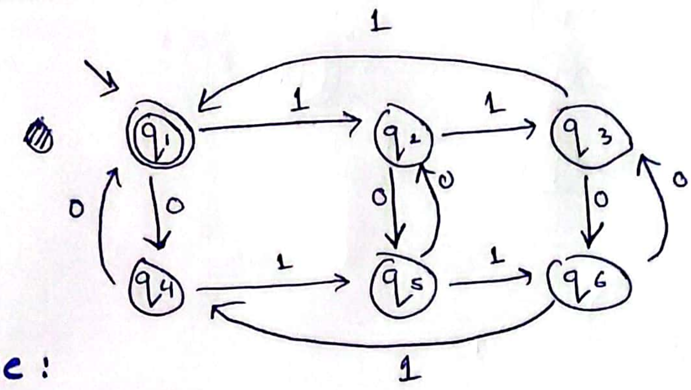
Question 1

a).



State table :

states	0	1
q_1	q_4	q_2
q_2	q_5	q_3
q_3	q_6	q_1
q_4	q_1	q_5
q_5	q_2	q_6
q_6	q_3	q_4

$\pi_0 = \{\overset{AA}{q_2}\ \overset{AB}{q_3}\ \overset{BA}{q_4}\ \overset{AA}{q_5}\ \overset{AA}{q_6}\}\ \{\overset{AA}{q_1}\}$ {seperating final and non final states}

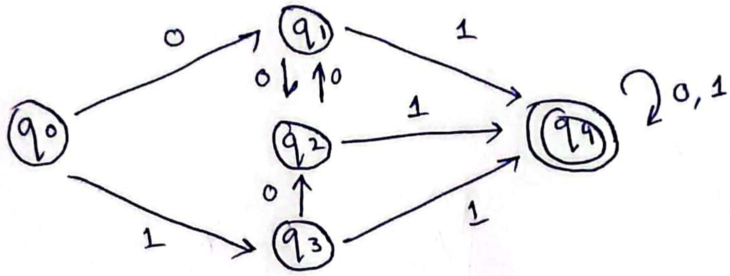
$\pi_1 = \overset{A}{\{\overset{CD}{q_2}\ \overset{CC}{q_5}\ \overset{DE}{q_6}\}}\ \{\overset{EF}{q_3}\}\ \{\overset{FC}{q_4}\}\ \overset{B}{\{q_1\}} \rightarrow EC$

C D E F

$\pi_2 = \{q_2\}\ \{q_5\}\ \{q_6\}\ \{q_3\}\ \{q_4\}\ \{q_1\}$

The DFA is already Minimised.

b) Ans.



State table:

states	0	1
q_0 -	q_1	q_3
q_1	q_2	q_4
q_2	q_1	q_4
q_3	q_2	q_4
q_4 +	q_4	q_4

→ Seperating final and non final states.

$$\pi_0 = \{\overset{AA}{q_0}\ \overset{AB}{q_1}\ \overset{AB}{q_2}\ \overset{AB}{q_3}\}\ \ \{\overset{BB}{q_4}\}$$

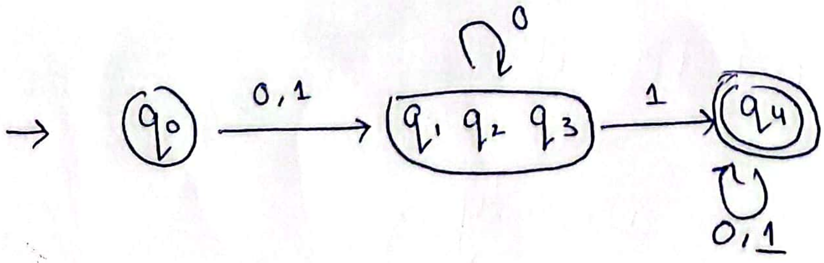
$\{q_0\ q_1\ q_2\ q_3\} \downarrow A$ $\quad \{q_4\} \downarrow B$

$$\pi_1 \quad \{\overset{DD}{q_0}\}\ \ \{\overset{DE}{q_1}\ \overset{DE}{q_2}\ \overset{DE}{q_3}\}\ \ \{\overset{EE}{q_4}\}$$

$\{q_0\} \downarrow C$ $\quad \{q_1\ q_2\ q_3\} \downarrow D$ $\quad \{q_4\} \downarrow E$

$$\pi_2 = \{q_0\}\ \ \{q_1\ q_2\ q_3\}\ \ \{q_4\}.$$

Minimized DFA:



c) Ans.

State table :

States	a	b
- A	B	F
B	A	F
C	G	A
D	H	B
+ E	A	G
+ F	H	C
+ G	A	D
H	A	C

→ Seperating final and non final states

$\pi_0 = \{\overset{PQ}{A}\ \overset{PQ}{B}\ \overset{QP}{C}\ \overset{PP}{D}\ \overset{PP}{H}\} \qquad \{\overset{PQ}{E}\ \overset{PP}{F}\ \overset{PP}{G}\}$

$\qquad\qquad \downarrow P \qquad\qquad\qquad\qquad \downarrow Q$

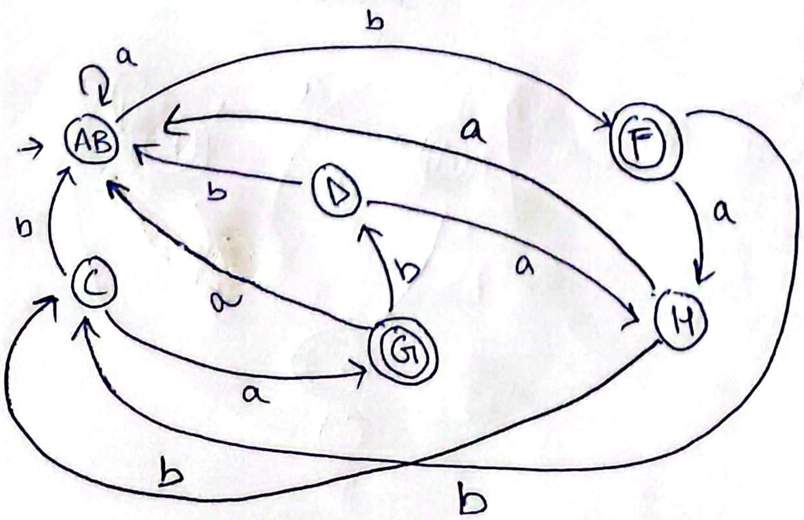
$\pi_1 = \{\overset{RV}{A}\ \overset{RV}{B}\} \quad \{\overset{VR}{C}\} \quad \{\overset{TR}{D}\ \overset{RS}{H}\} \quad \{E\} \quad \{\overset{TS}{F}\ \overset{RT}{G}\}$

$\qquad \downarrow R \qquad \downarrow S \qquad \downarrow T \qquad \downarrow U \qquad \downarrow V$

$\pi_2 = \{A\ B\} \quad \{C\} \quad \{D\} \quad \{H\} \quad \{E\} \quad \{F\} \quad \{G\}$

E has no incoming edge so remove it.

Minimised DFA :



Question 3

a) Ans·

Start state : A

→ λ closure of A : $\{A, B, D\} \rightarrow \boxed{A}$

$goto_a A = \{D\} \rightarrow (B)$

$goto_b A = \{C\} \rightarrow (C)$

$goto_c A = \{C\} \rightarrow (C)$

→ λ closure of $\{D\} = \{D\} \rightarrow \boxed{B}$

$goto_a B = \{D\} \rightarrow (B)$

$goto_b B = \{\ \}$

$goto_c B = \{\ \}$

→ λ closure of $\{C\} = \{C, B, D\} \rightarrow \boxed{C}$

$goto_a C = \{D\} \rightarrow (B)$

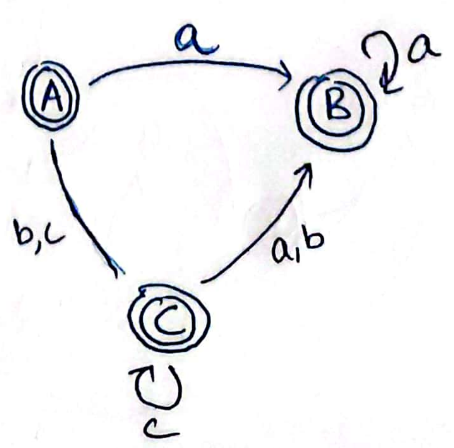
$goto_b C = \{D\} \rightarrow (B)$

$goto_c C = \{C\} \rightarrow (C)$

transition table :

states	a	b	c
A +	B	C	C
B +	B	–	–
C +	B	B	C

DFA :



2. b) Ans.

Start state = q_0

→ λ closure of q_0 = $\{q_0, q_1, q_3\}$ → $\boxed{A}$

goto a A = $\{q_1, q_2\}$ → (b)

goto b A = $\{q_3\}$ → (c)

→ λ closure of $\{q_1, q_2\}$ ⇒ $\{q_1, q_2, q_3, q_5\}$ → $\boxed{B}$

goto a B = $\{q_2\}$ → (d)

goto b B = $\{q_3, q_5\}$ → (e)

→ λ closure of $\{q_3\}$ = $\{q_3, q_1\}$ → $\boxed{C}$

goto a C = $\{q_2\}$ → (d)

goto b C = $\{q_3\}$ → (c)

→ λ closure of $\{q_2\}$ = $\{q_1, q_2, q_3, q_5\}$ → $\boxed{B}$

→ λ closure of $\{q_3, q_5\}$ = $\{q_3, q_1, q_5\}$ → $\boxed{D}$

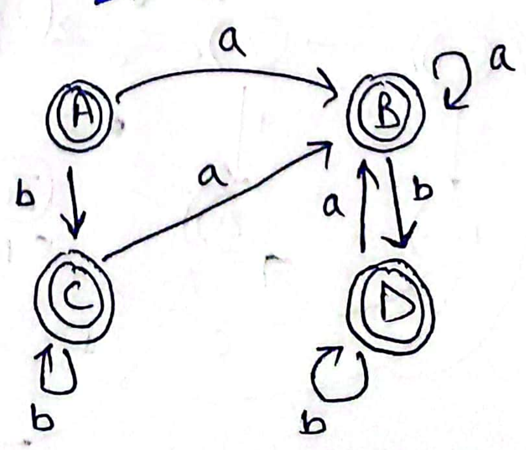
goto a D = $\{q_2\}$ → (d)

goto b D = $\{q_3, q_5\}$ → (e)

Transition table

states	a	b
A +	B	C
B +	B	D
C +	B	C
D +	B	D

DFA



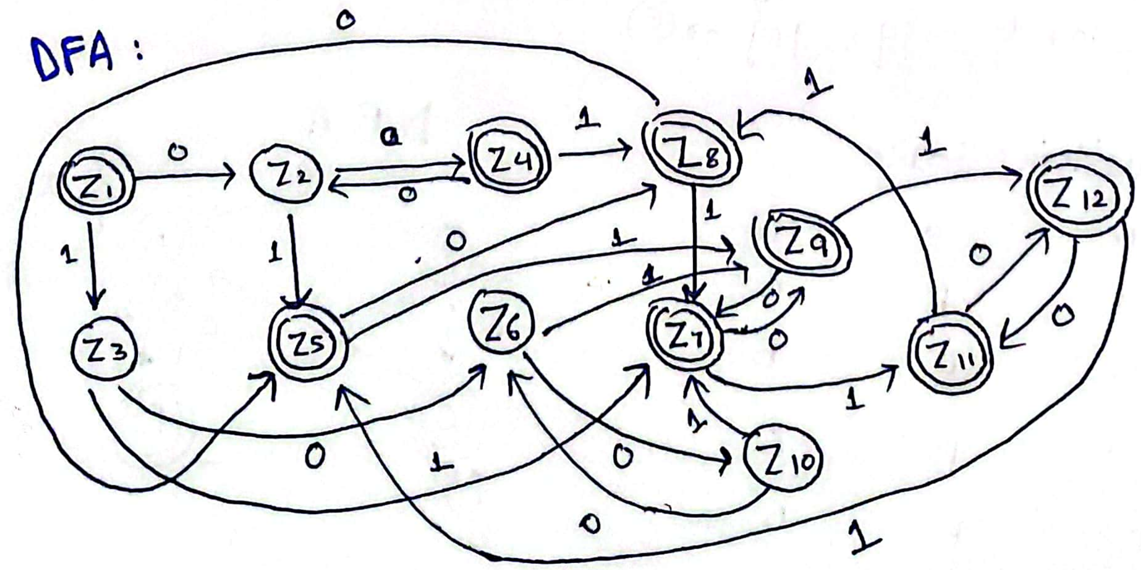
NOTE:
In first dfa, I have changed the name of states from q_1, q_2, q_3, q_4, q_5 and q_6 to x_1, x_2, x_3, x_4, x_5 and x_6 respectively.
DFA 2 is same.

Question 4

a) Ans: DFA 1 + DFA 2

Old states	New states 0	1
(x_1, q_0) $z_1^{\mp}$	(x_6, q_1) z_2	(x_2, q_3) Z_3
(x_6, q_1) Z_2	(x_1, q_2) Z_4^+	(x_5, q_4) Z_5^+
(x_2, q_3) Z_3	(x_5, q_2) Z_6	(x_3, q_4) Z_7^+
(x_1, q_2) Z_4^+	Z_2 (x_6, q_1)	(x_2, q_4) Z_8^+
(x_5, q_4) Z_5^+	(x_2, q_4) Z_8^+	(x_4, q_4) Z_9^+
(x_5, q_2) Z_6	(x_2, q_1) Z_{10}	(x_4, q_4) Z_9^+
(x_3, q_4) Z_7^+	(x_4, q_4) Z_9^+	(x_1, q_4) Z_{11}^+
(x_2, q_4) Z_8^+	(x_5, q_4) Z_5^+	(x_3, q_4) Z_7^+
(x_4, q_4) Z_9^+	(x_3, q_4) Z_7^+	(x_6, q_4) Z_{12}^+
(x_2, q_1) Z_{10}	(x_5, q_2) Z_6	(x_3, q_4) Z_7^+
(x_1, q_4) Z_{11}^+	(x_6, q_4) Z_{12}^+	(x_2, q_4) Z_8^+
(x_6, q_4) Z_{12}^+	(x_1, q_4) Z_{11}^+	(x_5, q_4) Z_5^+

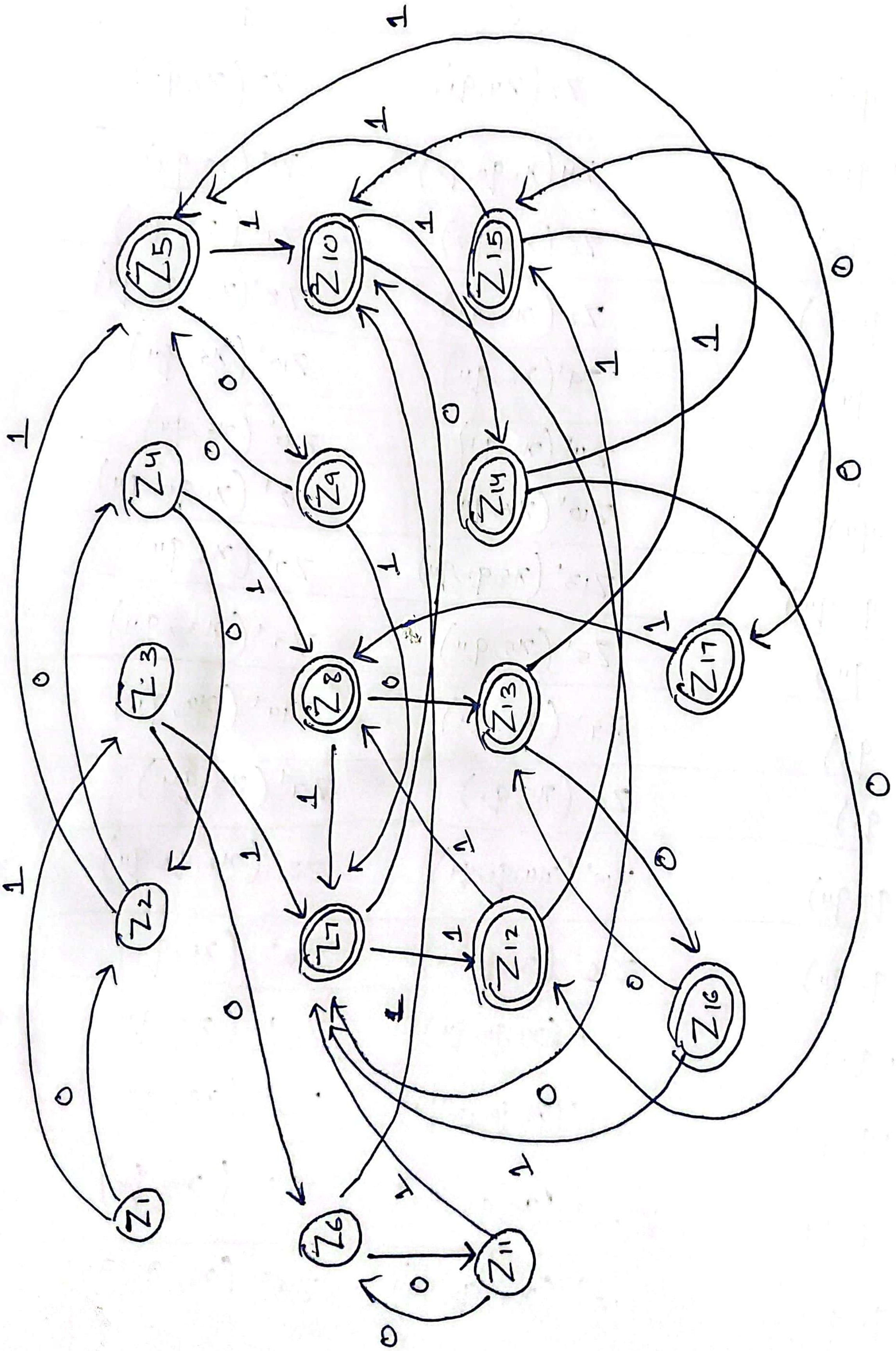
DFA :



b) Ans:

Old states	new states 0	new states 1
$Z_1^-(x_1, q_0)$	$Z_2(x_4, q_1)$	$Z_3(x_2, q_3)$
$Z_2(x_4, q_1)$	$Z_4(x_1, q_0, q_2)$	$Z_5^+(x_5, q_4)$
$Z_3(x_2, q_3)$	$Z_6(x_5, q_2)$	$Z_7^+(x_3, q_4)$
$Z_4(x_1, q_0, q_2)$	$Z_2(x_4, q_1)$	$Z_8^+(x_2, q_3, q_4)$
$Z_5^+(x_5, q_4)$	$Z_9^+(x_2, q_4)$	$Z_{10}^+(x_6, q_4)$
$Z_6(x_5, q_2)$	$Z_{11}^+(x_2, q_1)$	$Z_{10}^+(x_6, q_4)$
$Z_7^+(x_3, q_4)$	$Z_{10}^+(x_6, q_4)$	$Z_{12}^+(x_1, q_0, q_4)$
$Z_8^+(x_2, q_3, q_4)$	$Z_{13}^+(x_5, q_2, q_4)$	$Z_7^+(x_3, q_4)$
$Z_9^+(x_2, q_4)$	$Z_5^+(x_5, q_4)$	$Z_7^+(x_3, q_4)$
$Z_{10}^+(x_6, q_4)$	$Z_7^+(x_3, q_4)$	$Z_{14}^+(x_4, q_4)$
$Z_{11}^+(x_2, q_1)$	$Z_6(x_5, q_2)$	$Z_7^+(x_3, q_4)$
$Z_{12}^+(x_1, q_0, q_4)$	$Z_{15}^+(x_4, q_1, q_4)$	$Z_8^+(x_2, q_3, q_4)$
$Z_{13}^+(x_5, q_2, q_4)$	$Z_{16}^+(x_2, q_1, q_4)$	$Z_{10}^+(x_6, q_4)$
$Z_{14}^+(x_4, q_4)$	$Z_{12}^+(x_1, q_0, q_4)$	$Z_5^+(x_5, q_4)$
$Z_{15}^+(x_4, q_1, q_4)$	$Z_{17}^+(x_1, q_0, q_2, q_4)$	$Z_5^+(x_5, q_4)$
$Z_{16}^+(x_2, q_1, q_4)$	$Z_{13}^+(x_5, q_2, q_4)$	$Z_7^+(x_3, q_4)$
$Z_{17}^+(x_1, q_0, q_2, q_4)$	$Z_{15}^+(x_4, q_1, q_4)$	$Z_8^+(x_2, q_3, q_4)$

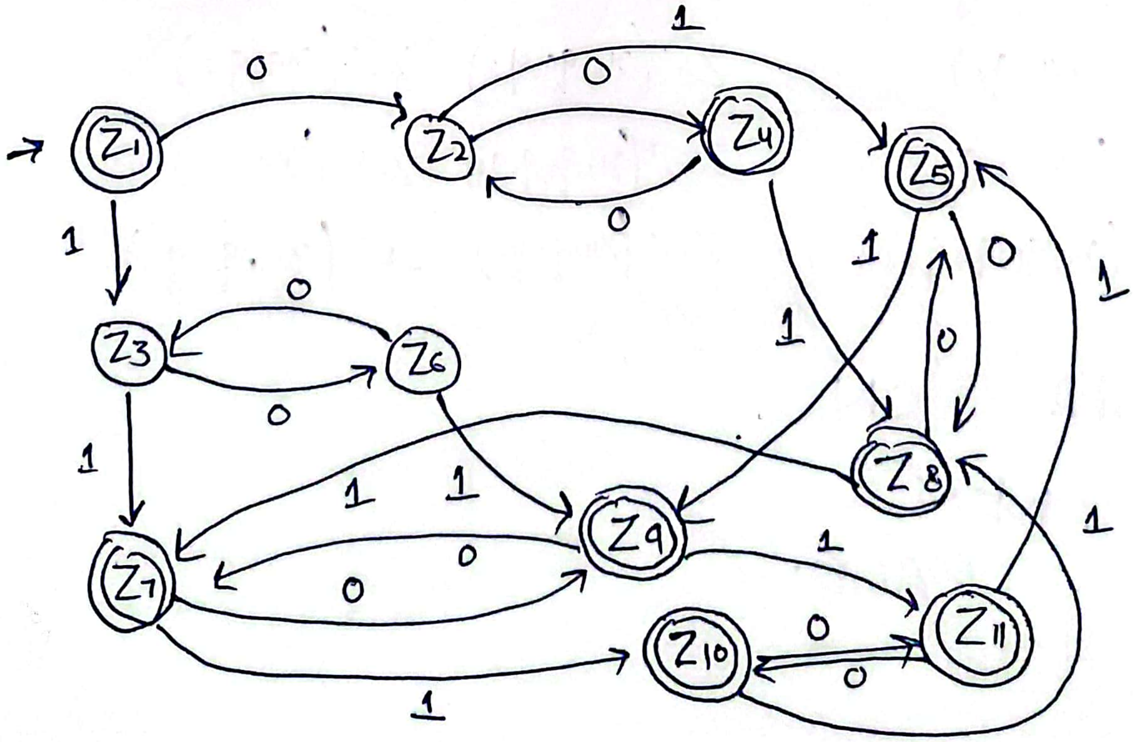
DFA



c) Ans. → For Minimised DFAs refer to Q 1a and Q1 b.
In 1a) instead of "q_1" i have used 'x_1' and so on respectively

Old state	New states 0	1
Z_1^+ (x_1, q_0)	$Z_2 (x_4, q_1)$	$Z_3 (x_2, q_1)$
Z_2 (x_4, q_1)	$Z_4^+ (x_1, q_1)$	$Z_5^+ (x_5, q_2)$
Z_3 (x_2, q_1)	$Z_6 (x_5, q_1)$	$Z_7^+ (x_3, q_2)$
Z_4^+ (x_1, q_1)	$Z_2 (x_4, q_1)$	$Z_8^+ (x_2, q_2)$
Z_5^+ (x_5, q_2)	$Z_8^+ (x_2, q_2)$	$Z_9^+ (x_6, q_2)$
Z_6 (x_5, q_1)	$Z_3 (x_2, q_1)$	$Z_9^+ (x_6, q_2)$
Z_7^+ (x_3, q_2)	$Z_9^+ (x_6, q_2)$	$Z_{10}^+ (x_1, q_2)$
Z_8^+ (x_2, q_2)	$Z_5^+ (x_5, q_2)$	$Z_7^+ (x_3, q_2)$
Z_9^+ (x_6, q_2)	$Z_7^+ (x_3, q_2)$	$Z_{11}^+ (x_4, q_2)$
Z_{10}^+ (x_1, q_2)	$Z_{11}^+ (x_4, q_2)$	$Z_8^+ (x_2, q_2)$
Z_{11}^+ (x_4, q_2)	$Z_{10}^+ (x_1, q_2)$	$Z_5^+ (x_5, q_2)$

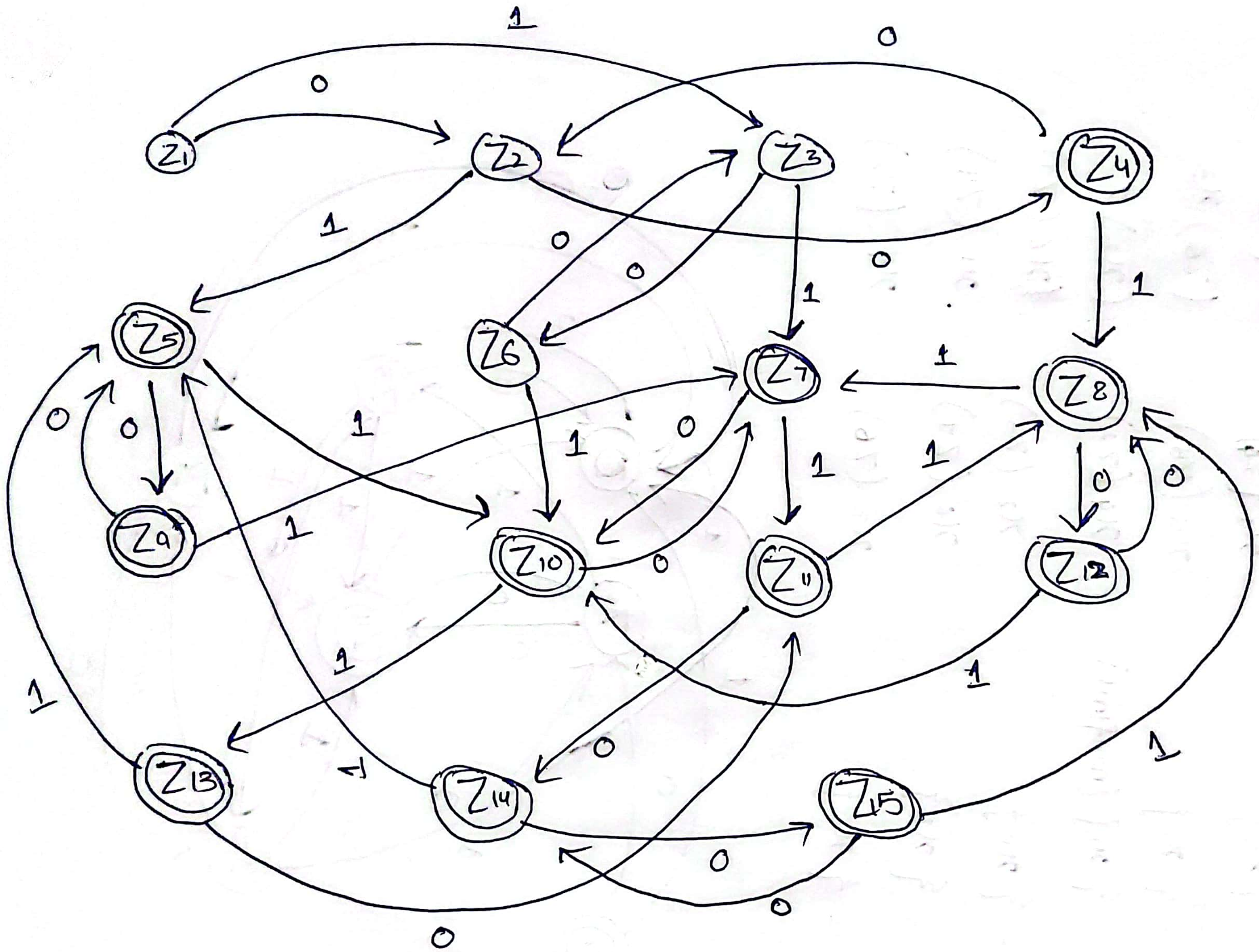
DFA :



d) Ans.

Old states	New states 0	1
$Z_1\ (x_1, q_0)$	$Z_2 (x_4, q_1)$	$Z_3 (x_2, q_1)$
$Z_2\ (x_4, q_1)$	$Z_4^+ (x_1, q_0, q_1)$	$Z_5^+ (x_5, q_2)$
$Z_3\ (x_2, q_1)$	$Z_6 (x_5, q_1)$	$Z_7^+ (x_3, q_2)$
$Z_4^+ (x_1, q_0, q_1)$	$Z_2 (x_4, q_1)$	$Z_8^+ (x_2, q_1, q_2)$
$Z_5^+ (x_5, q_2)$	$Z_9^+ (x_2, q_2)$	$Z_{10}^+ (x_6, q_2)$
$Z_6\ (x_5, q_1)$	$Z_3 (x_2, q_1)$	$Z_{10}^+ (x_6, q_2)$
$Z_7^+ (x_3, q_2)$	$Z_{10}^+ (x_6, q_2)$	$Z_{11}^+ (x_1, q_0, q_2)$
$Z_8^+ (x_2, q_1, q_2)$	$Z_{12}^+ (x_5, q_0, q_2)$	$Z_7^+ (x_3, q_2)$
$Z_9^+ (x_2, q_2)$	$Z_5^+ (x_5, q_2)$	$Z_7^+ (x_3, q_2)$
$Z_{10}^+ (x_6, q_2)$	$Z_7^+ (x_3, q_2)$	$Z_{13}^+ (x_4, q_2)$
$Z_{11}^+ (x_1, q_0, q_2)$	$Z_{14}^+ (x_4, q_1, q_2)$	$Z_8^+ (x_2, q_1, q_2)$
$Z_{12}^+ (x_5, q_0, q_2)$	$Z_8^+ (x_2, q_1, q_2)$	$Z_{10}^+ (x_6, q_2)$
$Z_{13}^+ (x_4, q_2)$	$Z_{11}^+ (x_1, q_0, q_2)$	$Z_5^+ (x_5, q_2)$
$Z_{14}^+ (x_4, q_1, q_2)$	$Z_{15}^+ (x_1, q_0, q_1, q_2)$	$Z_5^+ (x_5, q_2)$
$Z_{15}^+ (x_1, q_0, q_1, q_2)$	$Z_{14}^+ (x_4, q_1, q_2)$	$Z_8^+ (x_2, q_1, q_2)$

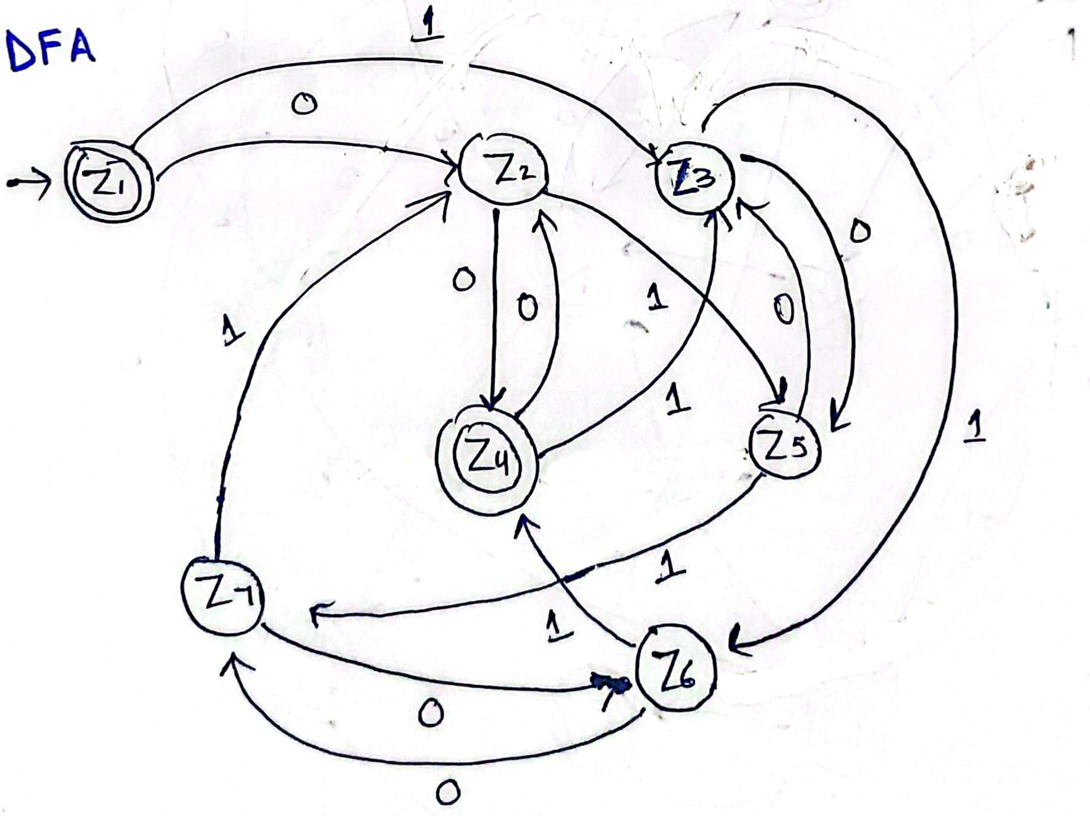
DFA ON NEXT PAGE.


Z1
Z2
Z3
Z4
Z5
Z6
Z7
Z8
Z9
Z10
Z11
Z12
Z13
Z14
Z15

e) Ans.

Old states	New states 0	New states 1
$Z_1^{\pm} = \{x_1\}$ (final)	$x_4 (Z_2)$	$x_2 (Z_3)$
$Z_2 = \{x_4\}$	$x_1 (Z_4)$	$x_5 (Z_5)$
$Z_3 = \{x_2\}$	$x_5 (Z_5)$	$(Z_6)\ x_3$
$Z_4^{+} = \{x_1\}$ (nonfinal)	$x_4 (Z_2)$	$(x_2)\ Z_3$
$Z_5 = \{x_5\}$	$x_2 (Z_3)$	$x_6 (Z_7)$
$Z_6 = \{x_3\}$	$x_6 (Z_7)$	$x_1 (Z_4)$
$Z_7 = \{x_6\}$	$x_3 (Z_6)$	$x_4 (Z_2)$

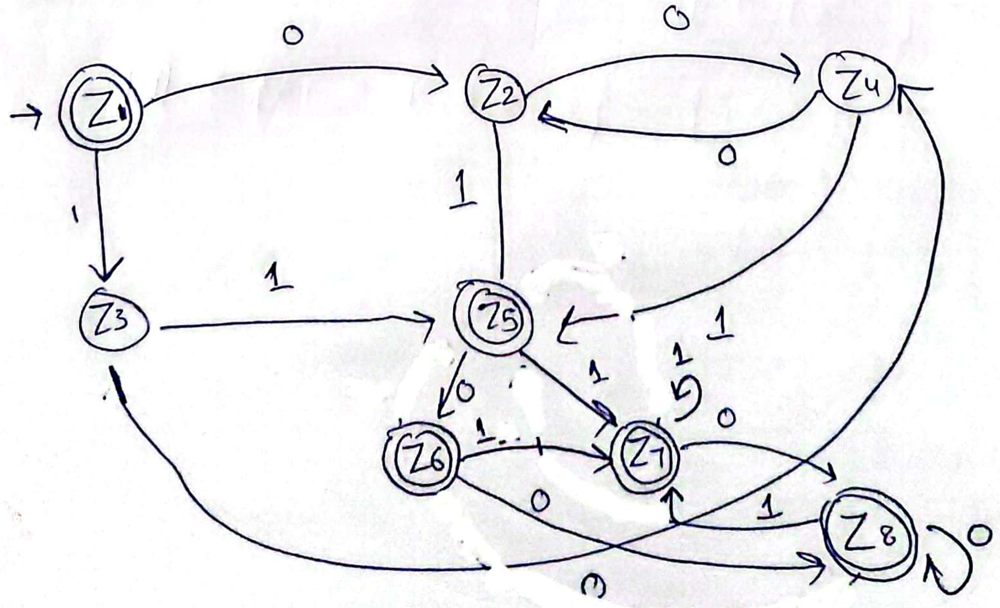
DFA



f) Ans.

Old states	New states 0	New states 1
$Z_1^+ = \{q_0\}$	$Z_2 = \{q_1\}$	$Z_3 = \{q_3\}$
$Z_2 = \{q_1\}$	$Z_4 = \{q_2\}$	$Z_5^+ = \{q_4 q_0\}$
$Z_3 = \{q_3\}$	$Z_4 = \{q_2\}$	$Z_5^+ = \{q_4 q_0\}$
$Z_4 = \{q_2\}$	$Z_2 = \{q_1\}$	$Z_5^+ = \{q_4 q_0\}$
$Z_5^+ = \{q_4 q_0\}$	$Z_6^+ = \{q_4, q_0, q_1\}$	$Z_7^+ = \{q_4 q_0 q_3\}$
$Z_6^+ = \{q_4, q_0, q_1\}$	$Z_8^+ = \{q_4, q_0, q_2, q_1\}$	$Z_7^+ = \{q_4, q_0, q_3\}$
$Z_7^+ = \{q_4, q_0, q_3\}$	$Z_8^+ = \{q_4, q_0, q_2, q_1\}$	$Z_7^+ = \{q_4, q_0, q_3\}$
$Z_8^+ = \{q_4, q_0, q_2, q_1\}$	$Z_8^+ = \{q_4, q_0, q_1, q_2\}$	$Z_7^+ = \{q_4, q_0, q_3\}$

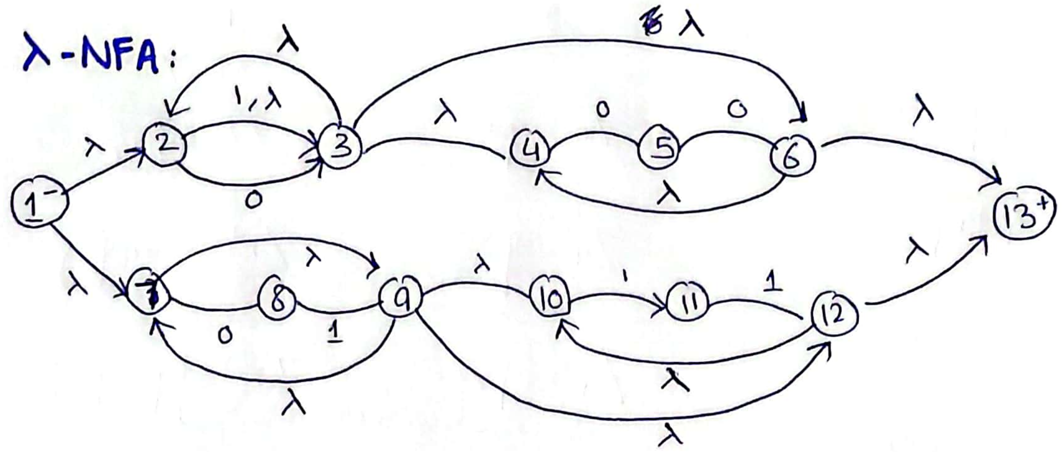
DFA :



Question 2

a) Ans. $(0+1)^*(00)^* + (01)^*(11)^*$

λ-NFA:



→ λ closure of 1 = {1, 2, 3, 4, 6, 7, 9, 10, 12, 13} → $\boxed{A}$

goto$_0$ A = {3, 5, 8} → (b)

goto$_1$ A = {3, 11} → (c)

→ λ closure of {3, 5, 8} = {3, 4, 5, 6, 2, 8, 13} → $\boxed{B}$

goto$_0$ B = {3, 5, 6} → (d)

goto$_1$ B = {3, 9} → (e)

→ λ closure of {3, 11} = {2, 3, 4, 6, 11, 13} → $\boxed{C}$

goto$_0$ C = {3, 5} → (f)

goto$_1$ C = {3, 12} → (g)

→ λ closure of {3, 5, 6} = {2, 3, 6, 4, 5, 13} → $\boxed{D}$

goto$_0$ D = {3, 5, 6} → (d)

goto$_1$ D = {3} → (h)

→ λ closure of {3, 9} = {2, 3, 4, 6, 7, 9, 10, 11, 12, 13} → $\boxed{E}$

goto$_0$ E = {3, 5, 8} → (b)

goto$_1$ E = {3, 11} → (c)

→ λ closure of $\{3,5\}$ = $\{2,3,4,6,13,5\}$ → $\boxed{D}$

→ λ closure of $\{3,12\}$ = $\{2,3,4,6,10,12,13\}$ → $\boxed{F}$

$goto_0 F = \{3,5\} \rightarrow (d)$

$goto_1 F = \{3,11\} \rightarrow (c)$

→ λ closure of $\{3\}$ = $\{2,3,4,6,13\}$ → $\boxed{G}$

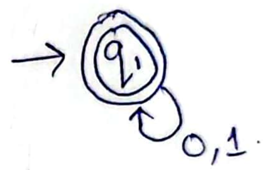
$goto_0 G = \{3,5\} \rightarrow (d)$

$goto_1 G = \{3\} \rightarrow (g)$

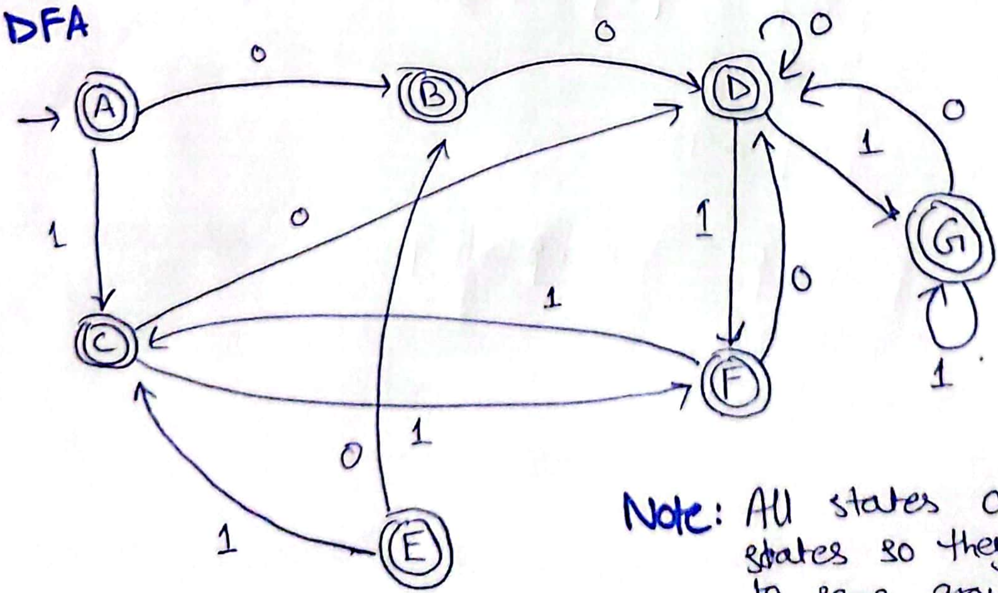
Transition table:

States	0	1
+A	B	C
+B	D	E
+C	D	F
+D	D	G
+E	B	C
+F	D	C
+G	D	G.

DFA (minimised):



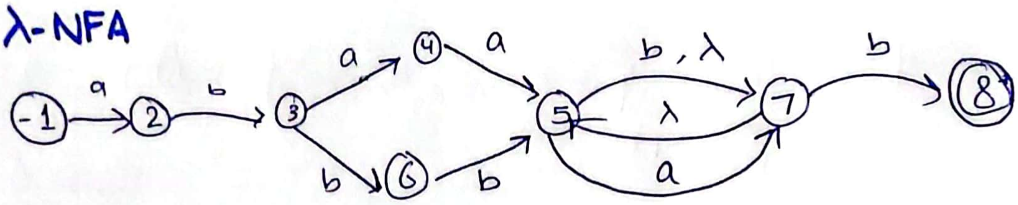
DFA



Note: All states are final states so they belong to same group.

b) Ans. $ab(aa+bb)(a+b)^* b$

λ-NFA



→ λ closure of {1} = {1} → $\boxed{A}$

goto$_a$ A = {2} → (b)

goto$_b$ A = { }

→ λ closure of {2} = {2} → $\boxed{B}$

goto$_a$ B = { }

goto$_b$ B = {3} → (c)

→ λ closure of {3} = {3} → $\boxed{C}$

goto$_a$ C = {4} → (d)

goto$_b$ C = {6} → (e)

→ λ closure of {4} = {4} → $\boxed{D}$

goto$_a$ D = {5} → (f)

goto$_b$ D = { }

→ λ closure of {6} = {6} → $\boxed{E}$

goto$_a$ E = { }

goto$_b$ E = {5} → (f)

→ λ closure of {5} = {5, 7} → $\boxed{F}$

goto$_a$ F = {7} → (g)

goto$_b$ F = {7, 8} → (h)

→ λ closure of $\{7\} \Rightarrow \{5,7\}$ → $\boxed{F}$

→ λ closure of $\{7,8\} = \{5,7,8\}$ → $\boxed{G}$

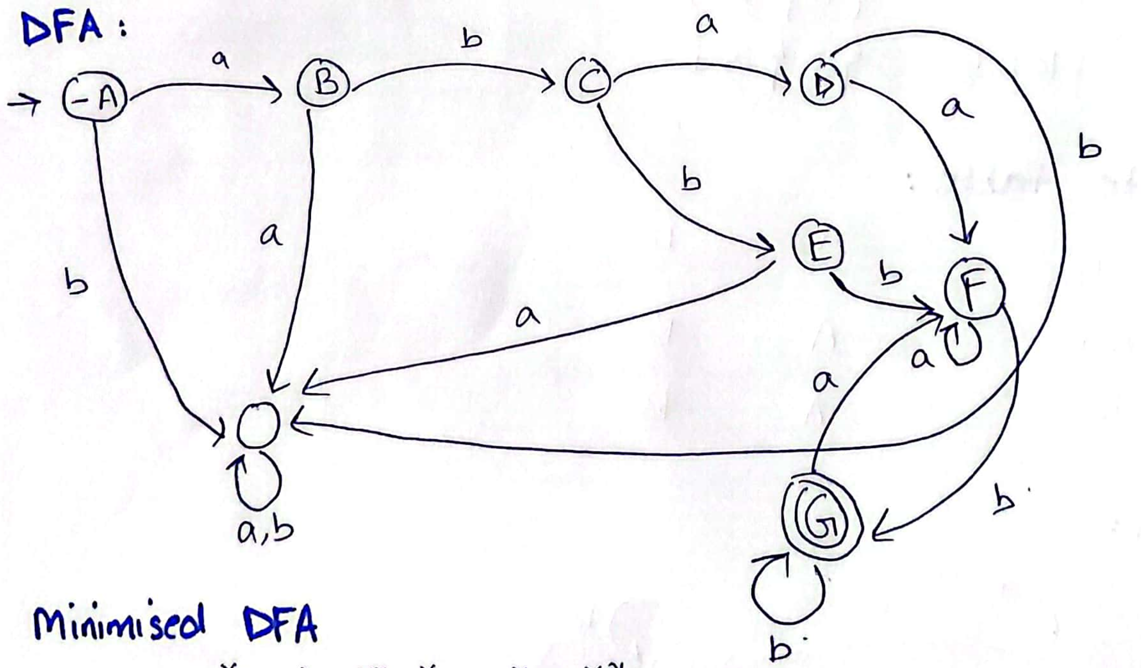
goto$_a$ G = $\{7\}$ → (g)

goto$_b$ G = $\{7,8\}$ → (h).

State table :

	a	b
- A	B	Φ
B	Φ	C
C	D	E
D	F	Φ
E	Φ	F
F	F	G
+ G	F	G

DFA :



Minimised DFA

$\pi_0 = \{\overset{X-}{A}\ \overset{-X}{B}\ \overset{XX}{C}\ \overset{X-}{D}\ \overset{-X}{E}\ \overset{XY}{F}\}$ $\{G\}$

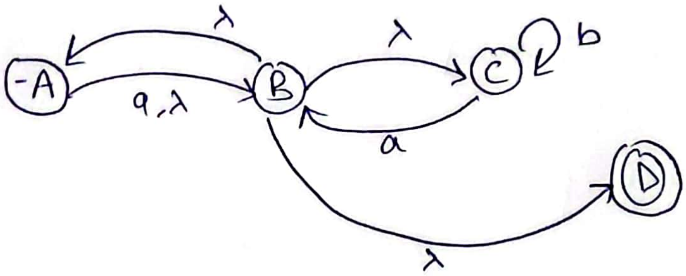
$\pi_1 = \{A\ D\}\ \{C\}\ \{B\ E\}\ \{F\}\ \{G\}$

$\pi_2 = \{A\}\ \{D\}\ \{C\}\ \{B\}\ \{E\}\ \{F\}\ \{G\}$.

It cannot be further minimised.

c) Ans. $a^*(b^+a)^*$

λ NFA :



→ λ closure of A = { A, B, C, D } → $\boxed{A}$

goto a A = { B } → (a)

goto b A = { C } → (b)

→ λ closure of { B } = {A B, C, D} → $\boxed{A}$

=

→ λ closure of { C } = { C } → $\boxed{B}$

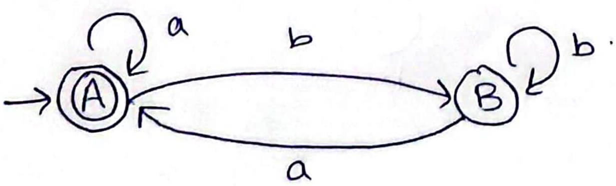
goto a B = { B } → (a)

goto b B = { C } → b.

State table :

states	a	b
A F	A	B
B	A	B

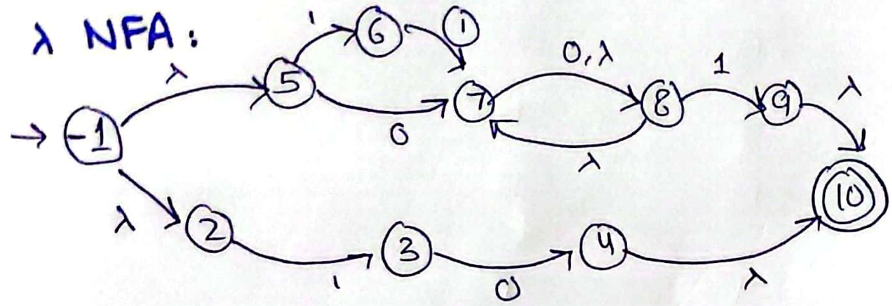
DFA :



This cannot be further minimised.

d) Ans $10 + (0+11)\,0^*\,1$

λ NFA :



→ λ closure of {1} = {1, 2, 5} → $\boxed{A}$

goto a A = {7} → (a)

goto b A {3,6} → (b)

→ λ closure of {7} = {7,8} → $\boxed{B}$

goto a B = {8} → (c)

goto b B = {9} → (d)

→ λ closure of {3,6} = {3,6} → $\boxed{C}$

goto a C = {4} → (e)

goto b C = {7} → (a)

→ λ closure of {8} = {7, 8} → $\boxed{B}$

→ λ closure of {9} = {9, 10} → $\boxed{D}$

goto a D = { }

goto b D = { }.

→ λ closure of {4} = {4, 10} → $\boxed{E}$

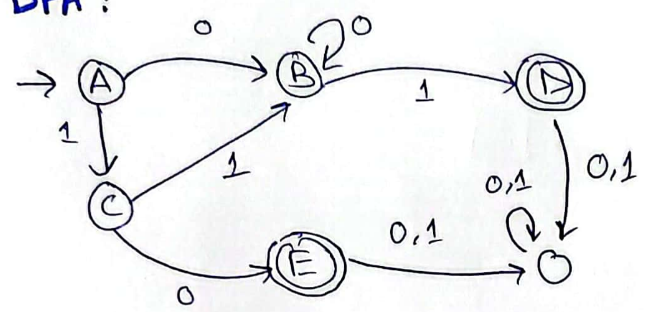
goto a E = { }

goto b E = { }

State table :

	0	1
- A	B	C
B	B	D
C	E	B
+ D	ϕ	ϕ
+ E	ϕ	ϕ

DFA :

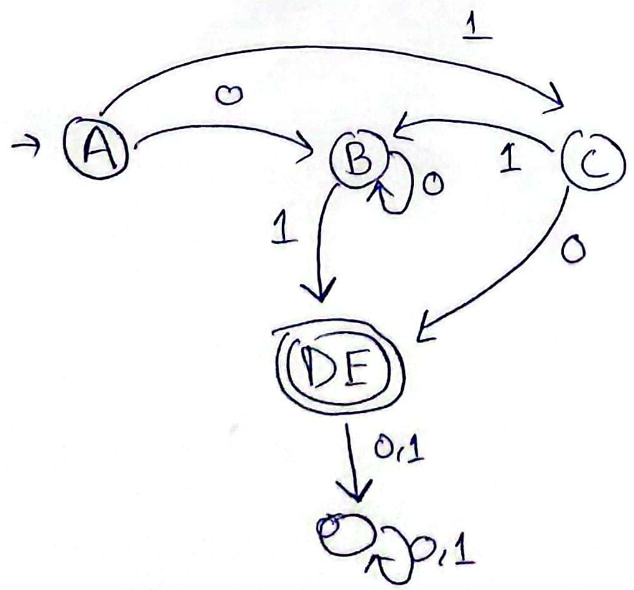


Minimised DFA

$$\pi_0 = \{\overset{PP}{A} \quad \overset{PQ}{B} \quad \overset{QP}{C}\} \qquad \{D \quad E\}$$

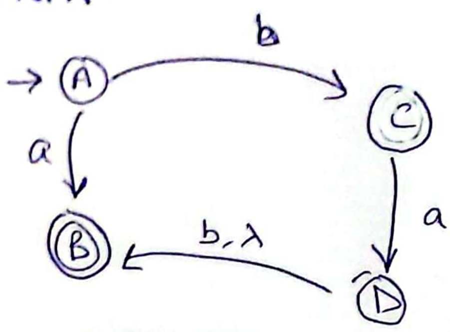
$$\pi_1 = \{A\} \ \{B\} \ \{C\} \qquad \{D\ E\}.$$

π_2 = same as π_1



Ans: $a + b(ab + a)$

λ NFA :



→ λ closure of A = {A} → $\boxed{A}$

goto a A = {B} → (B)

goto b A = {C} → (C)

→ λ closure of {B} = {B} → $\boxed{B}$

goto a B = { }

goto b B = { }

→ λ closure of {C} = {C} → $\boxed{C}$

goto a C = {D} → (d)

goto b C = { }

→ λ closure of {D} = {D, B} → $\boxed{D}$

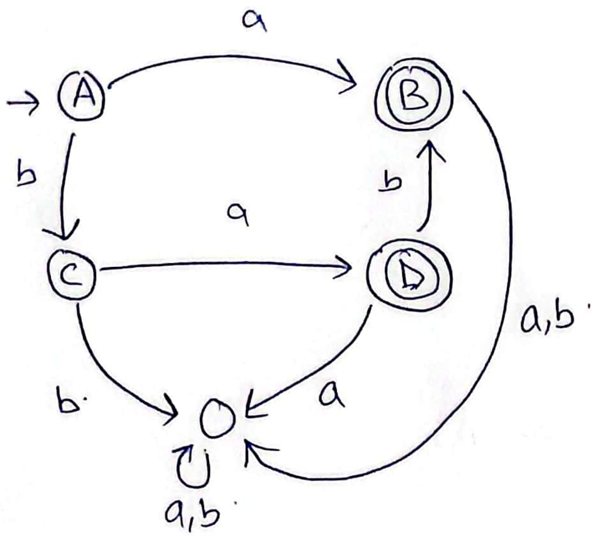
goto a D = { }

goto b D = {B} → (b)

State table :

states	a	b
- A	B	C
+ B	–	–
C	D	–
+ D	–	B

DFA :



Minimization :

$$\pi_0 = \{\overset{QP}{A}\ \overset{Q-}{C}\} \quad \{\overset{--}{B}\ \overset{-Q}{D}\}$$

P   Q

$$\pi_1 = \{A\}\ \{C\}\quad \{B\}\ \{D\}$$

$$\pi_2 = \{A\}\ \{C\}\quad \{B\}\ \{D\}.$$

Cannot be further minimised