB_Matrix_new (&A, GrB_FP64, n) ;
C = A ;                          // C is a shallow copy of A
```

Then `C` and `A` can be used interchangeably. However, only a pointer reference is made, and modifying one of them modifies both, and freeing one of them leaves the other as a dangling handle that should not be used. If two different matrices are needed, then this should be used instead:

```
GrB_Matrix A, C ;
GrB_Matrix_new (&A, GrB_FP64, n) ;
GrB_Matrix_dup (&C, A) ;         // like C = A, but making a deep copy
```

Then `C` and `A` are two different matrices that currently have the same set of values, but they do not depend on each other. Modifying one has no effect on the other.

4.8.3 GrB_Matrix_clear: **clear a matrix of all entries**

```
GrB_Info GrB_Matrix_clear    // clear a matrix of all entries;
(                            // type and dimensions remain unchanged
    GrB_Matrix A             // matrix to clear
) ;
```

`GrB_Matrix_clear` clears all entries from a matrix. All values `A(i,j)` are now equal to the implicit value, depending on what semiring ring is used to perform computations on the matrix. The pattern of `A` is empty, just as if it were created fresh with `GrB_Matrix_new`. Analogous with `A (:,:) = 0` in MATLAB. The type and dimensions of `A` do not change. In SuiteSparse:Graph-BLAS, any pending updates to the matrix are discarded.

4.8.4 GrB_Matrix_nrows: **return the number of rows of a matrix**

```
GrB_Info GrB_Matrix_nrows    // get the number of rows of a matrix
(
    GrB_Index *nrows,        // matrix has nrows rows
    const GrB_Matrix A       // matrix to query
) ;
```

`GrB_Matrix_nrows` returns the number of rows of a matrix (`nrow=size(A,1)` in MATLAB).

4.8.5 GrB_Matrix_ncols: **return the number of columns of a matrix**

```
GrB_Info GrB_Matrix_ncols    // get the number of columns of a matrix
(
    GrB_Index *ncols,        // matrix has ncols columns
    const GrB_Matrix A       // matrix to query
) ;
```

`GrB_Matrix_ncols` returns the number of columns of a matrix (`ncols=size(A,2)` in MATLAB).

4.8.6 GrB_Matrix_nvals: **return the number of entries in a matrix**

```
GrB_Info GrB_Matrix_nvals    // get the number of entries in a matrix
(
    GrB_Index *nvals,        // matrix has nvals entries
    const GrB_Matrix A       // matrix to query
) ;
```

`GrB_Matrix_nvals` returns the number of entries in a matrix. Roughly analogous to `nvals = nnz(A)` in MATLAB, except that the implicit value in GraphBLAS need not be zero and `nnz` (short for "number of nonzeros") in MATLAB is better described as "number of entries" in GraphBLAS.

Forced completion: All computations for the matrix `A` are guaranteed to be finished when `GrB_Matrix_nvals` returns. That is, it acts like an object-specific `GrB_wait` for just this particular matrix `A`. Other pending computations for other objects may remain. To ensure that all pending computations are complete for all GraphBLAS objects, used `GrB_wait` instead.

4.8.7 GxB_Matrix_type: **return the type of a matrix**

```
GrB_Info GxB_Matrix_type    // get the type of a matrix
(
    GrB_Type *type,         // returns the type of the matrix
    const GrB_Matrix A      // matrix to query
) ;
```

GxB_Matrix_type returns the type of a matrix, like `type=class(A)` in MATLAB.

SPEC: GxB_Matrix_type is an extension to the spec.

4.8.8 GrB_Matrix_build: **build a matrix from a set of tuples**

```
GrB_Info GrB_Matrix_build          // build a matrix from (I,J,X) tuples
(
    GrB_Matrix C,                  // matrix to build
    const GrB_Index *I,            // array of row indices of tuples
    const GrB_Index *J,            // array of column indices of tuples
    const <type> *X,               // array of values of tuples
    GrB_Index nvals,               // number of tuples
    const GrB_BinaryOp dup         // binary function to assemble duplicates
) ;
```

GrB_Matrix_build constructs a sparse matrix C from a set of tuples, I, J, and X, each of length nvals. The matrix C must have already been initialized with GrB_Matrix_new, and it must have no entries in it before calling GrB_Matrix_build. Thus the dimensions and type of C are not changed by this function, but are inherited from the prior call to GrB_Matrix_new or GrB_matrix_dup.

An error is returned (GrB_INDEX_OUT_OF_BOUNDS) if any row index in I is greater than or equal to the number of rows of C, or if any column index in J is greater than or equal to the number of columns of C

Any duplicate entries with identical indices are assembled using the binary dup operator provided on input. All three types (x, y, z for z=dup(x,y)) must be identical. The types of dup, C and X must all be compatible. See Section 2.4 regarding typecasting and compatibility. The values in X are type-casted, if needed, into the type of dup. Duplicates are then assembled into a matrix T of the same type as dup, using T(i,j) = dup (T (i,j), X (k)). After T is constructed, it is typecasted into the result C. That is, typecasting does not occur at the same time as the assembly of duplicates.

> **SPEC:** As an extension to the spec, results are defined even if dup is non-associative.

The GraphBLAS API requires dup to be associative so that entries can be assembled in any order, and states that the result is undefined if dup is not associative. However, SuiteSparse:GraphBLAS guarantees a well-defined order of assembly. Entries in the tuples [I,J,X] are first sorted in increasing order of row and column index, with ties broken by the position of the tuple in the [I,J,X] list. If duplicates appear, they are assembled in the order they appear in the [I,J,X] input. That is, if the same indices i and j appear

in positions `k1`, `k2`, `k3`, and `k4` in `[I,J,X]`, where `k1 < k2 < k3 < k4`, then the following operations will occur in order:

```
T (i,j) = X (k1) ;
T (i,j) = dup (T (i,j), X (k2)) ;
T (i,j) = dup (T (i,j), X (k3)) ;
T (i,j) = dup (T (i,j), X (k4)) ;
```

This is a well-defined order but the user should not depend upon it when using other GraphBLAS implementations since the GraphBLAS API does not require this ordering.

However, SuiteSparse:GraphBLAS guarantees this ordering, and with this well-defined order, several operators become very useful. In particular, the `SECOND` operator results in the last tuple overwriting the earlier ones. The `FIRST` operator means the value of the first tuple is used and the others are discarded.

The acronym `dup` is used here for the name of binary function used for assembling duplicates, but this should not be confused with the `_dup` suffix in the name of the function `GrB_Matrix_dup`. The latter function does not apply any operator at all, nor any typecasting, but simply makes a pure deep copy of a matrix.

The parameter `X` is a pointer to any C equivalent built-in type, or a `void *` pointer. The `GrB_Matrix_build` function uses the `_Generic` feature of ANSI C11 to detect the type of pointer passed as the parameter `X`. If `X` is a pointer to a built-in type, then the function can do the right typecasting. If `X` is a `void *` pointer, then it can only assume `X` to be a pointer to a user-defined type that is the same user-defined type of `C` and `dup`. This function has no way of checking this condition that the `void * X` pointer points to an array of the correct user-defined type, so behavior is undefined if the user breaks this condition.

The `GrB_Matrix_build` method is analogous to `C = sparse (I,J,X)` in MATLAB, with several important extensions that go beyond that which MATLAB can do. In particular, the MATLAB `sparse` function only provides one option for assembling duplicates (summation), and it can only build double, double complex, and logical sparse matrices.

4.8.9 GrB_Matrix_setElement: **add a single entry to a matrix**

```
GrB_Info GrB_Matrix_setElement          // C (i,j) = x
(
    GrB_Matrix C,                       // matrix to modify
    <type> x,                           // scalar to assign to C(i,j)
    GrB_Index i,                        // row index
    GrB_Index j                         // column index
) ;
```

`GrB_Matrix_setElement` sets a single entry in a matrix, `C(i,j)=x`. If the entry is already present in the pattern of `C`, it is overwritten with the new value. If the entry is not present, it is added to `C`. In either case, no entry is ever deleted by this function. Passing in a value of `x=0` simply creates an explicit entry at position `(i,j)` whose value is zero, even if the implicit value is assumed to be zero.

An error is returned (`GrB_INVALID_INDEX`) if the row index `i` is greater than or equal to the number of rows of `C`, or if the column index `j` is greater than or equal to the number of columns of `C`. Note that this error code differs from the same kind of condition in `GrB_Matrix_build`, which returns `GrB_INDEX_OUT_OF_BOUNDS`. This is because `GrB_INVALID_INDEX` is an API error, and is caught immediately even in non-blocking mode, whereas `GrB_INDEX_OUT_OF_BOUNDS` is an execution error whose detection may wait until the computation completes sometime later.

The scalar `x` is typecasted into the type of `C`. Any value can be passed to this function and its type will be detected, via the `_Generic` feature of ANSI C11. For a user-defined type, `x` is a `void *` pointer that points to a memory space holding a single entry of this user-defined type. This user-defined type must exactly match the user-defined type of `C` since no typecasting is done between user-defined types.

Performance considerations: SuiteSparse:GraphBLAS exploits the non-blocking mode to greatly improve the performance of this method. Refer to the example shown in Section 2.2. If the entry exists in the pattern already, it is updated right away and the work is not left pending. Otherwise, it is placed in a list of pending updates, and the later on the updates are done all at once, using the same algorithm used for `GrB_Matrix_build`. In other words, `setElement` in SuiteSparse:GraphBLAS builds its own internal list of tuples `[I,J,X]`, and then calls `GrB_Matrix_build` whenever the matrix is

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needed in another computation, or whenever `GrB_wait` is called.

As a result, if calls to `setElement` are mixed with calls to most other methods and operations (even `extractElement`) then the pending updates are assembled right away, which will be slow. Performance will be good if many `setElement` updates are left pending, and performance will be poor if the updates are assembled frequently.

A few methods and operations can be intermixed with `setElement`, in particular, some forms of the `GrB_assign` and `GxB_subassign` operations are compatible with the pending updates from `setElement`. Sections 7.10 gives more details on which `GxB_subassign` and `GrB_assign` operations can be interleaved with calls to `setElement` without forcing updates to be assembled. Other methods that do not access the existing entries may also be done without forcing the updates to be assembled, namely `GrB_Matrix_clear` (which erases all pending updates), `GrB_Matrix_free`, `GrB_Matrix_ncols`, `GrB_Matrix_nrows`, `GxB_Matrix_type`, and of course `GrB_Matrix_setElement` itself. All other methods and operations cause the updates to be assembled. Future versions of SuiteSparse:GraphBLAS may extend this list.

See Section 10.4 for an example of how to use `GrB_Matrix_setElement`.

4.8.10   GrB_Matrix_extractElement: **get a single entry from a matrix**

```
GrB_Info GrB_Matrix_extractElement      // x = A(i,j)
(
    <type> *x,                          // extracted scalar
    const GrB_Matrix A,                 // matrix to extract a scalar from
    GrB_Index i,                        // row index
    GrB_Index j                         // column index
) ;
```

`GrB_Matrix_extractElement` extracts a single entry from a matrix `x=A(i,j)`.

An error is returned (`GrB_INVALID_INDEX`) if the row index `i` is greater than or equal to the number of rows of `C`, or if column index `j` is greater than or equal to the number of columns of `C`.

NOTE: if no entry is present at `A(i,j)`, then `x` is not modified, and the return value of `GrB_Matrix_extractElement` is `GrB_NO_VALUE`.

If the entry is not present then GraphBLAS does not know its value, since its value depends on the implicit value, which is the identity value of the additive monoid of the semiring. It is not a characteristic of the matrix itself, but of the semiring it is used in. A matrix can be used in any

compatible semiring, and even a mixture of semirings, so the implicit value can change as the semiring changes.

As a result, if the entry is present, `x=A(i,j)` is performed and the scalar `x` is returned with this value. The method returns `GrB_SUCCESS`. If the entry is not present, `x` is not modified, and `GrB_NO_VALUE` is returned to the caller. What this means is up to the caller.

The function knows the type of the pointer `x`, so it can do typecasting as needed, from the type of `A` into the type of `x`. User-defined types cannot be typecasted, so if `A` has a user-defined type then `x` must be a `void *` pointer that points to a memory space the same size as a single scalar of the type of `A`.

Forced completion: All computations for the matrix `A` are guaranteed to be finished when the method returns. In particular, this method causes all pending updates from `GrB_setElement`, `GrB_assign`, or `GxB_subassign` to be assembled, so its use can have performance implications. Calls to this function should not be arbitrarily intermixed with calls to these other two functions. Everything will work correctly and results will be predictable, it will just be slow.

4.8.11 GrB_Matrix_extractTuples:get all entries from a matrix

```
GrB_Info GrB_Matrix_extractTuples          // [I,J,X] = find (A)
(
    GrB_Index *I,                 // array for returning row indices of tuples
    GrB_Index *J,                 // array for returning col indices of tuples
    <type> *X,                    // array for returning values of tuples
    GrB_Index *nvals,             // I,J,X size on input; # tuples on output
    const GrB_Matrix A            // matrix to extract tuples from
) ;
```

`GrB_Matrix_extractTuples` extracts all the entries from the matrix `A`, returning them as a list of tuples, analogous to `[I,J,X]=find(A)` in MAT-LAB. Entries in the tuples `[I,J,X]` are unique. No pair of row and column indices `(i,j)` appears more than once.

The GraphBLAS API states the tuples can be returned in any order. SuiteSparse:GraphBLAS chooses to always return them in sorted order, depending on whether the matrix is stored by row or by column.

The number of tuples in the matrix `A` is given by `GrB_Matrix_nvals(&anvals,A)`.
If `anvals` is larger than the size of the arrays (`nvals` in the parameter list),
an error `GrB_INSUFFICIENT_SIZE` is returned, and no tuples are extracted. If
`nvals` is larger than `anvals`, then only the first `anvals` entries in the arrays
`I J`, and `X` are modified, containing all the tuples of `A`, and the rest of `I J`,
and `X` are left unchanged. On output, `nvals` contains the number of tuples
extracted.

Forced completion: All computations for the matrix `A` are guaranteed to
be finished when the method returns.

4.8.12   GxB_Matrix_resize: **resize a matrix**

```
GrB_Info GxB_Matrix_resize        // change the size of a matrix
(
    GrB_Matrix A,                 // matrix to modify
    const GrB_Index nrows_new,    // new number of rows in matrix
    const GrB_Index ncols_new     // new number of columns in matrix
) ;
```

`GxB_Matrix_resize` changes the size of a matrix. If the dimensions de-
crease, entries that fall outside the resized matrix are deleted.

4.8.13   GrB_Matrix_free: **free a matrix**

```
GrB_Info GrB_free             // free a matrix
(
    GrB_Matrix *A             // handle of matrix to free
) ;
```

`GrB_Matrix_free` frees a matrix. Either usage:

```
GrB_Matrix_free (&A) ;
GrB_free (&A) ;
```

frees the matrix `A` and sets `A` to `NULL`. It safely does nothing if passed a `NULL`
handle, or if `A == NULL` on input. In SuiteSparse:GraphBLAS, any pending
updates to the matrix are abandoned.

4.9 GraphBLAS matrix and vector import/export

The import/export functions allow the user application to create a `GrB_Matrix` or `GrB_Vector` object, and to extract its contents, faster and with less memory overhead than the `GrB_*_build` and `GrB_*_extractTuples` functions.

The semantics of import/export are the same as the *move constructor* in C++. On import, the user provides a set of arrays that have been previously allocated via the ANSI C `malloc`, `calloc`, or `realloc` functions (by default), or by the corresponding functions passed to `GxB_init`. The arrays define the content of the matrix or vector. Unlike `GrB_*_build`, the GraphBLAS library then takes ownership of the user's input arrays and may either:

1. incorporate them into its internal data structure for the new `GrB_Matrix` or `GrB_Vector`, potentially creating the `GrB_Matrix` or `GrB_Vector` in constant time with no memory copying performed, or

2. if the library does not support the import format directly, then it may convert the input to its internal format, and then free the user's input arrays.

3. A GraphBLAS implementation may also choose to use a mix of the two strategies.

SuiteSparse:GraphBLAS takes the first approach, and so the import functions always take $O(1)$ time, and require $O(1)$ memory space to be allocated.

Regardless of the method chosen, as listed above, the input arrays are no longer owned by the user application. If `A` is a `GrB_Matrix` created by an import, the user input arrays are freed no later than `GrB_free(&A)`, and may be freed earlier, at the discretion of the GraphBLAS library. The data structure of the `GrB_Matrix` and `GrB_Vector` remain opaque.

The export of a `GrB_Matrix` or `GrB_Vector` is symmetric with the import operation. The export changes the ownership of the arrays, where the `GrB_Matrix` or `GrB_Vector` no longer exists when the export completes, and instead the user is returned several arrays that contain the matrix or vector in the requested format. Ownership of these arrays is given to the user application, which is then responsible for freeing them via the ANSI C `free` function (by default), or by the `free_function` that was passed in to `GxB_init`. Alternatively, these arrays can be re-imported into a `GrB_Matrix` or `GrB_Vector`, at which point they again become the responsibility of GraphBLAS.

For a matrix export, if the output format matches the current internal format of the matrix then these arrays are returned to the user application in $O(1)$ time and with no memory copying performed. Otherwise, the `GrB_Matrix` is first converted into the requested format, and then exported.

The vector import/export methods use a single format for a `GrB_Vector`. Four different formats are provided for the import/export of a `GrB_Matrix`. For each format, the `Ax` array has a C type corresponding to one of the 11 built-in types in GraphBLAS (`bool`, `int*_t`, `uint*_t`, `float`, and `double`), or that corresponds with the user-defined type. No typecasting is done on import or export.

The table below lists the methods presented in this section.

method	purpose	Section
GxB_Vector_import	import a vector	4.9.1
GxB_Vector_export	export a vector	4.9.2
GxB_Matrix_import_CSR	import a matrix in CSR form	4.9.3
GxB_Matrix_import_CSC	import a matrix in CSC form	4.9.4
GxB_Matrix_import_HyperCSR	import a matrix in HyperCSR form	4.9.5
GxB_Matrix_import_HyperCSC	import a matrix in HyperCSC form	4.9.6
GxB_Matrix_export_CSR	export a matrix in CSR form	4.9.7
GxB_Matrix_export_CSC	export a matrix in CSC form	4.9.8
GxB_Matrix_export_HyperCSR	export a matrix in HyperCSR form	4.9.9
GxB_Matrix_export_HyperCSC	export a matrix in HyperCSC form	4.9.10

SPEC: The import/export methods are extensions to the spec. However, they have been implemented in SuiteSparse:GraphBLAS at the request of the GraphBLAS C API Committee, as a prototype for future consideration for inclusion in a future specification. Their calling sequence may change if these functions are added to the specification as `GrB_*` functions. A GraphBLAS library need not implement these methods in constant time and memory. On import, a library may choose to copy the content of the user arrays into its internal data structure and then `free` the user arrays. On export, it may chose to `malloc` the output arrays, fill them with the requested data, and then `GrB_free` the GraphBLAS object being exported. The semantics of these options are the same as a move constructor; they just take more time and memory. The choice is up to the GraphBLAS implementation since the internal data structure is opaque to the user application.

4.9.1 GxB_Vector_import: **import a vector**

```
GrB_Info GxB_Vector_import  // import a vector in CSC format
(
    GrB_Vector *v,          // vector to create
    GrB_Type type,          // type of vector to create
    GrB_Index n,            // vector length
    GrB_Index nvals,        // number of entries in the vector
    GrB_Index **vi,         // indices, size nvals (in sorted order)
    void      **vx,         // values, size nvals
    const GrB_Descriptor desc    // currently unused
) ;
```

The `GxB_Vector_import` function is a fast way to construct a `GrB_Vector`, always taking just $O(1)$ time. Calling `GxB_Vector_import` with:

```
GxB_Vector_import (&v, type, n, nvals, &vi, &vx, desc) ;
```

is identical to the following:

```
int64_t *Ap = calloc (2, sizeof (int64_t)) ;
Ap [1] = nvals ;
GxB_Matrix_import_CSC (&A, type, n, 1, nvals, -1, &Ap, &vi, &vx, desc) ;
```

except that the latter creates an n-by-1 matrix instead. For the vector import, described here, the first argument is a `GrB_Vector`. The arguments `vi` and `vx` take the place of `Ai` and `Ax`, and the `Ap` array for the CSC matrix import is not provided for a vector import. Refer to the description of `GxB_Matrix_import_CSC` for details (Section 4.9.4).

If successful, `v` is created as a n-by-1 vector. Its entries are the row indices given by `vi`, with the corresponding values in `vx`. The two pointers `vi` and `vx` are returned as `NULL`, which denotes that they are no longer owned by the user application. They have instead been moved into the new vector `v`. The row indices in `vi` must appear in sorted order, and no duplicates can appear. These conditions are not checked, so results are undefined if they are not met exactly. The user application can check the resulting vector `v` with `GxB_print`, if desired, which will determine if these conditions hold.

If not successful, `v` is returned as `NULL` and `vi` and `vx` are not modified.

SPEC: `GxB_Vector_import` is an extension to the spec.

4.9.2 GxB_Vector_export: **export a vector**

```
GrB_Info GxB_Vector_export  // export and free a vector
(
    GrB_Vector *v,           // vector to export and free
    GrB_Type *type,          // type of matrix exported
    GrB_Index *n,            // length of the vector
    GrB_Index *nvals,        // number of entries in the vector
    GrB_Index **vi,          // indices, size nvals
    void      **vx,          // values, size nvals
    const GrB_Descriptor desc      // currently unused
) ;
```

The `GxB_Vector_export` function is a fast way to extract the contents of a `GrB_Vector`, always taking just $O(1)$ time. Using `GxB_Vector_export` with:

```
GxB_Vector_export (&v, &type, &n, &nvals, &vi, &vx, desc) ;
```

is analogous to:

```
GxB_Matrix_export_CSC (&A, &type, &n, &one, &nvals, &nonempty,
    &Ap, &Ai, &Ax, desc)
```

if `A` were an `n`-by-1 matrix. For the vector export, described here, the first argument is a `GrB_Vector`. The arguments `vi` and `vx` take the place of `Ai` and `Ax`, and the `Ap` array for the CSC matrix export is not returned from a vector export. Refer to the description of `GxB_Matrix_export_CSC` for details. (Section 4.9.8).

Exporting a vector forces completion of any pending operations on the vector.

If successful, `v` is returned as `NULL`, and its contents are returned to the user, with its `type`, dimension `n`, and number of entries `nvals`. A sorted list of row indices of entries that were in `v` is returned in `vi`, and the corresponding numerical values are returned in `vx`. If `nvals` is zero, the `vi` and `vx` arrays are returned as `NULL`; this is not an error condition.

If not successful, `v` is unmodified and `vi` and `vx` are not modified.

SPEC: `GxB_Vector_export` is an extension to the spec.

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4.9.3 GxB_Matrix_import_CSR: **import a CSR matrix**

```
GrB_Info GxB_Matrix_import_CSR      // import a CSR matrix
(
    GrB_Matrix *A,          // handle of matrix to create
    GrB_Type type,          // type of matrix to create
    GrB_Index nrows,        // matrix dimension is nrows-by-ncols
    GrB_Index ncols,
    GrB_Index nvals,        // number of entries in the matrix
    // CSR format:
    int64_t nonempty,       // number of rows with at least one entry:
                            // either < 0 if not known, or >= 0 if exact
    GrB_Index **Ap,         // row "pointers", size nrows+1
    GrB_Index **Aj,         // column indices, size nvals
    void      **Ax,         // values, size nvals
    const GrB_Descriptor desc     // currently unused
) ;
```

GxB_Matrix_import_CSR imports a matrix from 3 user arrays in CSR
format. In the resulting GrB_Matrix A, the CSR format is a matrix with a
format (GxB_FORMAT) of GxB_BY_ROW, in standard for instead of hypersparse
form (See Section 5.3).

The first four arguments of GxB_Matrix_import_CSR are the same as
all four arguments of GrB_Matrix_new, because this function is similar. It
creates a new GrB_Matrix A, with the given type and dimensions. The
GrB_Matrix A does not exist on input.

Unlike GrB_Matrix_new, this function also populates the new matrix A
with the three arrays Ap, Aj and Ax, provided by the user, all of which must
have been created with the ANSI C malloc, calloc, or realloc functions
(by default), or by the corresponding malloc_function, calloc_function,
or realloc_function provided to GxB_init. These arrays define the pattern
and values of the new matrix A:

- GrB_Index Ap [nrows+1] ; The Ap array is the row "pointer" array.
  It does not actual contain pointers. More precisely, it is an integer
  array that defines where the column indices and values appear in Aj
  and Ax, for each row. The number of entries in row i is given by the
  expression Ap [i+1] - Ap [i].

- GrB_Index Aj [nvals] ; The Aj array defines the column indices of
  entries in each row.

- ctype Aj [nvals] ; The `Ax` array defines the values of entries in each row. It is passed in as a (void *) pointer, but it must point to an array of size `nvals` values, each of size `sizeof(ctype)`, where `ctype` is the exact type in C that corresponds to the `GrB_Type type` parameter. That is, if `type` is `GrB_INT32`, then `ctype` is `int32_t`. User types may be used, just the same as built-in types.

The content of the three arrays `Ap Aj`, and `Ax` is very specific. This content is not checked, since this function takes only $O(1)$ time. Results are undefined if the following specification is not followed exactly.

The column indices of entries in the ith row of the matrix are held in `Aj [Ap [i] ... Ap[i+1]]`, and the corresponding values are held in the same positions in `Ax`. Column indices must be in the range 0 to `ncols`-1, and must appear in sorted order within each row. No duplicate column indices may appear in any row. `Ap [0]` must equal zero, and `Ap [nrows]` must equal nvals. The `Ap` array must be of size `nrows`+1 (or larger), and the `Aj` and `Ax` arrays must have size at least `nvals`.

If `nvals` is zero, then the content of the `Aj` and `Ax` arrays is not accessed and they may be `NULL` on input (if not `NULL`, they are still freed and returned as `NULL`, if the method is successful).

The `nonempty` parameter is optional. It states the number of rows that have at least one entry: if not known, use -1; if ≥ 0, it must be exact.

An example of the CSR format is shown below. Consider the following matrix with 10 nonzero entries, and suppose the zeros are not stored.

$$
A = \begin{bmatrix} 4.5 & 0 & 3.2 & 0 \\ 3.1 & 2.9 & 0 & 0.9 \\ 0 & 1.7 & 3.0 & 0 \\ 3.5 & 0.4 & 0 & 1.0 \end{bmatrix} \tag{1}
$$

The `Ap` array has length 5, since the matrix is 4-by-4. The first entry must always zero, and `Ap [5] = 10` is the number of entries. The content of the arrays is shown below:

```
int64_t Ap [ ] = { 0,        2,            5,        7,            10 } ;
int64_t Aj [ ] = { 0,   2,   0,   1,   3,   1,   2,   0,   1,   3   } ;
double  Ax [ ] = { 4.5, 3.2, 3.1, 2.9, 0.9, 1.7, 3.0, 3.5, 0.4, 1.0 } ;
```

Spaces have been added to the `Ap` array, just for illustration. Row zero is in `Aj [0..1]` (column indices) and `Ax [0..1]` (values), starting at

`Ap [0] = 0` and ending at `Ap [0+1]-1 = 1`. The list of column indices of row one is at `Aj [2..4]` and row two is in `Aj [5..6]`. The last row (three) appears `Aj [7..9]`, because `Ap [3] = 7` and `Ap [4]-1 = 10-1 = 9`. The corresponding numerical values appear in the same positions in `Ax`.

To iterate over the rows and entries of this matrix, the following code can be used:

```
int64_t nvals = Ap [nrows] ;
for (int64_t i = 0 ; i < nrows ; i++)
{
    // get A(i,:)
    for (int64_t p = Ap [i] ; p < Ap [i+1] ; p++)
    {
        // get A(i,j)
        int64_t  j = Aj [p] ;            // column index
        double aij = Ax [p] ;            // numerical value
    }
}
```

On successful creation of `A`, the three pointers `Ap`, `Aj`, and `Ax` are set to `NULL` on output. This denotes to the user application that it is no longer responsible for freeing these arrays. Internally, GraphBLAS has moved these arrays into its internal data structure. They will eventually be freed no later than when the user does `GrB_free(&A)`, but they may be freed or resized later, if the matrix changes.

If the matrix `A` is later exported in CSR form, and GraphBLAS has not yet reallocated these arrays, then these same three arrays are returned to the user by `GxB_Matrix_export_CSR` (see Section 4.9.7). If an export is performed, the freeing of these three arrays again becomes the responsibility of the user application.

The `GxB_Matrix_import_CSR` function will rarely fail, since it allocates just $O(1)$ space. If it does fail, it returns `GrB_OUT_OF_MEMORY`, and it leaves the three user arrays unmodified. They are still owned by the user application, which is eventually responsible for freeing them with `free(Ap)`, etc.

SPEC: `GxB_Matrix_import_CSR` is an extension to the spec.

4.9.4 GxB_Matrix_import_CSC: **import a CSC matrix**

```
GrB_Info GxB_Matrix_import_CSC      // import a CSC matrix
(
    GrB_Matrix *A,          // handle of matrix to create
    GrB_Type type,          // type of matrix to create
    GrB_Index nrows,        // matrix dimension is nrows-by-ncols
    GrB_Index ncols,
    GrB_Index nvals,        // number of entries in the matrix
    // CSC format:
    int64_t nonempty,       // number of columns with at least one entry:
                            // either < 0 if not known, or >= 0 if exact
    GrB_Index **Ap,         // column "pointers", size ncols+1
    GrB_Index **Ai,         // row indices, size nvals
    void      **Ax,         // values, size nvals
    const GrB_Descriptor desc       // currently unused
) ;
```

GxB_Matrix_import_CSC imports a matrix from 3 user arrays in CSC format. The GrB_Matrix A is created in the CSC format, which is a GxB_FORMAT of GxB_BY_COL. The arguments are identical to GxB_Matrix_import_CSR, except for how the 3 user arrays are interpreted. The column "pointer" array has size ncols+1. The row indices of the columns are in Ai, and must appear in ascending order in each column. The corresponding numerical values are held in Ax. The row indices of column j are held in Ai [Ap [j]...Ap [j+1]-1, and the corresponding numerical values are in the same locations in Ax.

The nonempty parameter is optional. It states the number of columns that have at least one entry: if not known, use -1; if ≥ 0, it must be exact.

The same matrix from Equation 1in the last section (repeated here):

$$A = \begin{bmatrix} 4.5 & 0 & 3.2 & 0 \\ 3.1 & 2.9 & 0 & 0.9 \\ 0 & 1.7 & 3.0 & 0 \\ 3.5 & 0.4 & 0 & 1.0 \end{bmatrix} \qquad (2)$$

is held in CSC form as follows:

```
int64_t Ap [ ] = { 0,             3,              6,        8,        10 } ;
int64_t Ai [ ] = { 0,   1,   3,   1,   2,   3,   0,   2,   1,   3   } ;
double  Ax [ ] = { 4.5, 3.1, 3.5, 2.9, 1.7, 0.4, 3.2, 3.0, 0.9, 1.0 } ;
```

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That is, the row indices of column 1 (the second column) are in `Ai [3..5]`, and the values in the same place in `Ax`, since `Ap [1] = 3` and `Ap [2]-1 = 5`.

To iterate over the columns and entries of this matrix, the following code can be used:

```
int64_t nvals = Ap [ncols] ;
for (int64_t j = 0 ; j < ncols ; j++)
{
    // get A(:,j)
    for (int64_t p = Ap [j] ; p < Ap [j+1] ; p++)
    {
        // get A(i,j)
        int64_t  i = Ai [p] ;          // row index
        double aij = Ax [p] ;          // numerical value
    }
}
```

The method is identical to `GxB_Matrix_import_CSR`; just the format is different. That is, if the method is successful, the 3 user arrays are imported into the new `GrB_Matrix A`, with the given type and dimensions, and returned as `NULL` pointers to the user application.

If `nvals` is zero, then the content of the `Ai` and `Ax` arrays is not accessed and they may be `NULL` on input (if not `NULL`, they are still freed and returned as `NULL`, if the method is successful).

SPEC: `GxB_Matrix_import_CSC` is an extension to the spec.

4.9.5 GxB_Matrix_import_HyperCSR: **import a HyperCSR matrix**

```
GrB_Info GxB_Matrix_import_HyperCSR     // import a hypersparse CSR matrix
(
    GrB_Matrix *A,          // handle of matrix to create
    GrB_Type type,          // type of matrix to create
    GrB_Index nrows,        // matrix dimension is nrows-by-ncols
    GrB_Index ncols,
    GrB_Index nvals,        // number of entries in the matrix
    // hypersparse CSR format:
    int64_t nonempty,       // number of rows in Ah with at least one entry,
                            // either < 0 if not known, or >= 0 if exact
    GrB_Index nvec,         // number of rows in Ah list
    GrB_Index **Ah,         // list of size nvec of rows that appear in A
    GrB_Index **Ap,         // row "pointers", size nvec+1
    GrB_Index **Aj,         // column indices, size nvals
    void      **Ax,         // values, size nvals
    const GrB_Descriptor desc     // currently unused
) ;
```

`GxB_Matrix_import_HyperCSR` imports a matrix in hypersparse CSR format in $O(1)$ time. In the hypersparse format, the `Ap` array itself becomes sparse, if the matrix has rows that are completely empty. An array `Ah` of size `nvec` provides a list of rows that appear in the data structure. For example, consider Equation 3, which is a sparser version of the matrix in Equation 1. Row 2 and column 1 of this matrix are all zero.

$$
A = \begin{bmatrix} 4.5 & 0 & 3.2 & 0 \\ 3.1 & 0 & 0 & 0.9 \\ 0 & 0 & 0 & 0 \\ 3.5 & 0 & 0 & 1.0 \end{bmatrix} \tag{3}
$$

The conventional CSR format would appear as follows. Since the third row (row 2) is all zero, accessing `Ai [Ap [2] ... Ap [3]-1]` gives an empty set (`[2..1]`), and the number of entries in this row is `Ap [i+1] - Ap [i]` = `Ap [3] - Ap [2] = 0`.

```
int64_t Ap [ ] = { 0,        2,2,      4,      5 } ;
int64_t Aj [ ] = { 0,   2,   0,   3,   0    3   }
double  Ax [ ] = { 4.5, 3.2, 3.1, 0.9, 3.5, 1.0 } ;
```

A hypersparse CSR format for this same matrix would discard these duplicate integers in `Ap`. Doing so requires another array, `Ah`, that keeps track of the rows that appear in the data structure.

```
int64_t nvec = 3 ;
int64_t Ah [ ] = { 0,          1,          3          } ;
int64_t Ap [ ] = { 0,          2,          4,          5 } ;
int64_t Aj [ ] = { 0,    2,    0,    3,    0    3    }
double  Ax [ ] = { 4.5, 3.2, 3.1, 0.9, 3.5, 1.0 } ;
```

Note that the `Aj` and `Ax` arrays are the same in the standard and hypersparse CSR formats. The row indices in `Ah` must appear in ascending order, and no duplicates can appear. To iterate over this data structure:

```
int64_t nvals = Ap [nvec] ;
for (int64_t k = 0 ; k < nvec ; k++)
{
    int64_t i = Ah [k] ;                    // row index
    // get A(i,:)
    for (int64_t p = Ap [k] ; p < Ap [k+1] ; p++)
    {
        // get A(i,j)
        int64_t  j = Aj [p] ;               // column index
        double aij = Ax [p] ;               // numerical value
    }
}
```

This is more complex than the standard CSR format, but it requires at most $O(e)$ space, where A is m-by-n with $e = $ `nvals` entries. The standard CSR format requires $O(m+e)$ space. If $e << m$, then the size $m+1$ of `Ap` can dominate the memory required. In the hypersparse form, `Ap` takes on size `nvec+1`, and `Ah` has size `nvec`, where `nvec` is the number of rows that appear in the data structure. The standard CSR format can be viewed as a dense array (of size `nrows`) of sparse row vectors. By contrast, the hypersparse CSR format is a sparse array (of size `nvec`) of sparse row vectors.

The import takes $O(1)$ time. If successful, the four arrays `Ah`, `Ap`, `Aj`, and `Ax` are returned as NULL, and the hypersparse `GrB_Matrix A` is created.

If `nvals` is zero, then the content of the `Aj` and `Ax` arrays is not accessed and they may be NULL on input (if not NULL, they are still freed and returned as NULL, if the method is successful). The `nonempty` parameter is optional. It states the number of rows that have at least one entry: if not known, use -1; if ≥ 0, it must be exact.

> **SPEC:** `GxB_Matrix_import_HyperCSR` is an extension to the spec.

4.9.6 GxB_Matrix_import_HyperCSC: import a HyperCSC matrix

```
GrB_Info GxB_Matrix_import_HyperCSC     // import a hypersparse CSC matrix
(
    GrB_Matrix *A,          // handle of matrix to create
    GrB_Type type,          // type of matrix to create
    GrB_Index nrows,        // matrix dimension is nrows-by-ncols
    GrB_Index ncols,
    GrB_Index nvals,        // number of entries in the matrix
    // hypersparse CSC format:
    int64_t nonempty,       // number of columns in Ah with at least one entry,
                            // either < 0 if not known, or >= 0 if exact
    GrB_Index nvec,         // number of columns in Ah list
    GrB_Index **Ah,         // list of size nvec of columns that appear in A
    GrB_Index **Ap,         // column "pointers", size nvec+1
    GrB_Index **Ai,         // row indices, size nvals
    void      **Ax,         // values, size nvals
    const GrB_Descriptor desc       // currently unused
) ;
```

GxB_Matrix_import_HyperCSC imports a matrix in hypersparse CSC format in $O(1)$ time. It is identical to GxB_Matrix_import_HyperCSR, except for the data structure defined by the four arrays Ah, Ap, Ai, and Ax. It is a sparse array of size nvec of sparse column vectors. The following code iterates over the matrix:

```
int64_t nvals = Ap [nvec] ;
for (int64_t k = 0 ; k < nvec ; k++)
{
    int64_t j = Ah [k] ;                // column index
    // get A(:,j)
    for (int64_t p = Ap [k] ; p < Ap [k+1] ; p++)
    {
        // get A(i,j)
        int64_t  i = Ai [p] ;           // row index
        double aij = Ax [p] ;           // numerical value
    }
}
```

The nonempty parameter is optional. It states the number of columns that have at least one entry: if not known, use -1; if ≥ 0, it must be exact.

> **SPEC:** GxB_Matrix_import_HyperCSC is an extension to the spec.

4.9.7 GxB_Matrix_export_CSR: **export a CSR matrix**

```
GrB_Info GxB_Matrix_export_CSR  // export and free a CSR matrix
(
    GrB_Matrix *A,            // handle of matrix to export and free
    GrB_Type *type,           // type of matrix exported
    GrB_Index *nrows,         // matrix dimension is nrows-by-ncols
    GrB_Index *ncols,
    GrB_Index *nvals,         // number of entries in the matrix
    // CSR format:
    int64_t *nonempty,        // number of rows with at least one entry
    GrB_Index **Ap,           // row "pointers", size nrows+1
    GrB_Index **Aj,           // column indices, size nvals
    void      **Ax,           // values, size nvals
    const GrB_Descriptor desc     // currently unused
) ;
```

GxB_Matrix_export_CSR exports a matrix in CSR form:

```
GxB_Matrix_export_CSR (&A, &type, &nrows, &ncols, &nvals, &nonempty,
                       &Ap, &Aj, &Ax, desc) ;
```

On successful output, the GrB_Matrix A is freed, and A is returned as NULL. Its type is returned in the type parameter, its dimensions in nrows and ncols, its number of entries in nvals, and the CSR format is in the three arrays Ap, Aj, and Ax. If nvals is zero, the Aj and Ax arrays are returned as NULL; this is not an error, and GxB_Matrix_import_CSR also allows these two arrays to be NULL on input when nvals is zero. After a successful export, the user application is responsible for freeing these three arrays via free (or the free function passed to GxB_init). The CSR format is described in Section 4.9.3.

This method takes $O(1)$ time if the matrix is already in standard (non-hypersparse) CSR format internally. If it is in hypersparse CSR form, the export must first convert the matrix to standard CSR form, taking $O(m)$ time and memory, where $m = \text{nrows}$. If the matrix is in CSC format, it is first transposed to convert it to CSR format, and then exported. This takes $O(m + n + e)$ or $O(m + e \log e)$ time and memory, whichever is less, where $n = \text{ncols}$ and $e = \text{nvals}$.

SPEC: GxB_Matrix_export_CSR is an extension to the spec.

4.9.8 GxB_Matrix_export_CSC: **export a CSC matrix**

```
GrB_Info GxB_Matrix_export_CSC  // export and free a CSC matrix
(
    GrB_Matrix *A,          // handle of matrix to export and free
    GrB_Type *type,         // type of matrix exported
    GrB_Index *nrows,       // matrix dimension is nrows-by-ncols
    GrB_Index *ncols,
    GrB_Index *nvals,       // number of entries in the matrix
    // CSC format:
    int64_t *nonempty,      // number of columns with at least one entry
    GrB_Index **Ap,         // column "pointers", size ncols+1
    GrB_Index **Ai,         // row indices, size nvals
    void       **Ax,        // values, size nvals
    const GrB_Descriptor desc      // currently unused
) ;
```

GxB_Matrix_export_CSC exports a matrix in CSC form:

```
GxB_Matrix_export_CSC (&A, &type, &nrows, &ncols, &nvals, &nonempty,
                       &Ap, &Ai, &Ax, desc) ;
```

On successful output, the GrB_Matrix A is freed, and A is returned as NULL. Its type is returned in the type parameter, its dimensions in nrows and ncols, its number of entries in nvals, and the CSC format is in the three arrays Ap, Ai, and Ax. If nvals is zero, the Ai and Ax arrays are returned as NULL; this is not an error, and GxB_Matrix_import_CSC also allows these two arrays to be NULL on input when nvals is zero. After a successful export, the user application is responsible for freeing these three arrays via free (or the free function passed to GxB_init). The CSC format is described in Section 4.9.4.

This method takes $O(1)$ time if the matrix is already in standard (non-hypersparse) CSC format internally. If it is in hypersparse CSC form, the export must first convert the matrix to standard CSC form, taking $O(n)$ time and memory, where $n = $ ncols. If the matrix is in CSR format, it is first transposed to convert it to CSC format, and then exported. This takes $O(m + n + e)$ or $O(n + e \log e)$ time and memory, whichever is less, where $m = $ nrows and $e = $ nvals.

> **SPEC:** GxB_Matrix_export_CSC is an extension to the spec.

4.9.9 GxB_Matrix_export_HyperCSR: **export a HyperCSR matrix**

```
GrB_Info GxB_Matrix_export_HyperCSR  // export and free a hypersparse CSR matrix
(
    GrB_Matrix *A,          // handle of matrix to export and free
    GrB_Type *type,         // type of matrix exported
    GrB_Index *nrows,       // matrix dimension is nrows-by-ncols
    GrB_Index *ncols,
    GrB_Index *nvals,       // number of entries in the matrix
    // hypersparse CSR format:
    int64_t *nonempty,      // number of rows in Ah with at least one entry
    GrB_Index *nvec,        // number of rows in Ah list
    GrB_Index **Ah,         // list of size nvec of rows that appear in A
    GrB_Index **Ap,         // row "pointers", size nvec+1
    GrB_Index **Aj,         // column indices, size nvals
    void      **Ax,         // values, size nvals
    const GrB_Descriptor desc      // currently unused
) ;
```

GxB_Matrix_export_HyperCSR exports a matrix in CSR form:

```
GxB_Matrix_export_HyperCSR (&A, &type, &nrows, &ncols, &nvals, &nonempty,
                            &nvec, &Ah, &Ap, &Aj, &Ax, desc) ;
```

On successful output, the GrB_Matrix A is freed, and A is returned as NULL. Its type is returned in the type parameter, its dimensions in nrows and ncols, its number of entries in nvals, and the number of non-empty rows in nvec. The hypersparse CSR format is in the four arrays Ah, Ap, Aj, and Ax. If nvals is zero, the Aj and Ax arrays are returned as NULL; this is not an error. After a successful export, the user application is responsible for freeing these three arrays via free (or the free function passed to GxB_init). The hypersparse CSR format is described in Section 4.9.5.

This method takes $O(1)$ time if the matrix is already in hypersparse CSR format internally. If it is in standard CSR form, the export must first convert the matrix to hypersparse CSR form, taking $O(m)$ time and memory, where $m = $ nrows. If the matrix is in CSC format, it is first transposed to convert it to hypersparse CSR format, and then exported. If in standard CSC form, the transpose takes $O(m + n + e)$ or $O(n + e \log e)$ time and memory, whichever is less. If in hypersparse CSC format, it takes $O(e \log e)$ time.

SPEC: GxB_Matrix_export_HyperCSR is an extension to the spec.

4.9.10   GxB_Matrix_export_HyperCSC: **export a HyperCSC matrix**

```
GrB_Info GxB_Matrix_export_HyperCSC  // export and free a hypersparse CSC matrix
(
    GrB_Matrix *A,          // handle of matrix to export and free
    GrB_Type *type,         // type of matrix exported
    GrB_Index *nrows,       // matrix dimension is nrows-by-ncols
    GrB_Index *ncols,
    GrB_Index *nvals,       // number of entries in the matrix
    // hypersparse CSC format:
    int64_t *nonempty,      // number of columns in Ah with at least one entry
    GrB_Index *nvec,        // number of columns in Ah list
    GrB_Index **Ah,         // list of size nvec of columns that appear in A
    GrB_Index **Ap,         // columns "pointers", size nvec+1
    GrB_Index **Ai,         // row indices, size nvals
    void      **Ax,         // values, size nvals
    const GrB_Descriptor desc      // currently unused
) ;
```

GxB_Matrix_export_HyperCSC exports a matrix in CSC form:

```
GxB_Matrix_export_HyperCSC (&A, &type, &nrows, &ncols, &nvals, &nonempty,
                            &nvec, &Ah, &Ap, &Ai, &Ax, desc) ;
```

On successful output, the `GrB_Matrix A` is freed, and `A` is returned as
NULL. Its type is returned in the `type` parameter, its dimensions in `nrows`
and `ncols`, its number of entries in `nvals`, and the number of non-empty rows
in `nvec`. The hypersparse CSC format is in the four arrays `Ah`, `Ap`, `Ai`, and
`Ax`. If `nvals` is zero, the `Ai` and `Ax` arrays are returned as NULL; this is not an
error. After a successful export, the user application is responsible for freeing
these three arrays via `free` (or the `free` function passed to `GxB_init`). The
hypersparse CSC format is described in Section 4.9.6.

This method takes $O(1)$ time if the matrix is already in hypersparse CSR
format internally. If it is in standard CSR form, the export must first convert
the matrix to hypersparse CSR form, taking $O(m)$ time and memory, where
$m = $ `nrows`. If the matrix is in CSC format, it is first transposed to convert it
to hypersparse CSR format, and then exported. If in standard CSC form, the
transpose takes $O(m + n + e)$ or $O(n + e \log e)$ time and memory, whichever
is less. If in hypersparse CSC format, it takes $O(e \log e)$ time.

> **SPEC:** `GxB_Matrix_export_HyperCSC` is an extension to the spec.

4.10   GraphBLAS descriptors: GrB_Descriptor

A GraphBLAS *descriptor* modifies the behavior of a GraphBLAS operation. If the descriptor is `GrB_NULL`, defaults are used.

> **SPEC:** `GxB_DEFAULT`, `GxB_NTHRADS`, `GxB_CHUNK`, `GxB_AxB_METHOD`, and `GxB_AxB_*` are extensions to the spec.

The access to these parameters and their values is governed by two `enum` types, `GrB_Desc_Field` and `GrB_Desc_Value`:

```
#define GxB_NTHREADS 5  // for both GrB_Desc_field and GxB_Option_field
#define GxB_CHUNK 7
typedef enum
{
    GrB_OUTP = 0,   // descriptor for output of a method
    GrB_MASK = 1,   // descriptor for the mask input of a method
    GrB_INP0 = 2,   // descriptor for the first input of a method
    GrB_INP1 = 3,   // descriptor for the second input of a method
    GxB_DESCRIPTOR_NTHREADS = GxB_NTHREADS,   // number of threads to use
    GxB_DESCRIPTOR_CHUNK = GxB_CHUNK,   // chunk size for small problems
    GxB_AxB_METHOD = 1000, // descriptor for selecting C=A*B algorithm
}
GrB_Desc_Field ;
```

```
typedef enum
{
    // for all GrB_Descriptor fields:
    GxB_DEFAULT = 0,    // default behavior of the method
    // for GrB_OUTP only:
    GrB_REPLACE = 1,    // clear the output before assigning new values to it
    // for GrB_MASK only:
    GrB_SCMP = 2,       // use the structural complement of the input
    // for GrB_INP0 and GrB_INP1 only:
    GrB_TRAN = 3,       // use the transpose of the input

    // for GxB_AxB_METHOD only:
    GxB_AxB_GUSTAVSON = 1001,   // gather-scatter saxpy method
    GxB_AxB_HEAP      = 1002,   // heap-based saxpy method
    GxB_AxB_DOT       = 1003    // dot product
}
GrB_Desc_Value ;
```

The internal representation is opaque to the user, but in this User Guide the five descriptor fields of a descriptor `desc` are illustrated as an array of five items, as described in the list below. The underlying implementation need not be an array:

- `desc [GrB_OUTP]` is a parameter that modifies the output of a Graph-BLAS operation. Currently, there are two possible settings. In the default case, the output is not cleared, and $\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{Z} = \mathbf{C} \odot \mathbf{T}$ is computed as-is, where $\mathbf{T}$ is the results of the particular GraphBLAS operation.

  In the non-default case, $\mathbf{Z} = \mathbf{C} \odot \mathbf{T}$ is first computed, using the results of $\mathbf{T}$ and the accumulator $\odot$. After this is done, if the `GrB_OUTP` descriptor field is set to `GrB_REPLACE`, then the output is cleared of its entries. Next, the assignment $\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{Z}$ is performed.

- `desc [GrB_MASK]` is a parameter that modifies the `Mask`, even if the mask is not present.

  If this parameter is set to its default value, and if the mask is not present (`Mask==NULL`) then implicitly `Mask(i,j)=1` for all `i` and `j`. If the mask is present then `Mask(i,j)=1` means that `C(i,j)` is to be modified by the $\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{Z}$ update. Otherwise, if `Mask(i,j)=0`, then `C(i,j)` is not modified, even if `Z(i,j)` is an entry with a different value; that value is simply discarded.

  If the `desc [GrB_MASK]` parameter is set to `GrB_SCMP`, then the use of the mask is complemented. In this case, if the mask is not present (`Mask==NULL`) then implicitly `Mask(i,j)=0` for all `i` and `j`. This means that none of $\mathbf{C}$ is modified and the entire computation of $\mathbf{Z}$ might as well have been skipped. That is, a complemented empty mask means no modifications are made to the output object at all, except perhaps to clear it in accordance with the `GrB_OUTP` descriptor. With a complemented mask, if the mask is present then `Mask(i,j)=0` means that `C(i,j)` is to be modified by the $\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{Z}$ update. Otherwise, if `Mask(i,j)=1`, then `C(i,j)` is not modified, even if `Z(i,j)` is an entry with a different value; that value is simply discarded.

  Using a parameter to complement the `Mask` is very useful because constructing the actual complement of a very sparse mask is impossible since it has too many entries. If the number of places in `C` that should

be modified is very small, then use a sparse mask without complementing it. If the number of places in `C` that should be protected from modification is very small, then use a sparse mask to indicate those places, and use a descriptor `GrB_MASK` that complements the use of the mask.

- `desc [GrB_INP0]` and `desc [GrB_INP1]` modify the use of the first and second input matrices `A` and `B` of the GraphBLAS operation.

  If the `desc [GrB_INP0]` is set to `GrB_TRAN`, then `A` is transposed before using it in the operation. Likewise, if `desc [GrB_INP1]` is set to `GrB_TRAN`, then the second input, typically called `B`, is transposed.

  Vectors are never transposed via the descriptor. If a method's first parameter is a matrix and the second a vector, then `desc [GrB_INP0]` modifies the matrix parameter and `desc [GrB_INP1]` is ignored. If a method's first parameter is a vector and the second a matrix, then `desc [GrB_INP1]` modifies the matrix parameter and `desc [GrB_INP0]` is ignored.

  To clarify this in each function, the inputs are labeled as `first input:` and `second input:` in the function signatures.

- `desc [GxB_AxB_METHOD]` suggests the method that should be used to compute `C=A*B`. All the methods compute the same result, except they may have different floating-point roundoff errors. This descriptor should be considered as a hint; SuiteSparse:GraphBLAS is free to ignore it. The current version always follows the hint, however.

  - `GxB_DEFAULT` means that a method is selected automatically.

  - `GxB_AxB_GUSTAVSON`: an extended version of Gustavson's method [Gus78], which is a very good general-purpose method, but sometimes the workspace can be too large. Assuming all matrices are stored by column, it computes `C(:,j)=A*B(:,j)` with a sequence of *saxpy* operations (`C(:,j)+=A(:,k)*B(k:,j)` for each nonzero `B(k,j)`). It requires workspace of size m, to the number of rows of `C`, which is not suitable if the matrices are extremely sparse.

  - `GxB_AxB_HEAP`: a heap-based method, computing `C(:,j)=A*B(:,j)` via a heap of size equal to the maximum number of entries in any column of `B`. The method is very good for hypersparse matrices,

particularly when $|\mathbf{B}|$ is less than the number of rows of C. The method used is similar to Algorithm II in [BG08] (see also [BG12]). It computes C in the same order as Gustavson's method, using a heap instead of a large gather/scatter workspace. The heap has size b, equal to the maximum number of entries in any one vector of B.

– GxB_AxB_DOT: computes C(i,j)=A(:,i)'*B(:,j), for each entry C(i,j). If the mask is present and not complemented, only entries for which M(i,j)=1 are computed. This is a very specialized method that works well only if the mask is present, very sparse, and not complemented, or when C is tiny. For example, it works very well when A and B are tall and thin, and C<M>=A'*B or C=A'*B are computed. It is impossibly slow if C is large and the mask is not present, since it takes $\Omega(mn)$ time if C is m-by-n in that case. It can work either without any workspace at all, or with workspace of size $O(k)$, where k is the inner dimension (the number of rows of B if the matrices are stored by column). It does not need any workspace at all if A or B are completely dense, and is the fastest method in this case. Otherwise, the method is faster if it has access to workspace, and thus it uses $O(k)$ workspace if $k < (|\mathbf{A}| + |\mathbf{B}|)$. If this condition does not hold, then the workspace could dominate the memory usage, so it foregoes the use of any workspace. Since it uses no workspace if $k \geq (|\mathbf{A}| + |\mathbf{B}|)$, it can work very well for extremely sparse or hypersparse matrices, when the mask is present and not complemented.

4.10.1   GrB_Descriptor_new: **create a new descriptor**

```
GrB_Info GrB_Descriptor_new     // create a new descriptor
(
    GrB_Descriptor *descriptor  // handle of descriptor to create
) ;
```

GrB_Descriptor_new creates a new descriptor, with all fields set to their defaults (output is not replaced, mask is not complemented, neither input matrix is transposed, and the method used in C=A*B is selected automatically).

4.10.2 GrB_Descriptor_set: **set a parameter in a descriptor**

```
GrB_Info GrB_Descriptor_set     // set a parameter in a descriptor
(
    GrB_Descriptor desc,        // descriptor to modify
    GrB_Desc_Field field,       // parameter to change
    GrB_Desc_Value val          // value to change it to
) ;
```

GrB_Descriptor_set sets a descriptor field (GrB_OUTP, GrB_MASK, GrB_INP0, GrB_INP1, or GxB_AxB_METHOD) to a particular value (GxB_DEFAULT, GrB_SCMP, GrB_TRAN, GrB_REPLACE, GxB_AxB_GUSTAVSON, GxB_AxB_HEAP, or GxB_AxB_DOT).

Descriptor field	Default	Non-default
GrB_OUTP	GxB_DEFAULT: The output matrix is not cleared. The operation computes $\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{C} \odot \mathbf{T}$.	GrB_REPLACE: After computing $\mathbf{Z} = \mathbf{C} \odot \mathbf{T}$, the output $\mathbf{C}$ is cleared of all entries. Then $\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{Z}$ is performed.
GrB_MASK	GxB_DEFAULT: The Mask is not complemented. Mask(i,j)=1 means the value C_{ij} can be modified by the operation, while Mask(i,j)=0 means the value C_{ij} shall not be modified by the operation.	GrB_SCMP: The Mask is complemented. Mask(i,j)=0 means the value C_{ij} can be modified by the operation, while Mask(i,j)=1 means the value C_{ij} shall not be modified by the operation.
GrB_INP0	GxB_DEFAULT: The first input is not transposed prior to using it in the operation.	GrB_TRAN: The first input is transposed prior to using it in the operation. Only matrices are transposed, never vectors.
GrB_INP1	GxB_DEFAULT: The second input is not transposed prior to using it in the operation.	GrB_TRAN: The second input is transposed prior to using it in the operation. Only matrices are transposed, never vectors.
GrB_AxB_METHOD	GxB_DEFAULT: The method used for computing C=A*B is selected automatically.	GxB_AxB_*method*: The selected method is used to compute C=A*B.

4.10.3 GxB_Desc_set: set a parameter in a descriptor

```
GrB_Info GxB_Desc_set           // set a parameter in a descriptor
(
    GrB_Descriptor desc,        // descriptor to modify
    GrB_Desc_Field field,       // parameter to change
    ...                         // value to change it to
) ;
```

GxB_Desc_set is identical to GrB_Descriptor_set, except that the type of the third parameter can vary with the field. All descriptor fields are currently of type GrB_Desc_Value, so currently this function is identical in all ways to GrB_Descriptor_set, except for the name of the function. Future versions of this function will allow for arbitrary types of the third parameter, depending on the field. For a simpler-to-use alternative, see GxB_set described in Section 5.

SPEC: GxB_Desc_set is an extension to the spec.

4.10.4 GxB_Desc_get: get a parameter from a descriptor

```
GrB_Info GxB_Desc_get           // get a parameter from a descriptor
(
    GrB_Descriptor desc,        // descriptor to query; NULL means defaults
    GrB_Desc_Field field,       // parameter to query
    ...                         // value of the parameter
) ;
```

GxB_Desc_get returns the value of a single field in a descriptor. The type of the third parameter is a pointer to a variable type, whose type depends on the field. Currently, all descriptor values are of type GrB_Desc_Value, so this third parameter is a pointer to a scalar value of type GrB_Desc_Value. For a simpler-to-use alternative, see GxB_get described in Section 5.

SPEC: GxB_Desc_get is an extension to the spec.

4.10.5 GrB_Descriptor_free: **free a descriptor**

```
GrB_Info GrB_free               // free a descriptor
(
    GrB_Descriptor *descriptor  // handle of descriptor to free
) ;
```

`GrB_Descriptor_free` frees a descriptor. Either usage:

```
GrB_Descriptor_free (&descriptor) ;
GrB_free (&descriptor) ;
```

frees the `descriptor` and sets `descriptor` to NULL. It safely does nothing if passed a `NULL` handle, or if `descriptor == NULL` on input.

There are currently no predefined descriptors, but if these are added in the future, this function will do nothing if passed a built-in descriptor.

4.11 GrB_free: free any GraphBLAS object

Each of the nine objects has `GrB_*_new` and `GrB_*_free` methods that are specific to each object. They can also be accessed by a generic function, `GrB_free`, that works for all nine objects. If `G` is any of the nine objects, the statement

```
GrB_free (&G) ;
```

frees the object and sets the variable `G` to `NULL`. It is safe to pass in a `NULL` handle, or to free an object twice:

```
GrB_free (NULL) ;        // SuiteSparse:GraphBLAS safely does nothing
GrB_free (&G) ;          // the object G is freed and G set to NULL
GrB_free (&G) ;          // SuiteSparse:GraphBLAS safely does nothing
```

However, the following sequence of operations is not safe. The first two are valid but the last statement will lead to undefined behavior.

```
H = G ;                  // valid; creates a 2nd handle of the same object
GrB_free (&G) ;          // valid; G is freed and set to NULL; H now undefined
GrB_some_method (H) ;    // not valid; H is undefined
```

Some objects are predefined, such as the built-in types. If a user application attempts to free a built-in object, SuiteSparse:GraphBLAS will safely do nothing. In all cases, the `GrB_free` function in SuiteSparse:GraphBLAS always returns `GrB_SUCCESS`.

5 SuiteSparse:GraphBLAS Options

> **SPEC:** GxB_set and GxB_get are extensions to the specification.

SuiteSparse:GraphBLAS includes two type-generic methods, `GxB_set` and `GxB_get`, that set and query various options and parameters settings, including a generic way to set values in the `GrB_Descriptor` object. Using these methods, the user application can provide hints to SuiteSparse:GraphBLAS on how it should store and operate on its matrices. These hints have no effect on the results of any GraphBLAS operation (except perhaps floating-point roundoff differences), but they can have a great impact on the amount of time or memory taken.

- `GxB_set (field, value)` provides hints to SuiteSparse:GraphBLAS on how it should store all matrices created after calling this function: by row, by column, and whether or not to use a *hypersparse* format [BG08, BG12]. These are global options that modify all matrices created after calling this method.

- `GxB_set (GrB_Matrix A, field, value)` provides hints to SuiteSparse: GraphBLAS on how to store a particular matrix. This method allows SuiteSparse:GraphBLAS to transform a specific matrix from one format to another. The format has no effect on the result computed by GraphBLAS; it only affects the time and memory taken to do the computations.

- `GxB_set (GrB_Descriptor desc, field, value)` is another way to set the value of a field in a `GrB_Descriptor`. It is identical to `GrB_Descriptor_set`, just with a generic name.

The `GxB_get` method queries a `GrB_Descriptor`, a `GrB_Matrix`, or the global options.

- `GxB_get (field, &value)` retrieves the current value of a global option.

- `GxB_get (GrB_Matrix A, field, &value)` retrieves the current value of an option from a particular matrix `A`.

- `GxB_get (GrB_Descriptor desc, field, &value)` retrieves the value of a field in a descriptor.

5.1 OpenMP parallelism

SuiteSparse:GraphBLAS Version 3 is a parallel library, based on OpenMP. By default, all GraphBLAS operations will use up to the maximum number of threads specified by the `omp_get_max_threads` OpenMP function. For small problems, GraphBLAS may choose to use fewer threads, using two parameters: the maximum number of threads to use (which may differ from the `omp_get_max_threads` value), and a parameter called the `chunk`. Suppose `work` is a measure of the work an operation needs to perform (say the number of nonzeros in the two input matrices for `GrB_eWiseAdd`). No more than `floor(work/chunk)` threads will be used (or one thread if the ratio is less than 1).

The default `chunk` value is 4096, but this may change in future versions, or it may be modified when GraphBLAS is installed on a particular machine.

Both parameters can be set in two ways:

- Globally: If the following methods are used, then all subsequent Graph-BLAS operations will use these settings. Note the typecast, `(double) chunk`. This is necessary if a literal constant such as `20000` is passed as this argument. The type of the constant must be double. This is true for all uses of `GxB_set`.

  ```
  int nthreads_max = 40 ;
  GxB_set (GxB_NTHREADS, nthreads_max) ;
  GxB_set (GxB_CHUNK, (double) 20000) ;
  ```

- Per operation: Most GraphBLAS operations take a `GrB_Descriptor` input, and this can be modified to set the number of threads and chunk size for the operation that uses this descriptor. Note that `chunk` is a `double`.

  ```
  GrB_Descriptor desc ;
  GrB_Descriptor_new (&desc)
  int nthreads_max = 40 ;
  GxB_set (desc, GxB_NTHREADS, nthreads_max) ;
  double chunk = 20000 ;
  GxB_set (desc, GxB_CHUNK, chunk) ;
  ```

The smaller of `nthreads_max` and `floor(work/chunk)` is used for any given GraphBLAS operation, except that a single thread is used if this value is zero or less.

If either parameter is set to `GxB_DEFAULT`, then default values are used. The default for `nthreads_max` is the return value from `omp_get_max_threads`, and the default chunk size is currently 4096.

If a descriptor value for either parameter is left at its default, or set to `GxB_DEFAULT`, then the global setting is used. This global setting may have been modified from its default, and this modified value will be used.

For example, suppose `omp_get_max_threads` reports 8 threads. If `GxB_set (GxB_NTHREADS, 4)` is used, then the global setting is four threads, not eight. If a descriptor is used but its `GxB_NTHREADS` is not set, or set to `GxB_DEFAULT`, then any operation that uses this descriptor will use the global setting of four threads.

5.2 Storing a matrix by row or by column

The GraphBLAS `GrB_Matrix` is entirely opaque to the user application, and the GraphBLAS API does not specify how the matrix should be stored. However, choices made in how the matrix is represented in a particular implementation, such as SuiteSparse:GraphBLAS, can have a large impact on performance.

Many graph algorithms are just as fast in any format, but some algorithms are much faster in one format or the other. For example, suppose the user application stores a directed graph as a matrix `A`, with the edge (i, j) represented as the value `A(i,j)`, and the application makes many accesses to the ith row of the matrix, with `GrB_Col_extract (w,...,A,GrB_ALL,...,i,desc)` with the transposed descriptor (`GrB_INP0` set to `GrB_TRAN`). If the matrix is stored by column this can be extremely slow, just like the expression `w=A(i,:)` in MATLAB, where `i` is a scalar. Since this is a typical use-case in graph algorithms, the default format in SuiteSparse:GraphBLAS is to store its matrices by row, in Compressed Sparse Row format (CSR).

MATLAB stores its sparse matrices by column, in "non-hypersparse" format, in what is called the Compressed Sparse Column format, or CSC for short. An `m`-by-`n` matrix in MATLAB is represented as a set of `n` column vectors, each with a sorted list of row indices and values of the nonzero entries in that column. As a result, `w=A(:,j)` is very fast, since the result is already held in the data structure a single list, the jth column vector. However, `w=A(i,:)` is very slow in MATLAB, since every column in the matrix has to be searched to see if it contains row `i`. In MATLAB, if many such accesses are made, it is much better to transpose the matrix (say `AT=A'`) and then use

`w=AT(:,i)` instead. This can have a dramatic impact on the performance of MATLAB.

Likewise, if `u` is a very sparse column vector and `A` is stored by column, then `w=u'*A` (via `GrB_vxm`) is slower than `w=A*u` (via `GrB_mxv`). The opposite is true if the matrix is stored by row.

An example of this can be found in Section B.1 of Version 1.2 of the GraphBLAS API Specification, where the breadth-first search `BFS` uses `GrB_vxm` to compute `q'=q'*A`. This method is not fast if the matrix `A` is stored by column. The `bfs5` and `bfs6` examples in the `Demo/` folder of Suite-Sparse:GraphBLAS use `GrB_vxm`, which is fast since the matrices are assumed to be stored in their default format, by row.

SuiteSparse:GraphBLAS stores its sparse matrices by row, by default. In Versions 2.1 and earlier, the matrices were stored by column, by default. However, it can also be instructed to store any selected matrices, or all matrices, by column instead (just like MATLAB), so that `w=A(:,j)` (via `GrB_Col_extract`) is very fast. The change in data format has no effect on the result, just the time and memory usage. To use a column-oriented format by default, the following can be done in a user application that tends to access its matrices by column.

```
GrB_init (...) ;
// just after GrB_init: do the following:
#ifdef GxB_SUITESPARSE_GRAPHBLAS
GxB_set (GxB_FORMAT, GxB_BY_COL) ;
#endif
```

If this is done, and no other `GxB_set` calls are made with `GxB_FORMAT`, all matrices will be stored by column. Alternatively, SuiteSparse:GraphBLAS can be compiled with `-DBYCOL`, which changes the default format to `GxB_BY_COL`, with no calls to any `GxB_*` function. **NOTE: the default format is now `GxB_BY_ROW` in this release; the default in Versions 2.1 and earlier was `GxB_BY_COL`.**

5.3   Hypersparse matrices

MATLAB can store an `m`-by-`n` matrix with a very large value of `m`, since a CSC data structure takes $O(n + |\mathbf{A}|)$ memory, independent of `m`, where $|\mathbf{A}|$ is the number of nonzeros in the matrix. It cannot store a matrix with a huge `n`, and this structure is also inefficient when $|\mathbf{A}|$ is much smaller than

n. In contrast, SuiteSparse:GraphBLAS can store its matrices in *hypersparse* format, taking only $O(|\mathbf{A}|)$ memory, independent of how it is stored (by row or by column) and independent of both m and n [BG08, BG12].

In both the CSR and CSC formats, the matrix is held as a set of sparse vectors. In non-hypersparse format, the set of sparse vectors is itself dense; all vectors are present, even if they are empty. For example, an m-by-n matrix in non-hypersparse CSC format contains n sparse vectors. Each column vector takes at least one integer to represent, even for a column with no entries. This allows for quick lookup for a particular vector, but the memory required is $O(n+|\mathbf{A}|)$. With a hypersparse CSC format, the set of vectors itself is sparse, and columns with no entries take no memory at all. The drawback of the hypersparse format is that finding an arbitrary column vector j, such as for the computation C=A(:,j), takes $O(\log k)$ time if there $k \leq n$ vectors in the data structure. One advantage of the hypersparse structure is the memory required for an m-by-n hypersparse CSC matrix is only $O(|\mathbf{A}|)$, independent of m and n. Algorithms that must visit all non-empty columns of a matrix are much faster when working with hypersparse matrices, since empty columns can be skipped.

The hyper_ratio parameter controls the hypersparsity of the internal data structure for a matrix. The parameter is typically in the range 0 to 1. The default is hyper_ratio = GxB_HYPER_DEFAULT, which is an extern const double value, currently set to 0.0625. This default ratio may change in the future.

The hyper_ratio determines how the matrix is converted between the hypersparse and non-hypersparse formats. Let n be the number of columns of a CSC matrix, or the number of rows of a CSR matrix. The matrix can have at most n non-empty vectors.

Let k be the actual number of non-empty vectors. That is, for the CSC format, $k \leq n$ is the number of columns that have at least one entry. Let h be the value of hyper_ratio.

If a matrix is currently hypersparse, it can be converted to non-hypersparse if the either condition $n \leq 1$ or $k > 2nh$ holds, or both. Otherwise, it stays hypersparse. Note that if $n \leq 1$ the matrix is always stored as non-hypersparse.

If currently non-hypersparse, it can be converted to hypersparse if both conditions $n > 1$ and $k \leq nh$ hold. Otherwise, it stays non-hypersparse. Note that if $n \leq 1$ the matrix always remains non-hypersparse.

The default value of hyper_ratio is assigned at startup by GrB_init,

and can then be modified globally with `GxB_set`. All new matrices are created with the same `hyper_ratio`, determined by the global value. Once a particular matrix `A` has been constructed, its hypersparsity ratio can be modified from the default with:

```
double hyper_ratio = 0.2 ;
GxB_set (A, GxB_HYPER, hyper_ratio) ;
```

To force a matrix to always be non-hypersparse, use `hyper_ratio` equal to `GxB_NEVER_HYPER`. To force a matrix to always stay hypersparse, set `hyper_ratio` to `GxB_ALWAYS_HYPER`.

A `GrB_Matrix` can thus be held in one of four formats: any combination of hyper/non-hyper and CSR/CSC. All `GrB_Vector` objects are always stored in non-hypersparse CSC format.

A new matrix created via `GrB_Matrix_new` starts with $k = 0$ and is created in hypersparse form by default unless $n \leq 1$ or if $h < 0$, where h is the global `hyper_ratio` value. The matrix is created in in either `GxB_BY_ROW` or `GxB_BY_COL` format, as determined by the last call to `GxB_set(GxB_FORMAT,...)` or `GrB_init`.

A new matrix `C` created via `GrB_dup (&C,A)` inherits the CSR/CSC format, hypersparsity format, and `hyper_ratio` from `A`.

Parameter types: The `GxB_Option_Field` enumerated type gives the type of the `field` parameter for the second argument of `GxB_set` and `GxB_get`, for setting global options or matrix options.

```
typedef enum
{
    GxB_HYPER = 0,       // defines switch to hypersparse format (double value)
    GxB_FORMAT = 1,      // defines CSR/CSC format: GxB_BY_ROW or GxB_BY_COL
    GxB_MODE = 2,        // mode passed to GrB_init (blocking or non-blocking)
    GxB_THREAD_SAFETY = 3,  // thread library for thread safety
    GxB_THREADING = 4,      // currently none (in progress)
    GxB_GLOBAL_NTHREADS = GxB_NTHREADS, // max number of threads to use
    GxB_GLOBAL_CHUNK = GxB_CHUNK,       // chunk size for small problems
    GxB_IS_HYPER = 6     // query a matrix to see if it hypersparse or not
                         // (GxB_Matrix_Option_get only)
}
GxB_Option_Field ;
```

The `GxB_FORMAT` field can be by row or by column, set to a value with the type `GxB_Format_Value`:

```
typedef enum
{
    GxB_BY_ROW = 0,       // CSR: compressed sparse row format
    GxB_BY_COL = 1        // CSC: compressed sparse column format
}
GxB_Format_Value ;
```

The default format (in SuiteSparse:GraphBLAS Version 2.2 and later) is by row. The format in SuiteSparse:GraphBLAS Version 2.1 and earlier was by column, just like MATLAB.

The default format is given by the predefined value `GxB_FORMAT_DEFAULT`, which is equal to `GxB_BY_ROW` if default compile-time options are used. To change the default at compile time to `GxB_BY_COL`, compile the SuiteSparse: GraphBLAS library with `-DBYCOL`. This changes `GxB_FORMAT_DEFAULT` to `GxB_BY_COL`. The default hypersparsity ratio is 0.0625 (1/16), but this value may change in the future.

Setting the `GxB_HYPER` field to `GxB_ALWAYS_HYPER` ensures a matrix always stays hypersparse. If set to `GxB_NEVER_HYPER`, it always stays non-hypersparse. At startup, `GrB_init` defines the following initial settings:

```
    GxB_set (GxB_HYPER, GxB_HYPER_DEFAULT) ;
    GxB_set (GxB_FORMAT, GxB_FORMAT_DEFAULT) ;
```

That is, by default, all new matrices are held by column in CSR format, unless `-DBYCOL` is used at compile time, in which case the default is to store all new matrices by row in CSC format. If a matrix has fewer than $n/16$ columns, it can be converted to hypersparse format. If it has more than $n/8$ columns, it can be converted to non-hypersparse format. These options can be changed for all future matrices with `GxB_set`. For example, to change all future matrices to be in non-hypersparse CSC when created, use:

```
    GxB_set (GxB_HYPER, GxB_NEVER_HYPER) ;
    GxB_set (GxB_FORMAT, GxB_BY_COL) ;
```

Then if a particular matrix needs a different format, then (as an example):

```
    GxB_set (A, GxB_HYPER, 0.1) ;
    GxB_set (A, GxB_FORMAT, GxB_BY_ROW) ;
```

This changes the matrix `A` so that it is stored by row, and it is converted from non-hypersparse to hypersparse format if it has fewer than 10% non-empty columns. If it is hypersparse, it is a candidate for conversion to non-hypersparse if has 20% or more non-empty columns. If it has between 10% and 20% non-empty columns, it remains in whatever format it is currently in.

MATLAB only supports a non-hypersparse CSC format. The format in SuiteSparse:GraphBLAS that is equivalent to the MATLAB format is given below:

```
GrB_init (...) ;
GxB_set (GxB_HYPER, GxB_NEVER_HYPER) ;
GxB_set (GxB_FORMAT, GxB_BY_COL) ;
// no subsequent use of GxB_HYPER or GxB_FORMAT
```

The `GxB_HYPER` and `GxB_FORMAT` options should be considered as suggestions from the user application as to how SuiteSparse:GraphBLAS can obtain the best performance for a particular application. SuiteSparse:GraphBLAS is free to ignore any of these suggestions, both now and in the future, and the available options and formats may be augmented in the future. Any prior options no longer needed in future versions of SuiteSparse:GraphBLAS will be silently ignored, so the use these options is safe for future updates.

The hypersparse status of a matrix can be queried with the following usage:

```
bool is_hyper ;
GxB_get (A, GxB_IS_HYPER, &is_hyper) ;
printf (is_hyper ? "A is hypersparse" : "A is standard sparse") ;
```

5.4   Other global options

`GxB_MODE`, `GxB_THREAD_SAFETY`, and `GxB_THREADING` can only be queried by `GxB_get`; they cannot be modified by `GxB_set`. The mode is the value passed to `GrB_init` (blocking or non-blocking). The `GxB_THREAD*` options are returned as an **enum** type with one of the following options:

```
typedef enum
{
    GxB_THREAD_NONE = 0,    // no threading
    GxB_THREAD_OPENMP = 1,  // OpenMP
```

```
    GxB_THREAD_POSIX = 2,   // POSIX pthreads
    GxB_THREAD_WINDOWS = 3, // Windows threads
    GxB_THREAD_ANSI = 4     // ANSI C11 threads
}
GxB_Thread_Model ;
```

SuiteSparse:GraphBLAS is not yet multi-threaded, but it is thread-safe if it is compiled with OpenMP or POSIX pthreads, and if the user application threads do not operate on the same matrices at the same time. Output matrices and vectors used by different threads must be different, and input matrices and vectors can be safely used only if any pending computations on them have finished, via `GrB_wait` or the per-matrix methods, `GrB_*_nvals`, `GrB_*_extractElement`, `GrB_*_extractTuples`, and reduction to a scalar via `GrB_*_reduce`.

The `GxB_THREAD_SAFETY` option returns the threading model used internally to synchronize user threads. This is determine during installation (see Section 11.3). Since `GxB_THREAD_NONE` is zero, the following can be used:

```
GxB_Thread_Model safe ;
GxB_get (GxB_THREAD_SAFETY, &safe) ;
if (safe)
{
    printf ("GraphBLAS is thread-safe\n") ;
}
else
{
    // neither OpenMP, POSIX pthreads, nor any other threading model
    // was available at compile-time
    printf ("GraphBLAS is not thread-safe!\n") ;
}
```

The `GxB_THREADING` option currently returns `GxB_THREAD_NONE`, but this will change when SuiteSparse:GraphBLAS exploits OpenMP for internal parallelism. In this case, the `GxB_THREADING` option will return either `GxB_THREAD_NONE` (if OpenMP is not available) or `GxB_THREAD_OPENMP` otherwise.

All threads in the same user application share the same global options, including hypersparsity and CSR/CSC format determined by `GxB_set`, the blocking mode determined by `GrB_init`, and the threading options. Specific format and hypersparsity parameters of each matrix are specific to that matrix and can be independently changed.

5.5   GxB_Global_Option_set: **set a global option**

```
GrB_Info GxB_set                        // set a global default option
(
    const GxB_Option_Field field,       // option to change
    ...                                 // value to change it to
) ;
```

This usage of `GxB_set` sets the value of a global option. The `field` parameter can be `GxB_HYPER`, `GxB_FORMAT`, `GxB_NTHREADS`, or `GxB_CHUNK`.

For example, the following usage sets the global hypersparsity ratio to 0.2, the format of future matrices to `GxB_BY_COL`, the maximum number of threads to 4, and the chunk size to 10000. No existing matrices are changed.

```
GxB_set (GxB_HYPER, 0.2) ;
GxB_set (GxB_FORMAT, GxB_BY_COL) ;
GxB_set (GxB_NTHREADS, 4) ;
GxB_set (GxB_CHUNK, 10000) ;
```

5.6   GxB_Matrix_Option_set: **set a matrix option**

```
GrB_Info GxB_set                        // set an option in a matrix
(
    GrB_Matrix A,                       // matrix to modify
    const GxB_Option_Field field,       // option to change
    ...                                 // value to change it to
) ;
```

This usage of `GxB_set` sets the value of a matrix option, for a particular matrix. The `field` parameter can be `GxB_HYPER` or `GxB_FORMAT`.

For example, the following usage sets the hypersparsity ratio to 0.2, and the format of `GxB_BY_COL`, for a particular matrix `A`. SuiteSparse:GraphBLAS currently applies these changes immediately, but since they are simply hints, future versions of SuiteSparse:GraphBLAS may delay the change in format if it can obtain better performance.

For performance, the matrix option should be set as soon as it is created with `GrB_Matrix_new`, so the internal transformation takes less time.

```
GxB_set (A, GxB_HYPER, 0.2) ;
GxB_set (A, GxB_FORMAT, GxB_BY_COL) ;
```

5.7 GxB_Desc_set: **set a GrB_Descriptor value**

```
GrB_Info GxB_set                // set a parameter in a descriptor
(
    GrB_Descriptor desc,        // descriptor to modify
    const GrB_Desc_Field field, // parameter to change
    ...                         // value to change it to
) ;
```

This usage is similar to `GrB_Descriptor_set`, just with a name that is
consistent with the other usages of this generic function. Unlike `GrB_Descriptor_set`,
the `field` may also be `GxB_NTHREADS`, or `GxB_CHUNK`. Refer to Sections 4.10.2 and 4.10.3
for details.

5.8 GxB_Global_Option_get: **retrieve a global option**

```
GrB_Info GxB_get                      // gets the current global default option
(
    const GxB_Option_Field field,     // option to query
    ...                               // return value of the global option
) ;
```

This usage of `GxB_get` retrieves the value of a global option. The `field`
parameter can be `GxB_HYPER`, `GxB_FORMAT`. `GxB_MODE`, `GxB_THREAD_SAFETY`,
`GxB_THREADING`, `GxB_NTHREADS`, or `GxB_CHUNK`. For example:

```
double h ;
GxB_get (GxB_HYPER, &h) ;
printf ("hyper_ratio = %g for all new matrices\n", h) ;

GxB_Format_Value s ;
GxB_get (GxB_FORMAT, &s) ;
if (s == GxB_BY_COL) printf ("all new matrices are stored by column\n") :
else printf ("all new matrices are stored by row\n") ;

GrB_mode mode ;
GxB_get (GxB_MODE, &mode) ;
if (mode == GrB_BLOCKING) printf ("GrB_init(GrB_BLOCKING) was called.\n") :
else printf ("GrB_init(GrB_NONBLOCK) was called.\n") ;

int nthreads_max ;
GxB_get (GxB_NTHREADS, &nthreads_max) ;
printf ("max # of threads to use: %d\n", nthreads_max) ;
```

```
    double chunk ;
    GxB_get (GxB_CHUNK, &chunk) ;
    printf ("chunk size: %g\n", chunk) ;

    // see Demo/Program/pthread_demo.c and openmp_demo.c for examples:
    GxB_Threading_Model thread_safety, threading ;
    GxB_get (GxB_THREAD_SAFETY, &thread_safey) ;
    GxB_get (GxB_THREADING, &threading) ;
```

5.9   GxB_Matrix_Option_get: **retrieve a matrix option**

```
GrB_Info GxB_get                      // gets the current option of a matrix
(
    GrB_Matrix A,                     // matrix to query
    GxB_Option_Field field,           // option to query
    ...                               // return value of the matrix option
) ;
```

This usage of `GxB_get` retrieves the value of a matrix option. The `field` parameter can be `GxB_HYPER` or `GxB_FORMAT`. For example:

```
    double h ;
    GxB_get (A, GxB_HYPER, &h) ;
    printf ("matrix A has hyper_ratio = %g\n", h) ;

    GxB_Format_Value s ;
    GxB_get (A, GxB_FORMAT, &s) ;
    if (s == GxB_BY_COL) printf ("matrix A is stored by column\n") :
    else printf ("matrix A is stored by row\n") ;
```

5.10   GxB_Desc_get: **retrieve a GrB_Descriptor value**

```
GrB_Info GxB_get                    // get a parameter from a descriptor
(
    GrB_Descriptor desc,            // descriptor to query; NULL means defaults
    GrB_Desc_Field field,           // parameter to query
    ...                             // value of the parameter
) ;
```

This usage is the same as `GxB_Desc_get`. The `field` parameter can be `GrB_OUTP`, `GrB_MASK`, `GrB_INP0`, `GrB_INP1`, `GxB_AxB_METHOD`, `GxB_NTHREADS`, or `GxB_CHUNK`. Refer to Section 4.10.4 for details.

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5.11 Summary of usage of GxB_set and GxB_get

The different usages of `GxB_set` and `GxB_get` are summarized below.
To set/get the global options:

```
GxB_set (GxB_HYPER, double h) ;
GxB_set (GxB_HYPER, GxB_ALWAYS_HYPER) ;
GxB_set (GxB_HYPER, GxB_NEVER_HYPER) ;
GxB_get (GxB_HYPER, double *h) ;

GxB_set (GxB_FORMAT, GxB_BY_ROW) ;
GxB_set (GxB_FORMAT, GxB_BY_COL) ;
GxB_get (GxB_FORMAT, GxB_Format_Value *s) ;

GxB_set (GxB_THREADS, int nthreads_max) ;
GxB_get (GxB_THREADS, int *nthreads_max) ;

GxB_set (GxB_CHUNK, double chunk) ;
GxB_get (GxB_CHUNK, double *chunk) ;
```

To get global options that can be queried but not modified:

```
GxB_get (GxB_MODE,          GrB_Mode *mode) ;
GxB_get (GxB_THREAD_SAFETY, GxB_Thread_Model *thread_safety) ;
GxB_get (GxB_THREADING,     GxB_Thread_Model *threading) ;
```

To set/get a matrix option:

```
GxB_set (GrB_Matrix A, GxB_HYPER, double h) ;
GxB_set (GrB_Matrix A, GxB_HYPER, GxB_ALWAYS_HYPER) ;
GxB_set (GrB_Matrix A, GxB_HYPER, GxB_NEVER_HYPER) ;
GxB_get (GrB_Matrix A, GxB_HYPER, double *h) ;

GxB_set (GrB_Matrix A, GxB_FORMAT, GxB_BY_ROW) ;
GxB_set (GrB_Matrix A, GxB_FORMAT, GxB_BY_COL) ;
GxB_get (GrB_Matrix A, GxB_FORMAT, GxB_Format_Value *s) ;
```

To get the hypersparse status of a matrix:

```
GxB_get (GrB_Matrix A, GxB_IS_HYPER, bool *is_hyper) ;
```

To set/get a descriptor field:

```
GxB_set (GrB_Descriptor d, GrB_OUTP, GxB_DEFAULT) ;
GxB_set (GrB_Descriptor d, GrB_OUTP, GrB_REPLACE) ;
GxB_get (GrB_Descriptor d, GrB_OUTP, GrB_Desc_Value *v) ;

GxB_set (GrB_Descriptor d, GrB_MASK, GxB_DEFAULT) ;
GxB_set (GrB_Descriptor d, GrB_MASK, GrB_SCMP) ;
GxB_get (GrB_Descriptor d, GrB_MASK, GrB_Desc_Value *v) ;

GxB_set (GrB_Descriptor d, GrB_INP0, GxB_DEFAULT) ;
GxB_set (GrB_Descriptor d, GrB_INP0, GrB_TRAN) ;
GxB_get (GrB_Descriptor d, GrB_INP0, GrB_Desc_Value *v) ;

GxB_set (GrB_Descriptor d, GrB_INP1, GxB_DEFAULT) ;
GxB_set (GrB_Descriptor d, GrB_INP1, GrB_TRAN) ;
GxB_get (GrB_Descriptor d, GrB_INP1, GrB_Desc_Value *v) ;

GxB_set (GrB_Descriptor d, GxB_AxB_METHOD, GxB_DEFAULT) ;
GxB_set (GrB_Descriptor d, GxB_AxB_METHOD, GxB_AxB_GUSTAVSON) ;
GxB_set (GrB_Descriptor d, GxB_AxB_METHOD, GxB_AxB_HEAP) ;
GxB_set (GrB_Descriptor d, GxB_AxB_METHOD, GxB_AxB_DOT) ;
GxB_get (GrB_Descriptor d, GrB_AxB_METHOD, GrB_Desc_Value *v) ;

GxB_set (GrB_Descriptor d, GxB_NTHREADS, int nthreads) ;
GxB_get (GrB_Descriptor d, GxB_NTHREADS, int *nthreads) ;

GxB_set (GrB_Descriptor d, GxB_CHUNK, double chunk) ;
GxB_get (GrB_Descriptor d, GxB_CHUNK, double *chunk) ;
```

6 SuiteSparse:GraphBLAS Colon and Index Notation

MATLAB uses a colon notation to index into matrices, such as `C=A(2:4,3:8)`, which extracts `C` as 3-by-6 submatrix from `A`, from rows 2 through 4 and columns 3 to 8 of the matrix `A`. A single colon is used to denote all rows, `C=A(:,9)`, or all columns, `C=A(12,:)`, which refers to the 9th column and 12th row of `A`, respectively. An arbitrary integer list can be given as well, such as the MATLAB statements:

```
I = [2 1 4] ;
J = [3 5] ;
C = A (I,J) ;
```

which creates the 3-by-2 matrix `C` as follows:

$$
C = \left[\begin{array}{cc} a_{2,3} & a_{2,5} \\ a_{1,3} & a_{1,5} \\ a_{4,3} & a_{4,5} \end{array} \right]
$$

The GraphBLAS API can do the equivalent of `C=A(I,J)`, `C=A(:,J)`, `C=A(I,:)`, and `C=A(:,:)`, by passing a parameter `const GrB_Index *I` as either an array of size `ni`, or as the special value `GrB_ALL`, which corresponds to the stand-alone colon `C=A(:,J)`, and the same can be done for J.. To compute `C=A(2:4,3:8)` in GraphBLAS requires the user application to create two explicit integer arrays `I` and `J` of size 3 and 5, respectively, and then fill them with the explicit values `[2,3,4]` and `[3,4,5,6,7,8]`. This works well if the lists are small, or if the matrix has more entries than rows or columns.

However, particularly with hypersparse matrices, the size of the explicit arrays `I` and `J` can vastly exceed the number of entries in the matrix. When using its hypersparse format, SuiteSparse:GraphBLAS allows the user application to create a `GrB_Matrix` with dimensions up to 2^{60}, with no memory constraints. The only constraint on memory usage in a hypersparse matrix is the number of entries in the matrix.

For example, creating a n-by-n matrix `A` of type `GrB_FP64` with $n = 2^{60}$ and one million entries is trivial to do in Version 2.1 (and later) of Suite-Sparse:GraphBLAS, taking at most 24MB of space. SuiteSparse:GraphBLAS

Version 2.1 (or later) could do this on an old smartphone. However, using just the pure GraphBLAS API, constructing `C=A(0:(n/2),0:(n/2))` in SuiteSparse Version 2.0 would require the creation of an integer array `I` of size 2^{59}, containing the sequence 0, 1, 2, 3, ..., requiring about 4 ExaBytes of memory (4 million terabytes). This is roughly 1000 times larger than the memory size of the world's largest computer in 2018.

SuiteSparse:GraphBLAS Version 2.1 and later extends the GraphBLAS API with a full implementation of the MATLAB colon notation for integers, `I=begin:inc:end`. This extension allows the construction of the matrix `C=A(0:(n/2),0:(n/2))` in this example, with dimension 2^{59}, probably taking just milliseconds on an old smartphone.

The `GrB_extract`, `GrB_assign`, and `GrB_subassign` operations (described in the Section 7) each have parameters that define a list of integer indices, using two parameters:

```
const GrB_Index *I ;      // an array, or a special value GrB_ALL
GrB_Index ni ;            // the size of I, or a special value
```

These two parameters define five kinds of index lists, which can be used to specify either an explicit or implicit list of row indices and/or column indices. The length of the list of indices is denoted `|I|`. This discussion applies equally to the row indices `I` and the column indices `J`. The five kinds are listed below.

1. An explicit list of indices, such as `I = [2 1 4 7 2]` in MATLAB notation, is handled by passing in `I` as a pointer to an array of size 5, and passing `ni=5` as the size of the list. The length of the explicit list is `ni=|I|`. Duplicates may appear.

2. To specify all rows of a matrix, use `I = GrB_ALL`. The parameter `ni` is ignored. This is equivalent to `C=A(:,J)` in MATLAB. In GraphBLAS, this is the sequence `0:(m-1)` if `A` has `m` rows, with length `|I|=m`. If `J` is used the columns of an `m`-by-`n` matrix, then `J=GrB_ALL` refers to all columns, and is the sequence `0:(n-1)`, of length `|J|=n`.

3. To specify a contiguous range of indices, such as `I=10:20` in MATLAB, the array `I` has size 2, and `ni` is passed to SuiteSparse:GraphBLAS as the special value `ni = GxB_RANGE`. The beginning index is `I[GxB_BEGIN]` and the ending index is `I[GxB_END]`. Both values must be non-negative

since `GrB_Index` is an unsigned integer (`uint64_t`). The value of `I[GxB_INC]` is ignored.

```
// to specify I = 10:20
GrB_Index I [2], ni = GxB_RANGE ;
I [GxB_BEGIN] = 10 ;      // the start of the sequence
I [GxB_END  ] = 20 ;      // the end of the sequence
```

Let $b = $ `I[GxB_BEGIN]`, let $e = $ `I[GxB_END]`, The sequence has length zero if $b > e$; otherwise the length is $|I| = (e - b) + 1$.

4. To specify a strided range of indices with a non-negative stride, such as `I=3:2:10`, the array `I` has size 3, and `ni` has the special value `GxB_STRIDE`. This is the sequence 3, 5, 7, 9, of length 4. Note that 10 does not appear in the list. The end point need not appear if the increment goes past it.

```
// to specify I = 3:2:10
GrB_Index I [3], ni = GxB_STRIDE ;
I [GxB_BEGIN ] = 3 ;      // the start of the sequence
I [GxB_INC   ] = 2 ;      // the increment
I [GxB_END   ] = 10 ;     // the end of the sequence
```

The `GxB_STRIDE` sequence is the same as the `List` generated by the following for loop:

```
int64_t k = 0 ;
GrB_Index *List = (a pointer to an array of large enough size)
for (int64_t i = I [GxB_BEGIN] ; i <= I [GxB_END] ; i += I [GxB_INC])
{
    // i is the kth entry in the sequence
    List [k++] = i ;
}
```

Then passing the explicit array `List` and its length `ni=k` has the same effect as passing in the array `I` of size 3, with `ni=GxB_STRIDE`. The latter is simply much faster to produce, and much more efficient for SuiteSparse:GraphBLAS to process.

Let $b = $ `I[GxB_BEGIN]`, let $e = $ `I[GxB_END]`, and let $\Delta = $ `I[GxB_INC]`. The sequence has length zero if $b > e$ or $\Delta = 0$. Otherwise, the length of the sequence is

$$|I| = \left\lfloor \frac{e - b}{\Delta} \right\rfloor + 1$$

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5. In MATLAB notation, if the stride is negative, the sequence is decreasing. For example, `10:-2:1` is the sequence 10, 8, 6, 4, 2, in that order. In SuiteSparse:GraphBLAS, use `ni = GxB_BACKWARDS`, with an array `I` of size 3. The following example specifies defines the equivalent of the MATLAB expression `10:-2:1` in SuiteSparse:GraphBLAS:

```
// to specify I = 10:-2:1
GrB_Index I [3], ni = GxB_BACKWARDS ;
I [GxB_BEGIN ] = 10 ;      // the start of the sequence
I [GxB_INC   ] = 2 ;       // the magnitude of the increment
I [GxB_END   ] = 1 ;       // the end of the sequence
```

The value -2 cannot be assigned to the `GrB_Index` array `I`, since that is an unsigned type. The signed increment is represented instead with the special value `ni = GxB_BACKWARDS`. The `GxB_BACKWARDS` sequence is the same as generated by the following for loop:

```
int64_t k = 0 ;
GrB_Index *List = (a pointer to an array of large enough size)
for (int64_t i = I [GxB_BEGIN] ; i >= I [GxB_END] ; i -= I [GxB_INC])
{
    // i is the kth entry in the sequence
    List [k++] = i ;
}
```

Let $b = $ `I[GxB_BEGIN]`, let $e = $ `I[GxB_END]`, and let $\Delta = $ `I[GxB_INC]` (note that Δ is not negative). The sequence has length zero if $b < e$ or $\Delta = 0$. Otherwise, the length of the sequence is

$$|I| = \left\lfloor \frac{b - e}{\Delta} \right\rfloor + 1$$

Since `GrB_Index` is an unsigned integer, all three values `I[GxB_BEGIN]`, `I[GxB_INC]`, and `I[GxB_END]` must be non-negative.

Just as in MATLAB, it is valid to specify an empty sequence of length zero. For example, `I = 5:3` has length zero in MATLAB and the same is true for a `GxB_RANGE` sequence in SuiteSparse:GraphBLAS, with `I[GxB_BEGIN]=5` and `I[GxB_END]=3`. This has the same effect as array `I` with `ni=0`.

SPEC: `GxB_RANGE`, `GxB_STRIDE`, and `GxB_BACKWARDS` are extensions to the specification.

---

7 GraphBLAS Operations

The next sections define each of the GraphBLAS operations, also listed in the table below. SuiteSparse:GraphBLAS extensions (`GxB_subassign`, `GxB_select` and `GxB_kron`) are included in the table.

`GrB_mxm`	matrix-matrix multiply	$\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{C} \odot \mathbf{AB}$
`GrB_vxm`	vector-matrix multiply	$\mathbf{w}^\mathsf{T}\langle\mathbf{m}^\mathsf{T}\rangle = \mathbf{w}^\mathsf{T} \odot \mathbf{u}^\mathsf{T}\mathbf{A}$
`GrB_mxv`	matrix-vector multiply	$\mathbf{w}\langle\mathbf{m}\rangle = \mathbf{w} \odot \mathbf{Au}$
`GrB_eWiseMult`	element-wise,	$\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{C} \odot (\mathbf{A} \otimes \mathbf{B})$
	set union	$\mathbf{w}\langle\mathbf{m}\rangle = \mathbf{w} \odot (\mathbf{u} \otimes \mathbf{v})$
`GrB_eWiseAdd`	element-wise,	$\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{C} \odot (\mathbf{A} \oplus \mathbf{B})$
	set intersection	$\mathbf{w}\langle\mathbf{m}\rangle = \mathbf{w} \odot (\mathbf{u} \oplus \mathbf{v})$
`GrB_extract`	extract submatrix	$\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{C} \odot \mathbf{A}(\mathbf{I}, \mathbf{J})$
		$\mathbf{w}\langle\mathbf{m}\rangle = \mathbf{w} \odot \mathbf{u}(\mathbf{i})$
`GxB_subassign`	assign submatrix,	$\mathbf{C}(\mathbf{I}, \mathbf{J})\langle\mathbf{M}\rangle = \mathbf{C}(\mathbf{I}, \mathbf{J}) \odot \mathbf{A}$
	with submask for $\mathbf{C}(\mathbf{I}, \mathbf{J})$	$\mathbf{w}(\mathbf{i})\langle\mathbf{m}\rangle = \mathbf{w}(\mathbf{i}) \odot \mathbf{u}$
`GrB_assign`	assign submatrix	$\mathbf{C}\langle\mathbf{M}\rangle(\mathbf{I}, \mathbf{J}) = \mathbf{C}(\mathbf{I}, \mathbf{J}) \odot \mathbf{A}$
	with submask for $\mathbf{C}$	$\mathbf{w}\langle\mathbf{m}\rangle(\mathbf{i}) = \mathbf{w}(\mathbf{i}) \odot \mathbf{u}$
`GrB_apply`	apply unary operator	$\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{C} \odot f(\mathbf{A})$
		$\mathbf{w}\langle\mathbf{m}\rangle = \mathbf{w} \odot f(\mathbf{u})$
`GxB_select`	apply select operator	$\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{C} \odot f(\mathbf{A}, \mathbf{k})$
		$\mathbf{w}\langle\mathbf{m}\rangle = \mathbf{w} \odot f(\mathbf{u}, \mathbf{k})$
`GrB_reduce`	reduce to vector	$\mathbf{w}\langle\mathbf{m}\rangle = \mathbf{w} \odot [\oplus_j \mathbf{A}(:, j)]$
	reduce to scalar	$s = s \odot [\oplus_{ij} \mathbf{A}(I, J)]$
`GrB_transpose`	transpose	$\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{C} \odot \mathbf{A}^\mathsf{T}$
`GxB_kron`	Kronecker product	$\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{C} \odot \mathrm{kron}(\mathbf{A}, \mathbf{B})$

7.1 The GraphBLAS specification in MATLAB

SuiteSparse:GraphBLAS includes a MATLAB implementation of nearly the entire GraphBLAS specification, including all built-in types and operators. The typecasting rules and integer operator rules from GraphBLAS are implemented in MATLAB via `mexFunctions` that call the GraphBLAS routines in C. All other functions are written purely in MATLAB `M`-files, and are given names of the form `GB_spec_*`. All of these MATLAB interfaces and `M`-file functions they are provided in the software distribution of Suite-Sparse:GraphBLAS. The purpose of this is two-fold:

- **Illustration and documentation:** MATLAB is so expressive, and so beautiful to read and write, that the `GB_spec_*` functions read almost like the exact specifications from the GraphBLAS API. Excerpts and condensed versions of these functions have already been used to this point in the User Guide, such as Figure 1, and the subsequent sections rely on them as well. This is why the discussion here is not just relegated to an Appendix on testing; the reader can benefit from studying the `GB_spec_*` functions to understand what a GraphBLAS operation is computing. For example, `GrB_mxm` (Section 7.2) includes a condensed and simplified version of `GB_spec_mxm`.

- **Testing:** Testing the C interface to SuiteSparse:GraphBLAS is a significant challenge since it supports so many different kinds of operations on a vast range of semirings. It is difficult to tell from looking at the result from a C function in GraphBLAS if the result is correct. Thus, each function has been written twice: once in a highly-optimized function in C, and again in a simple and elegant MATLAB function. The latter is almost a direct translation of all the mathematics behind the GraphBLAS API, so it is much easier to visually inspect the `GB_spec_*` version in MATLAB to ensure the correct mathematics are being computed.

The following functions are included in the SuiteSparse:GraphBLAS software distribution. Each has a name of the form `GB_spec_*`, and each of them is a "mimic" of a corresponding C function in GraphBLAS. Not all functions in the C API have a corresponding mimic; in particular, many of the vector functions can be computed directly with the corresponding matrix version in the MATLAB implementations. A list of these files is shown below:

MATLAB `GB_spec` function	corresponding GraphBLAS function or method	Section
`GB_spec_accum.m`	$\mathbf{Z} = \mathbf{C} \odot \mathbf{T}$	2.3
`GB_spec_mask.m`	$\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{Z}$	2.3
`GB_spec_accum_mask.m`	$\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{C} \odot \mathbf{T}$	2.3
`GB_spec_Vector_extractElement.m`	`GrB_Vector_extractElement`	4.7.9
`GB_spec_build.m`	`GrB_Matrix_build`	4.8.8
`GB_spec_Matrix_extractElement.m`	`GrB_Matrix_extractElement`	4.8.10
`GB_spec_extractTuples.m`	`GrB_Matrix_extractTuples`	4.8.11
`GB_spec_mxm.m`	`GrB_mxm`	7.2
`GB_spec_vxm.m`	`GrB_vxm`	7.3
`GB_spec_mxv.m`	`GrB_mxv`	7.4
`GB_spec_eWiseMult_Vector.m`	`GrB_eWiseMult_Vector`	7.5
`GB_spec_eWiseMult_Matrix.m`	`GrB_eWiseMult_Matrix`	7.5
`GB_spec_eWiseAdd_Vector.m`	`GrB_eWiseAdd_Vector`	7.6
`GB_spec_eWiseAdd_Matrix.m`	`GrB_eWiseAdd_Matrix`	7.6
`GB_spec_Vector_extract.m`	`GrB_Vector_extract`	7.7.1
`GB_spec_Matrix_extract.m`	`GrB_Matrix_extract`	7.7.2
`GB_spec_Col_extract.m`	`GrB_Col_extract`	7.7.3
`GB_spec_subassign.m`	`GxB_subassign`	7.8
`GB_spec_assign.m`	`GrB_assign`	7.9
`GB_spec_apply.m`	`GrB_apply`	7.11
`GB_spec_select.m`	`GxB_select`	7.12
`GB_spec_reduce_to_vector.m`	`GrB_reduce` (to vector)	7.13.1
`GB_spec_reduce_to_scalar.m`	`GrB_reduce` (to scalar)	7.13.3
`GB_spec_transpose.m`	`GrB_transpose`	7.14
`GB_spec_kron.m`	`GxB_kron`	7.15

Additional files are included for creating test problems and providing inputs to the above files, or supporting functions:

MATLAB `GB_spec` function	purpose
`GB_spec_compare.m`	Compares output of C and MATLAB functions
`GB_spec_random.m`	Generates a random matrix
`GB_spec_op.m`	MATLAB mimic of built-in operators
`GB_spec_operator.m`	Like `GrB_*Op_new`
`GB_spec_opsall.m`	List operators, types, and semirings
`GB_spec_semiring.m`	Like `GrB_Semiring_new`
`GB_spec_descriptor.m`	mimics a GraphBLAS descriptor
`GB_spec_identity.m`	returns the identity of a monoid
`GB_spec_matrix.m`	conforms a MATLAB sparse matrix to GraphBLAS
`GB_define*.m`	creates draft of `GraphBLAS.h`

An intensive test suite has been written that generates test graphs in MATLAB, then computes the result in both the C version of the Suite-Sparse:GraphBLAS and in the MATLAB `GB_spec_*` functions. Each C function in GraphBLAS has a direct `mexFunction` interface that allow the test suite in MATLAB to call both functions.

This approach has its limitations:

- **matrix classes:** MATLAB only supports sparse double, sparse double complex, and sparse logical matrices. MATLAB can represent dense matrices in all eleven built-in GraphBLAS data types, so in all these specification `M`-files, the matrices are either in dense format in the corresponding MATLAB class, or they are held as sparse double or sparse logical, and the actual GraphBLAS type is held with it as a string member of a MATLAB `struct`. To ensure the correct typecasting is computed, most of the MATLAB scripts work on dense matrices, not sparse ones. As a result, the MATLAB `GB_spec_*` function are not meant for production use, but just for testing and illustration.

- **integer operations:** MATLAB and GraphBLAS handle integer operations differently. In MATLAB, an integer result outside the range of the integer is set to maximum or minimum integer. For example, `int8(127)+1` is 127. This is useful for many computations such as image processing, but GraphBLAS follows the C rules instead, where integer values wrap, modulo style. For example, in GraphBLAS and in C, incrementing `(int8_t) 127` by one results in `-128`. Of course, an alternative would be for a MATLAB interface to create its own integer operators, each of which would follow the MATLAB integer rules of arithmetic. However, this would obscure the purpose of these `GB_spec_*` and `GB_mex_*` test functions, which is to test the C API of GraphBLAS. When the `GB_spec_*` functions need to perform integer computations and typecasting, they call GraphBLAS to do the work, instead doing the work in MATLAB. This ensures that the `GB_spec_*` functions obtain the same results as their GraphBLAS counterparts.

- **elegance:** to simplify testing, each MATLAB `mexFunction` interface a GraphBLAS function is a direct translation of the C API. For example, `GB_mex_mxm` is a direct interface to the GraphBLAS `GrB_mxm`, even down the order of parameters. This approach abandons some of the potential features of MATLAB for creating elegant `M`-file interfaces in a

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highly usable form, such as the ability to provide fewer parameters when optional parameters are not in use. These `mexFunctions`, as written, are not meant to be usable in a user application. They are not highly documented. They are meant to be fast, and direct, to accomplish the goal of testing SuiteSparse:GraphBLAS in MATLAB and comparing their results with the corresponding `GB_spec_*` function. They are not recommended for use in general applications in MATLAB.

- **generality:** the MATLAB `mexFunction` interface needs to test the C API directly, so it must access content of SuiteSparse:GraphBLAS objects that are normally opaque to an end user application. As a result, these `mexFunctions` do not serve as a general interface to any conforming GraphBLAS implementation, but only to SuiteSparse:GraphBLAS.

In the MATLAB mimic functions, `GB_spec_*`, a GraphBLAS matrix `A` is represented as a MATLAB `struct` with the following components:

- `A.matrix`: the values of the matrix. If `A.matrix` is a sparse double matrix, it holds a typecasted copy of the values of a GraphBLAS matrix, unless the GraphBLAS matrix is also double (`GrB_FP64`).

- `A.pattern`: a logical matrix holding the pattern; `A.pattern(i,j)=true` if `(i,j)` is in the pattern of `A`, and `false` otherwise.

- `A.class`: the MATLAB class of the matrix corresponding to one of the eleven built-in types. Normally this is simply `class(A.matrix)`.

- `A.values`: most of the GraphBLAS test `mexFunctions` return their result as a MATLAB sparse matrix, in the `double` class. This works well for all types except for the 64-bit integer types, since a double has about 54 bits of mantissa which is less than the 64 bits available in a long integer. To ensure no bits are lots, these values are also returned as a vector. This enables `GB_spec_compare` to ensure the test results are identical down to the very last bit, and not just to within roundoff error. Nearly all tests, even in double precision, check for perfect equality, not just for results accurate to within round-off error.

7.2 GrB_mxm: **matrix-matrix multiply**

```
GrB_Info GrB_mxm                        // C<Mask> = accum (C, A*B)
(
    GrB_Matrix C,                       // input/output matrix for results
    const GrB_Matrix Mask,              // optional mask for C, unused if NULL
    const GrB_BinaryOp accum,           // optional accum for Z=accum(C,T)
    const GrB_Semiring semiring,        // defines '+' and '*' for A*B
    const GrB_Matrix A,                 // first input:  matrix A
    const GrB_Matrix B,                 // second input: matrix B
    const GrB_Descriptor desc           // descriptor for C, Mask, A, and B
) ;
```

GrB_mxm multiplies two sparse matrices A and B using the semiring. The input matrices A and B may be transposed according to the descriptor, desc (which may be NULL) and then typecasted to match the multiply operator of the semiring. Next, T=A*B is computed on the semiring, precisely defined in the GB_spec_mxm.m script. The actual algorithm exploits sparsity and does not take $O(n^3)$ time, but what computes is the following:

```
[m s] = size (A.matrix) ;
[s n] = size (B.matrix) ;
T.matrix  = zeros (m, n, multiply.ztype) ;
T.pattern = zeros (m, n, 'logical') ;
T.matrix (:,:) = identity ;             % the identity of the semiring's monoid
T.class = multiply.ztype ;              % the ztype of the semiring's multiply op
A = cast (A.matrix, multiply.xtype) ;   % the xtype of the semiring's multiply op
B = cast (B.matrix, multiply.ytype) ;   % the ytype of the semiring's multiply op
for j = 1:n
    for i = 1:m
        for k = 1:s
            % T (i,j) += A (i,k) * B (k,j), using the semiring
            if (A.pattern (i,k) && B.pattern (k,j))
                z = multiply (A (i,k), B (k,j)) ;
                T.matrix  (i,j) = add (T.matrix (i,j),  z) ;
                T.pattern (i,j) = true ;
            end
        end
    end
end
```

Finally, T is typecasted into the type of C, and the results are written back into C via the accum and Mask, $\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{C} \odot \mathbf{T}$. The latter step is reflected in the MATLAB function GB_spec_accum_mask.m, discussed in Section 2.3.

Performance considerations: Suppose all matrices are in `GxB_BY_COL` format, and `B` is extremely sparse but `A` is not as sparse. Then computing `C=A*B` is very fast, and much faster then when `A` is extremely sparse. For example, if `A` is square and `B` is a column vector that is all nonzero except for one entry `B(j,0)=1`, then `C=A*B` is the same as extracting column `A(:,j)`. This is very fast if `A` is stored by column but slow if `A` is stored by row. If `A` is a sparse row with a single entry `A(0,i)=1`, then `C=A*B` is the same as extracting row `B(i,:)`. This is fast if `B` is stored by row but slow if `B` is stored by column.

If the user application needs to repeatedly extract rows and columns from a matrix, whether by matrix multiplication or by `GrB_extract`, then keep two copies: one stored by row, and other by column, and use the copy that results in the fastest computation.

7.3 GrB_vxm: **vector-matrix multiply**

```
GrB_Info GrB_vxm                        // w'<mask> = accum (w, u'*A)
(
    GrB_Vector w,                       // input/output vector for results
    const GrB_Vector mask,              // optional mask for w, unused if NULL
    const GrB_BinaryOp accum,           // optional accum for z=accum(w,t)
    const GrB_Semiring semiring,        // defines '+' and '*' for u'*A
    const GrB_Vector u,                 // first input:  vector u
    const GrB_Matrix A,                 // second input: matrix A
    const GrB_Descriptor desc           // descriptor for w, mask, and A
) ;
```

GrB_vxm multiplies a row vector u' times a matrix A. The matrix A may be first transposed according to desc (as the second input, GrB_INP1); the column vector u is never transposed via the descriptor. The inputs u and A are typecasted to match the xtype and ytype inputs, respectively, of the multiply operator of the semiring. Next, an intermediate column vector t=A'*u is computed on the semiring using the same method as GrB_mxm. Finally, the column vector t is typecasted from the ztype of the multiply operator of the semiring into the type of w, and the results are written back into w using the optional accumulator accum and mask.

The last step is $\mathbf{w}\langle\mathbf{m}\rangle = \mathbf{w} \odot \mathbf{t}$, as described in Section 2.3, except that all the terms are column vectors instead of matrices.

Performance considerations: If the GxB_FORMAT of A is GxB_BY_ROW, and the default descriptor is used (A is not transposed), then GrB_vxm is faster than than GrB_mxv with its default descriptor, when the vector u is very sparse. However, if the GxB_FORMAT of A is GxB_BY_COL, then GrB_mxv with its default descriptor is faster than GrB_vxm with its default descriptor, when the vector u is very sparse. Using the non-default GrB_TRAN descriptor for A makes the GrB_vxm operation equivalent to GrB_mxv with its default descriptor (with the operands reversed in the multiplier, as well). The reverse is true as well; GrB_mxv with GrB_TRAN is the same as GrB_vxm with a default descriptor.

The breadth-first search presented in Section 10.2 of this User Guide uses GrB_vxm instead of GrB_mxv, since the default format in SuiteSparse:GraphBLAS is GxB_BY_ROW. If the matrix is stored by column, then use GrB_mxv instead.

7.4 GrB_mxv: **matrix-vector multiply**

```
GrB_Info GrB_mxv                        // w<mask> = accum (w, A*u)
(
    GrB_Vector w,                       // input/output vector for results
    const GrB_Vector mask,              // optional mask for w, unused if NULL
    const GrB_BinaryOp accum,           // optional accum for z=accum(w,t)
    const GrB_Semiring semiring,        // defines '+' and '*' for A*B
    const GrB_Matrix A,                 // first input:  matrix A
    const GrB_Vector u,                 // second input: vector u
    const GrB_Descriptor desc           // descriptor for w, mask, and A
) ;
```

GrB_mxv multiplies a matrix A times a column vector u. The matrix A may be first transposed according to desc (as the first input); the column vector u is never transposed via the descriptor. The inputs A and u are typecasted to match the xtype and ytype inputs, respectively, of the multiply operator of the semiring. Next, an intermediate column vector t=A*u is computed on the semiring using the same method as GrB_mxm. Finally, the column vector t is typecasted from the ztype of the multiply operator of the semiring into the type of w, and the results are written back into w using the optional accumulator accum and mask.

The last step is $\mathbf{w}\langle\mathbf{m}\rangle = \mathbf{w} \odot \mathbf{t}$, as described in Section 2.3, except that all the terms are column vectors instead of matrices.

Performance considerations: Refer to the discussion of GrB_vxm. In SuiteSparse:GraphBLAS, GrB_mxv is very efficient when u is sparse or dense, when the default descriptor is used, and when the matrix is GxB_BY_COL. When u is very sparse and GrB_INP0 is set to its non-default GrB_TRAN, then this method is not efficient if the matrix is in GxB_BY_COL format. If an application needs to perform A'*u repeatedly where u is very sparse, then use the GxB_BY_ROW format for A instead.

7.5 GrB_eWiseMult: **element-wise operations, set intersection**

Element-wise "multiplication" is shorthand for applying a binary operator element-wise on two matrices or vectors `A` and `B`, for all entries that appear in the set intersection of the patterns of `A` and `B`. This is like `A.*B` for two sparse matrices in MATLAB, except that in GraphBLAS any binary operator can be used, not just multiplication.

The pattern of the result of the element-wise "multiplication" is exactly this set intersection. Entries in `A` but not `B`, or visa versa, do not appear in the result.

Let $\otimes$ denote the binary operator to be used. The computation $\mathbf{T} = \mathbf{A} \otimes \mathbf{B}$ is given below. Entries not in the intersection of $\mathbf{A}$ and $\mathbf{B}$ do not appear in the pattern of $\mathbf{T}$. That is:

for all entries (i, j) in $\mathbf{A} \cap \mathbf{B}$
$\quad\quad t_{ij} = a_{ij} \otimes b_{ij}$

Depending on what kind of operator is used and what the implicit value is assumed to be, this can give the Hadamard product. This is the case for `A.*B` in MATLAB since the implicit value is zero. However, computing a Hadamard product is not necessarily the goal of the `eWiseMult` operation. It simply applies any binary operator, built-in or user-defined, to the set intersection of `A` and `B`, and discards any entry outside this intersection. Its usefulness in a user's application does not depend upon it computing a Hadamard product in all cases. The operator need not be associative, commutative, nor have any particular property except for type compatibility with `A` and `B`, and the output matrix `C`.

The generic name for this operation is `GrB_eWiseMult`, which can be used for both matrices and vectors.

7.5.1 GrB_eWiseMult_Vector: **element-wise vector multiply**

```
GrB_Info GrB_eWiseMult              // w<mask> = accum (w, u.*v)
(
    GrB_Vector w,                   // input/output vector for results
    const GrB_Vector mask,          // optional mask for w, unused if NULL
    const GrB_BinaryOp accum,       // optional accum for z=accum(w,t)
    const <operator> multiply,      // defines '.*' for t=u.*v
    const GrB_Vector u,             // first input:  vector u
    const GrB_Vector v,             // second input: vector v
    const GrB_Descriptor desc       // descriptor for w and mask
) ;
```

GrB_eWiseMult_Vector computes the element-wise "multiplication" of two vectors u and v, element-wise using any binary operator (not just times). The vectors are not transposed via the descriptor. The vectors u and v are first typecasted into the first and second inputs of the multiply operator. Next, a column vector t is computed, denoted $\mathbf{t} = \mathbf{u} \otimes \mathbf{v}$. The pattern of t is the set intersection of u and v. The result t has the type of the output ztype of the multiply operator.

The operator is typically a GrB_BinaryOp, but the method is type-generic for this parameter. If given a monoid (GrB_Monoid), the additive operator of the monoid is used as the multiply binary operator. If given a semiring (GrB_Semiring), the multiply operator of the semiring is used as the multiply binary operator.

The next and final step is $\mathbf{w}\langle\mathbf{m}\rangle = \mathbf{w} \odot \mathbf{t}$, as described in Section 2.3, except that all the terms are column vectors instead of matrices. Note for all GraphBLAS operations, including this one, the accumulator $\mathbf{w} \odot \mathbf{t}$ is always applied in a set union manner, even though $\mathbf{t} = \mathbf{u} \otimes \mathbf{v}$ for this operation is applied in a set intersection manner.

7.5.2 GrB_eWiseMult_Matrix: **element-wise matrix multiply**

```
GrB_Info GrB_eWiseMult              // C<Mask> = accum (C, A.*B)
(
    GrB_Matrix C,                   // input/output matrix for results
    const GrB_Matrix Mask,          // optional mask for C, unused if NULL
    const GrB_BinaryOp accum,       // optional accum for Z=accum(C,T)
    const <operator> multiply,      // defines '.*' for T=A.*B
    const GrB_Matrix A,             // first input:  matrix A
    const GrB_Matrix B,             // second input: matrix B
    const GrB_Descriptor desc       // descriptor for C, Mask, A, and B
) ;
```

GrB_eWiseMult_Matrix computes the element-wise "multiplication" of two matrices A and B, element-wise using any binary operator (not just times). The input matrices may be transposed first, according to the descriptor desc. They are then typecasted into the first and second inputs of the multiply operator. Next, a matrix T is computed, denoted $\mathbf{T} = \mathbf{A} \otimes \mathbf{B}$. The pattern of T is the set intersection of A and B. The result T has the type of the output ztype of the multiply operator.

The multiply operator is typically a GrB_BinaryOp, but the method is type-generic for this parameter. If given a monoid (GrB_Monoid), the additive operator of the monoid is used as the multiply binary operator. If given a semiring (GrB_Semiring), the multiply operator of the semiring is used as the multiply binary operator.

The operation can be expressed in MATLAB notation as:

```
[nrows, ncols] = size (A.matrix) ;
T.matrix = zeros (nrows, ncols, multiply.ztype) ;
T.class = multiply.ztype ;
p = A.pattern & B.pattern ;
A = cast (A.matrix (p), multiply.xtype) ;
B = cast (B.matrix (p), multiply.ytype) ;
T.matrix (p) = multiply (A, B) ;
T.pattern = p ;
```

The final step is $\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{C} \odot \mathbf{T}$, as described in Section 2.3. Note for all GraphBLAS operations, including this one, the accumulator $\mathbf{C} \odot \mathbf{T}$ is always applied in a set union manner, even though $\mathbf{T} = \mathbf{A} \otimes \mathbf{B}$ for this operation is applied in a set intersection manner.

7.6   GrB_eWiseAdd: **element-wise operations, set union**

Element-wise "addition" is shorthand for applying a binary operator element-wise on two matrices or vectors `A` and `B`, for all entries that appear in the set intersection of the patterns of `A` and `B`. This is like `A+B` for two sparse matrices in MATLAB, except that in GraphBLAS any binary operator can be used, not just addition. The pattern of the result of the element-wise "addition" is the set union of the pattern of `A` and `B`. Entries in neither in `A` nor in `B` do not appear in the result.

Let $\oplus$ denote the binary operator to be used. The computation $\mathbf{T} = \mathbf{A} \oplus \mathbf{B}$ is exactly the same as the computation with accumulator operator as described in Section 2.3. It acts like a sparse matrix addition, except that any operator can be used. The pattern of $\mathbf{A} \oplus \mathbf{B}$ is the set union of the patterns of $\mathbf{A}$ and $\mathbf{B}$, and the operator is applied only on the set intersection of $\mathbf{A}$ and $\mathbf{B}$. Entries not in either the pattern of $\mathbf{A}$ or $\mathbf{B}$ do not appear in the pattern of $\mathbf{T}$. That is:

$$\text{for all entries } (i,j) \text{ in } \mathbf{A} \cap \mathbf{B}$$
$$t_{ij} = a_{ij} \oplus b_{ij}$$
$$\text{for all entries } (i,j) \text{ in } \mathbf{A} \setminus \mathbf{B}$$
$$t_{ij} = a_{ij}$$
$$\text{for all entries } (i,j) \text{ in } \mathbf{B} \setminus \mathbf{A}$$
$$t_{ij} = b_{ij}$$

The only difference between element-wise "multiplication" ($\mathbf{T} = \mathbf{A} \otimes \mathbf{B}$) and "addition" ($\mathbf{T} = \mathbf{A} \oplus \mathbf{B}$) is the pattern of the result, and what happens to entries outside the intersection. With $\otimes$ the pattern of $\mathbf{T}$ is the intersection; with $\oplus$ it is the set union. Entries outside the set intersection are dropped for $\otimes$, and kept for $\oplus$; in both cases the operator is only applied to those (and only those) entries in the intersection. Any binary operator can be used interchangeably for either operation.

Element-wise operations do not operate on the implicit values, even implicitly, since the operations make no assumption about the semiring. As a result, the results can be different from MATLAB, which can always assume the implicit value is zero. For example, `C=A-B` is the conventional matrix subtraction in MATLAB. Computing `A-B` in GraphBLAS with `eWiseAdd` will apply the `MINUS` operator to the intersection, entries in `A` but not `B` will be unchanged and appear in `C`, and entries in neither `A` nor `B` do not appear in `C`. For these cases, the results matches the MATLAB `C=A-B`. Entries in `B` but not `A` do appear in `C` but they are not negated; they cannot be subtracted

from an implicit value in `A`. This is by design. If conventional matrix subtraction of two sparse matrices is required, and the implicit value is known to be zero, use `GrB_apply` to negate the values in `B`, and then use `eWiseAdd` with the `PLUS` operator, to compute `A+(-B)`.

The generic name for this operation is `GrB_eWiseAdd`, which can be used for both matrices and vectors.

There is another minor difference in two variants of the element-wise functions. If given a `semiring`, the `eWiseAdd` functions use the binary operator of the semiring's monoid, while the `eWiseMult` functions use the multiplicative operator of the semiring.

7.6.1 GrB_eWiseAdd_Vector: **element-wise vector addition**

```
GrB_Info GrB_eWiseAdd               // w<mask> = accum (w, u+v)
(
    GrB_Vector w,                   // input/output vector for results
    const GrB_Vector mask,          // optional mask for w, unused if NULL
    const GrB_BinaryOp accum,       // optional accum for z=accum(w,t)
    const <operator> add,           // defines '+' for t=u+v
    const GrB_Vector u,             // first input:  vector u
    const GrB_Vector v,             // second input: vector v
    const GrB_Descriptor desc       // descriptor for w and mask
) ;
```

`GrB_eWiseAdd_Vector` computes the element-wise "addition" of two vectors `u` and `v`, element-wise using any binary operator (not just plus). The vectors are not transposed via the descriptor. Entries in the intersection of `u` and `v` are first typecasted into the first and second inputs of the `add` operator. Next, a column vector t is computed, denoted $\mathbf{t} = \mathbf{u} \oplus \mathbf{v}$. The pattern of t is the set union of `u` and `v`. The result t has the type of the output `ztype` of the `add` operator.

The `add` operator is typically a `GrB_BinaryOp`, but the method is type-generic for this parameter. If given a monoid (`GrB_Monoid`), the additive operator of the monoid is used as the `add` binary operator. If given a semiring (`GrB_Semiring`), the additive operator of the monoid of the semiring is used as the `add` binary operator.

The final step is $\mathbf{w}\langle\mathbf{m}\rangle = \mathbf{w} \odot \mathbf{t}$, as described in Section 2.3, except that all the terms are column vectors instead of matrices.

7.6.2 GrB_eWiseAdd_Matrix: **element-wise matrix addition**

```
GrB_Info GrB_eWiseAdd                 // C<Mask> = accum (C, A+B)
(
    GrB_Matrix C,                     // input/output matrix for results
    const GrB_Matrix Mask,            // optional mask for C, unused if NULL
    const GrB_BinaryOp accum,         // optional accum for Z=accum(C,T)
    const <operator> add,             // defines '+' for T=A+B
    const GrB_Matrix A,               // first input:  matrix A
    const GrB_Matrix B,               // second input: matrix B
    const GrB_Descriptor desc         // descriptor for C, Mask, A, and B
) ;
```

GrB_eWiseAdd_Matrix computes the element-wise "addition" of two matrices A and B, element-wise using any binary operator (not just plus). The input matrices may be transposed first, according to the descriptor desc. Entries in the intersection then typecasted into the first and second inputs of the add operator. Next, a matrix T is computed, denoted $\mathbf{T} = \mathbf{A} \oplus \mathbf{B}$. The pattern of T is the set union of A and B. The result T has the type of the output ztype of the add operator.

The add operator is typically a GrB_BinaryOp, but the method is type-generic for this parameter. If given a monoid (GrB_Monoid), the additive operator of the monoid is used as the add binary operator. If given a semiring (GrB_Semiring), the additive operator of the monoid of the semiring is used as the add binary operator.

The operation can be expressed in MATLAB notation as:

```
[nrows, ncols] = size (A.matrix) ;
T.matrix = zeros (nrows, ncols, add.ztype) ;
p = A.pattern & B.pattern ;
A = GB_mex_cast (A.matrix (p), add.xtype) ;
B = GB_mex_cast (B.matrix (p), add.ytype) ;
T.matrix (p) = add (A, B) ;
p =  A.pattern & ~B.pattern ; T.matrix (p) = cast (A.matrix (p), add.ztype) ;
p = ~A.pattern &  B.pattern ; T.matrix (p) = cast (B.matrix (p), add.ztype) ;
T.pattern = A.pattern | B.pattern ;
T.class = add.ztype ;
```

Except for when typecasting is performed, this is identical to how the accum operator is applied in Figure 1.

The final step is $\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{C} \odot \mathbf{T}$, as described in Section 2.3.

7.7 GrB_extract: **submatrix extraction**

The `GrB_extract` function is a generic name for three specific functions: `GrB_Vector_extract`, `GrB_Col_extract`, and `GrB_Matrix_extract`. The generic name appears in the function signature, but the specific function name is used when describing what each variation does.

7.7.1 GrB_Vector_extract: **extract subvector from vector**

```
GrB_Info GrB_extract                // w<mask> = accum (w, u(I))
(
    GrB_Vector w,                   // input/output vector for results
    const GrB_Vector mask,          // optional mask for w, unused if NULL
    const GrB_BinaryOp accum,       // optional accum for z=accum(w,t)
    const GrB_Vector u,             // first input:  vector u
    const GrB_Index *I,             // row indices
    const GrB_Index ni,             // number of row indices
    const GrB_Descriptor desc       // descriptor for w and mask
) ;
```

`GrB_Vector_extract` extracts a subvector from another vector, identical to `t = u (I)` in MATLAB where `I` is an integer vector of row indices. Refer to `GrB_Matrix_extract` for further details; vector extraction is the same as matrix extraction with n-by-1 matrices. See Section 6 for a description of `I` and `ni`. The final step is $\mathbf{w}\langle\mathbf{m}\rangle = \mathbf{w} \odot \mathbf{t}$, as described in Section 2.3, except that all the terms are column vectors instead of matrices.

7.7.2 GrB_Matrix_extract: **extract submatrix from matrix**

```
GrB_Info GrB_extract                // C<Mask> = accum (C, A(I,J))
(
    GrB_Matrix C,                   // input/output matrix for results
    const GrB_Matrix Mask,          // optional mask for C, unused if NULL
    const GrB_BinaryOp accum,       // optional accum for Z=accum(C,T)
    const GrB_Matrix A,             // first input:  matrix A
    const GrB_Index *I,             // row indices
    const GrB_Index ni,             // number of row indices
    const GrB_Index *J,             // column indices
    const GrB_Index nj,             // number of column indices
    const GrB_Descriptor desc       // descriptor for C, Mask, and A
) ;
```

GrB_Matrix_extract extracts a submatrix from another matrix, identical to T = A(I,J) in MATLAB where I and J are integer vectors of row and column indices, respectively, except that indices are zero-based in Graph-BLAS and one-based in MATLAB. The input matrix A may be transposed first, via the descriptor. The type of T and A are the same. The size of C is |I|-by-|J|. Entries outside A(I,J) are not accessed and do not take part in the computation. More precisely, assuming the matrix A is not transposed, the matrix T is defined as follows:

```
T.matrix  = zeros (ni, nj) ;    % a matrix of size ni-by-nj
T.pattern = false (ni, nj) ;
for i = 1:ni
    for j = 1:nj
        if (A (I(i),J(j)).pattern)
            T (i,j).matrix  = A (I(i),J(j)).matrix ;
            T (i,j).pattern = true ;
        end
    end
end
```

If duplicate indices are present in I or J, the above method defines the result in T. Duplicates result in the same values of A being copied into different places in T. See Section 6 for a description of the row indices I and ni, and the column indices J and nj. The final step is $\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{C} \odot \mathbf{T}$, as described in Section 2.3.

Performance considerations: If A is not transposed via input descriptor: if |I| is small, then it is fastest if A is GxB_BY_ROW; if |J| is small, then it is fastest if A is GxB_BY_COL. The opposite is true if A is transposed.

7.7.3 GrB_Col_extract: **extract column vector from matrix**

```
GrB_Info GrB_extract                // w<mask> = accum (w, A(I,j))
(
    GrB_Vector w,                   // input/output matrix for results
    const GrB_Vector mask,          // optional mask for w, unused if NULL
    const GrB_BinaryOp accum,       // optional accum for z=accum(w,t)
    const GrB_Matrix A,             // first input:  matrix A
    const GrB_Index *I,             // row indices
    const GrB_Index ni,             // number of row indices
    const GrB_Index j,              // column index
    const GrB_Descriptor desc       // descriptor for w, mask, and A
) ;
```

GrB_Col_extract extracts a subvector from a matrix, identical to `t = A (I,j)` in MATLAB where `I` is an integer vector of row indices and where `j` is a single column index. The input matrix `A` may be transposed first, via the descriptor, which results in the extraction of a single row `j` from the matrix `A`, the result of which is a column vector `w`. The type of `t` and `A` are the same. The size of `w` is `|I|`-by-1.

See Section 6 for a description of the row indices `I` and `ni`. The final step is $\mathbf{w}\langle\mathbf{m}\rangle = \mathbf{w} \odot \mathbf{t}$, as described in Section 2.3, except that all the terms are column vectors instead of matrices.

Performance considerations: If `A` is not transposed: it is fastest if the format of `A` is GxB_BY_COL. The opposite is true if `A` is transposed.

7.8 GxB_subassign: **submatrix assignment**

The methods described in this section are all variations of the form `C(I,J)=A`, which modifies a submatrix of the matrix `C`. All methods can be used in their generic form with the single name `GxB_subassign`. This is reflected in the prototypes. However, to avoid confusion between the different kinds of assignment, the name of the specific function is used when describing each variation. If the discussion applies to all variations, the simple name `GxB_subassign` is used.

See Section 6 for a description of the row indices `I` and `ni`, and the column indices `J` and `nj`.

`GxB_subassign` is very similar to `GrB_assign`, described in Section 7.9. The two operations are compared and contrasted in Section 7.10.

SPEC: All variants of `GxB_subassign` are extensions to the spec.

7.8.1 GxB_Vector_subassign: **assign to a subvector**

```
GrB_Info GxB_subassign              // w(I)<mask> = accum (w(I),u)
(
    GrB_Vector w,                   // input/output matrix for results
    const GrB_Vector mask,          // optional mask for w(I), unused if NULL
    const GrB_BinaryOp accum,       // optional accum for z=accum(w(I),t)
    const GrB_Vector u,             // first input:  vector u
    const GrB_Index *I,             // row indices
    const GrB_Index ni,             // number of row indices
    const GrB_Descriptor desc       // descriptor for w(I) and mask
) ;
```

`GxB_Vector_subassign` operates on a subvector `w(I)` of `w`, modifying it with the vector `u`. The method is identical to `GxB_Matrix_subassign` described in Section 7.8.2, where all matrices have a single column each. The `mask` has the same size as `w(I)` and `u`. The only other difference is that the input `u` in this method is not transposed via the `GrB_INP0` descriptor.

7.8.2 GxB_Matrix_subassign: **assign to a submatrix**

```
GrB_Info GxB_subassign              // C(I,J)<Mask> = accum (C(I,J),A)
(
    GrB_Matrix C,                   // input/output matrix for results
    const GrB_Matrix Mask,          // optional mask for C(I,J), unused if NULL
    const GrB_BinaryOp accum,       // optional accum for Z=accum(C(I,J),T)
    const GrB_Matrix A,             // first input:  matrix A
    const GrB_Index *I,             // row indices
    const GrB_Index ni,             // number of row indices
    const GrB_Index *J,             // column indices
    const GrB_Index nj,             // number of column indices
    const GrB_Descriptor desc       // descriptor for C(I,J), Mask, and A
) ;
```

GxB_Matrix_subassign operates only on a submatrix S of C, modifying it with the matrix A. For this operation, the result is not the entire matrix C, but a submatrix S=C(I,J) of C. The steps taken are as follows, except that **A** may be optionally transposed via the GrB_INP0 descriptor option.

Step	GraphBLAS notation	description
1	$\mathbf{S} = \mathbf{C(I,J)}$	extract the $\mathbf{C(I,J)}$ submatrix
2	$\mathbf{S}\langle\mathbf{M}\rangle = \mathbf{S} \odot \mathbf{A}$	apply the accumulator/mask to the submatrix $\mathbf{S}$
3	$\mathbf{C(I,J)} = \mathbf{S}$	put the submatrix $\mathbf{S}$ back into $\mathbf{C(I,J)}$

The accumulator/mask step in Step 2 is the same as for all other Graph-BLAS operations, described in Section 2.3, except that for GxB_subassign, it is applied to just the submatrix $\mathbf{S} = \mathbf{C(I,J)}$, and thus the Mask has the same size as $\mathbf{A}$, $\mathbf{S}$, and $\mathbf{C(I,J)}$.

The GxB_subassign operation is the reverse of matrix extraction:

- For submatrix extraction, GrB_Matrix_extract, the submatrix A(I,J) appears on the right-hand side of the assignment, C=A(I,J), and entries outside of the submatrix are not accessed and do not take part in the computation.

- For submatrix assignment, GxB_Matrix_subassign, the submatrix C(I,J) appears on the left-hand-side of the assignment, C(I,J)=A, and entries outside of the submatrix are not accessed and do not take part in the computation.

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In both methods, the accumulator and mask modify the submatrix of the assignment; they simply differ on which side of the assignment the submatrix resides on. In both cases, if the `Mask` matrix is present it is the same size as the submatrix:

- For submatrix extraction, $\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{C} \odot \mathbf{A}(\mathbf{I}, \mathbf{J})$ is computed, where the submatrix is on the right. The mask $\mathbf{M}$ has the same size as the submatrix $\mathbf{A}(\mathbf{I}, \mathbf{J})$.

- For submatrix assignment, $\mathbf{C}(\mathbf{I}, \mathbf{J})\langle\mathbf{M}\rangle = \mathbf{C}(\mathbf{I}, \mathbf{J}) \odot \mathbf{A}$ is computed, where the submatrix is on the left. The mask $\mathbf{M}$ has the same size as the submatrix $\mathbf{C}(\mathbf{I}, \mathbf{J})$.

In Step 1, the submatrix S is first computed by the `GrB_Matrix_extract` operation, `S=C(I,J)`.

Step 2 accumulates the results $\mathbf{S}\langle\mathbf{M}\rangle = \mathbf{S} \odot \mathbf{T}$, exactly as described in Section 2.3, but operating on the submatrix $\mathbf{S}$, not $\mathbf{C}$, using the optional `Mask` and `accum` operator. The matrix $\mathbf{T}$ is simply $\mathbf{T} = \mathbf{A}$, or $\mathbf{T} = \mathbf{A}^\mathsf{T}$ if $\mathbf{A}$ is transposed via the `desc` descriptor, `GrB_INP0`. The `GrB_REPLACE` option in the descriptor clears $\mathbf{S}$ after computing $\mathbf{Z} = \mathbf{T}$ or $\mathbf{Z} = \mathbf{C} \odot \mathbf{T}$, not all of $\mathbf{C}$ since this operation can only modify the specified submatrix of $\mathbf{C}$.

Finally, Step 3 writes the result (which is the modified submatrix S and not all of C) back into the C matrix that contains it, via the assignment `C(I,J)=S`, using the reverse operation from the method described for matrix extraction:

```
for i = 1:ni
    for j = 1:nj
        if (S (i,j).pattern)
            C (I(i),J(j)).matrix = S (i,j).matrix ;
            C (I(i),J(j)).pattern = true ;
        end
    end
end
```

Results are not defined for any `GxB_subassign` operation if duplicate indices appear in `I` or `J`.

Performance considerations: If `A` is not transposed: if `|I|` is small, then it is fastest if the format of `C` is `GxB_BY_ROW`; if `|J|` is small, then it is fastest if the format of `C` is `GxB_BY_COL`. The opposite is true if `A` is transposed.

7.8.3 GxB_Col_subassign: **assign to a sub-column of a matrix**

```
GrB_Info GxB_subassign              // C(I,j)<mask> = accum (C(I,j),u)
(
    GrB_Matrix C,                   // input/output matrix for results
    const GrB_Vector mask,          // optional mask for C(I,j), unused if NULL
    const GrB_BinaryOp accum,       // optional accum for z=accum(C(I,j),t)
    const GrB_Vector u,             // input vector
    const GrB_Index *I,             // row indices
    const GrB_Index ni,             // number of row indices
    const GrB_Index j,              // column index
    const GrB_Descriptor desc       // descriptor for C(I,j) and mask
) ;
```

GxB_Col_subassign modifies a single sub-column of a matrix C. It is the
same as GxB_Matrix_subassign where the index vector J[0]=j is a single
column index (and thus nj=1), and where all matrices in GxB_Matrix_subassign
(except C) consist of a single column. The mask vector has the same size as
u and the sub-column C(I,j). The input descriptor GrB_INP0 is ignored;
the input vector u is not transposed. Refer to GxB_Matrix_subassign for
further details.

Performance considerations: GxB_Col_subassign is much faster than
GxB_Row_subassign if the format of C is GxB_BY_COL. GxB_Row_subassign
is much faster than GxB_Col_subassign if the format of C is GxB_BY_ROW.

7.8.4 GxB_Row_subassign: **assign to a sub-row of a matrix**

```
GrB_Info GxB_subassign              // C(i,J)<mask'> = accum (C(i,J),u')
(
    GrB_Matrix C,                   // input/output matrix for results
    const GrB_Vector mask,          // optional mask for C(i,J), unused if NULL
    const GrB_BinaryOp accum,       // optional accum for z=accum(C(i,J),t)
    const GrB_Vector u,             // input vector
    const GrB_Index i,              // row index
    const GrB_Index *J,             // column indices
    const GrB_Index nj,             // number of column indices
    const GrB_Descriptor desc       // descriptor for C(i,J) and mask
) ;
```

GxB_Row_subassign modifies a single sub-row of a matrix C. It is the
same as GxB_Matrix_subassign where the index vector I[0]=i is a single

row index (and thus `ni=1`), and where all matrices in `GxB_Matrix_subassign` (except `C`) consist of a single row. The `mask` vector has the same size as `u` and the sub-column `C(I,j)`. The input descriptor `GrB_INP0` is ignored; the input vector `u` is not transposed. Refer to `GxB_Matrix_subassign` for further details.

Performance considerations: `GxB_Col_subassign` is much faster than `GxB_Row_subassign` if the format of `C` is `GxB_BY_COL`. `GxB_Row_subassign` is much faster than `GxB_Col_subassign` if the format of `C` is `GxB_BY_ROW`.

7.8.5   GxB_Vector_subassign_<type>: **assign a scalar to a subvector**

```
GrB_Info GxB_subassign              // w(I)<mask> = accum (w(I),x)
(
    GrB_Vector w,                   // input/output vector for results
    const GrB_Vector mask,          // optional mask for w(I), unused if NULL
    const GrB_BinaryOp accum,       // optional accum for z=accum(w(I),x)
    const <type> x,                 // scalar to assign to w(I)
    const GrB_Index *I,             // row indices
    const GrB_Index ni,             // number of row indices
    const GrB_Descriptor desc       // descriptor for w(I) and mask
) ;
```

`GxB_Vector_subassign_<type>` assigns a single scalar to an entire subvector of the vector `w`. The operation is exactly like setting a single entry in an n-by-1 matrix, `A(I,0) = x`, where the column index for a vector is implicitly `j=0`. For further details of this function, see `GxB_Matrix_subassign_<type>` in Section 7.8.6.

Unlike `GrB_Vector_assign_<type>` (see Section 7.9.5), results are not defined if `I` contains duplicate indices.

7.8.6   GxB_Matrix_subassign_<type>: **assign a scalar to a submatrix**

```
GrB_Info GxB_subassign                  // C(I,J)<Mask> = accum (C(I,J),x)
(
    GrB_Matrix C,                       // input/output matrix for results
    const GrB_Matrix Mask,              // optional mask for C(I,J), unused if NULL
    const GrB_BinaryOp accum,           // optional accum for Z=accum(C(I,J),x)
    const <type> x,                     // scalar to assign to C(I,J)
    const GrB_Index *I,                 // row indices
    const GrB_Index ni,                 // number of row indices
    const GrB_Index *J,                 // column indices
    const GrB_Index nj,                 // number of column indices
    const GrB_Descriptor desc           // descriptor for C(I,J) and Mask
) ;
```

GxB_Matrix_subassign_<type> assigns a single scalar to an entire sub-matrix of C, like the *scalar expansion* C(I,J)=x in MATLAB. The scalar x is implicitly expanded into a matrix A of size ni by nj, and then the matrix A is assigned to C(I,J) using the same method as in GxB_Matrix_subassign. Refer to that function in Section 7.8.2 for further details. For the accumulation step, the scalar x is typecasted directly into the type of C when the accum operator is not applied to it, or into the ytype of the accum operator, if accum is not NULL, for entries that are already present in C.

The <type> x notation is otherwise the same as GrB_Matrix_setElement (see Section 4.8.9). Any value can be passed to this function and its type will be detected, via the _Generic feature of ANSI C11. For a user-defined type, x is a void * pointer that points to a memory space holding a single entry of a scalar that has exactly the same user-defined type as the matrix C. This user-defined type must exactly match the user-defined type of C since no typecasting is done between user-defined types.

If a void * pointer is passed in and the type of the underlying scalar does not exactly match the user-defined type of C, then results are undefined. No error status will be returned since GraphBLAS has no way of catching this error. Unlike GrB_Matrix_assign_<type> (see Section 7.9.5), results are not defined if I or J contain duplicate indices.

Performance considerations: If A is not transposed: if |I| is small, then it is fastest if the format of C is GxB_BY_ROW; if |J| is small, then it is fastest if the format of C is GxB_BY_COL. The opposite is true if A is transposed.

7.9 GrB_assign: **submatrix assignment**

The methods described in this section are all variations of the form `C(I,J)=A`, which modifies a submatrix of the matrix `C`. All methods can be used in their generic form with the single name `GrB_assign`. These methods are very similar to their `GxB_subassign` counterparts in Section 7.8. They differ primarily in the size of the `Mask`, and how the `GrB_REPLACE` option works. Refer to Section 7.10 for a complete comparison of `GxB_subassign` and `GrB_assign`.

See Section 6 for a description of `I`, `ni`, `J`, and `nj`.

7.9.1 GrB_Vector_assign: **assign to a subvector**

```
GrB_Info GrB_assign                 // w<mask>(I) = accum (w(I),u)
(
    GrB_Vector w,                   // input/output matrix for results
    const GrB_Vector mask,          // optional mask for w, unused if NULL
    const GrB_BinaryOp accum,       // optional accum for z=accum(w(I),t)
    const GrB_Vector u,             // first input:  vector u
    const GrB_Index *I,             // row indices
    const GrB_Index ni,             // number of row indices
    const GrB_Descriptor desc       // descriptor for w and mask
) ;
```

`GrB_Vector_assign` operates on a subvector `w(I)` of `w`, modifying it with the vector `u`. The `mask` vector has the same size as `w`. The method is identical to `GrB_Matrix_assign` described in Section 7.9.2, where all matrices have a single column each. The only other difference is that the input `u` in this method is not transposed via the `GrB_INP0` descriptor.

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7.9.2 GrB_Matrix_assign: **assign to a submatrix**

```
GrB_Info GrB_assign                 // C<Mask>(I,J) = accum (C(I,J),A)
(
    GrB_Matrix C,                   // input/output matrix for results
    const GrB_Matrix Mask,          // optional mask for C, unused if NULL
    const GrB_BinaryOp accum,       // optional accum for Z=accum(C(I,J),T)
    const GrB_Matrix A,             // first input:  matrix A
    const GrB_Index *I,             // row indices
    const GrB_Index ni,             // number of row indices
    const GrB_Index *J,             // column indices
    const GrB_Index nj,             // number of column indices
    const GrB_Descriptor desc       // descriptor for C, Mask, and A
) ;
```

`GrB_Matrix_assign` operates on a submatrix `S` of `C`, modifying it with the matrix `A`. It may also modify all of `C`, depending on the input descriptor `desc` and the `Mask`.

Step	GraphBLAS notation	description
1	$\mathbf{S} = \mathbf{C}(\mathbf{I}, \mathbf{J})$	extract $\mathbf{C}(\mathbf{I}, \mathbf{J})$ submatrix
2	$\mathbf{S} = \mathbf{S} \odot \mathbf{A}$	apply the accumulator (but not the mask) to $\mathbf{S}$
3	$\mathbf{Z} = \mathbf{C}$	make a copy of $\mathbf{C}$
4	$\mathbf{Z}(\mathbf{I}, \mathbf{J}) = \mathbf{S}$	put the submatrix into $\mathbf{Z}(\mathbf{I}, \mathbf{J})$
5	$\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{Z}$	apply the mask/replace phase to all of $\mathbf{C}$

In contrast to `GxB_subassign`, the `Mask` has the same as `C`.

Step 1 extracts the submatrix and then Step 2 applies the accumulator (or $\mathbf{S} = \mathbf{A}$ if `accum` is NULL). The `Mask` is not yet applied.

Step 3 makes a copy of the $\mathbf{C}$ matrix, and then Step 4 writes the submatrix $\mathbf{S}$ into $\mathbf{Z}$. This is the same as Step 3 of `GxB_subassign`, except that it operates on a temporary matrix $\mathbf{Z}$.

Finally, Step 5 writes $\mathbf{Z}$ back into $\mathbf{C}$ via the `Mask`, using the Mask/Replace Phase described in Section 2.3. If `GrB_REPLACE` is enabled, then all of $\mathbf{C}$ is cleared prior to writing $\mathbf{Z}$ via the mask. As a result, the `GrB_REPLACE` option can delete entries outside the $\mathbf{C}(\mathbf{I}, \mathbf{J})$ submatrix.

Performance considerations: If `A` is not transposed: if `|I|` is small, then it is fastest if the format of `C` is `GxB_BY_ROW`; if `|J|` is small, then it is fastest if the format of `C` is `GxB_BY_COL`. The opposite is true if `A` is transposed.

7.9.3 GrB_Col_assign: **assign to a sub-column of a matrix**

```
GrB_Info GrB_assign                 // C<mask>(I,j) = accum (C(I,j),u)
(
    GrB_Matrix C,                   // input/output matrix for results
    const GrB_Vector mask,          // optional mask for C(:,j), unused if NULL
    const GrB_BinaryOp accum,       // optional accum for z=accum(C(I,j),t)
    const GrB_Vector u,             // input vector
    const GrB_Index *I,             // row indices
    const GrB_Index ni,             // number of row indices
    const GrB_Index j,              // column index
    const GrB_Descriptor desc       // descriptor for C(:,j) and mask
) ;
```

GrB_Col_assign modifies a single sub-column of a matrix C. It is the same as GrB_Matrix_assign where the index vector J[0]=j is a single column index, and where all matrices in GrB_Matrix_assign (except C) consist of a single column.

Unlike GrB_Matrix_assign, the mask is a vector with the same size as a single column of C.

The input descriptor GrB_INP0 is ignored; the input vector u is not transposed. Refer to GrB_Matrix_assign for further details.

Performance considerations: GrB_Col_assign is much faster than GrB_Row_assign if the format of C is GxB_BY_COL. GrB_Row_assign is much faster than GrB_Col_assign if the format of C is GxB_BY_ROW.

7.9.4 GrB_Row_assign: **assign to a sub-row of a matrix**

```
GrB_Info GrB_assign                    // C<mask'>(i,J) = accum (C(i,J),u')
(
    GrB_Matrix C,                      // input/output matrix for results
    const GrB_Vector mask,             // optional mask for C(i,:), unused if NULL
    const GrB_BinaryOp accum,          // optional accum for z=accum(C(i,J),t)
    const GrB_Vector u,                // input vector
    const GrB_Index i,                 // row index
    const GrB_Index *J,                // column indices
    const GrB_Index nj,                // number of column indices
    const GrB_Descriptor desc          // descriptor for C(i,:) and mask
) ;
```

GxB_Row_subassign modifies a single sub-row of a matrix C. It is the same as GxB_Matrix_subassign where the index vector I[0]=i is a single row index, and where all matrices in GxB_Matrix_subassign (except C) consist of a single row.

Unlike GrB_Matrix_assign, the mask is a vector with the same size as a single row of C.

The input descriptor GrB_INP0 is ignored; the input vector u is not transposed. Refer to GxB_Matrix_subassign for further details.

Performance considerations: GrB_Col_assign is much faster than GrB_Row_assign if the format of C is GxB_BY_COL. GrB_Row_assign is much faster than GrB_Col_assign if the format of C is GxB_BY_ROW.

7.9.5 GrB_Vector_assign_<type>: **assign a scalar to a subvector**

```
GrB_Info GrB_assign               // w<mask>(I) = accum (w(I),x)
(
    GrB_Vector w,                 // input/output vector for results
    const GrB_Vector mask,        // optional mask for w, unused if NULL
    const GrB_BinaryOp accum,     // optional accum for z=accum(w(I),x)
    const <type> x,               // scalar to assign to w(I)
    const GrB_Index *I,           // row indices
    const GrB_Index ni,           // number of row indices
    const GrB_Descriptor desc     // descriptor for w and mask
) ;
```

`GrB_Vector_assign_<type>` assigns a single scalar to an entire subvector of the vector `w`. The operation is exactly like setting a single entry in an `n`-by-1 matrix, `A(I,0) = x`, where the column index for a vector is implicitly `j=0`. The `mask` vector has the same size as `w`. For further details of this function, see `GrB_Matrix_assign_<type>` in the next section.

In contrast to `GxB_Vector_subassign_<type>`, results are well-defined if `I` contains duplicate indices. Duplicate indices are simply ignored.

7.9.6 GrB_Matrix_assign_<type>: **assign a scalar to a submatrix**

```
GrB_Info GrB_assign               // C<Mask>(I,J) = accum (C(I,J),x)
(
    GrB_Matrix C,                 // input/output matrix for results
    const GrB_Matrix Mask,        // optional mask for C, unused if NULL
    const GrB_BinaryOp accum,     // optional accum for Z=accum(C(I,J),x)
    const <type> x,               // scalar to assign to C(I,J)
    const GrB_Index *I,           // row indices
    const GrB_Index ni,           // number of row indices
    const GrB_Index *J,           // column indices
    const GrB_Index nj,           // number of column indices
    const GrB_Descriptor desc     // descriptor for C and Mask
) ;
```

`GrB_Matrix_assign_<type>` assigns a single scalar to an entire submatrix of C, like the *scalar expansion* `C(I,J)=x` in MATLAB. The scalar `x` is implicitly expanded into a matrix `A` of size `ni` by `nj`, and then the matrix `A` is assigned to `C(I,J)` using the same method as in `GrB_Matrix_assign`. Refer to that function in Section 7.9.2 for further details.

The `Mask` has the same size as `C`.

For the accumulation step, the scalar `x` is typecasted directly into the type of `C` when the `accum` operator is not applied to it, or into the `ytype` of the `accum` operator, if `accum` is not NULL, for entries that are already present in `C`.

The `<type>` x notation is otherwise the same as `GrB_Matrix_setElement` (see Section 4.8.9). Any value can be passed to this function and its type will be detected, via the `_Generic` feature of ANSI C11. For a user-defined type, `x` is a `void *` pointer that points to a memory space holding a single entry of a scalar that has exactly the same user-defined type as the matrix `C`. This user-defined type must exactly match the user-defined type of `C` since no typecasting is done between user-defined types.

If a `void *` pointer is passed in and the type of the underlying scalar does not exactly match the user-defined type of `C`, then results are undefined. No error status will be returned since GraphBLAS has no way of catching this error.

In contrast to `GxB_Matrix_subassign_<type>`, results are well-defined if `I` or `J` contain duplicate indices. Duplicate indices are simply ignored.

Performance considerations: If `A` is not transposed: if |`I`| is small, then it is fastest if the format of `C` is `GxB_BY_ROW`; if |`J`| is small, then it is fastest if the format of `C` is `GxB_BY_COL`. The opposite is true if `A` is transposed.

7.10 Comparing GrB_assign and GxB_subassign

The `GxB_subassign` and `GrB_assign` operations are very similar, but they differ in three ways:

1. **The Mask has a different size:** The mask in `GxB_subassign` has the same dimensions as `w(I)` for vectors and `C(I,J)` for matrices. In `GrB_assign`, the mask is the same size as `w` or `C`, respectively (except for the row/col variants). The two masks are related. If `M` is the mask for `GrB_assign`, then `M(I,J)` is the mask for `GxB_subassign`. If there is no mask, or if `I` and `J` are both `GrB_ALL`, the two masks are the same. For `GrB_Row_assign` and `GrB_Col_assign`, the `mask` vector is the same size as a row or column of `C`, respectively. For the corresponding `GxB_Row_subassign` and `GxB_Col_subassign` operations, the `mask` is the same size as the sub-row `C(i,J)` or subcolumn `C(I,j)`, respectively.

2. `GrB_REPLACE` **is different:** They differ in how `C` is affected in areas outside the `C(I,J)` submatrix. In `GxB_subassign`, the `C(I,J)` submatrix is the only part of `C` that can be modified, and no part of `C` outside the submatrix is ever modified. In `GrB_assign`, it is possible to delete entries in `C` outside the submatrix, but only in one specific manner. Suppose the mask `M` is present (or, suppose it is not present but `GrB_SCMP` is true). After (optionally) complementing the mask, the value of `M(i,j)` can be 0 for some entry outside the `C(I,J)` submatrix. If the `GrB_REPLACE` descriptor is true, `GrB_assign` deletes this entry.

3. **Scalar expansion when duplicates appear in `I` or `J`:**
They differ in how duplicate indices are treated in `I` and `J`. For both `assign` and `subassign`, results are not defined for `GrB_Matrix_*assign`, `GrB_Vector_*assign`, `GrB_Row_*assign`, and `GrB_Col_*assign` when duplicate indices appear in `I` or `J`. The scalar expansion operations, `GrB_*_assign_<type>`, are well-defined if duplicate indices appear (the results are the same as if duplicates are removed first from `I` and `J`). However, the scalar expansion operations `GxB_*_subassign_<type>` are not well-defined if duplicate indices appear in `I` or `J`.

 `GxB_subassign` and `GrB_assign` are identical if `GrB_REPLACE` is set to its default value of false, and if the masks happen to be the same. The two

masks can be the same in two cases: either the `Mask` input is `NULL` (and it is not complemented via `GrB_SCMP`), or `I` and `J` are both `GrB_ALL`. For scalar expansion, no duplicates can appear in the index lists `I` and `J`. If all these conditions hold, the two algorithms are identical and have the same performance. Otherwise, `GxB_subassign` is much faster than `GrB_assign` when the latter must examine the entire matrix `C` to delete entries (when `GrB_REPLACE` is true), and if it must deal with a much larger `Mask` matrix. However, both methods have specific uses.

Consider using `C(I,J)+=F` for many submatrices `F` (for example, when assembling a finite-element matrix). If the `Mask` is meant as a specification for which entries of `C` should appear in the final result, then use `GrB_assign`.

If instead the `Mask` is meant to control which entries of the submatrix `C(I,J)` are modified by the finite-element `F`, then use `GxB_subassign`. This is particularly useful is the `Mask` is a template that follows along with the finite-element `F`, independent of where it is applied to `C`. Using `GrB_assign` would be very difficult in this case since a new `Mask`, the same size as `C`, would need to be constructed for each finite-element `F`.

In GraphBLAS notation, the two methods can be described as follows:

matrix and vector subassign	$\mathbf{C(I,J)}\langle\mathbf{M}\rangle = \mathbf{C(I,J)} \odot \mathbf{A}$
matrix and vector assign	$\mathbf{C}\langle\mathbf{M}\rangle\mathbf{(I,J)} = \mathbf{C(I,J)} \odot \mathbf{A}$

This notation does not include the details of the `GrB_SCMP` and `GrB_REPLACE` descriptors, but it does illustrate the difference in the `Mask`. In the subassign, `Mask` is the same size as `C(I,J)` and `A`. If `I[0]=i` and `J[0]=j`, Then `Mask(0,0)` controls how `C(i,j)` is modified by the subassign, from the value `A(0,0)`. In the assign, `Mask` is the same size as `C`, and `Mask(i,j)` controls how `C(i,j)` is modified.

The `GxB_subassign` and `GrB_assign` functions have the same signatures; they differ only in how they consider the `Mask` and the `GrB_REPLACE` descriptor, and in how duplicate indices are treated for scalar expansion.

Details of each step of the two operations are listed below:

Step	`GrB_Matrix_assign`	`GxB_Matrix_subassign`
1	$\mathbf{S} = \mathbf{C(I,J)}$	$\mathbf{S} = \mathbf{C(I,J)}$
2	$\mathbf{S} = \mathbf{S} \odot \mathbf{A}$	$\mathbf{S}\langle\mathbf{M}\rangle = \mathbf{S} \odot \mathbf{A}$
3	$\mathbf{Z} = \mathbf{C}$	$\mathbf{C(I,J)} = \mathbf{S}$
4	$\mathbf{Z(I,J)} = \mathbf{S}$	
5	$\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{Z}$	

Step 1 is the same. In the Accumulator Phase (Step 2), the expression $\mathbf{S} \odot \mathbf{A}$, described in Section 2.3, is the same in both operations. The result is simply $\mathbf{A}$ if accum is NULL. It only applies to the submatrix $\mathbf{S}$, not the whole matrix. The result $\mathbf{S} \odot \mathbf{A}$ is used differently in the Mask/Replace phase.

The Mask/Replace Phase, described in Section 2.3 is different:

- For GrB_assign (Step 5), the mask is applied to all of $\mathbf{C}$. The mask has the same size as $\mathbf{C}$. Just prior to making the assignment via the mask, the GrB_REPLACE option can be used to clear all of $\mathbf{C}$ first. This is the only way in which entries in $\mathbf{C}$ that are outside the $\mathbf{C}(\mathbf{I}, \mathbf{J})$ submatrix can be modified by this operation.

- For GxB_subassign (Step 2b), the mask is applied to just $\mathbf{S}$. The mask has the same size as $\mathbf{C}(\mathbf{I}, \mathbf{J})$, $\mathbf{S}$, and $\mathbf{A}$. Just prior to making the assignment via the mask, the GrB_REPLACE option can be used to clear $\mathbf{S}$ first. No entries in $\mathbf{C}$ that are outside the $\mathbf{C}(\mathbf{I}, \mathbf{J})$ can be modified by this operation. Thus, GrB_REPLACE has no effect on entries in $\mathbf{C}$ outside the $\mathbf{C}(\mathbf{I}, \mathbf{J})$ submatrix.

The differences between GrB_assign and GxB_subassign can be seen in Tables 1 and 2. The first table considers the case when the entry c_{ij} is in the $\mathbf{C}(\mathbf{I}, \mathbf{J})$ submatrix, and it describes what is computed for both GrB_assign and GxB_subassign. They perform the exact same computation; the only difference is how the value of the mask is specified.

The first column of the table is *yes* if GrB_REPLACE is enabled, and a dash otherwise. The second column is *yes* if an accumulator operator is given, and a dash otherwise. The third column is c_{ij} if the entry is present in $\mathbf{C}$, and a dash otherwise. The fourth column is $a_{i'j'}$ if the corresponding entry is present in $\mathbf{A}$, where $i = \mathbf{I}(i')$ and $j = \mathbf{J}(i')$.

The *mask* column is 1 if the mask allows $\mathbf{C}$ to be modified, and 0 otherwise. This is m_{ij} for GrB_assign, and $m_{i'j'}$ for GxB_subassign, to reflect the difference in the mask, but this difference is not reflected in the table. The value 1 or 0 is the value of the entry in the mask after it is optionally complemented via the GrB_SCMP option.

Finally, the last column is the action taken in this case. It is left blank if no action is taken, in which case c_{ij} is not modified if present, or not inserted into $\mathbf{C}$ if not present.

repl	accum	**C**	**A**	mask	action taken by `GrB_assign` and `GxB_subassign`
-	-	c_{ij}	$a_{i'j'}$	1	$c_{ij} = a_{i'j'}$, update
-	-	-	$a_{i'j'}$	1	$c_{ij} = a_{i'j'}$, insert
-	-	c_{ij}	-	1	delete c_{ij} because $a_{i'j'}$ not present
-	-	-	-	1	
-	-	c_{ij}	$a_{i'j'}$	0	
-	-	-	$a_{i'j'}$	0	
-	-	c_{ij}	-	0	
-	-	-	-	0	
yes	-	c_{ij}	$a_{i'j'}$	1	$c_{ij} = a_{i'j'}$, update
yes	-	-	$a_{i'j'}$	1	$c_{ij} = a_{i'j'}$, insert
yes	-	c_{ij}	-	1	delete c_{ij} because $a_{i'j'}$ not present
yes	-	-	-	1	
yes	-	c_{ij}	$a_{i'j'}$	0	delete c_{ij} (because of `GrB_REPLACE`)
yes	-	-	$a_{i'j'}$	0	
yes	-	c_{ij}	-	0	delete c_{ij} (because of `GrB_REPLACE`)
yes	-	-	-	0	
-	yes	c_{ij}	$a_{i'j'}$	1	$c_{ij} = c_{ij} \odot a_{i'j'}$, apply accumulator
-	yes	-	$a_{i'j'}$	1	$c_{ij} = a_{i'j'}$, insert
-	yes	c_{ij}	-	1	
-	yes	-	-	1	
-	yes	c_{ij}	$a_{i'j'}$	0	
-	yes	-	$a_{i'j'}$	0	
-	yes	c_{ij}	-	0	
-	yes	-	-	0	
yes	yes	c_{ij}	$a_{i'j'}$	1	$c_{ij} = c_{ij} \odot a_{i'j'}$, apply accumulator
yes	yes	-	$a_{i'j'}$	1	$c_{ij} = a_{i'j'}$, insert
yes	yes	c_{ij}	-	1	
yes	yes	-	-	1	
yes	yes	c_{ij}	$a_{i'j'}$	0	delete c_{ij} (because of `GrB_REPLACE`)
yes	yes	-	$a_{i'j'}$	0	
yes	yes	c_{ij}	-	0	delete c_{ij} (because of `GrB_REPLACE`)
yes	yes	-	-	0	

Table 1: Results of assign and subassign for entries in the $\mathbf{C}(\mathbf{I}, \mathbf{J})$ submatrix

repl	accum	$\mathbf{C}$	$\mathbf{C} = \mathbf{Z}$	mask	action taken by `GrB_assign`
-	-	c_{ij}	c_{ij}	1	
-	-	-	-	1	
-	-	c_{ij}	c_{ij}	0	
-	-	-	-	0	
yes	-	c_{ij}	c_{ij}	1	
yes	-	-	-	1	
yes	-	c_{ij}	c_{ij}	0	delete c_{ij} (because of `GrB_REPLACE`)
yes	-	-	-	0	
-	yes	c_{ij}	c_{ij}	1	
-	yes	-	-	1	
-	yes	c_{ij}	c_{ij}	0	
-	yes	-	-	0	
yes	yes	c_{ij}	c_{ij}	1	
yes	yes	-	-	1	
yes	yes	c_{ij}	c_{ij}	0	delete c_{ij} (because of `GrB_REPLACE`)
yes	yes	-	-	0	

Table 2: Results of assign for entries outside the $\mathbf{C}(\mathbf{I}, \mathbf{J})$ submatrix. Sub-assign has no effect on these entries.

Table 2 illustrates how `GrB_assign` and `GxB_subassign` differ for entries outside the submatrix. `GxB_subassign` never modifies any entry outside the $\mathbf{C}(\mathbf{I}, \mathbf{J})$ submatrix, but `GrB_assign` can modify them in two cases listed in Table 2. When the `GrB_REPLACE` option is selected, and when the `Mask(i,j)` for an entry c_{ij} is false (or if the `Mask(i,j)` is true and `GrB_SCMP` is enabled via the descriptor), then the entry is deleted by `GrB_assign`.

The fourth column of Table 2 differs from Table 1, since entries in $\mathbf{A}$ never affect these entries. Instead, for all index pairs outside the $I \times J$ submatrix, $\mathbf{C}$ and $\mathbf{Z}$ are identical (see Step 3 above). As a result, each section of the table includes just two cases: either c_{ij} is present, or not. This in contrast to Table 1, where each section must consider four different cases.

The `GrB_Row_assign` and `GrB_Col_assign` operations are slightly different. They only affect a single row or column of $\mathbf{C}$. For `GrB_Row_assign`, Table 2 only applies to entries in the single row `C(i,J)` that are outside the list of indices, `J`. For `GrB_Col_assign`, Table 2 only applies to entries in the single column `C(I,j)` that are outside the list of indices, `I`.

7.10.1 Example

The difference between `GxB_subassign` and `GrB_assign` is illustrated in the following example. Consider the 2-by-2 matrix $\mathbf{C}$ where all entries are present.

$$\mathbf{C} = \begin{bmatrix} 11 & 12 \\ 21 & 22 \end{bmatrix}$$

Suppose `GrB_REPLACE` is true, and `GrB_SCMP` is false. Let the `Mask` be:

$$\mathbf{M} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}.$$

Let $\mathbf{A} = 100$, and let the index sets be $\mathbf{I} = 0$ and $\mathbf{J} = 1$. Consider the computation $\mathbf{C}\langle\mathbf{M}\rangle(0,1) = \mathbf{C}(0,1) + \mathbf{A}$, using the `GrB_assign` operation. The result is:

$$\mathbf{C} = \begin{bmatrix} 11 & 112 \\ - & 22 \end{bmatrix}.$$

The $(0,1)$ entry is updated and the $(1,0)$ entry is deleted because its `Mask` is zero. The other two entries are not modified since $\mathbf{Z} = \mathbf{C}$ outside the submatrix, and those two values are written back into $\mathbf{C}$ because their `Mask` values are 1. The $(1,0)$ entry is deleted because the entry $\mathbf{Z}(1,0) = 21$ is prevented from being written back into $\mathbf{C}$ since `Mask(1,0)=0`.

Now consider the analogous `GxB_subassign` operation. The `Mask` has the same size as $\mathbf{A}$, namely:

$$\mathbf{M} = \begin{bmatrix} 1 \end{bmatrix}.$$

After computing $\mathbf{C}(0,1)\langle\mathbf{M}\rangle = \mathbf{C}(0,1) + \mathbf{A}$, the result is

$$\mathbf{C} = \begin{bmatrix} 11 & 112 \\ 21 & 22 \end{bmatrix}.$$

Only the $\mathbf{C}(\mathbf{I},\mathbf{J})$ submatrix, the single entry $\mathbf{C}(0,1)$, is modified by `GxB_subassign`. The entry $\mathbf{C}(1,0) = 21$ is unaffected by `GxB_subassign`, but it is deleted by `GrB_assign`.

7.10.2 Performance of GxB_subassign, GrB_assign and GrB_*_setElement

When SuiteSparse:GraphBLAS uses non-blocking mode, the modifications to a matrix by `GxB_subassign`, `GrB_assign`, and `GrB_*_setElement` can postponed, and computed all at once later on. This has a huge impact on performance.

A sequence of assignments is fast if their completion can be postponed for as long as possible, or if they do not modify the pattern at all. Modifying the pattern can be costly, but it is fast if non-blocking mode can be fully exploited.

Consider a sequence of t submatrix assignments `C(I,J)=C(I,J)+A` to an n-by-n matrix `C` where each submatrix `A` has size a-by-a with s entries, and where `C` starts with c entries. Assume the matrices are all stored in non-hypersparse form, by row (`GxB_BY_ROW`).

If blocking mode is enabled, or if the sequence requires the matrix to be completed after each assignment, each of the t assignments takes $O(a + s \log n)$ time to process the `A` matrix and then $O(n + c + s \log s)$ time to complete `C`. The latter step uses `GrB_*_build` to build an update matrix and then merge it with `C`. This step does not occur if the sequence of assignments does not add new entries to the pattern of `C`, however. Assuming in the worst case that the pattern does change, the total time is $O(t [a + s \log n + n + c + s \log s])$.

If the sequence can be computed with all updates postponed until the end of the sequence, then the total time is no worse than $O(a + s \log n)$ to process each `A` matrix, for t assignments, and then a single `build` at the end, taking $O(n + c + st \log st)$ time. The total time is $O(t [a + s \log n] + (n + c + st \log st))$. If no new entries appear in `C` the time drops to $O(t [a + s \log n])$, and in this case, the time for both methods is the same; both are equally efficient.

A few simplifying assumptions are useful to compare these times. Consider a graph of n nodes with $O(n)$ edges, and with a constant bound on the degree of each node. The asymptotic bounds assume a worst-case scenario where `C` has a least some dense rows (thus the $\log n$ terms). If these are not present, if both t and c are $O(n)$, and if a and s are constants, then the total time with blocking mode becomes $O(n^2)$, assuming the pattern of `C` changes at each assignment. This very high for a sparse graph problem. In contrast, the non-blocking time becomes $O(n \log n)$ under these same assumptions, which is asymptotically much faster.

The difference in practice can be very dramatic, since n can be many millions for sparse graphs with n nodes and $O(n)$, which can be handled on a commodity laptop.

The following guidelines should be considered when using `GxB_subassign`, `GrB_assign` and `GrB_*_setElement`.

1. A sequence of assignments that does not modify the pattern at all is fast, taking as little as $\Omega(1)$ time per entry modified. The worst case time complexity is $O(\log n)$ per entry, assuming they all modify a dense row of `C` with `n` entries, which can occur in practice. It is more common, however, that most rows of `C` have a constant number of entries, independent of `n`. No work is ever left pending when the pattern of `C` does not change.

2. A sequence of assignments that modifies the entries that already exist in the pattern of a matrix, or adds new entries to the pattern (using the same `accum` operator), but does not delete any entries, is fast. The matrix is not completed until the end of the sequence.

3. Similarly, a sequence that modifies existing entries, or deletes them, but does not add new ones, is also fast. This sequence can also repeatedly delete pre-existing entries and then reinstate them and still be fast. The matrix is not completed until the end of the sequence.

4. A sequence that mixes assignments of types (2) and (3) above can be costly, since the matrix may need to be completed after each assignment. The time complexity can become quadratic in the worst case.

5. However, any single assignment takes no more than $O(a + s \log n + n + c + s \log s)$ time, even including the time for a matrix completion, where `C` is n-by-n with c entries and `A` is a-by-a with s entries. This time is essentially linear in the size of the matrix `C`, if `A` is relatively small and sparse compared with `C`. In this case, $n + c$ are the two dominant terms.

6. In general, `GxB_subassign` is faster than `GrB_assign`. If `GrB_REPLACE` is used with `GrB_assign`, the entire matrix `C` must be traversed. This is much slower than `GxB_subassign`, which only needs to examine the `C(I,J)` submatrix. Furthermore, `GrB_assign` must deal with a much larger `Mask` matrix, whereas `GxB_subassign` has a smaller mask. Since

its mask is smaller, `GxB_subassign` takes less time than `GrB_assign` to access the mask.

Submatrix assignment in SuiteSparse:GraphBLAS is extremely efficient, even without considering the advantages of non-blocking mode discussed in Section 7.10. Consider assigning a large submatrix `C(I,J)=A` where `C` is the `Freescale2` matrix from the SuiteSparse Collection [DH11], of size 3 million by 3 million, with 14.3 million nonzeros. With the vectors `I=randperm(n,5500)` and `J=randperm(n,7000)` and `A` a random sparse matrix with 38,500 nonzeros, `C(I,J)=A` takes 87 seconds in MATLAB.[1] The same computation takes 0.74 seconds in SuiteSparse:GraphBLAS, a speedup of over 100. This is after finishing all pending computations in GraphBLAS and returning result to MATLAB as a valid MATLAB sparse matrix. The dominant time complexity for GraphBLAS is $O(n + c)$, where n is the dimension of `C` and c is its number of nonzeros. As a comparison, MATLAB takes just 0.42 seconds to compute `C+C'` for this matrix, which also takes time linear in the size of the matrix data structure, $O(n + c)$.

The time for submatrix assignment can be greatly reduced if `C` is hypersparse. Let $\bar{n}$ be the number of non-empty rows, where $\bar{n} < n$, and sometimes $\bar{n} << n$. The $\log n$ terms remain since they reflect the binary search inside a row, but the term n by itself is replaced by $\bar{n}$ or $\bar{n} \log \bar{n}$. The term a is replaced by $a \log \bar{n}$. The total time for a sequence of t updates to a hypersparse matrix C takes $O(t [a \log \bar{n} + s \log n] + (\bar{n} + c + st \log st))$ time. This assumes that `C` could include one or more dense rows, with n entries. If instead the number of entries in each row of `C` is bounded by a constant, the $\log n$ term becomes a constant. If `A` is also hypersparse, the value a is removed from the expression, replaced with its number of non-empty rows, $\bar{a}$. Suppose also t is a constant, and s is $O(\bar{a})$. Suppose the pattern of `C` changes. Suppose the number of entries in any one row of `C` is bounded by $\bar{n}$, and likewise for `A`. Then the time for the non-hypersparse case simplifies to:

$$O(a + s \log \bar{n} + (n + c + s \log s)).$$

For the hypersparse case it becomes

$$O(\bar{a} \log \bar{n} + s \log \bar{n} + (\bar{n} + c + s \log s)).$$

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[1] All performance measurements in this document were done on a MacBook Pro, 2.8 GHz Intel Core i7, 16 GB Ram, OSX 10.11.6, clang 8.0.0, MATLAB R2017A.

Both of these times include the similar terms

$$O(s \log \bar{n} + (c + s \log s)),$$

which reflects the sorting of and searching of the nonzero entries themselves, in C and A. Excluding those terms and just considering the additional time, the non-hypersparse case takes an extra time of

$$O(a + n),$$

whereas the hypersparse case takes only:

$$O(\bar{a} \log \bar{n} + \bar{n}).$$

The difference is astonishing if $\bar{n} << n$ and $\bar{a} << a$. Hypersparse C and A matrices can be created with $a = n = 2^{60}$, but suppose $\bar{a} = \bar{n} = 2^{24}$, say, which is 4 million. Then $\bar{a} \log_2 \bar{n} + \bar{n}$ is only about 420 million. In stark contrast, updating the same matrices held in non-hypersparse form would take 2^{60} time and memory, a number currently beyond the reach of the world's largest supercomputer.

If the matrices C and A have no empty columns or rows, then $\bar{a} = a$ and $\bar{n} = n$. Hypersparse matrices are more costly in this case, since they still require an extra $\log n$ term to search for a specific row in C. As a result, Suite-Sparse:GraphBLAS supports both hypersparse and non-hypersparse data structures.

7.11   GrB_apply: **apply a unary operator**

The `GrB_apply` function is the generic name for two specific functions:
`GrB_Vector_apply` and `GrB_Matrix_apply`. The generic name appears in
the function prototypes, but the specific function name is used when describing each variation. When discussing features that apply to both versions, the
simple name `GrB_apply` is used.

7.11.1   GrB_Vector_apply: **apply a unary operator to a vector**

```
GrB_Info GrB_apply                  // w<mask> = accum (w, op(u))
(
    GrB_Vector w,                   // input/output vector for results
    const GrB_Vector mask,          // optional mask for w, unused if NULL
    const GrB_BinaryOp accum,       // optional accum for z=accum(w,t)
    const GrB_UnaryOp op,           // operator to apply to the entries
    const GrB_Vector u,             // first input:  vector u
    const GrB_Descriptor desc       // descriptor for w and mask
) ;
```

   `GrB_Vector_apply` applies a unary operator to the entries of a vector,
analogous to `t = op(u)` in MATLAB except the operator `op` is only applied
to entries in the pattern of `u`. Implicit values outside the pattern of `u` are not
affected. The entries in `u` are typecasted into the `xtype` of the unary operator.
The vector `t` has the same type as the `ztype` of the unary operator. The
final step is $\mathbf{w}\langle\mathbf{m}\rangle = \mathbf{w} \odot \mathbf{t}$, as described in Section 2.3, except that all the
terms are column vectors instead of matrices.

7.11.2 GrB_Matrix_apply: **apply a unary operator to a matrix**

```
GrB_Info GrB_apply                  // C<Mask> = accum (C, op(A)) or op(A')
(
    GrB_Matrix C,                   // input/output matrix for results
    const GrB_Matrix Mask,          // optional mask for C, unused if NULL
    const GrB_BinaryOp accum,       // optional accum for Z=accum(C,T)
    const GrB_UnaryOp op,           // operator to apply to the entries
    const GrB_Matrix A,             // first input:  matrix A
    const GrB_Descriptor desc       // descriptor for C, mask, and A
) ;
```

GrB_Matrix_apply applies a unary operator to the entries of a matrix, analogous to T = op(A) in MATLAB except the operator op is only applied to entries in the pattern of A. Implicit values outside the pattern of A are not affected. The input matrix A may be transposed first. The entries in A are typecasted into the xtype of the unary operator. The matrix T has the same type as the ztype of the unary operator. The final step is $\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{C} \odot \mathbf{T}$, as described in Section 2.3.

The built-in GrB_IDENTITY_T operators (one for each built-in type T) are very useful when combined with this function, enabling it to compute $\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{C} \odot \mathbf{A}$. This makes GrB_apply a direct interface to the accumulator/mask function for both matrices and vectors.

To compute $\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{A}$ or $\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{C} \odot \mathbf{A}$ for user-defined types, the user application would need to define an identity operator for the type. Since GraphBLAS cannot detect that it is an identity operator, it must call the operator to make the full copy T=A and apply the operator to each entry of the matrix or vector.

The other GraphBLAS operation that provides a direct interface to the accumulator/mask function is GrB_transpose, which does not require an operator to perform this task. As a result, GrB_transpose can be used as an efficient and direct interface to the accumulator/mask function for both built-in and user-defined types. However, it is only available for matrices, not vectors.

7.12 GxB_select: **apply a select operator**

The `GxB_select` function is the generic name for two specific functions:
`GxB_Vector_select` and `GxB_Matrix_select`. The generic name appears in
the function prototypes, but the specific function name is used when describing each variation. When discussing features that apply to both versions, the
simple name `GxB_select` is used.

> **SPEC:** The `GxB_select` operation and `GxB_SelectOp` operator are extensions to the spec.

7.12.1 GxB_Vector_select: **apply a select operator to a vector**

```
GrB_Info GxB_select                    // w<mask> = accum (w, op(u,k))
(
    GrB_Vector w,                      // input/output vector for results
    const GrB_Vector mask,             // optional mask for w, unused if NULL
    const GrB_BinaryOp accum,          // optional accum for z=accum(w,t)
    const GxB_SelectOp op,             // operator to apply to the entries
    const GrB_Vector u,                // first input:  vector u
    const GrB_Vector Thunk,            // optional input for the select operator
    const GrB_Descriptor desc          // descriptor for w and mask
) ;
```

`GxB_Vector_select` applies a select operator to the entries of a vector,
analogous to `t = u.*op(u)` in MATLAB except the operator `op` is only applied to entries in the pattern of `u`. Implicit values outside the pattern of `u`
are not affected. If the operator is not type-generic, the entries in `u` are typecasted into the `xtype` of the select operator. The vector `t` has the same type
and size as `u`. The final step is $\mathbf{w}\langle\mathbf{m}\rangle = \mathbf{w} \odot \mathbf{t}$, as described in Section 2.3,
except that all the terms are column vectors instead of matrices.

This operation operates on vectors just as if they were `m`-by-1 matrices,
except that GraphBLAS never transposes a vector via the descriptor. The
`op` is passed `n=1` as the number of columns. Refer to the next section on
`GxB_Matrix_select` for more details.

7.12.2 GxB_Matrix_select: **apply a select operator to a matrix**

```
GrB_Info GxB_select                 // C<Mask> = accum (C, op(A,k)) or op(A',k)
(
    GrB_Matrix C,                   // input/output matrix for results
    const GrB_Matrix Mask,          // optional mask for C, unused if NULL
    const GrB_BinaryOp accum,       // optional accum for Z=accum(C,T)
    const GxB_SelectOp op,          // operator to apply to the entries
    const GrB_Matrix A,             // first input:  matrix A
    const GrB_Vector Thunk,         // optional input for the select operator
    const GrB_Descriptor desc       // descriptor for C, mask, and A
) ;
```

`GxB_Matrix_select` applies a select operator to the entries of a matrix, analogous to `T = A .* op(A)` in MATLAB except the operator `op` is only applied to entries in the pattern of `A`. Implicit values outside the pattern of `A` are not affected. The input matrix `A` may be transposed first. If the operator is not type-generic, the entries in `A` are typecasted into the `xtype` of the select operator. The final step is $\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{C} \odot \mathbf{T}$, as described in Section 2.3.

The matrix `T` has the same size and type as `A` (or the transpose of `A` if the input is transposed via the descriptor). The entries of `T` are a subset of those of `A`. Each entry `A(i,j)` of `A` is passed to the `op`, as $z = f(i, j, m, n, a_{ij}, \text{thunk})$, where `A` is m-by-n. If `A` is transposed first then the operator is applied to entries in the transposed matrix, `A'`. If z is returned as true, then the entry is copied into `T`, unchanged. If it returns false, the entry does not appear in `T`.

If `Thunk` is not NULL, it must be a vector of length 1. If it has no entry, it is treated as if it had a single entry equal to zero (even for user-defined types).

For user-defined select operators, the entry is passed to the user-defined select operator, with no typecasting. Its type must be identical to `ttype` of the select operator.

For the `GxB_TRIL`, `GxB_TRIU`, `GxB_DIAG`, and `GxB_OFFDIAG`, the `Thunk` parameter may be NULL, or it may be present but contain no entry. In this case, these operators use the value of `k=0`, the main diagonal. If present, the `Thunk` can be any built-in type. The value of this entry is typecasted: `k = (int64_t) Thunk (0)`. The value `k=0` specifies the main diagonal of the matrix, `k=1` is the +1 diagonal (the entries just above the main diagonal), `k=-1` is the -1 diagonal, and so on.

For the `GxB_*ZERO` select operators, `Thunk` is ignored, and may be NULL.

For built-in types, with the `GxB_*THUNK` operators, the value of `Thunk(0)` is typecasted to the same type as the `A` matrix. For user-defined types, `Thunk` is passed to the select operator without typecasting.

The action of `GxB_select` with the built-in select operators is described in the table below. The MATLAB analogs are precise for `tril` and `triu`, but shorthand for the other operations. The MATLAB `diag` function returns a column with the diagonal, if `A` is a matrix, whereas the matrix `T` in `GxB_select` always has the same size as `A` (or its transpose if the `GrB_INP0` is set to `GrB_TRAN`). In the MATLAB analog column, `diag` is as if it operates like `GxB_select`, where `T` is a matrix.

The following operators may be used on matrices with a user-defined type: `GxB_TRIL`, `GxB_TRIU`, `GxB_DIAG`, `GxB_OFFIAG`, `GxB_NONZERO`, `GxB_EQ_ZERO`, `GxB_NE_THUNK`, and `GxB_EQ_THUNK`.

The comparators `GxB_GT_*` `GxB_GE_*` `GxB_LT_*`, and `GxB_LE_*` only work for built-in types. All other built-in select operators can be used for any type, both built-in and any user-defined type.

GraphBLAS name	MATLAB analog	
`GxB_TRIL`	`T=tril(A,k)`	Entries in `T` are the entries on and below the kth diagonal of `A`.
`GxB_TRIU`	`T=triu(A,k)`	Entries in `T` are the entries on and above the kth diagonal of `A`.
`GxB_DIAG`	`T=diag(A,k)`	Entries in `T` are the entries on the kth diagonal of `A`.
`GxB_OFFDIAG`	`T=A-diag(A,k)`	Entries in `T` are all entries not on the kth diagonal of `A`.
`GxB_NONZERO`	`T=A(A~=0)`	Entries in `T` are all entries in `A` that have nonzero value.
`GxB_EQ_ZERO`	`T=A(A==0)`	Entries in `T` are all entries in `A` that are equal to zero.
`GxB_GT_ZERO`	`T=A(A>0)`	Entries in `T` are all entries in `A` that are greater than zero.
`GxB_GE_ZERO`	`T=A(A<=0)`	Entries in `T` are all entries in `A` that are greater than or equal to zero.
`GxB_LT_ZERO`	`T=A(A<0)`	Entries in `T` are all entries in `A` that are less than zero.
`GxB_LE_ZERO`	`T=A(A<=0)`	Entries in `T` are all entries in `A` that are less than or equal to zero.
`GxB_NE_THUNK`	`T=A(A~=k)`	Entries in `T` are all entries in `A` that are not equal to `k`.
`GxB_EQ_THUNK`	`T=A(A==k)`	Entries in `T` are all entries in `A` that are equal to `k`.
`GxB_GT_THUNK`	`T=A(A>k)`	Entries in `T` are all entries in `A` that are greater than `k`.
`GxB_GE_THUNK`	`T=A(A>=k)`	Entries in `T` are all entries in `A` that are greater than or equal to `k`.
`GxB_LT_THUNK`	`T=A(A<k)`	Entries in `T` are all entries in `A` that are less than `k`.
`GxB_LE_THUNK`	`T=A(A<=k)`	Entries in `T` are all entries in `A` that are less than or equal to `k`.

7.13 GrB_reduce: **reduce to a vector or scalar**

The generic function name `GrB_reduce` may be used for all specific functions discussed in this section. When the details of a specific function are discussed, the specific name is used for clarity.

7.13.1 GrB_Matrix_reduce_<op>: **reduce a matrix to a vector**

```
GrB_Info GrB_reduce                 // w<mask> = accum (w,reduce(A))
(
    GrB_Vector w,                   // input/output vector for results
    const GrB_Vector mask,          // optional mask for w, unused if NULL
    const GrB_BinaryOp accum,       // optional accum for z=accum(w,t)
    const <operator> reduce,        // reduce operator for t=reduce(A)
    const GrB_Matrix A,             // first input:  matrix A
    const GrB_Descriptor desc       // descriptor for w, mask, and A
) ;
```

`GrB_Matrix_reduce_<op>` is a generic name for two specific methods. Both methods reduce a matrix to a column vector using an operator, roughly analogous to `t = sum (A')` in MATLAB, in the default case, where `t` is a column vector. By default, the method reduces across the rows to obtain a column vector; use `GrB_TRAN` to reduce down the columns.

`GrB_Matrix_reduce_BinaryOp` relies on a binary operator for the reduction: the fourth argument `reduce`, a `GrB_BinaryOp`. All three domains of the operator must be the same. `GrB_Matrix_reduce_Monoid` performs the same reduction using a `GrB_Monoid` as its fourth argument. In both cases the reduction operator must be commutative and associative. Otherwise the results are undefined.

The input matrix `A` may be transposed first. Its entries are then typecast into the type of the `reduce` operator or monoid. The reduction is applied to all entries in `A (i,:)` to produce the scalar `t (i)`. This is done without the use of the identity value of the monoid. If the `i`th row `A (i,:)` has no entries, then `(i)` is not an entry in `t` and its value is implicit. If `A (i,:)` has a single entry, then that is the result `t (i)` and `reduce` is not applied at all for the `i`th row. Otherwise, multiple entries in row `A (i,:)` are reduced via the `reduce` operator or monoid to obtain a single scalar, the result `t (i)`.

The final step is $\mathbf{w}\langle\mathbf{m}\rangle = \mathbf{w} \odot \mathbf{t}$, as described in Section 2.3, except that all the terms are column vectors instead of matrices.

7.13.2 GrB_Vector_reduce_\<type\>: **reduce a vector to a scalar**

```
GrB_Info GrB_reduce                    // c = accum (c, reduce_to_scalar (u))
(
    <type> *c,                         // result scalar
    const GrB_BinaryOp accum,          // optional accum for c=accum(c,t)
    const GrB_Monoid monoid,           // monoid to do the reduction
    const GrB_Vector u,                // vector to reduce
    const GrB_Descriptor desc          // descriptor (currently unused)
) ;
```

`GrB_Vector_reduce_<type>` reduces a vector to a scalar, analogous to `t = sum (u)` in MATLAB, except that in GraphBLAS any commutative and associative monoid can be used in the reduction.

The reduction operator is a commutative and associative monoid with an identity value. Results are undefined if the monoid does not have these properties. This function differs from `GrB_Matrix_reduce_BinaryOp` (which reduces a matrix to a vector) in that it requires a valid monoid additive identity value. If the vector `u` has no entries, that identity value is copied into the scalar `t`. Otherwise, all of the entries in the vector are reduced to a single scalar using the `reduce` operator.

The scalar type is any of the built-in types, or a user-defined type. In the function signature it is a C type: `bool`, `int8_t`, ... `float`, `double`, or `void *` for a user-defined type. The user-defined type must be identical to the type of the vector `u`. This cannot be checked by GraphBLAS and thus results are undefined if the types are not the same.

The descriptor is unused, but it appears in case it is needed in future versions of the GraphBLAS API. This function has no mask so its accumulator/mask step differs from the other GraphBLAS operations. It does not use the methods described in Section 2.3, but uses the following method instead.

If `accum` is `NULL`, then the scalar `t` is typecast into the type of `c`, and `c = t` is the final result. Otherwise, the scalar `t` is typecast into the `ytype` of the `accum` operator, and the value of `c` (on input) is typecast into the `xtype` of the `accum` operator. Next, the scalar `z = accum (c,t)` is computed, of the `ztype` of the `accum` operator. Finally, `z` is typecast into the final result, `c`.

Forced completion: All computations for the vector `u` are guaranteed to be finished when the method returns.

7.13.3 GrB_Matrix_reduce_<type>: reduce a matrix to a scalar

```
GrB_Info GrB_reduce                  // c = accum (c, reduce_to_scalar (A))
(
    <type> *c,                       // result scalar
    const GrB_BinaryOp accum,        // optional accum for c=accum(c,t)
    const GrB_Monoid monoid,         // monoid to do the reduction
    const GrB_Matrix A,              // matrix to reduce
    const GrB_Descriptor desc        // descriptor (currently unused)
) ;
```

`GrB_Matrix_reduce_<type>` reduces a matrix `A` to a scalar, roughly analogous to `t = sum (A (:))` in MATLAB. This function is identical to reducing a vector to a scalar, since the positions of the entries in a matrix or vector have no effect on the result. Refer to the reduction to scalar described in the previous Section 7.13.2.

Forced completion: All computations for the matrix `A` are guaranteed to be finished when the method returns.

7.14 GrB_transpose: transpose a matrix

```
GrB_Info GrB_transpose             // C<Mask> = accum (C, A')
(
    GrB_Matrix C,                  // input/output matrix for results
    const GrB_Matrix Mask,         // optional mask for C, unused if NULL
    const GrB_BinaryOp accum,      // optional accum for Z=accum(C,T)
    const GrB_Matrix A,            // first input:  matrix A
    const GrB_Descriptor desc      // descriptor for C, Mask, and A
) ;
```

GrB_transpose transposes a matrix A, just like the array transpose T = A.'
in MATLAB. The internal result matrix T = A' (or merely T = A if A is
transposed via the descriptor) has the same type as A. The final step is
$\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{C} \odot \mathbf{T}$, as described in Section 2.3, which typecasts T as needed
and applies the mask and accumulator.

To be consistent with the rest of the GraphBLAS API regarding the
descriptor, the input matrix A may be transposed first. It may seem counter-
intuitive, but this has the effect of not doing any transpose at all. As
a result, GrB_transpose is useful for more than just transposing a ma-
trix. It can be used as a direct interface to the accumulator/mask op-
eration, $\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{C} \odot \mathbf{A}$. This step also does any typecasting needed, so
GrB_transpose can be used to typecast a matrix A into another matrix C.
To do this, simply use NULL for the Mask and accum, and provide a non-
default descriptor desc that sets the transpose option:

```
// C = typecasted copy of A
GrB_Descriptor_set (desc, GrB_INP0, GrB_TRAN) ;
GrB_transpose (C, NULL, NULL, A, desc) ;
```

If the types of C and match, then the above two lines of code are the same
as GrB_Matrix_dup (&C, A), except that for GrB_transpose the matrix C
must already exist and be the right size. If C does not exist, the work of
GrB_Matrix_dup can be replicated with this:

```
// C = create an exact copy of A, just like GrB_Matrix_dup
GrB_Matrix C ;
GrB_Type type ;
GrB_Index nrows, ncols ;
GrB_Descriptor desc ;
GxB_Matrix_type (&type, A) ;
GrB_Matrix_nrows (&nrows, A) ;
```

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```
GrB_Matrix_ncols (&ncols, A) ;
GrB_Matrix_new (&C, type, nrows, ncols) ;
GrB_Descriptor_new (&desc) ;
GrB_Descriptor_set (desc, GrB_INP0, GrB_TRAN) ;
GrB_transpose (C, NULL, NULL, A, desc) ;
```

Since the input matrix `A` is transposed by the descriptor, SuiteSparse:Graph-BLAS does the right thing and does not transpose the matrix at all. Since `T = A` is not typecasted, SuiteSparse:GraphBLAS can construct `T` internally in $O(1)$ time and using no memory at all. This makes `Grb_transpose` a fast and direct interface to the accumulator/mask function in GraphBLAS.

This example is of course overkill, since the work can all be done by a single call to the `GrB_Matrix_dup` function. However, the `GrB_Matrix_dup` function can only create `C` as an exact copy of `A`, whereas variants of the code above can do many more things with these two matrices. For example, the `type` in the example can be replaced with any other type, perhaps selected from another matrix or from an operator.

Consider the following code excerpt, which uses `GrB_transpose` to remove all diagonal entries from a square matrix. It first creates a diagonal `Mask`, which is complemented so that $\mathbf{C}\langle\neg\mathbf{M}\rangle = \mathbf{A}$ does not modify the diagonal of $\mathbf{C}$. The `REPLACE` ensures that `C` is cleared first, and then $\mathbf{C}\langle\neg\mathbf{M}\rangle = \mathbf{A}$ modifies all entries in $\mathbf{C}$ where the mask $\mathbf{M}$ is false. These correspond to all the off-diagonal entries. The descriptor ensures that $\mathbf{A}$ is not transposed at all. The `Mask` can have any pattern, of course, and wherever it is set true, the corresponding entries in `A` are deleted from the copy `C`.

```
// remove all diagonal entries from the matrix A
// Mask = speye (n) ;
GrB_Matrix_new (&Mask, GrB_BOOL, n, n) ;
for (int64_t i = 0 ; i < n ; i++)
{
    GrB_Matrix_setElement (Mask, (bool) true, i, i) ;
}
// C<~Mask> = A, clearing C first.  No transpose.
GrB_Descriptor_new (&desc) ;
GrB_Descriptor_set (desc, GrB_INP0, GrB_TRAN) ;
GrB_Descriptor_set (desc, GrB_MASK, GrB_SCMP) ;
GrB_Descriptor_set (desc, GrB_OUTP, GrB_REPLACE) ;
GrB_transpose (A, Mask, NULL, A, desc) ;
```

7.15 GxB_kron: **Kronecker product**

```
GrB_Info GxB_kron                    // C<Mask> = accum (C, kron(A,B))
(
    GrB_Matrix C,                    // input/output matrix for results
    const GrB_Matrix Mask,           // optional mask for C, unused if NULL
    const GrB_BinaryOp accum,        // optional accum for Z=accum(C,T)
    const GrB_BinaryOp op,           // defines '*' for T=kron(A,B)
    const GrB_Matrix A,              // first input:  matrix A
    const GrB_Matrix B,              // second input: matrix B
    const GrB_Descriptor desc        // descriptor for C, Mask, A, and B
) ;
```

`GxB_kron` computes the Kronecker product, $\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{C} \odot \mathrm{kron}(\mathbf{A}, \mathbf{B})$ where

$$
\mathrm{kron}(\mathbf{A}, \mathbf{B}) = \left[\begin{array}{ccc} a_{00} \otimes \mathbf{B} & \dots & a_{0,n-1} \otimes \mathbf{B} \\ \vdots & \ddots & \vdots \\ a_{m-1,0} \otimes \mathbf{B} & \dots & a_{m-1,n-1} \otimes \mathbf{B} \end{array} \right]
$$

The $\otimes$ operator is defined by the `op` parameter. It is applied in an element-wise fashion (like `GrB_eWiseMult`), where the pattern of the submatrix $a_{ij}\otimes\mathbf{B}$ is the same as the pattern of $\mathbf{B}$ if a_{ij} is an entry in the matrix $\mathbf{A}$, or empty otherwise. The input matrices `A` and `B` can be of any dimension, and both matrices may be transposed first via the descriptor, `desc`. Entries in `A` and `B` are typecast into the input types of the `op`. The matrix `T=kron(A,B)` has the same type as the `ztype` of the binary operator, `op`. The final step is $\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{C} \odot \mathbf{T}$, as described in Section 2.3.

SPEC: `GxB_kron` is an extension to the spec.

8 Printing GraphBLAS objects

> **SPEC:** The GraphBLAS API has no mechanism for printing the contents of GraphBLAS objects. This entire section is an extension to the specification.

The 9 different objects handled by SuiteSparse:GraphBLAS are all opaque, although nearly all of their contents can be extracted via methods such as `GrB_Matrix_extractTuples`, `GrB_Matrix_extractElement`, `GxB_Matrix_type`, and so on. The GraphBLAS C API has no mechanism for printing all the contents of GraphBLAS objects, but this is helpful for debugging. Nine type-specific methods and two type-generic methods are provided, listed in the table below:

`GxB_Type_fprint`	print and check a `GrB_Type`
`GxB_UnaryOp_fprint`	print and check a `GrB_UnaryOp`
`GxB_BinaryOp_fprint`	print and check a `GrB_BinaryOp`
`GxB_SelectOp_fprint`	print and check a `GxB_SelectOp`
`GxB_Monoid_fprint`	print and check a `GrB_Monoid`
`GxB_Semiring_fprint`	print and check a `GrB_Semiring`
`GxB_Descriptor_fprint`	print and check a `GrB_Descriptor`
`GxB_Matrix_fprint`	print and check a `GrB_Matrix`
`GxB_Vector_fprint`	print and check a `GrB_Vector`
`GxB_fprint`	print/check any object to a file
`GxB_print`	print/check any object to `stdout`

These methods do not modify the status of any object. If a matrix or vector has not been completed, the pending computations are guaranteed to *not* be performed. The reason is simple. It is possible for a bug in the user application (such as accessing memory outside the bounds of an array) to mangle the internal content of a GraphBLAS object, and the `GxB_*print` methods can be helpful tools to track down this bug. If `GxB_*print` attempted to complete any computations prior to printing or checking the contents of the matrix or vector, then further errors could occur, including a segfault.

By contrast, all GraphBLAS methods and operations that return values into user-provided arrays or variables force the completion of pending operations (`GrB_*_nvals`, `GrB_*_extractElement`, `GrB_*_extractTuples`, and `GrB_reduce` (to scalar)). The `GxB_*print` methods provide a useful alternative for debugging, and for a quick understanding of what GraphBLAS is computing while developing a user application.

Each of the methods has a parameter of type `GxB_Print_Level` that specifies the amount to print:

```
typedef enum
{
    GxB_SILENT = 0,     // nothing is printed, just check the object
    GxB_SUMMARY = 1,    // print a terse summary
    GxB_SHORT = 2,      // short description, about 30 entries of a matrix
    GxB_COMPLETE = 3    // print the entire contents of the object
}
GxB_Print_Level ;
```

The nine type-specific functions include an additional argument, the `name` string. The `name` is printed at the beginning of the display (assuming the print level is not `GxB_SILENT`) so that the object can be more easily identified in the output. For the type-generic methods `GxB_fprint` and `GxB_print`, the `name` string is the variable name of the object itself.

If the file `f` is NULL, nothing is printed (`pr` is effectively `GxB_SILENT`) If `pr` is outside the bounds 0 to 3, negative values are treated as `GxB_SILENT`, and values larger than 3 are treated as `GxB_COMPLETE`. If `name` is NULL, it is treated as the empty string. None of these are error conditions.

The methods check their input objects carefully and extensively, even when `pr` is equal to `GxB_SILENT`. The following error codes can be returned:

- `GrB_SUCCESS`: object is valid
- `GrB_UNINITIALIZED_OBJECT`: object is not initialized
- `GrB_INVALID_OBJECT`: object is not valid
- `GrB_NULL_POINTER`: object is a NULL pointer
- `GrB_INVALID_VALUE`: `fprintf` returned an I/O error; see the ANSI C `errno` or `GrB_error( )` for details.

The content of any GraphBLAS object is opaque, and subject to change. As a result, the exact content and format of what is printed is implementation-dependent, and will change from version to version of SuiteSparse:GraphBLAS. Do not attempt to rely on the exact content or format by trying to parse the resulting output via another program. The intent of these functions is to produce a report of an object for visual inspection. If the user application needs to extract content from a GraphBLAS matrix or vector, use `GrB_*_extractTuples` instead.

8.1 GxB_fprint: **Print a GraphBLAS object to a file**

```
GrB_Info GxB_fprint             // print and check a GraphBLAS object
(
    GrB_<objecttype> object,    // object to print and check
    GxB_Print_Level pr,         // print level
    FILE *f                     // file for output
) ;
```

The `GxB_fprint` function prints the contents of any of the 9 Graph-BLAS objects to the file `f`. If `f` is `NULL`, the results are printed to `stdout`. For example, to print the entire contents of a matrix `A` to the file `f`, use `GxB_fprint (A, GxB_COMPLETE, f)`.

8.2 GxB_print: **Print a GraphBLAS object to** stdout

```
GrB_Info GxB_print              // print and check a GrB_Vector
(
    GrB_<objecttype> object,    // object to print and check
    GxB_Print_Level pr          // print level
) ;
```

`GxB_print` is the same as `GxB_fprint`, except that it prints the contents of the object to `stdout` instead of a file `f`. For example, to print the entire contents of a matrix `A`, use `GxB_print (A, GxB_COMPLETE)`.

8.3 GxB_Type_fprint: **Print a** GrB_Type

```
GrB_Info GxB_Type_fprint        // print and check a GrB_Type
(
    GrB_Type type,              // object to print and check
    const char *name,           // name of the object
    GxB_Print_Level pr,         // print level
    FILE *f                     // file for output
) ;
```

For example, `GxB_Type_fprint (GrB_BOOL, "boolean type", GxB_COMPLETE, f)` prints the contents of the `GrB_BOOL` object to the file `f`.

8.4 GxB_UnaryOp_fprint: **Print a** GrB_UnaryOp

```
GrB_Info GxB_UnaryOp_fprint          // print and check a GrB_UnaryOp
(
    GrB_UnaryOp unaryop,             // object to print and check
    const char *name,                // name of the object
    GxB_Print_Level pr,              // print level
    FILE *f                          // file for output
) ;
```

For example, `GxB_UnaryOp_fprint (GrB_LNOT, "not", GxB_COMPLETE, f)` prints the `GrB_LNOT` unary operator to the file `f`.

8.5 GxB_BinaryOp_fprint: **Print a** GrB_BinaryOp

```
GrB_Info GxB_BinaryOp_fprint         // print and check a GrB_BinaryOp
(
    GrB_BinaryOp binaryop,           // object to print and check
    const char *name,                // name of the object
    GxB_Print_Level pr,              // print level
    FILE *f                          // file for output
) ;
```

For example, `GxB_BinaryOp_fprint (GrB_PLUS_FP64, "plus", GxB_COMPLETE, f)` prints the `GrB_PLUS_FP64` binary operator to the file `f`.

8.6 GxB_SelectOp_fprint: **Print a** GxB_SelectOp

```
GrB_Info GxB_SelectOp_fprint         // print and check a GxB_SelectOp
(
    GxB_SelectOp selectop,           // object to print and check
    const char *name,                // name of the object
    GxB_Print_Level pr,              // print level
    FILE *f                          // file for output
) ;
```

For example, `GxB_SelectOp_fprint (GxB_TRIL, "tril", GxB_COMPLETE, f)` prints the `GxB_TRIL` select operator to the file `f`.

8.7 GxB_Monoid_fprint: **Print a** GrB_Monoid

```
GrB_Info GxB_Monoid_fprint          // print and check a GrB_Monoid
(
    GrB_Monoid monoid,              // object to print and check
    const char *name,               // name of the object
    GxB_Print_Level pr,             // print level
    FILE *f                         // file for output
) ;
```

For example, GxB_Monoid_fprint (GxB_PLUS_FP64_MONOID, "plus monoid", GxB_COMPLETE, f) prints the predefined GxB_PLUS_FP64_MONOID (based on the binary operator GrB_PLUS_FP64) to the file f.

8.8 GxB_Semiring_fprint: **Print a** GrB_Semiring

```
GrB_Info GxB_Semiring_fprint        // print and check a GrB_Semiring
(
    GrB_Semiring semiring,          // object to print and check
    const char *name,               // name of the object
    GxB_Print_Level pr,             // print level
    FILE *f                         // file for output
) ;
```

For example, GxB_Semiring_fprint (GxB_PLUS_TIMES_FP64, "standard", GxB_COMPLETE, f) prints the predefined GxB_PLUS_TIMES_FP64 semiring to the file f.

8.9 GxB_Descriptor_fprint: **Print a** GrB_Descriptor

```
GrB_Info GxB_Descriptor_fprint      // print and check a GrB_Descriptor
(
    GrB_Descriptor descriptor,      // object to print and check
    const char *name,               // name of the object
    GxB_Print_Level pr,             // print level
    FILE *f                         // file for output
) ;
```

For example, GxB_Descriptor_fprint (d, "descriptor", GxB_COMPLETE, f) prints the descriptor d to the file f.

8.10   GxB_Matrix_fprint: **Print a GrB_Matrix**

```
GrB_Info GxB_Matrix_fprint          // print and check a GrB_Matrix
(
    GrB_Matrix A,                   // object to print and check
    const char *name,               // name of the object
    GxB_Print_Level pr,             // print level
    FILE *f                         // file for output
) ;
```

For example, `GxB_Matrix_fprint (A, "my matrix", GxB_SHORT, f)` prints about 30 entries from the matrix `A` to the file `f`.

8.11   GxB_Vector_fprint: **Print a GrB_Vector**

```
GrB_Info GxB_Vector_fprint          // print and check a GrB_Vector
(
    GrB_Vector v,                   // object to print and check
    const char *name,               // name of the object
    GxB_Print_Level pr,             // print level
    FILE *f                         // file for output
) ;
```

For example, `GxB_Vector_fprint (v, "my vector", GxB_SHORT, f)` prints about 30 entries from the vector `v` to the file `f`.

8.12   Performance and portability considerations

Even when the print level is `GxB_SILENT`, these methods extensively check the contents of the objects passed to them, which can take some time. They should be considered debugging tools only, not for final use in production.

The return value of the `GxB_*print` methods can be relied upon, but the output to the file (or `stdout`) can change from version to version. If these methods are eventually added to the GraphBLAS C API Specification, a conforming implementation might never print anything at all, regardless of the `pr` value. This may be essential if the GraphBLAS library is installed in a dedicated device, with no file output, for example.

Some implementations may wish to print nothing at all if the matrix is not yet completed, or just an indication that the matrix has pending operations and cannot be printed, when non-blocking mode is employed. In this case, use `GrB_Matrix_nvals` or `GrB_wait` to finish all pending computations first.

If a matrix or vector has pending operations, SuiteSparse:GraphBLAS prints a list of the *pending tuples*, which are the entries not yet inserted into the primary data structure. It can also print out entries that remain in the data structure but are awaiting deletion; these are called *zombies* in the output report.

Most of the rest of the report is self-explanatory.

9 Creating user-defined objects at compile-time

> **SPEC:** The GraphBLAS API has no mechanism for constructing user-defined objects when GraphBLAS is compiled. This entire section, and the `GxB_*_define` macros, are extensions to the specification.

User-defined types, operators, monoids, and semirings provide a powerful and flexible mechanism for extending GraphBLAS functionality. For example, GraphBLAS does not support a built-in complex type, but it can be added in a user application with a few simple functions, and a few calls to `GrB_Type_new`, `GrB_BinaryOp_new`, `GrB_Monoid_new`, and `GrB_Semiring_new`. A complete example is given in Section 10.9. See in particular the example code in `Demo/Source/usercomplex.c`.

GraphBLAS does not need to be recompiled in order for the user application to add new types, operators, monoids, or semirings. This flexibility comes at the cost of performance, however. Since the multiply/add operations in a user-defined semiring must be accessed one at a time through a function pointer, a complex matrix multiply via `GrB_mxm` is about two or three times slower than it could be if GraphBLAS included a built-in complex type.

This performance gap could be solved in two ways. Complex operators and types could be added to SuiteSparse:GraphBLAS, or the GraphBLAS C API Specification itself, but this is not flexible. Other user-defined objects would still need to be created.

The mechanism described in this section is another solution. It allows the user to create an unlimited variety of user-defined types, operators, monoids, and semirings, and have them compiled into SuiteSparse:GraphBLAS. This is done by creating one or more files with the filename extension `*.m4`, and placing them in the `SuiteSparse/GraphBLAS/User` directory. Then when SuiteSparse:GraphBLAS is compiled via `cmake`, these new objects are compiled as well. This is illustrated by the example in Figure 2. It is a single file, `GraphBLAS/User/Example/my_complex.m4`, that defines the complex type `double complex` as a GraphBLAS `GrB_Type` called `My_Complex`.

If the file resides in its default location in `GraphBLAS/User/Example`, it is not compiled with GraphBLAS. Moving this file into `GraphBLAS/User`, or creating other `m4` files in the `GraphBLAS/User` directory, enables Suite-Sparse:GraphBLAS to incorporate these user-defined objects at compile-

time. Any number of `*.m4` files may be placed in the `GraphBLAS/User` directory; they are all included in SuiteSparse:GraphBLAS. There is no need to tell the `cmake` process what the file names are. All `*.m4` files in `GraphBLAS/User` will be found and included.

Of course, a C++ API for GraphBLAS could also create user-defined objects at compile-time via templates, but the API for GraphBLAS is in C, not C++. The solution described in this Section provides this functionality in a purely C interface.

In Figure 2, two inline functions are defined to perform complex addition and multiplication, and these are used as the basis for two GraphBLAS binary operators, `My_complex_plus` and `My_complex_times`. A monoid is constructed for complex addition, and then finally the complex plus-times semiring is defined. All of these user-defined objects are appended to the `GraphBLAS.h` include file.

Some user definitions such as the `static inline` functions `my_complex_plus` and `my_complex_times` in Figure 2 should appear in the `GraphBLAS.h` include file, so that they are available to any function in the user application. Other user declarations should appear only once, such as the declaration of global values used by user-defined functions.

To handle this, an `#ifdef GxB_USER_INCLUDE` mechanism is provided for use in the `User/*m4` file, in the following style:

```
#ifdef GxB_USER_INCLUDE

    // Part 1: any code here will be active in GraphBLAS.h.  Place all
    // declarations suitable for use in an #include file here.  These
    // declarations will be available to all user application files
    // and to all internal SuiteSparse/GraphBLAS codes.

#else

    // Part 2: declarations of user-defined variables, and executable
    // code that should be compiled just once appears here.

#endif
```

Placing user functions inside this `#ifdef` structure gives the user control over what declarations should be available to all of GraphBLAS and to all user application files (Part 1), and what definitions should appear just once (Part 2).

```
#ifdef GxB_USER_INCLUDE

    // Get complex.h but remove "I" since it is used elsewhere in GraphBLAS.
    #include <complex.h>
    #undef I

    // Not all complex.h definitions include the CMPLX macro
    #ifndef CMPLX
    #define CMPLX(real,imag) \
        ( (double complex)((double)(real)) + \
          (double complex)((double)(imag) * _Complex_I) )
    #endif

    // define a token so a user application can check for existence
    #define MY_COMPLEX

    static inline void my_complex_plus
    (
        double complex *z, const double complex *x, const double complex *y
    )
    {
        (*z) = (*x) + (*y) ;
    }

    static inline void my_complex_times
    (
        double complex *z, const double complex *x, const double complex *y
    )
    {
        (*z) = (*x) * (*y) ;
    }
#endif

// define the complex type, plus & times operators, plus monoid, and semiring
GxB_Type_define(My_Complex, double complex) ;
GxB_BinaryOp_define(My_Complex_plus,
    my_complex_plus,  My_Complex, My_Complex, My_Complex) ;
GxB_BinaryOp_define(My_Complex_times, my_complex_times,
    My_Complex, My_Complex, My_Complex) ;
GxB_Monoid_define(My_Complex_plus_monoid, My_Complex_plus, CMPLX(0,0)) ;
GxB_Semiring_define(My_Complex_plus_times, My_Complex_plus_monoid,
    My_Complex_times) ;
```

Figure 2: User-defined complex type and operators, defined at compile-time

For example, `User/Example/my_scale.m4` in Figure 3 defines a unary operator that computes `z=my_scalar*x`, where `my_scalar` is a global variable. The declaration of `my_scalar` should appear in `GraphBLAS.h`, but it should be defined only once.

The `GxB_*_define` macros must not appear inside either Part 1 or Part 2 of the `#ifdef GxB_USER_INCLUDE`.

When creating user-defined objects in a `User/*.m4` a useful (but optional) strategy is to define a token that can be used to discover whether or not a particular object is available at compile time. For example, for the `My_complex` objects in Figure 2, the following code snippet could appear in a user application:

```
#ifndef MY_COMPLEX
// construct complex type and operators at run-time
GrB_Type My_Complex ;
GrB_Type_new (&My_Complex, sizeof (double complex)) ;
... etc
#else
// use the pre-defined My_Complex type and corresponding objects
#endif
```

In either case, when the user application is finished, it can do `GrB_free(My_Complex)`. This safely does nothing if `My_Complex` is defined at compile-time with `GxB_Type_define`, or frees it if the type was created a run-time with `GrB_Type_new`.

These pre-defined objects can be used in the user application just as if they were created at run-time via the corresponding calls to `GrB_Type_new`, `GrB_BinaryOp_new`, `GrB_Monoid_new`, and `GrB_Semiring_new`. For a large matrix, computing `C=A*B` via `GrB_mxm` with the pre-compiled `My_Complex_plus_times` semiring is about 10% faster than the MATLAB statement `C=A*B` for sparse complex matrices. This is the same relative performance as when computing `C=A*B` for real matrices in MATLAB, versus using `GrB_mxm` with the built-in `GxB_PLUS_TIMES_FP64` semiring. If the complex plus-times semiring is defined at run-time instead, `GrB_mxm` is about two to three times slower.

User definitions in the `User/*m4` files are visible to all internal Suite-Sparse:GraphBLAS functions and thus must be given names that do not conflict with internal variables, functions, and macros.

Objects defined by `GxB_*_define`, user-defined static inline functions, typedefs, global variables, and macros (via `#define`) in the `User/*.m4` files must be given names with a unique prefix, such as `MY_`, `USER_`, `my_`, `user_`, or the name of the user application (say `RealCoolApp_`, or `PAGERANK_` as

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```
#ifdef GxB_USER_INCLUDE

    //------------------------------------------------------------------------
    // declarations: for GraphBLAS.h
    //------------------------------------------------------------------------

    // The following are declarations that are enabled in GraphBLAS.h and
    // appear in all user codes that #include "GraphBLAS.h", and also in all
    // internal GraphBLAS codes.  All user declarations (not definitions)
    // should appear here.

    #define MY_SCALE

    extern double my_scalar ;

    static inline void my_scale
    (
        double *z,
        const double *x
    )
    {
        (*z) = my_scalar * (*x) ;
    }

#else

    //------------------------------------------------------------------------
    // definitions: code appears just once, in Source/all_user_objects.c
    //------------------------------------------------------------------------

    // The following defintions are enabled in only a single place:
    // SuiteSparse/GraphBLAS/Source/all_user_objects.c.  This is the place
    // where all user-defined global variables should be defined.

    double my_scalar = 0 ;

#endif

// Unary operator to compute z = my_scalar*x
GxB_UnaryOp_define(My_scale, my_scale, GrB_FP64, GrB_FP64) ;
```

Figure 3: User-defined unary operator, dependent upon a global variable

exemplified in the `my_pagerank.m4` example). This will ensure that no name conflicts will occur.

An example macro name conflict occurs in the `User/Example/my_complex.m4` example in Figure 2, which includes the ANSI `complex.h` include file. The `complex.h` file defines an ANSI C11 macro `I` but that name conflicts with internal SuiteSparse:GraphBLAS variables, so `#undef I` is done immediately after the `complex.h` file is included. Undefining `I` is permitted in the ANSI C11 specification for `complex.h`.

The six `GxB_*_define` macros are presented below. They all have essentially the same parameters in the same order as the corresponding `GrB_*_new` methods, except that where the `GrB_*_new` methods use pointers to the new objects, the corresponding `GxB_*_define` macro uses just the name of the object. Unlike their `GrB_*_new` counterparts, the six `GxB_*_define` macros do not return an error code. Any errors will be detected by the compiler.

Since they are `m4` macros, no space can appear between the macro name `GxB_*_define` and the subsequent left parenthesis. This restriction may be relaxed in subsequent versions of SuiteSparse:GraphBLAS.

Since these objects are constructed at compile-time, they do not need to be freed with `GrB_free`. Attempting to free them is safe, however. Suite-Sparse:GraphBLAS will safely (and silently) do nothing if an attempt is made to free them.

The next sections describe the following 6 macros:

`GxB_Type_define`	define a `GrB_Type` at compile-time
`GxB_UnaryOp_define`	define a `GrB_UnaryOp` at compile-time
`GxB_BinaryOp_define`	define a `GrB_BinaryOp` at compile-time
`GxB_SelectOp_define`	define a `GxB_SelectOp` at compile-time
`GxB_Monoid_define`	define a `GrB_Monoid` at compile-time
`GxB_Monoid_terminal_define`	define a `GrB_Monoid` at compile-time (with a terminal value)
`GxB_Semiring_define`	define a `GrB_Semiring` at compile-time

9.1 GxB_Type_define: **define a GrB_Type at compile time**

```
GxB_Type_define(GrB_Type type, ctype) ;
```

GxB_Type_define is very similar to GrB_Type_new, except that it defines a type when SuiteSparse:GraphBLAS is compiled. Instead of the sizeof(ctype) second parameter of GrB_Type_new, the second parameter ctype of GxB_Type_define is a C type (a built-in one or from a typedef). As in all GxB_*_define macros, there is no & symbol in front of the GraphBLAS type parameter, since this macro defines the object instead of returning a pointer.

9.2 GxB_UnaryOp_define: **define a GrB_UnaryOp at compile time**

```
GxB_UnaryOp_define(GrB_UnaryOp op, func, GrB_Type ztype, GrB_Type xtype) ;
```

GxB_UnaryOp_define is identical to GrB_UnaryOp_new, except that it defines a unary operator when SuiteSparse:GraphBLAS is compiled. The function func is the name of a user-defined function, normally a static inline function in the user's *.m4 file. The ztype and xtype must be built-in types (GrB_BOOL, GrB_FP64, etc) or types defined with GxB_Type_define.

9.3 GxB_BinaryOp_define: **define a GrB_BinaryOp at compile time**

```
GxB_BinaryOp_define(GrB_BinaryOp op, func, GrB_Type ztype, GrB_Type xtype,
    GrB_Type ytype) ;
```

GxB_BinaryOp_define is identical to GrB_BinaryOp_new, except that it defines a binary operator when SuiteSparse:GraphBLAS is compiled. The function func is the name of a user-defined function, normally a static inline function in the user's *.m4 file. The ztype, xtype, and ytype must be built-in types (GrB_BOOL, GrB_FP64, etc) or types defined with GxB_Type_define.

9.4 GxB_SelectOp_define: **define a GxB_SelectOp at compile time**

```
GxB_SelectOp_define(GxB_SelectOp op, func, GrB_Type xtype, GrB_Type ttype) ;
```

GxB_SelectOp_define is identical to GxB_SelectOp_new, except that it defines a select operator when SuiteSparse:GraphBLAS is compiled. The function func is the name of a user-defined function, normally a static inline function in the user's *.m4 file. The xtype and/or ttype parameters may be GrB_NULL or NULL, which denotes a type-generic GxB_SelectOp operator. If not null, xtype and ttype must be a built-in types (GrB_BOOL, GrB_FP64, etc), or a type defined with GxB_Type_define.

9.5 GxB_Monoid_define: **define a GrB_Monoid at compile time**

```
GxB_Monoid_define(GrB_Monoid monoid, GrB_BinaryOp op, identity) ;
```

GxB_Monoid_define is identical to GrB_Monoid_new, except that it defines a monoid when SuiteSparse:GraphBLAS is compiled. The op is a built-in binary operator (GrB_PLUS_FP32, for example) or a binary operator defined by GxB_BinaryOp_define. The three types of the operator must be the same, but this cannot be checked by this method at compile time. Results are undefined if this condition does not hold.

Unlike GrB_Monoid_new, the identity parameter must be a compile-time constant expression. It must also be parsable as a valid argument to an m4 macro. For example, the following is a valid definition that appears in User/Example/my_complex.m4, It defines a plus monoid for the My_complex type, which is double complex in C.

```
GxB_Monoid_define(My_Complex_plus_monoid, My_Complex_plus, CMPLX(0,0)) ;
```

For user-defined types created from a C struct, another method must be used for the value of the identity parameter of GxB_Monoid_define. Consider the following excerpt from User/Example/my_pagerank.m4. A struct variable such as the pagerank_type can be initialized with the C expression identity = {0,0}, but the expression {0,0} cannot be passed to an m4 macro since it is interpreted by m4 as two arguments. The solution is to define a C preprocessor token, PAGERANK_ZERO, and pass that token as the third argument of GxB_Monoid_define.

```
#ifdef GxB_USER_INCLUDE
typedef struct
{
    double rank ;
    double invdegree ;
}
pagerank_type ;

// This is valid.  It defines the identity value of the monoid as a
// struct with identity.rank = 0 and identity.invdegree = 0.
#define PAGERANK_ZERO {0,0}
#endif

GxB_Monoid_define(PageRank_monoid, PageRank_add, PAGERANK_ZERO) ;
```

The following definition will fail to compile, since `m4` interprets the comma in the `identity` argument as the start of a fourth argument:

```
// This will fail:
GxB_Monoid_define(PageRank_monoid, PageRank_add, {0,0}) ;
```

9.6   GxB_Monoid_terminal_define: **define a GrB_Monoid at compile time, with a terminal value**

```
GxB_Monoid_terminal_define(monoid, op, identity, terminal) ;
```

`GxB_Monoid_terminal_define` is identical to `GxB_Monoid_define`, except that it allows the specification of a terminal value. See Section 4.5.2 for a description of the terminal value of a monoid.

9.7   GxB_Semiring_define: **define a GrB_Semiring at compile time**

```
GxB_Semiring_define(GrB_Semiring semiring, GrB_Monoid add, GrB_BinaryOp mult) ;
```

`GxB_Semiring_define` is identical to `GrB_Semiring_new`, except that it defines a semiring when SuiteSparse:GraphBLAS is compiled. The `add` parameter is a `GrB_Monoid` that is either predefined (such as `GxB_PLUS_TIMES_FP64`) or defined with `GxB_Monoid_define`. Similarly, the `mult` parameter is either a predefined binary operator (such as `GrB_TIMES_FP32`) or a binary operator defined with `GxB_BinaryOp_define`.

10 Examples

Six examples of how to use GraphBLAS are described below. They all appear in the `Demo` folder of SuiteSparse:GraphBLAS.

1. performing a breadth-first search,

2. finding a maximal independent set,

3. creating a random matrix,

4. creating a finite-element matrix,

5. reading a matrix from a file, and

6. complex numbers as a user-defined type.

Additional examples appear in the newly created LAGraph project, currently in progress. Finally, the `Extras` folder includes triangle counting and k-truss algorithms in GraphBLAS, and methods that do not GraphBLAS (both simple sequential methods, and methods that use OpenMP).

10.1 LAGraph

The LAGraph project is a community-wide effort to create graph algorithms based on GraphBLAS (any implementation of the API, not just SuiteSparse: GraphBLAS). As of July, 2019, the library includes the algorithms and utilities listed in the table below. Many additional algorithms are planned, such as betweenness centrality, PageRank, single-source shortest path (via delta stepping), minimum spanning trees, connected components, and many more. Refer to https://github.com/GraphBLAS/LAGraph for a current list of algorithms (the one below will soon be out of date). Most of the functions in the `Demo/` and the `Extras` folder in SuiteSparse:GraphBLAS will eventually be translated into algorithms or utilities for LAGraph.

To use LAGraph with SuiteSparse:GraphBLAS, place the two folders `LAGraph` and `GraphBLAS` in the same parent directory. This allows the `cmake` script in LAGraph to find the copy of GraphBLAS. Alternatively, the GraphBLAS source could be placed anywhere, as long as `sudo make install` is performed.

Build `GraphBLAS` first, then the `LAGraph` library, and then the tests in LAGraph/Test.

Algorithms	description
`LAGraph_bfs_pushpull`	a direction-optimized BFS [BAP12, YBO18], typically 2x faster than `bfs5m`
`LAGraph_bfs_simple`	a simple BFS (about the same as `bfs5m`)
`LACC_GraphBLAS.c`	connected components
`LAGraph_BF_*.c`	two variants of Bellman-Ford
`LAGraph_allktruss`	construct all k-trusses
`LAGraph_dnn`	sparse deep neural network
`LAGraph_ktruss`	construct a k-trusses
`LAGraph_lcc`	local clustering coefficient
`LAGraph_pagerank`	PageRank
`LAGraph_tricount`	triangle count

Utilities	description
`LAGraph_Vector_isall`	tests 2 vectors with a binary operator
`LAGraph_Vector_isequal`	tests if 2 vectors are equal
`LAGraph_Vector_to_dense`	converts a vector to dense
`LAGraph_alloc_global`	types, operators, monoids, and semirings
`LAGraph_finalize`	ends LAGraph
`LAGraph_free`	wrapper for `free`
`LAGraph_free_global`	frees objects created by `_alloc_global`
`LAGraph_init`	starts LAGraph
`LAGraph_isall`	tests 2 matrices with a binary operator
`LAGraph_isequal`	tests if 2 matrices are equal
`LAGraph_ispattern`	tests if all entries in a matrix are 1
`LAGraph_malloc`	wrapper for `malloc`
`LAGraph_mmread`	read a Matrix Market file
`LAGraph_mmwrite`	write a Matrix Market file
`LAGraph_pattern`	extracts the pattern of a matrix
`LAGraph_prune_diag`	diagonal entries from a matrix
`LAGraph_rand`	simple random number generator
`LAGraph_rand64`	`int64_t` random number generator
`LAGraph_random`	random matrix generator
`LAGraph_randx`	`double` random number generator
`LAGraph_tic`	start a timer
`LAGraph_toc`	end a timer
`LAGraph_tsvread`	read a TSV file
`LAGraph_xinit`	starts LAGraph, with different malloc

10.2 Breadth-first search

The `bfs` examples in the `Demo` folder provide several examples of how to compute a breadth-first search (BFS) in GraphBLAS. The `bfs5m` function starts at a given source node `s` of an undirected graph with `n` nodes. The graph is represented as an `n`-by-`n` Boolean matrix, `A`, where `A(i,j)` is the edge (i, j). The matrix `A` can actually have any type; if it is not Boolean (`bool` in C, or `GrB_BOOL` in GraphBLAS), it is typecasted to Boolean by the semiring, where zero is false and nonzero is true.

The vector `v` of size `n` holds the level of each node in the BFS, where `v(i)=0` if the node has not yet been seen. This particular value makes `v` useful for another role. It can be used as a Boolean mask, since `0` is `false` and nonzero is `true`. Initially the entire `v` vector is zero.

The vector `q` is the set of nodes just discovered at the current level, where `q(i)=true` if node `i` is in the current level. It starts out with just a single entry set to true, `q(s)`, the starting node.

Each iteration of the BFS consists of three calls to GraphBLAS. The first one uses `q` as a mask. It modifies all positions in `v` where `q` is true, setting them all to the current `level`. No accumulator or descriptor are used. Since `GrB_REPLACE` is not used and `I=GrB_ALL`, `GxB_subassign` and `GrB_assign` are identical; either can be used in this step:

```
// v<q> = level, using vector assign with q as the mask
GrB_assign (v, q, NULL, level, GrB_ALL, n, NULL) ;
```

The next call to GraphBLAS is the heart of the algorithm:

```
// q<!v> = q ||.&& A ; finds all the unvisited
// successors from current q, using !v as the mask
GrB_vxm (q, v, NULL, Boolean, q, A, desc) ;
```

The vector `q` is all the set of nodes at the current level. Suppose `q(j)` is true, and it has a neighbor `i`. Then `A(i,j)=1`, and the dot product of `A(i,:)*q` using the `OR-AND` semiring will use the `AND` multiplier on these two terms, `A(i,j) AND q(j)`, resulting in a value `true`. The `OR` monoid will "sum" up all the results in this single row `i`. If the result is a column vector `t=A*q`, then this `t(i)` will be true. The vector `t` will be true for any node adjacent to any node in the set `q`.

Some of these neighbors of the nodes in `q` have already been visited by the BFS, either in the current level or in a prior level. These results must

be discarded; what is desired is the set of all nodes `i` for which `t(i)` is true, and yet `v(i)` is still zero.

Enter the mask. The vector `v` is complemented for use a mask, via the `desc` descriptor. This means that wherever the vector is true, that position in the result is protected and will not be modified by the assignment. Only where `v` is false will the result be modified. This is exactly the desired result, since these represent newly seen nodes for the next level of the BFS. A node `k` already visited will have a nonzero `v(k)`, and thus `q(k)` will not be modified by the assignment.

The result `t` is written back into the vector `q`, through the mask, but to do this correctly, another descriptor parameter is used: `GrB_REPLACE`. The vector `q` was used to compute `t=A*q`, and after using it to compute `t`, the entire `q` vector needs to be cleared. Only new nodes are desired, for the next level. This is exactly what the `REPLACE` option does.

As a result, the vector `q` now contains the set of nodes at the new level of the BFS. It contains all those nodes (and only those nodes) that are neighbors of the prior set and that have not already been seen in any prior level.

Finally, a single call to GraphBLAS computes the `OR` for all entries in `q`, into a single scalar, `successor`. This value is true if `q` contains any value true, or false otherwise. If it is false, the BFS can terminate.

```
GrB_reduce (&successor, NULL, Lor, q, NULL) ;
```

Another method for computing the BFS is in the `bfs6` function in the `Demo` folder. It uses `GrB_apply` and a unary operator to set the levels of the newly discovered nodes, instead of `GrB_assign`.

```
GrB_Info bfs5m                   // BFS of a graph (using vector assign & reduce)
(
    GrB_Vector *v_output,    // v [i] is the BFS level of node i in the graph
    const GrB_Matrix A,      // input graph, treated as if boolean in semiring
    GrB_Index s              // starting node of the BFS
)
{

    GrB_Index n ;                               // # of nodes in the graph
    GrB_Vector q = NULL ;                       // nodes visited at each level
    GrB_Vector v = NULL ;                       // result vector
    GrB_Monoid Lor = NULL ;                     // Logical-or monoid
    GrB_Semiring Boolean = NULL ;               // Boolean semiring
    GrB_Descriptor desc = NULL ;                // Descriptor for vxm
    GrB_Matrix_nrows (&n, A) ;                  // n = # of rows of A
    GrB_Vector_new (&v, GrB_INT32, n) ;     // Vector<int32_t> v(n) = 0
    GrB_assign (v, NULL, NULL, 0, GrB_ALL, n, NULL) ;   // make v dense
    GrB_Vector_new (&q, GrB_BOOL, n) ;      // Vector<bool> q(n) = false
    GrB_Vector_setElement (q, true, s) ;    // q[s] = true, false elsewhere
    GrB_Monoid_new (&Lor, GrB_LOR, (bool) false) ;
    GrB_Semiring_new (&Boolean, Lor, GrB_LAND) ;
    GrB_Descriptor_new (&desc) ;
    GrB_Descriptor_set (desc, GrB_MASK, GrB_SCMP) ;     // invert the mask
    GrB_Descriptor_set (desc, GrB_OUTP, GrB_REPLACE) ;  // clear q first

    bool successor = true ; // true when some successor found
    for (int32_t level = 1 ; successor && level <= n ; level++)
    {
        // v<q> = level, using vector assign with q as the mask
        GrB_assign (v, q, NULL, level, GrB_ALL, n, NULL) ;
        // q<!v> = q ||.&& A ; finds all the unvisited successors from current
        // q, using !v as the mask
        GrB_vxm (q, v, NULL, Boolean, q, A, desc) ;
        // successor = ||(q)
        GrB_reduce (&successor, NULL, Lor, q, NULL) ;
    }

    GrB_Descriptor_set (desc, GrB_MASK, GxB_DEFAULT) ;  // mask not inverted
    GrB_assign (v, v, NULL, v, GrB_ALL, n, desc) ;      // make v sparse
    *v_output = v ;          // return result
    // free workspace
    GrB_free (&q) ; GrB_free (&Lor) ; GrB_free (&Boolean) ; GrB_free (&desc) ;
    return (GrB_SUCCESS) ;
}
```

10.3 Maximal independent set

The *maximal independent set* problem is to find a set of nodes S such that no two nodes in S are adjacent to each other (an independent set), and all nodes not in S are adjacent to at least one node in S (and thus S is maximal since it cannot be augmented by any node while remaining an independent set). The `mis` function in the `Demo` folder solves this problem using Luby's method [Lub86]. The key operations in the method are replicated on the next page.

The gist of the algorithm is this. In each phase, all candidate nodes are given a random score. If a node has a score higher than all its neighbors, then it is added to the independent set. All new nodes added to the set cause their neighbors to be removed from the set of candidates. The process must be repeated for multiple phases until no new nodes can be added. This is because in one phase, a node `i` might not be added because one of its neighbors `j` has a higher score, yet that neighbor `j` might not be added because one of its neighbors `k` is added to the independent set instead. The node `j` is no longer a candidate and can never be added to the independent set, but node `i` could be added to S in a subsequent phase.

The initialization step, before the `while` loop, computes the degree of each node with a `PLUS` reduction. The set of `candidates` is Boolean vector, the `i`th component is true if node `i` is a candidate. A node with no neighbors causes the algorithm to stall, so these nodes are not candidates. Instead, they are immediately added to the independent set, represented by another Boolean vector `iset`. Both steps are done with an `assign`, using the `degree` as a mask, except the assignment to `iset` uses the complement of the mask, via the `sr_desc` descriptor. Finally, the `GrB_Vector_nvals` statement counts how many candidates remain.

Each phase of Luby's algorithm consists of 11 calls to GraphBLAS operations, all of which are either parallel, or take $O(1)$ time. Not all of them are described here since they are commented in the code itself. The two matrix-vector multiplications are the important parts and also take the most time. They also make interesting use of semirings and masks. The first one computes the largest score of all the neighbors of each node in the candidate set:

```
// compute the max probability of all neighbors
GrB_vxm (neighbor_max, candidates, NULL, maxFirst, prob, A, r_desc) ;
```

```
    // compute the degree of each node
    GrB_reduce (degrees, NULL, NULL, GrB_PLUS_FP64, A, NULL) ;

    // singletons are not candidates; they are added to iset first instead
    // candidates[degree != 0] = 1
    GrB_assign (candidates, degrees, NULL, true, GrB_ALL, n, NULL);

    // add all singletons to iset
    // iset[degree == 0] = 1
    GrB_assign (iset, degrees, NULL, true, GrB_ALL, n, sr_desc) ;

    // Iterate while there are candidates to check.
    GrB_Index nvals ;
    GrB_Vector_nvals (&nvals, candidates) ;

    while (nvals > 0)
    {
        // sparsify the random number seeds (just keep it for each candidate)
        GrB_assign (Seed, candidates, NULL, Seed, GrB_ALL, n, r_desc) ;
        // compute a random probability scaled by inverse of degree
        prand_xget (X, Seed) ;  // two calls to GrB_apply
        GrB_eWiseMult (prob, candidates, NULL, set_random, degrees, X, r_desc) ;
        // compute the max probability of all neighbors
        GrB_vxm (neighbor_max, candidates, NULL, maxFirst, prob, A, r_desc) ;
        // select node if its probability is > than all its active neighbors
        GrB_eWiseAdd (new_members, NULL,NULL, GrB_GT_FP64, prob, neighbor_max,0);
        // add new members to independent set.
        GrB_eWiseAdd (iset, NULL, NULL, GrB_LOR, iset, new_members, NULL) ;
        // remove new members from set of candidates c = c & !new
        GrB_apply (candidates, new_members, NULL, GrB_IDENTITY_BOOL,
            candidates, sr_desc) ;
        GrB_Vector_nvals (&nvals, candidates) ;
        if (nvals == 0) { break ; }                  // early exit condition
        // Neighbors of new members can also be removed from candidates
        GrB_vxm (new_neighbors, candidates, NULL, Boolean,
            new_members, A, NULL) ;
        GrB_apply (candidates, new_neighbors, NULL, GrB_IDENTITY_BOOL,
            candidates, sr_desc) ;
        GrB_Vector_nvals (&nvals, candidates) ;
    }
```

A is a symmetric Boolean matrix and `prob` is a sparse real vector (of type FP32), where `prob(i)` is nonzero only if node `i` is a candidate. The `maxFirst` semiring uses `z=FIRST(x,y)` as the multiplier operator. The column `A(:,j)` is the adjacency of node `j`, and the dot product `prob'*A(:,j)` applies the

FIRST operator on all entries that appear in the intersection of `prob` and `A(:,j)`, where `z=FIRST(prob(i),A(i,j))` which is just `prob(i)` if `A(i,j)` is present. If `A(i,j)` not an explicit entry in the matrix, then this term is not computed and does not take part in the reduction by the `MAX` monoid.

Thus, each term `z=FIRST(prob(i),A(i,j))` is the score, `prob(i)`, of all neighbors `i` of node `j` that have a score. Node `i` does not have a score if it is not also a candidate and so this is skipped. These terms are then "summed" up by taking the maximum score, using `MAX` as the additive monoid.

Finally, the results of this matrix-vector multiply are written to the result, `neighbor_max`. The `r_desc` descriptor has the `REPLACE` option enabled. Since `neighbor_max` does not also take part in the computation `prob'*A`, it is simply cleared first. Next, is it modified only in those positions `i` where `candidates(i)` is true, using `candidates` as a mask. This sets the `neighbor_max` only for candidate nodes, and leaves the other components of `neighbor_max` as zero (implicit values not in the pattern of the vector).

All of the above work is done in a single matrix-vector multiply, with an elegant use of the `maxFirst` semiring coupled with a mask. The matrix-vector multiplication is described above as if it uses dot products of rows of `A` with the column vector `prob`, but SuiteSparse:GraphBLAS does not compute it that way. Sparse dot products are much slower the optimal method for multiplying a sparse matrix times a sparse vector. The result is the same, however.

The second matrix-vector multiplication is more straight-forward. Once the set of new members in the independent is found, it is used to remove all neighbors of those new members from the set of candidates.

The resulting method is very efficient. For the `Freescale2` matrix, the algorithm finds an independent set of size 1.6 million in 1.7 seconds (on the same MacBook Pro referred to in Section 10.2), taking four iterations of the `while` loop. For comparison, removing its diagonal entries (required for the algorithm to work) takes 0.3 seconds in GraphBLAS (see Section 7.14), and simply transposing the matrix takes 0.24 seconds in both MATLAB and GraphBLAS.

10.4 Creating a random matrix

The `random_matrix` function in the `Demo` folder generates a random matrix with a specified dimension and number of entries, either symmetric or unsymmetric, and with or without self-edges (diagonal entries in the matrix). It relies on `simple_rand*` functions in the `Demo` folder to provide a portable random number generator that creates the same sequence on any computer and operating system.

`random_matrix` can use one of two methods: `GrB_Matrix_setElement` and `GrB_Matrix_build`. The former method is very simple to use:

```
GrB_Matrix_new (&A, GrB_FP64, nrows, ncols) ;
for (int64_t k = 0 ; k < ntuples ; k++)
{
    GrB_Index i = simple_rand_i ( ) % nrows ;
    GrB_Index j = simple_rand_i ( ) % ncols ;
    if (no_self_edges && (i == j)) continue ;
    double x = simple_rand_x ( ) ;
    // A (i,j) = x
    GrB_Matrix_setElement (A, x, i, j) ;
    if (make_symmetric)
    {
        // A (j,i) = x
        GrB_Matrix_setElement (A, x, j, i) ;
    }
}
```

The above code can generate a million-by-million sparse `double` matrix with 200 million entries in 66 seconds (6 seconds of which is the time to generate the random i, j, and x), including the time to finish all pending computations. The user application does not need to create a list of all the tuples, nor does it need to know how many entries will appear in the matrix. It just starts from an empty matrix and adds them one at a time in arbitrary order. GraphBLAS handles the rest. This method is not feasible in MATLAB.

The next method uses `GrB_Matrix_build`. It is more complex to use than `setElement` since it requires the user application to allocate and fill the tuple lists, and it requires knowledge of how many entries will appear in the matrix, or at least a good upper bound, before the matrix is constructed. It is slightly faster, creating the same matrix in 60 seconds, 51 seconds of which is spent in `GrB_Matrix_build`.

```
GrB_Index *I, *J ;
double *X ;
int64_t s = ((make_symmetric) ? 2 : 1) * nedges + 1 ;
I = malloc (s * sizeof (GrB_Index)) ;
J = malloc (s * sizeof (GrB_Index)) ;
X = malloc (s * sizeof (double   )) ;
if (I == NULL || J == NULL || X == NULL)
{
    // out of memory
    if (I != NULL) free (I) :
    if (J != NULL) free (J) :
    if (X != NULL) free (X) :
    return (GrB_OUT_OF_MEMORY) ;
}
int64_t ntuples = 0 ;
for (int64_t k = 0 ; k < nedges ; k++)
{
    GrB_Index i = simple_rand_i ( ) % nrows ;
    GrB_Index j = simple_rand_i ( ) % ncols ;
    if (no_self_edges && (i == j)) continue ;
    double x = simple_rand_x ( ) ;
    // A (i,j) = x
    I [ntuples] = i ;
    J [ntuples] = j ;
    X [ntuples] = x ;
    ntuples++ ;
    if (make_symmetric)
    {
        // A (j,i) = x
        I [ntuples] = j ;
        J [ntuples] = i ;
        X [ntuples] = x ;
        ntuples++ ;
    }
}
GrB_Matrix_build (A, I, J, X, ntuples, GrB_SECOND_FP64) ;
```

The equivalent `sprandsym` function in MATLAB takes 150 seconds, but `sprandsym` uses a much higher-quality random number generator to create the tuples `[I,J,X]`. Considering just the time for `sparse(I,J,X,n,n)` in `sprandsym` (equivalent to `GrB_Matrix_build`), the time is 70 seconds. That is, each of these three methods, `setElement` and `build` in Suite-Sparse:GraphBLAS, and `sparse` in MATLAB, are equally fast.

10.5 Creating a finite-element matrix

Suppose a finite-element matrix is being constructed, with `k=40,000` finite-element matrices, each of size 8-by-8. The following operations (in pseudo-MATLAB notation) are very efficient in SuiteSparse:GraphBLAS.

```
A = sparse (m,n) ; % create an empty n-by-n sparse GraphBLAS matrix
for i = 1:k
    construct a 8-by-8 sparse or dense finite-element F
    I and J define where the matrix F is to be added:
    I = a list of 8 row indices
    J = a list of 8 column indices
    % using GrB_assign, with the 'plus' accum operator:
    A (I,J) = A (I,J) + F
end
```

If this were done in MATLAB or in GraphBLAS with blocking mode enabled, the computations would be extremely slow. This example is taken from Loren Shure's blog on MATLAB Central, *Loren on the Art of MAT-LAB* [Dav07], which discusses the built-in `wathen` function. In MATLAB, a far better approach is to construct a list of tuples `[I,J,X]` and to use `sparse(I,J,X,n,n)`. This is identical to creating the same list of tuples in GraphBLAS and using the `GrB_Matrix_build`, which is equally fast. The difference in time between using `sparse` or `GrB_Matrix_build`, and using submatrix assignment with blocking mode (or in MATLAB which does not have a nonblocking mode) can be extreme. For the example matrix discussed in [Dav07], using `sparse` instead of submatrix assignment in MATLAB cut the run time of `wathen` from 305 seconds down to 1.6 seconds.

In SuiteSparse:GraphBLAS, the performance of both methods is essentially identical, and roughly as fast as `sparse` in MATLAB. Inside Suite-Sparse:GraphBLAS, `GrB_assign` is doing the same thing. When performing `A(I,J)=A(I,J)+F`, if it finds that it cannot quickly insert an update into the `A` matrix, it creates a list of pending tuples to be assembled later on. When the matrix is ready for use in a subsequent GraphBLAS operation (one that normally cannot use a matrix with pending computations), the tuples are assembled all at once via `GrB_Matrix_build`.

GraphBLAS operations on other matrices have no effect on when the pending updates of a matrix are completed. Thus, any GraphBLAS method or operation can be used to construct the `F` matrix in the example above, without affecting when the pending updates to `A` are completed.

The MATLAB `wathen.m` script is part of Higham's `gallery` of matrices [Hig02]. It creates a finite-element matrix with random coefficients for a 2D mesh of size `nx`-by-`ny`, a matrix formulation by Wathen [Wat87]. The pattern of the matrix is fixed; just the values are randomized. The GraphBLAS equivalent can use either `GrB_Matrix_build`, or `GrB_assign`. Both methods have good performance. The `GrB_Matrix_build` version below is about 15% to 20% faster than the MATLAB `wathen.m` function, regardless of the problem size. It uses the identical algorithm as `wathen.m`.

```
int64_t ntriplets = nx*ny*64 ;
I = malloc (ntriplets * sizeof (int64_t)) ;
J = malloc (ntriplets * sizeof (int64_t)) ;
X = malloc (ntriplets * sizeof (double )) ;
if (I == NULL || J == NULL || X == NULL)
{
    FREE_ALL ;
    return (GrB_OUT_OF_MEMORY) ;
}
ntriplets = 0 ;
for (int j = 1 ; j <= ny ; j++)
{
    for (int i = 1 ; i <= nx ; i++)
    {
        nn [0] = 3*j*nx + 2*i + 2*j + 1 ;
        nn [1] = nn [0] - 1 ;
        nn [2] = nn [1] - 1 ;
        nn [3] = (3*j-1)*nx + 2*j + i - 1 ;
        nn [4] = 3*(j-1)*nx + 2*i + 2*j - 3 ;
        nn [5] = nn [4] + 1 ;
        nn [6] = nn [5] + 1 ;
        nn [7] = nn [3] + 1 ;
        for (int krow = 0 ; krow < 8 ; krow++) nn [krow]-- ;
        for (int krow = 0 ; krow < 8 ; krow++)
        {
            for (int kcol = 0 ; kcol < 8 ; kcol++)
            {
                I [ntriplets] = nn [krow] ;
                J [ntriplets] = nn [kcol] ;
                X [ntriplets] = em (krow,kcol) ;
                ntriplets++ ;
            }
        }
    }
}
```

```
// A = sparse (I,J,X,n,n) ;
GrB_Matrix_build (A, I, J, X, ntriplets, GrB_PLUS_FP64) ;
```

The `GrB_assign` version has the advantage of not requiring the user appli-
cation to construct the tuple list, and is almost as fast as using `GrB_Matrix_build`.
The code is more elegant than either the MATLAB `wathen.m` function or its
GraphBLAS equivalent above. Its performance is comparable with the other
two methods, but slightly slower, being about 5% slower than the MATLAB
`wathen`, and 20% slower than the GraphBLAS method above.

```
GrB_Matrix_new (&F, GrB_FP64, 8, 8) ;
for (int j = 1 ; j <= ny ; j++)
{
    for (int i = 1 ; i <= nx ; i++)
    {
        nn [0] = 3*j*nx + 2*i + 2*j + 1 ;
        nn [1] = nn [0] - 1 ;
        nn [2] = nn [1] - 1 ;
        nn [3] = (3*j-1)*nx + 2*j + i - 1 ;
        nn [4] = 3*(j-1)*nx + 2*i + 2*j - 3 ;
        nn [5] = nn [4] + 1 ;
        nn [6] = nn [5] + 1 ;
        nn [7] = nn [3] + 1 ;
        for (int krow = 0 ; krow < 8 ; krow++) nn [krow]-- ;
        for (int krow = 0 ; krow < 8 ; krow++)
        {
            for (int kcol = 0 ; kcol < 8 ; kcol++)
            {
                // F (krow,kcol) = em (krow, kcol)
                GrB_Matrix_setElement (F, em (krow,kcol), krow, kcol) ;
            }
        }
        // A (nn,nn) += F
        GrB_assign (A, NULL, GrB_PLUS_FP64, F, nn, 8, nn, 8, NULL) ;
    }
}
```

Since there is no `Mask`, and since `GrB_REPLACE` is not used, the call to
`GrB_assign` in the example above is identical to `GxB_subassign`. Either one
can be used, and their performance would be identical.

Refer to the `wathen.c` function in the `Demo` folder, which uses GraphBLAS
to implement the two methods above, and two additional ones.

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10.6   Reading a matrix from a file

NOTE: see also `LAGraph_mmread` and `LAGraph_mmwrite`, which can read and write any matrix in Matrix Market format.

The `read_matrix` function in the `Demo` reads in a triplet matrix from a file, one line per entry, and then uses `GrB_Matrix_build` to create the matrix. It creates a second copy with `GrB_Matrix_setElement`, just to test that method and compare the run times. A comparison of `build` versus `setElement` has already been discussed in Section 10.4.

The function can return the matrix as-is, which may be rectangular or unsymmetric. If an input parameter is set to make the matrix symmetric, `read_matrix` computes `A=(A+A')/2` if `A` is square (turning all directed edges into undirected ones. If `A` is rectangular, it creates a bipartite graph, which is the same as the augmented matrix, `A = [0 A ; A' 0]`. If `C` is an `n`-by-`n` matrix, then `C=(C+C')/2` can be computed as follows in GraphBLAS, (the `scale2` function divides an entry by 2):

```
GrB_Descriptor_new (&dt2) ;
GrB_Descriptor_set (dt2, GrB_INP1, GrB_TRAN) ;
GrB_Matrix_new (&A, GrB_FP64, n, n) ;
GrB_eWiseAdd (A, NULL, NULL, GrB_PLUS_FP64, C, C, dt2) ;    // A=C+C'
GrB_free (&C) ;
GrB_Matrix_new (&C, GrB_FP64, n, n) ;
GrB_UnaryOp_new (&scale2_op, scale2, GrB_FP64, GrB_FP64) ;
GrB_apply (C, NULL, NULL, scale2_op, A, NULL) ;            // C=A/2
GrB_free (&A) ;
GrB_free (&scale2_op) ;
```

This is of course not nearly as elegant as `A=(A+A')/2` in MATLAB, but with minor changes it can work on any type and use any built-in operators instead of `PLUS`, or it can use any user-defined operators and types. The above code in SuiteSparse:GraphBLAS takes 0.60 seconds for the `Freescale2` matrix, slightly slower than MATLAB (0.55 seconds).

Constructing the augmented system is more complicated using the Graph-BLAS C API Specification since it does not yet have a simple way of specifying a range of row and column indices, as in `A(10:20,30:50)` in MATLAB (`GxB_RANGE` is a SuiteSparse:GraphBLAS extension that is not in the Specification). Using the C API in the Specification, the application must instead build a list of indices first, `I=[10, 11 ... 20]`.

Thus, to compute the MATLAB equivalent of `A = [0 A ; A' 0]`, index lists `I` and `J` must first be constructed:

```
int64_t n = nrows + ncols ;
I = malloc (nrows * sizeof (int64_t)) ;
J = malloc (ncols * sizeof (int64_t)) ;
// I = 0:nrows-1
// J = nrows:n-1
if (I == NULL || J == NULL)
{
    if (I != NULL) free (I) ;
    if (J != NULL) free (J) ;
    return (GrB_OUT_OF_MEMORY) ;
}
for (int64_t k = 0 ; k < nrows ; k++) I [k] = k ;
for (int64_t k = 0 ; k < ncols ; k++) J [k] = k + nrows ;
```

Once the index lists are generated, however, the resulting GraphBLAS operations are fairly straightforward, computing `A=[0 C ; C' 0]`.

```
GrB_Descriptor_new (&dt1) ;
GrB_Descriptor_set (dt1, GrB_INP0, GrB_TRAN) ;
GrB_Matrix_new (&A, GrB_FP64, n, n) ;
// A (nrows:n-1, 0:nrows-1) = C'
GrB_assign (A, NULL, NULL, C, J, ncols, I, nrows, dt1) ;
// A (0:nrows-1, nrows:n-1) = C
GrB_assign (A, NULL, NULL, C, I, nrows, J, ncols, NULL) ;
```

This takes 1.38 seconds for the `Freescale2` matrix, almost as fast as `A=[sparse(m,m) C ; C' sparse(n,n)]` in MATLAB (1.25 seconds).

Both calls to `GrB_assign` use no accumulator, so the second one causes the partial matrix `A=[0 0 ; C' 0]` to be built first, followed by the final build of `A=[0 C ; C' 0]`. A better method, but not an obvious one, is to use the `GrB_FIRST_FP64` accumulator for both assignments. An accumulator enables SuiteSparse:GraphBLAS to determine that that entries created by the first assignment cannot be deleted by the second, and thus it need not force completion of the pending updates prior to the second assignment.

SuiteSparse:GraphBLAS also adds a `GxB_RANGE` mechanism that mimics the MATLAB colon notation. This speeds up the method and simplifies the code the user needs to write to compute `A=[0 C ; C' 0]`:

```
int64_t n = nrows + ncols ;
GrB_Matrix_new (&A, xtype, n, n) ;
GrB_Index I_range [3], J_range [3] ;
I_range [GxB_BEGIN] = 0 ;
I_range [GxB_END  ] = nrows-1 ;
```

```
J_range [GxB_BEGIN] = nrows ;
J_range [GxB_END  ] = ncols+nrows-1 ;
// A (nrows:n-1, 0:nrows-1) += C'
GrB_assign (A, NULL, GrB_FIRST_FP64, // or NULL,
    C, J_range, GxB_RANGE, I_range, GxB_RANGE, dt1) ;
// A (0:nrows-1, nrows:n-1) += C
GrB_assign (A, NULL, GrB_FIRST_FP64, // or NULL,
    C, I_range, GxB_RANGE, J_range, GxB_RANGE, NULL) ;
```

Any operator will suffice because it is not actually applied. An operator is only applied to the set intersection, and the two assignments do not overlap. If an `accum` operator is used, only the final matrix is built, and the time in GraphBLAS drops slightly to 1.25 seconds. This is a very small improvement because in this particular case, SuiteSparse:GraphBLAS is able to detect that no sorting is required for the first build, and the second one is a simple concatenation. In general, however, allowing GraphBLAS to postpone pending updates can lead to significant reductions in run time.

10.7   PageRank

The `Demo` folder contains three methods for computing the PageRank of the nodes of a graph. One uses floating-point arithmetic (`GrB_FP64`) and two user-defined unary operators (`dpagerank.c`). The second (`ipagerank.c`) is very similar, relying on integer arithmetic instead (`GrB_UINT64`). Neither method include a stopping condition. They simply compute a fixed number of iterations. The third example is more extensive (`dpagerank2.c`), and serves as an example of the power and flexibility of user-defined types, operators, monoids, and semirings. It creates a semiring for the entire PageRank computation. It terminates if the 2-norm of the change in the rank vector `r` is below a threshold.

If `my_pagerank.m4` is placed in `GraphBLAS/User/`, then the user objects are created at compile-time instead of at run-time, which leads to faster execution.

10.8 Triangle counting

A triangle in an undirected graph is a clique of size three: three nodes i, j, and k that are all pairwise connected. There are many ways of counting the number of triangles in a graph. Let `A` be a symmetric matrix with values 0 and 1, and no diagonal entries; this matrix is the adjacency matrix of the graph. Let `E` be the edge incidence matrix with exactly two 1's per column. A column of `E` with entries in rows `i` and `j` represents the edge (i, j) in the graph, `A(i,j)=1` where `i<j`. Let `L` and `U` be the strictly lower and upper triangular parts of `A`, respectively.

The methods are listed in the table below. Most of them use a form of masked matrix-matrix multiplication. The methods are implemented in MATLAB in the `tricount.m` file, and in GraphBLAS in the `tricount.c` file, both in the `GraphBLAS/Demo` folder. Refer to the comments in those two files for details and derivations on how these methods work.

When the matrix is stored by row, and a mask is present and not complemented, `GrB_INP1` is `GrB_TRAN`, and `GrB_INP0` is `GxB_DEFAULT`, the Suite-Sparse:GraphBLAS implementation of `GrB_mxm` always uses a dot-product formulation. Thus, the $\mathbf{C}\langle\mathbf{L}\rangle = \mathbf{LU}^\mathsf{T}$ method uses dot products. This provides a mechanism for the end-user to select a masked dot product matrix multiplication method in SuiteSparse:GraphBLAS, which is occasionally faster than the outer product method. The MATLAB form assumes the matrices are stored by column (the only option in MATLAB).

Each method is followed by a reduction to a scalar, via `GrB_reduce` in GraphBLAS or by `nnz` or `sum(sum(...))` in MATLAB.

method and citation	in MATLAB	in GraphBLAS
minitri [WBS15]	`nnz(A*E==2)/3`	$\mathbf{C} = \mathbf{AE}$, then `GrB_apply`
Burkhardt [Bur16]	`sum(sum((A^2).*A))/6`	$\mathbf{C}\langle\mathbf{A}\rangle = \mathbf{A}^2$
Cohen [ABG15, Coh09]	`sum(sum((L*U).*A))/2`	$\mathbf{C}\langle\mathbf{A}\rangle = \mathbf{LU}$
Sandia [WDB$^+$17]	`sum(sum((U*U).*U))`	$\mathbf{C}\langle\mathbf{L}\rangle = \mathbf{LL}$ (outer product)
SandiaDot	`sum(sum((U'*L).*L))`	$\mathbf{C}\langle\mathbf{U}\rangle = \mathbf{LU}^\mathsf{T}$ (dot product)
Sandia2	`sum(sum((L*L).*L))`	$\mathbf{C}\langle\mathbf{U}\rangle = \mathbf{UU}$ (outer product)

In general, the Sandia methods are the fastest of the 6 methods when implemented in GraphBLAS. For full details on the triangle counting and k-truss algorithms, and performance results, see [Dav18], a copy of which appears in the `SuiteSparse/GraphBLAS/Doc` folder. The code appears in `Extras`. That paper uses an earlier version of SuiteSparse:GraphBLAS in which all matrices are stored by column.

10.9 User-defined types and operators: double complex and struct-based

The `Demo` folder contains two working examples of user-defined types, first discussed in Section 4.1.1: `double complex`, and a user-defined `typedef` called `wildtype` with a `struct` containing a string and a 4-by-4 `float` matrix.

Double Complex: GraphBLAS does not have a native complex type, but this can be easily added as a user-defined type. The `Complex_init` function in the `usercomplex.c` file in the `Demo` folder creates the `Complex` type based on the ANSI C11 `double complex` type.

```
GrB_Type_new (&Complex, sizeof (double complex)) ;
```

Next, it creates a full suite of operators that correspond to every built-in GraphBLAS operator, both binary and unary. In addition, it creates the operators listed in the following table, where D is `double` and C is `Complex`.

name	types	MATLAB equivalent	description
`Complex_complex`	$D \times D \to C$	`z=complex(x,y)`	complex from real and imag.
`Complex_conj`	$C \to C$	`z=conj(x)`	complex conjugate
`Complex_real`	$C \to D$	`z=real(x)`	real part
`Complex_imag`	$C \to D$	`z=imag(x)`	imaginary part
`Complex_angle`	$C \to D$	`z=angle(x)`	phase angle
`Complex_complex_real`	$D \to C$	`z=complex(x,0)`	real to complex real
`Complex_complex_imag`	$D \to C$	`z=complex(0,x)`	real to complex imag.

The `Complex_init` function creates two monoids (`Complex_add_monoid` and `Complex_times_monoid`) and a semiring `Complex_plus_times` that corresponds to the conventional linear algebra for complex matrices. The include file `usercomplex.h` in the `Demo` folder is available so that this user-defined `Complex` type can easily be imported into any other user application. When the user application is done, the `Complex_finalize` function frees the `Complex` type and its operators, monoids, and semiring.

Struct-based: In addition, the `wildtype.c` program creates a user-defined `typedef` of a `struct` containing a dense 4-by-4 `float` matrix, and a 64-character string. It constructs an additive monoid that adds two 4-by-4 dense matrices, and a multiplier operator that multiplies two 4-by-4 matrices. Each of these 4-by-4 matrices is treated by GraphBLAS as a "scalar" value, and they can be manipulated in the same way any other GraphBLAS type can be manipulated. The purpose of this type is illustrate the endless possibilities of user-defined types and their use in GraphBLAS.

10.10   User applications using OpenMP or POSIX pthreads

Two example demo programs are included that illustrate how a multi-threaded user application can use GraphBLAS: `openmp_demo` uses OpenMP for its user threads and `pthread_demo` uses POSIX pthreads. **To be thread-safe, SuiteSparse:GraphBLAS must be compiled with a threading library, either OpenMP or POSIX**. Either option used inside GraphBLAS can typically be combined with any user threading model. See Section 11.

The `openmp_demo` can be compiled without OpenMP, in which case it becomes single-threaded. GraphBLAS can be compiled with OpenMP, POSIX pthreads, or no threading support (and is not thread-safe in this latter case). This gives 9 different combinations:

User applic.	GraphBLAS	`Demo/Output` file	comments
none	none	`user_none_gb_none.out`	OK
none	OpenMP	`user_none_gb_openmp.out`	OK
none	pthread	`user_none_gb_pthread.out`	OK
OpenMP	none	`user_openmp_gb_none.out`	fail
OpenMP	OpenMP	`user_openmp_gb_openmp.out`	OK, random
OpenMP	pthread	`user_openmp_gb_pthread.out`	OK, random
pthread	none	`user_pthread_gb_none.out`	fail
pthread	OpenMP	`user_pthread_gb_openmp.out`	OK, random
pthread	pthread	`user_pthread_gb_pthread.out`	OK, random

When the user application is multithreaded, GraphBLAS must be compiled with a threading library to be thread-safe. The results listed above as *OK, random* mean that the output of the program will appear out of order. This is by design, simply to show that the user application is running in parallel. The output of each thread should be the same. In particular, each thread generates an intentional error, and later on prints it with `GrB_error`. It will print its own error, not an error from another thread. When all the threads finish, the master thread prints out each matrix generated by each thread, and these results are identical for all 7 cases listed above as OK.

The GraphBLAS C API requires GraphBLAS to be thread-safe. If Suite-Sparse:GraphBLAS is not compiled with a threading library it will not be thread-safe (the two *fail* cases above). For these cases, a thread will not retrieve its own error, but the last error of any thread. In addition, since there is no critical section that SuiteSparse:GraphBLAS can use, the output will include errors about an invalid state of the global matrix queue. These errors are to be expected if SuiteSparse:GraphBLAS is not thread-safe.

11 Compiling and Installing SuiteSparse:GraphBLAS

11.1 Required compiler

GraphBLAS makes extensive use of features in the ANSI C11 standard, and thus a C compiler supporting this version of the C standard is required. On the Mac (OS X), `clang` 8.0.0 in `Xcode` version 8.2.1 is sufficient, although earlier versions of `Xcode` may work as well. For the GNU `gcc` compiler, version 4.9 or later is required. For the Intel `icc` compiler, version 18.0 or later is required. Version 2.8.12 or later of `cmake` is required; version 3.0.0 is preferred.

To compile SuiteSparse:GraphBLAS and the demo programs, simply type `make` in the main GraphBLAS folder, which compiles the library. To use a non-default compiler:

```
make CC=icc CXX=icc JOBS=4
```

After compiling the library, you can run the demos by typing `./demo` in the Demo folder.

If `cmake` or `make` fail, it might be that your default compiler does not support ANSI C11. Try another compiler. For example, try one of these options. Go into the `build` directory and type one of these:

```
CC=gcc cmake ..
CC=gcc-6 cmake ..
CC=xlc cmake ..
CC=icc cmake ..
```

You can also do the following in the top-level GraphBLAS folder instead:

```
CC=gcc make
CC=gcc-6 cmake
CC=xlc cmake
CC=icc cmake
```

For faster compilation, you can specify a parallel make. For example, to use 32 parallel jobs and the `gcc` compiler, do the following:

```
JOBS=32 CC=gcc make
```

11.2 Creating the Doxygen documentation

SuiteSparse:GraphBLAS is equipped with a `Doxyfile` that specifies how `doxygen` can be used to create detailed documentation of each file, including call-graphs. The `doxygen` documentation is not meant for end-users of GraphBLAS, but for GraphBLAS developers instead.

If you edit GraphBLAS, you must first run the MATLAB script `dox_headers.m` in `Doc/`, in MATLAB. This script parses all of the source code for structured comments, to create header files suitable for parsing by `doxygen`. Next, do the following in the top-level `GraphBLAS` folder:

```
make dox
```

This creates the `Doc/html/` folder, which is initially empty in the default distribution. Open the file `Doc/html/index.html` in any web browser. Alternatively, a copy of the `doxygen` documentation can be found at http://faculty.cse.tamu.edu/davis/GraphBLAS/html.

11.3 Thread-safety in multithreaded user applications

SuiteSparse:GraphBLAS is not yet parallel, but it is thread-safe if multiple simultaneous calls are made to GraphBLAS functions. The output variables of those calls to GraphBLAS must be unique; you cannot safely modify one GraphBLAS object in parallel, with two or more simultaneous GraphBLAS functions operating on the same output object. In addition, all pending operations of objects that appear in parallel calls to GraphBLAS must be complete. This can be done for all objects via `GrB_wait`, or it can be done by calling a method or operation that forces completion of a particular object (such as `GrB_*_nvals`). If multiple parallel calls to GraphBLAS functions operate on unique inputs, then those input objects can safely have pending operations.

To use GraphBLAS from a multithreaded user application, GraphBLAS requires access to a critical section for the `GrB_wait` queue of matrices with pending operations, and to a thread-local storage space so that each user thread can safely retrieve its own error message with `GrB_error`.

SuiteSparse:GraphBLAS supports the following user threading models. By default, the `cmake` script detects the presence of OpenMP and POSIX pthreads. If OpenMP is present, it uses OpenMP critical sections for `GrB_wait` and OpenMP `threadprivate(...)` for thread-local storage for `GrB_error`.

Otherwise, if POSIX pthreads are available, it uses a POSIX `mutex`, and POSIX thread-local storage via `pthread_key_create`.

These methods used inside GraphBLAS can typically inter-operate with any user threading model. That is, a user application that relies on POSIX threads, OpenMP, ANSI C11 threads, or Microsoft Windows threads will find GraphBLAS thread-safe, even though GraphBLAS uses OpenMP or POSIX internally to synchronize the user threads. However, for the most reliable results, the preferred approach is to use the same threading model in GraphBLAS as is used in the user application.

You can modify the automatic selection of a user thread synchronization model by adding the following settings for `cmake`. This setting does not determine how SuiteSparse:GraphBLAS creates and exploits multiple threads *inside* any given GraphBLAS operation. Rather, it determines which threading library it will use to synchronize multiple calls to GraphBLAS from more than one user thread.

- OpenMP: this is the default if your compiler supports OpenMP. It can also be specified with `cmake -DUSER_OPENMP=1` in the `cmake` command line. Internal parallelism in a future SuiteSparse:GraphBLAS version will be based on OpenMP, but this is typically safe to use with any user threading models.

- POSIX: this is used if OpenMP is not available. If OpenMP is available but you still want GraphBLAS to use POSIX synchronization, compile with `cmake -DUSER_POSIX=1`

- no user threading: compile with `cmake -DUSER_NONE=1`. **GraphBLAS will not be thread-safe**.

The following user-threading models are not yet supported, but may be in a future version.

- Microsoft Windows: `cmake -DUSER_WINDOWS=1`

- ANSI C11 threads: `cmake -DUSER_ANSI=1`

11.4   Default matrix format

By default, SuiteSparse:GraphBLAS stores its matrices by row, using the `GxB_BY_ROW` format. You can change the default at compile time to `GxB_BY_COL` using `cmake -DBYCOL=1`. For example:

```
cmake -DBYCOL=1 ..
```

The user application can also use `GxB_get` and `GxB_set` to set and query the global option (see also Sections 5.7 and 5.8):

```
GxB_Format_Value s ;
GxB_get (GxB_FORMAT, &s) ;
if (s == GxB_BY_COL) printf ("all new matrices are stored by column\n") :
else printf ("all new matrices are stored by row\n") ;
```

11.5   Setting the C flags and using CMake

The above options can also be combined. For example, to use the `gcc` compiler, to change the default format `GxB_FORMAT_DEFAULT` to `GxB_BY_COL`, and to use a POSIX mutex inside GraphBLAS to synchronize user threads, use the following `cmake` command while in the `GraphBLAS/build` directory:

```
CC=gcc cmake -DBYCOL=1 -DUSER_POSIX=1 ..
```

Then do `make` in the `build` directory. If this still fails, see the `CMakeLists.txt` file. You can edit that file to pass compiler-specific options to your compiler. Locate this section in the `CMakeLists.txt` file. Use the `set` command in `cmake`, as in the example below, to set the compiler flags you need.

```
# check which compiler is being used.  If you need to make
# compiler-specific modifications, here is the place to do it.
if ("${CMAKE_C_COMPILER_ID}" STREQUAL "GNU")
    # cmake 2.8 workaround: gcc needs to be told to do ANSI C11.
    # cmake 3.0 doesn't have this problem.
    set ( CMAKE_C_FLAGS  "${CMAKE_C_FLAGS} -std=c11 -lm " )
    ...
elseif ("${CMAKE_C_COMPILER_ID}" STREQUAL "Intel")
    ...
elseif ("${CMAKE_C_COMPILER_ID}" STREQUAL "Clang")
    ...
elseif ("${CMAKE_C_COMPILER_ID}" STREQUAL "MSVC")
    ...
endif ( )
```

To compile SuiteSparse:GraphBLAS without running the demos, use `make library` in the top-level directory, or `make` in the `build` directory.

Several compile-time options can be selected by editing the `Source/GB.h` file, but these are meant only for code development of SuiteSparse:GraphBLAS itself, not for end-users of SuiteSparse:GraphBLAS.

11.6   Using a plain makefile

The `GraphBLAS/alternative` directory contains a simple `Makefile` that can be used to compile SuiteSparse:GraphBLAS. This is a useful option if you do not have the required version of `cmake`.

11.7   Running the Demos

By default, `make` in the top-level directory compiles the library and runs the demos. You can also run the demos after compiling:

```
cd Demo
./demo
```

The `./demo` command is a script that runs the demos with various input matrices in the `Demo/Matrix` folder. The output of the demos will be compared with expected output files in `Demo/Output`.

11.8   Installing SuiteSparse:GraphBLAS

To install the library (typically in `/usr/local/lib` and `/usr/local/include` for Linux systems), go to the top-level GraphBLAS folder and type:

```
sudo make install
```

11.9   Running the tests

To perform the extensive tests in the `Test` folder, and the statement coverage tests in `Tcov`, MATLAB R2017A is required. See the `README.txt` files in those two folders for instructions on how to run the tests.

11.10 Cleaning up

To remove all compiled files, type `make distclean` in the top-level Graph-BLAS folder.

12 Acknowledgments

I would like to thank Jeremy Kepner (MIT Lincoln Laboratory Supercomputing Center), and the GraphBLAS API Committee: Aydın Buluç (Lawrence Berkeley National Laboratory), Timothy G. Mattson (Intel Corporation) Scott McMillan (Software Engineering Institute at Carnegie Mellon University), José Moreira (IBM Corporation), and Carl Yang (UC Davis), for creating the GraphBLAS specification and for patiently answering my many questions while I was implementing it.

I would like to thank Tim Mattson and Henry Gabb, Intel, Inc., for their collaboration and for the support of Intel.

I would like to thank John Gilbert (UC Santa Barbara) for our many discussions on GraphBLAS, and for our decades-long conversation and collaboration on sparse matrix computations, and sparse matrices in MATLAB in particular.

I would like to thank Cleve Moler (MathWorks) for our many discussions on MATLAB, and for creating MATLAB in the first place. Without MATLAB, SuiteSparse:GraphBLAS would have been impossible to implement and test.

I would like to thank Sébastien Villemot (Debian Developer, http://sebastien.villemot.name) for helping me with various build issues and other code issues with GraphBLAS (and all of SuiteSparse) for its packaging in Debian Linux.

I would like to thank Roi Lipman, Redis Labs (https://redislabs.com), for our many discussions on GraphBLAS and its use in RedisGraph (https://redislabs.com/redis-enterprise/technology/redisgraph/), a graph database module for Redis. Based on SuiteSparse:GraphBLAS, RedisGraph is up 600x faster than the fastest graph databases (https://youtu.be/9h3Qco_x0QE https://redislabs.com/blog/new-redisgraph-1-0-achieves-600x-faster-performance-graph-databases/).

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> Provides a basic overview of many sparse matrix algorithms and a simple sparse matrix data structure. The sparse data structure used in the book is much like the

one in both MATLAB and SuiteSparse:GraphBLAS. A series of 42 lectures are available on YouTube; see the link at http://faculty.cse.tamu.edu/davis/publications.html For the book, see https://dx.doi.org/10.1137/1.9780898718881

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Abstract: Wilkinson defined a sparse matrix as one with enough zeros that it pays to take advantage of them. This informal yet practical definition captures the essence of the goal of direct methods for solving sparse matrix problems. They exploit the sparsity of a matrix to solve problems economically: much faster and using far less memory than if all the entries of a matrix were stored and took part in explicit computations. These methods form the backbone of a wide range of problems in computational science. A glimpse of the breadth of applications relying on sparse solvers can be seen in the origins of matrices in published matrix benchmark collections (Duff and Reid 1979a, Duff, Grimes and Lewis 1989a, Davis and Hu 2011). The goal of this survey article is to impart a working knowledge of the underlying theory and practice of sparse direct methods for solving linear systems and least-squares problems, and to provide an overview of the algorithms, data structures, and software available to solve these problems, so that the reader can both

understand the methods and know how best to use them. DOI: https://dx.doi.org/10.1017/S0962492916000076

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From the preface: Graphs are among the most important abstract data types in computer science, and the algorithms that operate on them are critical to modern life. Graphs have been shown to be powerful tools for modeling complex problems because of their simplicity and generality. Graph algorithms are one of the pillars of mathematics, informing research in such diverse areas as combinatorial optimization, complexity theory, and topology. Algorithms on graphs are applied in many ways in today's worldfrom Web rankings to metabolic networks, from finite element meshes to semantic graphs. The current exponential growth in graph data has forced a shift to parallel computing for executing graph algorithms. Implementing parallel graph algorithms and achieving good parallel performance have proven difficult. This book addresses these challenges by exploiting the well-known duality between a canonical representation of graphs as abstract collections of vertices and edges and a sparse adjacency matrix representation. This linear algebraic approach is widely accessible to scientists and engineers who may not be formally trained in computer science. The authors show how to leverage existing parallel matrix computation techniques and the large amount of software infrastructure that exists for these computations to implement efficient and scalable parallel graph algorithms. The benefits of this approach are reduced algorithmic complexity, ease of implementation, and improved performance. DOI: https://dx.doi.org/10.1137/1.9780898719918

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             > important graph algorithms in network science. In this paper,
             > we address the IEEE HPEC Static Graph Challenge problem
             > of triangle counting, focusing on obtaining the best parallel
             > performance on a single multicore node. Our implementation
             > uses a linear algebra-based approach to triangle counting that
             > has grown out of work related to our miniTri data analyt-
             > ics miniapplication and our efforts to pose graph algorithms
             > in the language of linear algebra. We leverage KokkosKernels
             > to implement this approach efficiently on multicore architec-
             > tures. Our performance results are competitive with the fastest
             > known graph traversal-based approaches and are significantly
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