

Current	Best	2nd	3rd	4th	5th	6th	7th	8th
Draw 1	4.02	3.50	5.08	4.16	4.22	4.41	3.65	3.27
Draw 2	4.18	4.72	3.49	3.48	3.63	3.60	3.56	3.70
Draw 3	4.81	5.23	5.04	3.96	4.17	4.37	3.58	2.99

Table 1: Draws of utility scores after 100 iterations

1 Value Remaining

TO DO the naming convention in different. (Utility)

Add Matric besides hit rate. $\mu_S = \sum_{x \in S} e^{\beta x}$ is the utility for a given set S . Taken [2] and [1], The value remaining in the experiment is the posterior distribution of $\frac{\mu_{max} - \mu_{a^*}}{\mu_{a^*}}$ where μ_{max} is the largest value of the utility and μ_{a^*} is the utility of the set that is most likely to be optimal denoted a^* . This is constructed as follows, take n Monte Carlo draws from $p(\mu|y_t)$. Let μ_{max}^m be the max utility of draw m and $\mu_{a^*}^m$ be the utility using the draw m using the set a^* . Let $\Delta^m = \frac{\mu_{max}^m - \mu_{a^*}^m}{\mu_{a^*}^m}$.

As an example I took draws after respondent 100 in TS $\epsilon = .67$ $\delta = 10$ for 120 items see table 1 for draws of utility of a single item. I put the columns in current rank order and only show the top 8 for convenience. Say we were finding the value remaining for the the utility score using the top 3. Then $a^* = \{1, 2, 3\}$. Then $\mu_{a^*}^1 = 4.02 + 3.50 + 5.08 = 12.6$ and $\mu_{max}^1 = 5.08 + 4.41 + 4.16 = 13.65$ So $\Delta^1 = \frac{13.65 - 12.6}{12.6} = .083$. Likewise $\Delta^2 = \frac{12.6 - 12.39}{12.39} = .017$ and $\Delta^3 = \frac{15.08 - 15.08}{15.08} = 0$ (Note $\Delta^m = 0$ when the a^* contains the top utilities). The histogram of Δ after 100 iterations and 220 iterations is shown in figure 1.

The ‘potential value remaining’ (PVR) is the .95 quantile of the distribution Δ which in this case was .092. A way to interpret this number is “We do not know what the utility of a^* is, but whatever it is, a different set might beat it by as much as 9.2%.”

A good stopping rule is to stop when the PVR drops below a certain threshold, (We use .02 or .05). One advantage of this is it handles ties (two sets with close utility scores) really well.

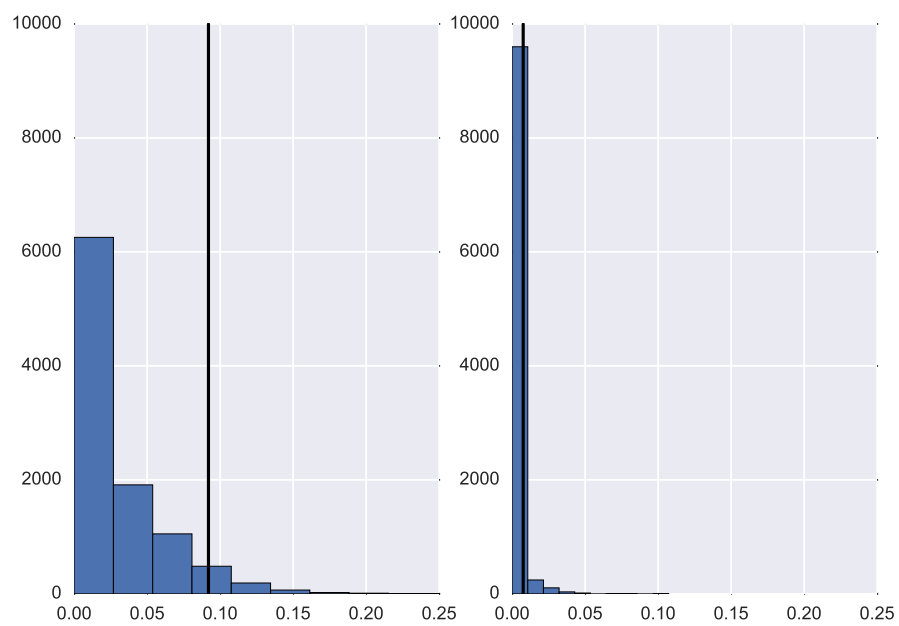


Figure 1: Two histograms of Δ . Left: After 100 iterations, the potential value remaining is .092. Right: After 220 iterations, the potential value remaining is .008

References

- [1] Steven L Scott. a modern bayesian look at the multi-armed banditby steven l. scott: Rejoinder. *Applied Stochastic Models in Business and Industry*, 26(6):665–667, 2010.
- [2] Steven L Scott. Multi-armed bandit experiments in the online service economy. *Applied Stochastic Models in Business and Industry*, 31(1):37–45, 2015.