Linear Kernel for Vertex Cover

Assignment Report

by

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Abstract

We study about a linear kernel for vertex cover. Our approach is based on the technique of linear programming. Here we obtain a $2k - kernel$ for the vertex cover problem. The current status of the problem as well as the applications are also included.

Contents

1 Introduction

In Parametrized Vertex Cover problem, given an undirected graph $G = (V, E)$ and a non negative integer k. The question is does there exists a subset of vertices $C \subseteq V$ with k or fewer vertices such that in E has atleast one endpoints in C? The output will be to return such a subset if it exists else return NO. Kernelization is a preprocessing technique in which inputs to the algorithm are replaced by a smaller input, known as kernel.

2 Problem Statement

To obtain a linear kernel for the parametrized vertex cover problem.

3 Technique for problem solving

Here we explore linear programming based kernels. It is well known that linear programming can be solved in polynomial time. Given a graph $G = (V, E)$ and a non-negative integer k, we make an integer linear programming ILP formulation for vertex cover. Suppose we have n variables, one variable x_v for each vertex $v \in V$ such that,

$$x_v = \begin{cases} 1, & \text{if } v \text{ is in vertex cover} \\ 0, & \text{otherwise} \end{cases}$$

The ILP formulation is as follows:

$$\begin{aligned} &Minimize \quad \textstyle\sum_{v\in V} x_v \geq 1 \\ &Subject\ \ to \quad x_u + x_v \geq 1\ \forall v \in V \\ &and\ x_v \in \{0,1\}\ \forall v \in V. \end{aligned}$$

Now we relax the ILP by replacing the constraint $x_v \in \{0, 1\}$ with constraint $0 \leq x_v \leq 1\ \forall v \in V$. The relaxed version $LPVC(G)\ isasfollows:$

$$\begin{aligned} &Minimize \quad \textstyle\sum_{v\in V} x_v \geq 1 \\ &Subject\ \ to \quad x_u + x_v \geq 1\ \forall v \in V \\ &and\ 0 \leq x_v \leq 1\ \forall v \in V. \end{aligned}$$

In this solution, we obtain vertices of G with some fractional values in $[0, 1]$. Let the definition of V_0, V_1, $V_{0.5}$ be as follows:

- V_0 is the set of vertices whose corresponding variables get factoral values smaller than 0.5.
- V_1 is the set of vertices whose corresponding variables get factoral values larger than 0.5.
- $V_{0.5}$ is the set of vertices whose corresponding variables get factoral values equal to 0.5.

Theorem 3.1. $Nemhauser\ trotter's\ theorem$: There is a minimum vertex cover VC of G such that $V_1 \leq VC \leq V_1 \cup V_{0.5}$

Proof. The proof is not covered in this document. □

4 Algorithm

To do

5 Analysis

We claim that using this technique, the vertex cover admits a kernel with atmost 2k vertices. Let graph $G = (V, E)$ and a non-negative integer k be the input to the algorithm. By solving the $LPVC(G)$ in polynomial time, we partition vertex set V into V_0, V_1, $V_{0.5}$, Let us define graph $G^{'}$=$G(V_{0.5})$, the graph induced by the vertex set $V_{0.5}$ on G. and $k^{'}$ = k - $|V_1|$.

We claim that (G,k), the input instance is a yes instance iff ($G^{'}$,$k^{'}$), the reduced instance is an yes instance. Let S be a vertex cover of G of size k. Then, by definiton $S^{'}$ = S $\cap V_{0.5}$ where $S^{'}$ is a vertex cover in $G^{'}$. By $Nemhauser\ trotter's\ theorem$, we can assume that $V_1 \leq VC \leq V_1 \cup V_{0.5}$. Using theorem we can say that size of $S^{'}$, $k^{'}$=k-V_1.

Now we need to show that if ($G^{'}$,$k^{'}$), the reduced instance is an yes instance then (G,k), the input instance is a yes instance. Let $S^{'}$ be a vertex cover in $G^{'}$. For every solution $LPVC(G)$, every edge with an endpoint from V_0 should have an endpoint in V_1. Hence, S=$S^{'}$ $\cup V_1$ is a vertex cover in G and the size of this vertex cover, k = $k^{'}$ + $|V_1|$.

Finally, we can say that

6 Current state of the Problem

The best kernel known is 2k-clogk found independently by x and y. Chetan et al showed that a kernelization algortithm that produces a kernel of size (2-e)k , then P=NP.

7 Applications of the problem

Applications include finding phlogenetic trees based on protein domain information. Significant improvement in running time was obtained after pre-processing with kernelization methods.

Another application arises in efficient dynamic deection of race conditions. In this case, each time global memory location is written into the information regarding the current thread and locks held by that thread are stored. Using the locked set based detection method, if any other thread need to write to that location, there will not be a race as it requires the lock to be set free. Thus the size of the hitting set, vertex cover for hypergraphs represents the minimum locked set size to be race free.

8 Conclusion

Vertex cover problem is studied. A $2k$-kernel is obtained for parametrized vertex cover problem using linear programming technique. Various applications for the problem is also identified.

References