

Design & Analysis of Algorithms I

Mid 1, Spring 2014

Date: 27th Feb. 2014

Time: 90 mins.

Q1. (5+5)

Below are the pseudo codes for insertion sort and bubble sort. It is assumed that data is stored in an array $A[1 \dots n]$. Determine the loop invariant for the inner loops of both the sorts and prove their correctness.

Insertion Sort	Bubble Sort
<pre>for j = 2 to A.length key = A[j] i = j - 1 while i > 0 and A[i] > key A[i+1] = A[i] i = i - 1 A[i+1] = key</pre>	<pre>for i = 1 to A.length-1 for j = 1 to A.length - i if (A[j] > A[j + 1]) temp = A[j] A[j]=A[j + 1] A[j + 1] = temp</pre>

Remarks: Looks good. No changes from me.

Modified Q2 (10)

Below is the pseudo code of count Sort. The indexes are 0-based for the C array, but 1-based for the arrays A and B. The below algorithm is stable.

If we however change the last for loop to go from 1 up to A.length, instead of A.length down to 1, it does not remain stable.

Your task is to change the Count sort code, so that with the new code, if we go from 1 to A.length in the last for loop, it still remains stable.

The modified algorithm must still be stable, and must still run in $O(n+k)$ time.

```
COUNT-SORT (A, B , k)
//Let C[0..k] be a new array
for i = 0 to k
    C[i] = 0
for j = 1 to A.length
    C[A[j]] = C[A[j]]+1
for i = 1 to k
    C[i] = C[i] + C[i-1]
for j = A.length down to 1
    B[C[A[j]]] = A[j]
    C[A[j]] = C[A[j]] - 1
```

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Modified Q3 (10)

Let array A be an array consisting of only zeros and ones. (0's and 1's). Suggest an algorithm to sort the records in $O(n)$ time and $O(1)$ additional space.