Number Systems

Course Title: Computer Organization & Architecture

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Lecture Outline

1. Number System
2. Conversion Between Number Systems
3. Addition and Subtraction
4. Integer Representation in computer
5. Character encoding and usage by computer (ASCII Code)

Number System

Topic sub heading..

Decimal Numbers

Binary Number system

Hexadecimal Number system

Number System

Decimal Numbers

In a positional number system

- Each digit in the number is associated with a power of 10.
- i.e.3245 = 3 thousands, 2 hundreds, 4 tens and 5 ones.

OR, 3245 = 3X(10^3)+2X(10^2)+4X(10^1)+5X(10^0)

In a positional number system

- A number b is selected as the base
- Symbols are assigned to numbers between 0 and b-1
- In decimal system symbols are 0,1,2,3,4,5,6,7,8,9.
- The base ten is represented as 10

Number System

Binary Number System

- The base is two
- There are only two digits, 0 and 1.
- i.e. binary string 11010 represent the number
  1X(2^4)+1X(2^3)+0X(2^2)+1X(2^1)+0X(2^0) = 26
- Base two is represented in binary as 2

Number System

Why Hexadecimal?

We have decimal and binary. Why Hex?

- **Binary:**
  - Numbers written in binary tend to be long and difficult to express
  - 16 bits are needed to represent a word in 8086-based computer.
- **Decimal:**
  - Difficult to convert into binary.

*** Thus a third number became necessary

Number System

Hexadecimal Number System

- The base is sixteen
- There are total sixteen digits: 0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F
- i.e. hex digit 4A represent the number:
  - 4X(16^1)+ AX(16^0) = 74
  - Base sixteen is represented in hex as 16
- Conversion between binary and hex is easy
- Sixteen is = 2^4, So each hex digit corresponds to a unique **four-bit number**.

Number System

Decimal, Binary and Hex Numbers

DEC	HEX	BIN	DEC	HEX	BIN	DEC	HEX	BIN
0	00	00000000	43	2B	00101011	86	56	01010110
1	01	00000001	44	2C	00101100	87	57	01010111
2	02	00000010	45	2D	00101101	88	58	01011000
3	03	00000011	46	2E	00101110	89	59	01011001
4	04	00000100	47	2F	00101111	90	5A	01011010
5	05	00000101	48	30	00110000	91	5B	01011011
6	06	00000110	49	31	00110001	92	5C	01011100
7	07	00000111	50	32	00110010	93	5D	01011101
8	08	00001000	51	33	00110011	94	5E	01011110
9	09	00001001	52	34	00110100	95	5F	01011111
10	0A	00001010	53	35	00110101	96	60	01100000
11	0B	00001011	54	36	00110110	97	61	01100001
12	0C	00001100	55	37	00110111	98	62	01100010
13	0D	00001101	56	38	00111000	99	63	01100011
14	0E	00001110	57	39	00111001	100	64	01100100
15	0F	00001111	58	3A	00111010	101	65	01100101
16	10	00010000	59	3B	00111011	102	66	01100110
17	11	00010001	60	3C	00111100	103	67	01100111
18	12	00010010	61	3D	00111101	104	68	01101000
19	13	00010011	62	3E	00111110	105	69	01101001
20	14	00010100	63	3F	00111111	106	6A	01101010
21	15	00010101	64	40	01000000	107	6B	01101011
22	16	00010110	65	41	01000001	108	6C	01101100
23	17	00010111	66	42	01000010	109	6D	01101101
24	18	00011000	67	43	01000011	110	6E	01101110
25	19	00011001	68	44	01000100	111	6F	01101111
26	1A	00011010	69	45	01000101	112	70	01110000
27	1B	00011011	70	46	01000110	113	71	01110001
28	1C	00011100	71	47	01000111	114	72	01110010
29	1D	00011101	72	48	01001000	115	73	01110011
30	1E	00011110	73	49	01001001	116	74	01110100
31	1F	00011111	74	4A	01001010	117	75	01110101
32	20	00100000	75	4B	01001011	118	76	01110110
33	21	00100001	76	4C	01001100	119	77	01110111
34	22	00100010	77	4D	01001101	120	78	01111000
35	23	00100011	78	4E	01001110	121	79	01111001
36	24	00100100	79	4F	01001111	122	7A	01111010
37	25	00100101	80	50	01010000	123	7B	01111011
38	26	00100110	81	51	01010001	124	7C	01111100
39	27	00100111	82	52	01010010	125	7D	01111101
40	28	00101000	83	53	01010011	126	7E	01111110
41	29	00101001	84	54	01010100	127	7F	01111111
42	2A	00101010	85	55	01010101			

Conversion Between Number Systems

- Binary to decimal
- Decimal to Binary
- Hexadecimal to Decimal
- Decimal to Hex
- Hex to Binary
- Binary to Hex

Conversion Between Number Systems

Decimal to Binary

Convert the Decimal number 25 in Binary.

Given a Decimal Number : 25_{10}

Divide it with 2 until the quotient becomes zero

2	25		
2	12	1	(remainder of 25/2)
2	6	0	(remainder of 12/2)
2	3	0	(remainder of 6/2)
2	1	1	(remainder of 3/2)
2	0	1	(remainder of 1/2)

Place the remainders in the reverse order to reach the equivalent Binary number : 11001_2

Conversion Between Number Systems

Hexadecimal to Decimal

- **Convert hex number 589 into decimal number.**

=5*(16^2)+8*(16^1)+9*(16^0)

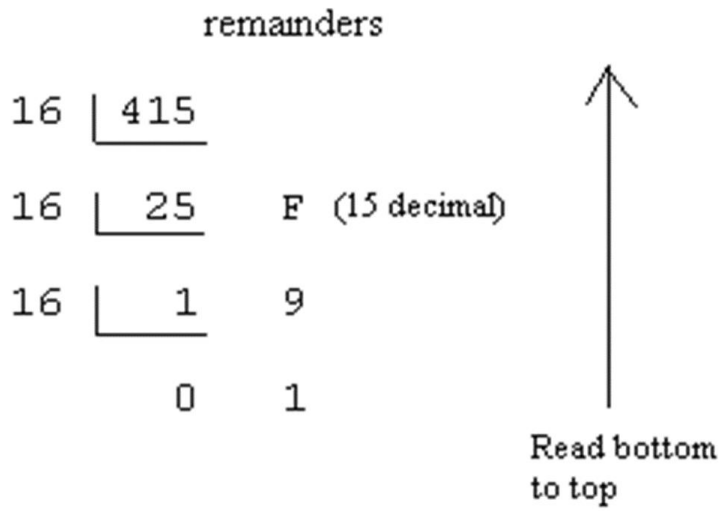
= 1280+128+9

= 1417

Conversion Between Number Systems

Decimal to Hex

➢ Convert 415 to Hex



Result: 415 in decimal is 19F in hexadecimal

Conversion Between Number Systems

Hex to Binary

- **Convert 39A2h to Binary**

Hexadecimal	3	9	A	2
Binary	0011	1001	1010	0010

Conversion Between Number Systems

Binary to Hex

- **Convert the Binary number 1000011001000011 into Hex**
- **1000 0110 0100 0011**
- **8 6 4 3**
- **Convert the Binary number 1001001001000100110101011011111 into Hex**

Conversion Between Number Systems

Conversion Task: Binary and Hex to Decimal

- **1110b**
- **100101011101b**
- **46Ah**
- **FAE2Ch**

Conversion Between Number Systems

Conversion Task: Decimal and Hex into Binary

- **97**
- **627**
- **A2Ch**
- **B34Dh**

Addition and Subtraction

- Hex Addition
- Binary Addition
- Hex Subtraction
- Binary Subtraction

Addition and Subtraction

Hex Multiplication Table

x	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F	10
1	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F	10
2	2	4	6	8	A	C	E	10	12	14	16	18	1A	1C	1E	20
3	3	6	9	C	F	12	15	18	1B	1E	21	24	27	2A	2D	30
4	4	8	C	10	14	18	1C	20	24	28	2C	30	34	38	3C	40
5	5	A	F	14	19	1E	23	28	2D	32	37	3C	41	46	4B	50
6	6	C	12	18	1E	24	2A	30	36	3C	42	48	4E	54	5A	60
7	7	E	15	1C	23	2A	31	38	3F	46	4D	54	5B	62	69	70
8	8	10	18	20	28	30	38	40	48	50	58	60	68	70	78	80
9	9	12	1B	24	2D	36	3F	48	51	5A	63	6C	75	7E	87	90
A	A	14	1E	28	32	3C	46	50	5A	64	6E	78	82	8C	96	A0
B	B	16	21	2C	37	42	4D	58	63	6E	79	84	8F	9A	A5	B0
C	C	18	24	30	3C	48	54	60	6C	78	84	90	9C	A8	B4	C0
D	D	1A	27	34	41	4E	5B	68	75	82	8F	9C	A9	B6	C3	D0
E	E	1C	2A	38	46	54	62	70	7E	8C	9A	A8	B6	C4	D2	E0
F	F	1E	2D	3C	4B	5A	69	78	87	96	A5	B4	C3	D2	E1	F0
10	10	20	30	40	50	60	70	80	90	A0	B0	C0	D0	E0	F0	100

Addition and Subtraction

Hex Addition

➢ Add hex number 5B39h with 7AF4h

- In Unit's column **9h+4h=13d=Dh**
- In next column, **3h+Fh=12h**, we **write 2 and carry 1**
- In next column, **Bh+Ah+ 1h** (from previous carry)= **16h** , we **write 6** and **carry 1**
- Finally, in last column, we compute, **5h+7h+1=Dh**

thus, if we combine it all,

5B39h
7AF4h
=====
D62Dh

Addition and Subtraction

Task: Hex Addition

Solve the following:

- **B23CDh + 17912h**
- **FEFFEh + FBCADh**

Addition and Subtraction

Binary Addition

- **Add binary number 100101111b with 110110b**
  - add some padding Zero with small number. So **110110b** becomes **000110110b**
  - In Unit's column **1b+0b=1b**
  - In next column, **1b+1b=10b,** we **write 0** and **carry 1**
  - In next column, **1b+1b+1b** (from previous carry)= **11b ,** we **write 1** and **carry 1**
  - In next column, **1b+0b+1b** (from previous carry)= **10b ,** we **write 0** and **carry 1**
  - In next column, **0b+1b+1b** (from previous carry)= **10b ,** we **write 0** and **carry 1**
  - In next column, **1b+1b+1b** (from previous carry)= **11b ,** we **write 1** and **carry 1**
  - In next column, **0b+0b+1b** (from previous carry)= **1b ,** we **write 1** and **No Carry**
  - In next column, **0b+0b+No carry = 0b**
  - In Last column, 1**b+0b = 1b**
  - **thus, if we combine it all,**

100101111b
000110110b
=========
101100101b

Addition and Subtraction

Task: Binary Addition

Solve the following:

- **100101b+10111b**
- **100111101b+10001111001b**

Addition and Subtraction

Subtraction: Hex

➢ Subtract hex number BA94h from D26Fh

- In Unit's column **Fh-4h=11d=Bh**
- In next column, **6<9** so, we must **borrow 1** from the third (right to left) column. Thus, **16h-9h=?**
    - **16h-9h = Dh,** (Looking at Hex addition table, D+9=16
    - **Or**, Hex numbers are A…F,10,11,12,13,14,15,**16h/23d - 9d/h = 14d/Dh**
- In next column, **12h-Ah-1**(previously lend one) **= 12h/18d-10d-1d/h = 7h**
- In next column, **Dh-Bh-1**(previously lend one) **= Dh/13d-11d-1d/h = 1h**

thus, if we combine it all,

D26Fh

- BA94h

=====

17DBh

Addition and Subtraction

Task: Hex Subtraction

Solve the following:

- **5FC12h-3ABD1h**
- **F001Ch-1FF3Fh**

Addition and Subtraction

Task: Binary Subtraction

Solve the following:

- **11011b-10110b**
- **10000101b-111011b**

Integer Representation in computer

- Hardware of a computer **restricts the size** of a number can be stored.
- **lsb** = least significant bit [**Right most bit or bit 0**]
- **msb** = most significant bit [**left most bit or bit 15**]

Integer Representation in computer

Unsigned integers

- **Unsigned integers:** Non-negative integer
- The largest unsigned integer of a byte= **11111111b** or **255d** or **FFh**
- The largest unsigned integer of a word= **1111111111111111b** or **65535d** or **FFFFh**
- *** if LSB of an integer is **1 then its ODD** . However, if it is **0 then Even**

Integer Representation in computer

Signed Integer

- **Signed Integer:** Can be **positive** or **negative** number
- The **MSB** is reserved for the **sign**
- **MSB =1 [Negative]**
- **MSB =0 [Positive]**
- *** Negative integers are stored in a computer using **TWO's complement**

Integer Representation in computer

One's Complement

- Ones complement is obtained by **complementing each bit.**
- **i.e. replacing 0 by a 1** and **1 by a 0.**
- **To find the one's complement of 5 or 0000000000000101** [16bit representation of 101]
- 1111111111111010 [complementing each bit]

Integer Representation in computer

Two's Complement

To get Two's Complement,

- Add 1 to the one's complement of a number
- Thus, Two's complement of 5 is

1111111111111010 [one's complement of 5]

+1

————————————

1111111111111011 [Two's complement of 5]

Integer Representation in computer

Two's Complement- Observation

- If we add two's complement of 5 with it, we get

1111111111111011

0000000000000101

==========

10000000000000000

- We get total 17bits here, however, word bits can hold only 16bits.
- Thus the carried out **MSB 1** is lost and final result is 0
- 5 +(-5) also results zero.
- Adding a number N and ones complement gives 16 one's
- Adding a number N and Two's complement gives 16 zero's

Integer Representation in computer

Two's Complement- Task: 8/16bit Representation

Find the two's complement of the two's complement of 5.

Show how the following decimal integer would be represented in

- **a) 8 bits b)16bits. [express the result in hex]**
- **-97**
- **-120**
- **-40000**
- **-128**
- **65536**

Integer Representation in computer

Two's Complement- Task

Give 16-bit representation of each of the following integers and express the result in hex:

- **234**
- **-16**
- **31634**
- **-32216**

Integer Representation in computer

Subtraction as Two's Complement Addition

- **If AX contains 21FCh and BX contains 5ABCh, find AX-BX**

AX=21FCh = 0010000111111100

BX= 5ABCh = 0101101010111100

O'C of 5ABCh = 1010010101000011 [one's complement]

+1

==========

1111111001111000 = A544h [1 is carried and lost]

Now, AX+BX=21FCh+A544h = C740h

ASCII Code

ASCII Table

Decimal	Hex	Char	Decimal	Hex	Char	Decimal	Hex	Char	Decimal	Hex	Char
0	0	[NULL]	32	20	[SPACE]	64	40	@	96	60	`
1	1	[START OF HEADING]	33	21	!	65	41	A	97	61	a
2	2	[START OF TEXT]	34	22	"	66	42	B	98	62	b
3	3	[END OF TEXT]	35	23	#	67	43	C	99	63	c
4	4	[END OF TRANSMISSION]	36	24	$	68	44	D	100	64	d
5	5	[ENQUIRY]	37	25	%	69	45	E	101	65	e
6	6	[ACKNOWLEDGE]	38	26	&	70	46	F	102	66	f
7	7	[BELL]	39	27	'	71	47	G	103	67	g
8	8	[BACKSPACE]	40	28	(	72	48	H	104	68	h
9	9	[HORIZONTAL TAB]	41	29	)	73	49	I	105	69	i
10	A	[LINE FEED]	42	2A	*	74	4A	J	106	6A	j
11	B	[VERTICAL TAB]	43	2B	+	75	4B	K	107	6B	k
12	C	[FORM FEED]	44	2C	,	76	4C	L	108	6C	l
13	D	[CARRIAGE RETURN]	45	2D	-	77	4D	M	109	6D	m
14	E	[SHIFT OUT]	46	2E	.	78	4E	N	110	6E	n
15	F	[SHIFT IN]	47	2F	/	79	4F	O	111	6F	o
16	10	[DATA LINK ESCAPE]	48	30	0	80	50	P	112	70	p
17	11	[DEVICE CONTROL 1]	49	31	1	81	51	Q	113	71	q
18	12	[DEVICE CONTROL 2]	50	32	2	82	52	R	114	72	r
19	13	[DEVICE CONTROL 3]	51	33	3	83	53	S	115	73	s
20	14	[DEVICE CONTROL 4]	52	34	4	84	54	T	116	74	t
21	15	[NEGATIVE ACKNOWLEDGE]	53	35	5	85	55	U	117	75	u
22	16	[SYNCHRONOUS IDLE]	54	36	6	86	56	V	118	76	v
23	17	[ENG OF TRANS. BLOCK]	55	37	7	87	57	W	119	77	w
24	18	[CANCEL]	56	38	8	88	58	X	120	78	x
25	19	[END OF MEDIUM]	57	39	9	89	59	Y	121	79	y
26	1A	[SUBSTITUTE]	58	3A	:	90	5A	Z	122	7A	z
27	1B	[ESCAPE]	59	3B	;	91	5B	[	123	7B	{
28	1C	[FILE SEPARATOR]	60	3C	<	92	5C	\	124	7C	|
29	1D	[GROUP SEPARATOR]	61	3D	=	93	5D	]	125	7D	}
30	1E	[RECORD SEPARATOR]	62	3E	>	94	5E	^	126	7E	~
31	1F	[UNIT SEPARATOR]	63	3F	?	95	5F	_	127	7F	[DEL]

ASCII Code

ASCII in detail

- ASCII code system Uses 7bits = 2^7 =128 to code each character.
- Codes from 32 to 126 = 95 characters are only printable.
- ** to display the character **A** on the screen, a program sends the ASCII code **41h** to the screen.
- If we type **“RG 2z”** in computer, it will process like following:

Address	Character	ASCII Code	Content
0	R	52	0101 0010
1	G	47	0100 0111
2	Space	20	0010 0000
3	2	32	0011 0010
4	z	7A	0111 1010

String Processing in a Computer

- If we want to print **“Hello World”** on the screen, How the memory will look like ?
- [**Hint**: Find ASCII code, content of string from the ASCII table and order them in a address sequence].

Books

- Assembly Language Programing and Organization of the IBM PC

Ytha Yu
Charles Marut

References

- Number Systems Introduction - Decimal, Binary, Octal, Hexadecimal & Conversions
    - https://www.youtube.com/watch?v=L2zsmYaI5ww&t=1435s
- Hex Addition, Subtraction
    - https://www.youtube.com/watch?v=y0B6tuC6niE
    - https://www.youtube.com/watch?v=3mV7q3_OAGM
- Binary Addition , Subtraction
    - https://www.youtube.com/watch?v=TaQS3qSDXV0
- Two's Complement
    - https://www.youtube.com/watch?v=_7RK3rsfQ8w
- ASCII Code
    - https://www.youtube.com/watch?v=KYEDJChGxh8
    - youtube.com/watch?v=5aJKKgSEUnY