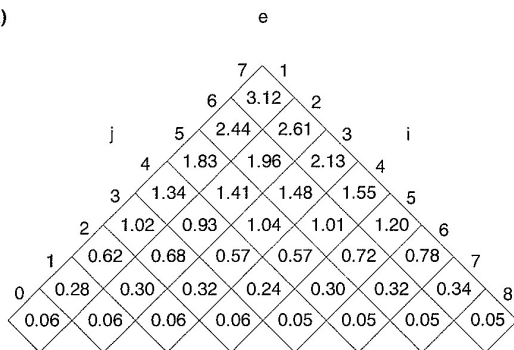# Quiz-6(A&B): Design and Analysis of Algorithms (Spring-2024)
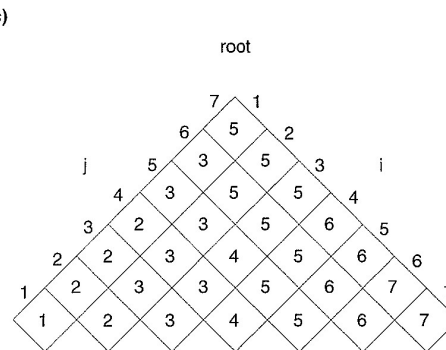
SOLUTION

Determine the cost and structure of an optimal binary search tree for a set of n = 7 keys with the following probabilities. Show all steps and tables.

i	0	1	2	3	4	5	6	7
p_i		0.04	0.06	0.08	0.02	0.10	0.12	0.14
q_i	0.06	0.06	0.06	0.06	0.05	0.05	0.05	0.05

(a) e

```
            7   1
          6  3.12  2
       j  5  2.44  2.61  3   i
        4  1.83  1.96  2.13  4
      3  1.34  1.41  1.48  1.55  5
    2  1.02  0.93  1.04  1.01  1.20  6
  1  0.62  0.68  0.57  0.57  0.72  0.78  7
0  0.28  0.30  0.32  0.24  0.30  0.32  0.34  8
 0.06  0.06  0.06  0.06  0.05  0.05  0.05  0.05
```

(c) root

```
            7   1
          6  5   2
       j  5  3   5   3   i
        4  3   5   5   4
      3  2   3   5   6   5
    2  2   3   4   5   6   6
  1  2   3   3   5   6   7   7
   1   2   3   4   5   6   7
```

(b) w

```
            7   1
          6  1.00  2
       j  5  0.81  0.90  3   i
        4  0.64  0.71  0.78  4
      3  0.49  0.54  0.59  0.64  5
    2  0.42  0.39  0.42  0.45  0.56  6
  1  0.28  0.32  0.27  0.28  0.37  0.41  7
0  0.16  0.18  0.20  0.13  0.20  0.22  0.24  8
 0.06  0.06  0.06  0.06  0.05  0.05  0.05  0.05
```

(d)

```
                k5
          k2          k7
       k1    k3     k6    d7
     d0 d1 d2 k4  d5 d6
              d3 d4
```
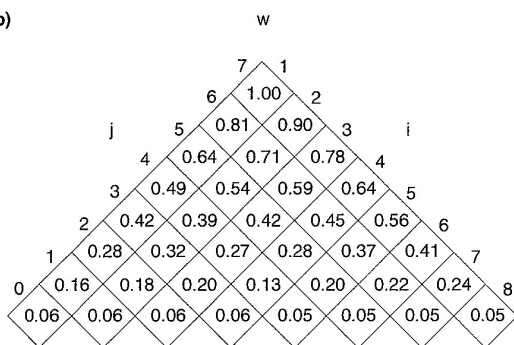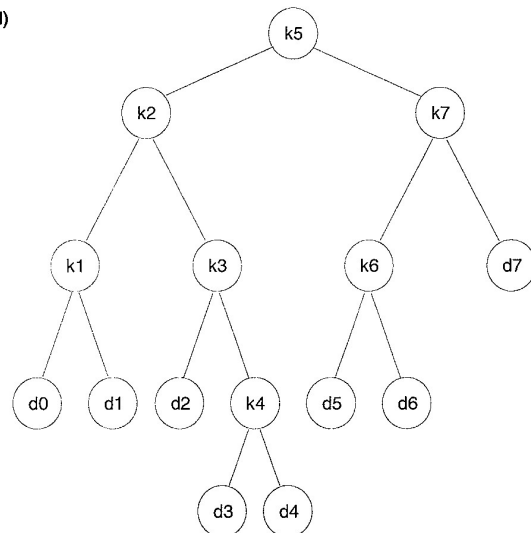
(a) $e[i,j]$ table, (b) $w[i,j]$ table, (c) $root[i,j]$ table, and (d) the optimal binary search tree