

Proof summary for **theory** top_group

Theory top_group totals: 0 formulas, 0 attempted, 0 succeeded (0.00 s)

Proof summary for **theory** commutative_semigroup

IMP_commutative_groupoid_TCC1.....proved - complete [shostak](0.31 s)
IMP_semigroup_TCC1.....proved - complete [shostak](0.31 s)
commutative_semigroup_TCC1.....proved - complete [shostak](0.32 s)
commutative_semigroup_is_semigroup....proved - complete [shostak](0.34 s)
commutative_semigroup_is_commutative_groupoid...proved - complete [shostak]

(0.34 s)

Theory commutative_semigroup totals: 5 formulas, 5 attempted, 5 succeeded

(1.62 s)

Proof summary for **theory** commutative_groupoid

commutative_groupoid_TCC1.....proved - complete [shostak](0.30 s)
commutative.....proved - complete [shostak](0.32 s)
commutative_groupoid_is_groupoid.....proved - complete [shostak](0.35 s)

Theory commutative_groupoid totals: 3 formulas, 3 attempted, 3 succeeded (0.97

s)

Proof summary for **theory** cyclic_monoid

IMP_monoid_TCC1.....proved - complete [shostak](0.30 s)
cyclic_monoid_TCC1.....proved - complete [shostak](0.32 s)
cyclic_monoid_is.....proved - complete [shostak](0.31 s)
cyclic_monoid_is_monoid.....proved - complete [shostak](0.36 s)
cyclic_monoid_is_commutative_monoid...proved - complete [shostak](0.40 s)

Theory cyclic_monoid totals: 5 formulas, 5 attempted, 5 succeeded (1.69 s)

Proof summary for **theory** cyclic_monoid_def

Theory cyclic_monoid_def totals: 0 formulas, 0 attempted, 0 succeeded (0.00 s)

Proof summary for **theory** subgroups

G_TCC1.....proved - complete [shostak](0.33 s)
pg64_1.....proved - complete [shostak](0.86 s)
center_normal_TCC1.....proved - complete [shostak](0.37 s)
center_normal.....proved - complete [shostak](0.45 s)

Theory subgroups totals: 4 formulas, 4 attempted, 4 succeeded (2.01 s)

Proof summary for **theory** symmetric_groups

op_TCC1.....proved - complete [shostak](0.33 s)
Sym_is_group.....proved - complete [shostak](0.97 s)

Theory symmetric_groups totals: 2 formulas, 2 attempted, 2 succeeded (1.30 s)

Proof summary for **theory** group_test

integer_plus_TCC1.....proved - complete [shostak](0.43 s)
nz_rational_mult_TCC1.....proved - complete [shostak](0.52 s)
pos_rational_mult_TCC1.....proved - complete [shostak](0.56 s)

Theory group_test totals: 3 formulas, 3 attempted, 3 succeeded (1.51 s)

Proof summary for **theory** infinite_cyclic_groups

Z_TCC1.....proved - complete [shostak](0.47 s)
F_TCC1.....proved - complete [shostak](0.32 s)
Z_gen.....proved - complete [shostak](0.86 s)
inf_cyclic_is_Z.....proved - complete [shostak](0.95 s)

Theory infinite_cyclic_groups totals: 4 formulas, 4 attempted, 4 succeeded (2.60 s)

Proof summary for **theory** cayleys

S_TCC1.....	proved - complete	[shostak](0.33 s)
cayley_prep_TCC1.....	proved - complete	[shostak](0.32 s)
cayley_prep_TCC2.....	proved - complete	[shostak](0.36 s)
cayley_prep.....	proved - complete	[shostak](0.59 s)
trans_is_group_TCC1.....	proved - complete	[shostak](0.32 s)
trans_is_group_TCC2.....	proved - complete	[shostak](0.40 s)
trans_is_group.....	proved - complete	[shostak](1.63 s)
Cayleys_TCC1.....	proved - complete	[shostak](0.33 s)
Cayleys_TCC2.....	proved - complete	[shostak](0.38 s)
Cayleys.....	proved - complete	[shostak](0.00 s)

Theory cayleys totals: 10 formulas, 10 attempted, 10 succeeded (4.67 s)

Proof summary for **theory** A_group

op_TCC1.....	proved - complete	[shostak](0.34 s)
A_is_group_TCC1.....	proved - complete	[shostak](0.33 s)
A_is_group.....	proved - complete	[shostak](0.80 s)

Theory A_group totals: 3 formulas, 3 attempted, 3 succeeded (1.47 s)

Proof summary for **theory** zn

floor_help.....	proved - complete	[shostak](0.43 s)
Z_group.....	proved - complete	[shostak](0.47 s)
Z_TCC1.....	proved - complete	[shostak](0.33 s)
Z_prep_TCC1.....	proved - complete	[shostak](0.34 s)
Z_prep_TCC2.....	proved - complete	[shostak](0.33 s)
Z_prep.....	proved - complete	[shostak](1.16 s)
Z__TCC1.....	proved - complete	[shostak](0.33 s)
nZ_plus_TCC1.....	proved - complete	[shostak](0.47 s)
nZ_prep_TCC1.....	proved - complete	[shostak](0.37 s)
nZ_prep_TCC2.....	proved - complete	[shostak](0.36 s)
nZ_prep.....	proved - complete	[shostak](0.57 s)
nZ_TCC1.....	proved - complete	[shostak](0.53 s)
nZ_TCC2.....	proved - complete	[shostak](0.55 s)
nZ_normal_TCC1.....	proved - complete	[shostak](0.69 s)
nZ_normal.....	proved - complete	[shostak](0.42 s)
Z_fact_test_TCC1.....	proved - complete	[shostak](1.01 s)
Z_fact_test_TCC2.....	proved - complete	[shostak](0.56 s)
Z_fact_test.....	proved - complete	[shostak](0.34 s)

Theory zn totals: 18 formulas, 18 attempted, 18 succeeded (9.25 s)

Proof summary for **theory** group_rew

IMP_group_TCC1.....	proved - complete	[shostak](0.31 s)
inv_left.....	proved - complete	[shostak](0.34 s)
inv_right.....	proved - complete	[shostak](0.32 s)
inv_inv.....	proved - complete	[shostak](0.33 s)
inv_one.....	proved - complete	[shostak](0.32 s)
inv_in.....	proved - complete	[shostak](0.34 s)
expt_0.....	proved - complete	[shostak](0.35 s)
expt_1.....	proved - complete	[shostak](0.35 s)
expt_m1.....	proved - complete	[shostak](0.35 s)
one_expt.....	proved - complete	[shostak](0.34 s)
one_left.....	proved - complete	[shostak](0.32 s)

one_right.....proved - complete [shostak](0.34 s)
Theory group_rew totals: 12 formulas, 12 attempted, 12 succeeded (4.00 s)

Proof summary for theory top_field

Theory top_field totals: 0 formulas, 0 attempted, 0 succeeded (0.00 s)

Proof summary for theory commutative_ring_with_one

IMP_ring_with_one_TCC1.....proved - complete [shostak](0.34 s)
IMP_commutative_ring_TCC1.....proved - complete [shostak](0.42 s)
commutative_ring_with_one_TCC1.....proved - complete [shostak](0.35 s)
commutative_ring_with_one_is.....proved - complete [shostak](0.35 s)
commutative_ring_with_one_is_commutative_ring...proved - complete [shostak]
(0.49 s)
commutative_ring_with_one_is_ring_with_one...proved - complete [shostak]
(0.50 s)
commutative_ring_with_one_is_commutative_monoid...proved - complete
[shostak](0.45 s)
Theory commutative_ring_with_one totals: 7 formulas, 7 attempted, 7 succeeded
(2.89 s)

Proof summary for theory top_sylow

Theory top_sylow totals: 0 formulas, 0 attempted, 0 succeeded (0.00 s)

Proof summary for theory sylow_theorems

IMP_finite_groups_TCC1.....proved - complete [shostak](0.35 s)
p_subgroup_sylow?_TCC1.....proved - complete [shostak](0.46 s)
p_subgroup_sylow?_TCC2.....proved - complete [shostak](0.37 s)
subgroup_is_factor_TCC1.....proved - complete [shostak](0.42 s)
subgroup_is_factor_TCC2.....proved - complete [shostak](0.43 s)
subgroup_is_factor_TCC3.....proved - complete [shostak](0.41 s)
subgroup_is_factor.....proved - complete [shostak](1.16 s)
First_Sylow_Theorem_TCC1.....proved - complete [shostak](0.41 s)
First_Sylow_Theorem_TCC2.....proved - incomplete [shostak](0.42 s)
First_Sylow_Theorem_TCC3.....proved - incomplete [shostak](0.43 s)
First_Sylow_Theorem_TCC4.....proved - incomplete [shostak](0.42 s)
First_Sylow_Theorem.....proved - incomplete [shostak](9.23 s)
p_group_is_subgroup_TCC1.....proved - complete [shostak](0.39 s)
p_group_is_subgroup_TCC2.....proved - incomplete [shostak](0.61 s)
p_group_is_subgroup_TCC3.....proved - incomplete [shostak](0.43 s)
p_group_is_subgroup.....proved - incomplete [shostak](1.00 s)
p_subgroup_sylow_order_TCC1.....proved - incomplete [shostak](0.43 s)
p_subgroup_sylow_order.....proved - incomplete [shostak](1.92 s)
conjugate_is_p_subgroup_sylow_TCC1...proved - incomplete [shostak](0.48 s)
conjugate_is_p_subgroup_sylow.....proved - incomplete [shostak](0.75 s)
unique_is_normal.....proved - incomplete [shostak](0.86 s)
Second_Sylow_Theorem_TCC1.....proved - incomplete [shostak](0.37 s)
Second_Sylow_Theorem_TCC2.....proved - incomplete [shostak](0.48 s)
Second_Sylow_Theorem.....proved - incomplete [shostak](1.50 s)
Third_Sylow_Theorem_TCC1.....proved - incomplete [shostak](0.47 s)
Third_Sylow_Theorem_TCC2.....proved - incomplete [shostak](0.47 s)
Third_Sylow_Theorem.....proved - incomplete [shostak](5.18 s)
Theory sylow_theorems totals: 27 formulas, 27 attempted, 27 succeeded (29.83
s)

Proof summary for theory isomorphism_theorems

G_TCC1.....	proved	- complete	[shostak](0.36 s)
GP_TCC1.....	proved	- complete	[shostak](0.36 s)
quotient_subgroup_TCC1.....	proved	- complete	[shostak](0.39 s)
quotient_subgroup_TCC2.....	proved	- complete	[shostak](0.42 s)
quotient_subgroup_TCC3.....	proved	- complete	[shostak](0.40 s)
quotient_subgroup_TCC4.....	proved	- complete	[shostak](0.39 s)
quotient_subgroup.....	proved	- complete	[shostak](1.59 s)
second_isomorphism_th_aux_TCC1.....	proved	- complete	[shostak](0.00 s)
second_isomorphism_th_aux_TCC2.....	proved	- complete	[shostak](0.52 s)
second_isomorphism_th_aux_TCC3.....	proved	- complete	[shostak](0.46 s)
second_isomorphism_th_aux_TCC4.....	proved	- complete	[shostak](0.64 s)
second_isomorphism_th_aux.....	proved	- complete	[shostak](2.12 s)
second_isomorphism_th_TCC1.....	proved	- complete	[shostak](0.41 s)
second_isomorphism_th_TCC2.....	proved	- complete	[shostak](0.40 s)
second_isomorphism_th_TCC3.....	proved	- complete	[shostak](0.46 s)
second_isomorphism_th_TCC4.....	proved	- complete	[shostak](0.42 s)
second_isomorphism_th_TCC5.....	proved	- complete	[shostak](0.63 s)
second_isomorphism_th.....	proved	- complete	[shostak](3.33 s)
third_isomorphism_th_aux_TCC1.....	proved	- complete	[shostak](0.40 s)
third_isomorphism_th_aux_TCC2.....	proved	- complete	[shostak](0.41 s)
third_isomorphism_th_aux_TCC3.....	proved	- complete	[shostak](0.42 s)
third_isomorphism_th_aux.....	proved	- complete	[shostak](1.85 s)
third_isomorphism_th_TCC1.....	proved	- complete	[shostak](0.45 s)
third_isomorphism_th_TCC2.....	proved	- complete	[shostak](0.40 s)
third_isomorphism_th_TCC3.....	proved	- complete	[shostak](0.89 s)
third_isomorphism_th_TCC4.....	proved	- complete	[shostak](0.40 s)
third_isomorphism_th_TCC5.....	proved	- complete	[shostak](0.78 s)
third_isomorphism_th_TCC6.....	proved	- complete	[shostak](0.42 s)
third_isomorphism_th.....	proved	- complete	[shostak](1.56 s)
correspondence_theorem.....	proved	- complete	[shostak](1.66 s)

Theory isomorphism_theorems totals: 30 formulas, 30 attempted, 30 succeeded
(22.93 s)

Proof summary for **theory** homomorphism_lemmas

G_TCC1.....	proved	- complete	[shostak](0.36 s)
GP_TCC1.....	proved	- complete	[shostak](0.35 s)
natural_homo_TCC1.....	proved	- complete	[shostak](0.40 s)
natural_homo_TCC2.....	proved	- complete	[shostak](0.41 s)
natural_homo.....	proved	- complete	[shostak](1.04 s)
homo_inv_TCC1.....	proved	- complete	[shostak](0.39 s)
homo_inv.....	proved	- complete	[shostak](0.61 s)
kernel_normal.....	proved	- complete	[shostak](0.91 s)
homo_image.....	proved	- complete	[shostak](0.69 s)
homo_image_normal_TCC1.....	proved	- complete	[shostak](0.42 s)
homo_image_normal.....	proved	- complete	[shostak](0.93 s)
homo_inv_image.....	proved	- complete	[shostak](0.67 s)
homo_inv_image_normal_TCC1.....	proved	- complete	[shostak](0.42 s)
homo_inv_image_normal.....	proved	- complete	[shostak](0.78 s)
kernel_in_inv_image.....	proved	- complete	[shostak](0.61 s)
homo_inv_image_image.....	proved	- complete	[shostak](1.08 s)
homo_inv_image_image_cor.....	proved	- complete	[shostak](0.89 s)
first_isomorphism_th_TCC1.....	proved	- complete	[shostak](0.42 s)
first_isomorphism_th_TCC2.....	proved	- complete	[shostak](0.49 s)
first_isomorphism_th_TCC3.....	proved	- complete	[shostak](0.38 s)
first_isomorphism_th_TCC4.....	proved	- complete	[shostak](0.43 s)

first_isomorphism_th.....proved - complete [shostak](3.32 s)
Theory homomorphism_lemmas totals: 22 formulas, 22 attempted, 22 succeeded
(16.00 s)

Proof summary for theory products_subgroups

IMP_normal_subgroups_TCC1.....proved - complete [shostak](0.37 s)
HK_subgroup.....proved - complete [shostak](0.69 s)
HK_subgroup_permute.....proved - complete [shostak](0.68 s)
H_K_are_subgroups.....proved - complete [shostak](0.44 s)
Theory products_subgroups totals: 4 formulas, 4 attempted, 4 succeeded (2.18 s)

Proof summary for theory homomorphisms

IMP_group_TCC1.....proved - complete [shostak](0.32 s)
IMP_group_TCC2.....proved - complete [shostak](0.36 s)
homomorphism?_TCC1.....proved - complete [shostak](0.44 s)
homo_one_TCC1.....proved - complete [shostak](0.36 s)
homo_one.....proved - complete [shostak](0.38 s)
kernel_TCC1.....proved - complete [shostak](0.47 s)
Theory homomorphisms totals: 6 formulas, 6 attempted, 6 succeeded (2.33 s)

Proof summary for theory p_groups

IMP_finite_groups_TCC1.....proved - complete [shostak](0.35 s)
alt_is_action_TCC1.....proved - complete [shostak](0.44 s)
alt_is_action.....proved - complete [shostak](0.40 s)
Fix_iff_subset.....proved - complete [shostak](0.49 s)
Fix_iff_subset_cor_TCC1.....proved - complete [shostak](0.42 s)
Fix_iff_subset_cor.....proved - incomplete [shostak](0.58 s)
subgroup_is_p_group_TCC1.....proved - complete [shostak](0.37 s)
subgroup_is_p_group.....proved - complete [shostak](0.39 s)
p_group_iff_power.....proved - incomplete [shostak](0.64 s)
p_divides_index.....proved - incomplete [shostak](0.81 s)
factor_cyclic_TCC1.....proved - complete [shostak](0.48 s)
factor_cyclic_TCC2.....proved - complete [shostak](0.38 s)
factor_cyclic_TCC3.....proved - complete [shostak](0.53 s)
factor_cyclic.....proved - complete [shostak](0.94 s)
normalizer_index_TCC1.....proved - complete [shostak](0.37 s)
normalizer_index_TCC2.....proved - complete [shostak](0.39 s)
normalizer_index_TCC3.....proved - complete [shostak](0.38 s)
normalizer_index.....proved - incomplete [shostak](0.95 s)
subgroup_proper.....proved - incomplete [shostak](0.81 s)
burside_theorem_TCC1.....proved - complete [shostak](0.41 s)
burside_theorem.....proved - incomplete [shostak](1.29 s)
p_square_is_abelian.....proved - incomplete [shostak](2.36 s)
Theory p_groups totals: 22 formulas, 22 attempted, 22 succeeded (14.16 s)

Proof summary for theory normalizer_centralizer

IMP_group_action_TCC1.....proved - complete [shostak](0.35 s)
normalizer_TCC1.....proved - complete [shostak](0.50 s)
centralizer_TCC1.....proved - complete [shostak](0.44 s)
a_by_c_TCC1.....proved - complete [shostak](0.39 s)
Cl_TCC1.....proved - complete [shostak](0.44 s)
normalizer_is_subgroup.....proved - complete [shostak](0.53 s)
subset_of_normalizer.....proved - complete [shostak](0.43 s)
normal_in_normalizer_TCC1.....proved - complete [shostak](0.37 s)

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normal_in_normalizer.....proved - complete [shostak](0.41 s)
centralizer_is_subgroup.....proved - complete [shostak](0.55 s)
singleton_iff_center.....proved - complete [shostak](0.66 s)
a_by_c_is_action.....proved - complete [shostak](0.43 s)
Fix_is_center_TCC1.....proved - complete [shostak](0.39 s)
Fix_is_center.....proved - complete [shostak](0.45 s)
stabilizer_is_centralizer.....proved - complete [shostak](0.44 s)
orbit_is_CL.....proved - complete [shostak](0.42 s)
orbits_is_CLs.....proved - complete [shostak](0.42 s)
orbits_nFix_is_CLs_nc.....proved - complete [shostak](0.47 s)
Cls_eq_index_TCC1.....proved - complete [shostak](0.37 s)
Cls_eq_index_TCC2.....proved - complete [shostak](0.41 s)
Cls_eq_index.....proved - incomplete [shostak](0.44 s)
class_equation_2_TCC1.....proved - complete [shostak](0.39 s)
class_equation_2_TCC2.....proved - incomplete [shostak](0.57 s)
class_equation_2.....proved - incomplete [shostak](0.51 s)
Theory normalizer_centralizer totals: 24 formulas, 24 attempted, 24 succeeded
(10.78 s)

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Proof summary for **theory** cauchy

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IMP_finite_cyclic_groups_TCC1.....proved - complete [shostak](0.36 s)
fseq_product_TCC1.....proved - complete [shostak](0.41 s)
fseq_product_TCC2.....proved - incomplete [shostak](0.52 s)
S_TCC1.....proved - complete [shostak](0.38 s)
fseq_product_in.....proved - incomplete [shostak](0.96 s)
fseq_product_o.....proved - incomplete [shostak](1.26 s)
fseq_product_one.....proved - incomplete [shostak](0.92 s)
fseq_product_power.....proved - incomplete [shostak](1.31 s)
one_in_SE.....proved - incomplete [shostak](0.46 s)
order_SE.....proved - incomplete [shostak](0.56 s)
S_bij_set_seq_TCC1.....proved - complete [shostak](0.39 s)
S_bij_set_seq.....proved - incomplete [shostak](1.69 s)
S_is_finite.....proved - incomplete [shostak](1.21 s)
S_card_TCC1.....proved - incomplete [shostak](0.38 s)
S_card.....proved - incomplete [shostak](0.00 s)
F_TCC1.....proved - incomplete [shostak](0.72 s)
F_1_TCC1.....proved - complete [shostak](0.36 s)
F_1_TCC2.....proved - complete [shostak](0.36 s)
F_2_TCC1.....proved - complete [shostak](0.36 s)
F_o_F12_TCC1.....proved - incomplete [shostak](0.54 s)
F_o_F12.....proved - incomplete [shostak](1.18 s)
fs_o_F21.....proved - incomplete [shostak](0.67 s)
F_in_S.....proved - incomplete [shostak](1.52 s)
F_is_action_TCC1.....proved - incomplete [shostak](0.38 s)
F_is_action_TCC2.....proved - complete [shostak](0.36 s)
F_is_action_TCC3.....proved - complete [shostak](0.36 s)
F_is_action_TCC4.....proved - complete [shostak](0.42 s)
F_is_action.....proved - incomplete [shostak](0.99 s)
Fixed_subset_TCC1.....proved - incomplete [shostak](0.45 s)
Fixed_subset_TCC2.....proved - incomplete [shostak](0.41 s)
Fixed_subset.....proved - incomplete [shostak](1.45 s)
cauchy.....proved - incomplete [shostak](1.75 s)
cauchy_cor_TCC1.....proved - complete [shostak](0.38 s)
cauchy_cor.....proved - incomplete [shostak](0.39 s)

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Theory cauchy totals: 34 formulas, 34 attempted, 34 succeeded (23.87 s)

Proof summary for **theory** finite_cyclic_groups

IMP_finite_groups_TCC1.....proved - complete [shostak](0.32 s)

prime_order_cycle.....proved - complete [shostak](0.49 s)

Theory finite_cyclic_groups totals: 2 formulas, 2 attempted, 2 succeeded (0.81

s)

Proof summary for **theory** group_action

IMP_lagrange_index_TCC1.....proved - complete [shostak](0.36 s)

group_action?_TCC1.....proved - complete [shostak](0.40 s)

group_action?_TCC2.....proved - complete [shostak](0.38 s)

stabilizer_TCC1.....proved - complete [shostak](0.44 s)

orbit_TCC1.....proved - complete [shostak](0.44 s)

Fix_TCC1.....proved - complete [shostak](0.44 s)

stabilizer_is_subgroup.....proved - complete [shostak](0.49 s)

singleton_iff_Fix.....proved - complete [shostak](0.55 s)

empty_iff_eq_Fix.....proved - complete [shostak](0.44 s)

orbits_nFix_disj_Fix.....proved - complete [shostak](0.60 s)

orbits_is_union.....proved - complete [shostak](0.56 s)

orbit_nonempty.....proved - complete [shostak](0.39 s)

orbits_nonempty.....proved - complete [shostak](0.39 s)

set_orbits_is.....proved - complete [shostak](0.43 s)

orbit_is_finite.....proved - complete [shostak](0.39 s)

orbits_disjoint.....proved - complete [shostak](0.64 s)

orbits_partition.....proved - complete [shostak](0.43 s)

orbits_nFix_partition.....proved - complete [shostak](0.47 s)

orbits_eq_index_aux_TCC1.....proved - complete [shostak](0.38 s)

orbits_eq_index_aux.....proved - complete [shostak](1.01 s)

orbits_eq_index_TCC1.....proved - complete [shostak](0.38 s)

orbits_eq_index_TCC2.....proved - complete [shostak](0.38 s)

orbits_eq_index.....proved - incomplete [shostak](0.47 s)

counting_formula_TCC1.....proved - incomplete [shostak](0.45 s)

counting_formula.....proved - incomplete [shostak](0.45 s)

class_equation_TCC1.....proved - complete [shostak](0.39 s)

class_equation_TCC2.....proved - incomplete [shostak](0.51 s)

class_equation.....proved - incomplete [shostak](1.10 s)

Fix_congruence_TCC1.....proved - complete [shostak](0.38 s)

Fix_congruence.....proved - incomplete [shostak](1.17 s)

Theory group_action totals: 30 formulas, 30 attempted, 30 succeeded (15.30 s)

Proof summary for **theory** lagrange_index

IMP_right_left_cosets_TCC1.....proved - complete [shostak](0.36 s)

Lagrange_index.....proved - incomplete [shostak](0.61 s)

index_divides.....proved - incomplete [shostak](0.37 s)

order_factor_TCC1.....proved - complete [shostak](0.39 s)

order_factor.....proved - incomplete [shostak](0.50 s)

Theory lagrange_index totals: 5 formulas, 5 attempted, 5 succeeded (2.23 s)

Proof summary for **theory** class_equation_scaf

card_rest_aux_TCC1.....proved - complete [shostak](0.38 s)

card_rest_aux_TCC2.....proved - complete [shostak](0.37 s)

card_rest_aux_TCC3.....proved - complete [shostak](0.41 s)

card_rest_aux.....proved - complete [shostak](0.64 s)

card_partition_TCC1.....proved - complete [shostak](0.37 s)

card_partition_TCC2.....proved - incomplete [shostak](1.14 s)

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card_partition.....proved - incomplete [shostak]( 8.84 s)
divide_sigma_TCC1.....proved - incomplete [shostak]( 0.39 s)
divide_sigma_TCC2.....proved - incomplete [shostak](19.18 s)
divide_sigma.....proved - incomplete [shostak](10.23 s)
Theory class_equation_scaf totals: 10 formulas, 10 attempted, 10 succeeded
(41.94 s)

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Proof summary for theory groups_scaf

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IMP_finite_groups_TCC1.....proved - complete [shostak](0.36 s)
divby_r.....proved - complete [shostak](0.65 s)
subgroup_transitive.....proved - complete [shostak](0.38 s)
normal_subgroup_tran.....proved - complete [shostak](0.38 s)
subgroup_intersection.....proved - complete [shostak](0.50 s)
conjugate_is_subgroup.....proved - complete [shostak](0.64 s)
center_is_normal_TCC1.....proved - complete [shostak](0.37 s)
center_is_normal.....proved - complete [shostak](0.50 s)
abelian_eq_center.....proved - complete [shostak](0.42 s)
order_gt_1.....proved - incomplete [shostak](0.45 s)
order_gt_p.....proved - incomplete [shostak](0.46 s)
exists_diff_one.....proved - complete [shostak](0.40 s)
one_iff_divides.....proved - complete [shostak](0.64 s)
order_power_TCC1.....proved - complete [shostak](0.38 s)
order_power_TCC2.....proved - complete [shostak](0.40 s)
order_power.....proved - incomplete [shostak](1.48 s)
coset_power_nat_TCC1.....proved - complete [shostak](0.37 s)
coset_power_nat_TCC2.....proved - complete [shostak](0.38 s)
coset_power_nat_TCC3.....proved - complete [shostak](0.37 s)
coset_power_nat_TCC4.....proved - complete [shostak](0.43 s)
coset_power_nat.....proved - complete [shostak](0.00 s)
coset_power_int.....proved - complete [shostak](0.75 s)
factor_of_cyclic_is_cyclic_TCC1.....proved - complete [shostak](0.45 s)
factor_of_cyclic_is_cyclic_TCC2.....proved - complete [shostak](0.37 s)
factor_of_cyclic_is_cyclic_TCC3.....proved - complete [shostak](0.52 s)
factor_of_cyclic_is_cyclic.....proved - complete [shostak](0.91 s)
Theory groups_scaf totals: 26 formulas, 26 attempted, 26 succeeded (12.94 s)

```

Proof summary for theory finite_groups

```

IMP_group_TCC1.....proved - complete [shostak](0.31 s)
finite_generated_by.....proved - complete [shostak](0.37 s)
finite_generated_by_def_TCC1.....proved - complete [shostak](0.33 s)
finite_generated_by_def.....proved - complete [shostak](1.46 s)
finite_generated_by_one.....proved - complete [shostak](0.97 s)
generated_by_card_1_TCC1.....proved - complete [shostak](0.32 s)
generated_by_card_1.....proved - complete [shostak](0.37 s)
finite_group_elements.....proved - complete [shostak](0.38 s)
period_TCC1.....proved - complete [shostak](0.37 s)
a_hat_period_TCC1.....proved - complete [shostak](0.34 s)
a_hat_period.....proved - complete [shostak](0.37 s)
finite_subgroup_def.....proved - complete [shostak](0.52 s)
orders_equal.....proved - complete [shostak](0.35 s)
period_is_generated_order_TCC1.....proved - complete [shostak](0.35 s)
period_is_generated_order.....proved - complete [shostak](1.39 s)
period_element_divides_group.....proved - complete [shostak](0.38 s)
period_element_divides_power.....proved - complete [shostak](0.50 s)
Theory finite_groups totals: 17 formulas, 17 attempted, 17 succeeded (9.09 s)

```


Proof summary for **theory** general_properties

seq_power_TCC1.....	proved	- incomplete	[shostak](0.38 s)
only_power_p_TCC1.....	proved	- incomplete	[shostak](0.36 s)
divides_element.....	proved	- complete	[shostak](0.45 s)
divides_rel_primes_TCC1.....	proved	- complete	[shostak](0.37 s)
divides_rel_primes.....	proved	- incomplete	[shostak](0.71 s)
divides_product.....	proved	- incomplete	[shostak](0.63 s)
product_power_TCC1.....	proved	- complete	[shostak](0.36 s)
product_power.....	proved	- incomplete	[shostak](1.28 s)
product_only_power_TCC1.....	proved	- incomplete	[shostak](0.36 s)
product_only_power.....	proved	- incomplete	[shostak](2.53 s)
divides_power.....	proved	- complete	[shostak](0.72 s)
divides_prime_power_TCC1.....	proved	- complete	[shostak](0.36 s)
divides_prime_power_TCC2.....	proved	- complete	[shostak](0.37 s)
divides_prime_power.....	proved	- incomplete	[shostak](1.64 s)
gcd_1_TCC1.....	proved	- complete	[shostak](0.37 s)
gcd_1.....	proved	- incomplete	[shostak](0.47 s)
gcd_1_nd_TCC1.....	proved	- complete	[shostak](0.36 s)
gcd_1_nd.....	proved	- incomplete	[shostak](0.43 s)
gcd_1_ndp.....	proved	- incomplete	[shostak](0.56 s)
gcd_1_gcd_1_TCC1.....	proved	- incomplete	[shostak](0.37 s)
gcd_1_gcd_1_TCC2.....	proved	- incomplete	[shostak](0.37 s)
gcd_1_gcd_1.....	proved	- incomplete	[shostak](0.42 s)

Theory general_properties totals: 22 formulas, 22 attempted, 22 succeeded
(13.85 s)

Proof summary for **theory** right_left_cosets

IMP_lagrange_TCC1.....	proved	- complete	[shostak](0.35 s)
nonempty_left_coset_TCC1.....	proved	- complete	[shostak](0.41 s)
nonempty_left_coset.....	proved	- complete	[shostak](0.37 s)
left_coset_finite_TCC1.....	proved	- complete	[shostak](0.38 s)
left_coset_finite.....	proved	- complete	[shostak](0.40 s)
left_coset_correspondence.....	proved	- complete	[shostak](0.42 s)
left_coset_correspondence_inv.....	proved	- complete	[shostak](0.45 s)
finite_left_coset_correspondence_TCC1...	proved	- complete	[shostak](0.37 s)
finite_left_coset_correspondence_TCC2...	proved	- complete	[shostak](0.39 s)
finite_left_coset_correspondence_TCC3...	proved	- complete	[shostak](0.37 s)
finite_left_coset_correspondence.....	proved	- incomplete	[shostak](0.55 s)
set_left_cosets_full.....	proved	- complete	[shostak](0.41 s)
left_cosets_disjoint.....	proved	- complete	[shostak](0.53 s)
left_cosets_partition.....	proved	- complete	[shostak](0.43 s)
set_right_cosets_full_1.....	proved	- complete	[shostak](0.42 s)
right_left_correspondence.....	proved	- complete	[shostak](1.02 s)
finite_right_left_correspondence_TCC1...	proved	- complete	[shostak](0.40 s)
finite_right_left_correspondence_TCC2...	proved	- complete	[shostak](0.39 s)
finite_right_left_correspondence.....	proved	- incomplete	[shostak](0.44 s)
index_TCC1.....	proved	- complete	[shostak](0.38 s)
index_gt1.....	proved	- complete	[shostak](0.40 s)
divide_TCC1.....	proved	- complete	[shostak](0.37 s)
divide_TCC2.....	proved	- complete	[shostak](0.36 s)
divide_TCC3.....	proved	- complete	[shostak](0.99 s)
card_factor_TCC1.....	proved	- complete	[shostak](0.65 s)
card_factor_TCC2.....	proved	- complete	[shostak](0.36 s)
card_factor.....	proved	- complete	[shostak](1.46 s)

Theory right_left_cosets totals: 27 formulas, 27 attempted, 27 succeeded (13.49 s)

Proof summary for **theory** lagrange

IMP_group_TCC1.....	proved - complete	[shostak](0.31 s)
right_coset_finite_TCC1.....	proved - complete	[shostak](0.39 s)
right_coset_finite.....	proved - complete	[shostak](0.43 s)
finite_right_coset_correspondence_TCC1...	proved - complete	[shostak](0.32 s)
finite_right_coset_correspondence_TCC2...	proved - complete	[shostak](0.33 s)
finite_right_coset_correspondence_TCC3...	proved - complete	[shostak](0.34 s)
finite_right_coset_correspondence.....	proved - complete	[shostak](0.52 s)
set_right_cosets_full.....	proved - complete	[shostak](0.38 s)
right_cosets_disjoint.....	proved - complete	[shostak](0.49 s)
right_cosets_partition.....	proved - complete	[shostak](0.42 s)
lagrange.....	proved - complete	[shostak](0.67 s)

Theory lagrange totals: 11 formulas, 11 attempted, 11 succeeded (4.60 s)

Proof summary for **theory** factor_groups

IMP_normal_subgroups_TCC1.....	proved - complete	[shostak](0.32 s)
p0.....	proved - complete	[shostak](0.38 s)
prep.....	proved - complete	[shostak](0.35 s)
mult_prep.....	proved - complete	[shostak](0.37 s)
mult_TCC1.....	proved - complete	[shostak](0.39 s)
mult_lem_TCC1.....	proved - complete	[shostak](0.33 s)
mult_lem_TCC2.....	proved - complete	[shostak](0.33 s)
mult_lem.....	proved - complete	[shostak](0.56 s)
mult_in.....	proved - complete	[shostak](0.38 s)
mult_is_coset.....	proved - complete	[shostak](0.35 s)
N_is_identity_TCC1.....	proved - complete	[shostak](0.35 s)
N_is_identity.....	proved - complete	[shostak](0.44 s)
left_cosets_group_TCC1.....	proved - complete	[shostak](0.36 s)
left_cosets_group_TCC2.....	proved - complete	[shostak](0.35 s)
left_cosets_group.....	proved - complete	[shostak](0.54 s)
over_TCC1.....	proved - complete	[shostak](0.35 s)

Theory factor_groups totals: 16 formulas, 16 attempted, 16 succeeded (6.14 s)

Proof summary for **theory** normal_subgroups

IMP_cosets_TCC1.....	proved - complete	[shostak](0.32 s)
normal_prep.....	proved - complete	[shostak](0.54 s)
normal_left_is_right.....	proved - complete	[shostak](0.41 s)
normal_subgroup_is_subgroup.....	proved - complete	[shostak](0.33 s)
nsg_prop.....	proved - complete	[shostak](0.36 s)
nsg_prop2.....	proved - complete	[shostak](0.36 s)
lc_gen_normal_TCC1.....	proved - complete	[shostak](0.36 s)
lc_gen_normal_TCC2.....	proved - complete	[shostak](0.33 s)
lc_gen_normal.....	proved - complete	[shostak](0.42 s)
abelian_normal.....	proved - complete	[shostak](0.37 s)

Theory normal_subgroups totals: 10 formulas, 10 attempted, 10 succeeded (3.82 s)

Proof summary for **theory** cosets

IMP_group_TCC1.....	proved - complete	[shostak](0.32 s)
congruence_is_equivalence.....	proved - complete	[shostak](0.45 s)
left_coset_subset.....	proved - complete	[shostak](0.36 s)
right_coset_subset.....	proved - complete	[shostak](0.33 s)

left_coset_one.....	proved	- complete	[shostak](0.35 s)
right_coset_one.....	proved	- complete	[shostak](0.34 s)
left_coset_assoc.....	proved	- complete	[shostak](0.37 s)
right_coset_assoc.....	proved	- complete	[shostak](0.38 s)
lr_coset_assoc.....	proved	- complete	[shostak](0.46 s)
subset_left_coset.....	proved	- complete	[shostak](0.33 s)
subset_right_coset.....	proved	- complete	[shostak](0.33 s)
right_coset_TCC1.....	proved	- complete	[shostak](0.32 s)
right_coset_image_TCC1.....	proved	- complete	[shostak](0.34 s)
right_coset_image.....	proved	- complete	[shostak](0.35 s)
right_coset_is.....	proved	- complete	[shostak](0.49 s)
right_coset_def.....	proved	- complete	[shostak](0.32 s)
nonempty_right_coset.....	proved	- complete	[shostak](0.33 s)
right_coset_correspondence_TCC1.....	proved	- complete	[shostak](0.35 s)
right_coset_correspondence.....	proved	- complete	[shostak](0.60 s)
left_coset_TCC1.....	proved	- complete	[shostak](0.32 s)
left_coset_image.....	proved	- complete	[shostak](0.36 s)
left_coset_def.....	proved	- complete	[shostak](0.33 s)
lc_gen_TCC1.....	proved	- complete	[shostak](0.34 s)
lc_gen_def_TCC1.....	proved	- complete	[shostak](1.04 s)
lc_gen_def.....	proved	- complete	[shostak](0.35 s)
rc_gen_TCC1.....	proved	- complete	[shostak](0.36 s)
rc_gen_def_TCC1.....	proved	- complete	[shostak](1.02 s)
rc_gen_def.....	proved	- complete	[shostak](0.35 s)
lc_eq.....	proved	- complete	[shostak](0.34 s)
lc_is_eq.....	proved	- complete	[shostak](0.38 s)
rc_eq.....	proved	- complete	[shostak](0.34 s)
rc_is_eq.....	proved	- complete	[shostak](0.38 s)

Theory cosets totals: 32 formulas, 32 attempted, 32 succeeded (13.01 s)

Proof summary for theory cyclic_group

IMP_group_TCC1.....	proved	- complete	[shostak](0.32 s)
generated_by_lem.....	proved	- complete	[shostak](0.34 s)
generated_is_subgroup.....	proved	- complete	[shostak](0.33 s)
generated_by_is_finite.....	proved	- complete	[shostak](0.44 s)
cyclic_abelian.....	proved	- complete	[shostak](0.38 s)
cyclic_subgroup.....	proved	- complete	[shostak](0.97 s)
is_cyclic.....	proved	- complete	[shostak](0.36 s)

Theory cyclic_group totals: 7 formulas, 7 attempted, 7 succeeded (3.15 s)

Proof summary for theory zp_group

Zn_group_TCC1.....	proved	- complete	[shostak](0.35 s)
Zn_group_TCC2.....	proved	- complete	[shostak](0.34 s)
Zn_group.....	proved	- complete	[shostak](0.81 s)
Zn_finite.....	proved	- complete	[shostak](0.37 s)
Zn_card_TCC1.....	proved	- complete	[shostak](0.34 s)
Zn_card.....	proved	- complete	[shostak](0.40 s)

Theory zp_group totals: 6 formulas, 6 attempted, 6 succeeded (2.62 s)

Proof summary for theory cauchy_scaf

set_seq_TCC1.....	proved	- complete	[shostak](0.37 s)
emptyset_gives_emptyset.....	proved	- incomplete	[shostak](0.36 s)
emptyset_gives_emptyset1.....	proved	- incomplete	[shostak](0.37 s)
set_seq_singleton.....	proved	- incomplete	[shostak](0.39 s)
set_seq_empty.....	proved	- complete	[shostak](0.37 s)

```

add_element_add_set.....proved - incomplete [shostak](0.71 s)
card_add_element_aux.....proved - incomplete [shostak](0.52 s)
card_add_element_TCC1.....proved - incomplete [shostak](0.40 s)
card_add_element.....proved - incomplete [shostak](0.53 s)
disjoint_add_set.....proved - incomplete [shostak](0.63 s)
add_set_is_add_ele.....proved - incomplete [shostak](0.63 s)
add_set_is_finite_aux.....proved - incomplete [shostak](0.52 s)
add_set_is_finite.....proved - incomplete [shostak](0.60 s)
card_add_set_TCC1.....proved - incomplete [shostak](0.39 s)
card_add_set.....proved - incomplete [shostak](0.90 s)
set_seq_is_finite.....proved - incomplete [shostak](1.10 s)
set_seq_is_add_set_TCC1.....proved - complete [shostak](0.35 s)
set_seq_is_add_set_TCC2.....proved - incomplete [shostak](0.37 s)
set_seq_is_add_set.....proved - incomplete [shostak](1.28 s)
card_set_seq_TCC1.....proved - incomplete [shostak](0.35 s)
card_set_seq_TCC2.....proved - complete [shostak](0.37 s)
card_set_seq.....proved - incomplete [shostak](0.84 s)
Theory cauchy_scaf totals: 22 formulas, 22 attempted, 22 succeeded (12.36 s)

```

Proof summary for theory top_rings

Theory top_rings totals: 0 formulas, 0 attempted, 0 succeeded (0.00 s)

Proof summary for theory boolean_ring_homomorphisms

```

S_TCC1.....proved - complete [shostak](0.35 s)
img_hom_bool_ring.....proved - complete [shostak](0.57 s)

```

Theory boolean_ring_homomorphisms totals: 2 formulas, 2 attempted, 2 succeeded (0.92 s)

Proof summary for theory boolean_ring_def

Theory boolean_ring_def totals: 0 formulas, 0 attempted, 0 succeeded (0.00 s)

Proof summary for theory chinese_remainder_theorem_Z

IMP_chinese_remainder_theorem_rings_TCC1...proved - complete [shostak](0.46 s)

```

nZ_mZ_comaximal_TCC1.....proved - complete [shostak](0.39 s)
nZ_mZ_comaximal.....proved - incomplete [shostak](0.62 s)
Intersection_add_first.....proved - incomplete [shostak](0.62 s)
nZ_fs_intersection.....proved - incomplete [shostak](1.48 s)
Chinese_Remainder_Theorem_for_int_TCC1...proved - incomplete [shostak](0.41 s)
Chinese_Remainder_Theorem_for_int_TCC2...proved - incomplete [shostak](0.39 s)
Chinese_Remainder_Theorem_for_int_TCC3...proved - incomplete [shostak](0.41 s)
Chinese_Remainder_Theorem_for_int_TCC4...proved - incomplete [shostak](0.42 s)
Chinese_Remainder_Theorem_for_int_TCC5...proved - incomplete [shostak](0.42 s)
Chinese_Remainder_Theorem_for_int_TCC6...proved - incomplete [shostak](0.43 s)
Chinese_Remainder_Theorem_for_int_TCC7...proved - incomplete [shostak](0.51 s)
Chinese_Remainder_Theorem_for_int_TCC8...proved - incomplete [shostak](0.61 s)
Chinese_Remainder_Theorem_for_int_TCC9...proved - incomplete [shostak](0.61 s)
Chinese_Remainder_Theorem_for_int_TCC10...proved - incomplete [shostak](0.49

```

```

s)
Chinese_Remainder_Theorem_for_int.....proved - incomplete [shostak](0.00 s)
gcd_lcm_property.....proved - incomplete [shostak](6.66 s)

```

Theory chinese_remainder_theorem_Z totals: 17 formulas, 17 attempted, 17 succeeded (14.92 s)

Proof summary for theory chinese_remainder_theorem_rings

```

IMP_product_finseq_sets_ring_TCC1....proved - complete [shostak]( 0.39 s)
oneSet_nonempty.....proved - complete [shostak]( 0.62 s)
surjective_aux_1_TCC1.....proved - complete [shostak]( 0.49 s)
surjective_aux_1.....proved - incomplete [shostak](14.00 s)
surjective_aux_2_TCC1.....proved - complete [shostak]( 0.43 s)
surjective_aux_2_TCC2.....proved - complete [shostak]( 0.59 s)
surjective_aux_2_TCC3.....proved - complete [shostak]( 0.60 s)
surjective_aux_2_TCC4.....proved - complete [shostak]( 0.44 s)
surjective_aux_2_TCC5.....proved - complete [shostak]( 0.62 s)
surjective_aux_2_TCC6.....proved - complete [shostak]( 0.61 s)
surjective_aux_2_TCC7.....proved - complete [shostak]( 0.61 s)
surjective_aux_2_TCC8.....proved - complete [shostak]( 0.43 s)
surjective_aux_2.....proved - incomplete [shostak]( 2.22 s)
CRT_aux_1_TCC1.....proved - complete [shostak]( 0.52 s)
CRT_aux_1_TCC2.....proved - complete [shostak]( 0.41 s)
CRT_aux_1_TCC3.....proved - complete [shostak]( 0.76 s)
CRT_aux_1_TCC4.....proved - complete [shostak]( 0.91 s)
CRT_aux_1_TCC5.....proved - complete [shostak]( 0.70 s)
CRT_aux_1_TCC6.....proved - complete [shostak]( 0.50 s)
CRT_aux_1.....proved - incomplete [shostak]( 0.81 s)
CRT_aux_2_TCC1.....proved - complete [shostak]( 0.47 s)
CRT_aux_2.....proved - incomplete [shostak]( 1.01 s)
Chinese_Remainder_Theorem_TCC1.....proved - complete [shostak]( 0.41 s)
Chinese_Remainder_Theorem_TCC2.....proved - incomplete [shostak]( 0.48 s)
Chinese_Remainder_Theorem_TCC3.....proved - incomplete [shostak]( 0.48 s)
Chinese_Remainder_Theorem_TCC4.....proved - incomplete [shostak]( 0.47 s)
Chinese_Remainder_Theorem_TCC5.....proved - incomplete [shostak]( 0.43 s)
Chinese_Remainder_Theorem_TCC6.....proved - complete [shostak]( 0.79 s)
Chinese_Remainder_Theorem_TCC7.....proved - complete [shostak]( 0.98 s)
Chinese_Remainder_Theorem_TCC8.....proved - complete [shostak]( 0.99 s)
Chinese_Remainder_Theorem_TCC9.....proved - complete [shostak]( 0.79 s)
Chinese_Remainder_Theorem.....proved - incomplete [shostak]( 4.27 s)

```

Theory chinese_remainder_theorem_rings totals: 32 formulas, 32 attempted, 32 succeeded (38.24 s)

Proof summary for **theory** product_finseq_sets_ring

```

IMP_cartesian_product_quot_ring_TCC1...proved - complete [shostak](0.41 s)
IMP_ring_with_one_TCC1.....proved - complete [shostak](0.46 s)
product_fs_rec_TCC1.....proved - complete [shostak](0.42 s)
product_fs_rec_TCC2.....proved - complete [shostak](0.43 s)
product_fs_rec_TCC3.....proved - complete [shostak](0.41 s)
product_fs_TCC1.....proved - complete [shostak](0.41 s)
product_of_sets_TCC1.....proved - complete [shostak](0.43 s)
product_of_sets_TCC2.....proved - complete [shostak](0.43 s)
product_fs_emptyseq.....proved - complete [shostak](0.41 s)
product_fs_1.....proved - complete [shostak](0.42 s)
product_fs_rec_caret_TCC1.....proved - complete [shostak](0.56 s)
product_fs_rec_caret.....proved - complete [shostak](0.63 s)
product_fs_rec_mult_TCC1.....proved - complete [shostak](0.48 s)
product_fs_rec_mult_TCC2.....proved - complete [shostak](0.52 s)
product_fs_rec_mult.....proved - complete [shostak](1.42 s)
product_fs_split_TCC1.....proved - complete [shostak](0.41 s)
product_fs_split_TCC2.....proved - complete [shostak](0.42 s)
product_fs_split.....proved - complete [shostak](0.53 s)
Product_fs_o.....proved - incomplete [shostak](0.65 s)

```

```

Product_fs_o_split_TCC1.....proved - complete [shostak](0.41 s)
Product_fs_o_split_TCC2.....proved - complete [shostak](0.41 s)
Product_fs_o_split.....proved - incomplete [shostak](1.30 s)
product_fs_rec_in_ring.....proved - complete [shostak](0.51 s)
product_fs_rec_in_each_TCC1.....proved - complete [shostak](0.70 s)
product_fs_rec_in_each.....proved - complete [shostak](1.38 s)
Intersection_of_ideals_is_ideal.....proved - incomplete [shostak](1.85 s)
product_of_ideals_subset_of_each.....proved - complete [shostak](0.65 s)
product_of_ideals_subset_intersection...proved - incomplete [shostak](0.53 s)
cartesian_product_fs_representative_TCC1...proved - complete [shostak](0.46
s)
cartesian_product_fs_representative_TCC2...proved - complete [shostak](0.47
s)
cartesian_product_fs_representative...proved - incomplete [shostak](3.01 s)
Theory product_finseq_sets_ring totals: 31 formulas, 31 attempted, 31
succeeded (21.53 s)

```

Proof summary for theory cartesian_product_quot_ring

```

IMP_quotient_rings_TCC1.....proved - complete [shostak](0.40 s)
Sfs_TCC1.....proved - complete [shostak](0.50 s)
Sfs_TCC2.....proved - complete [shostak](0.47 s)
Sfs_TCC3.....proved - complete [shostak](0.46 s)
Sfs_TCC4.....proved - complete [shostak](0.47 s)
cartesian_product_quot_ring_is_ring_TCC1...proved - complete [shostak](0.53
s)
cartesian_product_quot_ring_is_ring_TCC2...proved - complete [shostak](0.50
s)
cartesian_product_quot_ring_is_ring_TCC3...proved - complete [shostak](0.00
s)
cartesian_product_quot_ring_is_ring_TCC4...proved - complete [shostak](0.58
s)
cartesian_product_quot_ring_is_ring_TCC5...proved - complete [shostak](0.50
s)
cartesian_product_quot_ring_is_ring...proved - complete [shostak](5.89 s)
Theory cartesian_product_quot_ring totals: 11 formulas, 11 attempted, 11
succeeded (10.31 s)

```

Proof summary for theory cartesian_product_finite

```

cartesian_product_n_TCC1.....proved - complete [shostak](0.43 s)
cartesian_product_one_disjoint.....proved - incomplete [shostak](0.63 s)
cartesian_product_one_emptyset.....proved - incomplete [shostak](0.53 s)
cartesian_product_set_emptyset.....proved - incomplete [shostak](0.45 s)
cartesian_product_n_emptyset.....proved - complete [shostak](0.45 s)
cartesian_product_n_add_is_union.....proved - incomplete [shostak](1.12 s)
rest_card_fs.....proved - incomplete [shostak](0.89 s)
add_card_fs_TCC1.....proved - complete [shostak](0.41 s)
add_card_fs.....proved - incomplete [shostak](0.58 s)
cartesian_product_one_finite.....proved - incomplete [shostak](0.48 s)
cartesian_product_one_card_TCC1.....proved - incomplete [shostak](0.40 s)
cartesian_product_one_card.....proved - incomplete [shostak](0.59 s)
cartesian_product_set_finite_aux.....proved - incomplete [shostak](0.48 s)
cartesian_product_set_finite.....proved - incomplete [shostak](0.48 s)
cartesian_product_set_partition.....proved - incomplete [shostak](0.46 s)
cartesian_product_set_card_aux_TCC1...proved - incomplete [shostak](0.41 s)
cartesian_product_set_card_aux.....proved - incomplete [shostak](0.87 s)

```



```

cartesian_product_set_card_TCC1.....proved - incomplete [shostak](0.46 s)
cartesian_product_set_card.....proved - incomplete [shostak](0.56 s)
cartesian_product_n_finite.....proved - incomplete [shostak](1.12 s)
cartesian_product_n_degenerated_TCC1...proved - complete [shostak](0.41 s)
cartesian_product_n_degenerated.....proved - complete [shostak](0.56 s)
cartesian_product_n_card_degenerated_TCC1...proved - incomplete [shostak](0.41

```

s)

```

cartesian_product_n_card_degenerated...proved - incomplete [shostak](0.56 s)
cartesian_product_n_card_TCC1.....proved - incomplete [shostak](0.41 s)
cartesian_product_n_card_TCC2.....proved - complete [shostak](0.45 s)
cartesian_product_n_card.....proved - incomplete [shostak](1.77 s)

```

Theory cartesian_product_finite totals: 27 formulas, 27 attempted, 27

succeeded (16.40 s)

Proof summary for **theory** lagrange_scaf

```

partition_TCC1.....proved - complete [shostak](0.36 s)
finite_partition_TCC1.....proved - complete [shostak](0.40 s)
finite_partition_is_partition.....proved - complete [shostak](0.39 s)
card_Union_rest.....proved - complete [shostak](0.39 s)
card_equal_partition_TCC1.....proved - complete [shostak](0.34 s)
card_equal_partition_TCC2.....proved - complete [shostak](0.34 s)
card_equal_partition.....proved - complete [shostak](1.16 s)
card_eq_part_TCC1.....proved - complete [shostak](0.42 s)
card_eq_part_TCC2.....proved - complete [shostak](0.42 s)
card_eq_part_TCC3.....proved - complete [shostak](0.33 s)
card_eq_part_TCC4.....proved - complete [shostak](0.44 s)
card_eq_part.....proved - complete [shostak](0.38 s)

```

Theory lagrange_scaf totals: 12 formulas, 12 attempted, 12 succeeded (5.35 s)

Proof summary for **theory** sigma_R_below

```

IMP_ring_TCC1.....proved - complete [shostak](0.40 s)
R_sigma_below_TCC1.....proved - complete [shostak](0.46 s)
R_sigma_below_TCC2.....proved - complete [shostak](0.50 s)
R_sigma_below_TCC3.....proved - complete [shostak](0.42 s)
R_sigma_b_eq_arg.....proved - complete [shostak](0.41 s)
R_sigma_b_spl_TCC1.....proved - complete [shostak](0.51 s)
R_sigma_b_spl.....proved - complete [shostak](0.73 s)
R_sigma_b_split_TCC1.....proved - complete [shostak](0.43 s)
R_sigma_b_split.....proved - complete [shostak](0.54 s)
R_sigma_b_first_TCC1.....proved - complete [shostak](0.42 s)
R_sigma_b_first.....proved - complete [shostak](0.46 s)
R_sigma_b_last_TCC1.....proved - complete [shostak](0.42 s)
R_sigma_b_last_TCC2.....proved - complete [shostak](0.43 s)
R_sigma_b_last.....proved - complete [shostak](0.43 s)
R_sigma_b_middle_TCC1.....proved - complete [shostak](0.42 s)
R_sigma_b_middle.....proved - complete [shostak](0.47 s)
R_sigma_b_left_aux.....proved - complete [shostak](0.52 s)
R_sigma_b_left.....proved - complete [shostak](0.43 s)
R_sigma_b_right_aux.....proved - complete [shostak](0.52 s)
R_sigma_b_right.....proved - complete [shostak](0.43 s)
R_sigma_b_inv_aux.....proved - complete [shostak](0.55 s)
R_sigma_b_inv.....proved - complete [shostak](0.43 s)
R_sigma_b_eq_k_aux_TCC1.....proved - complete [shostak](0.41 s)
R_sigma_b_eq_k_aux_TCC2.....proved - complete [shostak](0.51 s)
R_sigma_b_eq_k_aux.....proved - complete [shostak](0.84 s)

```

```

R_sigma_b_eq_k_TCC1.....proved - complete [shostak](0.42 s)
R_sigma_b_eq_k_TCC2.....proved - complete [shostak](0.50 s)
R_sigma_b_eq_k.....proved - complete [shostak](0.51 s)
R_sigma_b_in_ideal_aux_TCC1.....proved - complete [shostak](0.42 s)
R_sigma_b_in_ideal_aux.....proved - complete [shostak](0.66 s)
R_sigma_b_in_ideal_TCC1.....proved - complete [shostak](0.43 s)
R_sigma_b_in_ideal.....proved - complete [shostak](0.47 s)
R_sigma_b_add_zero_aux.....proved - complete [shostak](0.46 s)
R_sigma_b_add_zero.....proved - complete [shostak](0.44 s)
Theory sigma_R_below totals: 34 formulas, 34 attempted, 34 succeeded (16.36 s)

```

Proof summary for theory comaximal_finseqs_ideals

```

IMP_ring_cosets_lemmas_TCC1.....proved - complete [shostak](0.41 s)
IMP_ring_with_one_TCC1.....proved - complete [shostak](0.46 s)
comaximal_ideals_equiv.....proved - complete [shostak](0.56 s)
Theory comaximal_finseqs_ideals totals: 3 formulas, 3 attempted, 3 succeeded

```

(1.43 s)

Proof summary for theory finite_integral_domain

```

IMP_integral_domain_TCC1.....proved - complete [shostak](0.41 s)
IMP_ring_nz_closed_TCC1.....proved - complete [shostak](0.47 s)
IMP_ring_with_one_basic_properties_TCC1...proved - complete [shostak](0.42

```

s)

```

surj_equiv_inj_fin_sets.....proved - complete [shostak](1.31 s)
zero_ring_is_fin_int_dom.....proved - complete [shostak](0.54 s)
nzx_member_S.....proved - complete [shostak](0.83 s)
auxiliar_map_TCC1.....proved - complete [shostak](0.44 s)
auxiliar_map.....proved - complete [shostak](0.50 s)
building_one.....proved - complete [shostak](0.53 s)
one_is_member_S.....proved - complete [shostak](0.47 s)
fin_int_domain_is_ring_with_one.....proved - complete [shostak](0.49 s)
fin_int_domain_is_mult_group.....proved - complete [shostak](0.58 s)
fin_int_domain_is_field.....proved - complete [shostak](0.44 s)
Theory finite_integral_domain totals: 13 formulas, 13 attempted, 13 succeeded

```

(7.43 s)

Proof summary for theory ring_binomial_theorem

```

IMP_ring_with_one_basic_properties_TCC1...proved - complete [shostak](0.42

```

s)

```

F_bino_TCC1.....proved - complete [shostak](0.59 s)
F_bino_TCC2.....proved - complete [shostak](0.44 s)
R_bino_theo.....proved - incomplete [shostak](2.78 s)
Theory ring_binomial_theorem totals: 4 formulas, 4 attempted, 4 succeeded

```

(4.23 s)

Proof summary for theory zero_ring

```

groupoid_plus_equiv.....proved - complete [shostak](0.47 s)
groupoid_times_equiv.....proved - complete [shostak](0.47 s)
zero_ring.....proved - complete [shostak](0.73 s)
zero_ring_with_one.....proved - complete [shostak](0.52 s)
Theory zero_ring totals: 4 formulas, 4 attempted, 4 succeeded (2.19 s)

```

Proof summary for theory ring_general_results_extras

```

IMP_ring_homomorphism_lemmas_TCC1.....proved - complete [shostak](0.43 s)
IMP_ring_homomorphism_lemmas_TCC2.....proved - complete [shostak](0.43 s)

```

```

IMP_ring_general_results_TCC1.....proved - complete [shostak](0.53 s)
no_prop_id_mono_TCC1.....proved - complete [shostak](0.48 s)
no_prop_id_mono_TCC2.....proved - complete [shostak](0.47 s)
no_prop_id_mono_TCC3.....proved - complete [shostak](0.48 s)
no_prop_id_mono_TCC4.....proved - complete [shostak](0.46 s)
no_prop_id_mono_TCC5.....proved - complete [shostak](0.47 s)
no_prop_id_mono.....proved - complete [shostak](1.66 s)
mono_no_prop_id.....proved - complete [shostak](0.52 s)
Theory ring_general_results_extras totals: 10 formulas, 10 attempted, 10
succeeded (5.93 s)

```

Proof summary for **theory** ring_general_results

```

IMP_ring_with_one_basic_properties_TCC1...proved - complete [shostak](0.43
s)
IMP_ring_characteristic_def_TCC1.....proved - complete [shostak](0.51 s)
homomorphism_Z_to_R_TCC1.....proved - complete [shostak](0.47 s)
homomorphism_Z_to_R.....proved - complete [shostak](1.24 s)
gen_times_char_one.....proved - complete [shostak](0.51 s)
nz_closed_char_prime.....proved - incomplete [shostak](0.76 s)
field_zero_maximal_ideal.....proved - complete [shostak](4.42 s)
maximal_ideal_iff_proper_id.....proved - complete [shostak](0.54 s)
proper_id_zero_maximal_ideal.....proved - complete [shostak](0.56 s)
Theory ring_general_results totals: 9 formulas, 9 attempted, 9 succeeded (9.45
s)

```

Proof summary for **theory** ring_with_one_homomorphism_extras

```

R_TCC1.....proved - complete [shostak](0.45 s)
S_TCC1.....proved - complete [shostak](0.44 s)
isomorphic_fields_charac.....proved - complete [shostak](0.84 s)
ring_w_one_isomorphic_groupoid.....proved - complete [shostak](1.11 s)
Theory ring_with_one_homomorphism_extras totals: 4 formulas, 4 attempted, 4
succeeded (2.84 s)

```

Proof summary for **theory** ring_with_one_homomorphism

```

IMP_ring_homomorphism_lemmas_TCC1....proved - complete [shostak](0.46 s)
IMP_ring_homomorphism_lemmas_TCC2....proved - complete [shostak](0.47 s)
R_TCC1.....proved - complete [shostak](0.54 s)
S_TCC1.....proved - complete [shostak](0.45 s)
epi_maps_ones_TCC1.....proved - complete [shostak](0.71 s)
epi_maps_ones.....proved - complete [shostak](0.69 s)
isomorphic_fields.....proved - complete [shostak](1.38 s)
Theory ring_with_one_homomorphism totals: 7 formulas, 7 attempted, 7 succeeded
(4.71 s)

```

Proof summary for **theory** ring_homomorphism_lemmas_extras

```

IMP_quotient_rings_TCC1.....proved - complete [shostak](0.43 s)
zero_natural_isomorphism_TCC1.....proved - complete [shostak](0.54 s)
zero_natural_isomorphism_TCC2.....proved - complete [shostak](0.49 s)
zero_natural_isomorphism_TCC3.....proved - complete [shostak](0.50 s)
zero_natural_isomorphism_TCC4.....proved - complete [shostak](0.49 s)
zero_natural_isomorphism.....proved - complete [shostak](1.02 s)
Theory ring_homomorphism_lemmas_extras totals: 6 formulas, 6 attempted, 6
succeeded (3.47 s)

```

Proof summary for **theory** ring_2nd_3rd_isomorphism_theorems

IMP_quotient_rings_TCC1.....	proved - complete	[shostak](0.46 s)
S_TCC1.....	proved - complete	[shostak](0.50 s)
second_isomorphism_th_ax_TCC1.....	proved - complete	[shostak](0.49 s)
second_isomorphism_th_ax_TCC2.....	proved - complete	[shostak](0.49 s)
second_isomorphism_th_ax_TCC3.....	proved - complete	[shostak](0.50 s)
second_isomorphism_th_ax_TCC4.....	proved - complete	[shostak](0.49 s)
second_isomorphism_th_ax_TCC5.....	proved - complete	[shostak](0.73 s)
second_isomorphism_th_ax.....	proved - complete	[shostak](2.22 s)
second_isomorphism_th_TCC1.....	proved - complete	[shostak](0.54 s)
second_isomorphism_th_TCC2.....	proved - complete	[shostak](0.52 s)
second_isomorphism_th_TCC3.....	proved - complete	[shostak](0.52 s)
second_isomorphism_th_TCC4.....	proved - complete	[shostak](0.51 s)
second_isomorphism_th_TCC5.....	proved - complete	[shostak](0.52 s)
second_isomorphism_th_TCC6.....	proved - complete	[shostak](0.51 s)
second_isomorphism_th_TCC7.....	proved - complete	[shostak](0.52 s)
second_isomorphism_th.....	proved - complete	[shostak](5.01 s)
third_isomorphism_th_ax_TCC1.....	proved - complete	[shostak](0.53 s)
third_isomorphism_th_ax_TCC2.....	proved - complete	[shostak](0.53 s)
third_isomorphism_th_ax_TCC3.....	proved - complete	[shostak](0.50 s)
third_isomorphism_th_ax_TCC4.....	proved - complete	[shostak](0.51 s)
third_isomorphism_th_ax_TCC5.....	proved - complete	[shostak](0.52 s)
third_isomorphism_th_ax_TCC6.....	proved - complete	[shostak](0.53 s)
third_isomorphism_th_ax_TCC7.....	proved - complete	[shostak](0.51 s)
third_isomorphism_th_ax_TCC8.....	proved - complete	[shostak](0.51 s)
third_isomorphism_th_ax_TCC9.....	proved - complete	[shostak](0.67 s)
third_isomorphism_th_ax.....	proved - complete	[shostak](0.00 s)
third_isomorphism_th_TCC1.....	proved - complete	[shostak](0.56 s)
third_isomorphism_th_TCC2.....	proved - complete	[shostak](0.65 s)
third_isomorphism_th_TCC3.....	proved - complete	[shostak](0.64 s)
third_isomorphism_th_TCC4.....	proved - complete	[shostak](0.48 s)
third_isomorphism_th_TCC5.....	proved - complete	[shostak](0.65 s)
third_isomorphism_th_TCC6.....	proved - complete	[shostak](0.66 s)
third_isomorphism_th_TCC7.....	proved - complete	[shostak](0.48 s)
third_isomorphism_th_TCC8.....	proved - complete	[shostak](0.48 s)
third_isomorphism_th_TCC9.....	proved - complete	[shostak](0.47 s)
third_isomorphism_th.....	proved - complete	[shostak](4.78 s)

Theory ring_2nd_3rd_isomorphism_theorems totals: 36 formulas, 36 attempted, 36 succeeded (29.20 s)

Proof summary for **theory** ring_1st_isomorphism_theorem

IMP_ring_homomorphism_lemmas_TCC1.....	proved - complete	[shostak](0.42 s)
IMP_ring_homomorphism_lemmas_TCC2.....	proved - complete	[shostak](0.41 s)
first_isomorphism_th_aux_1_TCC1.....	proved - complete	[shostak](0.53 s)
first_isomorphism_th_aux_1_TCC2.....	proved - complete	[shostak](0.51 s)
first_isomorphism_th_aux_1_TCC3.....	proved - complete	[shostak](0.44 s)
first_isomorphism_th_aux_1_TCC4.....	proved - complete	[shostak](0.45 s)
first_isomorphism_th_aux_1_TCC5.....	proved - complete	[shostak](0.44 s)
first_isomorphism_th_aux_1.....	proved - complete	[shostak](2.34 s)
first_isomorphism_th_aux_2.....	proved - complete	[shostak](0.49 s)
first_isomorphism_th_aux_3_TCC1.....	proved - complete	[shostak](0.70 s)
first_isomorphism_th_aux_3.....	proved - complete	[shostak](0.96 s)
first_isomorphism_th_aux_4_TCC1.....	proved - complete	[shostak](0.60 s)
first_isomorphism_th_aux_4.....	proved - complete	[shostak](0.51 s)
first_isomorphism_th_aux_5.....	proved - complete	[shostak](0.00 s)
first_isomorphism_th_aux_6.....	proved - complete	[shostak](0.49 s)

```

first_isomorphism_th_TCC1.....proved - complete [shostak](0.51 s)
first_isomorphism_th_TCC2.....proved - complete [shostak](0.46 s)
first_isomorphism_th_TCC3.....proved - complete [shostak](0.47 s)
first_isomorphism_th_TCC4.....proved - complete [shostak](0.45 s)
first_isomorphism_th.....proved - complete [shostak](3.09 s)
Theory ring_1st_isomorphism_theorem totals: 20 formulas, 20 attempted, 20
succeeded (14.27 s)

```

Proof summary for **theory** ring_homomorphism_lemmas

```

IMP_ring_basic_properties_TCC1.....proved - complete [shostak](0.35 s)
IMP_ring_basic_properties_TCC2.....proved - complete [shostak](0.40 s)
R_homo_plus_TCC1.....proved - complete [shostak](0.58 s)
R_homo_plus.....proved - complete [shostak](0.39 s)
R_homo_mult_TCC1.....proved - complete [shostak](0.92 s)
R_homo_mult.....proved - complete [shostak](0.38 s)
R_homo_equiv_TCC1.....proved - complete [shostak](0.50 s)
R_homo_equiv_TCC2.....proved - complete [shostak](0.74 s)
R_homo_equiv.....proved - complete [shostak](0.44 s)
zero_to_zero_TCC1.....proved - complete [shostak](0.40 s)
zero_to_zero.....proved - complete [shostak](0.44 s)
inv_to_inv_TCC1.....proved - complete [shostak](0.42 s)
inv_to_inv.....proved - complete [shostak](0.50 s)
epi_commutative.....proved - complete [shostak](0.44 s)
image_homo_is_subring.....proved - complete [shostak](0.99 s)
R_homo_image_subring.....proved - complete [shostak](0.97 s)
R_homo_inv_image_subring.....proved - complete [shostak](0.92 s)
R_kernel_is_subring.....proved - complete [shostak](0.78 s)
R_kernel_is_subgroup.....proved - complete [shostak](0.37 s)
monomorphism_charac.....proved - complete [shostak](0.85 s)
inv_iso_is_iso_TCC1.....proved - complete [shostak](1.83 s)
inv_iso_is_iso.....proved - complete [shostak](0.74 s)
R_isomorphic_groupoid_is_ring.....proved - complete [shostak](1.24 s)
R_kernel_is_ideal.....proved - complete [shostak](0.84 s)
R_epimorphism_image_ideal.....proved - complete [shostak](0.79 s)
R_homo_inv_image_ideal.....proved - complete [shostak](0.62 s)
R_kernel_in_inverse_image.....proved - complete [shostak](0.41 s)
inv_image_image_sum.....proved - complete [shostak](0.81 s)
inv_image_image_subring_TCC1.....proved - complete [shostak](0.54 s)
inv_image_image_subring.....proved - complete [shostak](0.75 s)
ring_natural_homo_TCC1.....proved - complete [shostak](0.38 s)
ring_natural_homo_TCC2.....proved - complete [shostak](0.39 s)
ring_natural_homo_TCC3.....proved - complete [shostak](0.49 s)
ring_natural_homo_TCC4.....proved - complete [shostak](0.38 s)
ring_natural_homo_TCC5.....proved - complete [shostak](0.38 s)
ring_natural_homo_TCC6.....proved - complete [shostak](0.49 s)
ring_natural_homo_TCC7.....proved - complete [shostak](0.39 s)
ring_natural_homo.....proved - complete [shostak](1.00 s)
Theory ring_homomorphism_lemmas totals: 38 formulas, 38 attempted, 38
succeeded (24.25 s)

```

Proof summary for **theory** ring_homomorphisms_def

```

Theory ring_homomorphisms_def totals: 0 formulas, 0 attempted, 0 succeeded
(0.00 s)

```

Proof summary for **theory** homomorphisms_def

homomorphism?_TCC1.....proved - complete [shostak](0.38 s)
Theory homomorphisms_def totals: 1 formulas, 1 attempted, 1 succeeded (0.38 s)

Proof summary for theory ring_euclidean_gcd_algorithm_Z

Z_TCC1.....proved - complete [shostak](0.49 s)
f_phi_Z_TCC1.....proved - complete [shostak](0.49 s)
f_phi_Z_TCC2.....proved - complete [shostak](0.50 s)
phi_Z_and_f_phi_Z_ok_TCC1.....proved - complete [shostak](0.67 s)
phi_Z_and_f_phi_Z_ok_TCC2.....proved - incomplete [shostak](2.19 s)
phi_Z_and_f_phi_Z_ok.....proved - incomplete [shostak](1.54 s)
euclidean_gcd_alg_correctness_in_Z_TCC1...proved - complete [shostak](0.56 s)
s) euclidean_gcd_alg_correctness_in_Z_TCC2...proved - complete [shostak](0.53 s)
s) euclidean_gcd_alg_correctness_in_Z_TCC3...proved - complete [shostak](0.53 s)
s) euclidean_gcd_alg_correctness_in_Z_TCC4...proved - incomplete [shostak](0.75 s)
s) euclidean_gcd_alg_correctness_in_Z_TCC5...proved - complete [shostak](0.64 s)
s) euclidean_gcd_alg_correctness_in_Z....proved - incomplete [shostak](0.69 s)
Theory ring_euclidean_gcd_algorithm_Z totals: 12 formulas, 12 attempted, 12 succeeded (9.60 s)

Proof summary for theory ring_euclidean_gcd_algorithm_Zi

Zi_is_ring.....proved - incomplete [shostak](1.34 s)
Zi_is_integral_domain_w_one.....proved - incomplete [shostak](1.25 s)
sq_abs_Re_Im_integer_rational_pred_TCC1...proved - incomplete [shostak](0.55 s)
s) sq_abs_Re_Im_integer_rational_pred....proved - incomplete [shostak](0.71 s)
times_conjugate_is_Zi.....proved - incomplete [shostak](1.20 s)
phi_Zi_TCC1.....proved - incomplete [shostak](0.66 s)
phi_Zi_is_multiplicative_TCC1.....proved - incomplete [shostak](0.62 s)
phi_Zi_is_multiplicative.....proved - incomplete [shostak](0.59 s)
div_rem_appx_TCC1.....proved - complete [shostak](0.60 s)
div_rem_appx_TCC2.....proved - complete [shostak](0.62 s)
div_rev_appx_correctness.....proved - incomplete [shostak](1.59 s)
f_phi_Zi_TCC1.....proved - incomplete [shostak](0.61 s)
f_phi_Zi_TCC2.....proved - incomplete [shostak](0.63 s)
f_phi_Zi_TCC3.....proved - incomplete [shostak](0.62 s)
f_phi_Zi_TCC4.....proved - incomplete [shostak](0.93 s)
f_phi_Zi_TCC5.....proved - incomplete [shostak](1.51 s)
phi_Zi_and_f_phi_Zi_ok_TCC1.....proved - incomplete [shostak](0.59 s)
phi_Zi_and_f_phi_Zi_ok_TCC2.....proved - incomplete [shostak](1.07 s)
phi_Zi_and_f_phi_Zi_ok_TCC3.....proved - incomplete [shostak](6.22 s)
phi_Zi_and_f_phi_Zi_ok.....proved - incomplete [shostak](8.11 s)
euclidean_gcd_alg_in_Zi_TCC1.....proved - incomplete [shostak](0.52 s)
euclidean_gcd_alg_in_Zi_TCC2.....proved - incomplete [shostak](0.50 s)
euclidean_gcd_alg_in_Zi_TCC3.....proved - incomplete [shostak](0.50 s)
euclidean_gcd_alg_in_Zi_TCC4.....proved - incomplete [shostak](0.51 s)
euclidean_gcd_alg_in_Zi_TCC5.....proved - incomplete [shostak](0.88 s)
euclidean_gcd_alg_in_Zi_TCC6.....proved - incomplete [shostak](0.50 s)
euclidean_gcd_alg_in_Zi.....proved - incomplete [shostak](0.66 s)
Theory ring_euclidean_gcd_algorithm_Zi totals: 27 formulas, 27 attempted, 27 succeeded (34.10 s)

Proof summary for **theory** ring_euclidean_algorithm

IMP_euclidean_domain_TCC1.....	proved - complete	[shostak](0.46 s)
euclidean_gcd_algorithm_TCC1.....	proved - complete	[shostak](0.85 s)
euclidean_gcd_algorithm_TCC2.....	proved - complete	[shostak](0.63 s)
euclidean_gcd_algorithm_TCC3.....	proved - complete	[shostak](0.63 s)
euclidean_gcd_algorithm_TCC4.....	proved - complete	[shostak](0.62 s)
euclidean_gcd_algorithm_TCC5.....	proved - complete	[shostak](0.70 s)
euclidean_gcd_algorithm_TCC6.....	proved - complete	[shostak](0.60 s)
euclidean_gcd_algorithm_TCC7.....	proved - complete	[shostak](0.73 s)
euclidean_gcd_algorithm_TCC8.....	proved - complete	[shostak](0.64 s)
euclidean_gcd_algorithm_TCC9.....	proved - complete	[shostak](11.49 s)
euclidean_gcd_algorithm_TCC10.....	proved - complete	[shostak](0.73 s)
euclidean_gcd_algorithm_TCC11.....	proved - complete	[shostak](0.63 s)
euclidean_gcd_algorithm_TCC12.....	proved - complete	[shostak](0.75 s)
Euclid_theorem_TCC1.....	proved - complete	[shostak](0.51 s)
Euclid_theorem_TCC2.....	proved - complete	[shostak](0.52 s)
Euclid_theorem.....	proved - complete	[shostak](0.00 s)
euclidean_gcd_alg_correctness_TCC1...	proved - complete	[shostak](0.58 s)
euclidean_gcd_alg_correctness.....	proved - complete	[shostak](4.91 s)

Theory ring_euclidean_algorithm totals: 18 formulas, 18 attempted, 18 succeeded (25.98 s)

Proof summary for **theory** euclidean_domain

IMP_ring_with_one_basic_properties_TCC1...	proved - complete	[shostak](0.47 s)
IMP_euclidean_ring_TCC1.....	proved - complete	[shostak](0.57 s)
integers_is_euclidean_domain.....	proved - complete	[shostak](1.03 s)
field_is_euclidean_domain.....	proved - complete	[shostak](0.82 s)
euclidean_is_unique_factorization_domain_TCC1...	proved - complete	[shostak](0.80 s)
euclidean_is_unique_factorization_domain...	proved - incomplete	[shostak](0.55 s)

Theory euclidean_domain totals: 6 formulas, 6 attempted, 6 succeeded (4.24 s)

Proof summary for **theory** euclidean_domain_def

Theory euclidean_domain_def totals: 0 formulas, 0 attempted, 0 succeeded (0.00 s)

Proof summary for **theory** ring_zn

IMP_quotient_rings_with_one_TCC1.....	proved - complete	[shostak](0.70 s)
nZ_add_TCC1.....	proved - complete	[shostak](0.49 s)
Z_ring.....	proved - complete	[shostak](0.39 s)
Z_TCC1.....	proved - complete	[shostak](0.38 s)
Z1_is_Z.....	proved - complete	[shostak](0.40 s)
Z_commutative_ring_w_one.....	proved - complete	[shostak](0.49 s)
nZ_ideal.....	proved - complete	[shostak](1.24 s)
Z_nz_closed.....	proved - complete	[shostak](0.60 s)
Z_integral_domain_w_one.....	proved - complete	[shostak](0.47 s)
Zn_finite_set.....	proved - complete	[shostak](1.10 s)
Zn_card_n_TCC1.....	proved - complete	[shostak](0.39 s)
Zn_card_n.....	proved - complete	[shostak](1.43 s)
Zn_commutative_ring_w_one_TCC1.....	proved - complete	[shostak](0.40 s)
Zn_commutative_ring_w_one_TCC2.....	proved - complete	[shostak](0.41 s)
Zn_commutative_ring_w_one_TCC3.....	proved - complete	[shostak](0.40 s)

```

Zn_commutative_ring_w_one_TCC4.....proved - complete [shostak](0.40 s)
Zn_commutative_ring_w_one_TCC5.....proved - complete [shostak](0.43 s)
Zn_commutative_ring_w_one.....proved - complete [shostak](1.88 s)
equal_cosets_div.....proved - complete [shostak](0.71 s)
nZ_mZ_sum_TCC1.....proved - complete [shostak](0.38 s)
nZ_mZ_sum.....proved - incomplete [shostak](1.02 s)
nZ_mZ_intersection_TCC1.....proved - incomplete [shostak](0.44 s)
nZ_mZ_intersection.....proved - incomplete [shostak](1.23 s)
nZ_mZ_rel_prime_intersection.....proved - incomplete [shostak](0.44 s)
Zn_charac.....proved - incomplete [shostak](1.52 s)
Z2_charac.....proved - incomplete [shostak](0.56 s)
Zp_prime_is_nz_closed.....proved - incomplete [shostak](1.53 s)
Zp_nz_closed_is_prime_or_one.....proved - incomplete [shostak](1.03 s)
Zp_prime_is_division_ring.....proved - incomplete [shostak](2.56 s)
Zp_prime_is_field.....proved - incomplete [shostak](0.72 s)
nZ_mZ_subset.....proved - complete [shostak](0.60 s)
power_sum_nat.....proved - complete [shostak](0.56 s)
power_sum_int.....proved - complete [shostak](0.53 s)
nZ_is_cyclic.....proved - complete [shostak](0.50 s)
mZ_nZ_is_cyclic_TCC1.....proved - complete [shostak](0.49 s)
mZ_nZ_is_cyclic_TCC2.....proved - complete [shostak](0.43 s)
mZ_nZ_is_cyclic_TCC3.....proved - complete [shostak](0.49 s)
mZ_nZ_is_cyclic_TCC4.....proved - complete [shostak](0.43 s)
mZ_nZ_is_cyclic_TCC5.....proved - complete [shostak](0.46 s)
mZ_nZ_is_cyclic.....proved - complete [shostak](0.83 s)
Theory ring_zn totals: 40 formulas, 40 attempted, 40 succeeded (29.41 s)

```

Proof summary for **theory** prop_primes_extra

```

fs_rel_prime?_TCC1.....proved - complete [shostak](0.40 s)
lcm_div_TCC1.....proved - complete [shostak](0.38 s)
lcm_div_TCC2.....proved - incomplete [shostak](0.65 s)
lcm_div.....proved - incomplete [shostak](1.03 s)
primes_lcm_div_TCC1.....proved - complete [shostak](0.39 s)
primes_lcm_div.....proved - incomplete [shostak](0.45 s)
fs_rel_prime_fixed_TCC1.....proved - complete [shostak](0.38 s)
fs_rel_prime_fixed_TCC2.....proved - incomplete [shostak](0.40 s)
fs_rel_prime_fixed.....proved - incomplete [shostak](0.97 s)
fs_rel_prime_i_TCC1.....proved - incomplete [shostak](0.46 s)
fs_rel_prime_i.....proved - incomplete [shostak](0.48 s)

```

Theory prop_primes_extra totals: 11 formulas, 11 attempted, 11 succeeded (5.99

s)

Proof summary for **theory** division_ring_extras

```

IMP_ring_unit_TCC1.....proved - complete [shostak](0.37 s)
xyx_division_ring.....proved - complete [shostak](0.70 s)
div_ring_nz_unit_TCC1.....proved - complete [shostak](0.40 s)
div_ring_nz_unit.....proved - complete [shostak](0.48 s)
no_prop_l_ideal_div_ring.....proved - complete [shostak](0.41 s)
no_prop_r_ideal_div_ring.....proved - complete [shostak](0.40 s)
div_ring_no_prop_ideal.....proved - complete [shostak](0.48 s)

```

Theory division_ring_extras totals: 7 formulas, 7 attempted, 7 succeeded (3.24

s)

Proof summary for **theory** ring_w_one_xyx_is_x

```

IMP_ring_xyx_is_x_TCC1.....proved - complete [shostak](0.39 s)

```

```

IMP_ring_with_one_nz_closed_TCC1.....proved - complete [shostak](0.43 s)
xyx_one_is_member.....proved - complete [shostak](0.45 s)
xyx_ring_with_one.....proved - complete [shostak](0.43 s)
xyx_R_unit.....proved - complete [shostak](0.78 s)
unit_xyx_R_TCC1.....proved - complete [shostak](0.48 s)
unit_xyx_R.....proved - complete [shostak](0.55 s)
unit_nz_closed.....proved - complete [shostak](0.41 s)
Theory ring_w_one_xyx_is_x totals: 8 formulas, 8 attempted, 8 succeeded (3.91
s)

```

```

Proof summary for theory ring_xyx_is_x
IMP_ring_nz_closed_aux_TCC1.....proved - complete [shostak](0.38 s)
xyx_is_x_nz_divisor.....proved - complete [shostak](0.52 s)
xyx_is_x_nz_closed.....proved - complete [shostak](0.44 s)
xyx_is_y.....proved - complete [shostak](0.58 s)
xyx_has_identity.....proved - complete [shostak](0.80 s)
Theory ring_xyx_is_x totals: 5 formulas, 5 attempted, 5 succeeded (2.73 s)

```

```

Proof summary for theory ring_xyx_is_x_def
Theory ring_xyx_is_x_def totals: 0 formulas, 0 attempted, 0 succeeded (0.00 s)

```

```

Proof summary for theory ring_with_one_nz_closed
IMP_ring_with_one_TCC1.....proved - complete [shostak](0.39 s)
IMP_ring_nz_closed_aux_TCC1.....proved - complete [shostak](0.47 s)
subring_nz_closed_one_TCC1.....proved - complete [shostak](0.40 s)
subring_nz_closed_one.....proved - complete [shostak](0.00 s)
Theory ring_with_one_nz_closed totals: 4 formulas, 4 attempted, 4 succeeded
(1.25 s)

```

```

Proof summary for theory lcm
lcm_1.....proved - incomplete [shostak](0.50 s)
lcm_same.....proved - incomplete [shostak](0.49 s)
lcm_sym.....proved - incomplete [shostak](0.64 s)
lcm_divides.....proved - incomplete [shostak](0.56 s)
lcm_is_min.....proved - incomplete [shostak](0.51 s)
lcm_times.....proved - incomplete [shostak](1.08 s)
lcm_rel_prime_TCC1.....proved - complete [shostak](0.40 s)
lcm_rel_prime.....proved - incomplete [shostak](0.72 s)
lcm_gdm_rel.....proved - incomplete [shostak](1.11 s)
lcm_absorption.....proved - incomplete [shostak](0.71 s)
divides_lcm.....proved - incomplete [shostak](0.87 s)
Theory lcm totals: 11 formulas, 11 attempted, 11 succeeded (7.60 s)

```

```

Proof summary for theory euclidean_ring
IMP_ring_principal_ideal_TCC1.....proved - complete [shostak](0.49 s)
euclidean_ring_ideal_is_gen.....proved - incomplete [shostak](0.95 s)
euclidean_ring_is_principal_ideal.....proved - incomplete [shostak](0.65 s)
euclidean_ring_has_one.....proved - incomplete [shostak](0.59 s)
Theory euclidean_ring totals: 4 formulas, 4 attempted, 4 succeeded (2.68 s)

```

```

Proof summary for theory euclidean_ring_def
euclidean_ring?_TCC1.....proved - complete [shostak](0.68 s)
euclidean_ring?_TCC2.....proved - complete [shostak](0.75 s)
euclidean_ring?_TCC3.....proved - complete [shostak](0.57 s)
euclidean_ring?_TCC4.....proved - complete [shostak](0.57 s)

```

```

euclidean_pair?_TCC1.....proved - complete [shostak](0.67 s)
euclidean_pair?_TCC2.....proved - complete [shostak](0.77 s)
euclidean_pair?_TCC3.....proved - complete [shostak](0.52 s)
euclidean_pair?_TCC4.....proved - complete [shostak](0.52 s)
euclidean_f_phi?_TCC1.....proved - complete [shostak](0.50 s)
euclidean_f_phi?_TCC2.....proved - complete [shostak](0.50 s)
Theory euclidean_ring_def totals: 10 formulas, 10 attempted, 10 succeeded
(6.06 s)

```

```

Proof summary for theory ring_unique_factorization_domain
IMP_ring_principal_ideal_domain_TCC1...proved - complete [shostak](0.50 s)
UFD_prime_iff_irreducible.....proved - incomplete [shostak](6.28 s)
PID_is_UFD_TCC1.....proved - complete [shostak](0.79 s)
PID_is_UFD.....proved - incomplete [shostak](0.75 s)
Theory ring_unique_factorization_domain totals: 4 formulas, 4 attempted, 4
succeeded (8.32 s)

```

```

Proof summary for theory ring_unique_factorization_domain_def
unique_factorization_domain?_TCC1....proved - complete [shostak](0.81 s)
unique_factorization_domain?_TCC2....proved - complete [shostak](1.95 s)
unique_factorization_domain?_TCC3....proved - complete [shostak](1.06 s)
Theory ring_unique_factorization_domain_def totals: 3 formulas, 3 attempted, 3
succeeded (3.82 s)

```

```

Proof summary for theory ring_principal_ideal_domain
IMP_ring_with_one_maximal_ideal_TCC1...proved - complete [shostak]( 0.45 s)
IMP_ring_principal_ideal_TCC1.....proved - complete [shostak]( 0.54 s)
PID_maximal_prime_ideal.....proved - complete [shostak]( 2.03 s)
el_max_iff_one_gen_maximal.....proved - complete [shostak]( 1.04 s)
PID_prime_el_iff_irreducible_TCC1...proved - complete [shostak]( 0.73 s)
PID_prime_el_iff_irreducible.....proved - complete [shostak]( 1.50 s)
nonzero_nonunit_irreducible_divides_TCC1...proved - complete [shostak]( 0.60
s)

```

```

nonzero_nonunit_irreducible_divides...proved - incomplete [shostak]( 0.94 s)
non_fact_el_set_TCC1.....proved - complete [shostak]( 0.54 s)
non_fact_el_set_TCC2.....proved - complete [shostak]( 0.74 s)
empty_non_fact_el_set_aux1_TCC1.....proved - complete [shostak]( 0.78 s)
empty_non_fact_el_set_aux1_TCC2.....proved - complete [shostak]( 0.78 s)
empty_non_fact_el_set_aux1.....proved - incomplete [shostak]( 1.44 s)
phi_TCC1.....proved - complete [shostak]( 0.00 s)
phi_TCC2.....proved - complete [shostak]( 0.66 s)
phi_TCC3.....proved - complete [shostak]( 0.49 s)
phi_TCC4.....proved - incomplete [shostak]( 0.51 s)
empty_non_fact_el_set_aux2_TCC1.....proved - complete [shostak]( 0.48 s)
empty_non_fact_el_set_aux2_TCC2.....proved - complete [shostak]( 0.52 s)
empty_non_fact_el_set_aux2.....proved - incomplete [shostak]( 0.57 s)
empty_non_fact_el_set_aux3_TCC1.....proved - complete [shostak]( 0.49 s)
empty_non_fact_el_set_aux3.....proved - incomplete [shostak]( 0.95 s)
empty_non_fact_el_set.....proved - incomplete [shostak]( 0.67 s)
PID_factorization_existence.....proved - incomplete [shostak]( 0.50 s)
PID_factorization_uniqueness_TCC1...proved - complete [shostak]( 0.71 s)
PID_factorization_uniqueness_TCC2...proved - complete [shostak](23.36 s)
PID_factorization_uniqueness_TCC3...proved - complete [shostak]( 5.87 s)
PID_factorization_uniqueness.....proved - incomplete [shostak]( 1.42 s)
Theory ring_principal_ideal_domain totals: 28 formulas, 28 attempted, 28

```

succeeded (49.32 s)

Proof summary for **theory** ring_with_one_maximal_ideal

IMP_ring_with_one_prime_ideal_TCC1.....	proved - complete	[shostak](0.44 s)
IMP_ring_maximal_ideal_TCC1.....	proved - complete	[shostak](0.51 s)
ring_one_maximal_prime_ideal.....	proved - complete	[shostak](0.57 s)
maximal_ideal_quot_field_TCC1.....	proved - complete	[shostak](0.47 s)
maximal_ideal_quot_field_TCC2.....	proved - complete	[shostak](0.48 s)
maximal_ideal_quot_field_TCC3.....	proved - complete	[shostak](0.48 s)
maximal_ideal_quot_field_TCC4.....	proved - complete	[shostak](0.47 s)
maximal_ideal_quot_field_TCC5.....	proved - complete	[shostak](0.47 s)
maximal_ideal_quot_field.....	proved - complete	[shostak](0.00 s)
quot_div_ring_maximal_ideal_TCC1.....	proved - complete	[shostak](0.47 s)
quot_div_ring_maximal_ideal_TCC2.....	proved - complete	[shostak](0.48 s)
quot_div_ring_maximal_ideal_TCC3.....	proved - complete	[shostak](0.49 s)
quot_div_ring_maximal_ideal_TCC4.....	proved - complete	[shostak](0.47 s)
quot_div_ring_maximal_ideal_TCC5.....	proved - complete	[shostak](0.47 s)
quot_div_ring_maximal_ideal.....	proved - complete	[shostak](1.46 s)
maximal_ideal_charac_TCC1.....	proved - complete	[shostak](0.46 s)
maximal_ideal_charac_TCC2.....	proved - complete	[shostak](0.48 s)
maximal_ideal_charac_TCC3.....	proved - complete	[shostak](0.48 s)
maximal_ideal_charac_TCC4.....	proved - complete	[shostak](0.47 s)
maximal_ideal_charac_TCC5.....	proved - complete	[shostak](0.47 s)
maximal_ideal_charac.....	proved - complete	[shostak](0.48 s)
nonzero_ring_exists_maximal_ideal_aux...	proved - incomplete	[shostak](0.54 s)
nonzero_ring_exists_maximal_ideal.....	proved - incomplete	[shostak](0.50 s)

Theory ring_with_one_maximal_ideal totals: 23 formulas, 23 attempted, 23 succeeded (11.61 s)

Proof summary for **theory** ring_with_one_basic_properties

IMP_ring_with_one_TCC1.....	proved - complete	[shostak](0.40 s)
IMP_ring_basic_properties_TCC1.....	proved - complete	[shostak](0.47 s)
power_commute_aux.....	proved - complete	[shostak](0.43 s)
power_commute.....	proved - complete	[shostak](0.42 s)
gen_times_int_one.....	proved - complete	[shostak](0.41 s)
ring_w_one_is_idempotent.....	proved - complete	[shostak](0.47 s)
one_diff_zero_monad.....	proved - complete	[shostak](0.45 s)

Theory ring_with_one_basic_properties totals: 7 formulas, 7 attempted, 7 succeeded (3.04 s)

Proof summary for **theory** ring_with_one_prime_ideal

IMP_ring_prime_ideal_TCC1.....	proved - complete	[shostak](0.45 s)
IMP_quotient_rings_with_one_TCC1.....	proved - complete	[shostak](0.51 s)
prime_ideal_charac_TCC1.....	proved - complete	[shostak](0.49 s)
prime_ideal_charac_TCC2.....	proved - complete	[shostak](0.49 s)
prime_ideal_charac_TCC3.....	proved - complete	[shostak](0.48 s)
prime_ideal_charac_TCC4.....	proved - complete	[shostak](0.47 s)
prime_ideal_charac_TCC5.....	proved - complete	[shostak](0.48 s)
prime_ideal_charac.....	proved - complete	[shostak](0.95 s)

Theory ring_with_one_prime_ideal totals: 8 formulas, 8 attempted, 8 succeeded (4.31 s)

Proof summary for **theory** quotient_rings_with_one

IMP_quotient_rings_TCC1.....	proved - complete	[shostak](0.37 s)
IMP_ring_with_one_ideal_TCC1.....	proved - complete	[shostak](0.41 s)

```

quotient_ring_with_one_TCC1.....proved - complete [shostak](0.42 s)
quotient_ring_with_one_TCC2.....proved - complete [shostak](0.40 s)
quotient_ring_with_one_TCC3.....proved - complete [shostak](0.40 s)
quotient_ring_with_one_TCC4.....proved - complete [shostak](0.39 s)
quotient_ring_with_one_TCC5.....proved - complete [shostak](0.40 s)
quotient_ring_with_one.....proved - complete [shostak](0.52 s)
fullset_quot_ring_with_one.....proved - complete [shostak](0.75 s)
one_diff_zero_coset.....proved - complete [shostak](0.40 s)
Theory quotient_rings_with_one totals: 10 formulas, 10 attempted, 10 succeeded
(4.47 s)

```

Proof summary for theory ring_maximal_ideal

```

IMP_ring_prime_ideal_TCC1.....proved - complete [shostak](0.47 s)
maximal_prime_ideal.....proved - complete [shostak](1.07 s)
Theory ring_maximal_ideal totals: 2 formulas, 2 attempted, 2 succeeded (1.54
s)

```

Proof summary for theory ring_maximal_ideal_def

```

Theory ring_maximal_ideal_def totals: 0 formulas, 0 attempted, 0 succeeded
(0.00 s)

```

Proof summary for theory ring_principal_ideal_domain_def

```

Theory ring_principal_ideal_domain_def totals: 0 formulas, 0 attempted, 0
succeeded (0.00 s)

```

Proof summary for theory ring_unit

```

IMP_ring_ideal_TCC1.....proved - complete [shostak](0.38 s)
IMP_ring_with_one_TCC1.....proved - complete [shostak](0.42 s)
proper_id_iff_no_unit_TCC1.....proved - complete [shostak](0.43 s)
proper_id_iff_no_unit.....proved - complete [shostak](0.52 s)
no_prop_l_ideal_nz_unit_TCC1.....proved - complete [shostak](0.49 s)
no_prop_l_ideal_nz_unit.....proved - complete [shostak](0.60 s)
no_prop_r_ideal_nz_unit_TCC1.....proved - complete [shostak](0.49 s)
no_prop_r_ideal_nz_unit.....proved - complete [shostak](0.59 s)
Theory ring_unit totals: 8 formulas, 8 attempted, 8 succeeded (3.92 s)

```

Proof summary for theory ring_prime_element

```

IMP_ring_with_id_one_generator_TCC1...proved - complete [shostak](0.47 s)
IMP_ring_prime_ideal_TCC1.....proved - complete [shostak](0.54 s)
prime_el_iff_prime_ideal.....proved - complete [shostak](1.00 s)
el_irred_iff_one_gen_maximal_TCC1....proved - complete [shostak](0.70 s)
el_irred_iff_one_gen_maximal.....proved - complete [shostak](8.08 s)
prime_el_is_irreducible_TCC1.....proved - complete [shostak](0.65 s)
prime_el_is_irreducible.....proved - complete [shostak](0.82 s)
assoc_irreducible_is_irreducible_TCC1...proved - complete [shostak](0.60 s)
assoc_irreducible_is_irreducible_TCC2...proved - complete [shostak](0.68 s)
assoc_irreducible_is_irreducible_TCC3...proved - complete [shostak](0.64 s)
assoc_irreducible_is_irreducible.....proved - complete [shostak](0.63 s)
assoc_prime_is_prime_TCC1.....proved - complete [shostak](0.65 s)
assoc_prime_is_prime.....proved - complete [shostak](0.61 s)
irreducible_el_divisors_charac_TCC1...proved - complete [shostak](0.58 s)
irreducible_el_divisors_charac_TCC2...proved - complete [shostak](0.63 s)
irreducible_el_divisors_charac.....proved - complete [shostak](0.51 s)
prime_el_divides_TCC1.....proved - complete [shostak](0.57 s)
prime_el_divides_TCC2.....proved - complete [shostak](0.55 s)

```



```

prime_el_divides.....proved - incomplete [shostak](2.17 s)
prime_el_divides_last_pos_TCC1.....proved - complete [shostak](0.74 s)
prime_el_divides_last_pos.....proved - incomplete [shostak](3.81 s)
irreducible_prod_not_unit_TCC1.....proved - complete [shostak](0.64 s)
irreducible_prod_not_unit_TCC2.....proved - complete [shostak](0.55 s)
irreducible_prod_not_unit.....proved - incomplete [shostak](1.09 s)
irreducible_prod_unit_length_0_TCC1...proved - complete [shostak](0.55 s)
irreducible_prod_unit_length_0.....proved - incomplete [shostak](0.50 s)
irreducible_prod_not_zero.....proved - complete [shostak](3.05 s)
prod_unit_irreducible_is_irreducible_TCC1...proved - complete [shostak](0.58
s)
prod_unit_irreducible_is_irreducible_TCC2...proved - complete [shostak](0.63
s)
prod_unit_irreducible_is_irreducible_TCC3...proved - complete [shostak](1.24
s)
prod_unit_irreducible_is_irreducible...proved - complete [shostak](0.75 s)
prod_unit_irreducible_is_associates_TCC1...proved - complete [shostak](2.44
s)
prod_unit_irreducible_is_associates...proved - complete [shostak](0.61 s)
Theory ring_prime_element totals: 33 formulas, 33 attempted, 33 succeeded
(38.28 s)

```

Proof summary for **theory** ring_prime_element_def

```

R_prime_element?_TCC1.....proved - complete [shostak](0.47 s)
R_prime_element?_TCC2.....proved - complete [shostak](0.49 s)
Theory ring_prime_element_def totals: 2 formulas, 2 attempted, 2 succeeded
(0.96 s)

```

Proof summary for **theory** ring_irreducible_element_def

```

Theory ring_irreducible_element_def totals: 0 formulas, 0 attempted, 0
succeeded (0.00 s)

```

Proof summary for **theory** ring_prime_ideal

```

IMP_ring_one_generator_TCC1.....proved - complete [shostak](0.45 s)
prime_ideal_prop1.....proved - complete [shostak](0.55 s)
prime_ideal_prop2.....proved - complete [shostak](0.67 s)
prime_ideal_prod_closed.....proved - complete [shostak](0.58 s)
prime_ideal_nz_closed_TCC1.....proved - complete [shostak](0.48 s)
prime_ideal_nz_closed_TCC2.....proved - complete [shostak](0.49 s)
prime_ideal_nz_closed_TCC3.....proved - complete [shostak](0.48 s)
prime_ideal_nz_closed_TCC4.....proved - complete [shostak](0.47 s)
prime_ideal_nz_closed.....proved - complete [shostak](0.63 s)
Theory ring_prime_ideal totals: 9 formulas, 9 attempted, 9 succeeded (4.80 s)

```

Proof summary for **theory** ring_prime_ideal_def

```

Theory ring_prime_ideal_def totals: 0 formulas, 0 attempted, 0 succeeded (0.00
s)

```

Proof summary for **theory** quotient_rings

```

IMP_ring_cosets_lemmas_TCC1.....proved - complete [shostak](0.37 s)
add_charac_TCC1.....proved - complete [shostak](0.42 s)
add_charac_TCC2.....proved - complete [shostak](0.38 s)
add_charac.....proved - complete [shostak](0.70 s)
add_is_coset.....proved - complete [shostak](0.50 s)
coset_add.....proved - complete [shostak](0.41 s)

```

product_charac.....	proved - complete	[shostak](0.69 s)
lprod_equal_rprod_TCC1.....	proved - complete	[shostak](0.37 s)
lprod_equal_rprod_TCC2.....	proved - complete	[shostak](0.38 s)
lprod_equal_rprod_TCC3.....	proved - complete	[shostak](0.38 s)
lprod_equal_rprod_TCC4.....	proved - complete	[shostak](0.37 s)
lprod_equal_rprod.....	proved - complete	[shostak](0.98 s)
product_is_coset.....	proved - complete	[shostak](0.70 s)
coset_product.....	proved - complete	[shostak](0.40 s)
quotient_group_is_abelian_group_TCC1...	proved - complete	[shostak](0.38 s)
quotient_group_is_abelian_group_TCC2...	proved - complete	[shostak](0.38 s)
quotient_group_is_abelian_group_TCC3...	proved - complete	[shostak](0.38 s)
quotient_group_is_abelian_group.....	proved - complete	[shostak](0.85 s)
quotient_group_is_ring_TCC1.....	proved - complete	[shostak](0.37 s)
quotient_group_is_ring.....	proved - complete	[shostak](0.67 s)
fullset_quot_group_is_ring.....	proved - complete	[shostak](2.21 s)
inv_charac_TCC1.....	proved - complete	[shostak](0.38 s)
inv_charac_TCC2.....	proved - complete	[shostak](0.40 s)
inv_charac.....	proved - complete	[shostak](0.58 s)
coset_subring_TCC1.....	proved - complete	[shostak](0.38 s)
coset_subring_TCC2.....	proved - complete	[shostak](0.38 s)
coset_subring_TCC3.....	proved - complete	[shostak](0.37 s)
coset_subring_TCC4.....	proved - complete	[shostak](0.37 s)
coset_subring_TCC5.....	proved - complete	[shostak](0.37 s)
coset_subring.....	proved - complete	[shostak](0.70 s)
coset_ideal_TCC1.....	proved - complete	[shostak](0.38 s)
coset_ideal_TCC2.....	proved - complete	[shostak](0.38 s)
coset_ideal_TCC3.....	proved - complete	[shostak](0.38 s)
coset_ideal_TCC4.....	proved - complete	[shostak](0.38 s)
coset_ideal.....	proved - complete	[shostak](0.88 s)
commutative_quotient_ring_TCC1.....	proved - complete	[shostak](0.38 s)
commutative_quotient_ring_TCC2.....	proved - complete	[shostak](0.38 s)
commutative_quotient_ring_TCC3.....	proved - complete	[shostak](0.37 s)
commutative_quotient_ring_TCC4.....	proved - complete	[shostak](0.44 s)
commutative_quotient_ring.....	proved - complete	[shostak](0.44 s)
lcoset_power_nat.....	proved - complete	[shostak](0.57 s)
lcoset_power_int.....	proved - complete	[shostak](0.44 s)

Theory quotient_rings totals: 42 formulas, 42 attempted, 42 succeeded (21.65

s)

Proof summary for **theory** ring_cosets_lemmas

IMP_ring_ideal_TCC1.....	proved - complete	[shostak](0.37 s)
lcoset_iff_rcoset.....	proved - complete	[shostak](0.48 s)
lcoset_iff_coset.....	proved - complete	[shostak](0.38 s)
lcos_eq_rcos.....	proved - complete	[shostak](0.40 s)
self_coset.....	proved - complete	[shostak](0.51 s)
gen_is_any.....	proved - complete	[shostak](0.52 s)
lcos_eq.....	proved - complete	[shostak](0.39 s)
lcos_eq2.....	proved - complete	[shostak](0.46 s)
lc_gen_eq_TCC1.....	proved - complete	[shostak](0.37 s)
lc_gen_eq.....	proved - complete	[shostak](0.45 s)
ring_lcos_subset.....	proved - complete	[shostak](0.39 s)
ring_rcos_subset.....	proved - complete	[shostak](0.40 s)
left_zero.....	proved - complete	[shostak](0.38 s)
right_zero.....	proved - complete	[shostak](0.38 s)
ideal_is_coset.....	proved - complete	[shostak](0.41 s)

```

sum_subring_ideal.....proved - complete [shostak](0.64 s)
sum_ideal_ideal.....proved - complete [shostak](0.46 s)
sum_is_ideal_TCC1.....proved - complete [shostak](0.38 s)
sum_is_ideal.....proved - complete [shostak](0.44 s)
Theory ring_cosets_lemmas totals: 19 formulas, 19 attempted, 19 succeeded
(8.21 s)

```

```

Proof summary for theory product_coset_def
product_TCC1.....proved - complete [shostak](0.40 s)
product_TCC2.....proved - complete [shostak](0.39 s)
Theory product_coset_def totals: 2 formulas, 2 attempted, 2 succeeded (0.79 s)

```

```

Proof summary for theory ring_with_one_ideal
IMP_ring_with_one_TCC1.....proved - complete [shostak](0.36 s)
IMP_ring_ideal_TCC1.....proved - complete [shostak](0.44 s)
l_ideal_w_one_is_R.....proved - complete [shostak](0.42 s)
r_ideal_w_one_is_R.....proved - complete [shostak](0.43 s)
ideal_w_one_is_R.....proved - complete [shostak](0.39 s)
no_prop_l_ideal_nz_closed.....proved - complete [shostak](0.51 s)
no_prop_r_ideal_nz_closed.....proved - complete [shostak](0.51 s)
set_of_ideals_bounded_above.....proved - incomplete [shostak](0.89 s)
set_of_ideals_has_maximal.....proved - incomplete [shostak](1.05 s)
Theory ring_with_one_ideal totals: 9 formulas, 9 attempted, 9 succeeded (5.01
s)

```

```

Proof summary for theory ring_principal_ideal
IMP_ring_one_generator_TCC1.....proved - complete [shostak](0.46 s)
gen_is_member.....proved - complete [shostak](0.53 s)
principal_ideal_is_ideal.....proved - complete [shostak](0.59 s)
principal_ideal_charac.....proved - complete [shostak](0.83 s)
comm_principal_ideal_charac.....proved - complete [shostak](0.49 s)
principal_ideal_subset.....proved - complete [shostak](0.50 s)
stable_chain.....proved - complete [shostak](0.56 s)
Theory ring_principal_ideal totals: 7 formulas, 7 attempted, 7 succeeded (3.96
s)

```

```

Proof summary for theory ring_principal_ideal_def
Theory ring_principal_ideal_def totals: 0 formulas, 0 attempted, 0 succeeded
(0.00 s)

```

```

Proof summary for theory ring_divides
IMP_ring_with_id_one_generator_TCC1...proved - complete [shostak](0.42 s)
IMP_ring_nz_closed_aux_TCC1.....proved - complete [shostak](0.50 s)
IMP_op_finseq_TCC1.....proved - complete [shostak](0.43 s)
divides_subset_TCC1.....proved - complete [shostak](0.45 s)
divides_subset.....proved - complete [shostak](0.54 s)
divides_equal_TCC1.....proved - complete [shostak](0.44 s)
divides_equal_TCC2.....proved - complete [shostak](0.44 s)
divides_equal.....proved - complete [shostak](0.57 s)
associates_equiv_relation.....proved - complete [shostak](0.53 s)
unit_divides_TCC1.....proved - complete [shostak](0.45 s)
unit_divides.....proved - complete [shostak](0.51 s)
one_gen_unit_R.....proved - complete [shostak](0.52 s)
quot_unit_associates.....proved - complete [shostak](0.46 s)
int_domain_assoc_quot_unit_TCC1.....proved - complete [shostak](0.65 s)

```

int_domain_assoc_quot_unit.....	proved - complete	[shostak](0.59 s)
x_divides_x_TCC1.....	proved - complete	[shostak](0.52 s)
x_divides_x.....	proved - complete	[shostak](0.45 s)
int_domain_assoc_unit_TCC1.....	proved - complete	[shostak](0.56 s)
int_domain_assoc_unit_TCC2.....	proved - complete	[shostak](0.64 s)
int_domain_assoc_unit_TCC3.....	proved - complete	[shostak](0.58 s)
int_domain_assoc_unit.....	proved - complete	[shostak](0.54 s)
div_member_fseq_div_op_finseq_TCC1....	proved - complete	[shostak](1.62 s)
div_member_fseq_div_op_finseq.....	proved - complete	[shostak](2.12 s)
Theory ring_divides totals: 23 formulas, 23 attempted, 23 succeeded (14.54 s)		

Proof summary for **theory** ring_with_id_one_generator

IMP_ring_one_generator_TCC1.....	proved - complete	[shostak](0.43 s)
IMP_ring_with_one_TCC1.....	proved - complete	[shostak](0.47 s)
member_center_r_prod_is_one_gen.....	proved - complete	[shostak](0.93 s)
member_center_l_prod_is_one_gen.....	proved - complete	[shostak](0.98 s)
member_center_l_prod_is_r_prod.....	proved - complete	[shostak](0.44 s)
commutative_id_one_gen_charac.....	proved - complete	[shostak](0.54 s)
Theory ring_with_id_one_generator totals: 6 formulas, 6 attempted, 6 succeeded (3.80 s)		

Proof summary for **theory** ring_one_generator

IMP_ring_basic_properties_TCC1.....	proved - complete	[shostak](0.43 s)
F_one_gen_TCC1.....	proved - complete	[shostak](0.49 s)
gen_is_member_one_gen.....	proved - complete	[shostak](0.46 s)
one_gen_is_sum_closed_TCC1.....	proved - complete	[shostak](0.53 s)
one_gen_is_sum_closed.....	proved - complete	[shostak](1.42 s)
inv_one_gen_TCC1.....	proved - complete	[shostak](0.45 s)
inv_one_gen.....	proved - complete	[shostak](0.92 s)
one_gen_is_ideal.....	proved - complete	[shostak](0.00 s)
R_sigma_of_comm_factor_of_gen.....	proved - complete	[shostak](0.58 s)
commutative_one_gen_charac.....	proved - complete	[shostak](0.58 s)
commutative_one_gen_is_ideal.....	proved - complete	[shostak](0.44 s)
F_one_gen_r_comm.....	proved - complete	[shostak](0.50 s)
R_sigma_over_center_l.....	proved - complete	[shostak](0.52 s)
F_one_gen_l_comm.....	proved - complete	[shostak](0.48 s)
R_sigma_over_center_r.....	proved - complete	[shostak](0.51 s)
subset_product_one_gen_TCC1.....	proved - complete	[shostak](0.66 s)
subset_product_one_gen.....	proved - complete	[shostak](0.84 s)
subset_prod_one_gen_ideal_prop1.....	proved - complete	[shostak](0.51 s)
subset_prod_one_gen_ideal_prop2.....	proved - complete	[shostak](1.10 s)
sum_strict_subset_one_gen.....	proved - complete	[shostak](0.53 s)
Theory ring_one_generator totals: 20 formulas, 20 attempted, 20 succeeded (11.96 s)		

Proof summary for **theory** ring_ideal

IMP_ring_basic_properties_TCC1.....	proved - complete	[shostak](0.35 s)
left_ideal_equiv.....	proved - complete	[shostak](0.48 s)
right_ideal_equiv.....	proved - complete	[shostak](0.43 s)
ideal_equiv.....	proved - complete	[shostak](0.44 s)
self_ideal.....	proved - complete	[shostak](0.42 s)
zero_ideal.....	proved - complete	[shostak](0.43 s)
ideal_transitive_TCC1.....	proved - complete	[shostak](0.60 s)
ideal_transitive.....	proved - complete	[shostak](0.41 s)
intersection_subring_ideal_TCC1.....	proved - complete	[shostak](0.58 s)

```

intersection_subring_ideal.....proved - complete [shostak](0.46 s)
r_prod_is_sum_closed.....proved - complete [shostak](0.39 s)
l_prod_is_sum_closed.....proved - complete [shostak](0.39 s)
inv_is_member_l_prod.....proved - complete [shostak](0.40 s)
inv_is_member_r_prod.....proved - complete [shostak](0.41 s)
l_prod_is_r_ideal.....proved - complete [shostak](0.47 s)
r_prod_is_l_ideal.....proved - complete [shostak](0.46 s)
ideal_iunion_ideal.....proved - complete [shostak](0.56 s)
chain_ideal_union_ideal.....proved - incomplete [shostak](0.67 s)
Theory ring_ideal totals: 18 formulas, 18 attempted, 18 succeeded (8.34 s)

```

Proof summary for theory ring_ideal_def

```

Theory ring_ideal_def totals: 0 formulas, 0 attempted, 0 succeeded (0.00 s)

```

Proof summary for theory cosets_def

```

lc_gen_TCC1.....proved - complete [shostak](0.38 s)
rc_gen_TCC1.....proved - complete [shostak](0.38 s)
gen_TCC1.....proved - complete [shostak](0.37 s)
lcos_is_left_coset.....proved - complete [shostak](0.37 s)
rcos_is_right_coset.....proved - complete [shostak](0.36 s)
add_TCC1.....proved - complete [shostak](0.37 s)
Theory cosets_def totals: 6 formulas, 6 attempted, 6 succeeded (2.23 s)

```

Proof summary for theory ring_center

```

IMP_ring_basic_properties_TCC1.....proved - complete [shostak](0.37 s)
center_subring.....proved - complete [shostak](0.63 s)
commutative_ring_equal_center.....proved - complete [shostak](0.42 s)
Theory ring_center totals: 3 formulas, 3 attempted, 3 succeeded (1.42 s)

```

Proof summary for theory ring_unit_def

```

l_r_inv_equal.....proved - complete [shostak](0.44 s)
Theory ring_unit_def totals: 1 formulas, 1 attempted, 1 succeeded (0.44 s)

```

Proof summary for theory integral_domain_with_one_def

```

Theory integral_domain_with_one_def totals: 0 formulas, 0 attempted, 0
succeeded (0.00 s)

```

Proof summary for theory ring_gcd_def

```

gcd?_TCC1.....proved - complete [shostak](0.49 s)
gcd?_TCC2.....proved - complete [shostak](0.61 s)
Theory ring_gcd_def totals: 2 formulas, 2 attempted, 2 succeeded (1.10 s)

```

Proof summary for theory ring_divides_def

```

associates?_TCC1.....proved - complete [shostak](0.50 s)
associates?_TCC2.....proved - complete [shostak](0.50 s)
Theory ring_divides_def totals: 2 formulas, 2 attempted, 2 succeeded (0.99 s)

```

Proof summary for theory quaternions_Hamilton

```

IMP_quaternions_TCC1.....proved - complete [shostak](0.49 s)
IMP_quaternions_TCC2.....proved - complete [shostak](2.14 s)
conversion_inv_TCC1.....proved - complete [shostak](0.71 s)
conversion_inv.....proved - complete [shostak](0.66 s)
conversion_quot.....proved - complete [shostak](0.72 s)
quat_is_Real_p_Vector_part.....proved - complete [shostak](0.58 s)
decompose_eq_Real_Vector_part.....proved - complete [shostak](0.57 s)

```

```

Vector_part_scalar.....proved - complete [shostak](0.63 s)
q_prod_Real_Vector_part.....proved - complete [shostak](1.83 s)
conjugate_Real_vector_part.....proved - complete [shostak](0.74 s)
T_q_Real_charac.....proved - incomplete [shostak](1.90 s)
r_angle_TCC1.....proved - complete [shostak](0.62 s)
r_angle_TCC2.....proved - complete [shostak](0.62 s)
n_rot_axis_TCC1.....proved - incomplete [shostak](0.61 s)
rot_quat_TCC1.....proved - incomplete [shostak](0.59 s)
rot_quat_TCC2.....proved - incomplete [shostak](0.60 s)
rot_quat_TCC3.....proved - incomplete [shostak](1.75 s)
rot_quat__TCC1.....proved - incomplete [shostak](0.63 s)
conj_quat_eq.....proved - complete [shostak](0.66 s)
rot_quat_eq.....proved - incomplete [shostak](0.71 s)
mult_QH_eq.....proved - complete [shostak](1.14 s)
T_q__TCC1.....proved - complete [shostak](0.83 s)
T_q_eq.....proved - complete [shostak](0.59 s)
Quat_Rot_Aux0.....proved - incomplete [shostak](1.82 s)
Quat_Rot_Aux1.....proved - incomplete [shostak](1.67 s)
Quat_Rot_Aux2.....proved - incomplete [shostak](2.66 s)
Quat_Rot_Aux3.....proved - incomplete [shostak](0.00 s)
Quaternions_Rotation.....proved - incomplete [shostak](0.90 s)
Quaternions_Rotation_Deform_TCC1.....proved - incomplete [shostak](0.57 s)
Quaternions_Rotation_Deform_TCC2.....proved - incomplete [shostak](0.61 s)
Quaternions_Rotation_Deform_TCC3.....proved - incomplete [shostak](0.62 s)
Quaternions_Rotation_Deform_TCC4.....proved - incomplete [shostak](3.02 s)
Quaternions_Rotation_Deform.....proved - incomplete [shostak](2.21 s)
Theory quaternions_Hamilton totals: 33 formulas, 33 attempted, 33 succeeded
(34.40 s)

```

```

Proof summary for theory vectors_3D_extra
angle_nnreal_le_pi_exists.....proved - incomplete [shostak](1.09 s)
angle_between_nnreal_le_pi.....proved - incomplete [shostak](1.08 s)
norm_cross_charac_aux.....proved - incomplete [shostak](0.96 s)
norm_cross_charac.....proved - incomplete [shostak](1.17 s)
orth_cross.....proved - incomplete [shostak](0.72 s)
Theory vectors_3D_extra totals: 5 formulas, 5 attempted, 5 succeeded (5.02 s)

```

```

Proof summary for theory quaternions_Hamilton_extra
IMP_quaternions_TCC1.....proved - complete [shostak]( 0.53 s)
IMP_quaternions_TCC2.....proved - complete [shostak]( 1.05 s)
sc_part_red_norm_nnereal.....proved - complete [shostak]( 1.37 s)
QH_norm_TCC1.....proved - complete [shostak]( 1.35 s)
sq_QHnorm_charac.....proved - incomplete [shostak]( 0.78 s)
red_norm_QH1.....proved - incomplete [shostak]( 0.73 s)
norm_decomp.....proved - incomplete [shostak]( 6.74 s)
norm_decomp_1.....proved - incomplete [shostak]( 0.99 s)
QH_nzquat_nznorm.....proved - incomplete [shostak]( 1.53 s)
QH_times_is_zero_q.....proved - incomplete [shostak]( 0.84 s)
QH_inv_red_norm_TCC1.....proved - incomplete [shostak]( 0.58 s)
QH_inv_red_norm_TCC2.....proved - incomplete [shostak]( 0.60 s)
QH_inv_red_norm_TCC3.....proved - incomplete [shostak]( 0.57 s)
QH_inv_red_norm_TCC4.....proved - incomplete [shostak]( 0.60 s)
QH_inv_red_norm_TCC5.....proved - incomplete [shostak]( 1.30 s)
QH_inv_red_norm_TCC6.....proved - incomplete [shostak]( 0.62 s)
QH_inv_red_norm.....proved - incomplete [shostak](14.20 s)

```


QH_inv_q_prod_charac.....proved - incomplete [shostak](1.68 s)
 quat_trig_form_aux.....proved - incomplete [shostak](1.78 s)
 quat_trig_form.....proved - incomplete [shostak](1.06 s)
 QH_norm_is_norm.....proved - incomplete [shostak](0.96 s)
 Theory quaternions_Hamilton_extra totals: 21 formulas, 21 attempted, 21
 succeeded (39.88 s)

Proof summary for theory quaternions

IMP_quaternions_def_TCC1.....	proved - complete	[shostak](0.56 s)
IMP_ring_characteristic_def_TCC1.....	proved - complete	[shostak](0.55 s)
IMP_pvs_strategies_lemmas_TCC1.....	proved - complete	[shostak](0.54 s)
nz_quat_TCC1.....	proved - complete	[shostak](0.63 s)
one_sc_times.....	proved - complete	[SHOSTAK](1.22 s)
sc_quat.....	proved - complete	[SHOSTAK](0.55 s)
sc_sqr_i.....	proved - complete	[SHOSTAK](0.53 s)
sc_sqr_j.....	proved - complete	[SHOSTAK](0.56 s)
sc_ji_prod.....	proved - complete	[SHOSTAK](0.53 s)
inv_charac.....	proved - complete	[SHOSTAK](0.57 s)
q_inv_inv.....	proved - complete	[SHOSTAK](0.58 s)
quat_negative_times.....	proved - complete	[SHOSTAK](0.58 s)
quat_times_negative.....	proved - complete	[SHOSTAK](0.58 s)
ji_prod_lem.....	proved - complete	[SHOSTAK](0.56 s)
kk_prod.....	proved - complete	[SHOSTAK](0.64 s)
ki_prod.....	proved - complete	[SHOSTAK](0.58 s)
kj_prod.....	proved - complete	[SHOSTAK](0.58 s)
ik_prod.....	proved - complete	[SHOSTAK](0.52 s)
jk_prod.....	proved - complete	[SHOSTAK](0.56 s)
basis_quat.....	proved - complete	[SHOSTAK](0.53 s)
zero_q_plus.....	proved - complete	[SHOSTAK](0.60 s)
plus_zero_q.....	proved - complete	[SHOSTAK](0.59 s)
q_is_zero_iff_conj.....	proved - complete	[SHOSTAK](0.61 s)
sc_one_q.....	proved - complete	[SHOSTAK](0.78 s)
sc_one_quat.....	proved - complete	[SHOSTAK](0.51 s)
q_prod_charac.....	proved - complete	[SHOSTAK](3.93 s)
zero_q_times.....	proved - complete	[SHOSTAK](0.53 s)
times_zero_q.....	proved - complete	[SHOSTAK](0.53 s)
quat_is_ring_w_one.....	proved - complete	[SHOSTAK](1.17 s)
quat_x_pure_part_commutates.....	proved - complete	[SHOSTAK](0.58 s)
pure_quat_charac.....	proved - complete	[SHOSTAK](4.89 s)
red_trace_charac.....	proved - complete	[SHOSTAK](0.51 s)
red_trace_plus.....	proved - complete	[SHOSTAK](0.56 s)
red_trace_times_commutates.....	proved - complete	[SHOSTAK](0.65 s)
red_norm_charac.....	proved - complete	[SHOSTAK](0.86 s)
red_norm_one_q.....	proved - complete	[SHOSTAK](0.52 s)
red_norm_is_scalar.....	proved - complete	[SHOSTAK](0.51 s)
conj_conj_quat.....	proved - complete	[SHOSTAK](0.52 s)
conj_product_quat.....	proved - complete	[SHOSTAK](0.00 s)
q_x_cq_commutates.....	proved - complete	[SHOSTAK](0.56 s)
conj_product_quat_scalar.....	proved - complete	[shostak](0.56 s)
red_norm_conj.....	proved - complete	[SHOSTAK](0.50 s)
sc_F_commutates.....	proved - complete	[SHOSTAK](0.56 s)
center_quat_is_sc_F.....	proved - complete	[SHOSTAK](1.19 s)
sc_F_product_charac.....	proved - complete	[SHOSTAK](0.55 s)
q_x_v_cq.....	proved - complete	[SHOSTAK](2.53 s)
T_q_TCC1.....	proved - complete	[shostak](0.50 s)

T_q_is_linear_TCC1.....	proved	- complete	[shostak](0.53 s)
T_q_is_linear.....	proved	- complete	[SHOSTAK](0.57 s)
red_norm_prod.....	proved	- complete	[SHOSTAK](0.58 s)
T_q_red_norm_invariant.....	proved	- complete	[SHOSTAK](0.54 s)
T_q_invariant_red_norm_TCC1.....	proved	- complete	[shostak](0.51 s)
T_q_invariant_red_norm.....	proved	- complete	[SHOSTAK](0.55 s)
nz_red_norm_iff_inv_exist.....	proved	- complete	[SHOSTAK](0.94 s)
div_ring_iff_nz_rednorm.....	proved	- complete	[SHOSTAK](0.71 s)
inv_q_prod_charac_TCC1.....	proved	- complete	[shostak](1.48 s)
inv_q_prod_charac_TCC2.....	proved	- complete	[shostak](1.09 s)
inv_q_prod_charac_TCC3.....	proved	- complete	[shostak](0.65 s)
inv_q_prod_charac_TCC4.....	proved	- complete	[shostak](0.51 s)
inv_q_prod_charac.....	proved	- complete	[SHOSTAK](4.84 s)
quat_div_ring_aux1.....	proved	- complete	[SHOSTAK](0.70 s)
quat_div_ring_aux2.....	proved	- complete	[SHOSTAK](3.47 s)
quat_div_ring_char.....	proved	- complete	[SHOSTAK](5.61 s)

Theory quaternions totals: 63 formulas, 63 attempted, 63 succeeded (61.06 s)

Proof summary for **theory** quaternions_def

IMP_group_TCC1.....	proved	- complete	[shostak](0.50 s)
nzpure_quat_TCC1.....	proved	- complete	[shostak](0.52 s)

Theory quaternions_def totals: 2 formulas, 2 attempted, 2 succeeded (1.02 s)

Proof summary for **theory** ring_characteristic_def

IMP_ring_basic_properties_TCC1.....	proved	- complete	[shostak](0.44 s)
charac_TCC1.....	proved	- complete	[shostak](0.53 s)
times_char.....	proved	- complete	[shostak](0.50 s)
member_N_or_zero_TCC1.....	proved	- complete	[shostak](0.51 s)
member_N_or_zero.....	proved	- complete	[shostak](0.44 s)
multiple_char.....	proved	- complete	[shostak](0.83 s)
char_1_zero_ring.....	proved	- complete	[shostak](0.53 s)

Theory ring_characteristic_def totals: 7 formulas, 7 attempted, 7 succeeded (3.79 s)

Proof summary for **theory** pvs_strategies_lemmas

IMP_field_TCC1.....	proved	- complete	[shostak](0.49 s)
times_heading_third.....	proved	- complete	[SHOSTAK](0.64 s)
times_heading_fourth.....	proved	- complete	[SHOSTAK](0.55 s)
times_heading_fifth.....	proved	- complete	[SHOSTAK](0.58 s)
times_heading_sixth.....	proved	- complete	[SHOSTAK](0.62 s)
times_heading_seventh.....	proved	- complete	[SHOSTAK](0.75 s)
plus_heading_third.....	proved	- complete	[SHOSTAK](0.54 s)
plus_heading_fourth.....	proved	- complete	[SHOSTAK](0.54 s)
plus_heading_fifth.....	proved	- complete	[SHOSTAK](0.56 s)
plus_heading_sixth.....	proved	- complete	[SHOSTAK](0.59 s)
plus_heading_seventh.....	proved	- complete	[SHOSTAK](0.63 s)

Theory pvs_strategies_lemmas totals: 11 formulas, 11 attempted, 11 succeeded (6.50 s)

Proof summary for **theory** center_def

center_def.....	proved	- complete	[shostak](0.39 s)
center_subset.....	proved	- complete	[shostak](0.38 s)

Theory center_def totals: 2 formulas, 2 attempted, 2 succeeded (0.77 s)

Proof summary for **theory** ring_nz_closed_aux

```

IMP_ring_basic_properties_TCC1.....proved - complete [shostak](0.37 s)
nz_times_is_zero.....proved - complete [shostak](0.49 s)
nzd_R_cancel_left.....proved - complete [shostak](0.46 s)
nzd_R_cancel_right.....proved - complete [shostak](0.46 s)
subring_nz_closed.....proved - complete [shostak](0.46 s)
Theory ring_nz_closed_aux totals: 5 formulas, 5 attempted, 5 succeeded (2.23

```

s)

Proof summary for theory ring_basic_properties

```

IMP_ring_TCC1.....proved - complete [shostak](0.36 s)
zero_is_member_R.....proved - complete [shostak](0.41 s)
inv_is_member_R.....proved - complete [shostak](0.39 s)
R_sum_star_closed.....proved - complete [shostak](0.38 s)
R_prod_star_closed.....proved - complete [shostak](0.37 s)
l_plus_zero.....proved - complete [shostak](0.38 s)
r_plus_zero.....proved - complete [shostak](0.38 s)
no_singleton_nzx.....proved - complete [shostak](0.39 s)
card_gt_one_nzx.....proved - complete [shostak](0.65 s)
no_singleton_card.....proved - complete [shostak](0.40 s)
subring_transitive_TCC1.....proved - complete [shostak](0.36 s)
subring_transitive.....proved - complete [shostak](0.38 s)
subring_equiv.....proved - complete [shostak](0.55 s)
times_member.....proved - complete [shostak](0.37 s)
left_times.....proved - complete [shostak](0.60 s)
right_times.....proved - complete [shostak](0.57 s)
inv_times_neg.....proved - complete [shostak](0.43 s)
inv_times_inv.....proved - complete [shostak](0.38 s)
times_inv_neg.....proved - complete [shostak](0.37 s)
times_int_zero.....proved - complete [shostak](0.37 s)
times_int_one.....proved - complete [shostak](0.37 s)
times_sum.....proved - complete [shostak](0.40 s)
times_o.....proved - complete [shostak](0.39 s)
times_product.....proved - complete [shostak](0.37 s)
R_sigma_TCC1.....proved - complete [shostak](0.38 s)
R_sigma_TCC2.....proved - complete [shostak](0.42 s)
R_sigma_first.....proved - complete [shostak](0.48 s)
R_sigma_eq_k.....proved - complete [shostak](0.66 s)
R_sigma_eq.....proved - complete [shostak](0.37 s)
R_sigma_eq2.....proved - complete [shostak](0.40 s)
R_sigma_sum.....proved - complete [shostak](0.47 s)
ast_R_sigma.....proved - complete [shostak](0.43 s)
R_sigma_ast.....proved - complete [shostak](0.44 s)
R_sigma_inv.....proved - complete [shostak](0.47 s)
R_sigma_o.....proved - complete [shostak](0.45 s)
R_sigma_R_sigma_TCC1.....proved - complete [shostak](0.40 s)
R_sigma_R_sigma_TCC2.....proved - complete [shostak](0.54 s)
R_sigma_R_sigma.....proved - complete [shostak](2.07 s)
R_sigma_is_member_R.....proved - complete [shostak](0.52 s)
nlzd_TCC1.....proved - complete [shostak](0.36 s)
nzd_cancel_left.....proved - complete [shostak](0.00 s)
nzd_cancel_right.....proved - complete [shostak](0.41 s)

```

Theory ring_basic_properties totals: 42 formulas, 42 attempted, 42 succeeded
(19.28 s)

Proof summary for theory op_finseq

```

IMP_monoid_TCC1.....proved - complete [shostak](0.43 s)
op_fseq_singleton.....proved - complete [shostak](0.60 s)
op_fseq_composition.....proved - incomplete [shostak](1.61 s)
op_subfseq_closed.....proved - complete [shostak](1.46 s)
op_fseq_closed.....proved - complete [shostak](0.70 s)
op_fseq_split_TCC1.....proved - complete [shostak](0.44 s)
op_fseq_split_TCC2.....proved - complete [shostak](0.44 s)
op_fseq_split.....proved - incomplete [shostak](1.47 s)
op_fseq_split_commute_TCC1.....proved - complete [shostak](0.45 s)
op_fseq_split_commute_TCC2.....proved - complete [shostak](0.44 s)
op_fseq_split_commute.....proved - incomplete [shostak](0.90 s)
op_fseq_split_delete.....proved - incomplete [shostak](2.49 s)
op_fseq_same_replace_first_TCC1.....proved - complete [shostak](0.47 s)
op_fseq_same_replace_first.....proved - incomplete [shostak](1.29 s)
op_fseq_same_replace_last_TCC1.....proved - complete [shostak](0.47 s)
op_fseq_same_replace_last.....proved - incomplete [shostak](1.18 s)
heading_fseq_TCC1.....proved - complete [shostak](0.46 s)
heading_fseq_TCC2.....proved - complete [shostak](0.45 s)
times_heading_fseq.....proved - incomplete [shostak](2.16 s)
Theory op_finseq totals: 19 formulas, 19 attempted, 19 succeeded (17.90 s)

```

Proof summary for theory op_finseq_def

```

op_fseq_TCC1.....proved - complete [shostak](0.44 s)
op_fseq_TCC2.....proved - complete [shostak](0.44 s)
op_fseq_TCC3.....proved - complete [shostak](0.51 s)
Theory op_finseq_def totals: 3 formulas, 3 attempted, 3 succeeded (1.39 s)

```

Proof summary for theory field

```

IMP_division_ring_TCC1.....proved - complete [shostak](0.35 s)
IMP_integral_domain_TCC1.....proved - complete [shostak](0.48 s)
field_TCC1.....proved - complete [shostak](0.36 s)
nz_star_TCC1.....proved - complete [shostak](0.40 s)
field_is_division_ring.....proved - complete [shostak](0.61 s)
field_is_integral_domain.....proved - complete [shostak](0.40 s)
field_is_abelian_group_TCC1.....proved - complete [shostak](0.37 s)
field_is_abelian_group_TCC2.....proved - complete [shostak](0.36 s)
field_is_abelian_group.....proved - complete [shostak](1.30 s)
mult_div_TCC1.....proved - complete [shostak](0.37 s)
mult_div.....proved - complete [shostak](0.76 s)
times_div_right.....proved - complete [shostak](0.76 s)
div_times_TCC1.....proved - complete [shostak](0.38 s)
div_times.....proved - complete [shostak](1.18 s)
cross_mult.....proved - complete [shostak](1.16 s)
add_div.....proved - complete [shostak](1.22 s)
minus_div1.....proved - complete [shostak](0.43 s)
sq_div.....proved - complete [shostak](0.43 s)
Theory field totals: 18 formulas, 18 attempted, 18 succeeded (11.33 s)

```

Proof summary for theory field_def

```

field?_TCC1.....proved - complete [shostak](0.38 s)
Theory field_def totals: 1 formulas, 1 attempted, 1 succeeded (0.38 s)

```

Proof summary for theory division_ring

```

IMP_ring_with_one_TCC1.....proved - complete [shostak](0.34 s)
IMP_ring_nz_closed_TCC1.....proved - complete [shostak](0.44 s)

```

```

IMP_group_TCC1.....proved - complete [shostak](0.40 s)
IMP_group_TCC2.....proved - complete [shostak](0.42 s)
IMP_group_TCC3.....proved - complete [shostak](0.39 s)
IMP_group_TCC4.....proved - complete [shostak](0.62 s)
division_ring_TCC1.....proved - complete [shostak](0.39 s)
division_ring_is.....proved - complete [shostak](0.37 s)
division_ring_is_ring_with_one.....proved - complete [shostak](0.60 s)
division_ring_is_ring_nz_closed.....proved - complete [shostak](0.39 s)
division_ring_is_group.....proved - complete [shostak](0.76 s)
one_ne_zero.....proved - complete [shostak](0.45 s)
cancel_times_right.....proved - complete [shostak](0.44 s)
cancel_times_left.....proved - complete [shostak](0.44 s)
idempotent_times.....proved - complete [shostak](0.37 s)
recip_ne_zero.....proved - complete [shostak](0.56 s)
nz_T_div_nz_T_is_nz_T.....proved - complete [shostak](0.56 s)
div_simplify.....proved - complete [shostak](0.42 s)
cancel_div_right.....proved - complete [shostak](0.75 s)
cancel_div_left.....proved - complete [shostak](1.11 s)
times_div_left.....proved - complete [shostak](0.39 s)
div_eq_zero.....proved - complete [shostak](0.56 s)
div_mult.....proved - complete [shostak](0.38 s)
div_mult_left.....proved - complete [shostak](0.41 s)
div_mult_right.....proved - complete [shostak](0.38 s)
div_distributes.....proved - complete [shostak](0.55 s)
div_distributes_minus.....proved - complete [shostak](0.42 s)
div_div1.....proved - complete [shostak](0.41 s)
div_div2_TCC1.....proved - complete [shostak](0.37 s)
div_div2.....proved - complete [shostak](0.73 s)
Theory division_ring totals: 30 formulas, 30 attempted, 30 succeeded (14.80 s)

```

Proof summary for **theory** ring_with_one

```

IMP_ring_TCC1.....proved - complete [shostak](0.34 s)
IMP_monoid_TCC1.....proved - complete [shostak](0.39 s)
ring_with_one_TCC1.....proved - complete [shostak](0.35 s)
one_times.....proved - complete [shostak](0.38 s)
times_one.....proved - complete [shostak](0.38 s)
unique_left_identity.....proved - complete [shostak](0.42 s)
unique_right_identity.....proved - complete [shostak](0.41 s)
minus_one_times.....proved - complete [shostak](0.36 s)
times_minus_one.....proved - complete [shostak](0.36 s)
minus_one_sq_is_one.....proved - complete [shostak](0.36 s)
ring_with_one_is_ring.....proved - complete [shostak](0.48 s)
ring_with_one_is_monoid.....proved - complete [shostak](0.44 s)
Theory ring_with_one totals: 12 formulas, 12 attempted, 12 succeeded (4.66 s)

```

Proof summary for **theory** integral_domain

```

IMP_commutative_ring_TCC1.....proved - complete [shostak](0.35 s)
integral_domain_TCC1.....proved - complete [shostak](0.39 s)
integral_domain_is.....proved - complete [shostak](0.34 s)
integral_domain_is_ring.....proved - complete [shostak](0.49 s)
Theory integral_domain totals: 4 formulas, 4 attempted, 4 succeeded (1.57 s)

```

Proof summary for **theory** integral_domain_def

```

Theory integral_domain_def totals: 0 formulas, 0 attempted, 0 succeeded (0.00 s)

```

Proof summary for **theory** commutative_ring

IMP_ring_TCC1.....	proved - complete	[shostak](0.33 s)
commutative_ring_TCC1.....	proved - complete	[shostak](0.39 s)
times_commutative.....	proved - complete	[shostak](0.36 s)
commutative_ring_is_ring.....	proved - complete	[shostak](0.46 s)
commutative_subrings.....	proved - complete	[shostak](0.37 s)
sq_times.....	proved - complete	[shostak](0.35 s)
Theory commutative_ring totals: 6 formulas, 6 attempted, 6 succeeded (2.25 s)		

Proof summary for **theory** division_ring_def

Theory division_ring_def totals: 0 formulas, 0 attempted, 0 succeeded (0.00 s)

Proof summary for **theory** ring_with_one_def

commutative_ring_with_one?_TCC1.....	proved - complete	[shostak](0.36 s)
finite_commutative_ring_with_one?_TCC1...	proved - complete	[shostak](0.37 s)
Theory ring_with_one_def totals: 2 formulas, 2 attempted, 2 succeeded (0.73 s)		

Proof summary for **theory** ring_nz_closed

IMP_ring_TCC1.....	proved - complete	[shostak](0.33 s)
ring_nz_closed_TCC1.....	proved - complete	[shostak](0.39 s)
ring_nz_closed_is.....	proved - complete	[shostak](0.37 s)
ring_nz_closed_is_ring.....	proved - complete	[shostak](0.52 s)
nz_T_times_nz_T_is_nz_T.....	proved - complete	[shostak](0.56 s)
negate_nz_T_is_nz_T.....	proved - complete	[shostak](0.35 s)
times_is_zero.....	proved - complete	[shostak](0.36 s)
nz_T_times.....	proved - complete	[shostak](0.35 s)
times_nz_T.....	proved - complete	[shostak](0.35 s)
nz_T_times_nz_T_is_not_zero.....	proved - complete	[shostak](0.35 s)
sq_nz_is_nz.....	proved - complete	[shostak](0.34 s)
sq_eq_zero.....	proved - complete	[shostak](0.35 s)
Theory ring_nz_closed totals: 12 formulas, 12 attempted, 12 succeeded (4.62 s)		

Proof summary for **theory** ring_nz_closed_def

Theory ring_nz_closed_def totals: 0 formulas, 0 attempted, 0 succeeded (0.00 s)

Proof summary for **theory** ring

IMP_abelian_group_TCC1.....	proved - complete	[shostak](0.36 s)
ring_TCC1.....	proved - complete	[shostak](0.38 s)
plus_associative.....	proved - complete	[shostak](0.35 s)
plus_commutative.....	proved - complete	[shostak](0.38 s)
times_associative.....	proved - complete	[shostak](0.35 s)
right_distributive.....	proved - complete	[shostak](0.36 s)
left_distributive.....	proved - complete	[shostak](0.35 s)
zero_plus.....	proved - complete	[shostak](0.33 s)
plus_zero.....	proved - complete	[shostak](0.33 s)
negate_is_left_inv.....	proved - complete	[shostak](0.34 s)
negate_is_right_inv.....	proved - complete	[shostak](0.34 s)
cancel_right_plus.....	proved - complete	[shostak](0.37 s)
cancel_left_plus.....	proved - complete	[shostak](0.35 s)
negate_negate.....	proved - complete	[shostak](0.33 s)
cancel_right_minus.....	proved - complete	[shostak](0.35 s)
cancel_left_minus.....	proved - complete	[shostak](0.36 s)
negate_zero.....	proved - complete	[shostak](0.34 s)


```

negate_plus.....proved - complete [shostak](0.34 s)
times_plus.....proved - complete [shostak](0.37 s)
idempotent_add_is_zero.....proved - complete [shostak](0.35 s)
zero_times.....proved - complete [shostak](0.34 s)
times_zero.....proved - complete [shostak](0.34 s)
negative_times.....proved - complete [shostak](0.35 s)
times_negative.....proved - complete [shostak](0.36 s)
negative_times_negative.....proved - complete [shostak](0.37 s)
ring_is_abelian_group.....proved - complete [shostak](0.44 s)
subring_is_ring.....proved - complete [shostak](0.35 s)
sq_rew.....proved - complete [shostak](0.34 s)
sq_neg.....proved - complete [shostak](0.34 s)
sq_plus.....proved - complete [shostak](0.37 s)
sq_minus.....proved - complete [shostak](0.36 s)
sq_neg_minus.....proved - complete [shostak](0.35 s)
sq_zero.....proved - complete [shostak](0.33 s)
Theory ring totals: 33 formulas, 33 attempted, 33 succeeded (11.66 s)

```

Proof summary for **theory** ring_def

```

ring?_TCC1.....proved - complete [shostak](0.35 s)
ring?_TCC2.....proved - complete [shostak](0.35 s)
commutative_ring?_TCC1.....proved - complete [shostak](0.35 s)
finite_commutative_ring?_TCC1.....proved - complete [shostak](0.35 s)
Theory ring_def totals: 4 formulas, 4 attempted, 4 succeeded (1.40 s)

```

Proof summary for **theory** abelian_group

```

IMP_group_TCC1.....proved - complete [shostak](0.31 s)
abelian_group_TCC1.....proved - complete [shostak](0.34 s)
abelian_group_is_group.....proved - complete [shostak](0.38 s)
abelian_group_is_commutative_monoid...proved - complete [shostak](0.37 s)
abelian_subgroups.....proved - complete [shostak](0.38 s)
finite_abelian_group_TCC1.....proved - complete [shostak](0.37 s)
finite_abelian_group_is_abelian_group...proved - complete [shostak](0.39 s)
finite_abelian_group_is_finite_group...proved - complete [shostak](0.40 s)
finite_abelian_subgroups.....proved - complete [shostak](0.33 s)
Theory abelian_group totals: 9 formulas, 9 attempted, 9 succeeded (3.27 s)

```

Proof summary for **theory** group

```

IMP_monoid_TCC1.....proved - complete [shostak](0.31 s)
group_TCC1.....proved - complete [shostak](0.33 s)
group_is_monoid.....proved - complete [shostak](0.36 s)
finite_group_TCC1.....proved - complete [shostak](0.39 s)
finite_group_is_group.....proved - complete [shostak](0.38 s)
finite_group_is_finite_monoid.....proved - complete [shostak](0.39 s)
finite_subgroups.....proved - complete [shostak](0.32 s)
one_is_group.....proved - complete [shostak](0.33 s)
one_finite_group.....proved - complete [shostak](0.32 s)
one_group_TCC1.....proved - complete [shostak](0.40 s)
group_card_gt_0.....proved - complete [shostak](0.33 s)
inv_exists.....proved - complete [shostak](0.33 s)
inv_TCC1.....proved - complete [shostak](0.33 s)
inv_left.....proved - complete [shostak](0.32 s)
inv_right.....proved - complete [shostak](0.31 s)
cancel_right.....proved - complete [shostak](0.37 s)
cancel_left.....proved - complete [shostak](0.33 s)

```

inv_inv.....	proved - complete	[shostak](0.33 s)
cancel_right_inv.....	proved - complete	[shostak](0.35 s)
cancel_left_inv.....	proved - complete	[shostak](0.37 s)
inv_one.....	proved - complete	[shostak](0.32 s)
inv_star.....	proved - complete	[shostak](0.34 s)
unique_inv.....	proved - complete	[shostak](0.34 s)
inv_member.....	proved - complete	[shostak](0.34 s)
inv_in.....	proved - complete	[shostak](0.32 s)
divby.....	proved - complete	[shostak](0.33 s)
product_in.....	proved - complete	[shostak](0.33 s)
one_is_subgroup.....	proved - complete	[shostak](0.40 s)
group_is_subgroup.....	proved - complete	[shostak](0.33 s)
subgroup_is_group.....	proved - complete	[shostak](0.33 s)
subgroup_def.....	proved - complete	[shostak](0.46 s)
inv_power.....	proved - complete	[shostak](0.43 s)
power_inv_right.....	proved - complete	[shostak](0.32 s)
power_inv_left.....	proved - complete	[shostak](0.32 s)
caret_TCC1.....	proved - complete	[shostak](0.32 s)
caret_TCC2.....	proved - complete	[shostak](0.32 s)
expt_0.....	proved - complete	[shostak](0.32 s)
expt_1.....	proved - complete	[shostak](0.31 s)
expt_m1.....	proved - complete	[shostak](0.33 s)
one_expt.....	proved - complete	[shostak](0.34 s)
expt_neg.....	proved - complete	[shostak](0.33 s)
inv_expt.....	proved - complete	[shostak](0.34 s)
expt_def1.....	proved - complete	[shostak](0.51 s)
expt_def2.....	proved - complete	[shostak](0.42 s)
expt_mult.....	proved - complete	[shostak](0.64 s)
expt_div.....	proved - complete	[shostak](0.35 s)
expt_expt.....	proved - complete	[shostak](0.47 s)
expt_commutes.....	proved - complete	[shostak](0.36 s)
expt_inv_right.....	proved - complete	[shostak](0.32 s)
expt_inv_left.....	proved - complete	[shostak](0.32 s)
expt_member.....	proved - complete	[shostak](0.37 s)
generated_by_TCC1.....	proved - complete	[shostak](0.45 s)
generated_by_lem.....	proved - complete	[shostak](0.32 s)
generated_is_subgroup.....	proved - complete	[shostak](0.33 s)
generated_by_is_finite.....	proved - complete	[shostak](0.44 s)
center_TCC1.....	proved - complete	[shostak](0.37 s)
center_def.....	proved - complete	[shostak](0.36 s)
center_subgroup.....	proved - complete	[shostak](0.48 s)
one_left.....	proved - complete	[shostak](0.32 s)
one_right.....	proved - complete	[shostak](0.32 s)
assoc.....	proved - complete	[shostak](0.32 s)

Theory group totals: 61 formulas, 61 attempted, 61 succeeded (21.85 s)

Proof summary for **theory** group_def

abelian_group?_TCC1.....	proved - complete	[shostak](0.32 s)
finite_abelian_group?_TCC1.....	proved - complete	[shostak](0.33 s)
finite_group_surj.....	proved - complete	[shostak](0.32 s)

Theory group_def totals: 3 formulas, 3 attempted, 3 succeeded (0.97 s)

Proof summary for **theory** monoid

IMP_monad_TCC1.....	proved - complete	[shostak](0.31 s)
IMP_semigroup_TCC1.....	proved - complete	[shostak](0.32 s)

```

monoid_TCC1.....proved - complete [shostak](0.31 s)
monoid_is_monad.....proved - complete [shostak](0.36 s)
monoid_is_semigroup.....proved - complete [shostak](0.35 s)
power_0.....proved - complete [shostak](0.31 s)
power_1.....proved - complete [shostak](0.31 s)
one_power.....proved - complete [shostak](0.33 s)
power_def.....proved - complete [shostak](0.40 s)
power_mult.....proved - complete [shostak](0.48 s)
power_power.....proved - complete [shostak](0.44 s)
power_commutes.....proved - complete [shostak](0.36 s)
power_member.....proved - complete [shostak](0.38 s)
one_is_monoid.....proved - complete [shostak](0.35 s)
generated_is_submonoid.....proved - complete [shostak](0.40 s)
generated_set_card_1.....proved - complete [shostak](0.37 s)
finite_monoid_TCC1.....proved - complete [shostak](0.44 s)
finite_monoid_is_monoid.....proved - complete [shostak](0.36 s)
finite_monoid_is_finite_monad.....proved - complete [shostak](0.37 s)
finite_submonoids.....proved - complete [shostak](0.33 s)
commutative_monoid_TCC1.....proved - complete [shostak](0.40 s)
commutative_monoid_is_monoid.....proved - complete [shostak](0.37 s)
commutative_monoid_is_commutative_monad...proved - complete [shostak](0.36

```

s)

```

commutative_submonoids.....proved - complete [shostak](0.35 s)
Theory monoid totals: 24 formulas, 24 attempted, 24 succeeded (8.74 s)

```

Proof summary for theory monoid_def

```

power_TCC1.....proved - complete [shostak](0.32 s)
power_TCC2.....proved - complete [shostak](0.31 s)
generated_set_lem.....proved - complete [shostak](0.31 s)
monoid?_TCC1.....proved - complete [shostak](0.32 s)
commutative_monoid?_TCC1.....proved - complete [shostak](0.33 s)
finite_commutative_monoid?_TCC1.....proved - complete [shostak](0.33 s)
Theory monoid_def totals: 6 formulas, 6 attempted, 6 succeeded (1.92 s)

```

Proof summary for theory monad

```

monad_TCC1.....proved - complete [shostak](0.30 s)
one_member.....proved - complete [shostak](0.33 s)
one_in.....proved - complete [shostak](0.32 s)
left_identity.....proved - complete [shostak](0.32 s)
right_identity.....proved - complete [shostak](0.32 s)
unique_left_identity.....proved - complete [shostak](0.32 s)
unique_right_identity.....proved - complete [shostak](0.32 s)
one_is_monad.....proved - complete [shostak](0.36 s)
trivial_monad_TCC1.....proved - complete [shostak](0.36 s)
monad_is_groupoid.....proved - complete [shostak](0.33 s)
sing_one_finite_monad.....proved - complete [shostak](0.34 s)
finite_monad_TCC1.....proved - complete [shostak](0.35 s)
commutative_monad_TCC1.....proved - complete [shostak](0.38 s)
finite_commutative_monad_TCC1.....proved - complete [shostak](0.36 s)
order_TCC1.....proved - complete [shostak](0.37 s)
order_is_1.....proved - complete [shostak](0.49 s)
finite_monad_is_monad.....proved - complete [shostak](0.35 s)
commutative_monad_is_monad.....proved - complete [shostak](0.35 s)
finite_commutative_monad_is_commutative_monad...proved - complete [shostak]

```

(0.36 s)

finite_commutative_monad_is_finite_monad...proved - complete [shostak](0.36 s)

Theory monad totals: 20 formulas, 20 attempted, 20 succeeded (7.01 s)

Proof summary for **theory** monad_def

monad?_TCC1.....proved - complete [shostak](0.32 s)

monad?_TCC2.....proved - complete [shostak](0.31 s)

commutative_monad?_TCC1.....proved - complete [shostak](0.32 s)

finite_commutative_monad?_TCC1.....proved - complete [shostak](0.32 s)

Theory monad_def totals: 4 formulas, 4 attempted, 4 succeeded (1.27 s)

Proof summary for **theory** semigroup

fullset_is_semigroup_TCC1.....proved - complete [shostak](0.31 s)

semigroup_TCC1.....proved - complete [shostak](0.33 s)

semigroup_TCC2.....proved - complete [shostak](0.30 s)

associative.....proved - complete [shostak](0.34 s)

semigroup_is_groupoid.....proved - complete [shostak](0.31 s)

Theory semigroup totals: 5 formulas, 5 attempted, 5 succeeded (1.58 s)

Proof summary for **theory** semigroup_def

semigroup?_TCC1.....proved - complete [shostak](0.31 s)

finite_commutative_semigroup?_TCC1....proved - complete [shostak](0.32 s)

Theory semigroup_def totals: 2 formulas, 2 attempted, 2 succeeded (0.63 s)

Proof summary for **theory** groupoid

fullset_is_groupoid.....proved - complete [shostak](0.35 s)

groupoid_TCC1.....proved - complete [shostak](0.32 s)

closed.....proved - complete [shostak](0.32 s)

star_closed.....proved - complete [shostak](0.32 s)

Theory groupoid totals: 4 formulas, 4 attempted, 4 succeeded (1.31 s)

Proof summary for **theory** groupoid_def

commutative_groupoid?_TCC1.....proved - complete [shostak](0.31 s)

finite_commutative_groupoid?_TCC1....proved - complete [shostak](0.31 s)

Theory groupoid_def totals: 2 formulas, 2 attempted, 2 succeeded (0.62 s)

Proof summary for **theory** operator_defs_more

Theory operator_defs_more totals: 0 formulas, 0 attempted, 0 succeeded (0.00 s)

Proof summary for **theory** top

Theory top totals: 0 formulas, 0 attempted, 0 succeeded (0.00 s)

Grand Totals: 1744 proofs, 1744 attempted, 1744 succeeded (1196.66 s)