

Backpropagation of Syntax Errors in Context-Sensitive Languages

Breandan Considine, Jin Guo, Xujie Si

Main Idea

- Matrices over \mathbb{Z}_2^n are useful structures for studying finite state machines
- The operators $\{\text{XOR}, \wedge, \top\}$ are *functionally complete* logical primitives
- We use them to implement probabilistic context-sensitive program repair

Algebraic Parsing

Given a CFG, $\mathcal{G}' : \langle \Sigma, V, P, S \rangle$ in Chomsky Normal Form (CNF), we can define a *recognizer*, $R : \mathcal{G}' \rightarrow \Sigma^n \rightarrow \mathbb{B}$ for bounded strings $\sigma : \Sigma^n$ using the following construction. Let 2^V be our domain, 0 be \emptyset , \oplus be \cup , and \otimes :

$$X \otimes Y := \{ w \mid \langle x, y \rangle \in X \times Y, (w \rightarrow xy) \in P \}$$

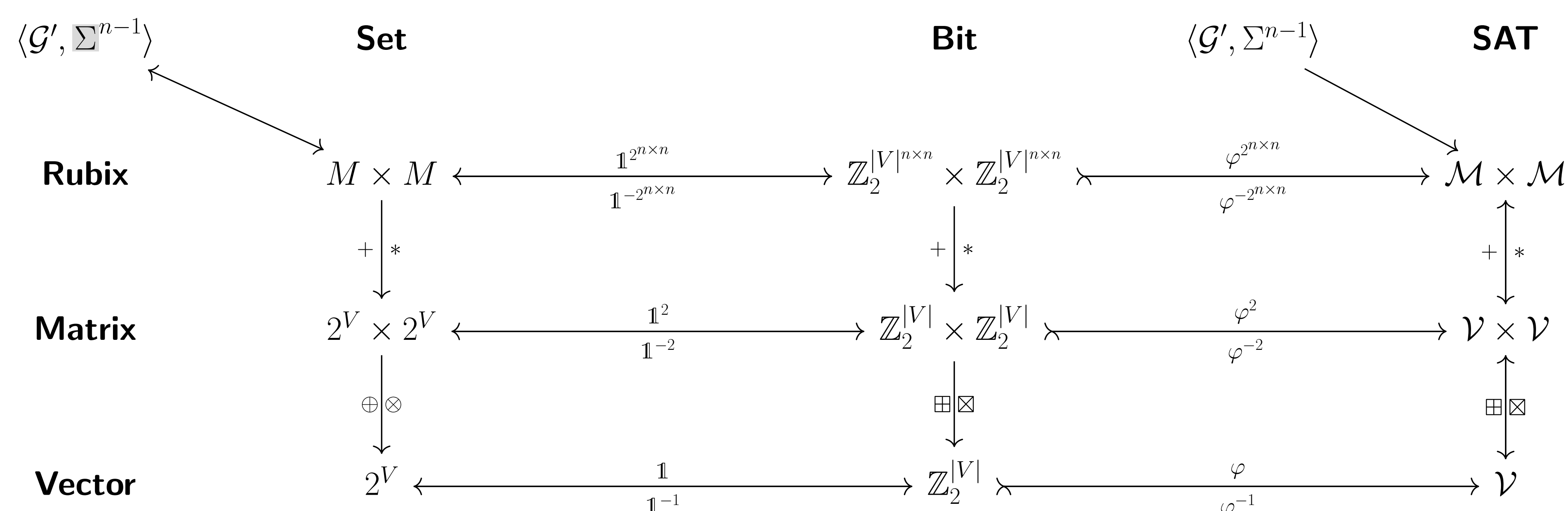
Valiant (1975) shows that if we let $\sigma_r^\dagger := \{ V \mid (V \rightarrow \sigma_r^\dagger) \in P \}$, initialize the matrix $M_{r+1=c}^0(\mathcal{G}', e) := \sigma_r^\dagger$ and solve for its fixpoint $M^* = M + M^2$,

$$M^0 := \begin{pmatrix} \emptyset & \sigma_1^\dagger & \emptyset & \dots & \emptyset \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \emptyset & \dots & \sigma_n^\dagger & \dots & \emptyset \end{pmatrix} \Rightarrow M^* = \begin{pmatrix} \emptyset & \sigma_1^\dagger & \mathcal{V} & \dots & \mathcal{V}^* \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \emptyset & \dots & \sigma_n^\dagger & \dots & \emptyset \end{pmatrix}$$

the recognizer is then defined as: $R(\mathcal{G}', \sigma) := S \in \mathcal{V}^*? \iff \sigma \in \mathcal{L}(\mathcal{G})?$

Galois Connection

- CYK parsers can be lowered onto $\mathbb{Z}_2^{|V| \times n \times n}$ or $\mathcal{M} : (\mathbb{Z}_2^{|V|} \rightarrow \mathbb{Z}_2)^{|V| \times n \times n}$
- \mathcal{M}^* can be solved for directly using Gaussian elimination or XOR-SAT
- Enables sketch-based synthesis in σ or \mathcal{G} : just use variables for holes!
- We can encode using the characteristic function, i.e., $\mathbb{1}_{\subseteq V} : V \rightarrow \mathbb{Z}_2^{|V|}$
- \oplus, \otimes are defined as \boxplus, \boxtimes , so that the following diagram commutes:



Brozowski's Derivative

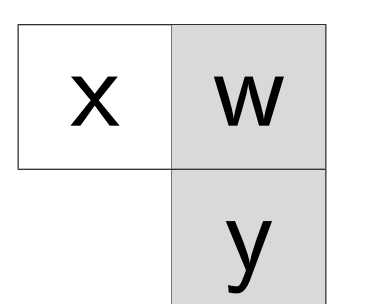
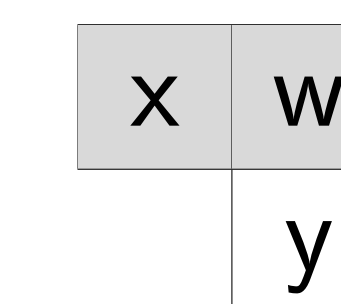
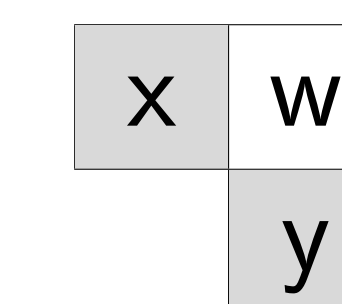
Valiant's \otimes operator, which unifies known factors in a binary CFG, implies a left- and right-quotient, which yield the set of nonterminal forests that may appear to either side of a known factor and its corresponding root.

Valiant's \otimes

Left Quotient

Right Quotient

$$x \otimes y = \{ w \mid (w \rightarrow xy) \} \quad \frac{\partial f}{\partial x} = \{ y \mid (w \rightarrow xy) \} \quad \frac{\partial f}{\partial y} = \{ x \mid (w \rightarrow xy) \}$$



The left quotient coincides with Brzozowski's derivative (1964) over regular languages, here lifted into the context-sensitive setting (our work).

Context Sensitivity

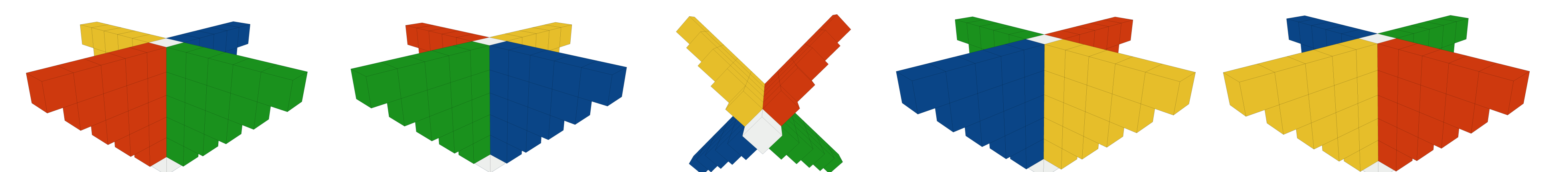
It is well-known that the family of CFLs is not closed under intersection. For example, consider $\mathcal{L}_\cap := \mathcal{L}(\mathcal{G}_1) \cap \mathcal{L}(\mathcal{G}_1)$ defined in the following way:

$$P_1 := \{ S \rightarrow LR, L \rightarrow ab \mid aLb, R \rightarrow c \mid cR \}$$

$$P_2 := \{ S \rightarrow LR, R \rightarrow bc \mid bRc, L \rightarrow a \mid aL \}$$

\mathcal{L}_\cap is equivalent to the language $\{ a^d b^d c^d \mid d > 0 \}$, which is not a CFL. We can encode $\bigcap_{i=1}^c \mathcal{L}(\mathcal{G}_i)$ as a polygonal prism with upper-triangular matrices adjoined to each rectangular face. Specifically, we intersect all terminals $\Sigma_\cap := \bigcap_{i=1}^c \Sigma_i$, then for each $t \in \Sigma_\cap$, construct an equivalence class $E(t, \mathcal{G}_i) = \{ w_i \mid (w_i \rightarrow t) \in P_i \}$ and glue them together at each σ_i :

$$\bigwedge_{t \in \Sigma_\cap} \bigwedge_{j=1}^{c-1} \bigwedge_{i=1}^{|\sigma|} E(t, \mathcal{G}_j) \equiv_{\sigma_i} E(t, \mathcal{G}_{j+1})$$



Orientations of a $\bigcap_{i=1}^4 \mathcal{L}(\mathcal{G}_i) \cap \Sigma^6$ configuration, reprojected into 2-space.

As $c \rightarrow \infty$, this shape approximates a circular cone whose symmetric axis intersects orthonormal CNF unit productions $w_i \rightarrow t$, with $S_i \in \mathcal{T}_i?$ encoded by bitvectors on the base perimeter. Equations of this form are equiexpressive with the family of CSLs realizable by finite CFL intersection.

