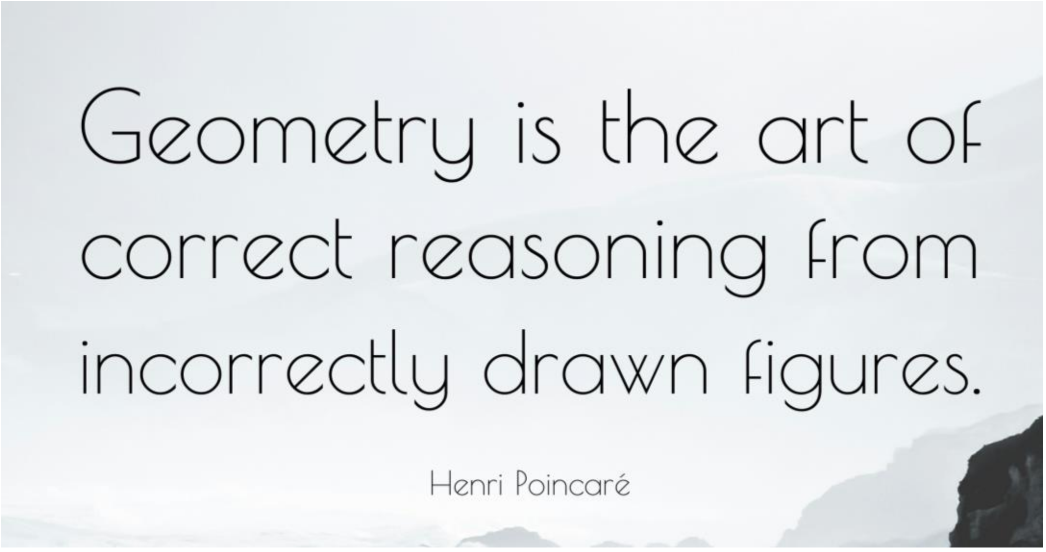
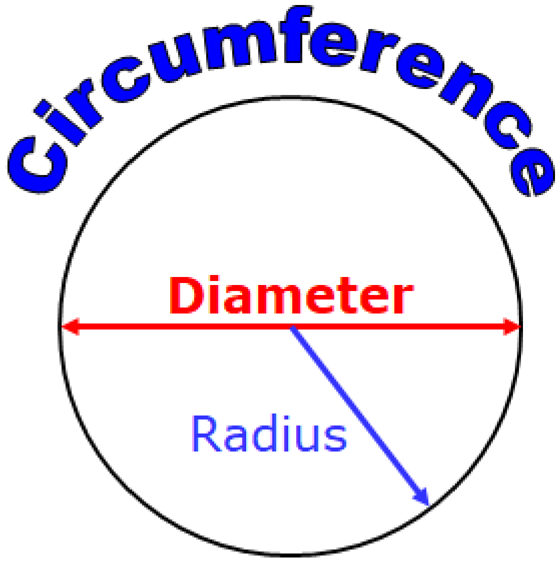

Geometry is the art of correct reasoning from incorrectly drawn figures.
Henri Poincaré

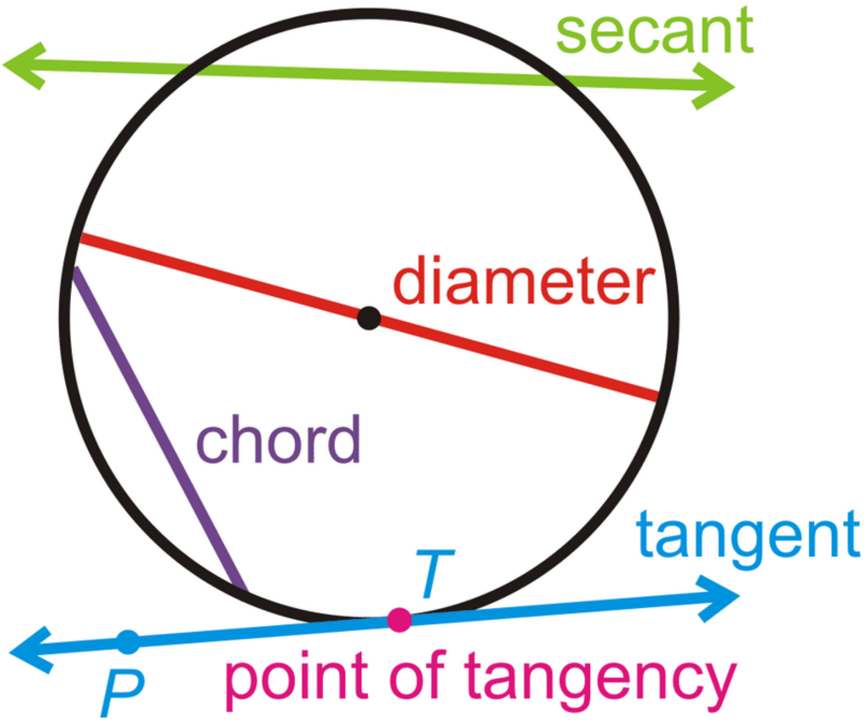
Circle



Circumference = $2\pi R$
Area = πR^2

A circle is set of all those points that are at a constant distance from a fixed point.
The fixed point is its **Centre** and fixed distance is its **radius.**

Fun: A circle is just a round straight line with a hole in the middle.



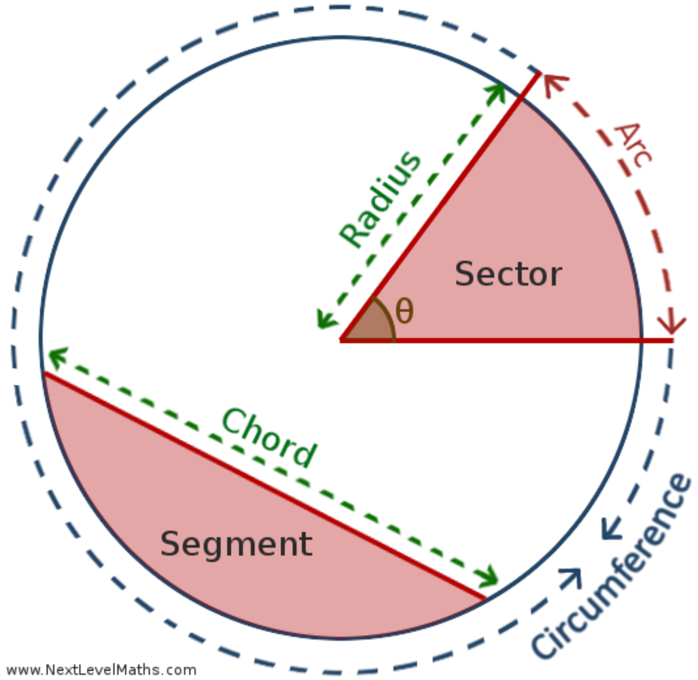
Chord: A chord is a line segment whose endpoints lie on the circle.

Diameter: The diameter is the chord passing through the centre of the circle.

Secant: It is a line that intersects the circle in two distinct points.

Tangent: It is a line in the plane of the circle, which has one and only one point common with the circle.

Radius is always perpendicular to the tangent.



Arc length = $\dfrac{\theta}{360} \times 2\pi r$

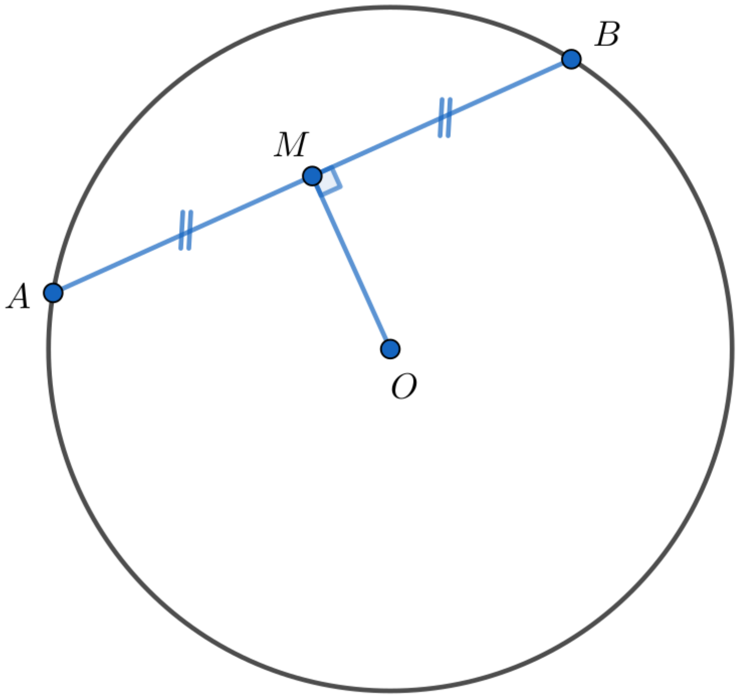
Area of sector = $\dfrac{\theta}{360} \times \pi r^2$

Perimeter of sector = 2r+ Arc length = $2r + \dfrac{\theta}{360} \times 2\pi r$

Area of Segment = Area of sector- Area of triangle

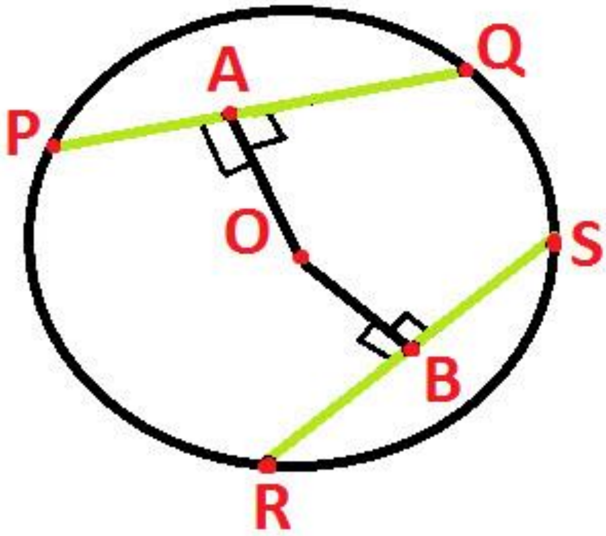
$$= \frac{\theta}{360} \times \pi r^2 - \frac{1}{2} r^2 \sin\theta$$

Where θ is angle at the center



OM ⊥ AB then AM= BM

Reverse: ⊥ bisector of a chord passes through the center of the circle.



Equal chords are equidistant from the center and reverse is also true.

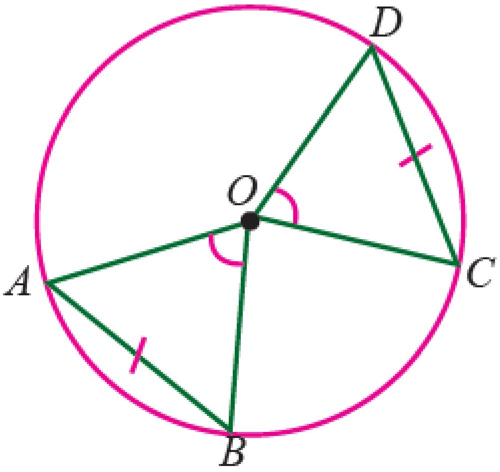
If OA = OB then PQ = RS

Or

If PQ = RS then OA = OB.

Chord closer to the center is longer.

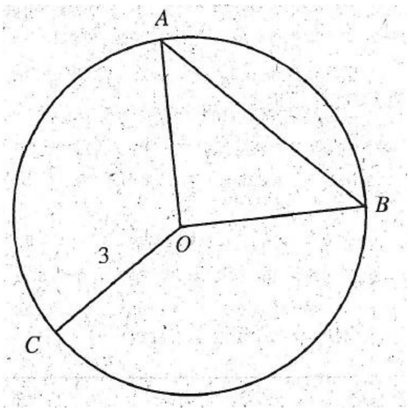
If OA> OB then RS > PQ and reverse is also true.

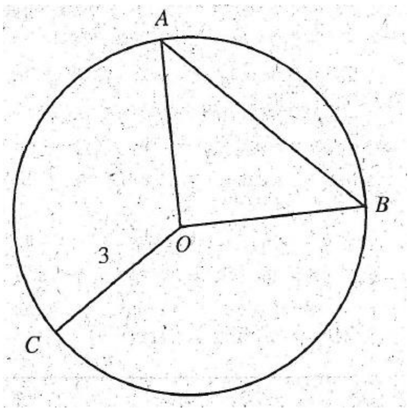


Equal equal arcs subtend equal angles at the centre and reverse is also true.

If ∠AOB = ∠COD then arc(AB) =arc (CD)

Problem: In the circle shown in the figure the length of the arc ACB is 3 times the length of the arc AB. What is the length of the line segment AB?





Arc length AB = $\frac{1}{4} \times 2\pi R$

Arc length ACB = $\frac{3}{4} \times 2\pi R$

3×Angle subtended by arc AB at center = Angle subtended by arc ACB at center

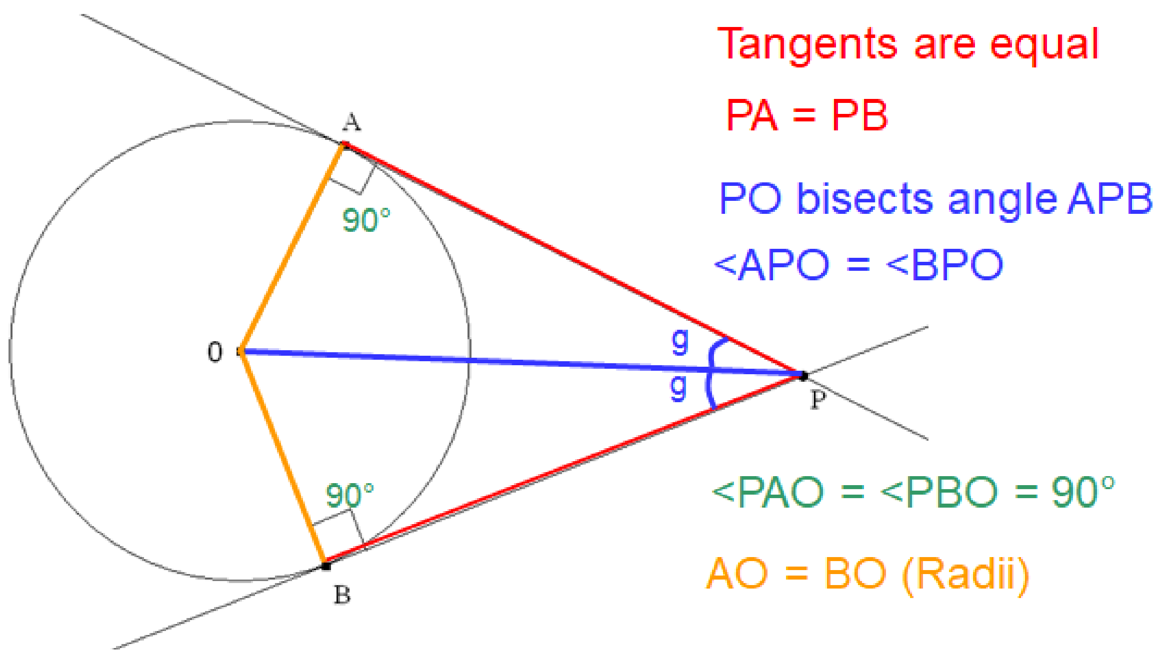
Therefore $\angle AOB = 90$

Triangle AOB is 45, 45, 90.

Sides are 3,3, $3\sqrt{2}$

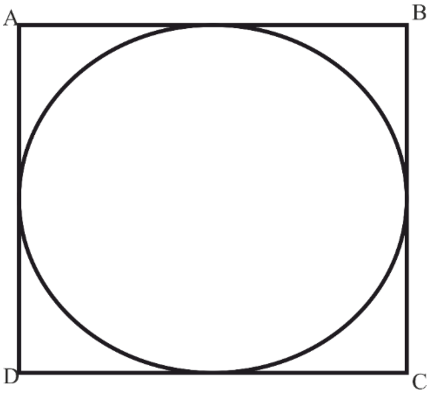
AB = $3\sqrt{2}$

Two tangents from a point outside circle

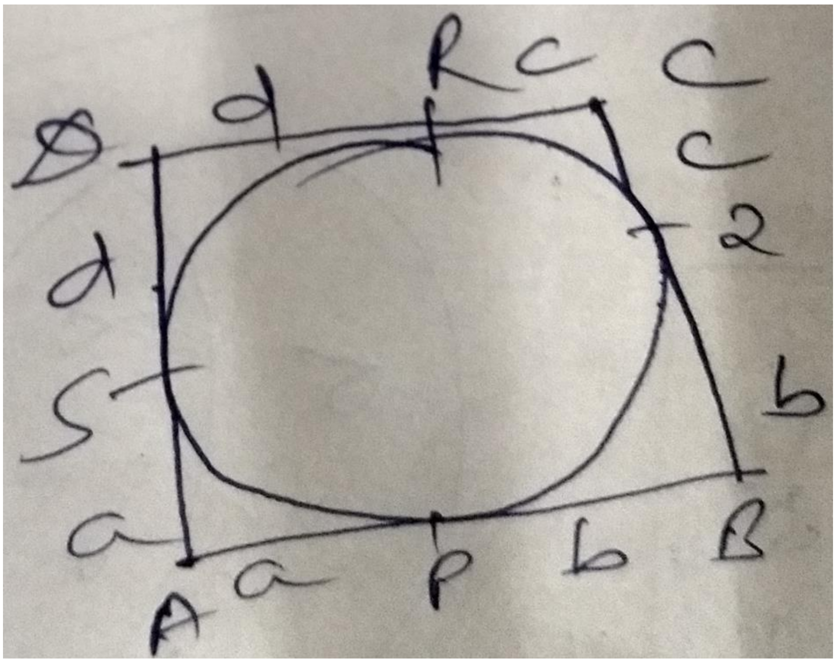


The two Triangles APO and BPO are Congruent

Problem: AB = 20cm, CD = 15cm, BC = 17cm then find AD, if a circle is inscribed in a Quadrilateral ABCD.



AB = 20cm, CD = 15cm, BC = 17cm then find AD, if a circle is inscribed in a Quadrilateral ABCD



AP & AS are 2 tangents from the same point and therefore

AP =AS

Similarly

BP = BQ, CQ = CR & RD = DS

AB+CD = a+b+c+d

BC+AD= a+b+c+d

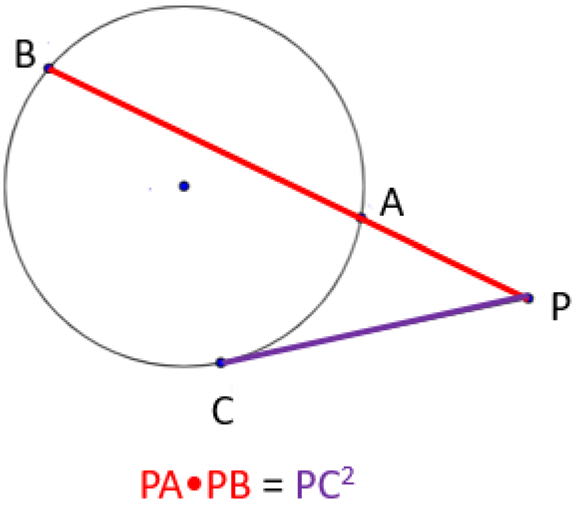
Sum of opposite sides is same.

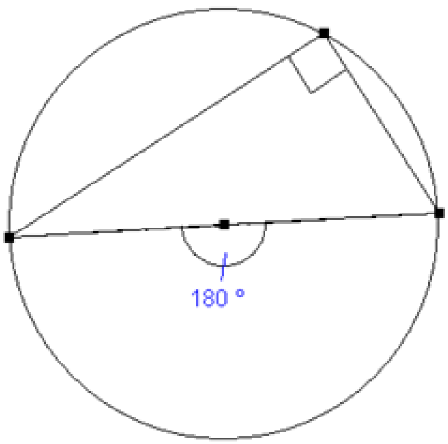
AB+CD = BC+AD

20+15 = 17+AD

⇨ AD = 18 cm

If PBA is a secant intersecting the circle at A and B, and PC is a tangent, then $PA \times PB = PC^2$.





When the angle stands on the diameter, what is the size of angle a?

The diameter is a straight line so the angle at the centre is 180°

Angle a = 90°

"The angle in a semi-circle is a Right Angle"

Problem: Find the maximum area of a quadrilateral which is inscribed in a circle of radius 8cm?

Find the maximum area of a quadrilateral which is inscribed in a circle of radius 8cm

Note: When a polygon is inscribed in a circle its area is maximum when it is a regular polygon.

Therefore, this quadrilateral should be Square.

Angle in semi-circle is 90°.

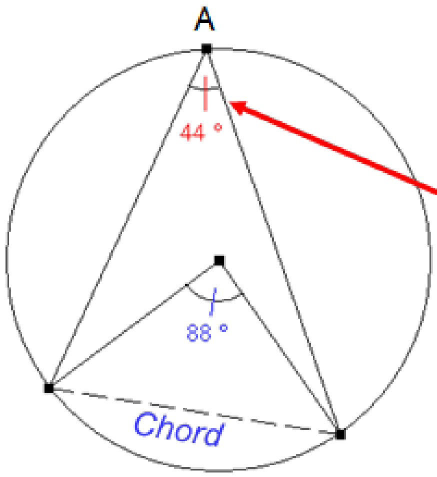
Diagonal of Square = Diameter of the circle

$\sqrt{2} \times a = 2R$

⇨ $a = \dfrac{16}{\sqrt{2}} = 8\sqrt{2}$

⇨ Area of Square = $\left(8\sqrt{2}\right)^2 = 128$

Angle at the centre



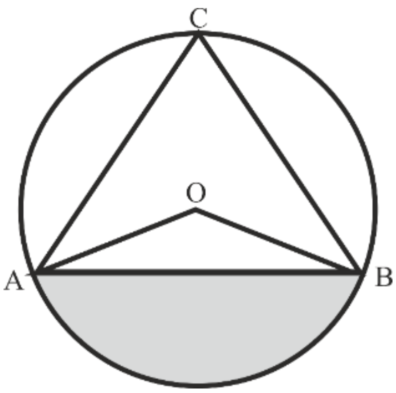
Consider the two angles which stand on this same chord

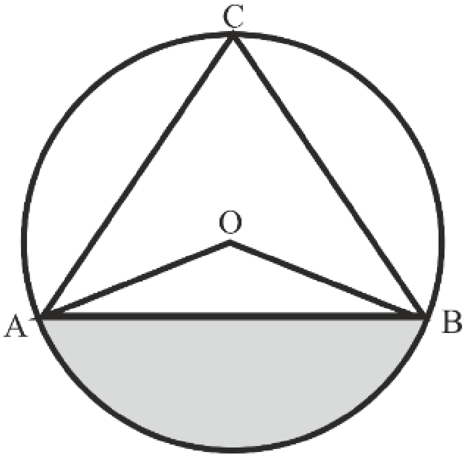
What do you notice about the angle at the circumference?

It is half the angle at the centre

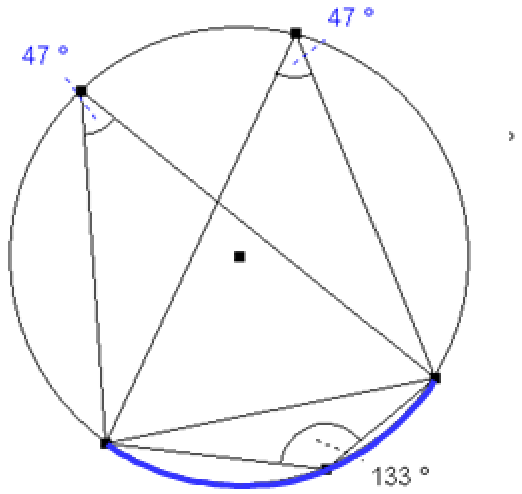
"If two angles stand on the same chord, then the angle at the centre is twice the angle at the circumference"

Problem: In ΔABC $\angle ACB = 15°$ then find the area of shaded region if radius of the circle is 42cm.



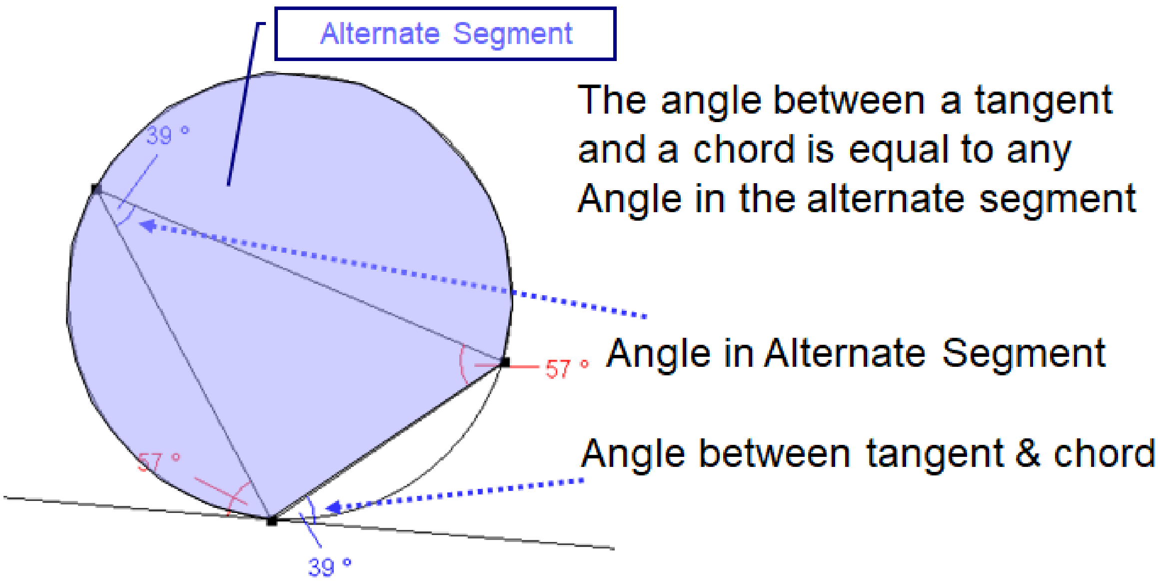


$$\text{Area of sector} = \frac{30}{360} \times \frac{22}{7} \times 42 \times 42 = 462$$



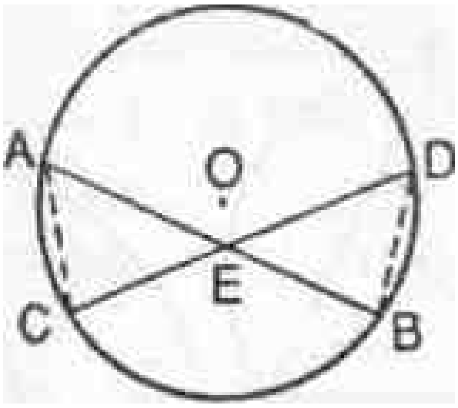
Angle made by same arc in the same segment are equal.

Alternate Segment Theorem



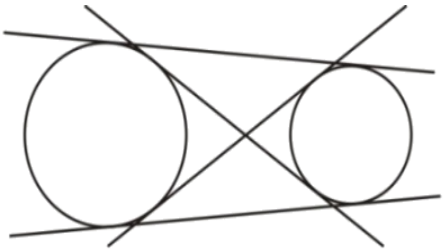
"The angle between a tangent and a chord is equal to any Angle in the alternate (opposite) segment"

If two chords AB and CD of a circle intersect at a point E, then in both the cases, AE x EB = DE x EC.

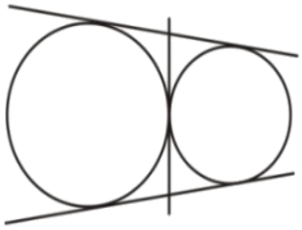


Common Tangents to Two Circles

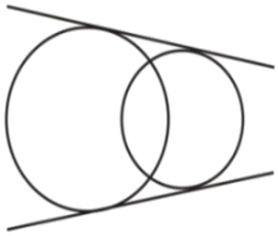
The number of common tangents to two circles could range from 4 to 0 depending on the relative placement of the circles.



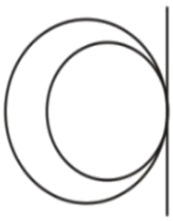
Case (i): Circles external to each other. 2 Direct and 2 Transverse Tangent



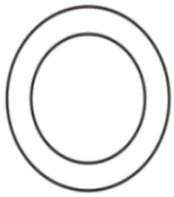
Case (ii): Circles touch externally. 2 Direct and 1 Transverse Tangent



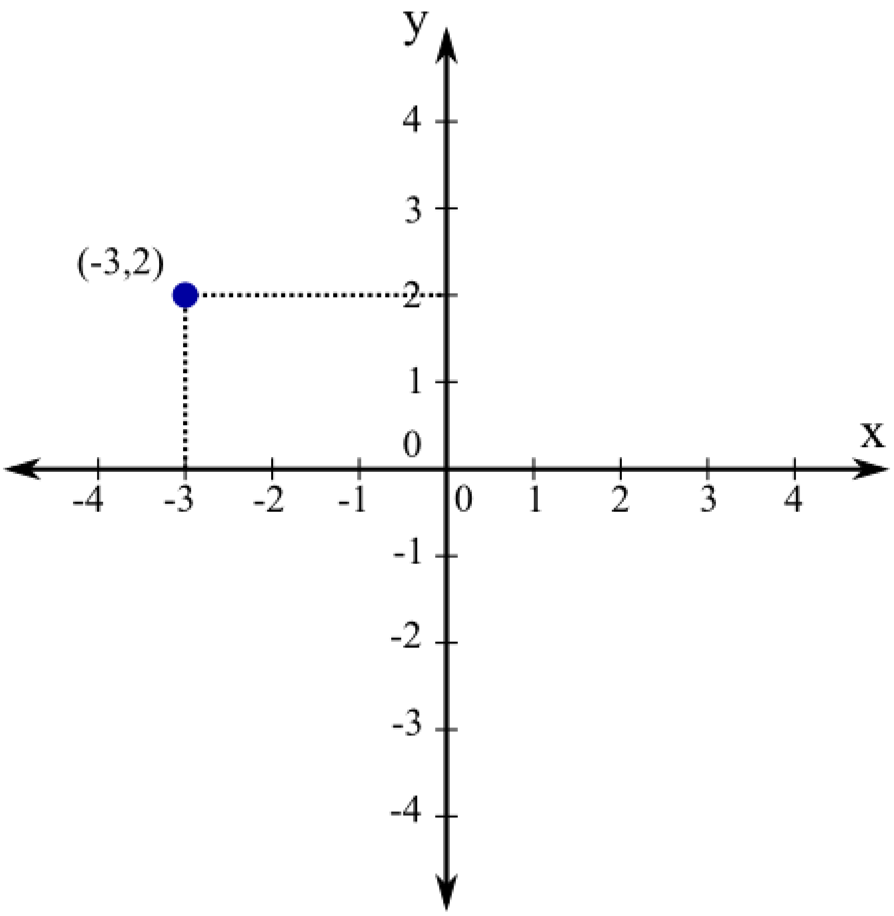
Case (iii): Circles intersect. 2 Direct Tangent



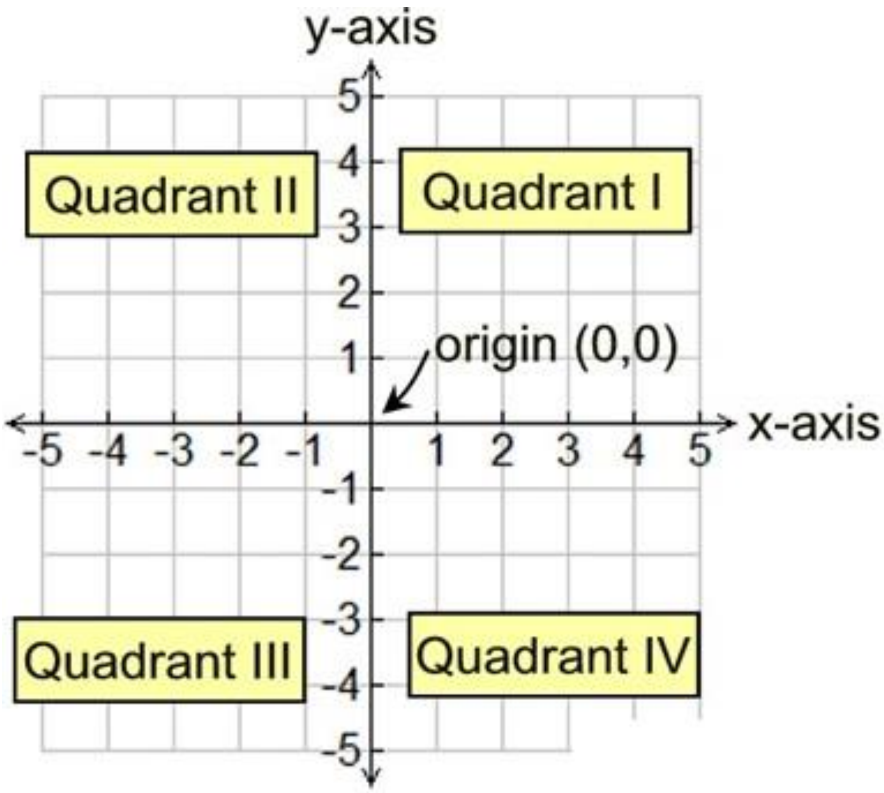
Case (iv):Circles touch internally. 1 Transverse Tangent



Case (v): Circle within other. No common tangnet



- Distance of a point from X axis is its y coordinate.
- Distance of a point from Y axis is its x coordinate.



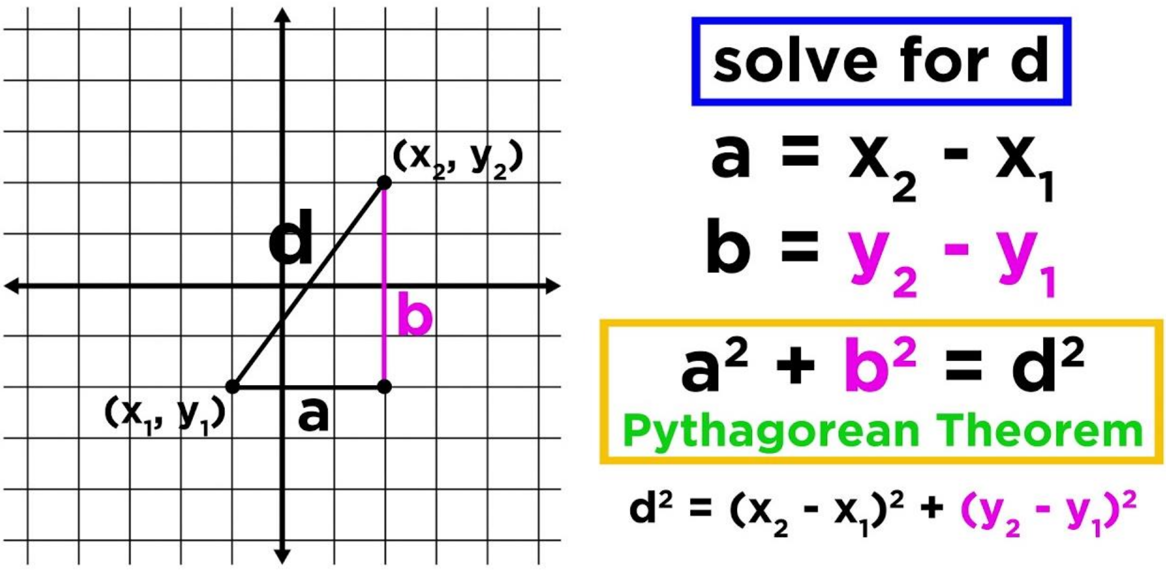
Problem: If point A(a,b) lies in the second quadrant then point B (a,-b) lies in which quadrant?

If point A(a,b) lies in the second quadrant then point B (a,-b) lies in which quadrant?

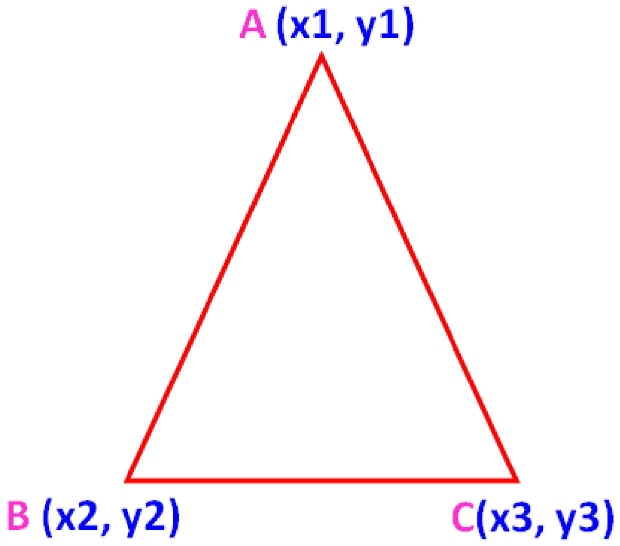
In second quadrant x is –ve i.e. a is –ve and y is +ve i.e. b is +ve.

Point B (a,-b), a is –ve and –b is –ve and therefore point B is in 3rd quadrant.

Distance between 2 points



$$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

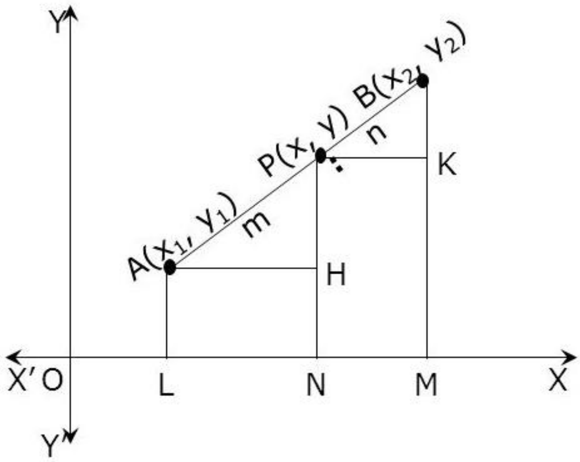


$$\boxed{\text{Area of } \Delta = \frac{1}{2}\left\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right\}}$$

Co-ordinates of Centroid G(x,y)

$$x = \frac{x_1 + x_2 + x_3}{3} \text{ and } y = \frac{y_1 + y_2 + y_3}{3}$$

Section Formula – Internal Division

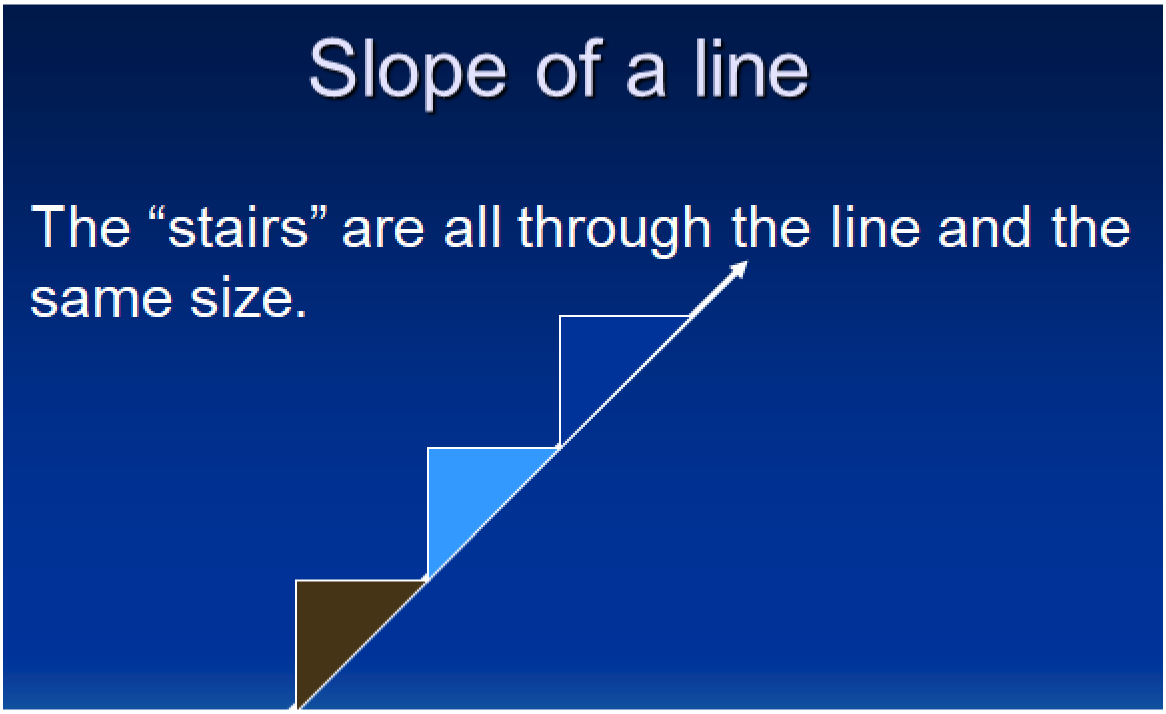


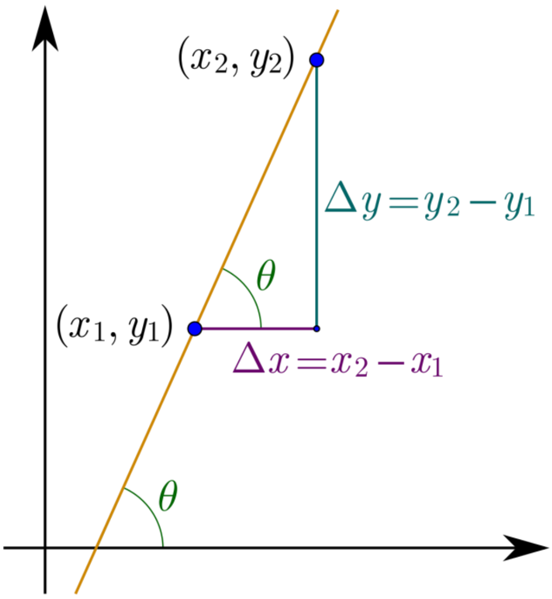
Clearly $\Delta AHP \sim \Delta PKB$

$$\therefore \frac{AP}{BP} = \frac{AH}{PK} = \frac{PH}{BK}$$

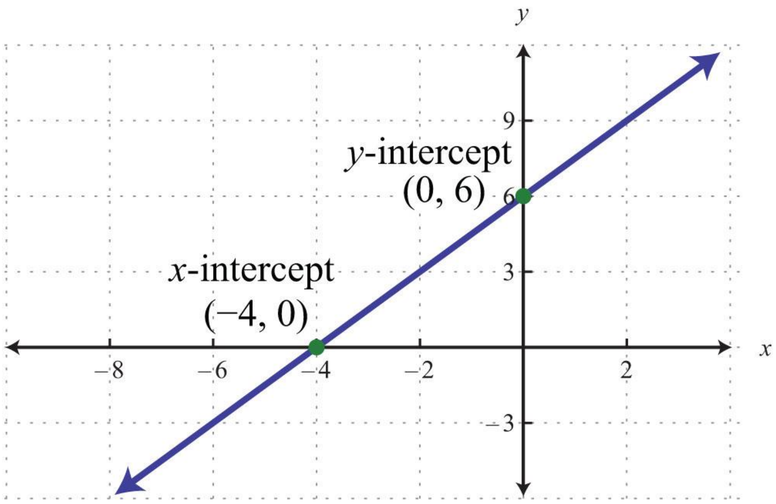
$$\therefore \frac{m}{n} = \frac{x - x_1}{x_2 - x} = \frac{y - y_1}{y_2 - y}$$

$$\therefore P \equiv \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n} \right)$$





$$\text{Slope (m)} = \tan\theta = \frac{\text{change in y}}{\text{change in x}} = \frac{y_2 - y_1}{x_2 - x_1}$$



Intercept

X –intercept: where y is 0

Y – intercept: where x is 0

$$\text{Slope(m)} = \frac{-\text{ y intercept}}{\text{xintercept}}$$

If equation of line is ax+by+c = 0 then slope of line = $\frac{-a}{b}$

Problem

Find the slope, x –intercept and y-intercept of a line x+3y = 63

Find the slope, x –intercept and y-intercept of a line x+3y = 63

For x-intercept put y =0

$x+3\times0= 63$

- ⇨ $x = 63$

For y – intercept put x = 0

$0+3y = 63$

- ⇨ $y = 21$

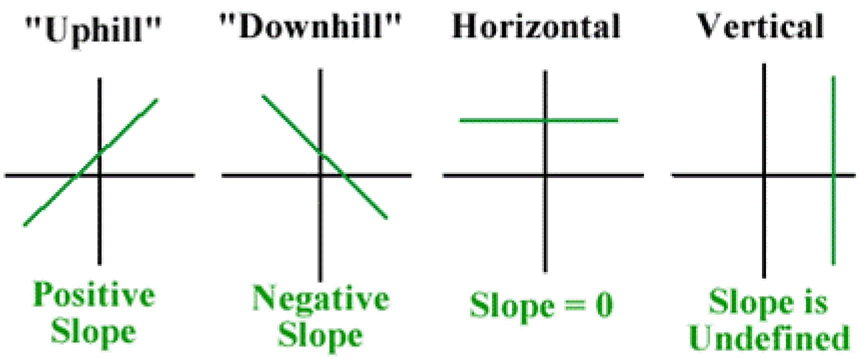
Slope = $\frac{-1}{3}$

If lines are || then their slopes are equal and reverse is also true.

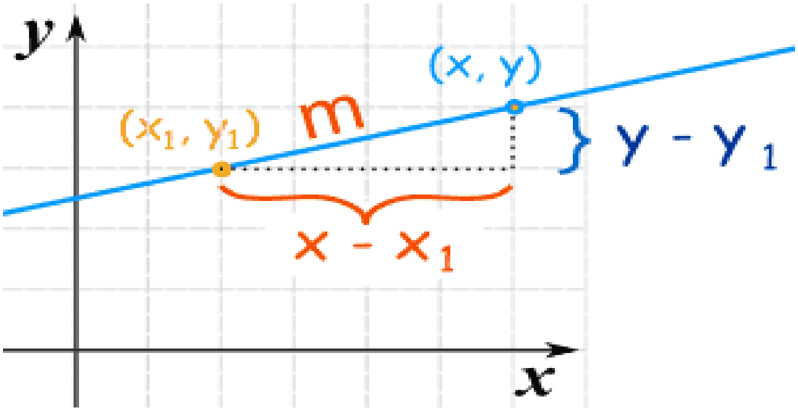
Example:
ax+by +c=0 & ax+by+d = 0

If 2 lines are ⊥ then product of their slopes is -1 i.e.

$m_1\times m_2 = -1$



Equation of line



$$\frac{y - y_1}{x - x_1} = m$$

$$\frac{y - y_1}{\boxed{x - x_1}} = m(x - x_1)$$

$$y - y_1 = m(x - x_1)$$

Problem:

Lines l_1 and l_2 are parallel and equation of l_1 is $2x+9y=30$, then find the equation of l_2 if it passes through the point $(2,7)$?

Lines l_1 and l_2 are parallel and equation of l_1 is 2x+9y=30, then find the equation of l_2 if it passes through the point (2,7)?

Slope of l_1 = Slope of l_2 = $\frac{-2}{9}$

Equation of line is $y-y_1=m(x-x_1)$

$y-7=\frac{-2}{9}(x-2)$

or

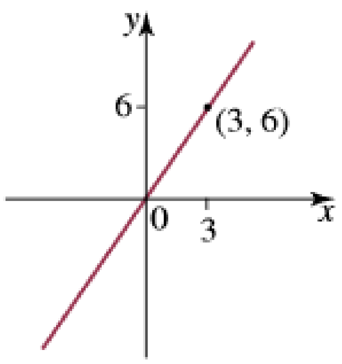
As we know only constant term is different in the equations of the || lines

Let equation of l_2 is 2x+9y=c

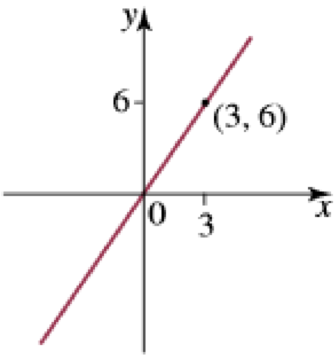
Since it is passing through the point (2,7) and therefore it has to saisfy the equation.

$2\times2+9\times7=c$

Problem: Find the equation of the given line?



Find the equation of the given line?



$$\frac{6-0}{3-0} = \frac{y-0}{x-0}$$

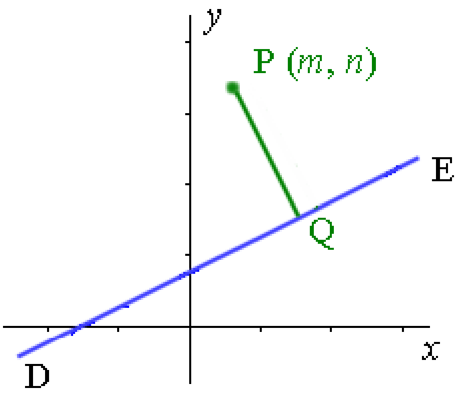
Perpendicular Distance between 2 || lines

Let equations of the lines are

$ax+by+c_1 = 0$ and $ax+by+c_2=0$

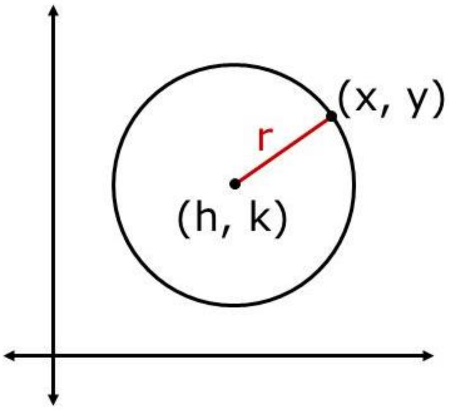
$$\text{Distance} = \frac{c_1 - c_2}{\sqrt{a^2+b^2}}$$

Perpendicular distance between a point and line



Length of perpendicular to line DE with equation ax + by + c=0 from the point P(m,n) is

$$\frac{am+bn+c}{\sqrt{a^2+b^2}}$$



Use the Distance Formula to write this.

$$r = \sqrt{(x-h)^2 + (y-k)^2}$$

General equation of circle is $x^2+y^2+2fx+2gy+c = 0$

Center is (-f,-g) and radius $=\sqrt{f^2+g^2-c}$

Problem:

Find the radius and center of the circle if equation of circle is

$x^2+y^2+8x-10y-23 = 0$

$x^2+y^2+8x-10y-23 = 0$

$2f = 8, \Rightarrow f = 4$

$2g = -10 \Rightarrow g = -5$

Center(-4,5)

Radius = $\sqrt{4^2+5^2+23} = 8$