Backpropagation of Syntax Errors in Context-Sensitive Languages

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Main Idea

- $GF(2^n)$ matrices are useful structures for studying finite state machines
- The operators $\{\text{XOR}, \wedge, \top\}$ are *functionally complete* logical primitives
- We use them to implement probabilistic context-sensitive program repair

Algebraic Parsing

Given a CFG, $\mathcal{G}' : \langle \Sigma, V, P, S \rangle$ in Chomsky Normal Form (CNF), we can define a *recognizer*, $R : \mathcal{G}' \to \Sigma^n \to \mathbb{B}$ for bounded strings $\sigma : \Sigma^n$ using the following construction. Let 2^V be our domain, 0 be $\varnothing$, $\oplus$ be $\cup$, and $\otimes$:

$$x \otimes y := \{\, W \mid \langle X, Y \rangle \in x \times y, (W \to XY) \in P \,\}$$

Valiant (1975) shows that if we let $\sigma_r^{\uparrow} := \{V \mid (V \to \sigma_r^{\uparrow}) \in P\}$, initialize the matrix $M^0_{r+1=c}(\mathcal{G}', e) := \ \sigma_r^{\uparrow}$ and solve for its fixpoint $M^* = M + M^2$,

$$M^0 := \begin{pmatrix} \varnothing & \sigma_1^{\uparrow} & \varnothing & \cdots & \varnothing \\ & & \ddots & \ddots & \vdots \\ & & & & \varnothing \\ & & & & \sigma_n^{\uparrow} \\ \varnothing & & \cdots & & \varnothing \end{pmatrix} \Rightarrow M^* = \begin{pmatrix} \varnothing & \sigma_1^{\uparrow} & \cdots & \mathcal{T} \\ & \ddots & \ddots & \vdots \\ & & & \sigma_n^{\uparrow} \\ \varnothing & \cdots & & \varnothing \end{pmatrix}$$

the recognizer is then defined as: $R(\mathcal{G}', \sigma) := S \in \mathcal{T}? \iff \sigma \in \mathcal{L}(\mathcal{G})?$

Galois Connection

- CYK parser can be lowered onto a tensor $\mathbb{Z}_2^{n \times n \times |V|}$ and $GF(2^{|V|})^{n \times n}$
- Binarized CYK parser can be compiled to SAT to solve for $\mathbf{M}^*$ directly
- Enables sketch-based synthesis in σ or $\mathcal{G}$: just use variables for holes!
- We simply encode the characteristic function, i.e. $1_{\subseteq V} : V \to \mathbb{B}^{|V|}$
- $\oplus, \otimes$ are defined as $\boxplus, \boxtimes$, so that the following diagram commutes:

$$\begin{tikzcd}
\langle \mathcal{G}', \Sigma^{n-1} \rangle \arrow[drr, leftrightarrow] & \textbf{Set} & \textbf{Bit} & \langle \mathcal{G}', \Sigma^{n-1} \rangle \arrow[dr] & \textbf{SAT} \\
\textbf{Rubix} & M \times M \arrow[r, leftrightarrow, "1^{2^{n\times n}}", "1^{-2^{n\times n}}"'] \arrow[d, "\oplus \mid \otimes"] & \mathbb{Z}_2^{|V|^{n\times n}} \times \mathbb{Z}_2^{|V|^{n\times n}} \arrow[rr, tail, "\varphi^{2^{n\times n}}", "\varphi^{-2^{n\times n}}"'] \arrow[d, "\boxplus \mid \boxtimes"] & & \mathcal{M} \times \mathcal{M} \arrow[d, leftrightarrow, "+ \mid *"] \\
\textbf{Matrix} & 2^V \times 2^V \arrow[r, leftrightarrow, "1^2", "1^{-2}"'] \arrow[d, "\oplus \mid \otimes"] & \mathbb{Z}_2^{|V|} \times \mathbb{Z}_2^{|V|} \arrow[rr, tail, "\varphi^2", "\varphi^{-2}"'] \arrow[d, "\boxplus \mid \boxtimes"] & & \mathcal{V} \times \mathcal{V} \arrow[d, leftrightarrow, "\boxplus \mid \boxtimes"] \\
\textbf{Vector} & 2^V \arrow[r, leftrightarrow, "1", "1^{-1}"'] & \mathbb{Z}_2^{|V|} \arrow[rr, tail, "\varphi", "\varphi^{-1}"'] & & \mathcal{V}
\end{tikzcd}$$

Probabilistic Programming

Let $\mathbf{R}$: GF$(2^{n \times n})$ be a matrix $\mathbf{M}_{0,c} = P_c + \mathbf{M}_{r+1=c} = \top$, where P is a feedback polynomial with coefficients $P_{1\ldots n}$ and $\oplus := \veebar, \otimes := \wedge$:

$$\mathbf{M}^t V = \begin{pmatrix} P_1 & \cdots & \cdots & P_n \\ \top & \circ & \cdots & \circ \\ \circ & \ddots & \ddots & \vdots \\ \circ & \cdots & \top & \circ \end{pmatrix}^t \begin{pmatrix} V_1 \\ \vdots \\ V_n \end{pmatrix}$$

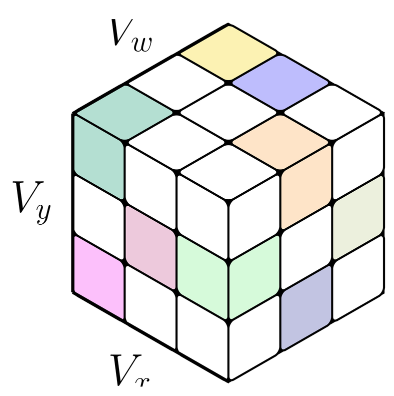
Selecting any $V \neq \mathbf{0}$ and coefficients P_j from a *primitive polynomial*, then powering the matrix $\mathbf{M}$ generates an ergodic sequence over GF(2^n):

$$\mathbf{S} = \left(V \; \mathbf{M}V \; \mathbf{M}^2 V \; \mathbf{M}^3 V \cdots \mathbf{M}^{2^n - 1} V \right)$$

This sequence has *full periodicity*, i.e., $\forall i, j \in [0, 2^n), \mathbf{S}_i = \mathbf{S}_j \Rightarrow i = j$.

Brozozowski's Derivative

$$\begin{aligned} o &\to so \mid rs \mid rr \mid oo \\ r &\to so \mid ss \mid rr \mid os \\ s &\to so \mid rs \mid or \mid oo \end{aligned} \qquad \mathcal{H}_{\{o\}} = \begin{pmatrix} \frac{\partial^2 o}{\partial \overleftarrow{o} \partial \overrightarrow{o}} & \frac{\partial^2 o}{\partial \overleftarrow{o} \partial \overrightarrow{r}} & \frac{\partial^2 o}{\partial \overleftarrow{o} \partial \overrightarrow{s}} \\ \frac{\partial^2 o}{\partial \overleftarrow{r} \partial \overrightarrow{o}} & \frac{\partial^2 o}{\partial \overleftarrow{r} \partial \overrightarrow{r}} & \frac{\partial^2 o}{\partial \overleftarrow{r} \partial \overrightarrow{s}} \\ \frac{\partial^2 o}{\partial \overleftarrow{s} \partial \overrightarrow{o}} & \frac{\partial^2 o}{\partial \overleftarrow{s} \partial \overrightarrow{r}} & \frac{\partial^2 o}{\partial \overleftarrow{s} \partial \overrightarrow{s}} \end{pmatrix}$$



$$\mathcal{H}_{\{r\}} = \begin{pmatrix} \frac{\partial^2 r}{\partial \overleftarrow{o} \partial \overrightarrow{o}} & \frac{\partial^2 r}{\partial \overleftarrow{o} \partial \overrightarrow{r}} & \frac{\partial^2 r}{\partial \overleftarrow{o} \partial \overrightarrow{s}} \\ \frac{\partial^2 r}{\partial \overleftarrow{r} \partial \overrightarrow{o}} & \frac{\partial^2 r}{\partial \overleftarrow{r} \partial \overrightarrow{r}} & \frac{\partial^2 r}{\partial \overleftarrow{r} \partial \overrightarrow{s}} \\ \frac{\partial^2 r}{\partial \overleftarrow{s} \partial \overrightarrow{o}} & \frac{\partial^2 r}{\partial \overleftarrow{s} \partial \overrightarrow{r}} & \frac{\partial^2 r}{\partial \overleftarrow{s} \partial \overrightarrow{s}} \end{pmatrix}$$

$$\mathcal{H}_{\{s\}} = \begin{pmatrix} \frac{\partial^2 s}{\partial \overleftarrow{o} \partial \overrightarrow{o}} & \frac{\partial^2 s}{\partial \overleftarrow{o} \partial \overrightarrow{r}} & \frac{\partial^2 s}{\partial \overleftarrow{o} \partial \overrightarrow{s}} \\ \frac{\partial^2 s}{\partial \overleftarrow{r} \partial \overrightarrow{o}} & \frac{\partial^2 s}{\partial \overleftarrow{r} \partial \overrightarrow{r}} & \frac{\partial^2 s}{\partial \overleftarrow{r} \partial \overrightarrow{s}} \\ \frac{\partial^2 s}{\partial \overleftarrow{s} \partial \overrightarrow{o}} & \frac{\partial^2 s}{\partial \overleftarrow{s} \partial \overrightarrow{r}} & \frac{\partial^2 s}{\partial \overleftarrow{s} \partial \overrightarrow{s}} \end{pmatrix}$$

It is well-known that the family of CFLs is not closed under intersection. For example, consider $\mathcal{L}_\cap := \mathcal{L}(\mathcal{G}_1) \cap \mathcal{L}(\mathcal{G}_1)$:
$\mathcal{L}_\cap$ generates the language $\{\, a^d b^d c^d \mid d > 0 \,\}$, which is not context free.

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