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Bandit MaxDiff: Large-scale adaptive MaxDiff with ϵ-Diffuse Thompson Sampling PRELIMINARY! DO NOT DISTRIBUTE

For large MaxDiff studies whose main purpose is identifying the top few items for the sample, a new adaptive approach called Adaptive MaxDiff may increase efficiency fourfold over standard non-adaptive MaxDiff. Adaptive MaxDiff leverages information from previous respondents via aggregate logit and Thompson Sampling so later respondents receive designs that oversample the topmost items that are most likely to turn out to be the overall winners. Our approach applies beyond MaxDiff problems to a more general set of bandit problems. We propose a flexible algorithm, ϵ-Diffuse Thompson Sampling (TS), which nests traditional TS. Blending ideas from ϵ-greedy in machine learning and Bayesian approaches, this is a more robust version of TS, which a manager can control with tuning parameters. For instance, being less risk averse, one may drawn more items from diffuse posteriors, making the algorithm robust to changing environments, even extreme non-stationarity. We implement the methods using MaxDiff survey from a large consumer packaged goods manufacturer. Beyond showing our approach outperforms better than current methods, we show under which conditions it performs even better than (larger problems) or just as well as existing methods (smaller problems).

Key words: idea screening, maximum-difference surveys, adaptive conjoint, active learning, multi-armed bandit, Bayesian bootstrap, best-worst scaling, Thompson Sampling, uncertainty minimization

1. Introduction

Firms collect more customer data than ever. While much of that is behavioral data (e.g., purchases, clicks, visits, etc.), the online market research relies on surveys (e.g., preference measurement) at an increasingly large scale. And as firms push preference measurement methods into a larger scale, market researchers seek to collect data more efficiently – more

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One common managerial decision driving such data collection is to select which are most preferred among a large set of hundreds of possible items. For instance, consider these illustrative managerial decisions:

1. *Idea screening.* Technology companies have long invited consumers to provide ideas for new products and improvements. Automotive companies Complex products, such as, self-driving cars are may contain hundreds of novel features and are sold with different add-ons, which are bundles of features. So consumer surveys could reveal how the best features are preferred, to help the firm decide which features can be included in different models.
2. *Marketing communications.* Consumer packaged goods companies use marketing research, after the products are developed, to decide which consumer benefits they should emphasize. Out of hundreds of potential benefits, they have to select a small number of them for messages.
3. *Product offerings.* Retailers are increasingly able to respond to demand and consumer preferences quickly by changing their available product offerings. In anticipation of a new fashion season, an ecommerce retailer may survey customers to identify most preferred styles and lines, and then select the range of items to make available for purchase.

These settings have a common challenge: identify the best set of items out of many. This problem setting is distinct from closely related problems but existing methods do not suffice due to scale and objective.

Typical choice experiment methods, such as conjoint, are infeasible, too costly, or simply inefficient at this scale. Instead, researchers often use pretest or screening phases to select a smaller set, and use that for the main analysis. Yet there is no systematic method to smoothly transition out of that screening phase, even when the ultimate objective is learning precisely the set of most preferred items.

We focus on settings where the objective is identifying a top set of items as quickly, cheaply, and accurately as possible. Identifying a top set is not the same objective as estimating parameters as precisely as possible, as in conjoint, as we will illustrate later. So even with improving computation and larger budgets, this problem has a distinct objective and demands a tailored methods.

Choice experiment methods for preference measurement continue to be among the most widely adopted market research methods. Choice-based conjoint (CBC) analysis and maximum difference (MaxDiff), for instance, are common choice tasks that continue to be used in both academic and industry settings.

Methodological advances, such as adaptive conjoint, have long sought to improve efficiency, obtaining more information from fewer respondents. Those methods will use past responses to select the next question to improve precision of all parameters. For instance, adaptive choice-based conjoint (ACBC) methods present respondents with questions to reduce the uncertainty where it is greatest. It chooses the next question to best reduce overall uncertainty.

But the market researcher's goal is typically not to obtain precise estimates of utility of every single alternative or attribute level. If their goal is to identify the most preferred items, why do they need to spend resources (questions, respondents, time) to learn how precisely poor the least-preferred item is compared to the second-least preferred item?

With that objective in mind, we propose several new adaptive choice experiment methods. Our approach departs from traditional adaptive survey methods like adaptive conjoint analysis (ACA). They aim to improve the precision of each of the parameters; we estimate the best items more precisely.

We propose novel methods to address the problem setting that stated choice data collection, such as adaptive idea screening and adaptive MaxDiff surveys. The focus of our research is a broader framework to align the choice data collection process with managerial objectives. By aligning the two, we aim to improve efficiency and scalability for big data collection via choice experiments. We find we can "get more for less," by improving precision only where it matters for decision making, using a smaller sample size compared to existing methods.

While we propose this framework broadly, we demonstrate empirically with MaxDiff, as it is an increasingly popular and important choice experiment method. The idea screening problem is sufficiently similar. But we will consider the general choice-based conjoint case in our discussion section.

Our approach draws on state-of-the-art active learning and multi-armed bandit algorithms in statistical machine learning.

We contribute three ideas to the literature and practice. First, we frame choice task data collection as an earning-and-learning problem. During the process of data collection, we simultaneously balance the desire to learn preferences of all items and earn a reward towards achieving objective, by only serving questions the truly best subset of items. In this case, the objective is to identify the best set of items, for instance, the most desirable levels of attributes. The faster we identify the best items, the more we include them in questions, improving the precision of estimates. And if we need to reach a desired level of precision, we can do so faster, saving money in the form of number of respondents and time.

The preceding paragraph needs to be changed. We are not doing any earning just learning Pure exploration

Multi-armed bandit problems appear in marketing experiments for online advertising Schwartz et al. (2015), Urban et al. (2013) and website design Hauser et al. (2009). One particular MAB method amenable to multi-variable parametric models is Thompson Sampling.

While we use bandit algorithms, our approach differs from extant bandit applications in a number of ways. For a bandit problem the reward is observed immediately (click, acquisition, purchase). While we have immediate observations (choice), that is not our reward. Instead, our goal is to correctly identify the truly best items as precisely as possible. These are not evaluable in real-time.

While our problem is not a bandit problem exactly, it is a bandit-inspired problem. However we find bandit algorithms form a heuristic to achieve our objective better than existing MaxDiff and conjoint methods.

Consider what would occur if the reward was whether an item is selected as best alternative, the arm's reward would be completely dependent on all of the other items presented in the set (e.g., was it presented alongside poor items). Second, the need to optimize this process is greater for large-scale problems with extremely large number of possible attribute levels or items. In settings with hundreds of items, there is greater opportunity cost of focusing on items that are not important, calling for a need to improve efficiency.

Third, we propose an adaptive method for best-worst scaling. Existing adaptive methods have been largely limited to conjoint, adapting at both the aggregate level, Arora and Huber (2001), and the individual level, Toubia et al. (2004). Yet best-worst methods, such as MaxDiff, have continued to emerge as important and commonly used in areas including marketing research and public health. We introduce a first step to make MaxDiff adaptive in a principled manner by using MAB methods. We accommodate an increasingly accepted method of solving multi-armed bandit problems, Thompson Sampling. We introduce two versions: MaxDiff-Thompson Sampling and its generalization MaxDiff ϵ-diffuse Thompson Sampling.

The ϵ-diffuse Thompson Sampling nests traditional TS. We combine ideas from ϵ-greedy in machine learning and Bayesian approaches to produce a more robust version of TS, which a manager can control with tuning parameters. For instance, if a manager wants to be extra conservative and maintaining more samples drawn from diffuse posteriors, she will make the algorithm robust to changing environments, even extreme non-stationarity. We also draw on ideas in closely related area, active learning, such as, Follow The Perturbed Leader (FTPL) Kalai and Vempala (2005). Recent theoretical advances prove that adding perturbations of Gumbel distributed noise leads to an optimal MAB strategy Abernethy et al. (2015); Kujala and Elomaa (2005). The perturbation methods provide an intuitive randomization decision strategy, which is appropriate for our application, yet their connection to other approaches and applications are limited. Perturbation not only resolves the explore-exploit (learn-earn) tradeoff for the stochastic (iid) MAB setting, but it also does so in the adversarial setting, suggesting it is an MAB strategy more robust to changes in the non-random changes in the environment. We shed light on this methodological link as it has consequences beyond our method.

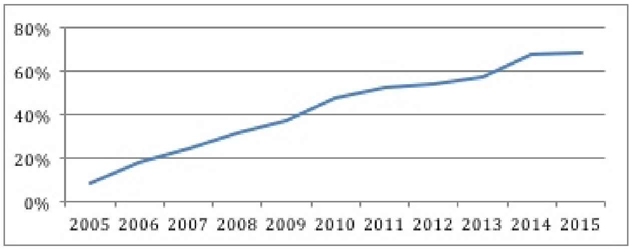
The rest of the paper is structured as follows. In the next section we introduce the illustrative problem of maximum difference scaling focusing on its use in practice with large numbers of items. We then briefly introduce the multi-armed bandit framework. After formalizing the discrete choice model of maxdiff analysis, we introduce our two algorithms that implement this method for Maxdiff Surveys. Then we present our empirical results in regular setting and then present variants of our main results. We implement the methods using MaxDiff survey implemented by Sawtooth Software with a large consumer packaged goods manufacturer.

2. Framework and literature

2.1. Maximum Difference Scaling

MaxDiff, Maximum Difference Scaling, is a preference measurement and item scaling method. In a MaxDiff questionnaire the researcher asks respondents for their best and

Figure 1 **MaxDiff is becoming more popular over time with Sawtooth Software users**



worst item out of a set, then repeats this choice task. Initially proposed by Louviere and Woodworth (1991), MaxDiff was first released as a software system in 2004 by Sawtooth Software, a top marketing research software company in North America. Since its release its popularity increased steadily with penetration of the technique now reaching 68% of all Sawtooth users in 2015 (Figure 1).

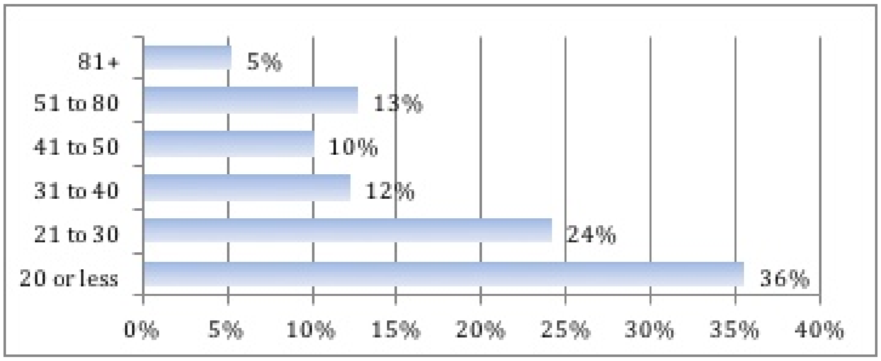
MaxDiff offers benefits over alternative methods. MaxDiff provides more discrimination among items and between respondents on the items than traditional rating scales Cohen and Orme (2004). Besides enhanced discrimination, it avoids the scale use bias so problematic with traditional ratings scales (CITE).

MaxDiff, while it is a distinct type of choice experiment, is closely related to conjoint. A key difference is that it involves both best and worst choices instead of only one. But in its most common form, MaxDiff may be thought of as a one-attribute CBC study with many levels.

2.2. Studying More Items with MaxDiff: Survey of Market Research Practitioners

MaxDiff has proven so useful that market researchers increasingly find reasons to use MaxDiff for a large number of items. How many is a large number of items? In their 2007 paper, Hendrix and Drucker described “large sets” as about 40 to 60 items, proposing variants to MaxDiff called Augmented and Tailored MaxDiff to handle such large problems Hendrix and Drucker (2007). In their 2012 paper, Wirth and Wolfrath also investigated variants to MaxDiff called Express and Sparse MaxDiff for handling what they described as “very large sets” of items Wirth and Wolfrath (2012). Very large to these authors meant potentially more than 100 items. To support their findings, they conducted a study among synthetic robotic respondents with 120 items and a real study among humans with 60 items.

Figure 2 Maximum Number of Items Studied via MaxDiff during 2015



N = ??, Mean= 40, Median=30, Maximum=400

For Hendrix and Drucker 40 to 60 items was large, for Wirth and Wolfrath 120 items was very large. For this current paper, we're referring to huge numbers of items as potentially 300 or more. The motivation of our research is beyond academic curiosity, as marketing researchers are seeking such applications pushing MaxDiff further than it was perhaps ever intended.

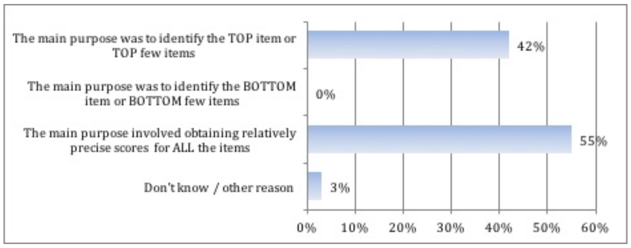
In Sawtooth Software's 2015 Customer Feedback Survey, we asked respondents to tell us the largest number of items they had included in a MaxDiff study during the last 12 months (Figure 2). Nearly one-fifth of respondents indicated their firms had conducted a study with 51 or more items. The maximum number of items studied was 400!

To some it may seem bizarre and overwhelming that some researchers are conducting MaxDiff studies with 81+ or even 400 items. However, when we consider that individual MaxDiff items may actually represent conjoined elements that constitute a profile (say, a combination of packaging style, color, claims, and highlighted ingredients), then it can make much more sense to do 400-item studies. If the profiles involve multiple highly interactive attributes that pose challenges for CBC, then MaxDiff with huge numbers of items could be a viable alternative (given the new approach we demonstrate further below).

We also asked Sawtooth Software customers what the main purpose was for that study with the reported maximum number of items. For studies involving 41 or more items, the main reasons are displayed in Figure 3.

For 42% of these large MaxDiff studies, the main purpose was to identify the TOP item or TOP few items. Our research shows that if this is the main goal, then traditional design strategies are very wasteful. An adaptive approach using Thompson Sampling can be about 4x more efficient. Without the Thompson Sampling approach, you are potentially wasting

Figure 3 **Main Purpose for MaxDiff Study with 41+ Items**



75 cents of every dollar you are spending on MaxDiff data collection.

The problem is that current MaxDiff approaches don't scale well to increasing the number of items. More items require commensurately longer questionnaires, larger sample sizes, and larger data collection costs with more tired respondents. If the researcher is concerned about obtaining robust individual-level estimates for all the items, then the current methodologies especially don't scale well to large lists of items. Respondents just tire out with such long surveys. In contrast, our approach employs an adaptive divide-and-conquer aggregate approach that leverages prior learning to create more efficient questionnaires and more precise aggregate score estimates.

2.3. Overview of approach: Bandit MaxDiff with Thompson Sampling

Thompson Sampling has been proposed as an efficient solution for solving the multi-armed bandit problem. Thompson Sampling involves allocating resources to an action in proportion to the probability that it is the best action Thompson (1933). Any bandit method must find an appropriate balance between exploring to gain information and exploiting that knowledge.

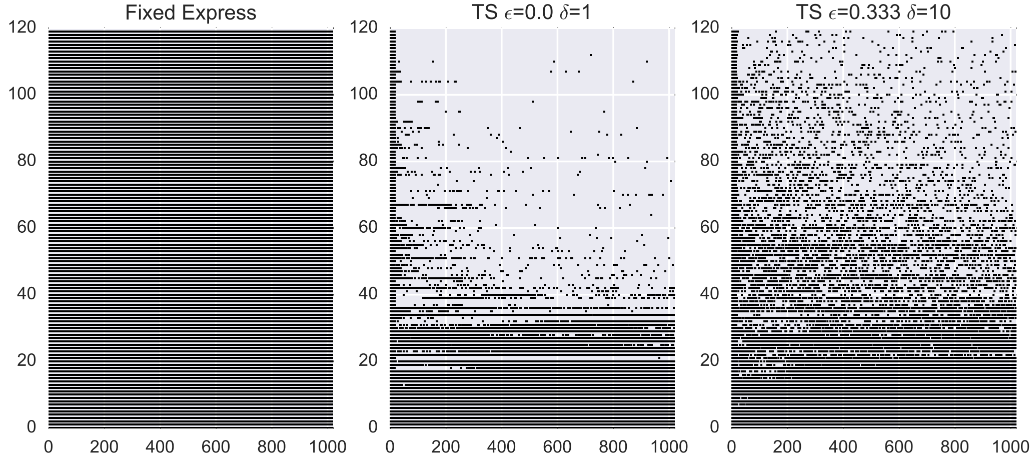
On the one hand, we want to learn about the relative scores of a large number of items within a MaxDiff problem. On the other hand, we want to utilize what we have learned so far to focus our efforts on a targeted set of actions that will likely yield greater precision regarding the items of most interest to the researcher. While there are many methods to accomplish this, Thompson Sampling has proven very useful for these types of problems. For a marketing application of Thompson Sampling and a review of the literature, see Schwartz et al. (2016).

The traditional MaxDiff design approach shows each item an equal number of times across all respondents x tasks. However, if the main goal is to identify the top few items for the sample, after the first, say 20, respondents it seems reasonable to start paying attention to the already-collected MaxDiff responses and oversampling the items that are already viewed as most preferred (the stars). We can use aggregate logit to estimate both preference scores and standard errors at any point during data collection (say, after the 20th, 40th, ... respondent has completed the survey).

Thompson Sampling makes a new draw from the vector of item preferences using the estimated population preferences (aggregate logit scores) plus normally distributed error, with standard deviations equal to the standard errors of the logit weights. As the sample size increases, the standard errors of course tighten.

To understand the logic of the algorithm, consider a snapshot in time during the data collection. Imagine we have just collected data from 100 respondents, and we decide to summarize their preferences (for each of 100+ items) with aggregate logit. Then, to generate a MaxDiff task for the 101st respondent, we could generate a draw from the population preferences leveraging the population means and normal errors with standard deviations equal to the empirically estimated standard errors. We then can sort that newly sampled vector of preference scores from the most to the least preferred item. The five most preferred items might be taken into the first task to show to the 101st respondent. The process (with or without updating the logit weights after recording the first task's answer) could be repeated to choose the five items to show in the second task for the 101st respondent, etc. To reduce the load on the server managing the data collection, perhaps only after every

Figure 4 Respondent-by-item counts



20th respondent has completed the survey, the logit weights and standard errors would be updated.

We note that the translation of MaxDiff into a bandit problem is not obvious. Our goal is identifying the utilities of the top set of items with maximum precision. Selecting an arm corresponds to including it in the survey for the next respondent. The reward, however, is not as clear. On the one hand, suppose each task included all possible items. Then the reward would be clear: whether the item is chosen as the "best." But it is not feasible to show so many items to respondents, so that's not a relevant scenario. On the other hand, given we only select a subset of items, the item receiving the "best" label doesn't translate directly to a reward. Indirectly, that choice data does enable us to infer how it ranks among all items, exactly aligning with our goal. We use the alignment of the managerial goal and the MAB algorithm's balancing of learning and earning to our advantage.

Is our goal to identify top items with maximum precision? Because we do not show any data that we do that or that is even important for problem (identify top items)

3. Adaptive MaxDiff

We formalize the proposed procedure in two stages. First, we introduce MaxDiff as a best-worst scaling discrete choice task, and we relate it to the standard multinomial logit model. Next, we introduce adaptive techniques as the natural method for solving the pure exploration problem.

3.1. MaxDiff choice model

Every respondent selects both the best and the worst option from an available set of options in each discrete choice task. The model for that data comes from a class of probability models known as best-worst scaling, and MaxDiff is one such model. We adopt the framework from the best-worst scaling literature. For a review, see Marley and Pihlens (2012) Marley 2010, Marley and Louviere (2005). We have K possible items, and we select a set S for each choice task. To describe the items, we define two random variables, best B_z and worst W_z, for each $z \in S$. We then define a third random variable, best-worst $BW_{r,s}$, for any $r, s \in S$.

Following a random utility framework, these utilities have deterministic and stochastic components.

A consistent extreme value random utility model is a Thurstone random utility model where each ε_z has the extreme value distribution. Consistent model means $B_z = -W_z = U_z$ and $BW_{r,s} = U_r - U_s$, then

$$\begin{aligned}
&B_z = v_z + \varepsilon_z \\
&W_z = -v_z - \varepsilon_z \\
&BW_{r,s} = v_r - v_s + \varepsilon_r - \varepsilon_s
\end{aligned}$$

We can write the probability that an item is the best and the probability that the item is the worst.

$$\begin{aligned}
B_S(x) &= Pr(B_x = \max_{z \in S} B_z) \\
W_S(y) &= Pr(W_y = \max_{z \in S} W_z)
\end{aligned}$$

Without even using the particular utility or scale of items we can derive choice probabilities. In the most general, form we suppose $b()$ and $w()$ are separate interval scales. Then the resulting probability of an item being best or worst is

$$\begin{aligned}
B_S(x) &= \frac{b(x)}{\sum_{z \in S} b(z)} \\
W_S(y) &= \frac{w(y)}{\sum_{z \in S} w(z)}
\end{aligned}$$

For any pair of items, $x, y \in S$, we can write the joint probability that x is best and y is worst. However, when considering the best and worst jointly the scales b and w are not separately identified. So we fix their ratio for the same item by setting $w(z) = \frac{c}{b(z)}$. The resulting joint probability is a function of the ratios of scales for pairs of items,

$$\begin{aligned}
BW_S(x, y) &= Pr(U_x > U_z > U_y | z \in S - \{x, y\}, x \neq y) \\
BW_S(x, y) &= \frac{b(x)/b(y)}{\sum_{r,s \in S, r \neq s} b(r)/b(s)}
\end{aligned}$$

To accommodate the standard utility structure, we let utility $u(z) = \log(b(z))$ Then each of the probabilities

$$\begin{aligned}
B_S(x) &= \frac{e^{u(x)}}{\sum_{z\in S} e^{u(z)}} \\
W_S(y) &= \frac{e^{u(y)}}{\sum_{z\in S} e^{u(z)}} \\
WB_S(x,y) &= \frac{e^{u(x)-u(y)}}{\sum_{r,s\in S, r\neq s} e^{u(r)-u(s)}}
\end{aligned}$$

We can derive the same representation from the random utility model Marley and Louviere (2005).

We can view best-worst choice as a generalization of the classic multinomial logit for the choice of the best only.

3.2. Estimation

From our perspective as researchers, all of the utilities are unknown. We can estimate the MaxDiff choice model in a variety of ways. One way to do this exactly is to enumerate all possible pairs of items x and y and then we describe their joint probability of being best and worst, which is the probability of being having the largest difference $BW_S(x,y)$. The pairwise approach scales quadratically in the number of items. We adopt an alternative approach, which reflects the literature and practice and is shown to be a near exact approximation Cohen et al. (2003). This allows us to estimate the best model and worst model independently, without explicitly estimating the best-worst probability. We describe the data for any individual-task combination. Let $Y_{B_S}(z)$ be the binary choice variable, which equals 1 if the item is selected as best in the set S, and 0 otherwise. Then $Y_{W_S}(z)$ is the indicator of whether item $z \in S$ is selected as the worst. The design matrix X_{B_S} (of size $|S|$-by-N) contains indicator variables taking on value of 1 for each item in the current set S and 0 otherwise. To signal the item as worst, we set $X_{W_S} = -X_{B_S}$, so X_{W_S} contains values of 0 or -1. Taken together, we express the negative likelihood of the choice data as a multinomial logit with choice probabilities in vector notation as follows,

$$-\Pi \frac{\exp\left(\begin{bmatrix} Y_B \\ Y_W \end{bmatrix} \theta\right)}{\exp\left(\begin{bmatrix} X_B \\ X_W \end{bmatrix} \theta\right)}$$

The rows in this matrix representation represent every respondent-task-item combination, $N * J * |S|$, repeated twice. The link between the models of best choice and worst choice is the parameter $\theta = \{\theta_1, \dots, \theta_m\}$. This common parameter vector represents the overall utility of each item $1, \dots, m$. For a more positive θ_i, the item i has a larger probability of being chosen as best; the more negative, the more likely the item will be chosen as worst. We clarify language about utility, which may diverge from conjoint language or language for multi-attribute profiles. Since X is an indicator, the θ only represents the utility of item i being included versus excluded. If $\theta_i > \theta_{i'}$, then we say item i is "more preferred," "more important," or simply, "better."

Due to the sparse nature of MaxDiff for huge numbers of items plus the desire for rapid real time updates, we decided to use aggregate MNL rather than a Bayesian approach. The log likelihood can be written in summation notation where S_n denotes the nth set of choices as follows

$$LL(\theta) = -\sum_{n=1}^{N} \sum_{x \in S_n} (Y_{B_{S_n}}(x) \log \frac{e^{\theta_x}}{\sum_{z \in S_n} e^{\theta_z}} + Y_{W_{S_n}}(x) \log \frac{e^{-\theta_x}}{\sum_{z \in S_n} e^{-\theta_z}})$$

In our work we find θ by minimizing the negative log likelihood using Newton-Rapson.

4. Adaptive MaxDiff

For each respondent we select L items, and we generate a MaxDiff design of J best-worst tasks, each with a choice set of $|S|$ items. We use $L = 20$, $J = 12$, $|S| = 5$ as a default since it is a standard number of items used in MaxDiff studies Wirth and Wolfrath (2012). We begin by selecting the L items uniformly. After an initial number of respondents, we are still uncertain about the each parameter value, so we continue collecting data. But we do not need to reduce that uncertainty equally for each one, so we begin adapting. To translate our current beliefs about parameters into action, we can draw from an exact or approximate posterior distribution and use an adaptive method, explained in detail in the following sections and summarized in table 1, to decide what questions to show the next respondent. In the case of the multinomial logit, we can use MCMC to obtain samples, sample from asymptotic distribution via MLE implied by the estimated mean and standard errors, or as we do in the empirical application, use Bayesian bootstrapping. Using Bayesian Bootstrap a draw u is made in the following way.

$$\beta_1, \beta_2, \dots, \beta_N \sim \exp(1)$$

$$LL(\theta;\beta) = -\sum_{n=1}^{N} \beta_n \sum_{x\in S_n} (Y_{B_{S_n}}(x) \log \frac{e^{\theta_x}}{\sum_{z\in S_n} e^{\theta_z}} + Y_{W_{S_n}}(x) \log \frac{e^{-\theta_x}}{\sum_{z\in S_n} e^{-\theta_z}})$$
$$u = \arg\min_{\theta} LL(\theta;\beta)$$

The adaptive method then use these draws to decide the next L items to show.

There are two practical issue we highlight. First, We want to avoid repeating extremely similar questions to the same respondent. A natural consequence of adaptive methods is convergence: as the sample size grows, certain items achieve high preference scores with smaller standard errors. Without any additional restrictions, the same few items will eventually tend to be drawn into adjacent MaxDiff tasks for the same respondent, causing much annoyance due to the severe degree of item repetition. Although this is statistically most efficient, it would drive human respondents mad. To avoid this, after drawing a fixed number of items (e.g., 20 or 30) to show each respondent, those draws of 20 items are shown to each respondent in a balanced, near-orthogonal design, leading to a palatable low degree of repetition of items across adjacent sets. The attentive reader will notice that our approach is quite similar to Wirth's Express MaxDiff approach, except that the logic for selecting the 20 items for each respondent is adaptive leveraging information from the previous respondents-focusing the most recent respondent's efforts on discriminating among items that already have been judged likely to be the stars.

One issue this approach avoids is the respondent tiring out. Instead of trying to create a MaxDiff design with 120 items, which would lead to an unreasonable number of best-worst tasks per respondent, we use the smaller subset of 20. We also avoid repetition. A particularly annoying alternative is to draw independent samples ranking each choice task for the same respondent. This results in very repetitive choice tasks as parameter estimates.

The second is frequency of updating the data. If people are taking the survey sequentially then all the data from the previously respondents can be used to generate the surveys for the newcomer. Often this is not the case, as an alternative you can update the data once every b people and decide the questions for the next b people. We call b the batch size and in our empirical analysis we let $b = 20$.

Both of these considerations avoid is latency during a survey adn across respondents. Since all the questions for the next b people are set, no computation takes place before or inbetween questions for a respondant. This allows for these methods to be put into practice.

Algorithm 1 General adaptive method

```
Initialize

Collect new data from respondents
Estimate model parameters
Select next questions for respondents
Check stopping rule
```

```
given: $K, L, J, S, b$
Initialize first set of questions by sampling $L$ items uniformly
Design MaxDiff questionnaire covering $L$ items with $J$ tasks of $S$ questions each, MDDesign$(L, S, J)$, to the next batch of $b$ respondents.
Collect new data from respondents, currently with $n = b * t$ respondents
Bayesian Bootstrap sample weight replacement using weights $(\alpha_1, ...., \alpha_n) \sim \text{expon}(1)$ obtaining some subset of $n$ previous respondents.
Estimate model parameters. $\theta_t$. Obtain estimated utilities $u = (u_1, ..., u_K)$.
Choose $L$ items using draws from the empirical posterior
Act Select next questions for respondents
```

In summary, given a adaptive method we serve up questions in the following way $S = \#\{\mathcal{S}\}$

We briefly describe to simple benchmarks before moving on to describe the more sophisticated proposed methods.

4.0.1. Fixed Express MaxDiff . We uniformly randomly drew L of K items (without replacement) to show to each respondent. This achieves balance across items and respondents. For instance, in a problem with $L = 20$ of the $K = 120$, then each item appears $\frac{J*S}{L} = \frac{12*5}{20} = 3$ times per respondent, on average.

4.0.2. Algorithm ϵ-greedy As a baseline for the adaptive methods will we test the ϵ-greedy. Let θ be the current estimated parameters. Take the $(1-\epsilon)L$ items with the greatest θ value and choose the remaining ϵL uniformly from the remaining items. For our empirical analysis we use $\epsilon = \frac{1}{4}$ but tested others. Note that greedy alone $\epsilon = 0$ would always serve the top L items with the highest estimated average utility to the next respondents.

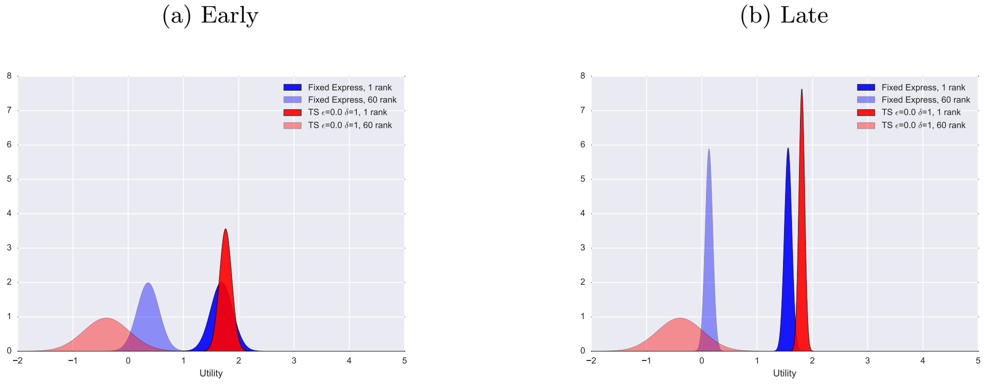
Table 1 Summary of Adaptive MaxDiff Algorithms

Algorithm	Short Description
MaxDiff TS	Sample from the current posterior distribution and serve up the L items with the highest sampled utility.
MaxDiff ϵ-Diffuse TS	Sample from the current posterior distribution serve up the $(1-\epsilon)L$ items with the highest sampled utility and sample from the current diffuse posterior distribution and serve up the ϵL items with the highest sampled utility not in $(1-\epsilon)L$.
ϵ-greedy	Take the $(1-\epsilon)L$ with the greatest current estimated θ. Take the remaining ϵL uniformily from the remaining items.
TS closest to the threshold	Take the L items that have sampled utility closest to $\frac{u_k+u_{k+1}}{2}$.
ϵ-Diffuse TS closest to the threshold	Take the $(1-\epsilon)L$ (from posterior) and ϵL (from diffuse posterior) items that have sampled utility closest to $\frac{u_k+u_{k+1}}{2}$.
ϵ-greedy closest to the threshold	Take the $(1-\epsilon)L$ items that have estimated utility closest to $\frac{\theta_k+\theta_{k+1}}{2}$ and ϵL items uniformly from the remaining.
Misclassification Minimization with random perturbation	Take the L items that have most likely been misclassified (bottom items that should be top items and vice-versa). Add perturbation to the that probability.
Greatest Uncertainty with random perturbation	Take the L items whose probabilities of being a top item are closest to 50%. Add perturbation to the that probability.

4.0.3. Algorithm MaxDiff TS While typical Bayesian or numerical integration methods call for large numbers of draws (or sufficient number) to achieve coverage of the full distribution of parameter values, this approach relies on the variability of a single sample. There is value in the sample to sample differences in the value of parameters and their relative rank ordering.

The algorithm learn over time to intuitively achieve the goal of identifying the items with truly high utility with high precision. Early on, we still have substantial uncertainty. The independent samples will differ substantially in rank order of item utilities. This yields MaxDiff designs across respondents with less overlap in items. Later, the uncertainty is reduced most around the truly high-utility items. Across independent samples, the ranking of items will be highly correlated near the top of the ranking (but not near the bottom

Figure 5 MaxDiff TS Illustration



(a) Early

(b) Late

where uncertainty remains large). As a result, the top subset of J items selected converges to the same group for each respondent.

One practical issue with Thompson Sampling is robustness to changes overtime. On the one hand, there is built-in robustness. Recall the algorithm is stochastic and adapts continuously. If the early data leads the algorithm astray, then it will self-correct, eventually finding and converging to the truly best items. Suppose the early respondents made choices, by chance, leading us to believe certain items were the best when they were not. The respondents that immediately follow will start receiving these truly poor items. However, as this continues, the uncertainty is reduced around these poor items to reveal there are many other items with probability of being better. By sampling from the joint belief distribution of item utilities, we will be less likely to draw those poor items. One concern is that such sampling could be too aggressive.

4.0.4. Variant-MaxDiff exploration-diffuse (ϵ-δ) TS Key: Distinguish Point Estimate vs. Bayesian Bootstrap. With point estimate δ is directly variation inflation factor for the estimator. But with Bayesian Bootstrap, δ is a percentage of the data, indirectly inflating the estimator's variance. Perhaps the natural parameter uncertainty is not enough or is perhaps too slow to adjust, so we propose an algorithm MaxDiff epsilon-diffuse TS, which has an extra layer of self-correction, making it more robust to non-stationarity or respondent self-selection. The epsilon comes from the popular ϵ-greedy, which mixes greedy with uniform sampling. But instead of being greedy for $1 - \epsilon$ samples, this algorithm, follows regular Thompson Sampling. And on ϵ samples, it does not necessarily sample items uniformly, but it does explore more than regular Thompson

Sampling, by drawing parameters a more diffuse posterior distribution, where the variance inflation is controlled by δ.

The ϵ-δ-**diffuse TS** is a generalized version of **TS** , which directs nests regular **TS** . This is analogous to the way a two-component mixture model nests a pooled model without segments. As the $\epsilon \to 0$, the diffuse distribution is never used, so the algorithm collapses to regular **TS** . As $\delta \to 1$, the diffuse distribution becomes equivalent to the regular (non-diffuse) distribution.

The ϵ-δ-**diffuse TS** version intuitively hedges its bets on the best items . As in illustration, instead of sampling 20 items all from **TS** , we only sample 15 of 20 items using standard **TS** (hence, $L = 20$, $\epsilon = 1/4$), and the remaining 5 of 20 items are drawn using **TS** with a much more diffuse prior (subsampling δ=.25 of the data with replacement and drawing u as a Bayes bootstrap of the subsampled data).

Figure 6 illustrates how ϵ-δ-**diffuse TS** differs from **TS** .

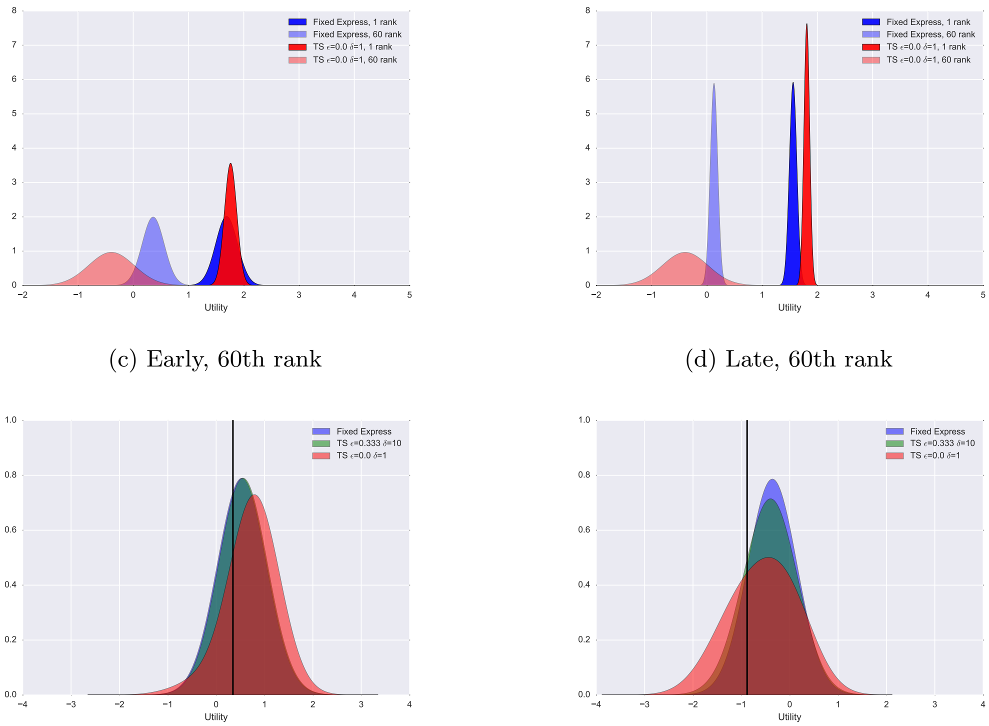
For ϵ-δ case, its density does not become a spike as quickly as it does without ϵ-δ . This speed is controlled through two parameters: epsilon, the proportion of items sampled from the diffuse distribution, and δ, factor increasing variance for the diffuse distribution. The larger the ϵ and the smaller the δ, the more exploration and slower the algorithm settles on its set of items.

We show how ϵ-δ-**diffuse TS** performance varies with different values of ϵ and δ in Section 5. For our empirical analysis, we use ($\epsilon = \frac{1}{4}$, $\delta = \frac{1}{4}$). Also tested were ($\epsilon = \frac{1}{2}$, $\delta = \frac{1}{4}$) and ($\epsilon = 1$, $\delta = \frac{1}{4}$) which performed on par with ($\epsilon = \frac{1}{4}$, $\delta = \frac{1}{4}$). Our recommended ranges for the parameters are $\frac{1}{5} \leq \epsilon \leq \frac{1}{2}$, $\frac{1}{4} \leq \delta \leq \frac{1}{2}$.

(eric) What range did we actually test? Where did we get $\epsilon = 1/5$? Did we test $\delta < 1/4$ ever?

4.0.5. Variant Closest to the Threshold In Toubia and Florès (2007), they introduce Closest to Threshold based on Bradlow and Wainers (1998) recommendation "select ideas near the cutoff". We also will do that to find the top k. For a draw u calculate the cutoff $c = \frac{u_k + u_{k+1}}{2}$ a score $s_i = |c - u^i|$ for every item. Then take the L items with the lowest score. Similarly, we can define ϵ diffuse version here. For two draws u_R (posterior) and u^D (diffuse posterior) and calculate scores s_R and s_D. Take $(1-\epsilon)L$ items with lowest $s_R$scores and the ϵL items with the lowest s_D scores not already included. We will also test the ϵ

Figure 6 **Illustration of TS v ϵ-δ-diffuse TS for two items at two points in time. (eric) Show a 4 plots in a 2x2 in this figure. Show the express, TS , ϵ-δ-diffuse TS belief distributions and show the underlying diffuse distribution that is one component of ϵ-δ-diffuse TS . Show this for 1st ranked item (top row of plots) and 60th ranked item (bottom row) to show . Show for early beliefs after respondent 40 (left column of plots) and late beliefs after respondent 500 (right) show learning.**



(a) Early, 1st rank

(b) Late, 1st rank

(c) Early, 60th rank

(d) Late, 60th rank

- greedy version where the scores are $s_i = |c - \theta^i|$ for $c = \frac{\theta_k + \theta_{k+1}}{2}$ you take the $((1-\epsilon)L)$ lowest scores and the rest uniformly.

```
       utilities          rank ordering
item : a b c d e f   max(1)(2)(3)(4)(5)(6)min
draw1: 6 7 7 8 8 9  -->  f  d  e  c  b  a
draw2: 7 5 5 9 8 6  -->  d  e  a  f  b  c
draw3: 6 4 5 8 7 5  -->  d  e  a  c  f  b
```

4.0.6. Algorithm Misclassification Minimization Adapted from Toubia and Florès (2007), define the score, s_i for each item as follows. If the ith item is estimated to be a bottom item the score is $s_i = \mathbb{P}(\text{item i is in the top k})$. If the ith item is estimated to be in the top k then the score is $s_i = \mathbb{P}(\text{item i is not in the top k})$. You can estimate this quantity by taking a large number of draws from the posterior distribution. You take the L items with the highest score.

4.0.7. Algorithm Greatest Uncertainty You define the score, s_i for each item as follows. $s_i = |.5 - \mathbb{P}(\text{item i is in the top k})|$. You can estimate this quantity by taking a large number of draws from the posterior distribution. You take the L items with the lowest score.

4.0.8. Variant Random Perturbation From Toubia and Florès (2007), after calculating the scores for a respondent, you perturb the scores by a normal vector with mean zero and variance $c\frac{1}{n_i}$, Where n_i is the number of questions that has had item i and c is some constant, we take it to be $\frac{1}{10}$ (also tested ws $c = \frac{1}{2}$ which preformed similar). Using the pertubed scores Serve up the L items accordingly. Our recommended range for the parameter is $.01 \le c \le 1$

The reason for using a variance that decreases over time rather than a fixed variance can be found in Toubia and Florès (2007) where the author derived insight from genetic algorithms. This method avoids misclassifying an item that will not be sampled further.

Additionally if you are using a batch size greater than one, Misclassification Minimization and Greatest Uncertainty would show the same items to all of the respondents. After getting scores that are the same for the next b respondents adding the perturbation vector adds variety to the items shown. This is useful because using Bayes bootstrapping for estimating probabilities required in Misclassification Minimization and Greatest Uncertainty takes a good deal of time since you are optimizing the negative log likelihood a large number of times.

5. Empirical Analysis: Main Results

We compared our proposed set of adaptive approaches to existing adaptive and non-adaptive strategies. We use simulated choice data based on inferred preferences from an actual MaxDiff survey.

5.1. Data

The data come from from a survey conducted by a Procter & Gamble with Sawtooth Software. The study involved 981 respondents and 120 items. The subject matter and exact item text was hidden for confidentiality purposes. As is common in MaxDiff surveys, the items represented product features and benefits. The questions were came from a sparse

MaxDiff study, so this was a fixed non-adaptive design, with balance across all items and respondents.

Instead of raw choice data, they provided us with the individual-level posterior mean utilities for all respondents and items, which were obtained via Markov Chain Monte Carlo sampling for a hierarchical Bayes (HB) logit model. We call those individual-level utilities the true HB utilities. These HB utilities offered realistic patterns of preferences across the items and respondents and became our data-generating process for use in our respondent simulations. Therefore Our simulated respondents mimicked the actual respondents' preferences, on average: to answer each new MaxDiff task, we perturbed those true HB utilities by iid Gumbel distributed error.

5.2. Simulation Setup

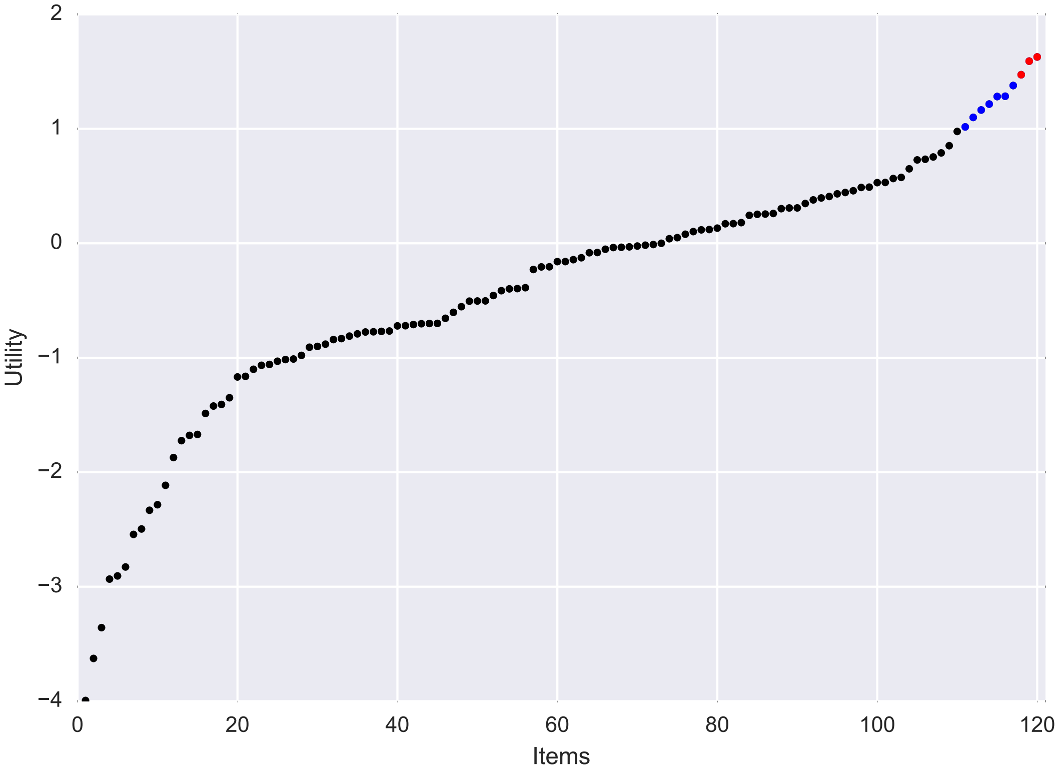
To measure performance, we consider a range of metrics: the estimated rank order of items' utilities, the hit rate of whether the estimated top-ranked set of items is correct, the value of the utilities of that estimated top set, and the variability around the estimated utilities. We obtain these measures by comparing estimates to true values. For each batch of respondents, we ran aggregate logit and compare the current rank order of the estimated utilities to the true rank order for the known true utilities, which are the aggregate average across all respondents' true individual-level utilities. We ran the simulations 100 independent runs to obtain a distribution of measures.

Since we motivated the problem with identifying a top set of items, our primary performance measure is **top k hit rate**.**Top k hit rate**: What percent of the true top k items appear in the estimated top k set? For instance, for the top-10 hit rate, if the estimated scores identified 7 of the true top 10 items, irrespective of order, the hit rate was 70%. We use $k = 3, 10, 20, 40$.

Hit rate is a natural consideration in an active learning problem. It evaluates the quality of the adaptive learning procedures with respect to the eventual decision. Hit rate is also related to regret in a typical multi-armed bandit problem. It reflects how far we are from always selecting the truly best set of arms for all time periods. Again, this is only related but not equivalent to a bandit setting, since it is not exactly an observed reward in the sense of the usual bandit setting (e.g., clicks, purchases).

The difficulty of identifying the truly best items is related to the differences among those top item utilities (Figure 7). With 120 items in the dataset, we observed the true

Figure 7 True utilities of each item learn. The plot shows the rank ordering and values of the utilities on the logit scale for all 120 items, highlighting the top 10 (blue) and top 3 (red). These are the means for unobserved data-generating process.



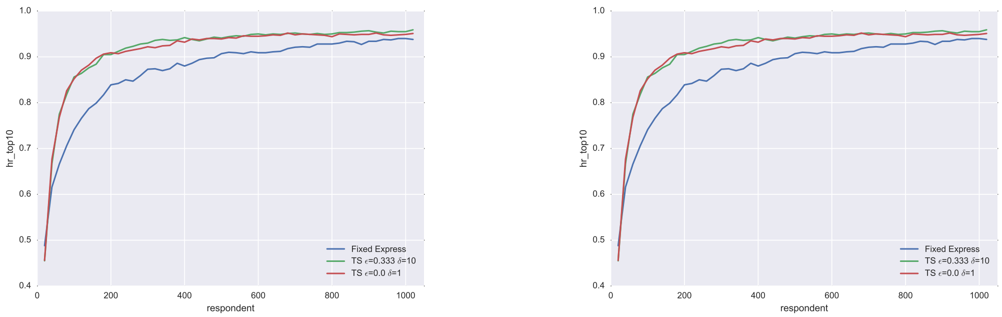
preferences for the approximately the top 15 were close in terms of utility (within 1.0 on logit scale). There were no runaway winners. Due to how tightly the top items are clustered, the hit rate measures we employed were quite discriminating between competing methods.

We describe the *base setting*. We simulate the process of collecting survey data from $N = 500$ respondents, in batches of $b = 20$ respondents per period, using bootstrap sampling, with replacement from the original 981 individuals. We consider the first 20 respondents to be the initial group, which always received $L = 20$ items uniformly selected for all methods. All adaptive methods only begin after the 20th respondent. Each simulated respondent completed $J = 12$ choice sets (best-worst tasks), where each set included $S = 5$ items.

As described earlier, we tested the different adaptive approaches found in Table 1. For our base settings, we use ($\epsilon = \frac{1}{4}$, $\delta = \frac{1}{4}$) for $\epsilon - \delta$TS methods. The natural benchmark is the existing fixed MaxDiff approach.

Figure 8 **Adaptive vs Static: TS and ϵ-δ-diffuse TS improve hit rate. (eric) Only show express, ϵ-greedy, TS , ϵ-δ-diffuse TS here for k=3, K=120, L=20. Performance is hit rate over respondents. This is a placeholder plot.**

Figure 9 **How ϵ-δ-diffuse TS varies with ϵ and δ. (eric) Show typical plot of hit rate over respondents. Show many ϵ-δ-diffuse TS versions: eps = 1/2 (left) and eps = 1/4 (right), with multiple lines in each plot, delta = 1/2 (one color) and delta = 1/4 (another color). And in each of the plots also show regular TS (eps=0,delta=1). The plots below are just placeholders.**



(a) $\epsilon = 1/2$

(b) $\epsilon = 1/4$

5.3. Results: Greedy, Thompson Sampling (TS), $\epsilon - \delta$ TS vs Fixed Express

The first result shows adaptive methods, even simple ones, are better than static ones. But a simple greedy algorithm, which adapts, but does not explicitly incorporate learning, is not good enough. Table 8 shows how TS improves substantially over greedy, and $\epsilon - \delta$ TS adds even more improvement in hit rate.

For example, after the first 200 respondents, the Fixed Express Design obtains a top 3 hit rate of 60% whereas the TS-based approaches achieve a top 3 hit rate of about 80%. Practically, if a marketing researcher wants a hit rate of 80%, for example, then these methods can achieve that with a smaller sample size. A good adaptive method is at least *three times more efficient* than the standard Fixed Express MaxDiff approach. This holds for the different values tested in the basic setting.

Figure 9 shows the way the ϵ and δ affect the performance.

We will later show the performance gap between **TS** and ϵ-δ-**diffuse TS** widens dramatically when considering non-stationary settings, such as with a misinformed starts in Section 6.

Table 2 **Top k Hit Rate for Various Algorithms after the 260th, 500th Respondent with 120 Items (eric) Check the values to make sure they correspond with the Figures. I'm looking at TSe4 and TSthres columns and see discrepancies with the figures. (eric) And for tables, reorder columns: fixed_express, greedy, greedy_thres, TS, TSed, TS_thres, TSed**

(a) After 260 respondents

k	fixed_express	greedy	greedythres	mismin	TS	TSe4	TSregthres	TSthres	uncert
3	0.650	0.853	0.860	0.883	0.863	0.873	0.850	0.893	0.840
10	0.825	0.880	0.902	0.907	0.923	0.909	0.909	0.907	0.901
20	0.824	0.784	0.863	0.885	0.825	0.820	0.869	0.879	0.886
40	0.857	NA	0.884	0.908	NA	NA	0.895	0.888	0.897

(b) After 500 respondents

k	fixed_express	greedy	greedythres	mismin	TS	TSe4	TSregthres	TSthres	uncert
3	0.780	0.950	0.933	0.950	0.940	**0.957**	0.917	0.947	0.943
10	0.877	0.925	0.928	0.939	0.933	**0.952**	0.940	0.947	0.945
20	0.862	0.827	0.914	**0.928**	0.863	0.868	0.916	0.914	0.919
40	0.887	NA	0.926	0.937	NA	NA	0.929	0.927	**0.938**

5.4. Results: Thresholding, Uncertainty reduction (max-uncert), Misclassification (max-misclass)

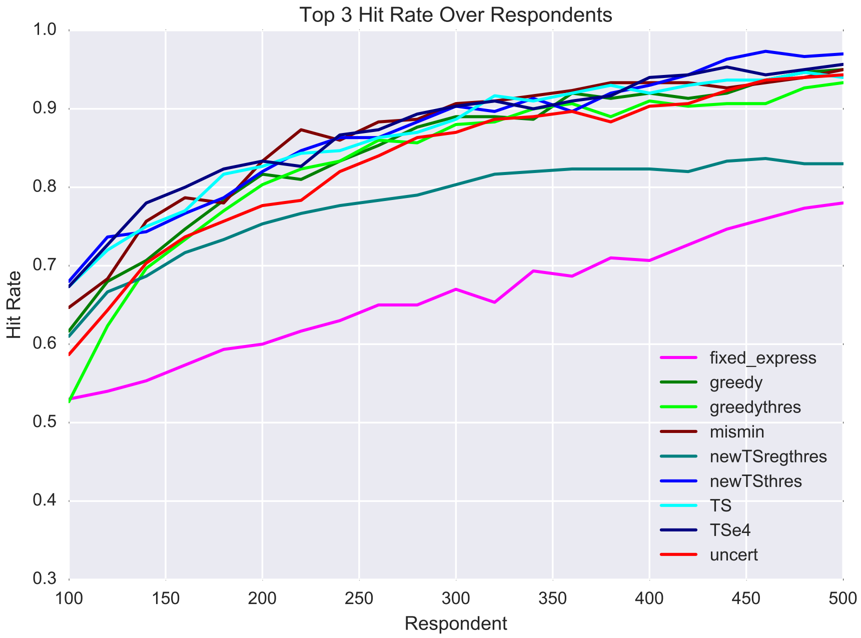
However, we next examine the other proposed methods. Recall each algorithm selects $L = 20$ items to serve to every respondent. But we examine hit rate for the top k items, for $k = \{3, 10, 20, 40\}$. Overall, a smaller k makes the problem more challenging to do well earlier, since a hit rate for the top 3 out of 120 is a higher bar than the hit rate for 40 out of 120, especially in light of the distribution of true utility values (Figure 7).

Figures 10 and 10 and Table 2 show the results for the $K = 120$-item dataset serving $L = 20$ questions per respondent, where all methods were tested.

We find performance depends on the hit rate measure relative to the number of items selected (k vs L). For $k \geq L$ the better performers are Greatest Uncertainty with random perturbations (**max-uncert**) and Misclassification Minimization with random perturbations (**max-misclass**). Indeed these two methods also perform nearly as well as the other winners in the $k < L$ condition. But the ϵ-diffuse Thompson sampling with thresholding (ϵ-δ-**TS-thres**) and without thresholding (ϵ-δ-**diffuse TS**) perform best when $k < L$.

Figure 10 **Hit Rates for Top $k = \{3, 10\}$ with 120 items. The cumulative hit rate obtained (y-axis) improves with the number of respondents interviewed (x-axis), but it does so at different rates for each algorithm. (eric) Please make y-axes the same across all k for the K120 L20 problem.**

(a) Top 3



(b) Top 10

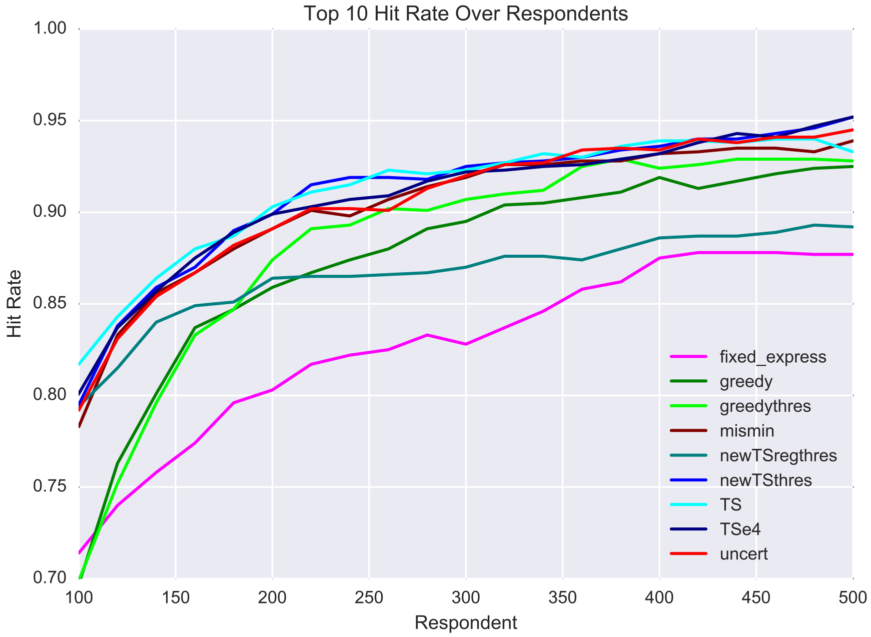
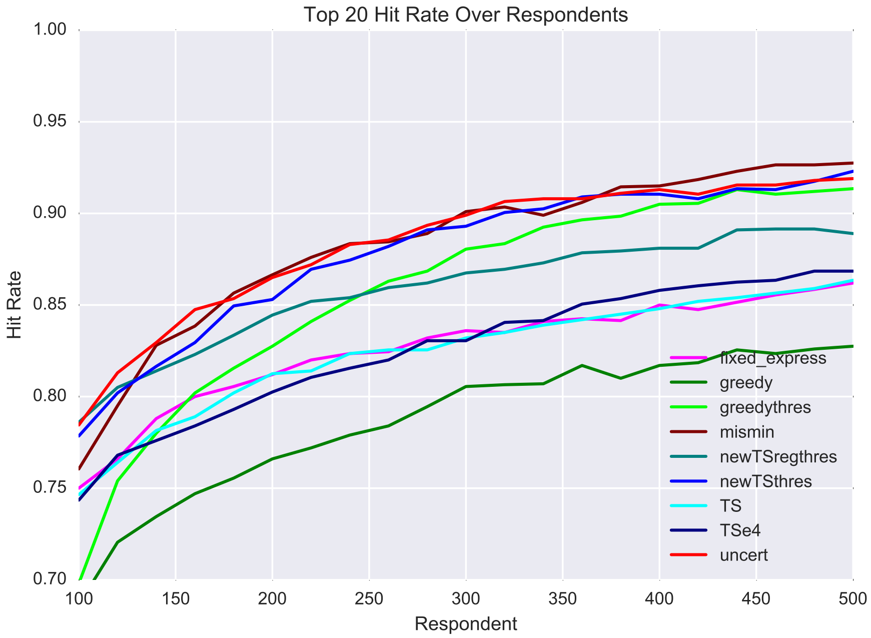
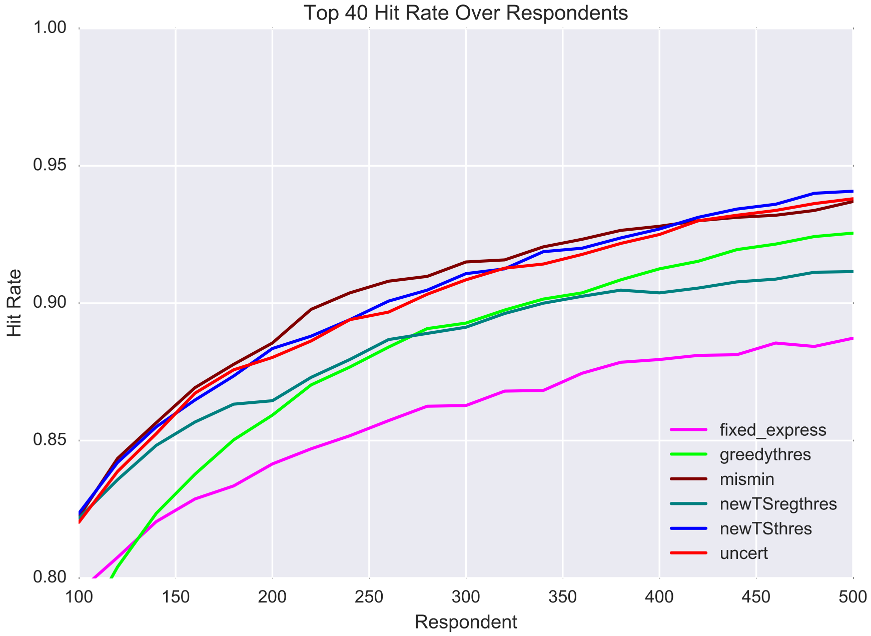


Figure 11 **Hit Rates for Top $k = \{20, 40\}$ with 120 items.(eric) Please make y-axes the same across all k for the K120 L20 problem.**

(a) Top 20



(b) Top 40



But the differences among the other algorithms are also telling. We find that Thompson Sampling (**TS**) alone has its limits. It is constructed to select the best items (and eventually, only the best item), so it can do quite well if we only care about identifying a small number of items (the cases $k = 3, 10$). But is not suited for the active learning problem generally, especially when being evaluated on identifying a larger top set, for instance, when we care about the hit rate for all items in the survey (the case $k = L = 20$). While the ϵ-δ-**diffuse TS** adds exploration to improve its performance for the other cases, that extra exploration does not direct resources toward the places that matter most in this active learning problem: the areas of high uncertainty at the decision boundary, not just near the top of the list and not scattered randomly.

Therefore, why do the threshold-based policies, ϵ-δ-**TS-thres** , **max-uncert** and **max-misclass**, outperform those without thresholds? **(eric)** Add plots highlighting these differences under a few conditions

(eric) Add a Takeaway from each section beginning here. In conclusion, **we find ...**

6. Robustness tests

6.1. Misinformed Starts (When Early Responders Are Horribly Non-Representative)

What would happen if the first 50 respondents we interviewed were actually not very representative of the average preferences for the sample? What if we tried to throw Adaptive MaxDiff off the scent? In fact, let's consider a worst-case scenario: the first responders actually believe *nearly the opposite* from the rest of the sample

For the simulations reported in table 3, the first 50 robotic respondents mimicked randomly drawn human vectors of utilities as before but were diabolically manipulated to behave as if the top 3 true items were actually nearly the worst in preference (we set the utilities for the top 3 true items for the population equal to the bottom 25th percentile utility item for each respondent). After this misinformed start, the remaining respondents represented well-behaved respondents drawn using bootstrap sampling as before, with true individual-level preferences as given in the original dataset donated by Procter & Gamble.

Our diabolical simulation of a misinformed start is worse than anything you would realistically see in practice, so it is a strong test of the robustness of the Adaptive MaxDiff

Resp	fixed_express	greedy	greedythres	mismin	TS	TSe4	TSregthres	TSthres	uncert
260	0.09	0.297	0.237	0.253	0.067	0.347	0.067	0.357	0.27
500	0.247	0.57	0.49	0.54	0.127	0.537	0.1	0.64	0.503

Table 3 Top 3 Hit Rate for Various Algorithms at the 260th and 500th Respondent for the 120 item data set with Misinformed Start

k	fixed_express	greedy	greedythres	mismin	TS	TSe4	TSregthres	TSthres	uncert
3	0.307	0.593	0.477	0.543	0.597	0.640	0.537	0.647	0.580
10	0.462	0.612	0.540	0.682	0.630	0.662	0.576	0.674	0.694
20	0.550	0.599	0.593	0.722	0.606	0.658	0.590	0.714	0.719
40	0.617	NA	0.616	0.705	NA	NA	0.638	0.702	0.721

Table 4 Top k Hit Rate for Various Algorithms at the 260th Respondent for the 300 item data set

approach. This suggests robustness to non-stationarity in preference or self-selection of respondents during the sampling window. We created 50 misinforming early responders because in practice we are never guaranteed that the first responders represent a fair and representative draw from the population. In fact, depending on how rapidly we invite a panel of respondents to take the survey, the first 50 respondents may share some atypical characteristics (e.g. anxious and available to take the survey at your launch time). It would be a bad thing if our adaptive approaches performed well in simulations with well-behaved respondents, but fell apart under more realistic conditions.

We illustrate the robustness in Table 3. One critical point to note is that the standard Bandit MaxDiff TS approaches preform worst than Fixed Express MaxDiff under Misinformed Starts. The extra randomness Bandit MaxDiff ϵ-diffuse TS approach as well as in the other methods is essential in allowing them to continue investigating the value of some lesser chosen items with enough frequency among later respondents, even if the prior respondents seem to have generally rejected them.

6.2. Effect of increasing number of items

Would the benefits of Adaptive MaxDiff we observed with 120 items continue for 300 items? While we did not have a dataset of utilities from human respondents on 300 items, we did our best to generate such a data set by leveraging the 120-item data set Procter & Gamble shared with us. To generate preferences across an additional set of 180 items, we

k	fixed_express	greedy	greedythres	mismin	TS	TSe4	TSregthres	TSthres	uncert
3	0.387	0.763	0.747	0.773	0.720	0.793	0.617	0.800	0.777
10	0.578	0.759	0.718	0.794	0.702	0.784	0.626	0.792	0.792
20	0.676	0.718	0.764	0.842	0.673	0.764	0.633	0.821	0.846
40	0.717	NA	0.751	0.813	NA	NA	0.693	0.799	0.828

Table 5 Top k Hit Rate for Various Algorithms at the 500th Respondent for the 300 item data set

k	fixed_express	greedy	greedythres	mismin	TS	TSe4	TSregthres	TSthres	uncert
3	0.897	0.973	0.970	0.940	0.973	0.973	0.973	0.960	0.943
10	0.869	0.897	0.912	0.907	0.926	0.905	0.930	0.911	0.916

Table 6 Top k Hit Rate for Various Algorithms at the 260th Respondent for the 40 item data set

k	fixed_express	greedy	greedythres	mismin	TS	TSe4	TSregthres	TSthres	uncert
3	0.930	0.990	0.990	0.983	0.990	0.987	1.000	0.997	0.990
10	0.903	0.919	0.933	0.925	0.936	0.936	0.941	0.930	0.932

Table 7 Top k Hit Rate for Various Algorithms at the 500th Respondent for the 40 item data set

randomly combined pairs of existing items according to a randomly distributed weighting scheme, with additional random variation added. The result was a 300-item MaxDiff data set based on the original preferences of the 981 respondents.

The advantages seen in the 120-item results are improved upon in the results with 300 items. Tables 4 and 5 shows the well-informed start results. The adaptive approaches show substantial gains over the Fixed Express MaxDiff approach on the top-3,10,20,40 hit rate criterion.

6.3. What about a smaller set of 40 Items?

Our adaptive models have great advantage over fixed designs for very large numbers of items, but what happens if we have a more traditional “large” MaxDiff list of 40 items. Using a random 40-item subset from our original set of 120 items, we reran our simulations. By reducing the number of items to 40, the Express MaxDiff design can now show each item to every other respondent on average, which is much more than in larger item cases. Nevertheless, our results for Adaptive MaxDiff are still better than traditional MaxDiff.

k	fixed_express	greedy	greedythres	mismin	TS	TSe4	TSregthres	TSthres	uncert
10	0.212	0.232	0.220	0.260	0.236	0.228	0.244	0.220	0.244
20	0.337	0.343	0.349	0.319	0.335	0.340	0.348	0.337	0.349

Table 8 **Top k Hit Rate for Various Algorithms at the 260th Respondent for the uncorrelated data set**

k	fixed_express	greedy	greedythres	mismin	TS	TSe4	TSregthres	TSthres	uncert
10	0.248	0.324	0.308	0.344	0.318	0.268	0.332	0.296	0.320
20	0.363	0.399	0.412	0.394	0.377	0.402	0.410	0.401	0.428

Table 9 **Top k Hit Rate for Various Algorithms at the 500th Respondent for the uncorrelated data set**

k	fixed_express	greedy	greedythres	mismin	TS	TSe4	TSregthres	TSthres	uncert
10	0.772	0.878	0.884	0.888	0.870	0.892	0.882	0.892	0.878
20	0.874	0.862	0.922	0.929	0.875	0.904	0.925	0.935	0.933

Table 10 **Top k Hit Rate for Various Algorithms at the 260th Respondent for the correlated data set**

k	fixed_express	greedy	greedythres	mismin	TS	TSe4	TSregthres	TSthres	uncert
10	0.846	0.922	0.924	0.932	0.930	0.938	0.916	0.940	0.934
20	0.902	0.895	0.954	0.957	0.896	0.927	0.947	0.961	0.963

Table 11 **Top k Hit Rate for Various Algorithms at the 500th Respondent for the correlated data set**

6.4. Simulated data sets

We created two types of simulated preferences to see if these results would generalize to other data sets. The first, labeled the uncorrelated data set, each respondents preference utility for 100 items are sampled from a uniform distribution. So each respondent's preferences are uncorrelated with every other respondent (the worst possible environment). We take the top k highest mean preference scores across the respondents to be the top k true items. The results are in tables 8 and 9.

The second, labeled the correlated data set, we sample a θ_i from a uniform distribution for $i = 1, \dots, 100$. Then each respondent preference utility is sampled from a normal distribution with mean θ_i and variance 1. The rankings for each respondent are highly correlated with each other. Once again we take the top k highest mean preference scores across the respondents to be the top k true items. The results are in tables **??** and **??**.

The take away is that our previous results generalize to other data sets.

S	Percent True Utility
{1,2,3}	1.0
{1,2,4}	.973
{1,2,5}	.948
{1,3,4}	.934
{1,3,5}	.910
{1,4,5}	.882
{2,3,4}	.922
{2,3,5}	.897
{2,4,5}	.870
{3,4,5}	.831

Table 12 Percent true utility for Various S for Top 3 from the Procter & Gamble data

7. Alternative Performance Measure: Using True Utility as a Generalization of Hit-Rate

One might like to differentiate between an algorithm puts the top 9 items and the 11th item in the 10 top and one puts the top 9 items and the 40th item in the 10 top. A measure that differentiates the two is Percent True Utility (PTU).

Exponential Weighted Utility: For a set S the exponential weighted utility is

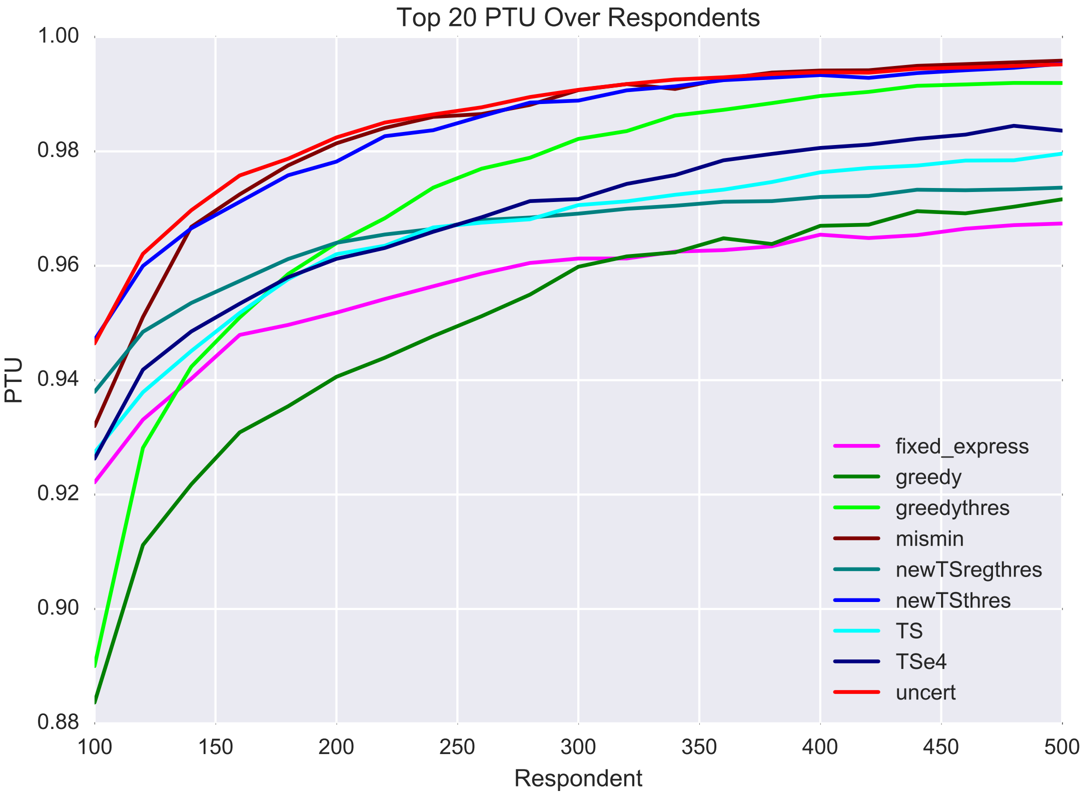
$$\mu_S = \sum_{x \in S} b(x) = \sum_{x \in S} e^{u(x)}$$

Percent True Utility of the Top k: The Percent True Utility (PTU) is the exponential weighted true utility of the top k items that the robotic respondents identified over the maximum exponential weighted true utility that could be attained with k items. Mathematically, if $\tilde{S}$ is the top k items and S is the current estimate of the k top items then PTU is

$$\frac{\mu_S}{\mu_{\tilde{S}}}$$

This can be see as a generalization of Hit-Rate. It has the advantage of differentiating as in the case the section started with though one loses the ability to easy interpret the result. In Table 12 we show the PTU for various subsets of the data. Recall that for $k = 20$ using the hit rate metric that Fixed Express, TS, and ϵ diffuse TS were on par with each other. Using $k = 20$ PTU, in figure 12 we see that ϵ diffuse TS has a slight edge over TS which is

Figure 12 Percent True Utility of the Top 20 with 120 items



better than Fixed Express. This means that both TS approaches put better ranked items in the top 20 then Fixed Express. In this wider consideration the methods are better than Fixed Express even though the top 20 hit rate was about the same.

8. Stopping Rules: Posterior Distribution Regret and Value Remaining

Almost all of the results show diminishing results as we give more surveys. Thus it becomes inefficient to keep giving surveys after a certain point. In practice we do not know this point *a priori* but we would still like to know when we should stop.

Adapted from Scott (2015) and Scott (2010) for MAB, The value remaining in the experiment is the posterior distribution of $\frac{\mu_{S^*}-\mu_S}{\mu_S}$ where μ_{S^*} is the largest value of the exponential weighted utility and μ_S is the exponential weighted utility of the set that is most likely to be optimal, denoted S. This is constructed as follows, take n Bayes Bootstrap draws from the posterior. Let $\mu^m_{S^*}$ be the max exponential weighted utility of draw m and μ^m_S be the

	Current belief: rank order of items by utility							
	1st	2nd	3rd	4th	5th	6th	7th	8th
Draw 1	4.02	3.50	5.08	4.16	4.22	4.41	3.65	3.27
Draw 2	4.18	4.72	3.49	3.48	3.63	3.60	3.56	3.70
Draw 3	4.81	5.23	5.04	3.96	4.17	4.37	3.58	2.99

Table 13 Draws of the exponential utility of the items after 100 iterations

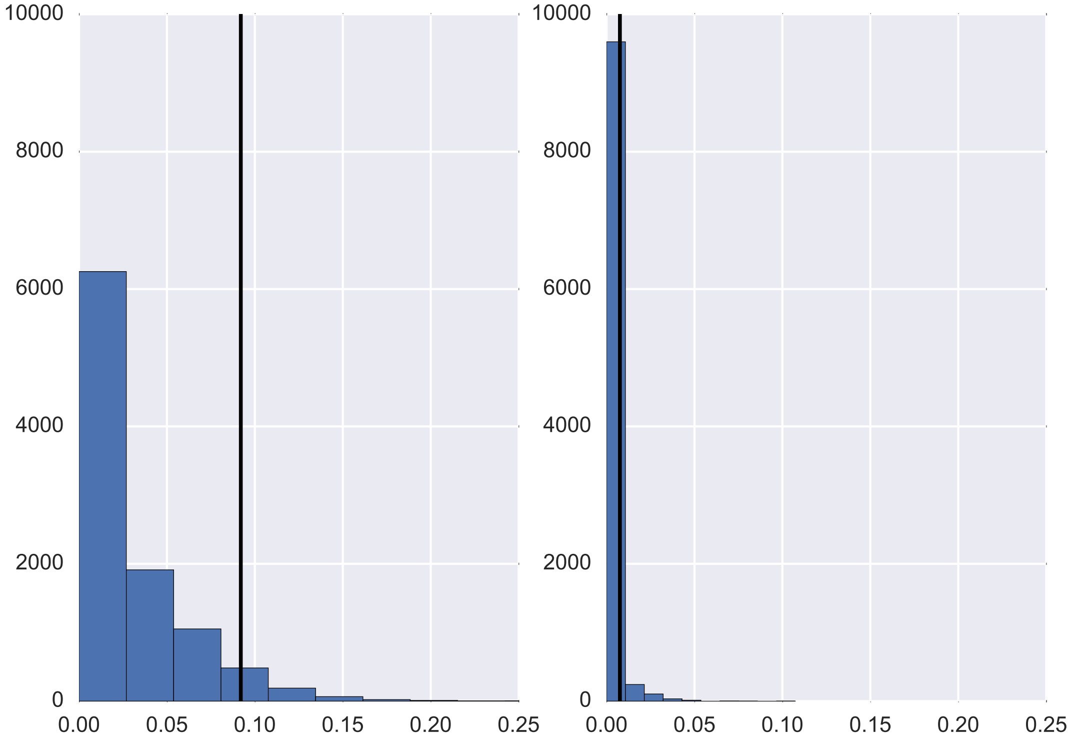
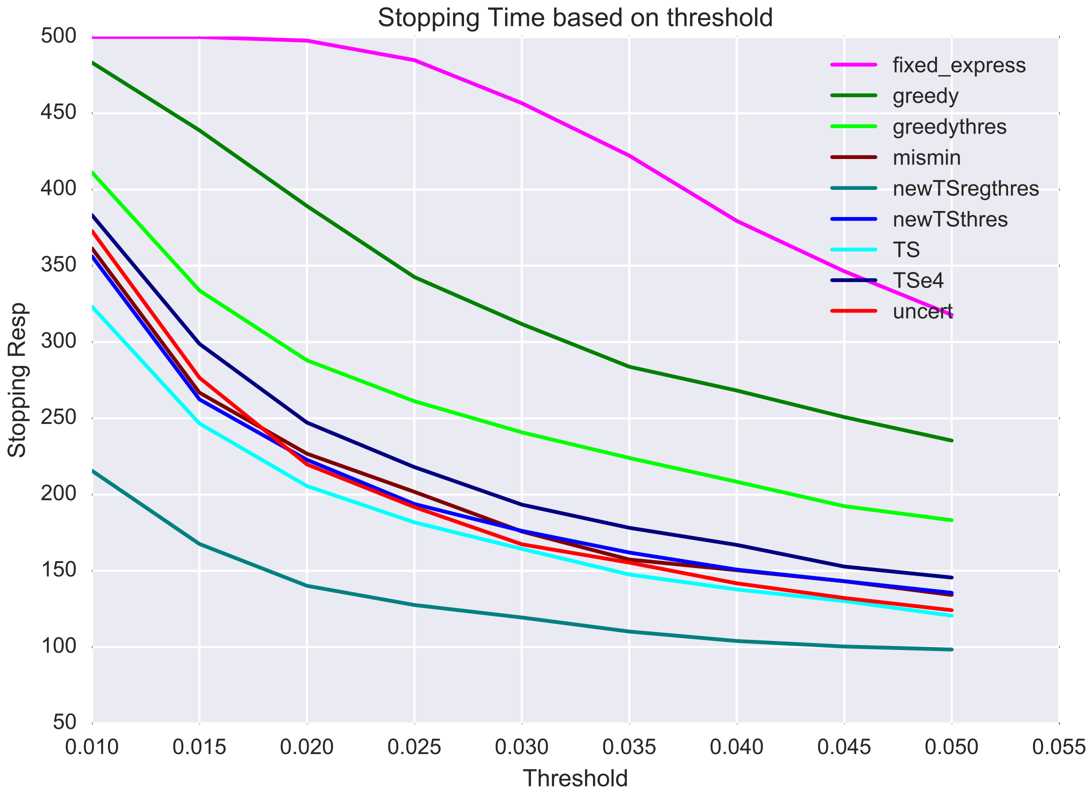


Figure 13 Two histograms of Δ. Left: After 100 iterations, the potential value remaining is .092. Right: After 220 iterations, the potential value remaining is .008

utility using the draw m using the set S. Let $\Delta^m = \frac{\mu^m_{S*} - \mu^m_S}{\mu^m_S}$.

As an example see table 13 for draws of exponential weighted utility of a single item. I put the columns in current rank order and only show the top 8 for convenience. Then S is the set containing what is currently ranked as the first, second and third ranked item. Then $\mu^1_S = 4.02 + 3.50 + 5.08 = 12.6$ and $\mu^1_{S*} = 5.08 + 4.41 + 4.16 = 13.65$ So $\Delta^1 = \frac{13.65-12.6}{12.6} = .083$. Likewise $\Delta^2 = \frac{12.6-12.39}{12.39} = .017$ and $\Delta^3 = \frac{15.08-15.08}{15.08} = 0$ (Note $\Delta^m = 0$

Figure 14 **Average stopping time with defferent thresholds**



	fixed_express	greedy	greedythres	mismin	TS	TSe4	TSregthres	TSthres	uncert
Avg ST	318	235	183	134	121	146	98	136	124
Avg hr	0.854	0.88	0.852	0.848	0.846	0.866	0.795	0.857	0.837

Table 14 **Average Stopping Time and Average Hit Rate With a threshold of .02 PVR for top 10 items**

when the S contains the top utilities). The histogram of Δ after 100 iterations and 220 iterations is shown in figure 13.

The ‘potential value remaining’ (PVR) is the 95 quantile of the distribution Δ see figure 13. After 100 iterations the PVR was 0.092. Scott’s way to interpret this number is “we do not know what the utility of S is, but whatever it is, a different set might beat it by as much as 9.2%.”

A good stopping rule is to stop when the PVR drops below a certain threshold. See figure 14 for the average stopping time for different algorithms based on threshold for finding the top 10 items. Tables 14 and 15 show the average stopping time and average hit

	fixed_express	greedy	greedythres	mismin	TS	TSe4	TSregthres	TSthres	uncert
Avg ST	498	389	288	227	206	247	140	223	220
Avg hr	0.893	0.918	0.902	0.911	0.907	0.912	0.822	0.911	0.909

Table 15 Average Stopping Time and Average Hit Rate With a threshold of .05 PVR for top 10 items

rate at those stopping time for threshold values 0.02 and 0.05. The trade-off is accuracy for how many respondents you survey. If you are using regret as a stopping rule, keep in mind that in general higher k gives smaller PVR values.

9. Other Sampling Schemes and Methods

9.1. Asking for Bests instead of Best-Worst

Because a key assumption for using an adaptive MaxDiff approach is that the researcher is mainly interested in identifying the top few items, we wondered about the value of spending time asking respondents to identify the worst item within each MaxDiff set. What would happen if we asked our robotic respondents only to select the best item within each set? The results somewhat surprised us. The value of asking respondents to indicate both best and worst within each set more than compensated for the 40% additional effort we suppose these "worst" questions add to the total interview time when interviewing human respondents. In a five item set (A,B,C,D and E) there are 10 possible 2-way comparisons. If we assume A is preferred to B and B is preferred to C and so on, then asking about only the best item will let us know A>B, A>C, A>D and A>E (4/10 comparisons). By asking about worsts as well, for only one additional question we also add B>E, C>E, and D>E (7/10 comparisons), leaving only the order relationship between B, C, and D unknown.
The case of asking for bests when all respondents have same preferences that turns into the marked-bandit problem in Simchowitz et al. (2016). In that paper the authors give different algorithms for pulling the arms and upper and lower bounds on how many queries it takes to identify the top k items with high probability.

9.2. What about Double Adaptivity?

In Orme (2006), one of the authors presented a paper on Adaptive MaxDiff that featured within-respondent adaptation rather than what we have shown here in Bandit MaxDiff based on Thompson Sampling, which is an across-respondent adaptive approach. For the within-respondent adaptive procedure, items that a respondent indicates are worst are dropped from further consideration by that same respondent through a round-robin tournament until eventually that respondent's best item is identified. We thought adding this additional layer of within-respondent adaptivity on top of the Bandit MaxDiff approach could additionally lift its performance. To our surprise, this double-adaptive approach actually performed worse than Bandit MaxDiff alone in terms of hit rates for the top 3 or 10 items for the sample. After some head-scratching (and much code checking), we determined that the lack of improvement was due to degree of heterogeneity across the robotic respondents. For example, if we are interviewing a respondent who doesn't agree much with the overall population regarding which are the top items, it is detrimental to allow that respondent to drop from further consideration (due to judging them worst) what actually are among the globally most preferred items. It serves the greater good for each respondent to spend increased effort judging among the items that previous respondents on average have judged as potentially best.

9.3. Drawing from the Asymptotic Distribution

As an alternative to Bayesian bootstrapping to get draws from the posterior, one can sample from asymptotic distribution via MLE implied by the estimated mean and standard errors, sampling $u \sim N(\theta, H^{-1})$, where θ is the minimizer of the negative loglikehood and H is the Hessian of the negative log likehood evaluated at θ. The sampling methods preformed about the same using draws from either Bayesian bootstrapping or the asymptotic distribution.

9.4. What about Sparse MaxDiff vs. Express MaxDiff?

In Wirth and Wolfrath (2012), the authors compared non-adaptive Sparse MaxDiff and Express MaxDiff at the Sawtooth Software Conference.

Fixed Sparse MaxDiff: we showed each item to each respondent an equal number of times (if possible). With 120 items, 12 sets, and 5 items per set each item appeared on average $\frac{12*5}{120} = 0.5$ times per respondent.

We compared the results using our simulation and found a modest edge in performance for Express MaxDiff (Sparse ending at 85.6% for top 10 hit rate and Express ending at 87.7%).

10. Conclusions and Future Research

Our results suggest that if your main purpose in using large item lists in MaxDiff is to identify the top items for the population (not individual-level estimates), then adaptive MaxDiff approaches can be 3x more efficient than standard Express MaxDiff designs. You are potentially wasting 66 cents of each dollar spent on data collection by not using adaptive MaxDiff.

Adaptive MaxDiff leverages information from prior respondents to show more effective trade-offs to later respondents (tending to oversample the stars). Greatest Uncertainty with random perturbations, Misclassification Minimization with random perturbations, ϵ-diffuse Thompson sampling with thresholding, and ϵ-greedy with thresholding under any choice of k. Additionally ϵ-diffuse Thompson Sampling works well when $k < L$ and ϵ-greedy works well when $k << L$. One author prefers Misclassification Minimization with random perturbations and ϵ-diffuse Thompson sampling with thresholding in all cases.

Even in the face of diabolically imposed misinformed starts (horribly unrepresentative first responders), these adaptive MaxDiff approaches are robust and self-correcting.

Although our simulations involve 120-item and 300-item tests, we expect that even greater efficiency gains (compared to standard Express MaxDiff designs) may occur with 500-item (or more) MaxDiff studies. For studies using 40 respondents, our simulation showed a 2x advantage in efficiency over fixed MaxDiff designs. Though not as dramatic, this is still a sizable boost.

Future research should test our findings using human respondents. Using an adaptive process that focuses on comparing best items may result in a more cognitively difficult task than a standard level-balanced, near-orthogonal approach. The greater expected within-set utility balance may lead to higher response error which may counteract some of the benefits of the adaptive approaches. However, based on previous research Orme (2006) that employed within-respondent adaptivity, the additional degree of difficulty that the Bandit adaptive approach could impose upon individual respondents (owing to utility balance) would probably not counteract the lion share of the benefits we've demonstrated using simulated respondents.

This paper also introduces a framework that links conjoint methods and multi-armed bandit methods. Much like adaptive conjoint methods began with aggregate adaptation and then progressed to individual-level adaptive, so we propose an aggregate adaptive approach.

But future could explore methods using fully heterogeneous models, and adapting within each individual. A partially pooled model will be useful here, as it is with other adaptive conjoint methods.

Our method relates to existing adaptive conjoint just as M-efficiency criterion is to D-efficiency. Unlike M-efficiency designs where the researcher decides the managerial weight of different factors a priori, we know which of the items should receive more weight. Instead, that is exactly what we want to learn actively.

We should note that as of this article's publication date, Sawtooth Software does not offer Adaptive MaxDiff as a commercial tool. Sawtooth Software may perhaps one day soon offer Adaptive MaxDiff as an option within its commercially available MaxDiff software. As for the authors, we look forward to this possibility as we've been especially impressed by the potential cost savings and increased accuracy!

Acknowledgments

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