

Proof summary for **theory** boolean\_ring\_homomorphisms

S\_TCC1.....proved - complete [shostak](0.27 s)

img\_hom\_bool\_ring.....proved - complete [shostak](0.48 s)

**Theory** boolean\_ring\_homomorphisms totals: 2 formulas, 2 attempted, 2 succeeded (0.74 s)

Proof summary for **theory** boolean\_ring\_def

**Theory** boolean\_ring\_def totals: 0 formulas, 0 attempted, 0 succeeded (0.00 s)

Proof summary for **theory** chinese\_remainder\_theorem\_Z

IMP\_chinese\_remainder\_theorem\_rings\_TCC1...proved - complete [shostak](0.40 s)

s)

nZ\_mZ\_comaximal\_TCC1.....proved - complete [shostak](0.29 s)

nZ\_mZ\_comaximal.....proved - incomplete [shostak](0.52 s)

Intersection\_add\_first.....proved - incomplete [shostak](0.51 s)

nZ\_fs\_intersection.....proved - incomplete [shostak](1.36 s)

Chinese\_Remainder\_Theorem\_for\_int\_TCC1...proved - incomplete [shostak](0.31 s)

Chinese\_Remainder\_Theorem\_for\_int\_TCC2...proved - incomplete [shostak](0.29 s)

Chinese\_Remainder\_Theorem\_for\_int\_TCC3...proved - incomplete [shostak](0.30 s)

Chinese\_Remainder\_Theorem\_for\_int\_TCC4...proved - incomplete [shostak](0.31 s)

Chinese\_Remainder\_Theorem\_for\_int\_TCC5...proved - incomplete [shostak](0.31 s)

Chinese\_Remainder\_Theorem\_for\_int\_TCC6...proved - incomplete [shostak](0.31 s)

Chinese\_Remainder\_Theorem\_for\_int\_TCC7...proved - incomplete [shostak](0.40 s)

Chinese\_Remainder\_Theorem\_for\_int\_TCC8...proved - incomplete [shostak](0.51 s)

Chinese\_Remainder\_Theorem\_for\_int\_TCC9...proved - incomplete [shostak](0.51 s)

Chinese\_Remainder\_Theorem\_for\_int\_TCC10...proved - incomplete [shostak](0.38 s)

s)

Chinese\_Remainder\_Theorem\_for\_int.....proved - incomplete [shostak](1.58 s)

gcd\_lcm\_property.....proved - incomplete [shostak](6.60 s)

**Theory** chinese\_remainder\_theorem\_Z totals: 17 formulas, 17 attempted, 17 succeeded (14.91 s)

Proof summary for **theory** chinese\_remainder\_theorem\_rings

IMP\_product\_fineseq\_sets\_ring\_TCC1...proved - complete [shostak]( 0.29 s)

oneSet\_nonempty.....proved - complete [shostak]( 0.51 s)

surjective\_aux\_1\_TCC1.....proved - complete [shostak]( 0.38 s)

surjective\_aux\_1.....proved - incomplete [shostak](14.15 s)

surjective\_aux\_2\_TCC1.....proved - complete [shostak]( 0.32 s)

surjective\_aux\_2\_TCC2.....proved - complete [shostak]( 0.49 s)

surjective\_aux\_2\_TCC3.....proved - complete [shostak]( 0.49 s)

surjective\_aux\_2\_TCC4.....proved - complete [shostak]( 0.34 s)

surjective\_aux\_2\_TCC5.....proved - complete [shostak]( 0.51 s)

surjective\_aux\_2\_TCC6.....proved - complete [shostak]( 0.50 s)

surjective\_aux\_2\_TCC7.....proved - complete [shostak]( 0.50 s)

surjective\_aux\_2\_TCC8.....proved - complete [shostak]( 0.32 s)

surjective\_aux\_2.....proved - incomplete [shostak]( 0.00 s)

CRT\_aux\_1\_TCC1.....proved - complete [shostak]( 0.40 s)

CRT\_aux\_1\_TCC2.....proved - complete [shostak]( 0.30 s)

CRT\_aux\_1\_TCC3.....proved - complete [shostak]( 0.65 s)

CRT\_aux\_1\_TCC4.....proved - complete [shostak]( 0.80 s)

CRT\_aux\_1\_TCC5.....proved - complete [shostak]( 0.60 s)

CRT\_aux\_1\_TCC6.....proved - complete [shostak]( 0.40 s)

CRT\_aux\_1.....proved - incomplete [shostak]( 5.73 s)

CRT\_aux\_2\_TCC1.....proved - complete [shostak]( 0.36 s)

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CRT_aux_2.....proved - incomplete [shostak]( 0.89 s)
Chinese_Remainder_Theorem_TCC1.....proved - complete [shostak]( 0.31 s)
Chinese_Remainder_Theorem_TCC2.....proved - incomplete [shostak]( 0.37 s)
Chinese_Remainder_Theorem_TCC3.....proved - incomplete [shostak]( 0.37 s)
Chinese_Remainder_Theorem_TCC4.....proved - incomplete [shostak]( 0.38 s)
Chinese_Remainder_Theorem_TCC5.....proved - incomplete [shostak]( 0.33 s)
Chinese_Remainder_Theorem_TCC6.....proved - complete [shostak]( 0.70 s)
Chinese_Remainder_Theorem_TCC7.....proved - complete [shostak]( 0.84 s)
Chinese_Remainder_Theorem_TCC8.....proved - complete [shostak]( 0.87 s)
Chinese_Remainder_Theorem_TCC9.....proved - complete [shostak]( 0.68 s)
Chinese_Remainder_Theorem.....proved - incomplete [shostak]( 4.52 s)
Theory chinese_remainder_theorem_rings totals: 32 formulas, 32 attempted, 32
succeeded (38.29 s)

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Proof summary for theory product\_finseq\_sets\_ring

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IMP_cartesian_product_quot_ring_TCC1...proved - complete [shostak](0.40 s)
IMP_ring_with_one_TCC1.....proved - complete [shostak](0.34 s)
product_fs_rec_TCC1.....proved - complete [shostak](0.32 s)
product_fs_rec_TCC2.....proved - complete [shostak](0.32 s)
product_fs_rec_TCC3.....proved - complete [shostak](0.30 s)
product_fs_TCC1.....proved - complete [shostak](0.31 s)
product_of_sets_TCC1.....proved - complete [shostak](0.31 s)
product_of_sets_TCC2.....proved - complete [shostak](0.34 s)
product_fs_emptyseq.....proved - complete [shostak](0.30 s)
product_fs_1.....proved - complete [shostak](0.32 s)
product_fs_rec_caret_TCC1.....proved - complete [shostak](0.47 s)
product_fs_rec_caret.....proved - complete [shostak](0.53 s)
product_fs_rec_mult_TCC1.....proved - complete [shostak](0.39 s)
product_fs_rec_mult_TCC2.....proved - complete [shostak](0.41 s)
product_fs_rec_mult.....proved - complete [shostak](1.40 s)
product_fs_split_TCC1.....proved - complete [shostak](0.30 s)
product_fs_split_TCC2.....proved - complete [shostak](0.31 s)
product_fs_split.....proved - complete [shostak](0.46 s)
Product_fs_o.....proved - incomplete [shostak](0.63 s)
Product_fs_o_split_TCC1.....proved - complete [shostak](0.34 s)
Product_fs_o_split_TCC2.....proved - complete [shostak](0.34 s)
Product_fs_o_split.....proved - incomplete [shostak](1.35 s)
product_fs_rec_in_ring.....proved - complete [shostak](0.42 s)
product_fs_rec_in_each_TCC1.....proved - complete [shostak](0.64 s)
product_fs_rec_in_each.....proved - complete [shostak](1.40 s)
Intersection_of_ideals_is_ideal.....proved - incomplete [shostak](0.00 s)
product_of_ideals_subset_of_each.....proved - complete [shostak](0.60 s)
product_of_ideals_subset_intersection...proved - incomplete [shostak](0.46 s)
cartesian_product_fs_representative_TCC1...proved - complete [shostak](0.36
s)
cartesian_product_fs_representative_TCC2...proved - complete [shostak](0.38
s)
cartesian_product_fs_representative...proved - incomplete [shostak](2.98 s)
Theory product_finseq_sets_ring totals: 31 formulas, 31 attempted, 31
succeeded (17.44 s)

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Proof summary for theory cartesian\_product\_quot\_ring

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IMP_quotient_rings_TCC1.....proved - complete [shostak](0.31 s)
Sfs_TCC1.....proved - complete [shostak](0.41 s)
Sfs_TCC2.....proved - complete [shostak](0.38 s)

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Sfs_TCC3.....proved - complete [shostak](0.35 s)
Sfs_TCC4.....proved - complete [shostak](0.36 s)
cartesian_product_quot_ring_is_ring_TCC1...proved - complete [shostak](0.44
s)
cartesian_product_quot_ring_is_ring_TCC2...proved - complete [shostak](0.39
s)
cartesian_product_quot_ring_is_ring_TCC3...proved - complete [shostak](0.55
s)
cartesian_product_quot_ring_is_ring_TCC4...proved - complete [shostak](0.47
s)
cartesian_product_quot_ring_is_ring_TCC5...proved - complete [shostak](0.39
s)
cartesian_product_quot_ring_is_ring...proved - complete [shostak](5.99 s)
Theory cartesian_product_quot_ring totals: 11 formulas, 11 attempted, 11
succeeded (10.04 s)

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Proof summary for theory cartesian_product_finite
cartesian_product_n_TCC1.....proved - complete [shostak](0.31 s)
cartesian_product_one_disjoint.....proved - incomplete [shostak](0.55 s)
cartesian_product_one_emptyset.....proved - incomplete [shostak](0.42 s)
cartesian_product_set_emptyset.....proved - incomplete [shostak](0.33 s)
cartesian_product_n_emptyset.....proved - complete [shostak](0.34 s)
cartesian_product_n_add_is_union.....proved - incomplete [shostak](1.03 s)
rest_card_fs.....proved - incomplete [shostak](0.78 s)
add_card_fs_TCC1.....proved - complete [shostak](0.30 s)
add_card_fs.....proved - incomplete [shostak](0.48 s)
cartesian_product_one_finite.....proved - incomplete [shostak](0.37 s)
cartesian_product_one_card_TCC1.....proved - incomplete [shostak](0.30 s)
cartesian_product_one_card.....proved - incomplete [shostak](0.47 s)
cartesian_product_set_finite_aux.....proved - incomplete [shostak](0.37 s)
cartesian_product_set_finite.....proved - incomplete [shostak](0.37 s)
cartesian_product_set_partition.....proved - incomplete [shostak](0.36 s)
cartesian_product_set_card_aux_TCC1...proved - incomplete [shostak](0.30 s)
cartesian_product_set_card_aux.....proved - incomplete [shostak](0.76 s)
cartesian_product_set_card_TCC1.....proved - incomplete [shostak](0.36 s)
cartesian_product_set_card.....proved - incomplete [shostak](0.47 s)
cartesian_product_n_finite.....proved - incomplete [shostak](1.10 s)
cartesian_product_n_degenerated_TCC1...proved - complete [shostak](0.30 s)
cartesian_product_n_degenerated.....proved - complete [shostak](0.44 s)
cartesian_product_n_card_degenerated_TCC1...proved - incomplete [shostak](0.30
s)
cartesian_product_n_card_degenerated...proved - incomplete [shostak](0.45 s)
cartesian_product_n_card_TCC1.....proved - incomplete [shostak](0.30 s)
cartesian_product_n_card_TCC2.....proved - complete [shostak](0.34 s)
cartesian_product_n_card.....proved - incomplete [shostak](1.69 s)
Theory cartesian_product_finite totals: 27 formulas, 27 attempted, 27
succeeded (13.59 s)

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Proof summary for theory lagrange_scaf
partition_TCC1.....proved - complete [shostak](0.34 s)
finite_partition_TCC1.....proved - complete [shostak](0.35 s)
finite_partition_is_partition.....proved - complete [shostak](0.36 s)
card_Union_rest.....proved - complete [shostak](0.36 s)
card_equal_partition_TCC1.....proved - complete [shostak](0.30 s)
card_equal_partition_TCC2.....proved - complete [shostak](0.00 s)

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card_equal_partition.....proved - complete [shostak](1.13 s)
card_eq_part_TCC1.....proved - complete [shostak](0.40 s)
card_eq_part_TCC2.....proved - complete [shostak](0.39 s)
card_eq_part_TCC3.....proved - complete [shostak](0.31 s)
card_eq_part_TCC4.....proved - complete [shostak](0.41 s)
card_eq_part.....proved - complete [shostak](0.35 s)
Theory lagrange_scaf totals: 12 formulas, 12 attempted, 12 succeeded (4.70 s)

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Proof summary for theory sigma\_R\_below

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IMP_ring_TCC1.....proved - complete [shostak](0.29 s)
R_sigma_below_TCC1.....proved - complete [shostak](0.35 s)
R_sigma_below_TCC2.....proved - complete [shostak](0.38 s)
R_sigma_below_TCC3.....proved - complete [shostak](0.31 s)
R_sigma_b_eq_arg.....proved - complete [shostak](0.31 s)
R_sigma_b_spl_TCC1.....proved - complete [shostak](0.40 s)
R_sigma_b_spl.....proved - complete [shostak](0.62 s)
R_sigma_b_split_TCC1.....proved - complete [shostak](0.33 s)
R_sigma_b_split.....proved - complete [shostak](0.43 s)
R_sigma_b_first_TCC1.....proved - complete [shostak](0.31 s)
R_sigma_b_first.....proved - complete [shostak](0.34 s)
R_sigma_b_last_TCC1.....proved - complete [shostak](0.30 s)
R_sigma_b_last_TCC2.....proved - complete [shostak](0.31 s)
R_sigma_b_last.....proved - complete [shostak](0.31 s)
R_sigma_b_middle_TCC1.....proved - complete [shostak](0.30 s)
R_sigma_b_middle.....proved - complete [shostak](0.34 s)
R_sigma_b_left_aux.....proved - complete [shostak](0.42 s)
R_sigma_b_left.....proved - complete [shostak](0.32 s)
R_sigma_b_right_aux.....proved - complete [shostak](0.42 s)
R_sigma_b_right.....proved - complete [shostak](0.31 s)
R_sigma_b_inv_aux.....proved - complete [shostak](0.43 s)
R_sigma_b_inv.....proved - complete [shostak](0.32 s)
R_sigma_b_eq_k_aux_TCC1.....proved - complete [shostak](0.31 s)
R_sigma_b_eq_k_aux_TCC2.....proved - complete [shostak](0.39 s)
R_sigma_b_eq_k_aux.....proved - complete [shostak](0.74 s)
R_sigma_b_eq_k_TCC1.....proved - complete [shostak](0.30 s)
R_sigma_b_eq_k_TCC2.....proved - complete [shostak](0.40 s)
R_sigma_b_eq_k.....proved - complete [shostak](0.41 s)
R_sigma_b_in_ideal_aux_TCC1.....proved - complete [shostak](0.31 s)
R_sigma_b_in_ideal_aux.....proved - complete [shostak](0.55 s)
R_sigma_b_in_ideal_TCC1.....proved - complete [shostak](0.31 s)
R_sigma_b_in_ideal.....proved - complete [shostak](0.36 s)
R_sigma_b_add_zero_aux.....proved - complete [shostak](0.34 s)
R_sigma_b_add_zero.....proved - complete [shostak](0.33 s)
Theory sigma_R_below totals: 34 formulas, 34 attempted, 34 succeeded (12.59 s)

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Proof summary for theory comaximal\_finseqs\_ideals

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IMP_ring_cosets_lemmas_TCC1.....proved - complete [shostak](0.30 s)
IMP_ring_with_one_TCC1.....proved - complete [shostak](0.34 s)
comaximal_ideals_equiv.....proved - complete [shostak](0.44 s)

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Theory comaximal\_finseqs\_ideals totals: 3 formulas, 3 attempted, 3 succeeded (1.08 s)

Proof summary for theory finite\_integral\_domain

```

IMP_integral_domain_TCC1.....proved - complete [shostak](0.30 s)
IMP_ring_nz_closed_TCC1.....proved - complete [shostak](0.36 s)

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IMP_ring_with_one_basic_properties_TCC1...proved - complete [shostak](0.31
s)
surj_equiv_inj_fin_sets.....proved - complete [shostak](1.18 s)
zero_ring_is_fin_int_dom.....proved - complete [shostak](0.43 s)
nzx_member_S.....proved - complete [shostak](0.71 s)
auxiliar_map_TCC1.....proved - complete [shostak](0.32 s)
auxiliar_map.....proved - complete [shostak](0.39 s)
building_one.....proved - complete [shostak](0.42 s)
one_is_member_S.....proved - complete [shostak](0.36 s)
fin_int_domain_is_ring_with_one.....proved - complete [shostak](0.36 s)
fin_int_domain_is_mult_group.....proved - complete [shostak](0.48 s)
fin_int_domain_is_field.....proved - complete [shostak](0.33 s)
Theory finite_integral_domain totals: 13 formulas, 13 attempted, 13 succeeded
(5.95 s)

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Proof summary for **theory** ring\_binomial\_theorem

```

IMP_ring_with_one_basic_properties_TCC1...proved - complete [shostak](0.31
s)
F_bino_TCC1.....proved - complete [shostak](0.50 s)
F_bino_TCC2.....proved - complete [shostak](0.33 s)
R_bino_theo.....proved - incomplete [shostak](2.68 s)
Theory ring_binomial_theorem totals: 4 formulas, 4 attempted, 4 succeeded
(3.82 s)

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Proof summary for **theory** zero\_ring

```

groupoid_plus_equiv.....proved - complete [shostak](0.35 s)
groupoid_times_equiv.....proved - complete [shostak](0.36 s)
zero_ring.....proved - complete [shostak](0.63 s)
zero_ring_with_one.....proved - complete [shostak](0.42 s)
Theory zero_ring totals: 4 formulas, 4 attempted, 4 succeeded (1.76 s)

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Proof summary for **theory** ring\_general\_results\_extras

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IMP_ring_homomorphism_lemmas_TCC1....proved - complete [shostak](0.33 s)
IMP_ring_homomorphism_lemmas_TCC2....proved - complete [shostak](0.32 s)
IMP_ring_general_results_TCC1.....proved - complete [shostak](0.43 s)
no_prop_id_mono_TCC1.....proved - complete [shostak](0.39 s)
no_prop_id_mono_TCC2.....proved - complete [shostak](0.37 s)
no_prop_id_mono_TCC3.....proved - complete [shostak](0.37 s)
no_prop_id_mono_TCC4.....proved - complete [shostak](0.35 s)
no_prop_id_mono_TCC5.....proved - complete [shostak](0.37 s)
no_prop_id_mono.....proved - complete [shostak](1.55 s)
mono_no_prop_id.....proved - complete [shostak](0.43 s)
Theory ring_general_results_extras totals: 10 formulas, 10 attempted, 10
succeeded (4.91 s)

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Proof summary for **theory** ring\_general\_results

```

IMP_ring_with_one_basic_properties_TCC1...proved - complete [shostak](0.34
s)
IMP_ring_characteristic_def_TCC1.....proved - complete [shostak](0.40 s)
homomorphism_Z_to_R_TCC1.....proved - complete [shostak](0.37 s)
homomorphism_Z_to_R.....proved - complete [shostak](1.10 s)
gen_times_char_one.....proved - complete [shostak](0.42 s)
nz_closed_char_prime.....proved - incomplete [shostak](0.68 s)
field_zero_maximal_ideal.....proved - complete [shostak](4.07 s)
maximal_ideal_iff_proper_id.....proved - complete [shostak](0.45 s)

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proper\_id\_zero\_maximal\_ideal.....proved - complete [shostak](0.45 s)  
 Theory ring\_general\_results totals: 9 formulas, 9 attempted, 9 succeeded (8.28 s)

Proof summary for theory ring\_with\_one\_homomorphism\_extras  
 R\_TCC1.....proved - complete [shostak](0.34 s)  
 S\_TCC1.....proved - complete [shostak](0.34 s)  
 isomorphic\_fields\_charac.....proved - complete [shostak](0.73 s)  
 ring\_w\_one\_isomorphic\_groupoid.....proved - complete [shostak](0.99 s)  
 Theory ring\_with\_one\_homomorphism\_extras totals: 4 formulas, 4 attempted, 4 succeeded (2.40 s)

Proof summary for theory ring\_with\_one\_homomorphism  
 IMP\_ring\_homomorphism\_lemmas\_TCC1....proved - complete [shostak](0.37 s)  
 IMP\_ring\_homomorphism\_lemmas\_TCC2....proved - complete [shostak](0.37 s)  
 R\_TCC1.....proved - complete [shostak](0.44 s)  
 S\_TCC1.....proved - complete [shostak](0.35 s)  
 epi\_maps\_ones\_TCC1.....proved - complete [shostak](0.62 s)  
 epi\_maps\_ones.....proved - complete [shostak](0.59 s)  
 isomorphic\_fields.....proved - complete [shostak](1.28 s)  
 Theory ring\_with\_one\_homomorphism totals: 7 formulas, 7 attempted, 7 succeeded (4.02 s)

Proof summary for theory ring\_homomorphism\_lemmas\_extras  
 IMP\_quotient\_rings\_TCC1.....proved - complete [shostak](0.33 s)  
 zero\_natural\_isomorphism\_TCC1.....proved - complete [shostak](0.44 s)  
 zero\_natural\_isomorphism\_TCC2.....proved - complete [shostak](0.40 s)  
 zero\_natural\_isomorphism\_TCC3.....proved - complete [shostak](0.40 s)  
 zero\_natural\_isomorphism\_TCC4.....proved - complete [shostak](0.40 s)  
 zero\_natural\_isomorphism.....proved - complete [shostak](0.92 s)  
 Theory ring\_homomorphism\_lemmas\_extras totals: 6 formulas, 6 attempted, 6 succeeded (2.89 s)

Proof summary for theory ring\_2nd\_3rd\_isomorphism\_theorems  
 IMP\_quotient\_rings\_TCC1.....proved - complete [shostak](0.35 s)  
 S\_TCC1.....proved - complete [shostak](0.41 s)  
 second\_isomorphism\_th\_ax\_TCC1.....proved - complete [shostak](0.40 s)  
 second\_isomorphism\_th\_ax\_TCC2.....proved - complete [shostak](0.40 s)  
 second\_isomorphism\_th\_ax\_TCC3.....proved - complete [shostak](0.40 s)  
 second\_isomorphism\_th\_ax\_TCC4.....proved - complete [shostak](0.39 s)  
 second\_isomorphism\_th\_ax\_TCC5.....proved - complete [shostak](0.63 s)  
 second\_isomorphism\_th\_ax.....proved - complete [shostak](2.02 s)  
 second\_isomorphism\_th\_TCC1.....proved - complete [shostak](0.39 s)  
 second\_isomorphism\_th\_TCC2.....proved - complete [shostak](0.39 s)  
 second\_isomorphism\_th\_TCC3.....proved - complete [shostak](0.39 s)  
 second\_isomorphism\_th\_TCC4.....proved - complete [shostak](0.39 s)  
 second\_isomorphism\_th\_TCC5.....proved - complete [shostak](0.40 s)  
 second\_isomorphism\_th\_TCC6.....proved - complete [shostak](0.39 s)  
 second\_isomorphism\_th\_TCC7.....proved - complete [shostak](0.39 s)  
 second\_isomorphism\_th.....proved - complete [shostak](4.57 s)  
 third\_isomorphism\_th\_ax\_TCC1.....proved - complete [shostak](0.37 s)  
 third\_isomorphism\_th\_ax\_TCC2.....proved - complete [shostak](0.39 s)  
 third\_isomorphism\_th\_ax\_TCC3.....proved - complete [shostak](0.40 s)  
 third\_isomorphism\_th\_ax\_TCC4.....proved - complete [shostak](0.37 s)  
 third\_isomorphism\_th\_ax\_TCC5.....proved - complete [shostak](0.38 s)



third_isomorphism_th_ax_TCC6.....	proved	- complete	[shostak](0.38 s)
third_isomorphism_th_ax_TCC7.....	proved	- complete	[shostak](0.38 s)
third_isomorphism_th_ax_TCC8.....	proved	- complete	[shostak](0.38 s)
third_isomorphism_th_ax_TCC9.....	proved	- complete	[shostak](0.52 s)
third_isomorphism_th_ax.....	proved	- complete	[shostak](2.47 s)
third_isomorphism_th_TCC1.....	proved	- complete	[shostak](0.46 s)
third_isomorphism_th_TCC2.....	proved	- complete	[shostak](0.55 s)
third_isomorphism_th_TCC3.....	proved	- complete	[shostak](0.55 s)
third_isomorphism_th_TCC4.....	proved	- complete	[shostak](0.37 s)
third_isomorphism_th_TCC5.....	proved	- complete	[shostak](0.54 s)
third_isomorphism_th_TCC6.....	proved	- complete	[shostak](0.54 s)
third_isomorphism_th_TCC7.....	proved	- complete	[shostak](0.39 s)
third_isomorphism_th_TCC8.....	proved	- complete	[shostak](0.38 s)
third_isomorphism_th_TCC9.....	proved	- complete	[shostak](0.38 s)
third_isomorphism_th.....	proved	- complete	[shostak](4.61 s)

**Theory** ring\_2nd\_3rd\_isomorphism\_theorems totals: 36 formulas, 36 attempted, 36 succeeded (27.11 s)

Proof summary for **theory** ring\_1st\_isomorphism\_theorem

IMP_ring_homomorphism_lemmas_TCC1....	proved	- complete	[shostak](0.29 s)
IMP_ring_homomorphism_lemmas_TCC2....	proved	- complete	[shostak](0.29 s)
first_isomorphism_th_aux_1_TCC1.....	proved	- complete	[shostak](0.42 s)
first_isomorphism_th_aux_1_TCC2.....	proved	- complete	[shostak](0.39 s)
first_isomorphism_th_aux_1_TCC3.....	proved	- complete	[shostak](0.34 s)
first_isomorphism_th_aux_1_TCC4.....	proved	- complete	[shostak](0.34 s)
first_isomorphism_th_aux_1_TCC5.....	proved	- complete	[shostak](0.33 s)
first_isomorphism_th_aux_1.....	proved	- complete	[shostak](2.20 s)
first_isomorphism_th_aux_2.....	proved	- complete	[shostak](0.38 s)
first_isomorphism_th_aux_3_TCC1.....	proved	- complete	[shostak](0.59 s)
first_isomorphism_th_aux_3.....	proved	- complete	[shostak](0.85 s)
first_isomorphism_th_aux_4_TCC1.....	proved	- complete	[shostak](0.49 s)
first_isomorphism_th_aux_4.....	proved	- complete	[shostak](0.40 s)
first_isomorphism_th_aux_5.....	proved	- complete	[shostak](1.48 s)
first_isomorphism_th_aux_6.....	proved	- complete	[shostak](0.39 s)
first_isomorphism_th_TCC1.....	proved	- complete	[shostak](0.40 s)
first_isomorphism_th_TCC2.....	proved	- complete	[shostak](0.36 s)
first_isomorphism_th_TCC3.....	proved	- complete	[shostak](0.35 s)
first_isomorphism_th_TCC4.....	proved	- complete	[shostak](0.34 s)
first_isomorphism_th.....	proved	- complete	[shostak](2.91 s)

**Theory** ring\_1st\_isomorphism\_theorem totals: 20 formulas, 20 attempted, 20 succeeded (13.53 s)

Proof summary for **theory** ring\_homomorphism\_lemmas

IMP_ring_basic_properties_TCC1.....	proved	- complete	[shostak](0.24 s)
IMP_ring_basic_properties_TCC2.....	proved	- complete	[shostak](0.29 s)
R_homo_plus_TCC1.....	proved	- complete	[shostak](0.47 s)
R_homo_plus.....	proved	- complete	[shostak](0.29 s)
R_homo_mult_TCC1.....	proved	- complete	[shostak](0.82 s)
R_homo_mult.....	proved	- complete	[shostak](0.27 s)
R_homo_equiv_TCC1.....	proved	- complete	[shostak](0.39 s)
R_homo_equiv_TCC2.....	proved	- complete	[shostak](0.62 s)
R_homo_equiv.....	proved	- complete	[shostak](0.36 s)
zero_to_zero_TCC1.....	proved	- complete	[shostak](0.29 s)
zero_to_zero.....	proved	- complete	[shostak](0.34 s)
inv_to_inv_TCC1.....	proved	- complete	[shostak](0.30 s)

```

inv_to_inv.....proved - complete [shostak](0.39 s)
epi_commutative.....proved - complete [shostak](0.34 s)
image_homo_is_subring.....proved - complete [shostak](0.88 s)
R_homo_image_subring.....proved - complete [shostak](0.85 s)
R_homo_inv_image_subring.....proved - complete [shostak](0.81 s)
R_kernel_is_subring.....proved - complete [shostak](0.68 s)
R_kernel_is_subgroup.....proved - complete [shostak](0.27 s)
monomorphism_charac.....proved - complete [shostak](0.74 s)
inv_iso_is_iso_TCC1.....proved - complete [shostak](1.73 s)
inv_iso_is_iso.....proved - complete [shostak](0.62 s)
R_isomorphic_groupoid_is_ring.....proved - complete [shostak](1.14 s)
R_kernel_is_ideal.....proved - complete [shostak](0.71 s)
R_epimorphism_image_ideal.....proved - complete [shostak](0.68 s)
R_homo_inv_image_ideal.....proved - complete [shostak](0.51 s)
R_kernel_in_inverse_image.....proved - complete [shostak](0.31 s)
inv_image_image_sum.....proved - complete [shostak](0.69 s)
inv_image_image_subring_TCC1.....proved - complete [shostak](0.44 s)
inv_image_image_subring.....proved - complete [shostak](0.63 s)
ring_natural_homo_TCC1.....proved - complete [shostak](0.27 s)
ring_natural_homo_TCC2.....proved - complete [shostak](0.28 s)
ring_natural_homo_TCC3.....proved - complete [shostak](0.39 s)
ring_natural_homo_TCC4.....proved - complete [shostak](0.27 s)
ring_natural_homo_TCC5.....proved - complete [shostak](0.27 s)
ring_natural_homo_TCC6.....proved - complete [shostak](0.36 s)
ring_natural_homo_TCC7.....proved - complete [shostak](0.29 s)
ring_natural_homo.....proved - complete [shostak](0.87 s)
Theory ring_homomorphism_lemmas totals: 38 formulas, 38 attempted, 38
succeeded (20.12 s)

Proof summary for theory ring_homomorphisms_def
Theory ring_homomorphisms_def totals: 0 formulas, 0 attempted, 0 succeeded
(0.00 s)

Proof summary for theory homomorphisms_def
homomorphism?_TCC1.....proved - complete [shostak](0.28 s)
Theory homomorphisms_def totals: 1 formulas, 1 attempted, 1 succeeded (0.28 s)

Proof summary for theory ring_euclidean_gcd_algorithm_Z
Z_TCC1.....proved - complete [shostak](0.37 s)
f_phi_Z_TCC1.....proved - complete [shostak](0.38 s)
f_phi_Z_TCC2.....proved - complete [shostak](0.39 s)
phi_Z_and_f_phi_Z_ok_TCC1.....proved - complete [shostak](0.58 s)
phi_Z_and_f_phi_Z_ok_TCC2.....proved - incomplete [shostak](2.11 s)
phi_Z_and_f_phi_Z_ok.....proved - incomplete [shostak](1.44 s)
euclidean_gcd_alg_correctness_in_Z_TCC1...proved - complete [shostak](0.47
s)
euclidean_gcd_alg_correctness_in_Z_TCC2...proved - complete [shostak](0.43
s)
euclidean_gcd_alg_correctness_in_Z_TCC3...proved - complete [shostak](0.45
s)
euclidean_gcd_alg_correctness_in_Z_TCC4...proved - incomplete [shostak](0.66
s)
euclidean_gcd_alg_correctness_in_Z_TCC5...proved - complete [shostak](0.54
s)
euclidean_gcd_alg_correctness_in_Z....proved - incomplete [shostak](0.59 s)

```



**Theory** ring\_euclidean\_gcd\_algorithm\_Z totals: 12 formulas, 12 attempted, 12 succeeded (8.40 s)

Proof summary for **theory** ring\_euclidean\_gcd\_algorithm\_Zi

Zi\_is\_ring.....proved - incomplete [shostak](1.19 s)  
Zi\_is\_integral\_domain\_w\_one.....proved - incomplete [shostak](1.09 s)  
sq\_abs\_Re\_Im\_integer\_rational\_pred\_TCC1...proved - incomplete [shostak](0.44 s)

s)  
sq\_abs\_Re\_Im\_integer\_rational\_pred....proved - incomplete [shostak](0.61 s)  
times\_conjugate\_is\_Zi.....proved - incomplete [shostak](1.05 s)  
phi\_Zi\_TCC1.....proved - incomplete [shostak](0.55 s)  
phi\_Zi\_is\_multiplicative\_TCC1.....proved - incomplete [shostak](0.50 s)  
phi\_Zi\_is\_multiplicative.....proved - incomplete [shostak](0.47 s)  
div\_rem\_appx\_TCC1.....proved - complete [shostak](0.48 s)  
div\_rem\_appx\_TCC2.....proved - complete [shostak](0.52 s)  
div\_rev\_appx\_correctness.....proved - incomplete [shostak](1.48 s)  
f\_phi\_Zi\_TCC1.....proved - incomplete [shostak](0.51 s)  
f\_phi\_Zi\_TCC2.....proved - incomplete [shostak](0.53 s)  
f\_phi\_Zi\_TCC3.....proved - incomplete [shostak](0.51 s)  
f\_phi\_Zi\_TCC4.....proved - incomplete [shostak](0.83 s)  
f\_phi\_Zi\_TCC5.....proved - incomplete [shostak](1.43 s)  
phi\_Zi\_and\_f\_phi\_Zi\_ok\_TCC1.....proved - incomplete [shostak](0.50 s)  
phi\_Zi\_and\_f\_phi\_Zi\_ok\_TCC2.....proved - incomplete [shostak](6.22 s)  
phi\_Zi\_and\_f\_phi\_Zi\_ok\_TCC3.....proved - incomplete [shostak](6.16 s)  
phi\_Zi\_and\_f\_phi\_Zi\_ok.....proved - incomplete [shostak](8.09 s)  
euclidean\_gcd\_alg\_in\_Zi\_TCC1.....proved - incomplete [shostak](0.41 s)  
euclidean\_gcd\_alg\_in\_Zi\_TCC2.....proved - incomplete [shostak](0.38 s)  
euclidean\_gcd\_alg\_in\_Zi\_TCC3.....proved - incomplete [shostak](0.40 s)  
euclidean\_gcd\_alg\_in\_Zi\_TCC4.....proved - incomplete [shostak](0.41 s)  
euclidean\_gcd\_alg\_in\_Zi\_TCC5.....proved - incomplete [shostak](0.78 s)  
euclidean\_gcd\_alg\_in\_Zi\_TCC6.....proved - incomplete [shostak](0.42 s)  
euclidean\_gcd\_alg\_in\_Zi.....proved - incomplete [shostak](0.55 s)

**Theory** ring\_euclidean\_gcd\_algorithm\_Zi totals: 27 formulas, 27 attempted, 27 succeeded (36.53 s)

Proof summary for **theory** ring\_euclidean\_algorithm

IMP\_euclidean\_domain\_TCC1.....proved - complete [shostak](0.36 s)  
euclidean\_gcd\_algorithm\_TCC1.....proved - complete [shostak](0.76 s)  
euclidean\_gcd\_algorithm\_TCC2.....proved - complete [shostak](0.54 s)  
euclidean\_gcd\_algorithm\_TCC3.....proved - complete [shostak](0.52 s)  
euclidean\_gcd\_algorithm\_TCC4.....proved - complete [shostak](0.52 s)  
euclidean\_gcd\_algorithm\_TCC5.....proved - complete [shostak](0.61 s)  
euclidean\_gcd\_algorithm\_TCC6.....proved - complete [shostak](0.52 s)  
euclidean\_gcd\_algorithm\_TCC7.....proved - complete [shostak](0.63 s)  
euclidean\_gcd\_algorithm\_TCC8.....proved - complete [shostak](0.56 s)  
euclidean\_gcd\_algorithm\_TCC9.....proved - complete [shostak](7.01 s)  
euclidean\_gcd\_algorithm\_TCC10.....proved - complete [shostak](0.63 s)  
euclidean\_gcd\_algorithm\_TCC11.....proved - complete [shostak](0.53 s)  
euclidean\_gcd\_algorithm\_TCC12.....proved - complete [shostak](0.67 s)  
Euclid\_theorem\_TCC1.....proved - complete [shostak](0.43 s)  
Euclid\_theorem\_TCC2.....proved - complete [shostak](0.42 s)  
Euclid\_theorem.....proved - complete [shostak](2.46 s)  
euclidean\_gcd\_alg\_correctness\_TCC1...proved - complete [shostak](0.41 s)  
euclidean\_gcd\_alg\_correctness.....proved - complete [shostak](4.50 s)

**Theory** ring\_euclidean\_algorithm totals: 18 formulas, 18 attempted, 18

succeeded (22.07 s)

Proof summary for **theory** euclidean\_domain

IMP\_ring\_with\_one\_basic\_properties\_TCC1...proved - complete [shostak](0.39 s)  
IMP\_euclidean\_ring\_TCC1.....proved - complete [shostak](0.73 s)  
integers\_is\_euclidean\_domain.....proved - complete [shostak](0.85 s)  
field\_is\_euclidean\_domain.....proved - complete [shostak](0.68 s)  
euclidean\_is\_unique\_factorization\_domain\_TCC1...proved - complete [shostak](0.65 s)  
euclidean\_is\_unique\_factorization\_domain...proved - incomplete [shostak](0.42 s)  
**Theory** euclidean\_domain totals: 6 formulas, 6 attempted, 6 succeeded (3.72 s)

Proof summary for **theory** euclidean\_domain\_def

**Theory** euclidean\_domain\_def totals: 0 formulas, 0 attempted, 0 succeeded (0.00 s)

Proof summary for **theory** ring\_zn

IMP\_quotient\_rings\_with\_one\_TCC1.....proved - complete [shostak](0.58 s)  
nZ\_add\_TCC1.....proved - complete [shostak](0.38 s)  
Z\_ring.....proved - complete [shostak](0.28 s)  
Z\_TCC1.....proved - complete [shostak](0.28 s)  
Z1\_is\_Z.....proved - complete [shostak](0.30 s)  
Z\_commutative\_ring\_w\_one.....proved - complete [shostak](0.38 s)  
nZ\_ideal.....proved - complete [shostak](1.13 s)  
Z\_nz\_closed.....proved - complete [shostak](0.51 s)  
Z\_integral\_domain\_w\_one.....proved - complete [shostak](0.37 s)  
Zn\_finite\_set.....proved - complete [shostak](1.01 s)  
Zn\_card\_n\_TCC1.....proved - complete [shostak](0.28 s)  
Zn\_card\_n.....proved - complete [shostak](1.30 s)  
Zn\_commutative\_ring\_w\_one\_TCC1.....proved - complete [shostak](0.30 s)  
Zn\_commutative\_ring\_w\_one\_TCC2.....proved - complete [shostak](0.30 s)  
Zn\_commutative\_ring\_w\_one\_TCC3.....proved - complete [shostak](0.29 s)  
Zn\_commutative\_ring\_w\_one\_TCC4.....proved - complete [shostak](0.29 s)  
Zn\_commutative\_ring\_w\_one\_TCC5.....proved - complete [shostak](0.33 s)  
Zn\_commutative\_ring\_w\_one.....proved - complete [shostak](1.72 s)  
equal\_cosets\_div.....proved - complete [shostak](0.62 s)  
nZ\_mZ\_sum\_TCC1.....proved - complete [shostak](0.29 s)  
nZ\_mZ\_sum.....proved - incomplete [shostak](0.93 s)  
nZ\_mZ\_intersection\_TCC1.....proved - incomplete [shostak](0.33 s)  
nZ\_mZ\_intersection.....proved - incomplete [shostak](1.13 s)  
nZ\_mZ\_rel\_prime\_intersection.....proved - incomplete [shostak](0.33 s)  
Zn\_charac.....proved - incomplete [shostak](1.43 s)  
Z2\_charac.....proved - incomplete [shostak](0.46 s)  
Zp\_prime\_is\_nz\_closed.....proved - incomplete [shostak](1.40 s)  
Zp\_nz\_closed\_is\_prime\_or\_one.....proved - incomplete [shostak](0.94 s)  
Zp\_prime\_is\_division\_ring.....proved - incomplete [shostak](0.00 s)  
Zp\_prime\_is\_field.....proved - incomplete [shostak](0.61 s)  
nZ\_mZ\_subset.....proved - complete [shostak](0.49 s)  
power\_sum\_nat.....proved - complete [shostak](0.45 s)  
power\_sum\_int.....proved - complete [shostak](0.43 s)  
nZ\_is\_cyclic.....proved - complete [shostak](0.39 s)  
mZ\_nZ\_is\_cyclic\_TCC1.....proved - complete [shostak](0.38 s)  
mZ\_nZ\_is\_cyclic\_TCC2.....proved - complete [shostak](0.32 s)

```

mZ_nZ_is_cyclic_TCC3.....proved - complete [shostak](0.38 s)
mZ_nZ_is_cyclic_TCC4.....proved - complete [shostak](0.32 s)
mZ_nZ_is_cyclic_TCC5.....proved - complete [shostak](0.35 s)
mZ_nZ_is_cyclic.....proved - complete [shostak](0.73 s)
Theory ring_zn totals: 40 formulas, 40 attempted, 40 succeeded (22.74 s)

```

Proof summary for theory prop\_primes\_extra

```

fs_rel_prime?_TCC1.....proved - complete [shostak](0.31 s)
lcm_div_TCC1.....proved - complete [shostak](0.28 s)
lcm_div_TCC2.....proved - incomplete [shostak](0.56 s)
lcm_div.....proved - incomplete [shostak](0.93 s)
primes_lcm_div_TCC1.....proved - complete [shostak](0.27 s)
primes_lcm_div.....proved - incomplete [shostak](0.34 s)
fs_rel_prime_fixed_TCC1.....proved - complete [shostak](0.28 s)
fs_rel_prime_fixed_TCC2.....proved - incomplete [shostak](0.29 s)
fs_rel_prime_fixed.....proved - incomplete [shostak](0.87 s)
fs_rel_prime_i_TCC1.....proved - incomplete [shostak](0.35 s)
fs_rel_prime_i.....proved - incomplete [shostak](0.38 s)
Theory prop_primes_extra totals: 11 formulas, 11 attempted, 11 succeeded (4.85

```

s)

Proof summary for theory division\_ring\_extras

```

IMP_ring_unit_TCC1.....proved - complete [shostak](0.27 s)
xyx_division_ring.....proved - complete [shostak](0.59 s)
div_ring_nz_unit_TCC1.....proved - complete [shostak](0.29 s)
div_ring_nz_unit.....proved - complete [shostak](0.38 s)
no_prop_l_ideal_div_ring.....proved - complete [shostak](0.30 s)
no_prop_r_ideal_div_ring.....proved - complete [shostak](0.30 s)
div_ring_no_prop_ideal.....proved - complete [shostak](0.38 s)
Theory division_ring_extras totals: 7 formulas, 7 attempted, 7 succeeded (2.51

```

s)

Proof summary for theory ring\_w\_one\_xyx\_is\_x

```

IMP_ring_xyx_is_x_TCC1.....proved - complete [shostak](0.27 s)
IMP_ring_with_one_nz_closed_TCC1.....proved - complete [shostak](0.33 s)
xyx_one_is_member.....proved - complete [shostak](0.35 s)
xyx_ring_with_one.....proved - complete [shostak](0.33 s)
xyx_R_unit.....proved - complete [shostak](0.67 s)
unit_xyx_R_TCC1.....proved - complete [shostak](0.38 s)
unit_xyx_R.....proved - complete [shostak](0.43 s)
unit_nz_closed.....proved - complete [shostak](0.30 s)
Theory ring_w_one_xyx_is_x totals: 8 formulas, 8 attempted, 8 succeeded (3.05

```

s)

Proof summary for theory ring\_xyx\_is\_x

```

IMP_ring_nz_closed_aux_TCC1.....proved - complete [shostak](0.29 s)
xyx_is_x_nz_divisor.....proved - complete [shostak](0.42 s)
xyx_is_x_nz_closed.....proved - complete [shostak](0.34 s)
xyx_is_y.....proved - complete [shostak](0.48 s)
xyx_has_identity.....proved - complete [shostak](0.67 s)
Theory ring_xyx_is_x totals: 5 formulas, 5 attempted, 5 succeeded (2.20 s)

```

Proof summary for theory ring\_xyx\_is\_x\_def

```

Theory ring_xyx_is_x_def totals: 0 formulas, 0 attempted, 0 succeeded (0.00 s)

```

Proof summary for **theory** ring\_with\_one\_nz\_closed  
 IMP\_ring\_with\_one\_TCC1.....proved - complete [shostak](0.28 s)  
 IMP\_ring\_nz\_closed\_aux\_TCC1.....proved - complete [shostak](0.36 s)  
 subring\_nz\_closed\_one\_TCC1.....proved - complete [shostak](0.30 s)  
 subring\_nz\_closed\_one.....proved - complete [shostak](0.42 s)  
**Theory** ring\_with\_one\_nz\_closed totals: 4 formulas, 4 attempted, 4 succeeded  
 (1.36 s)

Proof summary for **theory** lcm  
 lcm\_1.....proved - incomplete [shostak](0.40 s)  
 lcm\_same.....proved - incomplete [shostak](0.38 s)  
 lcm\_sym.....proved - incomplete [shostak](0.54 s)  
 lcm\_divides.....proved - incomplete [shostak](0.44 s)  
 lcm\_is\_min.....proved - incomplete [shostak](0.39 s)  
 lcm\_times.....proved - incomplete [shostak](0.99 s)  
 lcm\_rel\_prime\_TCC1.....proved - complete [shostak](0.28 s)  
 lcm\_rel\_prime.....proved - incomplete [shostak](0.63 s)  
 lcm\_gdm\_rel.....proved - incomplete [shostak](1.00 s)  
 lcm\_absorption.....proved - incomplete [shostak](0.60 s)  
 divides\_lcm.....proved - incomplete [shostak](0.76 s)  
**Theory** lcm totals: 11 formulas, 11 attempted, 11 succeeded (6.40 s)

Proof summary for **theory** euclidean\_ring  
 IMP\_ring\_principal\_ideal\_TCC1.....proved - complete [shostak](0.37 s)  
 euclidean\_ring\_ideal\_is\_gen.....proved - incomplete [shostak](0.84 s)  
 euclidean\_ring\_is\_principal\_ideal.....proved - incomplete [shostak](0.54 s)  
 euclidean\_ring\_has\_one.....proved - incomplete [shostak](0.47 s)  
**Theory** euclidean\_ring totals: 4 formulas, 4 attempted, 4 succeeded (2.21 s)

Proof summary for **theory** euclidean\_ring\_def  
 euclidean\_ring?\_TCC1.....proved - complete [shostak](0.55 s)  
 euclidean\_ring?\_TCC2.....proved - complete [shostak](0.63 s)  
 euclidean\_ring?\_TCC3.....proved - complete [shostak](0.46 s)  
 euclidean\_ring?\_TCC4.....proved - complete [shostak](0.46 s)  
 euclidean\_pair?\_TCC1.....proved - complete [shostak](0.57 s)  
 euclidean\_pair?\_TCC2.....proved - complete [shostak](0.66 s)  
 euclidean\_pair?\_TCC3.....proved - complete [shostak](0.40 s)  
 euclidean\_pair?\_TCC4.....proved - complete [shostak](0.40 s)  
 euclidean\_f\_phi?\_TCC1.....proved - complete [shostak](0.38 s)  
 euclidean\_f\_phi?\_TCC2.....proved - complete [shostak](0.39 s)  
**Theory** euclidean\_ring\_def totals: 10 formulas, 10 attempted, 10 succeeded  
 (4.89 s)

Proof summary for **theory** ring\_unique\_factorization\_domain  
 IMP\_ring\_principal\_ideal\_domain\_TCC1...proved - complete [shostak](0.37 s)  
 UFD\_prime\_iff\_irreducible.....proved - incomplete [shostak](1.54 s)  
 PID\_is\_UFD\_TCC1.....proved - complete [shostak](0.66 s)  
 PID\_is\_UFD.....proved - incomplete [shostak](0.63 s)  
**Theory** ring\_unique\_factorization\_domain totals: 4 formulas, 4 attempted, 4  
 succeeded (3.21 s)

Proof summary for **theory** ring\_unique\_factorization\_domain\_def  
 unique\_factorization\_domain?\_TCC1....proved - complete [shostak](0.68 s)  
 unique\_factorization\_domain?\_TCC2....proved - complete [shostak](1.86 s)  
 unique\_factorization\_domain?\_TCC3....proved - complete [shostak](0.91 s)

Theory ring\_unique\_factorization\_domain\_def totals: 3 formulas, 3 attempted, 3 succeeded (3.45 s)

Proof summary for theory ring\_principal\_ideal\_domain

```
IMP_ring_with_one_maximal_ideal_TCC1...proved - complete [shostak]( 0.35 s)
IMP_ring_principal_ideal_TCC1.....proved - complete [shostak]( 0.43 s)
PID_maximal_prime_ideal.....proved - complete [shostak]( 0.00 s)
el_max_iff_one_gen_maximal.....proved - complete [shostak]( 0.93 s)
PID_prime_el_iff_irreducible_TCC1...proved - complete [shostak]( 0.63 s)
PID_prime_el_iff_irreducible.....proved - complete [shostak]( 1.38 s)
nonzero_nonunit_irreducible_divides_TCC1...proved - complete [shostak]( 0.50
```

s)

```
nonzero_nonunit_irreducible_divides...proved - incomplete [shostak]( 0.83 s)
non_fact_el_set_TCC1.....proved - complete [shostak]( 0.45 s)
non_fact_el_set_TCC2.....proved - complete [shostak]( 0.63 s)
empty_non_fact_el_set_aux1_TCC1.....proved - complete [shostak]( 0.68 s)
empty_non_fact_el_set_aux1_TCC2.....proved - complete [shostak]( 0.68 s)
empty_non_fact_el_set_aux1.....proved - incomplete [shostak]( 1.33 s)
phi_TCC1.....proved - complete [shostak]( 0.69 s)
phi_TCC2.....proved - complete [shostak]( 0.55 s)
phi_TCC3.....proved - complete [shostak]( 0.40 s)
phi_TCC4.....proved - incomplete [shostak]( 0.40 s)
empty_non_fact_el_set_aux2_TCC1.....proved - complete [shostak]( 0.37 s)
empty_non_fact_el_set_aux2_TCC2.....proved - complete [shostak]( 0.41 s)
empty_non_fact_el_set_aux2.....proved - incomplete [shostak]( 0.47 s)
empty_non_fact_el_set_aux3_TCC1.....proved - complete [shostak]( 0.37 s)
empty_non_fact_el_set_aux3.....proved - incomplete [shostak]( 0.83 s)
empty_non_fact_el_set.....proved - incomplete [shostak]( 0.55 s)
PID_factorization_existence.....proved - incomplete [shostak]( 0.37 s)
PID_factorization_uniqueness_TCC1...proved - complete [shostak]( 0.60 s)
PID_factorization_uniqueness_TCC2...proved - complete [shostak](18.49 s)
PID_factorization_uniqueness_TCC3...proved - complete [shostak]( 5.70 s)
PID_factorization_uniqueness.....proved - incomplete [shostak]( 6.43 s)
```

Theory ring\_principal\_ideal\_domain totals: 28 formulas, 28 attempted, 28 succeeded (45.45 s)

Proof summary for theory ring\_with\_one\_maximal\_ideal

```
IMP_ring_with_one_prime_ideal_TCC1...proved - complete [shostak](0.34 s)
IMP_ring_maximal_ideal_TCC1.....proved - complete [shostak](0.42 s)
ring_one_maximal_prime_ideal.....proved - complete [shostak](0.46 s)
maximal_ideal_quot_field_TCC1.....proved - complete [shostak](0.37 s)
maximal_ideal_quot_field_TCC2.....proved - complete [shostak](0.37 s)
maximal_ideal_quot_field_TCC3.....proved - complete [shostak](0.38 s)
maximal_ideal_quot_field_TCC4.....proved - complete [shostak](0.37 s)
maximal_ideal_quot_field_TCC5.....proved - complete [shostak](0.39 s)
maximal_ideal_quot_field.....proved - complete [shostak](2.08 s)
quot_div_ring_maximal_ideal_TCC1.....proved - complete [shostak](0.38 s)
quot_div_ring_maximal_ideal_TCC2.....proved - complete [shostak](0.37 s)
quot_div_ring_maximal_ideal_TCC3.....proved - complete [shostak](0.38 s)
quot_div_ring_maximal_ideal_TCC4.....proved - complete [shostak](0.37 s)
quot_div_ring_maximal_ideal_TCC5.....proved - complete [shostak](0.38 s)
quot_div_ring_maximal_ideal.....proved - complete [shostak](1.33 s)
maximal_ideal_charac_TCC1.....proved - complete [shostak](0.36 s)
maximal_ideal_charac_TCC2.....proved - complete [shostak](0.00 s)
maximal_ideal_charac_TCC3.....proved - complete [shostak](0.38 s)
```

```

maximal_ideal_charac_TCC4.....proved - complete [shostak](0.37 s)
maximal_ideal_charac_TCC5.....proved - complete [shostak](0.37 s)
maximal_ideal_charac.....proved - complete [shostak](0.37 s)
nonzero_ring_exists_maximal_ideal_aux...proved - incomplete [shostak](0.44 s)
nonzero_ring_exists_maximal_ideal.....proved - incomplete [shostak](0.38 s)
Theory ring_with_one_maximal_ideal totals: 23 formulas, 23 attempted, 23
succeeded (11.07 s)

```

Proof summary for **theory** ring\_with\_one\_basic\_properties

```

IMP_ring_with_one_TCC1.....proved - complete [shostak](0.29 s)
IMP_ring_basic_properties_TCC1.....proved - complete [shostak](0.35 s)
power_commute_aux.....proved - complete [shostak](0.32 s)
power_commute.....proved - complete [shostak](0.31 s)
gen_times_int_one.....proved - complete [shostak](0.29 s)
ring_w_one_is_idempotent.....proved - complete [shostak](0.35 s)
one_diff_zero_monad.....proved - complete [shostak](0.33 s)
Theory ring_with_one_basic_properties totals: 7 formulas, 7 attempted, 7
succeeded (2.24 s)

```

Proof summary for **theory** ring\_with\_one\_prime\_ideal

```

IMP_ring_prime_ideal_TCC1.....proved - complete [shostak](0.34 s)
IMP_quotient_rings_with_one_TCC1.....proved - complete [shostak](0.40 s)
prime_ideal_charac_TCC1.....proved - complete [shostak](0.39 s)
prime_ideal_charac_TCC2.....proved - complete [shostak](0.38 s)
prime_ideal_charac_TCC3.....proved - complete [shostak](0.37 s)
prime_ideal_charac_TCC4.....proved - complete [shostak](0.36 s)
prime_ideal_charac_TCC5.....proved - complete [shostak](0.39 s)
prime_ideal_charac.....proved - complete [shostak](0.84 s)
Theory ring_with_one_prime_ideal totals: 8 formulas, 8 attempted, 8 succeeded
(3.47 s)

```

Proof summary for **theory** quotient\_rings\_with\_one

```

IMP_quotient_rings_TCC1.....proved - complete [shostak](0.27 s)
IMP_ring_with_one_ideal_TCC1.....proved - complete [shostak](0.31 s)
quotient_ring_with_one_TCC1.....proved - complete [shostak](0.31 s)
quotient_ring_with_one_TCC2.....proved - complete [shostak](0.29 s)
quotient_ring_with_one_TCC3.....proved - complete [shostak](0.29 s)
quotient_ring_with_one_TCC4.....proved - complete [shostak](0.28 s)
quotient_ring_with_one_TCC5.....proved - complete [shostak](0.30 s)
quotient_ring_with_one.....proved - complete [shostak](0.41 s)
fullset_quot_ring_with_one.....proved - complete [shostak](0.64 s)
one_diff_zero_coset.....proved - complete [shostak](0.29 s)
lcoset_power_prod_nat_TCC1.....proved - complete [shostak](0.28 s)
lcoset_power_prod_nat.....proved - complete [shostak](0.67 s)
Theory quotient_rings_with_one totals: 12 formulas, 12 attempted, 12 succeeded
(4.34 s)

```

Proof summary for **theory** ring\_maximal\_ideal

```

IMP_ring_prime_ideal_TCC1.....proved - complete [shostak](0.35 s)
maximal_prime_ideal.....proved - complete [shostak](0.96 s)
Theory ring_maximal_ideal totals: 2 formulas, 2 attempted, 2 succeeded (1.31
s)

```

Proof summary for **theory** ring\_maximal\_ideal\_def

```

Theory ring_maximal_ideal_def totals: 0 formulas, 0 attempted, 0 succeeded

```



(0.00 s)

Proof summary for **theory** ring\_principal\_ideal\_domain\_def

**Theory** ring\_principal\_ideal\_domain\_def totals: 0 formulas, 0 attempted, 0 succeeded (0.00 s)

Proof summary for **theory** ring\_unit

IMP\_ring\_ideal\_TCC1.....proved - complete [shostak](0.27 s)  
IMP\_ring\_with\_one\_TCC1.....proved - complete [shostak](0.32 s)  
proper\_id\_iff\_no\_unit\_TCC1.....proved - complete [shostak](0.32 s)  
proper\_id\_iff\_no\_unit.....proved - complete [shostak](0.41 s)  
no\_prop\_l\_ideal\_nz\_unit\_TCC1.....proved - complete [shostak](0.39 s)  
no\_prop\_l\_ideal\_nz\_unit.....proved - complete [shostak](0.48 s)  
no\_prop\_r\_ideal\_nz\_unit\_TCC1.....proved - complete [shostak](0.39 s)  
no\_prop\_r\_ideal\_nz\_unit.....proved - complete [shostak](0.48 s)  
**Theory** ring\_unit totals: 8 formulas, 8 attempted, 8 succeeded (3.06 s)

Proof summary for **theory** ring\_prime\_element

IMP\_ring\_with\_id\_one\_generator\_TCC1...proved - complete [shostak](0.36 s)  
IMP\_ring\_prime\_ideal\_TCC1.....proved - complete [shostak](0.44 s)  
prime\_el\_iff\_prime\_ideal.....proved - complete [shostak](0.90 s)  
el\_irred\_iff\_one\_gen\_maximal\_TCC1....proved - complete [shostak](0.60 s)  
el\_irred\_iff\_one\_gen\_maximal.....proved - complete [shostak](3.53 s)  
prime\_el\_is\_irreducible\_TCC1.....proved - complete [shostak](0.55 s)  
prime\_el\_is\_irreducible.....proved - complete [shostak](0.71 s)  
assoc\_irreducible\_is\_irreducible\_TCC1...proved - complete [shostak](0.52 s)  
assoc\_irreducible\_is\_irreducible\_TCC2...proved - complete [shostak](0.58 s)  
assoc\_irreducible\_is\_irreducible\_TCC3...proved - complete [shostak](0.55 s)  
assoc\_irreducible\_is\_irreducible.....proved - complete [shostak](0.54 s)  
assoc\_prime\_is\_prime\_TCC1.....proved - complete [shostak](0.56 s)  
assoc\_prime\_is\_prime.....proved - complete [shostak](0.48 s)  
irreducible\_el\_divisors\_charac\_TCC1...proved - complete [shostak](0.47 s)  
irreducible\_el\_divisors\_charac\_TCC2...proved - complete [shostak](0.54 s)  
irreducible\_el\_divisors\_charac.....proved - complete [shostak](0.40 s)  
prime\_el\_divides\_TCC1.....proved - complete [shostak](0.47 s)  
prime\_el\_divides\_TCC2.....proved - complete [shostak](0.44 s)  
prime\_el\_divides.....proved - incomplete [shostak](2.03 s)  
prime\_el\_divides\_last\_pos\_TCC1.....proved - complete [shostak](0.63 s)  
prime\_el\_divides\_last\_pos.....proved - incomplete [shostak](8.71 s)  
irreducible\_prod\_not\_unit\_TCC1.....proved - complete [shostak](0.54 s)  
irreducible\_prod\_not\_unit\_TCC2.....proved - complete [shostak](0.46 s)  
irreducible\_prod\_not\_unit.....proved - incomplete [shostak](1.00 s)  
irreducible\_prod\_unit\_length\_0\_TCC1...proved - complete [shostak](0.47 s)  
irreducible\_prod\_unit\_length\_0.....proved - incomplete [shostak](0.39 s)  
irreducible\_prod\_not\_zero.....proved - complete [shostak](2.84 s)  
prod\_unit\_irreducible\_is\_irreducible\_TCC1...proved - complete [shostak](0.49 s)  
s)  
prod\_unit\_irreducible\_is\_irreducible\_TCC2...proved - complete [shostak](0.52 s)  
s)  
prod\_unit\_irreducible\_is\_irreducible\_TCC3...proved - complete [shostak](1.12 s)  
s)  
prod\_unit\_irreducible\_is\_irreducible...proved - complete [shostak](0.66 s)  
prod\_unit\_irreducible\_is\_associates\_TCC1...proved - complete [shostak](2.39 s)  
s)  
prod\_unit\_irreducible\_is\_associates...proved - complete [shostak](0.50 s)

**Theory** ring\_prime\_element totals: 33 formulas, 33 attempted, 33 succeeded (35.38 s)

Proof summary for **theory** ring\_prime\_element\_def

R\_prime\_element?\_TCC1.....proved - complete [shostak](0.37 s)

R\_prime\_element?\_TCC2.....proved - complete [shostak](0.39 s)

**Theory** ring\_prime\_element\_def totals: 2 formulas, 2 attempted, 2 succeeded (0.76 s)

Proof summary for **theory** ring\_irreducible\_element\_def

**Theory** ring\_irreducible\_element\_def totals: 0 formulas, 0 attempted, 0 succeeded (0.00 s)

Proof summary for **theory** ring\_prime\_ideal

IMP\_ring\_one\_generator\_TCC1.....proved - complete [shostak](0.35 s)

prime\_ideal\_prop1.....proved - complete [shostak](0.45 s)

prime\_ideal\_prop2.....proved - complete [shostak](0.58 s)

prime\_ideal\_prod\_closed.....proved - complete [shostak](0.48 s)

prime\_ideal\_nz\_closed\_TCC1.....proved - complete [shostak](0.37 s)

prime\_ideal\_nz\_closed\_TCC2.....proved - complete [shostak](0.38 s)

prime\_ideal\_nz\_closed\_TCC3.....proved - complete [shostak](0.38 s)

prime\_ideal\_nz\_closed\_TCC4.....proved - complete [shostak](0.38 s)

prime\_ideal\_nz\_closed.....proved - complete [shostak](0.54 s)

**Theory** ring\_prime\_ideal totals: 9 formulas, 9 attempted, 9 succeeded (3.90 s)

Proof summary for **theory** ring\_prime\_ideal\_def

**Theory** ring\_prime\_ideal\_def totals: 0 formulas, 0 attempted, 0 succeeded (0.00 s)

Proof summary for **theory** quotient\_rings

IMP\_ring\_cosets\_lemmas\_TCC1.....proved - complete [shostak](0.25 s)

add\_charac\_TCC1.....proved - complete [shostak](0.30 s)

add\_charac\_TCC2.....proved - complete [shostak](0.27 s)

add\_charac.....proved - complete [shostak](0.59 s)

add\_is\_coset.....proved - complete [shostak](0.40 s)

coset\_add.....proved - complete [shostak](0.29 s)

product\_charac.....proved - complete [shostak](0.58 s)

lprod\_equal\_rprod\_TCC1.....proved - complete [shostak](0.26 s)

lprod\_equal\_rprod\_TCC2.....proved - complete [shostak](0.27 s)

lprod\_equal\_rprod\_TCC3.....proved - complete [shostak](0.27 s)

lprod\_equal\_rprod\_TCC4.....proved - complete [shostak](0.26 s)

lprod\_equal\_rprod.....proved - complete [shostak](0.86 s)

product\_is\_coset.....proved - complete [shostak](0.61 s)

coset\_product.....proved - complete [shostak](0.29 s)

quotient\_group\_is\_abelian\_group\_TCC1...proved - complete [shostak](0.28 s)

quotient\_group\_is\_abelian\_group\_TCC2...proved - complete [shostak](0.27 s)

quotient\_group\_is\_abelian\_group\_TCC3...proved - complete [shostak](0.28 s)

quotient\_group\_is\_abelian\_group.....proved - complete [shostak](0.76 s)

quotient\_group\_is\_ring\_TCC1.....proved - complete [shostak](0.26 s)

quotient\_group\_is\_ring.....proved - complete [shostak](0.57 s)

fullset\_quot\_group\_is\_ring.....proved - complete [shostak](2.11 s)

inv\_charac\_TCC1.....proved - complete [shostak](0.27 s)

inv\_charac\_TCC2.....proved - complete [shostak](0.30 s)

inv\_charac.....proved - complete [shostak](0.53 s)

coset\_subring\_TCC1.....proved - complete [shostak](0.29 s)

```

coset_subring_TCC2.....proved - complete [shostak](0.29 s)
coset_subring_TCC3.....proved - complete [shostak](0.28 s)
coset_subring_TCC4.....proved - complete [shostak](0.29 s)
coset_subring_TCC5.....proved - complete [shostak](0.28 s)
coset_subring.....proved - complete [shostak](0.62 s)
coset_ideal_TCC1.....proved - complete [shostak](0.29 s)
coset_ideal_TCC2.....proved - complete [shostak](0.00 s)
coset_ideal_TCC3.....proved - complete [shostak](0.27 s)
coset_ideal_TCC4.....proved - complete [shostak](0.27 s)
coset_ideal.....proved - complete [shostak](0.81 s)
commutative_quotient_ring_TCC1.....proved - complete [shostak](0.28 s)
commutative_quotient_ring_TCC2.....proved - complete [shostak](0.28 s)
commutative_quotient_ring_TCC3.....proved - complete [shostak](0.29 s)
commutative_quotient_ring_TCC4.....proved - complete [shostak](0.37 s)
commutative_quotient_ring.....proved - complete [shostak](0.35 s)
lcoset_power_nat.....proved - complete [shostak](0.47 s)
lcoset_power_int.....proved - complete [shostak](0.34 s)
Theory quotient_rings totals: 42 formulas, 42 attempted, 42 succeeded (17.22

```

s)

Proof summary for theory ring\_cosets\_lemmas

```

IMP_ring_ideal_TCC1.....proved - complete [shostak](0.26 s)
lcoset_iff_rcoset.....proved - complete [shostak](0.40 s)
lcoset_iff_coset.....proved - complete [shostak](0.27 s)
lcos_eq_rcos.....proved - complete [shostak](0.30 s)
self_coset.....proved - complete [shostak](0.41 s)
gen_is_any.....proved - complete [shostak](0.41 s)
lcos_eq.....proved - complete [shostak](0.28 s)
lcos_eq2.....proved - complete [shostak](0.35 s)
lc_gen_eq_TCC1.....proved - complete [shostak](0.27 s)
lc_gen_eq.....proved - complete [shostak](0.36 s)
ring_lcos_subset.....proved - complete [shostak](0.29 s)
ring_rcos_subset.....proved - complete [shostak](0.29 s)
left_zero.....proved - complete [shostak](0.28 s)
right_zero.....proved - complete [shostak](0.28 s)
ideal_is_coset.....proved - complete [shostak](0.31 s)
sum_subring_ideal.....proved - complete [shostak](0.54 s)
sum_ideal_ideal.....proved - complete [shostak](0.35 s)
sum_is_ideal_TCC1.....proved - complete [shostak](0.27 s)
sum_is_ideal.....proved - complete [shostak](0.32 s)
Theory ring_cosets_lemmas totals: 19 formulas, 19 attempted, 19 succeeded

```

(6.25 s)

Proof summary for theory product\_coset\_def

```

product_TCC1.....proved - complete [shostak](0.28 s)
product_TCC2.....proved - complete [shostak](0.29 s)
Theory product_coset_def totals: 2 formulas, 2 attempted, 2 succeeded (0.57 s)

```

Proof summary for theory ring\_with\_one\_ideal

```

IMP_ring_with_one_TCC1.....proved - complete [shostak](0.27 s)
IMP_ring_ideal_TCC1.....proved - complete [shostak](0.36 s)
l_ideal_w_one_is_R.....proved - complete [shostak](0.33 s)
r_ideal_w_one_is_R.....proved - complete [shostak](0.33 s)
ideal_w_one_is_R.....proved - complete [shostak](0.30 s)
no_prop_l_ideal_nz_closed.....proved - complete [shostak](0.41 s)

```

```

no_prop_r_ideal_nz_closed.....proved - complete [shostak](0.40 s)
set_of_ideals_bounded_above.....proved - incomplete [shostak](0.79 s)
set_of_ideals_has_maximal.....proved - incomplete [shostak](0.97 s)
Theory ring_with_one_ideal totals: 9 formulas, 9 attempted, 9 succeeded (4.16

```

s)

Proof summary for theory ring\_principal\_ideal

```

IMP_ring_one_generator_TCC1.....proved - complete [shostak](0.35 s)
gen_is_member.....proved - complete [shostak](0.42 s)
principal_ideal_is_ideal.....proved - complete [shostak](0.49 s)
principal_ideal_charac.....proved - complete [shostak](0.74 s)
comm_principal_ideal_charac.....proved - complete [shostak](0.38 s)
principal_ideal_subset.....proved - complete [shostak](0.40 s)
stable_chain.....proved - complete [shostak](0.45 s)
Theory ring_principal_ideal totals: 7 formulas, 7 attempted, 7 succeeded (3.23

```

s)

Proof summary for theory ring\_principal\_ideal\_def

```

Theory ring_principal_ideal_def totals: 0 formulas, 0 attempted, 0 succeeded
(0.00 s)

```

Proof summary for theory ring\_divides

```

IMP_ring_with_id_one_generator_TCC1...proved - complete [shostak](0.31 s)
IMP_ring_nz_closed_aux_TCC1.....proved - complete [shostak](0.39 s)
IMP_op_finseq_monoid_TCC1.....proved - complete [shostak](0.34 s)
divides_subset_TCC1.....proved - complete [shostak](0.34 s)
divides_subset.....proved - complete [shostak](0.42 s)
divides_equal_TCC1.....proved - complete [shostak](0.33 s)
divides_equal_TCC2.....proved - complete [shostak](0.33 s)
divides_equal.....proved - complete [shostak](0.47 s)
associates_equiv_relation.....proved - complete [shostak](0.42 s)
unit_divides_TCC1.....proved - complete [shostak](0.33 s)
unit_divides.....proved - complete [shostak](0.40 s)
one_gen_unit_R.....proved - complete [shostak](0.40 s)
quot_unit_associates.....proved - complete [shostak](0.35 s)
int_domain_assoc_quot_unit_TCC1.....proved - complete [shostak](0.54 s)
int_domain_assoc_quot_unit.....proved - complete [shostak](0.49 s)
x_divides_x_TCC1.....proved - complete [shostak](0.42 s)
x_divides_x.....proved - complete [shostak](0.34 s)
int_domain_assoc_unit_TCC1.....proved - complete [shostak](0.46 s)
int_domain_assoc_unit_TCC2.....proved - complete [shostak](0.54 s)
int_domain_assoc_unit_TCC3.....proved - complete [shostak](0.48 s)
int_domain_assoc_unit.....proved - complete [shostak](0.43 s)
div_member_fseq_div_op_finseq_TCC1...proved - complete [shostak](1.49 s)
div_member_fseq_div_op_finseq.....proved - complete [shostak](0.00 s)
Theory ring_divides totals: 23 formulas, 23 attempted, 23 succeeded (10.00 s)

```

Proof summary for theory ring\_with\_id\_one\_generator

```

IMP_ring_one_generator_TCC1.....proved - complete [shostak](0.33 s)
IMP_ring_with_one_TCC1.....proved - complete [shostak](0.37 s)
member_center_r_prod_is_one_gen.....proved - complete [shostak](0.85 s)
member_center_l_prod_is_one_gen.....proved - complete [shostak](0.87 s)
member_center_l_prod_is_r_prod.....proved - complete [shostak](0.35 s)
commutative_id_one_gen_charac.....proved - complete [shostak](0.44 s)
Theory ring_with_id_one_generator totals: 6 formulas, 6 attempted, 6 succeeded

```

(3.21 s)

Proof summary for **theory** ring\_one\_generator

IMP_ring_basic_properties_TCC1.....	proved - complete	[shostak](0.33 s)
F_one_gen_TCC1.....	proved - complete	[shostak](0.39 s)
gen_is_member_one_gen.....	proved - complete	[shostak](0.36 s)
one_gen_is_sum_closed_TCC1.....	proved - complete	[shostak](0.43 s)
one_gen_is_sum_closed.....	proved - complete	[shostak](1.33 s)
inv_one_gen_TCC1.....	proved - complete	[shostak](0.35 s)
inv_one_gen.....	proved - complete	[shostak](0.82 s)
one_gen_is_ideal.....	proved - complete	[shostak](1.55 s)
R_sigma_of_comm_factor_of_gen.....	proved - complete	[shostak](0.47 s)
commutative_one_gen_charac.....	proved - complete	[shostak](0.48 s)
commutative_one_gen_is_ideal.....	proved - complete	[shostak](0.34 s)
F_one_gen_r_comm.....	proved - complete	[shostak](0.39 s)
R_sigma_over_center_l.....	proved - complete	[shostak](0.42 s)
F_one_gen_l_comm.....	proved - complete	[shostak](0.38 s)
R_sigma_over_center_r.....	proved - complete	[shostak](0.42 s)
subset_product_one_gen_TCC1.....	proved - complete	[shostak](0.57 s)
subset_product_one_gen.....	proved - complete	[shostak](0.74 s)
subset_prod_one_gen_ideal_prop1.....	proved - complete	[shostak](0.40 s)
subset_prod_one_gen_ideal_prop2.....	proved - complete	[shostak](0.99 s)
sum_strict_subset_one_gen.....	proved - complete	[shostak](0.43 s)

**Theory** ring\_one\_generator totals: 20 formulas, 20 attempted, 20 succeeded

(11.59 s)

Proof summary for **theory** ring\_ideal

IMP_ring_basic_properties_TCC1.....	proved - complete	[shostak](0.26 s)
left_ideal_equiv.....	proved - complete	[shostak](0.39 s)
right_ideal_equiv.....	proved - complete	[shostak](0.33 s)
ideal_equiv.....	proved - complete	[shostak](0.33 s)
self_ideal.....	proved - complete	[shostak](0.30 s)
zero_ideal.....	proved - complete	[shostak](0.32 s)
ideal_transitive_TCC1.....	proved - complete	[shostak](0.48 s)
ideal_transitive.....	proved - complete	[shostak](0.28 s)
intersection_subring_ideal_TCC1.....	proved - complete	[shostak](0.48 s)
intersection_subring_ideal.....	proved - complete	[shostak](0.35 s)
r_prod_is_sum_closed.....	proved - complete	[shostak](0.28 s)
l_prod_is_sum_closed.....	proved - complete	[shostak](0.28 s)
inv_is_member_l_prod.....	proved - complete	[shostak](0.30 s)
inv_is_member_r_prod.....	proved - complete	[shostak](0.29 s)
l_prod_is_r_ideal.....	proved - complete	[shostak](0.37 s)
r_prod_is_l_ideal.....	proved - complete	[shostak](0.37 s)
ideal_iunion_ideal.....	proved - complete	[shostak](0.47 s)
chain_ideal_union_ideal.....	proved - incomplete	[shostak](0.56 s)

**Theory** ring\_ideal totals: 18 formulas, 18 attempted, 18 succeeded (6.44 s)

Proof summary for **theory** ring\_ideal\_def

**Theory** ring\_ideal\_def totals: 0 formulas, 0 attempted, 0 succeeded (0.00 s)

Proof summary for **theory** cosets\_def

lc_gen_TCC1.....	proved - complete	[shostak](0.27 s)
rc_gen_TCC1.....	proved - complete	[shostak](0.27 s)
gen_TCC1.....	proved - complete	[shostak](0.26 s)
lcos_is_left_coset.....	proved - complete	[shostak](0.25 s)

rcos\_is\_right\_coset.....proved - complete [shostak](0.25 s)  
add\_TCC1.....proved - complete [shostak](0.27 s)  
Theory cosets\_def totals: 6 formulas, 6 attempted, 6 succeeded (1.57 s)

Proof summary for theory ring\_center

IMP\_ring\_basic\_properties\_TCC1.....proved - complete [shostak](0.26 s)  
center\_subring.....proved - complete [shostak](0.52 s)  
commutative\_ring\_equal\_center.....proved - complete [shostak](0.30 s)  
Theory ring\_center totals: 3 formulas, 3 attempted, 3 succeeded (1.08 s)

Proof summary for theory ring\_unit\_def

l\_r\_inv\_equal.....proved - complete [shostak](0.34 s)  
Theory ring\_unit\_def totals: 1 formulas, 1 attempted, 1 succeeded (0.34 s)

Proof summary for theory integral\_domain\_with\_one\_def

Theory integral\_domain\_with\_one\_def totals: 0 formulas, 0 attempted, 0  
succeeded (0.00 s)

Proof summary for theory ring\_gcd\_def

gcd?\_TCC1.....proved - complete [shostak](0.39 s)  
gcd?\_TCC2.....proved - complete [shostak](0.48 s)  
Theory ring\_gcd\_def totals: 2 formulas, 2 attempted, 2 succeeded (0.87 s)

Proof summary for theory ring\_divides\_def

associates?\_TCC1.....proved - complete [shostak](0.40 s)  
associates?\_TCC2.....proved - complete [shostak](0.39 s)  
Theory ring\_divides\_def totals: 2 formulas, 2 attempted, 2 succeeded (0.79 s)

Proof summary for theory quaternions\_Hamilton

IMP\_quaternions\_TCC1.....proved - complete [shostak](0.39 s)  
IMP\_quaternions\_TCC2.....proved - complete [shostak](0.89 s)  
conversion\_quot\_TCC1.....proved - complete [shostak](0.62 s)  
conversion\_quot.....proved - complete [shostak](0.73 s)  
quat\_is\_Real\_p\_Vector\_part.....proved - complete [shostak](0.48 s)  
decompose\_eq\_Real\_Vector\_part.....proved - complete [shostak](0.49 s)  
Vector\_part\_scalar.....proved - complete [shostak](0.55 s)  
q\_prod\_Real\_Vector\_part.....proved - complete [shostak](1.64 s)  
conjugate\_Real\_vector\_part.....proved - complete [shostak](0.60 s)  
T\_q\_Real\_charac.....proved - incomplete [shostak](0.00 s)  
r\_angle\_TCC1.....proved - complete [shostak](0.49 s)  
r\_angle\_TCC2.....proved - complete [shostak](0.50 s)  
n\_rot\_axis\_TCC1.....proved - incomplete [shostak](0.48 s)  
rot\_quat\_TCC1.....proved - incomplete [shostak](0.48 s)  
rot\_quat\_TCC2.....proved - incomplete [shostak](0.48 s)  
rot\_quat\_TCC3.....proved - incomplete [shostak](1.43 s)  
conj\_quat\_eq.....proved - complete [shostak](0.49 s)  
mult\_QH\_eq.....proved - complete [shostak](0.99 s)  
T\_q\_TCC1.....proved - complete [shostak](0.72 s)  
T\_q\_eq.....proved - complete [shostak](0.48 s)  
Quat\_Rot\_Aux0.....proved - incomplete [shostak](0.63 s)  
Quat\_Rot\_Aux1.....proved - incomplete [shostak](0.65 s)  
Quat\_Rot\_Aux2.....proved - incomplete [shostak](1.42 s)  
Quat\_Rot\_Aux3.....proved - incomplete [shostak](2.18 s)  
Quaternions\_Rotation.....proved - incomplete [shostak](0.82 s)  
Quaternions\_Rotation\_Deform\_TCC1.....proved - incomplete [shostak](0.46 s)



Quaternions\_Rotation\_Deform\_TCC2.....proved - incomplete [shostak](0.50 s)  
 Quaternions\_Rotation\_Deform\_TCC3.....proved - incomplete [shostak](0.46 s)  
 Quaternions\_Rotation\_Deform\_TCC4.....proved - incomplete [shostak](0.60 s)  
 Quaternions\_Rotation\_Deform.....proved - incomplete [shostak](1.73 s)  
 Theory quaternions\_Hamilton totals: 30 formulas, 30 attempted, 30 succeeded  
 (22.37 s)

Proof summary for theory vectors\_3D\_extra  
 angle\_nnreal\_le\_pi\_exists.....proved - incomplete [shostak](1.00 s)  
 angle\_between\_nnreal\_le\_pi.....proved - incomplete [shostak](0.99 s)  
 LI\_implies\_nzvector.....proved - incomplete [shostak](0.41 s)  
 norm\_cross\_charac\_aux.....proved - incomplete [shostak](0.90 s)  
 norm\_cross\_charac.....proved - incomplete [shostak](1.10 s)  
 orth\_cross.....proved - incomplete [shostak](0.63 s)  
 Theory vectors\_3D\_extra totals: 6 formulas, 6 attempted, 6 succeeded (5.03 s)

Proof summary for theory quaternions\_Hamilton\_extra  
 IMP\_quaternions\_TCC1.....proved - complete [shostak](0.41 s)  
 IMP\_quaternions\_TCC2.....proved - complete [shostak](0.92 s)  
 sc\_part\_red\_norm\_nnegreal.....proved - complete [shostak](1.24 s)  
 QH\_norm\_TCC1.....proved - complete [shostak](1.29 s)  
 sq\_QHnorm\_charac.....proved - incomplete [shostak](0.65 s)  
 red\_norm\_QH1.....proved - incomplete [shostak](0.55 s)  
 norm\_decomp.....proved - incomplete [shostak](6.46 s)  
 norm\_decomp\_1.....proved - incomplete [shostak](0.88 s)  
 QH\_nzquat\_nznorm.....proved - incomplete [shostak](1.39 s)  
 QH\_times\_is\_zero\_q.....proved - incomplete [shostak](0.73 s)  
 QH\_inv\_red\_norm\_TCC1.....proved - incomplete [shostak](0.48 s)  
 QH\_inv\_red\_norm\_TCC2.....proved - incomplete [shostak](0.50 s)  
 QH\_inv\_red\_norm\_TCC3.....proved - incomplete [shostak](0.48 s)  
 QH\_inv\_red\_norm\_TCC4.....proved - incomplete [shostak](0.49 s)  
 QH\_inv\_red\_norm\_TCC5.....proved - incomplete [shostak](1.20 s)  
 QH\_inv\_red\_norm\_TCC6.....proved - incomplete [shostak](0.50 s)  
 QH\_inv\_red\_norm.....proved - incomplete [shostak](9.13 s)  
 QH\_inv\_q\_prod\_charac.....proved - incomplete [shostak](1.60 s)  
 quat\_trig\_form\_aux.....proved - incomplete [shostak](1.71 s)  
 quat\_trig\_form.....proved - incomplete [shostak](0.98 s)  
 QH\_norm\_is\_norm.....proved - incomplete [shostak](0.89 s)  
 Theory quaternions\_Hamilton\_extra totals: 21 formulas, 21 attempted, 21  
 succeeded (32.46 s)

Proof summary for theory quaternions  
 IMP\_quaternions\_def\_TCC1.....proved - complete [shostak]( 0.42 s)  
 IMP\_ring\_characteristic\_def\_TCC1....proved - complete [shostak]( 0.42 s)  
 IMP\_pvs\_strategies\_lemmas\_TCC1.....proved - complete [shostak]( 0.41 s)  
 nz\_quat\_TCC1.....proved - complete [shostak]( 0.46 s)  
 one\_sc\_times.....proved - complete [SHOSTAK]( 0.96 s)  
 sc\_quat.....proved - complete [SHOSTAK]( 0.44 s)  
 sc\_sqr\_i.....proved - complete [SHOSTAK]( 0.41 s)  
 sc\_sqr\_j.....proved - complete [SHOSTAK]( 0.41 s)  
 sc\_ji\_prod.....proved - complete [SHOSTAK]( 0.41 s)  
 inv\_charac.....proved - complete [SHOSTAK]( 0.45 s)  
 q\_inv\_inv.....proved - complete [SHOSTAK]( 0.41 s)  
 quat\_negative\_times.....proved - complete [SHOSTAK]( 0.41 s)  
 quat\_times\_negative.....proved - complete [SHOSTAK]( 0.42 s)

ji_prod_lem.....	proved - complete	[SHOSTAK]( 0.40 s)
kk_prod.....	proved - complete	[SHOSTAK]( 0.48 s)
ki_prod.....	proved - complete	[SHOSTAK]( 0.43 s)
kj_prod.....	proved - complete	[SHOSTAK]( 0.47 s)
ik_prod.....	proved - complete	[SHOSTAK]( 0.40 s)
jk_prod.....	proved - complete	[SHOSTAK]( 0.44 s)
basis_quat.....	proved - complete	[SHOSTAK]( 0.43 s)
zero_q_plus.....	proved - complete	[SHOSTAK]( 0.49 s)
plus_zero_q.....	proved - complete	[SHOSTAK]( 0.50 s)
q_is_zero_iff_conj.....	proved - complete	[SHOSTAK]( 0.52 s)
sc_one_q.....	proved - complete	[SHOSTAK]( 0.65 s)
sc_one_quat.....	proved - complete	[SHOSTAK]( 0.41 s)
q_prod_charac.....	proved - complete	[SHOSTAK]( 3.70 s)
zero_q_times.....	proved - complete	[SHOSTAK]( 0.44 s)
times_zero_q.....	proved - complete	[SHOSTAK]( 0.43 s)
quat_is_ring_w_one.....	proved - complete	[SHOSTAK]( 1.03 s)
quat_x_pure_part_commutates.....	proved - complete	[SHOSTAK]( 0.49 s)
pure_quat_charac.....	proved - complete	[SHOSTAK]( 4.55 s)
red_trace_charac.....	proved - complete	[SHOSTAK]( 0.42 s)
red_trace_plus.....	proved - complete	[SHOSTAK]( 0.45 s)
red_trace_times_commutates.....	proved - complete	[SHOSTAK]( 0.54 s)
red_norm_charac.....	proved - complete	[SHOSTAK]( 0.78 s)
red_norm_one_q.....	proved - complete	[SHOSTAK]( 0.45 s)
red_norm_is_scalar.....	proved - complete	[SHOSTAK]( 0.41 s)
conj_conj_quat.....	proved - complete	[SHOSTAK]( 0.43 s)
conj_product_quat.....	proved - complete	[SHOSTAK]( 1.92 s)
q_x_cq_commutates.....	proved - complete	[SHOSTAK]( 0.47 s)
conj_product_quat_scalar.....	proved - complete	[shostak]( 0.47 s)
red_norm_conj.....	proved - complete	[SHOSTAK]( 0.40 s)
sc_F_commutates.....	proved - complete	[SHOSTAK]( 0.46 s)
center_quat_is_sc_F.....	proved - complete	[SHOSTAK]( 1.07 s)
sc_F_product_charac.....	proved - complete	[SHOSTAK]( 0.45 s)
q_x_v_cq.....	proved - complete	[SHOSTAK]( 0.00 s)
T_q_TCC1.....	proved - complete	[shostak]( 0.40 s)
T_q_is_linear_TCC1.....	proved - complete	[shostak]( 0.43 s)
T_q_is_linear.....	proved - complete	[SHOSTAK]( 0.47 s)
red_norm_prod.....	proved - complete	[SHOSTAK]( 0.46 s)
T_q_red_norm_invariant.....	proved - complete	[SHOSTAK]( 0.43 s)
T_q_invariant_red_norm_TCC1.....	proved - complete	[shostak]( 0.42 s)
T_q_invariant_red_norm.....	proved - complete	[SHOSTAK]( 0.45 s)
nz_red_norm_iff_inv_exist.....	proved - complete	[SHOSTAK]( 0.83 s)
div_ring_iff_nz_rednorm.....	proved - complete	[SHOSTAK]( 0.63 s)
inv_q_prod_charac_TCC1.....	proved - complete	[shostak]( 1.33 s)
inv_q_prod_charac_TCC2.....	proved - complete	[shostak]( 0.95 s)
inv_q_prod_charac_TCC3.....	proved - complete	[shostak]( 0.54 s)
inv_q_prod_charac_TCC4.....	proved - complete	[shostak]( 0.40 s)
inv_q_prod_charac.....	proved - complete	[SHOSTAK]( 4.50 s)
quat_div_ring_aux1.....	proved - complete	[SHOSTAK]( 0.60 s)
quat_div_ring_aux2.....	proved - complete	[SHOSTAK]( 3.26 s)
quat_div_ring_char.....	proved - complete	[SHOSTAK](10.45 s)
Theory quaternions totals: 63 formulas, 63 attempted, 63 succeeded (57.54 s)		

Proof summary for theory quaternions\_def

IMP_group_TCC1.....	proved - complete	[shostak](0.39 s)
nzpure_quat_TCC1.....	proved - complete	[shostak](0.42 s)

Theory quaternions\_def totals: 2 formulas, 2 attempted, 2 succeeded (0.81 s)

Proof summary for theory ring\_characteristic\_def

IMP_ring_basic_properties_TCC1.....	proved - complete	[shostak](0.34 s)
charac_TCC1.....	proved - complete	[shostak](0.43 s)
times_char.....	proved - complete	[shostak](0.40 s)
member_N_or_zero_TCC1.....	proved - complete	[shostak](0.43 s)
member_N_or_zero.....	proved - complete	[shostak](0.34 s)
multiple_char.....	proved - complete	[shostak](0.74 s)
char_1_zero_ring.....	proved - complete	[shostak](0.43 s)

Theory ring\_characteristic\_def totals: 7 formulas, 7 attempted, 7 succeeded (3.11 s)

Proof summary for theory pvs\_strategies\_lemmas

IMP_field_TCC1.....	proved - complete	[shostak](0.38 s)
times_heading_third.....	proved - complete	[SHOSTAK](0.52 s)
times_heading_fourth.....	proved - complete	[SHOSTAK](0.44 s)
times_heading_fifth.....	proved - complete	[SHOSTAK](0.46 s)
times_heading_sixth.....	proved - complete	[SHOSTAK](0.50 s)
times_heading_seventh.....	proved - complete	[SHOSTAK](0.56 s)
plus_heading_third.....	proved - complete	[SHOSTAK](0.42 s)
plus_heading_fourth.....	proved - complete	[SHOSTAK](0.43 s)
plus_heading_fifth.....	proved - complete	[SHOSTAK](0.48 s)
plus_heading_sixth.....	proved - complete	[SHOSTAK](0.49 s)
plus_heading_seventh.....	proved - complete	[SHOSTAK](0.55 s)

Theory pvs\_strategies\_lemmas totals: 11 formulas, 11 attempted, 11 succeeded (5.24 s)

Proof summary for theory center\_def

center_def.....	proved - complete	[shostak](0.28 s)
center_subset.....	proved - complete	[shostak](0.27 s)

Theory center\_def totals: 2 formulas, 2 attempted, 2 succeeded (0.55 s)

Proof summary for theory ring\_nz\_closed\_aux

IMP_ring_basic_properties_TCC1.....	proved - complete	[shostak](0.27 s)
nz_times_is_zero.....	proved - complete	[shostak](0.37 s)
nzd_R_cancel_left.....	proved - complete	[shostak](0.35 s)
nzd_R_cancel_right.....	proved - complete	[shostak](0.35 s)
subring_nz_closed.....	proved - complete	[shostak](0.36 s)

Theory ring\_nz\_closed\_aux totals: 5 formulas, 5 attempted, 5 succeeded (1.71 s)

Proof summary for theory ring\_basic\_properties

IMP_ring_TCC1.....	proved - complete	[shostak](0.25 s)
zero_is_member_R.....	proved - complete	[shostak](0.29 s)
inv_is_member_R.....	proved - complete	[shostak](0.27 s)
R_sum_star_closed.....	proved - complete	[shostak](0.27 s)
R_prod_star_closed.....	proved - complete	[shostak](0.26 s)
l_plus_zero.....	proved - complete	[shostak](0.28 s)
r_plus_zero.....	proved - complete	[shostak](0.27 s)
no_singleton_nzx.....	proved - complete	[shostak](0.28 s)
card_gt_one_nzx.....	proved - complete	[shostak](0.55 s)
no_singleton_card.....	proved - complete	[shostak](0.30 s)
subring_transitive_TCC1.....	proved - complete	[shostak](0.26 s)
subring_transitive.....	proved - complete	[shostak](0.27 s)

```

subring_equiv.....proved - complete [shostak](0.44 s)
times_member.....proved - complete [shostak](0.27 s)
left_times.....proved - complete [shostak](0.48 s)
right_times.....proved - complete [shostak](0.47 s)
inv_times_neg.....proved - complete [shostak](0.33 s)
inv_times_inv.....proved - complete [shostak](0.27 s)
times_inv_neg.....proved - complete [shostak](0.27 s)
times_int_zero.....proved - complete [shostak](0.26 s)
times_int_one.....proved - complete [shostak](0.26 s)
times_sum.....proved - complete [shostak](0.29 s)
times_o.....proved - complete [shostak](0.29 s)
times_product.....proved - complete [shostak](0.26 s)
R_sigma_TCC1.....proved - complete [shostak](0.26 s)
R_sigma_TCC2.....proved - complete [shostak](0.33 s)
R_sigma_first.....proved - complete [shostak](0.37 s)
R_sigma_eq_k.....proved - complete [shostak](0.56 s)
R_sigma_eq.....proved - complete [shostak](0.27 s)
R_sigma_eq2.....proved - complete [shostak](0.28 s)
R_sigma_sum.....proved - complete [shostak](0.36 s)
ast_R_sigma.....proved - complete [shostak](0.33 s)
R_sigma_ast.....proved - complete [shostak](0.33 s)
R_sigma_inv.....proved - complete [shostak](0.37 s)
R_sigma_o.....proved - complete [shostak](0.36 s)
R_sigma_R_sigma_TCC1.....proved - complete [shostak](0.30 s)
R_sigma_R_sigma_TCC2.....proved - complete [shostak](0.42 s)
R_sigma_R_sigma.....proved - complete [shostak](1.98 s)
R_sigma_is_member_R.....proved - complete [shostak](0.41 s)
nlzd_TCC1.....proved - complete [shostak](0.25 s)
nzd_cancel_left.....proved - complete [shostak](0.30 s)
nzd_cancel_right.....proved - complete [shostak](0.32 s)

```

**Theory** ring\_basic\_properties totals: 42 formulas, 42 attempted, 42 succeeded (15.25 s)

Proof summary for **theory** op\_finseq\_monoid

```

IMP_monoid_TCC1.....proved - complete [shostak](0.33 s)
op_fseq_singleton.....proved - complete [shostak](0.50 s)
op_fseq_composition.....proved - incomplete [shostak](1.51 s)
op_subfseq_closed.....proved - complete [shostak](1.36 s)
op_fseq_closed.....proved - complete [shostak](0.60 s)
op_fseq_split_TCC1.....proved - complete [shostak](0.34 s)
op_fseq_split_TCC2.....proved - complete [shostak](0.34 s)
op_fseq_split.....proved - incomplete [shostak](1.38 s)
op_fseq_split_commute_TCC1.....proved - complete [shostak](0.34 s)
op_fseq_split_commute_TCC2.....proved - complete [shostak](0.34 s)
op_fseq_split_commute.....proved - incomplete [shostak](0.79 s)
op_fseq_split_delete.....proved - incomplete [shostak](2.42 s)
op_fseq_same_replace_first_TCC1.....proved - complete [shostak](0.38 s)
op_fseq_same_replace_first.....proved - incomplete [shostak](0.00 s)
op_fseq_same_replace_last_TCC1.....proved - complete [shostak](0.38 s)
op_fseq_same_replace_last.....proved - incomplete [shostak](1.08 s)
heading_fseq_TCC1.....proved - complete [shostak](0.36 s)
heading_fseq_TCC2.....proved - complete [shostak](0.35 s)
times_heading_fseq.....proved - incomplete [shostak](2.07 s)

```

**Theory** op\_finseq\_monoid totals: 19 formulas, 19 attempted, 19 succeeded (14.86 s)

Proof summary for **theory** op\_finseq\_monoid\_def

```
op_fseq_TCC1.....proved - complete [shostak](0.33 s)
op_fseq_TCC2.....proved - complete [shostak](0.34 s)
op_fseq_TCC3.....proved - complete [shostak](0.41 s)
Theory op_finseq_monoid_def totals: 3 formulas, 3 attempted, 3 succeeded (1.08 s)
```

s)

Proof summary for **theory** field

```
IMP_division_ring_TCC1.....proved - complete [shostak](0.38 s)
IMP_integral_domain_TCC1.....proved - complete [shostak](0.52 s)
field_TCC1.....proved - complete [shostak](0.41 s)
nz_star_TCC1.....proved - complete [shostak](0.43 s)
field_is_division_ring.....proved - complete [shostak](0.65 s)
field_is_integral_domain.....proved - complete [shostak](0.43 s)
field_is_abelian_group_TCC1.....proved - complete [shostak](0.41 s)
field_is_abelian_group_TCC2.....proved - complete [shostak](0.41 s)
field_is_abelian_group.....proved - complete [shostak](1.34 s)
mult_div_TCC1.....proved - complete [shostak](0.42 s)
mult_div.....proved - complete [shostak](0.80 s)
times_div_right.....proved - complete [shostak](0.80 s)
div_times_TCC1.....proved - complete [shostak](0.42 s)
div_times.....proved - complete [shostak](1.19 s)
cross_mult.....proved - complete [shostak](1.18 s)
add_div.....proved - complete [shostak](1.23 s)
minus_div1.....proved - complete [shostak](0.51 s)
sq_div.....proved - complete [shostak](0.48 s)
Theory field totals: 18 formulas, 18 attempted, 18 succeeded (12.01 s)
```

Proof summary for **theory** field\_def

```
field?_TCC1.....proved - complete [shostak](0.30 s)
Theory field_def totals: 1 formulas, 1 attempted, 1 succeeded (0.30 s)
```

Proof summary for **theory** division\_ring

```
IMP_ring_with_one_TCC1.....proved - complete [shostak](0.40 s)
IMP_ring_nz_closed_TCC1.....proved - complete [shostak](0.50 s)
IMP_group_TCC1.....proved - complete [shostak](0.47 s)
IMP_group_TCC2.....proved - complete [shostak](0.49 s)
IMP_group_TCC3.....proved - complete [shostak](0.46 s)
IMP_group_TCC4.....proved - complete [shostak](0.67 s)
division_ring_TCC1.....proved - complete [shostak](0.43 s)
division_ring_is.....proved - complete [shostak](0.40 s)
division_ring_is_ring_with_one.....proved - complete [shostak](0.62 s)
division_ring_is_ring_nz_closed.....proved - complete [shostak](0.43 s)
division_ring_is_group.....proved - complete [shostak](0.82 s)
one_ne_zero.....proved - complete [shostak](0.47 s)
cancel_times_right.....proved - complete [shostak](0.49 s)
cancel_times_left.....proved - complete [shostak](0.50 s)
idempotent_times.....proved - complete [shostak](0.42 s)
recip_ne_zero.....proved - complete [shostak](0.59 s)
nz_T_div_nz_T_is_nz_T.....proved - complete [shostak](0.59 s)
div_simplify.....proved - complete [shostak](0.47 s)
cancel_div_right.....proved - complete [shostak](0.83 s)
cancel_div_left.....proved - complete [shostak](1.16 s)
times_div_left.....proved - complete [shostak](0.43 s)
```

```

div_eq_zero.....proved - complete [shostak](0.60 s)
div_mult.....proved - complete [shostak](0.43 s)
div_mult_left.....proved - complete [shostak](0.46 s)
div_mult_right.....proved - complete [shostak](0.44 s)
div_distributes.....proved - complete [shostak](0.59 s)
div_distributes_minus.....proved - complete [shostak](0.47 s)
div_div1.....proved - complete [shostak](0.46 s)
div_div2_TCC1.....proved - complete [shostak](0.42 s)
div_div2.....proved - complete [shostak](0.80 s)
Theory division_ring totals: 30 formulas, 30 attempted, 30 succeeded (16.33 s)

```

Proof summary for theory ring\_with\_one

```

IMP_ring_TCC1.....proved - complete [shostak](0.26 s)
IMP_monoid_TCC1.....proved - complete [shostak](0.31 s)
ring_with_one_TCC1.....proved - complete [shostak](0.28 s)
one_times.....proved - complete [shostak](0.30 s)
times_one.....proved - complete [shostak](0.30 s)
unique_left_identity.....proved - complete [shostak](0.34 s)
unique_right_identity.....proved - complete [shostak](0.35 s)
minus_one_times.....proved - complete [shostak](0.28 s)
times_minus_one.....proved - complete [shostak](0.28 s)
minus_one_sq_is_one.....proved - complete [shostak](0.27 s)
ring_with_one_is_ring.....proved - complete [shostak](0.41 s)
ring_with_one_is_monoid.....proved - complete [shostak](0.37 s)
Theory ring_with_one totals: 12 formulas, 12 attempted, 12 succeeded (3.75 s)

```

Proof summary for theory integral\_domain

```

IMP_commutative_ring_TCC1.....proved - complete [shostak](0.30 s)
integral_domain_TCC1.....proved - complete [shostak](0.35 s)
integral_domain_is.....proved - complete [shostak](0.31 s)
integral_domain_is_ring.....proved - complete [shostak](0.45 s)
Theory integral_domain totals: 4 formulas, 4 attempted, 4 succeeded (1.41 s)

```

Proof summary for theory integral\_domain\_def

Theory integral\_domain\_def totals: 0 formulas, 0 attempted, 0 succeeded (0.00 s)

Proof summary for theory commutative\_ring

```

IMP_ring_TCC1.....proved - complete [shostak](0.31 s)
commutative_ring_TCC1.....proved - complete [shostak](0.35 s)
times_commutative.....proved - complete [shostak](0.32 s)
commutative_ring_is_ring.....proved - complete [shostak](0.44 s)
commutative_subrings.....proved - complete [shostak](0.34 s)
sq_times.....proved - complete [shostak](0.33 s)
Theory commutative_ring totals: 6 formulas, 6 attempted, 6 succeeded (2.09 s)

```

Proof summary for theory division\_ring\_def

Theory division\_ring\_def totals: 0 formulas, 0 attempted, 0 succeeded (0.00 s)

Proof summary for theory ring\_with\_one\_def

```

commutative_ring_with_one?_TCC1.....proved - complete [shostak](0.29 s)
finite_commutative_ring_with_one?_TCC1...proved - complete [shostak](0.28 s)
Theory ring_with_one_def totals: 2 formulas, 2 attempted, 2 succeeded (0.57 s)

```

Proof summary for theory ring\_nz\_closed



```

IMP_ring_TCC1.....proved - complete [shostak](0.31 s)
ring_nz_closed_TCC1.....proved - complete [shostak](0.35 s)
ring_nz_closed_is.....proved - complete [shostak](0.32 s)
ring_nz_closed_is_ring.....proved - complete [shostak](0.45 s)
nz_T_times_nz_T_is_nz_T.....proved - complete [shostak](0.52 s)
negate_nz_T_is_nz_T.....proved - complete [shostak](0.31 s)
times_is_zero.....proved - complete [shostak](0.32 s)
nz_T_times.....proved - complete [shostak](0.32 s)
times_nz_T.....proved - complete [shostak](0.32 s)
nz_T_times_nz_T_is_not_zero.....proved - complete [shostak](0.31 s)
sq_nz_is_nz.....proved - complete [shostak](0.33 s)
sq_eq_zero.....proved - complete [shostak](0.32 s)
Theory ring_nz_closed totals: 12 formulas, 12 attempted, 12 succeeded (4.18 s)

```

Proof summary for **theory** ring\_nz\_closed\_def

**Theory** ring\_nz\_closed\_def totals: 0 formulas, 0 attempted, 0 succeeded (0.00 s)

Proof summary for **theory** ring

```

IMP_abelian_group_TCC1.....proved - complete [shostak](0.25 s)
ring_TCC1.....proved - complete [shostak](0.27 s)
plus_associative.....proved - complete [shostak](0.26 s)
plus_commutative.....proved - complete [shostak](0.30 s)
times_associative.....proved - complete [shostak](0.26 s)
right_distributive.....proved - complete [shostak](0.28 s)
left_distributive.....proved - complete [shostak](0.27 s)
zero_plus.....proved - complete [shostak](0.26 s)
plus_zero.....proved - complete [shostak](0.25 s)
negate_is_left_inv.....proved - complete [shostak](0.25 s)
negate_is_right_inv.....proved - complete [shostak](0.25 s)
cancel_right_plus.....proved - complete [shostak](0.28 s)
cancel_left_plus.....proved - complete [shostak](0.28 s)
negate_negate.....proved - complete [shostak](0.25 s)
cancel_right_minus.....proved - complete [shostak](0.28 s)
cancel_left_minus.....proved - complete [shostak](0.27 s)
negate_zero.....proved - complete [shostak](0.25 s)
negate_plus.....proved - complete [shostak](0.25 s)
times_plus.....proved - complete [shostak](0.28 s)
idempotent_add_is_zero.....proved - complete [shostak](0.27 s)
zero_times.....proved - complete [shostak](0.26 s)
times_zero.....proved - complete [shostak](0.26 s)
negative_times.....proved - complete [shostak](0.27 s)
times_negative.....proved - complete [shostak](0.28 s)
negative_times_negative.....proved - complete [shostak](0.26 s)
ring_is_abelian_group.....proved - complete [shostak](0.35 s)
subring_is_ring.....proved - complete [shostak](0.27 s)
sq_rew.....proved - complete [shostak](0.25 s)
sq_neg.....proved - complete [shostak](0.25 s)
sq_plus.....proved - complete [shostak](0.29 s)
sq_minus.....proved - complete [shostak](0.29 s)
sq_neg_minus.....proved - complete [shostak](0.27 s)
sq_zero.....proved - complete [shostak](0.26 s)
Theory ring totals: 33 formulas, 33 attempted, 33 succeeded (8.89 s)

```

Proof summary for **theory** ring\_def

```

ring?_TCC1.....proved - complete [shostak](0.25 s)
ring?_TCC2.....proved - complete [shostak](0.27 s)
commutative_ring?_TCC1.....proved - complete [shostak](0.25 s)
finite_commutative_ring?_TCC1.....proved - complete [shostak](0.27 s)
Theory ring_def totals: 4 formulas, 4 attempted, 4 succeeded (1.04 s)

```

Proof summary for **theory** abelian\_group

```

IMP_group_TCC1.....proved - complete [shostak](0.25 s)
abelian_group_TCC1.....proved - complete [shostak](0.27 s)
abelian_group_is_group.....proved - complete [shostak](0.32 s)
abelian_group_is_commutative_monoid...proved - complete [shostak](0.32 s)
abelian_subgroups.....proved - complete [shostak](0.33 s)
finite_abelian_group_TCC1.....proved - complete [shostak](0.30 s)
finite_abelian_group_is_abelian_group...proved - complete [shostak](0.34 s)
finite_abelian_group_is_finite_group...proved - complete [shostak](0.34 s)
finite_abelian_subgroups.....proved - complete [shostak](0.27 s)
Theory abelian_group totals: 9 formulas, 9 attempted, 9 succeeded (2.74 s)

```

Proof summary for **theory** group

```

IMP_monoid_TCC1.....proved - complete [shostak](0.26 s)
group_TCC1.....proved - complete [shostak](0.25 s)
group_is_monoid.....proved - complete [shostak](0.31 s)
finite_group_TCC1.....proved - complete [shostak](0.34 s)
finite_group_is_group.....proved - complete [shostak](0.33 s)
finite_group_is_finite_monoid.....proved - complete [shostak](0.34 s)
finite_subgroups.....proved - complete [shostak](0.28 s)
one_is_group.....proved - complete [shostak](0.28 s)
one_finite_group.....proved - complete [shostak](0.27 s)
one_group_TCC1.....proved - complete [shostak](0.36 s)
group_card_gt_0.....proved - complete [shostak](0.29 s)
inv_exists.....proved - complete [shostak](0.27 s)
inv_TCC1.....proved - complete [shostak](0.28 s)
inv_left.....proved - complete [shostak](0.25 s)
inv_right.....proved - complete [shostak](0.26 s)
cancel_right.....proved - complete [shostak](0.30 s)
cancel_left.....proved - complete [shostak](0.28 s)
inv_inv.....proved - complete [shostak](0.27 s)
cancel_right_inv.....proved - complete [shostak](0.28 s)
cancel_left_inv.....proved - complete [shostak](0.31 s)
inv_one.....proved - complete [shostak](0.28 s)
inv_star.....proved - complete [shostak](0.30 s)
unique_inv.....proved - complete [shostak](0.28 s)
inv_member.....proved - complete [shostak](0.30 s)
inv_in.....proved - complete [shostak](0.27 s)
divby.....proved - complete [shostak](0.29 s)
product_in.....proved - complete [shostak](0.28 s)
one_is_subgroup.....proved - complete [shostak](0.35 s)
group_is_subgroup.....proved - complete [shostak](0.27 s)
subgroup_is_group.....proved - complete [shostak](0.25 s)
subgroup_def.....proved - complete [shostak](0.41 s)
inv_power.....proved - complete [shostak](0.37 s)
power_inv_right.....proved - complete [shostak](0.27 s)
power_inv_left.....proved - complete [shostak](0.28 s)
caret_TCC1.....proved - complete [shostak](0.27 s)
caret_TCC2.....proved - complete [shostak](0.28 s)

```

expt_0.....	proved	- complete	[shostak](0.26 s)
expt_1.....	proved	- complete	[shostak](0.28 s)
expt_m1.....	proved	- complete	[shostak](0.27 s)
one_expt.....	proved	- complete	[shostak](0.29 s)
expt_neg.....	proved	- complete	[shostak](0.29 s)
inv_expt.....	proved	- complete	[shostak](0.28 s)
expt_def1.....	proved	- complete	[shostak](0.47 s)
expt_def2.....	proved	- complete	[shostak](0.36 s)
expt_mult.....	proved	- complete	[shostak](0.58 s)
expt_div.....	proved	- complete	[shostak](0.30 s)
expt_expt.....	proved	- complete	[shostak](0.44 s)
expt_commutes.....	proved	- complete	[shostak](0.31 s)
expt_inv_right.....	proved	- complete	[shostak](0.27 s)
expt_inv_left.....	proved	- complete	[shostak](0.27 s)
expt_member.....	proved	- complete	[shostak](0.31 s)
generated_by_TCC1.....	proved	- complete	[shostak](0.40 s)
generated_by_lem.....	proved	- complete	[shostak](0.26 s)
generated_is_subgroup.....	proved	- complete	[shostak](0.26 s)
generated_by_is_finite.....	proved	- complete	[shostak](0.39 s)
center_TCC1.....	proved	- complete	[shostak](0.32 s)
center_def.....	proved	- complete	[shostak](0.31 s)
center_subgroup.....	proved	- complete	[shostak](0.44 s)
one_left.....	proved	- complete	[shostak](0.27 s)
one_right.....	proved	- complete	[shostak](0.26 s)
assoc.....	proved	- complete	[shostak](0.27 s)

Theory group totals: 61 formulas, 61 attempted, 61 succeeded (18.75 s)

Proof summary for theory group\_def

abelian_group?_TCC1.....	proved	- complete	[shostak](0.26 s)
finite_abelian_group?_TCC1.....	proved	- complete	[shostak](0.26 s)
finite_group_surj.....	proved	- complete	[shostak](0.27 s)

Theory group\_def totals: 3 formulas, 3 attempted, 3 succeeded (0.79 s)

Proof summary for theory monoid

IMP_monad_TCC1.....	proved	- complete	[shostak](0.25 s)
IMP_semigroup_TCC1.....	proved	- complete	[shostak](0.26 s)
monoid_TCC1.....	proved	- complete	[shostak](0.24 s)
monoid_is_monad.....	proved	- complete	[shostak](0.30 s)
monoid_is_semigroup.....	proved	- complete	[shostak](0.30 s)
power_0.....	proved	- complete	[shostak](0.27 s)
power_1.....	proved	- complete	[shostak](0.27 s)
one_power.....	proved	- complete	[shostak](0.28 s)
power_def.....	proved	- complete	[shostak](0.33 s)
power_mult.....	proved	- complete	[shostak](0.41 s)
power_power.....	proved	- complete	[shostak](0.37 s)
power_commutes.....	proved	- complete	[shostak](0.32 s)
power_member.....	proved	- complete	[shostak](0.34 s)
one_is_monoid.....	proved	- complete	[shostak](0.30 s)
generated_is_submonoid.....	proved	- complete	[shostak](0.35 s)
generated_set_card_1.....	proved	- complete	[shostak](0.32 s)
finite_monoid_TCC1.....	proved	- complete	[shostak](0.39 s)
finite_monoid_is_monoid.....	proved	- complete	[shostak](0.30 s)
finite_monoid_is_finite_monad.....	proved	- complete	[shostak](0.32 s)
finite_submonoids.....	proved	- complete	[shostak](0.26 s)
commutative_monoid_TCC1.....	proved	- complete	[shostak](0.35 s)

commutative\_monoid\_is\_monoid.....proved - complete [shostak](0.32 s)  
 commutative\_monoid\_is\_commutative\_monad...proved - complete [shostak](0.32 s)

commutative\_submonoids.....proved - complete [shostak](0.31 s)  
**Theory** monoid totals: 24 formulas, 24 attempted, 24 succeeded (7.49 s)

Proof summary for **theory** monoid\_def

power\_TCC1.....proved - complete [shostak](0.27 s)  
 power\_TCC2.....proved - complete [shostak](0.25 s)  
 generated\_set\_lem.....proved - complete [shostak](0.25 s)  
 monoid?\_TCC1.....proved - complete [shostak](0.26 s)  
 commutative\_monoid?\_TCC1.....proved - complete [shostak](0.26 s)  
 finite\_commutative\_monoid?\_TCC1.....proved - complete [shostak](0.29 s)  
**Theory** monoid\_def totals: 6 formulas, 6 attempted, 6 succeeded (1.57 s)

Proof summary for **theory** monad

monad\_TCC1.....proved - complete [shostak](0.26 s)  
 one\_member.....proved - complete [shostak](0.28 s)  
 one\_in.....proved - complete [shostak](0.25 s)  
 left\_identity.....proved - complete [shostak](0.26 s)  
 right\_identity.....proved - complete [shostak](0.26 s)  
 unique\_left\_identity.....proved - complete [shostak](0.25 s)  
 unique\_right\_identity.....proved - complete [shostak](0.27 s)  
 one\_is\_monad.....proved - complete [shostak](0.31 s)  
 trivial\_monad\_TCC1.....proved - complete [shostak](0.32 s)  
 monad\_is\_groupoid.....proved - complete [shostak](0.28 s)  
 sing\_one\_finite\_monad.....proved - complete [shostak](0.30 s)  
 finite\_monad\_TCC1.....proved - complete [shostak](0.30 s)  
 commutative\_monad\_TCC1.....proved - complete [shostak](0.32 s)  
 finite\_commutative\_monad\_TCC1.....proved - complete [shostak](0.31 s)  
 order\_TCC1.....proved - complete [shostak](0.30 s)  
 order\_is\_1.....proved - complete [shostak](0.44 s)  
 finite\_monad\_is\_monad.....proved - complete [shostak](0.30 s)  
 commutative\_monad\_is\_monad.....proved - complete [shostak](0.30 s)  
 finite\_commutative\_monad\_is\_commutative\_monad...proved - complete [shostak]

(0.31 s)

finite\_commutative\_monad\_is\_finite\_monad...proved - complete [shostak](0.32 s)

s)

**Theory** monad totals: 20 formulas, 20 attempted, 20 succeeded (5.95 s)

Proof summary for **theory** monad\_def

monad?\_TCC1.....proved - complete [shostak](0.26 s)  
 monad?\_TCC2.....proved - complete [shostak](0.26 s)  
 commutative\_monad?\_TCC1.....proved - complete [shostak](0.26 s)  
 finite\_commutative\_monad?\_TCC1.....proved - complete [shostak](0.25 s)  
**Theory** monad\_def totals: 4 formulas, 4 attempted, 4 succeeded (1.03 s)

Proof summary for **theory** semigroup

fullset\_is\_semigroup\_TCC1.....proved - complete [shostak](0.26 s)  
 semigroup\_TCC1.....proved - complete [shostak](0.26 s)  
 semigroup\_TCC2.....proved - complete [shostak](0.25 s)  
 associative.....proved - complete [shostak](0.29 s)  
 semigroup\_is\_groupoid.....proved - complete [shostak](0.27 s)  
**Theory** semigroup totals: 5 formulas, 5 attempted, 5 succeeded (1.33 s)

Proof summary for **theory** semigroup\_def

semigroup?\_TCC1.....proved - complete [shostak](0.26 s)

finite\_commutative\_semigroup?\_TCC1....proved - complete [shostak](0.25 s)

**Theory** semigroup\_def totals: 2 formulas, 2 attempted, 2 succeeded (0.51 s)

Proof summary for **theory** groupoid

fullset\_is\_groupoid.....proved - complete [shostak](0.26 s)

groupoid\_TCC1.....proved - complete [shostak](0.25 s)

closed.....proved - complete [shostak](0.27 s)

star\_closed.....proved - complete [shostak](0.26 s)

**Theory** groupoid totals: 4 formulas, 4 attempted, 4 succeeded (1.04 s)

Proof summary for **theory** groupoid\_def

commutative\_groupoid?\_TCC1.....proved - complete [shostak](0.26 s)

finite\_commutative\_groupoid?\_TCC1....proved - complete [shostak](0.26 s)

**Theory** groupoid\_def totals: 2 formulas, 2 attempted, 2 succeeded (0.52 s)

Proof summary for **theory** operator\_defs\_more

**Theory** operator\_defs\_more totals: 0 formulas, 0 attempted, 0 succeeded (0.00 s)

Proof summary for **theory** top\_rings

**Theory** top\_rings totals: 0 formulas, 0 attempted, 0 succeeded (0.00 s)

Grand Totals: 1256 proofs, 1256 attempted, 1256 succeeded (766.70 s)