THE DIFFERENTIAL GEOMETRY DIFFERENTIAL TOPOLOGY DUMP

ERNEST YEUNG ERNESTYALUMNI@GMAIL.COM

Contents

ABSTRACT. Everything about Differential Geometry, Differential Topology

Part 1.

Theorem 1 ((Implicit Function Thm.)). *Let open subset $U \subseteq \mathbb{R}^n \times \mathbb{R}^d$, $(x,y) = (x^1 \ldots x^n, y^1 \ldots y^k)$ on U.*
Suppose smooth $\Phi : U \to \mathbb{R}^k$, $(a,b) \in U$, $c = \Phi(a,b)$

If $k \times k$ matrix $\frac{\partial \Phi^i}{\partial y^j}(a,b)$ nonsingular, then $\exists$ neighborhoods $V_0 \subseteq \mathbb{R}^n$ of a and smooth $F : V_0 \to W_0$ s.t.
$$W_0 \subseteq \mathbb{R}^k \text{ of } b$$

$\Phi^{-1}(c) \bigcap (V_0 \times W_0)$ is graph of F, i.e.
$\Phi(x,y) = c$ for $(x,y) \in V_0 \times W_0$ iff $y = F(x)$.

References

[1]