Syntax Repair as Language Intersection

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We introduce a new technique for correcting syntax errors in arbitrary context-free languages. Our work comes from the observation that syntax errors with a small repair typically have very few unique small repairs, which can usually be enumerated within a small edit distance then ranked within a short amount of time. Furthermore, we place a heavy emphasis on precision: the enumerated set must contain every possible repair within a few edits and no invalid repairs. To do so, we reduce CFL recognition onto Boolean tensor completion, then model error correction as a language intersection problem between a Levenshtein automaton and a context-free grammar. To decode the solutions, we then sample trees without replacement from the intersection grammar, which yields valid repairs within a certain Levenshtein distance. Finally, we rank all repairs discovered within 60 seconds by a Markov chain.

1 INTRODUCTION

Syntax repair is the problem of modifying an invalid sentence so it conforms to some grammar. Prior work has been devoted to fixing syntax errors using handcrafted heuristics. This work features a variety of approaches including rule-based systems and statistical language models. However, these techniques are often brittle, and are susceptible to misgeneralization. In a prior paper published in SPLASH, the authors sample production rules from an error correcting grammar. While theoretically sound, this technique is incomplete, i.e., not guaranteed to sample all edits within a certain Levenshtein distance, and no more. In this paper, we demonsrate it is possible to attain a significant advantage by synthesizing and scoring all repairs within a certain Levenshtein distance. Not only does this technique guarantee perfect generalization, but also helps with precision.

We take a first-principles approach making no assumptions about the sentence or grammar and focuses on correctness and end-to-end latency. Our technique is simple:

(1) We first reduce the problem of CFL recognition to Boolean tensor completion, then use that to compute the Parikh image of the CFL. This follows from a straightforward extension of the Chomsky-Schützenberger enumeration theorem.
(2) We then model syntax correction as a language intersection problem between a Levenshtein automaton and a context-free grammar, which we explicitly materialize using a specialized version of the Bar-Hillel construction to Levenshtein intersections. This greatly reduces the number of generated productions.
(3) To decode the members from the intersection grammar, we sample trees without replacement by constructing a bijection between syntax trees and the integers, then sampling integers uniformly without replacement from a finite range. This yields concrete repairs within a certain Levenshtein distance.
(4) Finally, we rank all repairs found within 60 seconds by a Markov chain.

Though simple, this technique outperforms SoTA syntax repair techniques. Its efficacy owes to the fact it does not sample edits or nor productions, but unique, fully formed repairs within a certain Levenshtein distance. It is sound and complete up to a Levenshtein bound - i.e., it will find all repairs within an arbitrary Levenshtein distance, and no more. Often the langauge of small repairs is surpisingly small compared with the langauge of all possible edits, enabling us to efficiently synthesize and score every possible solution. This offers a significant advantage over memoryless sampling techniques.

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2 EXAMPLE

Consider the following Python snippet, which contains a small syntax error:

```
def prepend(i, k, L=[]) n and [prepend(i - 1, k, [b] + L) for b in range(k)]
```

We can fix it by inserting a colon after the function definition, yielding:

```
def prepend(i, k, L=[]): n and [prepend(i - 1, k, [b] + L) for b in range(k)]
```

A careful observer will note that there is only one way to repair this Python snippet by making a single edit. In fact, many programming languages share this curious property: syntax errors with a small repair have few uniquely small repairs. Valid sentences corrupted by a few small errors rarely have many small corrections. We call such sentences *metastable*, since they are relatively stable to small perturbations, as likely to be incurred by a careless typist or novice programmer.

Let us consider a slightly more ambiguous error: `v = df.iloc(5:, 2:)`. Assuming an alphabet of just one hundred lexical tokens, this tiny statement has millions of possible two-token edits, yet only six of those possibilities are accepted by the Python parser:

(1) `v = df.iloc(5:, 2,)`	(3) `v = df.iloc(5[:, 2:])`	(5) `v = df.iloc[5:, 2:]`
(2) `v = df.iloc(5), 2()`	(4) `v = df.iloc(5:, 2:)`	(6) `v = df.iloc(5[:, 2])`

With some typing information we could easily narrow the results, but even in the absence of semantic constraints, one can probably rule out (2, 4, 6) given that `5[` and `2(` are rare bigrams in the Python language, and knowing `df.iloc` is often followed by `[`, determine (3) is most natural. This is the key insight behind our approach: we can usually locate the intended fix by exhaustively searching small repairs. As the set of small repairs is itself often small, if only we had some procedure to distinguish valid repairs, the resulting solutions could be simply ranked by naturalness.

The trouble is that any such procedure must be highly sample-efficient. We cannot afford to sample the universe of possible d-token edits, then reject invalid ones – assuming it takes just 10ms to generate and check each sample, (1-6) could take 24+ hours to find. The hardness of brute-force search grows superpolynomially with edit distance, sentence length and alphabet size. We need a more efficient procedure for sampling all and only small valid repairs.

3 PROBLEM

We can model syntax repair as a language intersection problem between a context-free language (CFL) and a regular language.

Definition 3.1 (Bounded Levenshtein-CFL reachability). Given a CFL ℓ and an invalid string $\underset{\sim}{\sigma} : \ell^{\complement}$, the BCFLR problem is to find every valid string reachable within d edits of $\underset{\sim}{\sigma}$, i.e., letting Δ be the Levenshtein metric and $L(\underset{\sim}{\sigma}, d) \coloneqq \{\sigma \mid \Delta(\underset{\sim}{\sigma}, \sigma) \leq d\}$, we seek to find $L(\underset{\sim}{\sigma}, d) \cap \ell$.

To solve this problem, we will first pose a simpler problem that only considers intersections with a finite language, then turn our attention back to BCFLR.

Definition 3.2 (Porous completion). Let $\underline{\Sigma} \coloneqq \Sigma \cup \{\_\}$, where _ denotes a hole. We denote $\sqsubseteq : \Sigma^n \times \underline{\Sigma}^n$ as the relation $\{\langle \sigma', \sigma \rangle \mid \sigma_i \in \Sigma \implies \sigma'_i = \sigma_i\}$ and the set of all inhabitants $\{\sigma' : \Sigma^+ \mid \sigma' \sqsubseteq \sigma\}$ as $\text{H}(\sigma)$. Given a *porous string*, $\sigma : \underline{\Sigma}^*$ we seek all syntactically valid inhabitants, i.e., $A(\sigma) \coloneqq \text{H}(\sigma) \cap \ell$.

As $A(\sigma)$ can be a large-cardinality set, we want a procedure which prioritizes natural solutions:

Definition 3.3 (Ranked repair). Given a finite language $A = L(\underset{\sim}{\sigma}, d) \cap \ell$ and a probabilistic language model $P : \Sigma^* \to [0, 1] \subset \mathbb{R}$, the ranked repair problem is to find the top-k maximum likelihood repairs under the language model. That is,

$$R(A, P) \coloneqq \underset{\{\boldsymbol{\sigma} \mid \boldsymbol{\sigma} \subseteq A, |\boldsymbol{\sigma}| \leq k\}}{\text{argmax}} \sum_{\sigma \in \boldsymbol{\sigma}} \mathrm{P}(\sigma \mid \underset{\sim}{\sigma}, \theta) \tag{1}$$

The problem of ranked repair is typically solved by learning a distribution over string edits, however this approach can be sample-inefficient and generalize poorly to new langauges. Modeling the distribution over Σ^* forces the model to learn both syntax and stylistics. As we demonstrate, this problem can be decomposed into a bilevel objective: first retrieval, then ranking. By ensuring retrieval is sufficiently precise and exhaustive, maximizing likliehood over the set of proximal repairs can be achieved with a much weaker, syntax-oblivious language model.

Even with an extremely efficient approximate sampler for $\sigma \sim \ell_\cap$, due to the size of A, it would be intractable to sample either $\ell \cap \Sigma^n$ or $L(\underset{\sim}{\sigma}, d)$, then reject invalid ($\sigma \notin \ell$) or unreachable ($\sigma \notin L(\underset{\sim}{\sigma}, d)$) edits, and completely out of the question to sample $\sigma \sim \Sigma^*$ as do many large language models.

Instead, we will explicitly construct a grammar generating $\ell \cap L(\underset{\sim}{\sigma}, d)$, sample from it, then rerank all repairs after a fixed timeout. So long as $|\ell_\cap|$ is sufficiently small and recognizes all and only small repairs, our sampler is sure to retrieve the most natural repair and terminate quickly. Then, the problem becomes one of ranking only sampled repairs which can be completed quickly using a Markov chain.

4 METHOD

The syntax of most programming languages is context-free. Our proposed method is simple. We construct a context-free grammar representing the intersection between the langauge syntax and an automaton recognizing the Levenshtein ball of a given radius. Since CFLs are closed under intersection with regular languages, this is admissible. Three outcomes are possible:

(1) $\mathcal{G}_\cap$ is empty, in which case there is no repair within the given radius. In this case, we simply increase the radius and try again.
(2) $\mathcal{L}(\mathcal{G}_\cap)$ is small, in which case we enumerate all possible repairs. Enumeration is tractable for $\sim 80\%$ of the dataset in ≤ 90s.
(3) $\mathcal{L}(\mathcal{G}_\cap)$ is too large to enumerate, so we sample from the intersection grammar $\mathcal{G}_\cap$. Sampling is necessary for $\sim 20\%$ of the dataset.

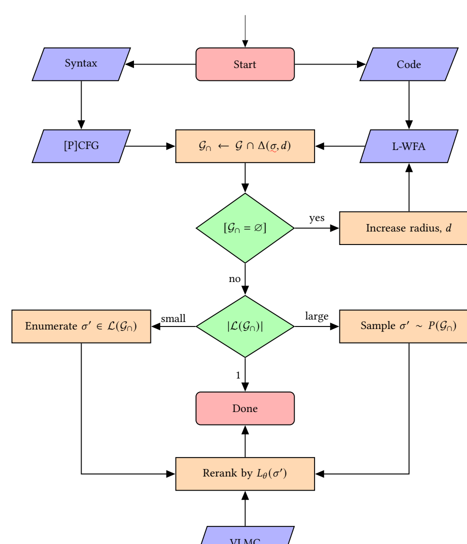


Fig. 1. Flowchart of our proposed method.

When ambiguous, we use an n-gram model to rank and return the top-k results by likelihood. This procedure is depicted in Fig. 1.

4.1 The Nominal Levenshtein Automaton

Levenshtein edits are recognized by a certain kind of automaton, known as the Levenshtein automaton. Since the original approach used by Schultz and Mihov contains cycles and epsilon transitions, we propose a modified variant which is epsilon-free, acyclic and monotone. Furthermore, we use a nominal automaton, allowing for infinite alphabets. This considerably simplifies the langauge intersection. We give an example of a small Levenshtein automaton recognizing $\Delta(\sigma : \Sigma^5, 3)$ in Fig. 2. Unlabeled arcs accept any terminal.

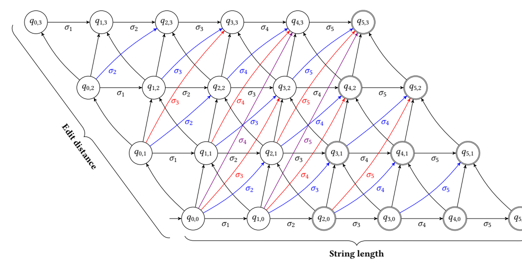


Fig. 2. NFA recognizing Levenshtein $\Delta(\sigma : \Sigma^5, 3)$.

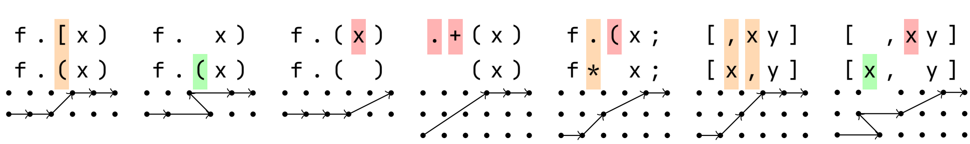
Alternatively, this transition system can be viewed as a kind of proof system. This is equivalent to the Levenshtein automaton used by Schultz and Mihov, but is more amenable to our purposes as it does not any contain ε-arcs, and uses skip connections to represent consecutive deletions.

$$\frac{s \in \Sigma \quad i \in [0, n] \quad j \in [1, k]}{(q_{i,j-1} \overset{s}{\rightarrow} q_{i,j}) \in \delta} \qquad \frac{s \in \Sigma \quad i \in [1, n] \quad j \in [1, k]}{(q_{i-1,j-1} \overset{s}{\rightarrow} q_{i,j}) \in \delta}$$

$$\frac{i \in [1, n] \quad j \in [0, k]}{(q_{i-1,j} \overset{\sigma_i}{\rightarrow} q_{i,j}) \in \delta} \qquad \frac{d \in [1, d_{\max}] \quad i \in [d+1, n] \quad j \in [d, k]}{(q_{i-d-1,j-d} \overset{\sigma_i}{\rightarrow} q_{i,j}) \in \delta}$$

$$\frac{}{q_{0,0} \in I}\ \textsc{Init} \qquad \frac{q_{i,j} \qquad |n - i + j| \leq k}{q_{i,j} \in F}\ \textsc{Done}$$

Each arc plays a specific role. handles insertions, handles substitutions, handles insertions and handles [consecutive] deletions of various lengths. Let us consider some illustrative cases.



Note that the same patch can have multiple Levenshtein alignments. Done constructs the final states, which are all states accepting strings σ' such that Levenshtein distance of $\Delta(\sigma, \sigma') \leq d_{\max}$.

To avoid creating multiple arcs for each insertion or deletion, we alter the following rules:

$$\frac{S(s : \Sigma) \mapsto [s \neq \sigma_i] \quad i \in [0, n] \quad j \in [1, k]}{(q_{i,j-1} \overset{S}{\rightarrow} q_{i,j}) \in \delta} \qquad \frac{S(s : \Sigma) \mapsto [s \neq \sigma_i] \quad i \in [1, n] \quad j \in [1, k]}{(q_{i-1,j-1} \overset{S}{\rightarrow} q_{i,j}) \in \delta}$$

By nominalizing the NFA, we can eliminate the creation of $e = |\Sigma| \cdot |\sigma| \cdot 2d_{\max}$ unnecessary arcs across the entire Levenshtein automaton. Thus, it is absolutely essential to first nominalize the automaton before proceeding.

4.2 Levenshtein-Bar-Hillel Construction

We now describe the Bar-Hillel construction, which generates a grammar recognizing the intersection between a regular and a context-free language, then specialize it to Levenshtein intersections.

Lemma 4.1. *For any context-free language ℓ and finite state automaton α, there exists a context-free grammar $G_\cap$ such that $\mathcal{L}(G_\cap) = \ell \cap \mathcal{L}(\alpha)$. See Bar-Hillel [1].*

Although Bar-Hillel [1] lacks an explicit construction, Beigel and Gasarch [2] construct $G_\cap$ like so:

$$\frac{q \in I \quad r \in F}{\big(S \rightarrow qSr\big) \in P_\cap} \qquad \frac{(A \rightarrow a) \in P \qquad (q \overset{a}{\rightarrow} r) \in \delta}{\big(qAr \rightarrow a\big) \in P_\cap} \qquad \frac{(w \rightarrow xz) \in P \qquad p, q, r \in Q}{\big(pwr \rightarrow (pxq)(qzr)\big) \in P_\cap} \bowtie$$

This, now standard, Bar-Hillel construction applies to any CFL and REG language intersection, but generates a grammar whose cardinality is approximately $|P_\cap| = |I| \cdot |F| + |P| \cdot |\Sigma| \cdot |\sigma| \cdot 2d_{\max} + |P| \cdot |Q|^3$. Applying the BH construction directly to programming language syntax and Levenshtein automata can generate hundreds of trillions of productions for moderately sized grammars and Levenshtein automata. In this section, we describe a kind of reachability analysis that elides many unreachable productions in the case of Levenshtein intersection. We will now describe this reduction in detail.

Consider $\bowtie$, the most expensive rule. What $\bowtie$ tells us is each nonterminal in the intersection grammar, $P_\cap$, matches a substring simultaneously recognized by (1) a pair of states in the original NFA and (2) a nonterminal in the original CFG. A key observation is that $\bowtie$ considers the intersection between every possible triple, but this is a huge overapproximation for most NFAs and CFGs, as the vast majority of all state pairs and nonterminals will recognize no strings in common.

To identify these triples, we define an interval domain that soundly overapproximates the Parikh image, encoding the minimum and maximum number of terminals each nonterminal can represent. Since some intervals may be right-unbounded, we write $\mathbb{N}^* = \mathbb{N} \cup \{\infty\}$ to capture the upper bound.

Definition 4.2 (Parikh mapping of a nonterminal). Let $p : \Sigma^* \rightarrow \mathbb{N}^{|\Sigma|}$ be the Parikh operator [4], which counts the frequency of terminals in a string. Let $\pi : V \rightarrow \{[a, b] \in \mathbb{N} \times \mathbb{N}^* \mid a \leq b\}^{|\Sigma|}$ be a function returning the smallest interval such that $\forall \sigma : \Sigma^*, \forall v : V, v \Rightarrow^* \sigma \vdash p(s) \in \pi(v)$.

In other words, the Parikh mapping computes the greatest lower and least upper bound of the Parikh image over all strings in the language of a nonterminal. The infimum of a nonterminal's Parikh interval tells us how many of each terminal a nonterminal *must* generate, and the supremum tells us how many it *can* generate. Likewise, we define a similar relation over NFA state pairs:

Definition 4.3 (Parikh mapping of NFA states). We define $\pi : Q \times Q \rightarrow \{[a, b] \in \mathbb{N} \times \mathbb{N}^* \mid a \leq b\}^{|\Sigma|}$ as returning the smallest interval such that $\forall \sigma : \Sigma^*, \forall q, q' : Q, q \overset{\sigma}{\Longrightarrow} q' \vdash p(\sigma) \in \pi(q, q')$.

Next, we will define a measure on Parikh intervals, which will represent the minimum total edits required to transform a string in one Parikh interval to a string in another.

Definition 4.4 (Parikh divergence). Given two Parikh intervals $\pi, \pi' : \{[a, b] \in \mathbb{N} \times \mathbb{N}^* \mid a \leq b\}^{|\Sigma|}$, we define the divergence between them as $\pi \parallel \pi' = \sum_{n=1}^{|\Sigma|} \min_{(i,i') \in \pi[n] \times \pi'[n]} |i - i'|$.

Now, we know that if the Parikh divergence between two intervals exceeds the Levenshtein margin between two states in a Lev-NFA, those intervals must be incompatible as no two strings, one from each Parikh interval, can be transformed into the other in fewer than $\pi \parallel \pi'$ edits.

Definition 4.5 (Levenshtein-Parikh compatibility). Let $q = q_{h,i}, q' = q_{j,k}$ be two states in a Lev-NFA and V be a CFG nonterminal. We say that $(q, v, q') : Q \times V \times Q$ are compatible iff the Parikh divergence is bounded by the Levenshtein margin $k - i$, i.e., $v \triangleleft qq' \iff (\pi(v) \parallel \pi(q, q')) \leq k - i$.

Finally, we define the modified Bar-Hillel construction for Levenshtein intersections as follows:

$$\frac{(A \rightarrow a) \in P \quad S(a) \quad (q \overset{S}{\rightarrow} r) \in \delta}{\big(qAr \rightarrow a\big) \in P_\cap} \qquad \frac{w \triangleleft pr \quad x \triangleleft pq \quad z \triangleleft qr \quad (w \rightarrow xz) \in P \quad p, q, r \in Q}{\big(pwr \rightarrow (pxq)(qzr)\big) \in P_\cap} \bowtie$$

4.3 Parsing as idempotent matrix completion

Recall that a CFG is a quadruple consisting of terminals (Σ), nonterminals (V), productions ($P\colon V \rightarrow (V \mid \Sigma)^*$), and a start symbol, ($S$). Every CFG is reducible to *Chomsky Normal Form*, $P'\colon V \rightarrow (V^2 \mid \Sigma)$, in which every P takes one of two forms, either $w \rightarrow xz$, or $w \rightarrow t$, where $w, x, z : V$ and $t : \Sigma$. For example:

$$G := \left\{\, S \rightarrow S\,S \mid (\,S\,) \mid (\,) \,\right\} \Longrightarrow \left\{\, S \rightarrow Q\,R \mid S\,S \mid L\,R,\quad R \rightarrow\,),\quad L \rightarrow (,\quad Q \rightarrow L\,S \,\right\}$$

Given a CFG, $G' : \mathbb{G} = \langle \Sigma, V, P, S \rangle$ in CNF, we can construct a recognizer $R : \mathbb{G} \rightarrow \Sigma^n \rightarrow \mathbb{B}$ for strings $\sigma : \Sigma^n$ as follows. Let 2^V be our domain, 0 be $\varnothing$, $\oplus$ be $\cup$, and $\otimes$ be defined as:

$$X \otimes Z := \left\{\, w \mid \langle x, z \rangle \in X \times Z, (w \rightarrow xz) \in P \,\right\} \tag{2}$$

If we define $\hat{\sigma}_r := \{w \mid (w \rightarrow \sigma_r) \in P\}$, then construct a matrix with nonterminals on the superdiagonal representing each token, $M_0[r + 1 = c](G', \sigma) := \hat{\sigma}_r$ and solve for the fixpoint $M_{i+1} = M_i + M_i^2$,

$$M_0 := \begin{pmatrix} \varnothing & \hat{\sigma}_1 & \varnothing & \cdots & \varnothing \\ \vdots & & \ddots & \ddots & \vdots \\ & & & & \varnothing \\ & & & & \hat{\sigma}_n \\ \varnothing & \cdots & & & \varnothing \end{pmatrix} \Rightarrow \begin{pmatrix} \varnothing & \hat{\sigma}_1 & \Lambda & \cdots & \varnothing \\ \vdots & & \ddots & \ddots & \vdots \\ & & & & \Lambda \\ & & & & \hat{\sigma}_n \\ \varnothing & \cdots & & & \varnothing \end{pmatrix} \Rightarrow \ldots \Rightarrow M_\infty = \begin{pmatrix} \varnothing & \hat{\sigma}_1 & \Lambda & \cdots & \Lambda_\sigma^* \\ \vdots & & \ddots & \ddots & \vdots \\ & & & & \Lambda \\ & & & & \hat{\sigma}_n \\ \varnothing & \cdots & & & \varnothing \end{pmatrix}$$

we obtain the recognizer, $R(G', \sigma) := [S \in \Lambda_\sigma^*] \Leftrightarrow [\sigma \in \mathcal{L}(G)]$ [1].

Since $\bigoplus_{c=1}^{n} M_{r,c} \otimes M_{c,r}$ has cardinality bounded by $|V|$, it can be represented as $\mathbb{Z}_2^{|V|}$ using the characteristic function, $\mathbb{1}$. A concrete example is shown in § ??.

Let us consider an example with two holes, $\sigma = 1$ _ _, and the grammar being $G := \{S \rightarrow NON, O \rightarrow + \mid \times, N \rightarrow 0 \mid 1\}$. This can be rewritten into CNF as $G' := \{S \rightarrow NL, N \rightarrow 0 \mid 1, O \rightarrow \times \mid +, L \rightarrow ON\}$. Using the algebra where $\oplus = \cup$, $X \otimes Z = \left\{\, w \mid \langle x, z \rangle \in X \times Z, (w \rightarrow xz) \in P \,\right\}$, the fixpoint $M' = M + M^2$ can be computed as follows, shown in the leftmost column:

	2^V	$\mathbb{Z}_2^{\lvert V\rvert}$	$\mathbb{Z}_2^{\lvert V\rvert} \rightarrow \mathbb{Z}_2^{\lvert V\rvert}$
M_0	$\begin{pmatrix} \{N\} & & \\ & \{N,O\} & \\ & & \{N,O\} \end{pmatrix}$	$\begin{pmatrix} \square\blacksquare\square\square & & \\ & \square\blacksquare\blacksquare\square & \\ & & \square\blacksquare\blacksquare\square \end{pmatrix}$	$\begin{pmatrix} V_{0,1} & & \\ & V_{1,2} & \\ & & V_{2,3} \end{pmatrix}$
M_1	$\begin{pmatrix} \{N\} & \varnothing & \\ & \{N,O\} & \{L\} \\ & & \{N,O\} \end{pmatrix}$	$\begin{pmatrix} \square\blacksquare\square\square & \square\square\square\square & \\ & \square\blacksquare\blacksquare\square & \blacksquare\square\square\square \\ & & \square\blacksquare\blacksquare\square \end{pmatrix}$	$\begin{pmatrix} V_{0,1} & V_{0,2} & \\ & V_{1,2} & V_{1,3} \\ & & V_{2,3} \end{pmatrix}$
M_∞	$\begin{pmatrix} \{N\} & \varnothing & \{S\} \\ & \{N,O\} & \{L\} \\ & & \{N,O\} \end{pmatrix}$	$\begin{pmatrix} \square\blacksquare\square\square & \square\square\square\square & \square\square\square\blacksquare \\ & \square\blacksquare\blacksquare\square & \blacksquare\square\square\square \\ & & \square\blacksquare\blacksquare\square \end{pmatrix}$	$\begin{pmatrix} V_{0,1} & V_{0,2} & V_{0,3} \\ & V_{1,2} & V_{1,3} \\ & & V_{2,3} \end{pmatrix}$

The same procedure can be translated, without loss of generality, into the bit domain ($\mathbb{Z}_2^{|V|}$) using a lexicographic ordering, however these both are recognizers. That is to say, $[S \in V_{0,3}] \Leftrightarrow [V_{0,3,3} =$

[1]Hereinafter, we use Iverson brackets to denote the indicator function of a predicate with free variables, i.e., $[P] \Leftrightarrow \mathbb{1}(P)$.

■] $\Leftrightarrow [A(\sigma) \neq \varnothing]$. Since $V_{0,3} = \{S\}$, we know there is at least one $\sigma' \in A(\sigma)$, but M_∞ does not reveal its identity.

In order to extract the inhabitants, we can translate the bitwise procedure into an equation with free variables. Here, we can encode the idempotency constraint directly as $M = M^2$. We first define $X \boxtimes Z = [X_2 \land Z_1, \bot, \bot, X_1 \land Z_0]$ and $X \boxplus Z = [X_i \lor Z_i]_{i \in [0,|V|)}$. Since the unit nonterminals O, N can only occur on the superdiagonal, they may be safely ignored by $\otimes$. To solve for M_∞, we proceed by first computing $V_{0,2}, V_{1,3}$ as follows:

$$\begin{aligned}
V_{0,2} &= V_{0,j} \cdot V_{j,2} = V_{0,1} \boxtimes V_{1,2} && (3)\\
&= [L \in V_{0,2}, \bot, \bot, S \in V_{0,2}] && (4)\\
&= [O \in V_{0,1} \land N \in V_{1,2}, \bot, \bot, N \in V_{0,1} \land L \in V_{1,2}] && (5)\\
&= [V_{0,1,2} \land V_{1,2,1}, \bot, \bot, V_{0,1,1} \land V_{1,2,0}] && (6)
\end{aligned}$$

$$\begin{aligned}
V_{1,3} &= V_{1,j} \cdot V_{j,3} = V_{1,2} \boxtimes V_{2,3} && (7)\\
&= [L \in V_{1,3}, \bot, \bot, S \in V_{1,3}] && (8)\\
&= [O \in V_{1,2} \land N \in V_{2,3}, \bot, \bot, N \in V_{1,2} \land L \in V_{2,3}] && (9)\\
&= [V_{1,2,2} \land V_{2,3,1}, \bot, \bot, V_{1,2,1} \land V_{2,3,0}] && (10)
\end{aligned}$$

Now we solve for the corner entry $V_{0,3}$ by taking the bitwise dot product between the first row and last column, yielding:

$$\begin{aligned}
V_{0,3} &= V_{0,j} \cdot V_{j,3} = V_{0,1} \boxtimes V_{1,3} \boxplus V_{0,2} \boxtimes V_{2,3} && (11)\\
&= [V_{0,1,2} \land V_{1,3,1} \lor V_{0,2,2} \land V_{2,3,1}, \bot, \bot, V_{0,1,1} \land V_{1,3,0} \lor V_{0,2,1} \land V_{2,3,0}] && (12)
\end{aligned}$$

Since we only care about $V_{0,3,3} \Leftrightarrow [S \in V_{0,3}]$, so we can ignore the first three entries and solve for:

$$\begin{aligned}
V_{0,3,3} &= V_{0,1,1} \land V_{1,3,0} \lor V_{0,2,1} \land V_{2,3,0} && (13)\\
&= V_{0,1,1} \land (V_{1,2,2} \land V_{2,3,1}) \lor V_{0,2,1} \land \bot && (14)\\
&= V_{0,1,1} \land V_{1,2,2} \land V_{2,3,1} && (15)\\
&= [N \in V_{0,1}] \land [O \in V_{1,2}] \land [N \in V_{2,3}] && (16)
\end{aligned}$$

Now we know that $\sigma = 1\ \underline{\text{O}}\ \underline{\text{N}}$ is a valid solution, and therefor we can take the product $\{1\} \times \hat{\sigma}_r^{-1}(O) \times \hat{\sigma}_r^{-1}(N)$ to recover the admissible set, yielding $A(\sigma) = \{1 + 0, 1 + 1, 1 \times 0, 1 \times 1\}$. In this case, since G is unambiguous, there is only one parse tree satisfying $V_{0,|\sigma|,|\sigma|}$, but in general, there can be multiple valid parse trees, in which case we can decode them incrementally.

4.4 A Pairing Function for Breadth-Bounded Binary Trees

Now that we have a method for parsing with matrix completion, we will translate this to the tree domain, where the semiring algebra will be defined over indexed forests. Then we will show how to sample trees.

We will now describe a technique for sampling trees from the intersection grammar, representing distinct, fully formed repairs. When the number of choices is sufficiently constrained, this can be sampled without replacement, or otherwise with replacement using a PCFG. Once we have obtained the intersection grammar, we solve for all inhabitants using a matrix fixedpoint recurrence.

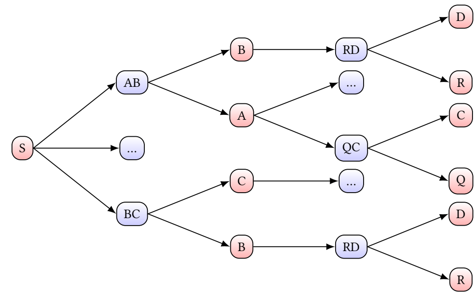


Fig. 3. A partial $\mathbb{T}_2$ for the grammar $P = \{S \to BC \mid \ldots \mid AB, B \to RD \mid \ldots, A \to QC \mid \ldots\}$.

We define an algebraic data type $\mathbb{T}_3 = (V \cup \Sigma) \rightharpoonup \mathbb{T}_2$ where $\mathbb{T}_2 = (V \cup \Sigma) \times (\mathbb{N} \rightharpoonup \mathbb{T}_2 \times \mathbb{T}_2)^2$. Morally, we can think of $\mathbb{T}_2$ as an implicit set of possible trees sharing the same root, and $\mathbb{T}_3$ as a dictionary of possible $\mathbb{T}_2$ values indexed by possible roots, given by a specific CFG under a finite-length porous string. We construct $\hat{\sigma}_r = \Lambda(\sigma_r)$ as follows:

$$\Lambda(s : \underline{\Sigma}) \mapsto \begin{cases} \bigoplus_{s \in \Sigma} \Lambda(s) & \text{if } s \text{ is a hole,} \\ \big\{\mathbb{T}_2\big(w, \big[\langle \mathbb{T}_2(s), \mathbb{T}_2(\varepsilon)\rangle\big]\big) \mid (w \to s) \in P\big\} & \text{otherwise.} \end{cases}$$

This initializes the superdiagonal entries, enabling us to compute the fixpoint M_∞ by redefining $\oplus, \otimes : \mathbb{T}_3 \times \mathbb{T}_3 \to \mathbb{T}_3$ as:

$$X \oplus Z \mapsto \bigcup_{k \in \pi_1(X \cup Z)} \big\{k \Rightarrow \mathbb{T}_2(k, x \cup z) \mid x \in \pi_2(X \circ k), z \in \pi_2(Z \circ k)\big\}$$

$$X \otimes Z \mapsto \bigoplus_{(w \to xz) \in P} \Big\{\mathbb{T}_2\Big(w, [\langle X \circ x, Z \circ z\rangle]\Big) \mid x \in \pi_1(X), z \in \pi_1(Z)\Big\}$$

These operators group subtrees by their root nonterminal, then aggregate their children. Instead of tracking sets, each Λ now becomes a dictionary indexed by the root nonterminal, which can be sampled by obtaining $(\Lambda_\sigma^* \circ S) : \mathbb{T}_2$, then recursively choosing twins as we describe in § **??**, or without replacement via enumeration as described in § 4.4.

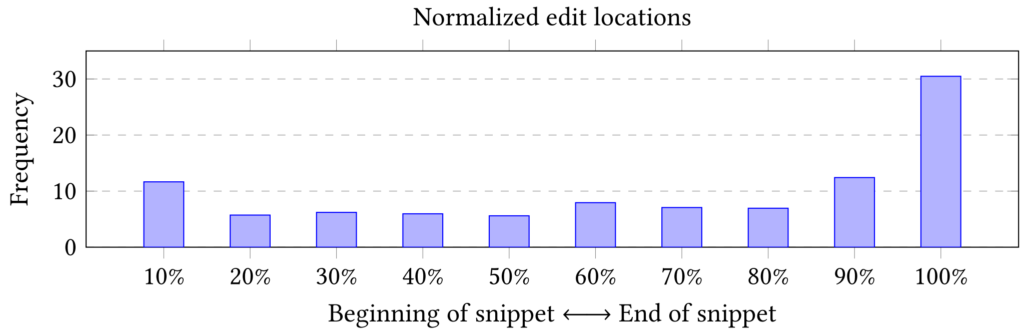
Given a probabilistic CFG whose productions indexed by each nonterminal are decorated with a probability vector $\mathbf{p}$ (this may be uniform in the non-probabilistic case), we define a tree sampler $\Gamma : (\mathbb{T}_2 \mid \mathbb{T}_2^2) \rightsquigarrow \mathbb{T}$ which recursively samples children according to a Multinoulli distribution:

$$\Gamma(T) \mapsto \begin{cases} \Gamma\big(\text{Multi}\big(\texttt{children}(T), \mathbf{p}\big)\big) & \text{if } T : \mathbb{T}_2 \\ \big\langle \Gamma\big(\pi_1(T)\big), \Gamma\big(\pi_2(T)\big)\big\rangle & \text{if } T : \mathbb{T}_2 \times \mathbb{T}_2 \end{cases}$$

This is closely related to the generating function for the ordinary Boltzmann sampler from analytic combinatorics,

$$\Gamma C(x) \mapsto \begin{cases} \text{Bern}\left(\frac{A(x)}{A(x)+B(x)}\right) \to \Gamma A(x) \mid \Gamma B(x) & \text{if } \mathcal{C} = \mathcal{A} + \mathcal{B} \\ \big\langle \Gamma A(x), \Gamma B(x) \big\rangle & \text{if } \mathcal{C} = \mathcal{A} \times \mathcal{B} \end{cases}$$

[2]Given a $T : \mathbb{T}_2$, we may also refer to $\pi_1(T), \pi_2(T)$ as $\texttt{root}(T)$ and $\texttt{children}(T)$ respectively, where children are pairs of conjoined twins.



however unlike Duchon et al. [?], our work does not depend on rejection to guarantee exact-size sampling, as all trees contained in $\mathbb{T}_2$ will necessarily be the same width.

The type $\mathbb{T}_2$ of all possible trees that can be generated by a CFG in Chomksy Normal Form corresponds to the fixpoints of the following recurrence, which tells us that each $\mathbb{T}_2$ can be a terminal, nonterminal, or a nonterminal and a sequence consisting of nonterminal pairs and their two children:

$$L(p) = 1 + pL(p) \qquad\qquad P(a) = \Sigma + V + VL\big(V^2P(a)^2\big)$$

Given a $\sigma : \underline{\Sigma}$, we construct $\mathbb{T}_2$ from the bottom-up, and sample from the top-down. Depicted below is a partial $\mathbb{T}_2$, where red nodes are `roots` and blue nodes are `children`:

The number of binary trees inhabiting a single instance of $\mathbb{T}_2$ is sensitive to the number of nonterminals and rule expansions in the grammar. To obtain the total number of trees with breadth n, we abstractly parse the porous string using the algebra defined in § ??, letting $T = \Lambda^*_{\underline{\sigma}} \circ S$, and compute the total number of trees using the recurrence:

$$|T : \mathbb{T}_2| \mapsto \begin{cases} 1 & \text{if } T \text{ is a leaf,} \\ \sum_{\langle T_1, T_2\rangle \in \texttt{children}(T)} |T_1| \cdot |T_2| & \text{otherwise.} \end{cases}$$

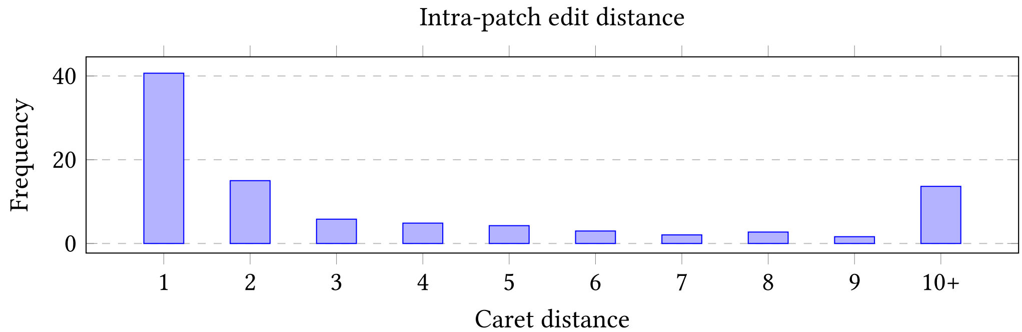
To sample all trees in a given $T : \mathbb{T}_2$ uniformly without replacement, we then construct a modular pairing function $\varphi : \mathbb{T}_2 \to \mathbb{Z}_{|T|} \to \texttt{BTree}$, which is defined as follows:

$$\varphi(T : \mathbb{T}_2, i : \mathbb{Z}_{|T|}) \mapsto \begin{cases} \Big\langle \texttt{BTree}\big(\texttt{root}(T)\big), i \Big\rangle & \text{if } T \text{ is a leaf,} \\ \text{Let } b = |\texttt{children}(T)|, & \\ \quad q_1, r = \big\langle \lfloor \frac{i}{b} \rfloor, i \pmod b \big\rangle, & \\ \quad lb, rb = \texttt{children}[r], & \\ \quad T_1, q_2 = \varphi(lb, q_1), & \\ \quad T_2, q_3 = \varphi(rb, q_2) \text{ in} & \\ \Big\langle \texttt{BTree}\big(\texttt{root}(T), T_1, T_2\big), q_3 \Big\rangle & \text{otherwise.} \end{cases}$$

Then, instead of sampling trees, we can simply sample integers uniformly without replacement from $\mathbb{Z}_{|T|}$ using the construction defined in § ??, and lazily decode them into trees.

5 DATASET

The StackOverflow dataset is comprised of 500k Python code snippets, each of which has been annotated with a human repair. We depict the normalized edit loations relative to the snippet length below.



Likewise, we can plot the number of tokens between edits within each patch:

6 EVALUATION

We consider the following research questions for our evaluation:

- **RQ 1**: What are the characteristics of the human repair? (Levenshtein edit distance, distance between edits etc)
- **RQ 2**: How do we compare with SoTA approaches? (e.g., Seq2Parse, BIFI, OrdinalFix, et al.)
- **RQ 3**: Ablation study over design choices? (search vs. sampling, timing cutoffs, ngram models, etc.)

For our evaluation, we use the StackOverflow dataset from [3]. We preprocess the dataset to lexicalize both the broken and fixed code snippets, then filter the dataset by length and edit distance, in which all Python snippets whose broken form is fewer than 80 lexical tokens and whose human fix is under four Levenshtein edits is retained.

For our first experiment, we run the sampler until the human repair is detected, then measure the number of samples required to draw the exact human repair across varying Levenshtein radii.

Fig. 4. Sample efficiency of LBH sampler at varying Levenshtein radii.

Next, measure the precision at various ranking cutoffs for varying wall-clock timeouts. Here, P@{k=1, 5, 10, All} indicates the percentage of syntax errors with a human repair of $\Delta = \{1, 2, 3, 4\}$ edits found in $\leq p$ seconds that were matched within the top-k results, using an n-gram likelihood model.

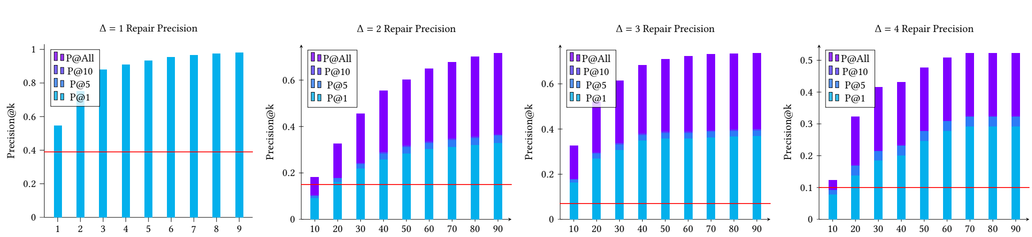


Fig. 5. Human repair benchmark. Note the y-axis across different edit distance plots has varying ranges.

6.1 Old Evaluation

We evaluate Tidyparse along three primary axes: latency, throughput, and accuracy on a dataset of human repairs. Our intention here is to show that Tidyparse is competitive with a large language model (roughly, a deep circuit) that is slow but highly sample-efficient with a small language model (roughly, a shallow circuit) that is fast but less sample-efficient.

Large language models typically take between several hundred milliseconds and several seconds to infer a repair. The output is not guaranteed to be syntactically valid, and may require more than one sample to discover a valid repair. In contrast, Tidyparse can discover thousands of repairs in the same duration, all of which are guaranteed to be syntactically valid. Furthermore, if a valid repair exists within a certain number of edits, it will eventually be found.

To substantiate these claims, we conduct experiments measuring:

- the average worst-case time to discover a human repair across varying sizes, i.e., average latency to discover a repair with edit distance d.
- the average accuracy at varying latency cutoffs, i.e., average precision@k at latency cutoff t.
- the average repair throughput across varying CPU cores, i.e., average number of admissible repairs discovered per second over the repair length.
- the relative throughput versus a uniform sampler, i.e., average number of admissible repairs discovered per second divided by the uniform sampler's throughput

6.2 Uniform sampling benchmark

Below, we plot the precision of the uniform sampling procedure described in §?? against human repairs of varying edit distances and latency cutoffs. Repairs discovered before timeout expiration are reranked by tokenwise perplexity then compared using an exact lexical match with the human repair at or below rank k. We note that the uniform sampling procedure is not intended to be used in practice, but rather provides a baseline for the empirical density of the admissible set, and an upper bound on the latency required to attain a given precision.

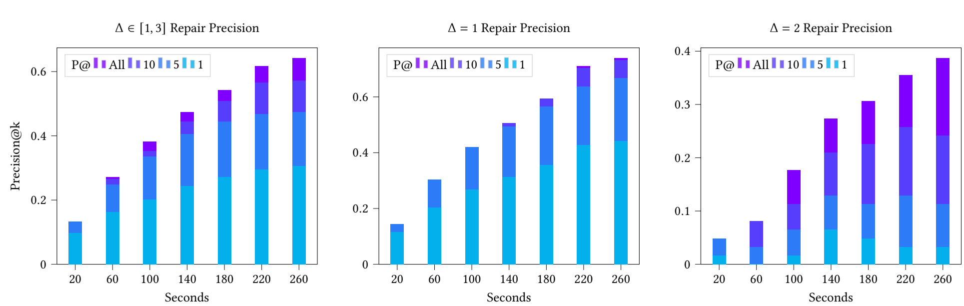


Fig. 6. Human repair benchmark. Note the y-axis across different edit distance plots has varying ranges.

Despite the high-latency, this demonstrates a uniform prior with post-timeout reranking is still able to achieve competitive precision@k using a relatively cheap ranking metric. This suggests that we can use the metric to bias the sampler towards more likely repairs, which we will now do.

6.3 Repair with an adaptive sampler

In the following benchmark, we measure the precision@k of our repair procedure against human repairs of varying edit distances and latency cutoffs, using an adaptive resampling procedure described in §??. This sampler maintains a buffer of successful repairs ranked by perplexity and uses stochastic local search to resample edits within a neighborhood. Initially, edits are sampled uniformly at random. Over time and as the admissible set grows, it prioritizes edits nearby low-perplexity repairs. This technique offers a significant advantage in the low-latency setting.

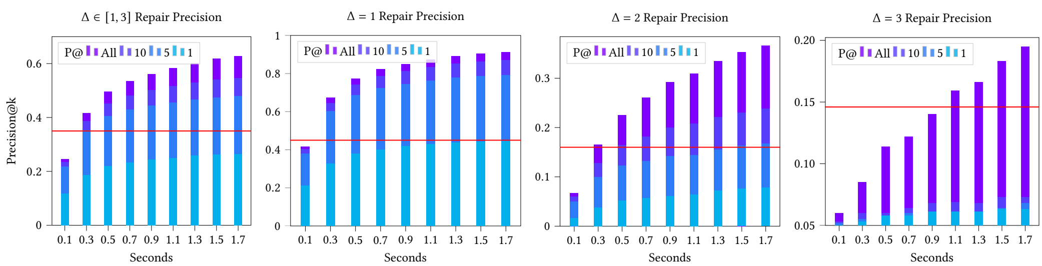


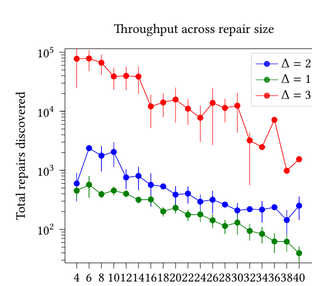
Fig. 7. Adaptive sampling repairs. The red line indicates Seq2Parse precision@1 on the same dataset. Since it only supports generating one repair, we do not report precision@k or the intermediate latency cutoffs.

We also evaluate Seq2Parse on the same dataset. Seq2Parse only supports precision@1 repairs, and so we only report Seq2parse precision@1 from the StackOverflow benchmark for comparison. Unlike our approach which only produces syntactically correct repairs, Seq2Parse also produces syntactically incorrect repairs and so we report the percentage of repairs matching the human repair for both our method and Seq2Parse. Seq2Parse latency varies depending on the length of the repair, averaging 1.5s for $\Delta = 1$ to 2.7s for $\Delta = 3$, across the entire StackOverflow dataset.

While adapting sampling is able to saturate the admissible set for 1- and 2-edit repairs before the timeout elapses, 3-edit throughput is heavily constrained by compute around 16 lexical tokens, when Python's Levenshtein ball has a volume of roughly 6×10^8 edits. This bottleneck can be relaxed with a longer timeout or additional CPU cores. Despite the high computational cost of sampling multi-edit repairs, our precision@all remains competitive with the Seq2Parse neurosymbolic baseline at the same latency. We provide some qualitative examples of repairs in Table **??**.

6.4 Throughput benchmark

End-to-end throughput varies significantly with the edit distance of the repair. Some errors are trivial to fix, while others require a large number of edits to be sampled before a syntactically valid edit is discovered. We evaluate throughput by sampling edits across invalid strings $|\sigma| \leq 40$ from the StackOverflow dataset of varying length, and measure the total number of syntactically valid edits discovered, as a function of string length and language edit distance $\Delta \in [1, 3]$. Each trial is terminated after 10 seconds, and the experiment is repeated across 7.3k total repairs. Note the y-axis is log-scaled, as the number of admissible repairs increases sharply with language edit distance. Our approach discovers a large number of syntactically valid repairs in a relatively short amount of time, and is able to quickly saturate the admissible set for 1- and 2-edit repairs before timeout. As the Seq2Parse baseline is unable to generate more than one syntactically valid repair per string, we do not report its throughput.



6.5 Synthetic repair benchmark

In addition to the StackOverflow dataset, we also evaluate our approach on two datasets containing synthetic strings generated by a Dyck language, and bracketing errors of synthetic and organic provenance in organic source code. The first dataset contains length-50 strings sampled from various Dyck languages, i.e., the Dyck language containing n different types of balanced parentheses. The second contains abstracted Java and Python source code mined from GitHub repositories. The Dyck languages used in the remaining experiments are defined by the following context-free grammar(s):

```
Dyck-1 -> ( ) | ( Dyck-1 ) | Dyck-1 Dyck-1
    Dyck-2 -> Dyck-1 | [ ] | ( Dyck-2 ) | [ Dyck-2 ] | Dyck-2 Dyck-2
    Dyck-3 -> Dyck-2 | { } | ( Dyck-3 ) | [ Dyck-3 ] | { Dyck-3 } | Dyck-3 Dyck
         -3
```

In experiment (1a), we sample a random valid string $\sigma \sim \Sigma^{50} \cap \mathcal{L}_{\text{Dyck-n}}$, then replace a fixed number of indices in $[0, |\sigma|)$ with holes and measure the average time required to decode ten syntactically-admissible repairs across 100 trial runs. In experiment (1b), we sample a random valid string as before, but delete p tokens at random and rather than provide their location(s), ask our model to solve for both the location(s) and repair by sampling uniformly from all n-token HCs, then measure the total time required to decode the first admissible repair. Note the logarithmic scale on the y-axis.

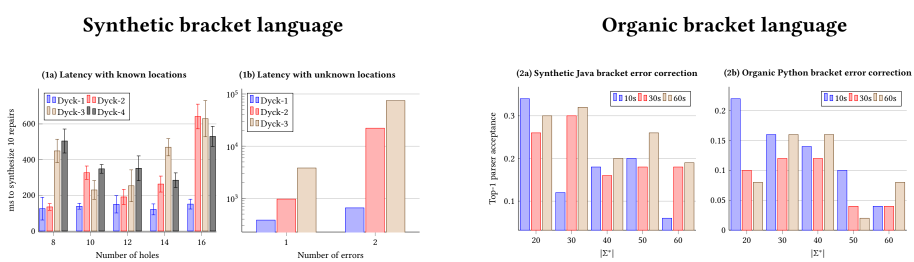


Fig. 8. Benchmarking bracket correction latency and accuracy across two bracketing languages, one generated from Dyck-n, and the second uses an abstracted source code snippet with imbalanced parentheses.

In the second set of experiments, we analyze bracketing errors in a dataset of Java and Python code snippets mined from open-source repositories on GitHub using the Dyck-nw[3], in which all source code tokens except brackets are replaced with a `w` token. For Java (2a), we sample valid single-line statements with bracket nesting more than two levels deep, synthetically delete one bracket uniformly at random, and repair using Tidyparse, then take the top-1 repair after t seconds, and validate using ANTLR's Java 8 parser. For Python (2b), we sample invalid code fragments uniformly from the imbalanced bracket category of the Break-It-Fix-It (BIFI) dataset [5], a dataset of organic Python errors, which we repair using Tidyparse, take the top-1 repair after t seconds, and validate repairs using Python's `ast.parse()` method. Since the Java and Python datasets do not have a ground-truth human fix, we report the percentage of repairs that are accepted by the language's official parser for repairs generated under a fixed time cutoff. Although the Java and Python datasets are not directly comparable, we observe that Tidyparse can detect and repair a significant fraction of bracket errors in both languages with a relatively unsophisticated grammar.

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