ELECTROMAGNETISM, ELECTRODYNAMICS DUMP; ELECTRICITY AND MAGNETISM DUMP (INCLUDES NOTES AND SOLUTIONS TO PURCELL'S ELECTRICITY AND MAGNETISM

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Abstract. Electricity and Magnetism notes "dump" - Everything about or involving electricity and magnetism, electrodynamics.

Part 1. Maxwell's Equations; My version of Maxwell's Equations

1. My version of Maxwell's equations

1.1. **Maxwell's Equations, my version, in "vector calculus" form.** If $\nabla \cdot \mathbf{B} = 0$, then

$$\nabla \times \mathbf{E} = \frac{-1}{c}\left(\frac{\partial \mathbf{B}}{\partial t}\right) \quad (1)$$

If $\nabla \cdot \mathbf{E} = 4\pi\rho_{\text{total}}$, then

$$\nabla \times \mathbf{B} = \frac{1}{c}\left(\frac{\partial \mathbf{E}}{\partial t} + 4\pi\frac{\partial \mathbf{P}}{\partial t} + 4\pi\mathbf{J}_{\text{free}} + 4\pi c\nabla \times \mathbf{M}\right) \quad (2)$$

1.2. **Maxwell's Equations, my version, over spacetime manifold** M**.** For spacetime manifold M, of dimensions $\dim M = d+1$, and for

$$E \in \Omega^1(M)$$
$$B \in \Omega^2(M)$$

If $\mathbf{d}B = 0$, then

$$\boxed{\mathbf{d}E + \frac{\partial B}{\partial t} = 0} \quad (3)$$

If $\delta E = *\mathbf{d}*E = 4\pi\rho_{\text{total}}$,

$$\boxed{\delta B = *\mathbf{d}*B = \frac{\partial E}{\partial t} + 4\pi\frac{\partial P}{\partial t} + 4\pi J_{\text{free}} + 4\pi c\delta\mathbf{M}} \quad (4)$$

with $\mathbf{M} \in \Omega^2(M)$, magnetization in matter (i.e. matter magnetization) is *necessarily* a 2-form.

1.2.1. *Some of the algebra (scratch) work/explicit calculations, for Maxwell's Equations, my version, over spacetime manifold* M.

$$\mathbf{d}B \Longleftrightarrow \nabla \cdot B$$

since component-wise,

$$\mathbf{d}B = \frac{\partial}{\partial x^k}B_{ij}dx^k \wedge dx^i \wedge dx^j \Longleftrightarrow \nabla \cdot B$$

$$\mathbf{d}E = -\frac{\partial B}{\partial t} \Longleftrightarrow \nabla \times E \equiv \text{curl}E = \frac{-1}{c}\frac{\partial \mathbf{B}}{\partial t}$$

since, component-wise,

$$\mathbf{d}E = \frac{\partial}{\partial x^k}E_i dx^k \wedge dx^i = \frac{\partial}{\partial x^j}E_k dx^j \wedge dx^k = \frac{-\partial}{\partial t}B_{jk}dx^j \wedge dx^k$$

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For $\delta E = *\mathbf{d}*E = 4\pi\rho_{\text{total}}$, consider

$$*E = \frac{1}{(d-1)!}\sqrt{\mathbf{g}}\epsilon_{i_1 i_2 \dots i_{d-1} j_1} E_j g^{jj_1} e^{i_1}\wedge e^{i_2}\wedge \dots \wedge e^{i_{d-1}} = \frac{1}{2}\sqrt{\mathbf{g}}\epsilon_{ijk}E_{k'}g^{k'k}dx^i\wedge dx^j$$

Further,

$$\begin{aligned} \mathbf{d}*E &= \frac{1}{(d-1)!}\frac{\partial}{\partial x^k}(\sqrt{\mathbf{g}}E_j g^{jj_1})\epsilon_{i_1 i_2\dots i_{d-1}j_1}dx^k\wedge dx^{i_1}\wedge dx^{i_2}\wedge\dots\wedge dx^{i_{d-1}} = \\ &= \frac{1}{(d-1)!}\frac{\partial}{\partial x^k}(\sqrt{\mathbf{g}}E^{j_1})\epsilon_{i_1 i_2\dots i_{d-1}j_1}\epsilon^{ki_1 i_2\dots i_{d-1}}\frac{\text{vol}^d}{\sqrt{|\mathbf{g}|}} = \\ &= \frac{1}{(d-1)!}\frac{\partial}{\partial x^k}(\sqrt{|\mathbf{g}|}E^{j_1})\delta^k_{j_1}(d-1)!\frac{\text{vol}^d}{\sqrt{|\mathbf{g}|}} = \frac{1}{\sqrt{|\mathbf{g}|}}\frac{\partial}{\partial x^k}(\sqrt{|\mathbf{g}|}E^k)\text{vol}^d \end{aligned}$$

where this (generalized) Kronecker delta relation was used:

$$\frac{1}{p!}\delta^{\mu_1\dots\mu_p}_{\nu_1\dots\nu_p}\delta^{\nu_1\dots\nu_p}{}_{\rho_1\dots\rho_p} = \delta^{\mu_1\dots\mu_p}_{\rho_1\dots\rho_p}$$

where

$$\delta^{\mu_1\dots\mu_n}_{\nu_1\dots\nu_n} = \epsilon^{\mu_1\dots\mu_n}\epsilon_{\nu_1\dots\nu_n}$$

Note that

$$*1 = \text{vol}$$
$$**1 = (-1)^{0(n-0)}1 = 1 = *\text{vol}$$

and so

$$*\mathbf{d}*E = \delta E = \frac{1}{\sqrt{|\mathbf{g}|}}\frac{\partial}{\partial x^k}(\sqrt{|\mathbf{g}|}E^k)$$

Indeed, we had generalized the divergence, but on a 1-form:

$$\begin{gathered} \delta : \Omega^1(M)\to C^\infty(M) \\ \delta E = \delta(E_k dx^k) = \frac{1}{\sqrt{|\mathbf{g}|}}\frac{\partial}{\partial x^k}(\sqrt{|\mathbf{g}|}E^k) \equiv \frac{1}{\sqrt{|\mathbf{g}|}}\frac{\partial}{\partial x^k}(\sqrt{|\mathbf{g}|}g^{kk_1}E_{k_1}) \end{gathered} \tag{5}$$

2. Magnetostatics, macroscopic Magnetism, Magnetic permeability, magnetic susceptibility, field H, free currents and field H

Keywords: magnetic permeability, magnetic susceptibility

Suppose we have matter (i.e. the "macroscopic problem", referred to from Jackson (1998), Sec. 5.8 "Macroscopic Equations, Boundary Conditions on B and H", [1]), *not* a vacuum.

Atoms in matter have electrons, e^- in orbit, contributing to (rapidly) fluctuating magnetic moments $\mathbf{m}$, along with e^-'s intrinsic $\mathbf{m}$.

Consider an average macroscopic magnetization or magnetic moment density $\mathbf{M}(\mathbf{x})$ defined in a "vector calculus" manner by Jackson (1998) [1],

$$\mathbf{M}(\mathbf{x}) = \sum_I N_I\langle \mathbf{m}_I\rangle, \qquad I\equiv \text{ index of a particle}$$

Recalling Maxwell's Equations, Eq. 4,

$$\delta B = \frac{\partial E}{\partial t} + 4\pi\frac{\partial P}{\partial t} + 4\pi J_{\text{free}} + 4\pi c\delta\mathbf{M}$$

Consider a time-independent E and negligible P. Then

$$\Longrightarrow \delta B = 4\pi J_{\text{free}} + 4\pi c\delta\mathbf{M}$$

Jackson (1998) [1] considers this magnetization $\mathbf{M}$ as contributing to an *effective current density* by vector calculus arguments of it having a vector potential form, and so he proceeds to write it as (Jackson (1998), Eqn. (5.80) [1])

$$\text{curl}\mathbf{B} = \mu_0(\mathbf{J} + \text{curl}\mathbf{M}) \qquad (SI)$$

Then Jackson *defines* the macroscopic field $\mathbf{H}$, in Jackson (1998), Eqn. (5.81) [1],

$$\mathbf{H} := \frac{1}{\mu_0}\mathbf{B} - \mathbf{H}$$

However, Purcell's treatment is both more lucid, and more grounded in what B field really is physically, less relying upon artificial artifices.

2.1. Free currents $\mathbf{J}_{\text{free}}$ and the field H, magnetic susceptibility.

cf. Purcell (1984) [2], Sec. 11.10 Free Currents, and the Field $\mathbf{H}$

Keywords: $\mathbf{H}$, volume magnetic susceptibility

Bound current $\mathbf{J}_{\text{bound}}$ are current associated with molecular or atomic magnetic moments, including the intrinsic magnetic moment of particles with spin.

Free currents $\mathbf{J}_{\text{free}}$ are ordinary conduction currents.

$$\mathbf{J}_{\text{bound}} = c\nabla\times\mathbf{M} \tag{6}$$

cf. Purcell (1984), Eq. (44) of Ch. 11 [2]

At a surface, where $\mathbf{M}$ is discontinuous, we have a surface current density $\mathcal{J}$.

By superposition,

$$\nabla\times\mathbf{B} = \frac{4\pi}{c}(\mathbf{J}_{\text{bound}} + \mathbf{J}_{\text{free}}) = \frac{4\pi}{c}\mathbf{J}_{\text{total}} \tag{7}$$

cf. Purcell (1984), Eq. (50) of Ch. 11 [2]

Thus,

$$\begin{aligned} \nabla\times\mathbf{B} &= \frac{4\pi}{c}(c\nabla\times\mathbf{M}) + \frac{4\pi}{c}\mathbf{J}_{\text{free}} = \\ &= \nabla\times(\mathbf{B}-4\pi\mathbf{M}) = \frac{4\pi}{c}\mathbf{J}_{\text{free}} \end{aligned}$$

cf. Purcell (1984), Eq. (51) of Ch. 11 [2]

Purcell also defines

$$\mathbf{H} := \mathbf{B} - 4\pi\mathbf{M} \tag{8}$$

cf. Purcell (1984), Eq. (52) of Ch. 11 [2]; and so

$$\nabla\times\mathbf{H} = \frac{4\pi}{c}\mathbf{J}_{\text{free}} \quad (cgs) \qquad \nabla\times\mathbf{H} = \mathbf{J}_{\text{free}} \quad (SI)$$

cf. Purcell (1984), Eq. (53), (53'), respectively, of Ch. 11 [2].

In magnetic systems, it is precisely the free currents that we can control. So $\mathbf{H}$ is useful:

$$\int_C \mathbf{H}\cdot d\mathbf{l} = \frac{4\pi}{c}\int_S \mathbf{J}_{\text{free}}\cdot d\mathbf{a} = \frac{4\pi}{c}I_{\text{free}} \quad (cgs) \qquad \int_C\mathbf{H}\cdot d\mathbf{l} = \int_S \mathbf{J}_{\text{free}}\cdot d\mathbf{a} = I_{\text{free}} \quad (SI) \tag{9}$$

where in SI, $H\sim\frac{\text{amps}}{\text{meter}}$. cf. Purcell (1984), Eq. (54), (54'), respectively, of Ch. 11 [2].

$\mathbf{B}$ is the *fundamental magnetic field vector*; it is **only B** s.t. $\nabla\cdot\mathbf{B} = 0$ or $\mathbf{d}B = 0$

References

[1] J.D. Jackson. **Classical Electrodynamics** Third Edition. Wiley. 1998. ISBN-13: 978-0471309321
[2] Edward M. Purcell. **Electricity and Magnetism** (Berkeley Physics Course, Vol. 2) Second Edition. McGraw-Hill Science/Engineering/Math. 1984. ISBN-13: 978-0070049086