

Quality Assurance ECSA Report

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Contents

List of tables.....	ii
Table of figures.....	ii
Introduction	1
Phase 1: Descriptive statistics and Exploratory analysis	2
1.1.1 Investigating products.....	2
1.1.2 Investigating customers.....	3
1.1.3 Investigating sales from 2022 and 2023.....	4
Phase 2: Control charts and process capabilities parameters	6
2.1 Statistical Process Control (SPC) Analysis of Delivery Times	6
2.2 Results	6
Process Capability	7
Control Issues.....	7
Phase 3: Risk / Data correction and optimising for maximum profit.....	8
3.1 Risk and Data correction	8
3.1.1 Data Errors.....	8
3.1.2 Data Corrections	9
3.2 Profit Optimisation for Coffee Shops.....	12
Shop 1	12
Shop 2	13
Phase 4: Design of Experiments and System reliability.....	14
4.1 ANOVA Table	14
4.2 The Fisher's Least significant difference (LSD) test	15
4.3 Reliability of service.....	16
Expected reliable service days.....	16
Profit optimisation and staffing	16
Conclusion	17
Appendix A.....	18
References	21

List of tables

Table 1: Process Capability	7
Table 2: Control Issues.....	8
Table 3: H0 1a Standard ANOVA table	14
Table 4: H0 1b Standard ANOVA table	14
Table 5: H0 2a Standard ANOVA table	15
Table 6: H0 2b Standard ANOVA table	15
Table 7: Differences between averages.....	15
Table 8: Significant differences.....	16

Table of figures

Figure 1: Distribution of selling prices by category	2
Figure 2: Average selling and cost prices by category.....	2
Figure 3: Number of customers per city	3
Figure 4: Total quantity sold per city	3
Figure 5: Scatterplot of Age vs Income.....	4
Figure 6: Monthly quantity trend by year	4
Figure 7: Distribution of order quantity	5
Figure 8: Distribution of picking hours	5
Figure 9: Distribution of delivery hours	5
Figure 10: Corrected Average selling price and cost price by category.....	9
Figure 11: Total Profit by category	10
Figure 12: Cost vs selling price with profit % by category.....	10
Figure 13: Total quantity ordered per category	11
Figure 14: Average service time per barista (shop 1)	12
Figure 15: Reliability vs number of baristas (shop 1).....	12
Figure 16: Daily net profit per barista (shop 1)	12
Figure 17: Average service time per barista (shop 2)	13
Figure 18: Reliability vs number of baristas (shop 2).....	13
Figure 19: Daily net profit per barista (shop 2)	13
Figure 20: Net loss given number of employees.....	16
Figure 21: Cloud Subscription X-bar Chart and S-Chart.....	18
Figure 22: Keyboard X-bar Chart and S-Chart	18
Figure 23: Laptop X-bar Chart and S-Chart.....	19
Figure 24: Monitor X-bar Chart and S-Chart.....	19
Figure 26: Mouse X-bar Chart and S-Chart.....	20
Figure 25: Software X-bar Chart and S-Chart	20

Introduction

This report presents a structured quality assurance analysis of a technology retailer's operations, applying key concepts from descriptive statistics, statistical process control (SPC), risk analysis, and design of experiments (DOE). The study aims to evaluate data integrity, process stability, and operational performance, while identifying opportunities for improvement through statistical evidence.

The first phase explored products, customers, and 2022 & 2023 sales data to identify purchasing patterns, product performance, and overall sales activity. Several data quality issues were noted, raising concerns about the reliability of some initial conclusions.

Phase 2 focused on monitoring delivery performance using SPC methods and process capability analysis to assess whether the process consistently meets the Voice of the Customer (VOC) requirement.

Phase 3 introduced the concepts of Type I and Type II errors to assess the accuracy of control decisions, followed by a data correction process and a profit optimisation exercise for a coffee shop, demonstrating how service reliability and profitability can be balanced through quantitative decision-making.

Finally, Phase 4 applied Design of Experiments (DOE) and Analysis of Variance (ANOVA) to statistically evaluate demand, purchasing, and profitability variations, using Fisher's Least Significant Difference (LSD) testing to identify where specific differences occur. This phase also included an optimisation analysis for a car rental agency, using a binomial probability model to evaluate staffing reliability and maximise profit.

Together, these analyses demonstrate the practical integration of statistical and engineering tools to support data-driven decision-making that enhances quality, consistency, and operational efficiency across diverse business contexts.

Phase 1: Descriptive statistics and Exploratory analysis

1.1.1 Investigating products

The analysis of products was based solely on the *products_headoffice.csv* dataset, which contained a larger number of records compared to the *products_data.csv* dataset. Although *Products Head-office* included some quality issues, such as product IDs recorded as “NA” (e.g. NA011), it was deemed more suitable for analysis, since *Products data* had fewer entries and several mismatched categories that made product identification unreliable.

Products Head-office provided information on product categories, descriptions, selling prices, and markup percentages for six product types (cloud subscriptions, keyboards, laptops, monitors, mice, and software) each available in 60 different descriptions.

Figure 2 shows that all products have broadly similar cost and selling price levels, with average markups consistently around 20%. This indicates a standardised pricing strategy where selling prices are proportionally set above costs across product types. The uniform markup indicates the company applies a consistent approach to value addition, rather than adjusting prices based on product category or perceived market value.

The selling prices for all products display a positively skewed distribution (Figure 1), with most prices concentrated at the lower end and a few higher-priced items extending the upper tail, thereby increasing the mean. The spread of data is particularly evident in the upper tail, where products such as cloud subscriptions, keyboards, and laptops show greater variability in selling prices. This suggests that these higher-value products have more diverse pricing structures or market segments. In contrast, items like monitors, mice, and software exhibit narrower distributions with limited upper-tail spread, indicating more consistent and uniform pricing. Overall, while lower prices dominate across all categories, the variation in upper tail spread highlights differences in price dispersion, product value, and pricing dynamics across product types.



Figure 2: Average selling and cost prices by category



Figure 1: Distribution of selling prices by category

1.1.2 Investigating customers

The customer dataset covers seven major cities, with ages ranging from 16 to just over 100 and incomes spanning from 5,000 to 140,000. The average age is approximately 51 years, and the mean income is around 80,000, indicating a relatively mature and moderately high-income customer base.

The graph shown in Figure 3, shows that most cities have similar customer counts, reflecting an even distribution across locations. San Francisco stands out with around 780 customers, suggesting a slightly stronger market presence or customer engagement compared to other cities, while Miami has slightly fewer at 647, indicating a marginally smaller presence. When looking at total quantities sold, in Figure 4, Los Angeles and San Francisco lead, whereas Miami has the lowest totals, which aligns with the slightly smaller customer base in that city.

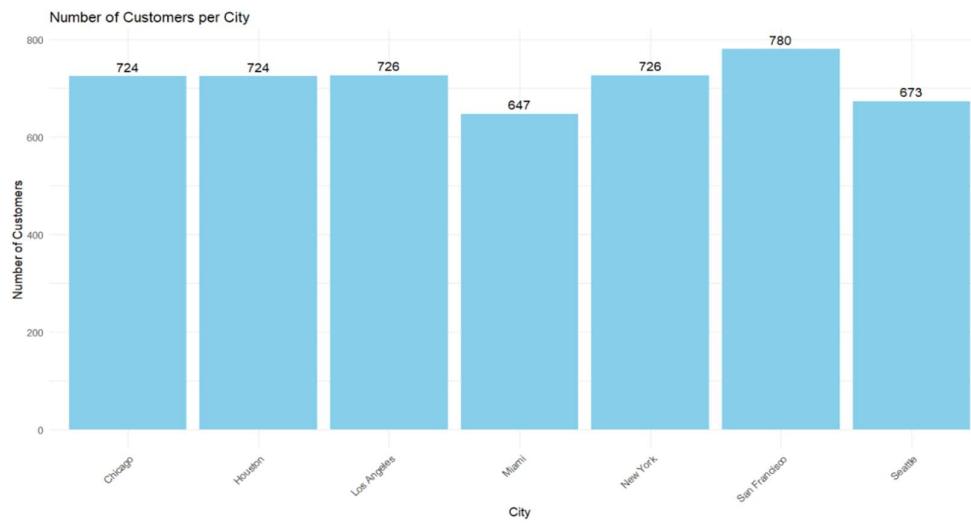


Figure 3: Number of customers per city

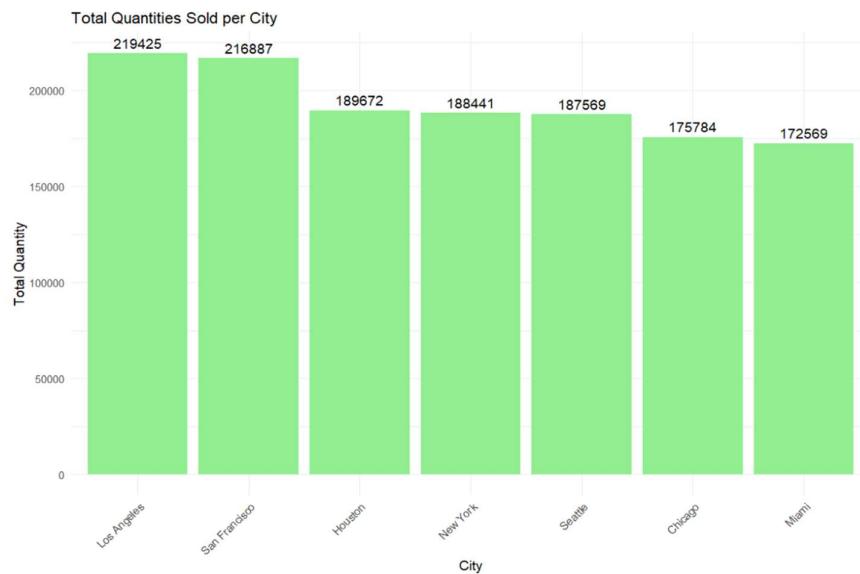


Figure 4: Total quantity sold per city

The scatter plot of customer age versus income (see Figure 5) reveals three distinct groups with differing income patterns. Young customers (ages roughly 16–30) have incomes ranging from 5,000 to 105,000. Middle-aged customers (30–65) show a wider income spread, from 5,000 to 140,000. Older customers (65 and above) have incomes between 20,000 and 120,000. These patterns suggest that income generally increases from early-career to mid-career stages, before slightly declining in retirement, reflecting typical income progression across life stages.

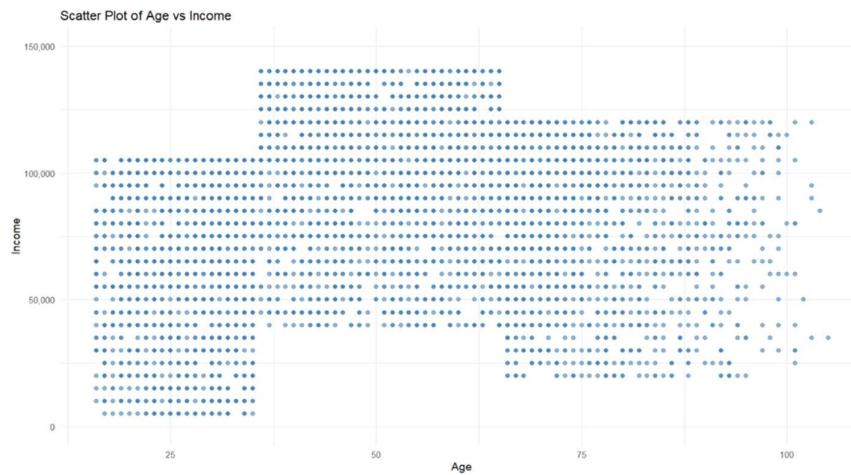


Figure 5: Scatterplot of Age vs Income

1.1.3 Investigating sales from 2022 and 2023

The sales datasets for 2022 and 2023 capture detailed information on customer purchases, including quantities and order timestamps. Picking hours ranged from 0 to 45 hours, and delivery hours from 0 to about 38 hours, with mean values of approximately 14.7 and 17.5 hours respectively. Orders were placed throughout all months and days of the year, with average order times around midday, indicating continuous sales activity across the full calendar period.

The line graph of monthly quantities sold, as in Figure 6, shows similar trends across both years, with overall quantities slightly lower in 2023. Notably, sales in January and December are considerably lower, which may reflect seasonal effects such as holiday periods, reduced customer activity, or changes in store operations during these months. Mid-year months maintain relatively high sales with some random variation, indicating consistent demand throughout the rest of the year.

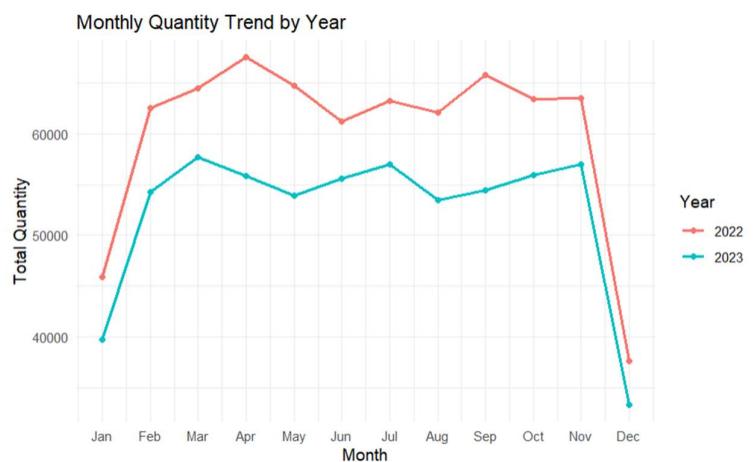


Figure 6: Monthly quantity trend by year

The histogram of quantities sold (Figure 7) is left-skewed, with most transactions concentrated at low quantities (0–10 units) and a relatively consistent, mild decline as quantities increase. This indicates that most orders are relatively small, which is typical for retail or tech stores, while larger orders are less frequent but still occur, suggesting occasional bulk purchases or business clients.

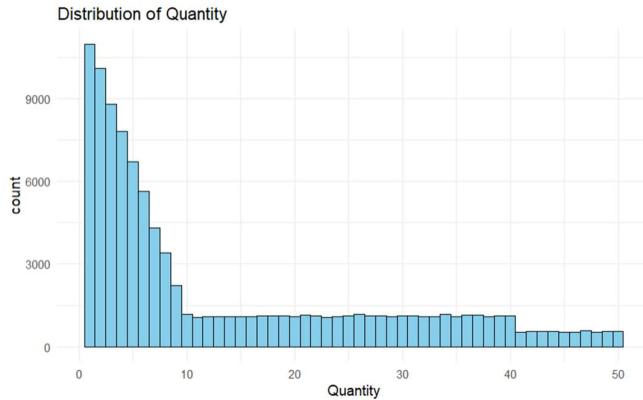


Figure 7: Distribution of order quantity

The picking hours distribution (Figure 8) shows a pronounced spike at 0–1 hour, which likely represents orders that are digital and requiring no physical picking (Cloud subscription and software). Beyond this, the distribution is bimodal: the first and dominant peak occurs between 5–30 hours, likely reflecting standard orders with moderate complexity or multiple items. The smaller second peak between 30–45 hours suggest a minority of very complex or large orders that require significantly more handling time.

Similarly, the delivery hours histogram (Figure 9) shows a steep spike at 0–1 hour yet again representing digital orders (Cloud subscription and software). After this, delivery times follow a normal distribution from 6 to 38 hours. This likely reflects the typical delivery process for standard orders, while very fast or very slow deliveries are less common.

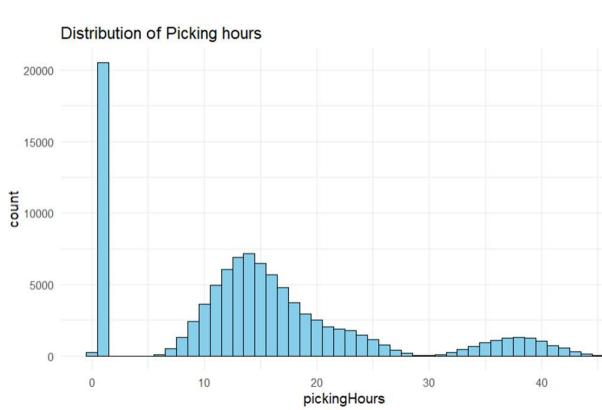


Figure 8: Distribution of picking hours

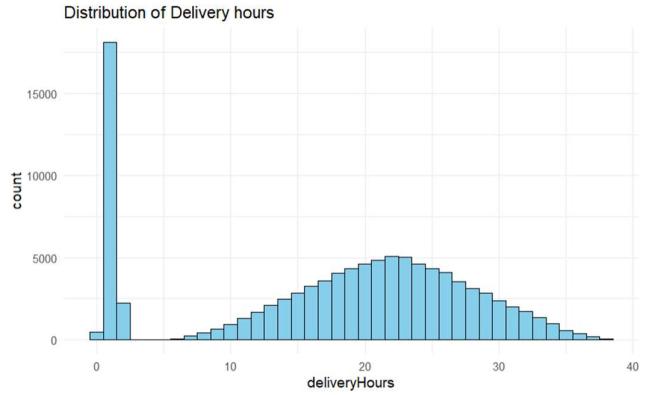


Figure 9: Distribution of delivery hours

Overall, these patterns suggest that order size and complexity directly influence operational workload, with larger or more complex orders requiring more picking time and slightly longer deliveries, while the majority of small or digital orders are processed very quickly.

Phase 2: Control charts and process capabilities parameters

2.1 Statistical Process Control (SPC) Analysis of Delivery Times

The sales data for 2026 and 2027 were analysed using Statistical Process Control (SPC), a method for monitoring process performance and identifying sources of unwanted variation. The goal of SPC is to ensure that process outputs are stable, predictable, and within acceptable limits.

In this study, delivery times for each product type were grouped into samples of 24, ordered chronologically by order year, month, day, and time. The first 30 samples (representing 720 deliveries per product type) were used to calculate control limits, including centre lines, 1σ and 2σ zones, and the traditional $\pm 3\sigma$ limits. Subsequent samples were monitored to simulate real-time process control in line with standard SPC practice.

Control charts were constructed for both the sample mean (\bar{X}) and the sample standard deviation (s). The s -charts reveal the stability of process variation, while the \bar{X} -charts highlight trends or shifts in the process mean. Process capability indices (C_p , C_{pu} , C_{pl} , and C_{pk}) were calculated for the first 1,000 deliveries per product, assuming a lower specification limit (LSL) of 0 hours and an upper specification limit (USL) of 32 hours.

2.2 Results

The SPC charts, shown in Appendix A, display similar patterns across all product types. The \bar{X} charts indicate a gradual upward trend in delivery times over the year, followed by a return to the mean at the start of the next year. This cyclical trend suggests seasonal or annual influences, such as demand fluctuations, workforce changes or logistical constraints, affecting performance.

Because control limits were calculated using only the first 720 observations, they do not account for these recurring patterns. As a result, later increases in delivery time appear as mean shifts or drifts in the \bar{X} chart, even though they represent systematic seasonal variation rather than random instability.

Despite these shifts, most data points remained within control limits, showing that the process is predictable and that variation is well-managed. The s -charts confirm that process variability remained consistent, even as mean delivery times fluctuated. Overall, the process is stable in terms of variability but periodically influenced by external factors affecting average performance.

Process Capability

Process capability indices were calculated for the first 1,000 deliveries across all product categories (Table 1). The indices C_p and C_{pk} were used to assess how well each process meets the upper specification limit (USL) of 32 hours for delivery time. C_p measures the potential capability assuming the process is centred, while C_{pk} reflects the actual capability, considering both variation and centring relative to specification limits (SixSigma.us, 2024).

The C_p values ranged between 0.888 and 0.917 for most products, indicating that the process spread is too wide relative to the customer's tolerance. The C_{pk} values ranged between 0.697 and 0.729 for most categories, which are all below the recommended benchmark of 1.33 for a capable process (SixSigma.us, 2024).

Software displays an unusually high $C_p = 18.118$ and $C_{pk} = 1.083$, suggesting minimal variation due to its smaller delivery scale. However, even this category falls short of the desired capability threshold. Overall, all product types can be classified as not capable based on $C_{pk} < 1.3$. To achieve consistent conformance, process variation must be reduced (increasing C_p) and the process re-centred (improving C_{pk}).

Product	N_used	Mean	SD	C_p	C_{pu}	C_{pl}	C_{pk}	Capable ($C_{pk} \geq 1.3$)
Cloud subscription	1000	19.206	5.928	0.900	0.719	1.080	0.719	FALSE
Keyboard	1000	19.268	5.818	0.917	0.729	1.104	0.729	FALSE
Laptop	1000	19.609	5.927	0.900	0.697	1.103	0.697	FALSE
Monitor	1000	19.405	6.004	0.888	0.699	1.077	0.699	FALSE
Mouse	1000	19.306	5.828	0.915	0.726	1.104	0.726	FALSE
Software	1000	0.956	0.294	18.118	35.153	1.083	1.083	FALSE

Table 1: Process Capability

Control Issues

Three standard SPC tests were applied to assess special-cause variation (Table 2):

Test A checks for points outside the $\pm 3\sigma$ control limits for the s-charts, which indicate extreme variation that falls outside normal process behaviour. A value of zero across all products means that none of the samples exceeded these limits, suggesting there were no severe outliers or uncontrolled shifts.

Test B measures the maximum number of consecutive samples within the $\pm 1\sigma$ zone. A high count can suggest the process is consistently producing similar values, which might indicate reduced variability or, in some cases, potential instrument sensitivity or data clustering. Among the products, cloud subscription shows the highest run length (39), which could warrant further review for possible process consistency or lack of variation.

Test C identifies instances where four consecutive averages fall beyond the $\pm 2\sigma$ limit, which can suggest a systematic trend or gradual drift in the process mean. Software and Mice recorded the highest runs (30 and 28 respectively), implying that their processes may exhibit consistent mean shifts that need to be monitored to prevent potential out-of-control conditions.

Product	Test A	Test B	Test C
Cloud subscription	0	39	16
Keyboard	0	14	25
Laptop	0	24	14
Monitor	0	18	19
Mouse	0	14	28
Software	0	15	30

Table 2: Control Issues

The SPC analysis indicates that the delivery process is statistically stable, with consistent variation and no evidence of uncontrolled behaviour. However, mean delivery times periodically drift upward due to predictable seasonal influences. While the process remains under control, it is not yet capable of meeting the VOC target ($C_{pk} \geq 1.3$). Continued improvement in process speed, standardisation, and reduction of variability is needed to achieve consistent customer satisfaction.

Phase 3: Risk / Data correction and optimising for maximum profit

3.1 Risk and Data correction

3.1.1 Data Errors

Type I (Manufacturer's) Error

A Type I error, or Manufacturer's Risk, represents the probability of incorrectly identifying a process as being out of control when it is in fact stable. (Bhandari, 2021) Under Statistical Process Control (SPC), the null hypothesis (H_0) assumes that the process remains centred on the established control chart centreline derived from the initial 30 samples. A Type I error therefore reflects a false alarm caused by normal random variation rather than an actual process shift.

In a normal distribution, the probability that any single sample falls above the centreline is 0.5. Therefore, the probability of finding seven consecutive samples above the centreline is $(0.5)^7 = 0.0078$, or approximately 0.78%. This indicates that even in a well-controlled process, occasional false signals may occur purely due to natural variation.

Accordingly, the likelihood of a Type I error across Tests A, B, and C is expected to be low but not insignificant, as each test applies criteria that can detect unusual sequences or patterns. While stricter rules enhance sensitivity to potential shifts, they simultaneously increase the risk of unnecessary interventions. Maintaining balance between responsiveness and stability is therefore essential to prevent over-adjustment and maintain process efficiency.

Type II (Consumer's) Error

A Type II error, or Consumer's Risk, occurs when a process that has shifted away from its intended target is incorrectly accepted as being in control (Bhandari, 2021). In this case, the null hypothesis (H_0) assumes that the process remains centred at the original mean of 25.05 litres with established control limits between 25.011 litres and 25.089 litres. However, the process mean has shifted to 25.028 litres, and the standard deviation has increased from 0.013 litres to 0.017 litres.

Using these parameters, the probability that a sample mean from the shifted process still falls within the original control limits can be expressed as:

$$\beta = P(LCL < \bar{X} < UCL)$$

Substituting the values yields a Type II error probability (β) of approximately 0.861, meaning there is an 86.1% chance that the control chart will fail to detect this process shift. This high probability demonstrates the risk of relying on outdated control limits once process variability changes.

In practical terms, this result highlights the importance of regularly recalculating control limits when shifts in the process mean or variation occur. Failure to do so increases the likelihood of undetected deviations, potentially allowing substandard performance to persist and affecting product quality or customer satisfaction.

3.1.2 Data Corrections

The initial analysis was based on the *products_headoffice.csv* dataset, as it contained more records than *products_data.csv*. However, management later confirmed several data quality issues, including incorrectly assigned product identifiers (e.g. NA011 instead of SOF011) and inaccurate selling prices and markups beyond the first ten entries. Verified values from *products_data.csv* were used to correct these errors, and the dataset was also updated to ensure that each product category correctly corresponded to its ID, ensuring consistency and accuracy before re-analysis.

After the corrections, pricing and markup values across all product categories became more consistent and realistic. Mean selling prices showed only minor adjustments, while Laptops and Monitors experienced the most noticeable changes, with mean selling prices increasing by approximately 2.8% and 1.4%, respectively. Markup values were standardised across categories, reducing previous inconsistencies and centring around 20%. The corrected average prices and cost prices are shown in Figure 10.



Figure 10: Corrected Average selling price and cost price by category

With the product ID and pricing issues resolved, a more reliable analysis of product sales was possible. Figure 11 shows the total profit generated by each category. Laptops and Monitors clearly lead, with total profits of 455,643,479 and 297,175,938 respectively, while Cloud Subscriptions and Keyboards perform moderately, and Mice and Software are the lowest-performing categories.

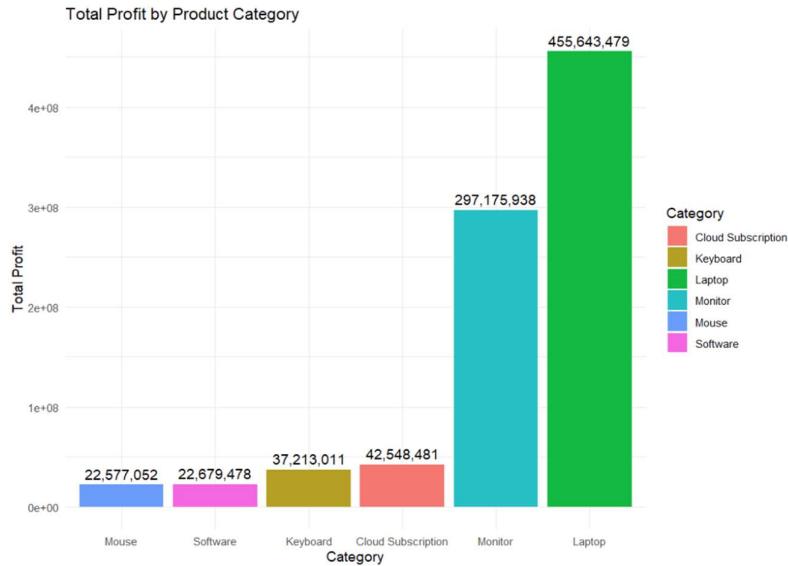


Figure 11: Total Profit by category

Figure 12 provides a detailed per-unit perspective based on products that were sold in 2022 and 2023, showing average cost, selling price, and realised profit percentages. Laptops dominate in revenue, with an average cost of roughly 15 311 and a selling price of 18 082, yielding a profit margin of 18.5%. Monitors follow with lower costs (5 109) but higher margins (24%). Keyboards, Cloud Subscriptions, and Mice show smaller revenues and moderate profits, while Software has the lowest profit percentage (16%) despite high sales.

Compared to Figure 10, which reflects the corrected prices and markups from the full product dataset, Figure 12 shows the realised profits per unit based on actual sales in 2022 and 2023. This highlights that while high-value items like laptops drive total revenue, mid-range products such as monitors can yield higher per-unit profit margins.



Figure 12: Cost vs selling price with profit % by category

Figure 13 shows total quantities purchased per product per year. Overall, all products experienced a slight decrease in sales from 2022 to 2023. Laptops were the least purchased, with only 136,721 units sold across both years, yet they generated the highest profit due to their high selling price. In contrast, products like mice and software had much higher sales volumes but lower unit prices, resulting in smaller per-unit profits.



Figure 13: Total quantity ordered per category

In summary, the data corrections improved the reliability and consistency of product information, enabling meaningful analysis of pricing, markups, and sales performance.

The establishment appears to have three product ranges: high-value items with lower sales (laptops, monitors), mid-value items with moderate sales (keyboards, cloud subscriptions), and lower-value items with higher sales (software, mice).

High-value items emerged as the top performers in terms of total profit despite lower sales volumes, while lower-value items, though sold in larger quantities, contributed less to overall profit. Mid-value items demonstrated that per-unit profit can be substantial even when total sales are moderate. Overall, the analysis highlights the importance of both unit price and sales volume in driving profitability across product categories.

3.2 Profit Optimisation for Coffee Shops

The optimisation analysis for the two coffee shops, focused on determining the optimal number of baristas needed, to balance service reliability, customer satisfaction, and profitability. Customer satisfaction was assessed by targeting the smallest average service time per order, reflecting faster service. Each customer generates a material profit of R30, while each barista incurs a daily cost of R1 000. The shops operate with a minimum of two baristas and a maximum of six.

Shop 1

Service performance is heavily dependent on the number of baristas. With only one barista, service reliability is 0% (Figure 15) and average service time is approximately 200 seconds per customer (Figure 14). Adding a second barista dramatically improves reliability to 99.7% and reduces the average service time to around 100 seconds.

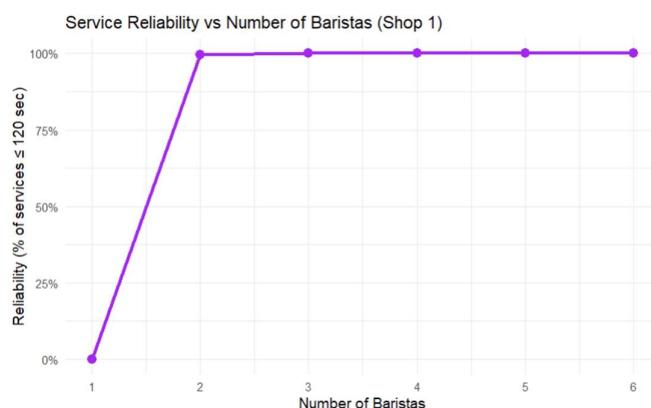


Figure 15: Reliability vs number of baristas (shop 1)

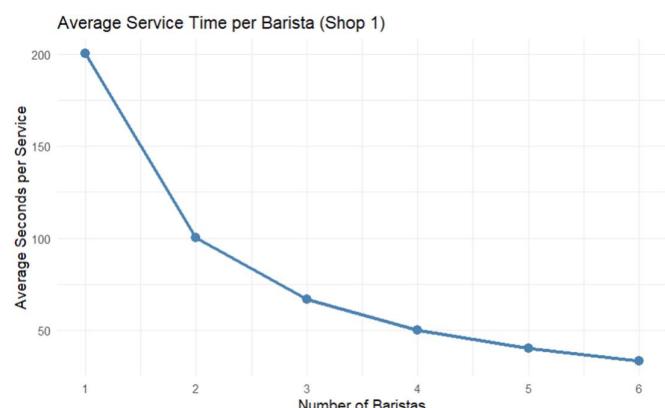


Figure 14: Average service time per barista (shop 1)

Beyond two baristas, reliability reaches 100%, but daily profit declines as the cost of additional staff outweighs the revenue gains (Figure 16).

The optimal staffing level for Shop 1 is therefore 2 baristas per day, balancing near-perfect reliability with the highest achievable daily profit of R14 438.



Figure 16: Daily net profit per barista (shop 1)

Shop 2

Shop 2 exhibits a slightly different pattern due to its service dynamics. With one or two baristas, reliability is almost zero (Figure 18), and service times remain high (Figure 17). Three baristas increase reliability to approximately 79%, with an average service time of 115 seconds, while four baristas ensure near-perfect reliability (100%) and reduce service time to roughly 100 seconds.

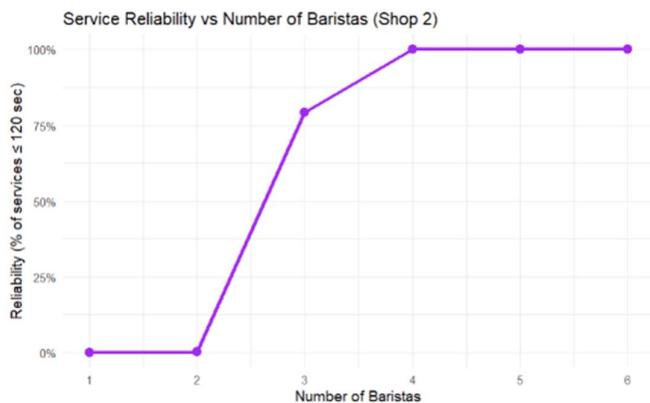


Figure 18: Reliability vs number of baristas (shop 2)

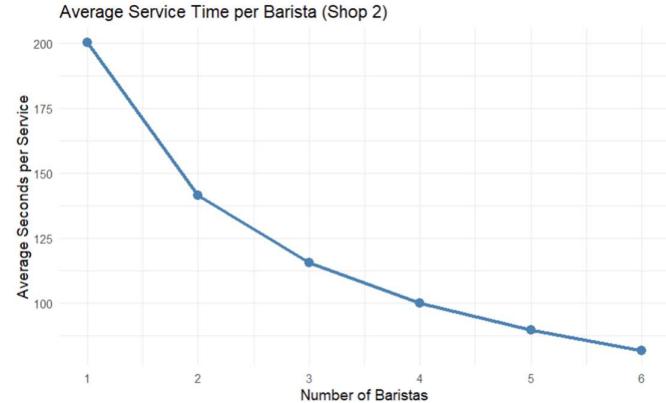


Figure 17: Average service time per barista (shop 2)

Further increases in staffing maintain perfect reliability but progressively reduce daily profit due to higher personnel costs (Figure 19).

For Shop 2, the optimal choice is 4 baristas per day, prioritising consistent service quality over maximum profit, resulting in a daily net profit of R12 438.

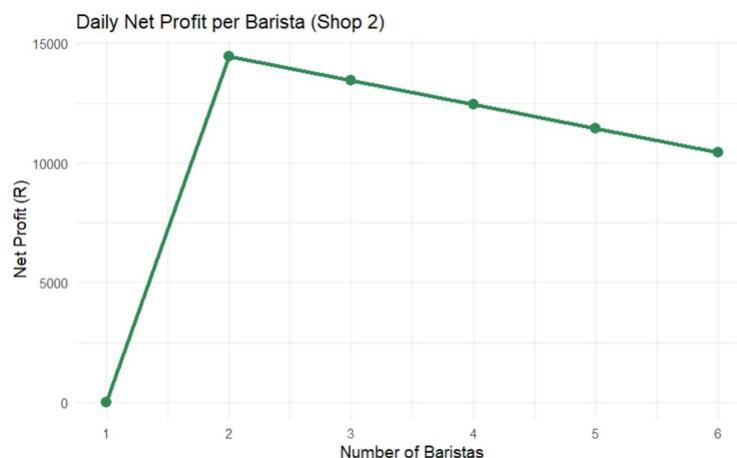


Figure 19: Daily net profit per barista (shop 2)

Overall, the optimisation analysis demonstrates that the number of baristas significantly impacts service reliability, speed, and profitability. Shop 1 achieves the best balance with 2 baristas, while Shop 2 requires 4 baristas to ensure consistent service. This highlights that optimal staffing depends on both service dynamics and customer satisfaction targets, rather than simply maximising profit.

Phase 4: Design of Experiments and System reliability

4.1 ANOVA Table

To evaluate demand fluctuations, purchasing behaviour, and profitability between 2022 and 2023, four null hypotheses were formulated and tested using ANOVA (Analysis of Variance). A 5% significance level ($\alpha = 0.05$) was used for all tests.

A custom R function was developed to perform ANOVA manually. This function demonstrated how Design of Experiments (DOE) and ANOVA operate. DOE provides a structured approach to study how changes in input factors influence output responses, helping to identify and control variation under different conditions. The ANOVA component then tests whether the means of multiple groups (treatments) are equal, assessing whether any observed differences arise from random variation or reflect true statistical differences between treatments. The function accepted a matrix as input, where rows represented treatments and columns represented observations per treatment, ensuring a balanced design. It calculated the total variation (SST), the variation between treatments (SST_r), and the error variation (SSE), along with their corresponding degrees of freedom (DoF). The F-statistic (F_0) was then computed as the ratio of mean square for treatments to mean square for error (MS_r/MS_e).

H₀ 1a: The demand between 2022 and 2023 will remain the same for product categories. The ANOVA results ($F_0 = 37.97 > F\text{-critical} = 1.048$) and ($p = 0 < 0.05$) show a statistically significant difference, leading to the rejection of H₀. This indicates that demand varied notably across product categories between 2022 and 2023.

Source	SS	Dof	MS	F	P_value
Treatment	857376493	59	14531805.0	37.97	0
Error	22961963	60	382699.4	NA	NA
Total	880338456	119	NA	NA	NA

Table 3: H₀ 1a Standard ANOVA table

H₀ 1b: The purchases between 2022 and 2023 will remain the same for re-occurring customers.

The results ($F_0 = 0.99 < F\text{-critical} = 1.048$) and ($p = 0.588 > 0.05$) are not significantly different, so H₀ cannot be rejected. This indicates that purchase behaviour among re-occurring customers remained consistent across the two years.

Source	SS	Dof	MS	F	P_value
Treatment	462752840	4998	92587.60	0.99	0.5879342
Error	465764512	4999	93171.54	NA	NA
Total	928517351	9997	NA	NA	NA

Table 4: H₀ 1b Standard ANOVA table

H₀ 2a: The profit between 2022 and 2023 will remain the same for product categories.

The results ($F_0 = 24.36 > F\text{-critical} = 1.536$) and ($p = 0 < 0.05$) are significantly different, so H₀ is rejected. This shows that profit varied significantly across product categories between 2022 and 2023.

Source	SS	DoF	MS	F	P_value
	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>
Treatment	9.834591e...	59	1.666880e...	24.36	0
Error	4.105828e...	60	6.843046e...	NA	NA
Total	1.024517e...	119	NA	NA	NA

Table 5: H0 2a Standard ANOVA table

H₀ 2b: The profit between 2022 and 2023 will remain the same for re-occurring customers. The results ($F_0 = 1.01 < F\text{-critical} = 1.536$) and ($p = 0.383 > 0.05$) are not significant, so H_0 cannot be rejected. Profit from re-occurring customers remained relatively stable over the two years.

Source	SS	DoF	MS	F	P_value
	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>
Treatment	2.135328e...	4998	42723645421	1.01	0.3825494
Error	2.117778e...	4999	42364033628	NA	NA
Total	4.253106e...	9997	NA	NA	NA

Table 6: H0 2b Standard ANOVA table

The ANOVA analysis revealed that demand and profit varied significantly across product categories between 2022 and 2023, while purchases and profit from re-occurring customers remained largely stable. This suggests that changes in sales performance are driven more by product category differences than by shifts in behaviour of existing customers. These insights can guide targeted strategies for product management and marketing.

4.2 The Fisher's Least significant difference (LSD) test

The Fisher's Least Significant Difference (LSD) test was used to determine whether product demand differed significantly across cities. While the ANOVA procedure identifies whether there are overall differences among group means, the LSD test isolates which specific pairs of cities differ when more than two treatments ($a > 2$) are present.

The null hypothesis stated that the mean demand is equal across all cities. The dataset included seven cities (Chicago, Houston, Los Angeles, Miami, New York, San Francisco, and Seattle) and six product categories per city. Before analysis, missing category-city combinations were filled with a demand value of 0 to ensure a balanced dataset.

The calculated LSD value was 9386.29 units ($\alpha = 0.025$). Differences in average demand between city pairs (Table 7) ranged from 205.17 to 7809.33 units, all of which were below the LSD threshold. This indicates that none of the observed differences were statistically significant, as seen in Table 8, which contains only 0's.

Therefore, the null hypothesis (H_0), that mean demand is consistent across all cities, cannot be rejected. These results suggest that demand levels are relatively uniform across regions, implying consistent customer purchasing behaviour and that no city demonstrates a statistically higher or lower demand compared to others.

[,1]	[,2]	[,3]	[,4]	[,5]	[,6]	[,7]	[,8]	
[1,] "0"	"2314.6667"	"7273.5"	"535.8333"	"2109.5"	"6850.5"	"1964.1667"	"Comparing 1 with 2 etc."	
[2,] "0"	"0"	"4958.8333"	"2850.5"	"205.1667"	"4535.8333"	"350.5"	"Comparing 2 with 3 etc."	
[3,] "0"	"0"	"0"	"7809.3333"	"5164"	"423"	"5309.3333"	"Comparing 3 with 4 etc."	
[4,] "0"	"0"	"0"	"0"	"2645.3333"	"7386.3333"	"2500"	"Comparing 4 with 5 etc."	
[5,] "0"	"0"	"0"	"0"	"0"	"4741"	"145.3333"	"Comparing 5 with 6 etc."	
[6,] "0"	"0"	"0"	"0"	"0"	"0"	"4886.3333"	"Comparing 6 with 7 etc."	
[7,] "0"	"0"	"0"	"0"	"0"	"0"	"0"		

Table 7: Differences between averages

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]	[,7]	[,8]
[1,]	"0"	"0"	"0"	"0"	"0"	"0"	"Comparing 1 with 2 etc."	
[2,]	"0"	"0"	"0"	"0"	"0"	"0"	"Comparing 2 with 3 etc."	
[3,]	"0"	"0"	"0"	"0"	"0"	"0"	"Comparing 3 with 4 etc."	
[4,]	"0"	"0"	"0"	"0"	"0"	"0"	"Comparing 4 with 5 etc."	
[5,]	"0"	"0"	"0"	"0"	"0"	"0"	"Comparing 5 with 6 etc."	
[6,]	"0"	"0"	"0"	"0"	"0"	"0"	"Comparing 6 with 7 etc."	
[7,]	"0"	"0"	"0"	"0"	"0"	"0"	"0"	

Table 8: Significant differences

4.3 Reliability of service

Expected reliable service days

Based on the available data on staffing levels, the probability of having at least 15 workers on duty was calculated. Using a binomial-style approach, the expected number of days with reliable service (≥ 15 workers) is approximately 336.5 days per year, while the expected number of days with insufficient staffing (< 15 workers) is approximately 28.5 days per year. This indicates that, on average, the agency experiences reliable service for most of the year, with only a few weeks of potential understaffing.

Profit optimisation and staffing

The car rental agency incurs an average daily loss of R20 000 whenever staffing levels fall below 15 employees, resulting in unreliable service. Based on historical data, the expected annual loss under current conditions (no extra staff) is approximately R570 025, corresponding to only 336 reliable service days per year.

By hiring one additional worker, reliability increases to 359 days per year, reducing expected losses from R570 025 to R110 327. After accounting for the annual salary cost of 300 000, the total expected net loss decreases to R410 327, representing a significant improvement in profitability and service consistency. Hiring two extra workers further enhances reliability to 364 days per year, leaving an expected loss of only R18 388, but total personnel costs rise to R600 000, resulting in a higher overall net loss of R618 388.

Adding three or more workers eliminates the probability of unreliable service (365 reliable days per year), but the additional salary costs (R900 000 and above) outweigh the marginal benefits, causing the total net loss to increase again.

Therefore, hiring one additional worker is the most cost-effective solution, providing the lowest total net loss of R410 327, as seen in Figure 20, and substantially improving service reliability without incurring unnecessary staffing expenses.

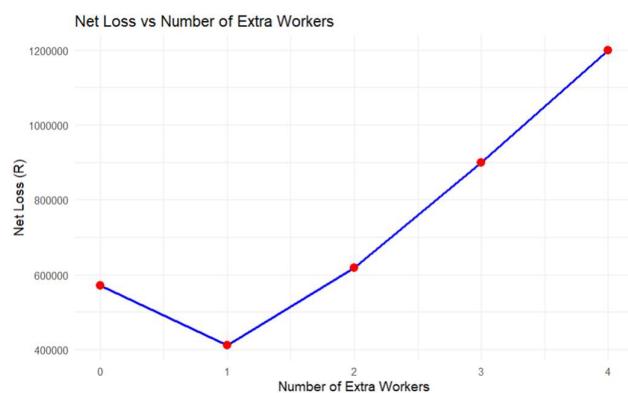


Figure 20: Net loss given number of employees

Conclusion

This report applied a structured quality assurance framework to evaluate and improve business and operational performance through data analytics and engineering-based decision tools.

Descriptive analysis in Phase 1 established key insights into pricing, customer behaviour, and sales trends, revealing that while lower-priced products dominate in quantity sold, higher-value categories such as laptops and monitors generate the highest overall profit. The findings also highlighted consistent pricing strategies and minor data inconsistencies, which were subsequently corrected to improve analytical accuracy.

Phase 2 demonstrated that delivery processes are statistically stable and well-controlled, though not yet capable of fully meeting customer expectations ($Cpk < 1.3$). This indicates that while variation is predictable, delivery performance must be improved through process re-centring and variability reduction. These findings underscore the importance of continuous process monitoring and improvement to align performance capability with customer requirements.

Phase 3 expanded on statistical reliability by quantifying the risks of false and missed process alarms (Type I and Type II errors) and applying corrective data cleaning to enhance analytical precision. The subsequent coffee shop profit optimisation analysis showed that service reliability can be maximised with minimal staff increases, illustrating how quantitative modelling supports efficient resource allocation and improved profitability.

Phase 4 extended the analysis to experimental design and hypothesis testing. ANOVA confirmed significant differences in demand and profitability across product categories, but no significant differences in purchasing behaviour or profitability among returning customers. Fisher's LSD test further showed that mean demand across cities did not differ significantly, indicating consistent customer behaviour across regions. The additional car rental optimisation study demonstrated how probabilistic modelling can balance service reliability and staffing costs, identifying the most cost-effective staffing configuration.

Overall, the findings confirm that the company operates with generally stable processes but limited capability in consistently meeting performance targets. The integration of SPC, DOE, and optimisation techniques provides a data-driven foundation for improving efficiency, reliability, and profitability. This comprehensive approach demonstrates that statistical quality assurance not only diagnoses current performance but also enables proactive, evidence-based decision-making that strengthens organisational competitiveness and customer satisfaction.

Appendix A

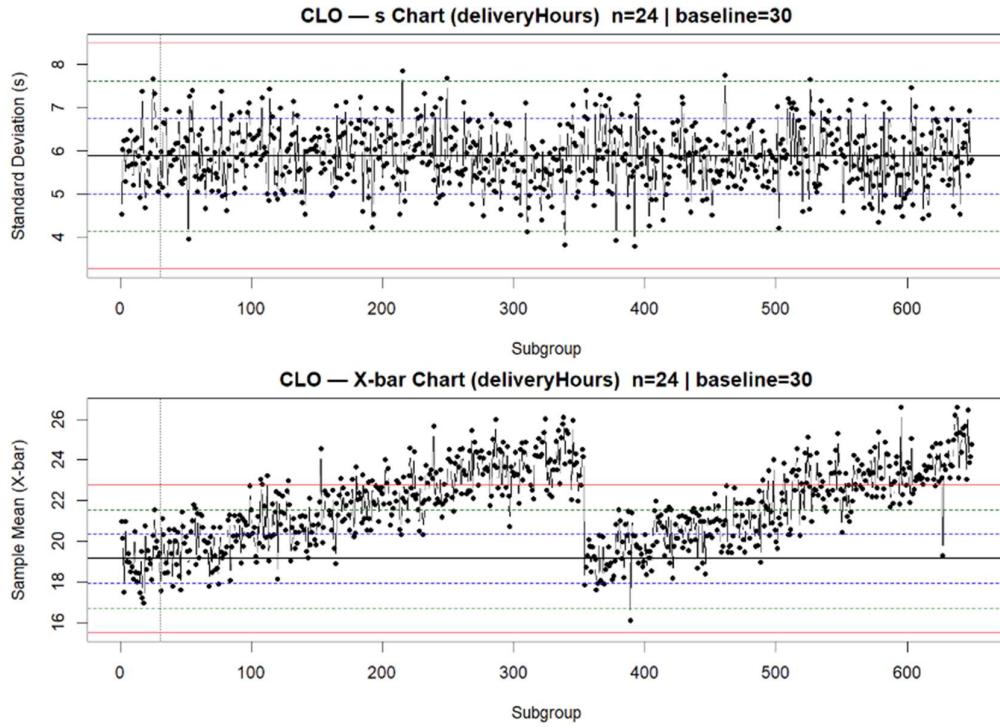


Figure 21: Cloud Subscription X-bar Chart and S-Chart

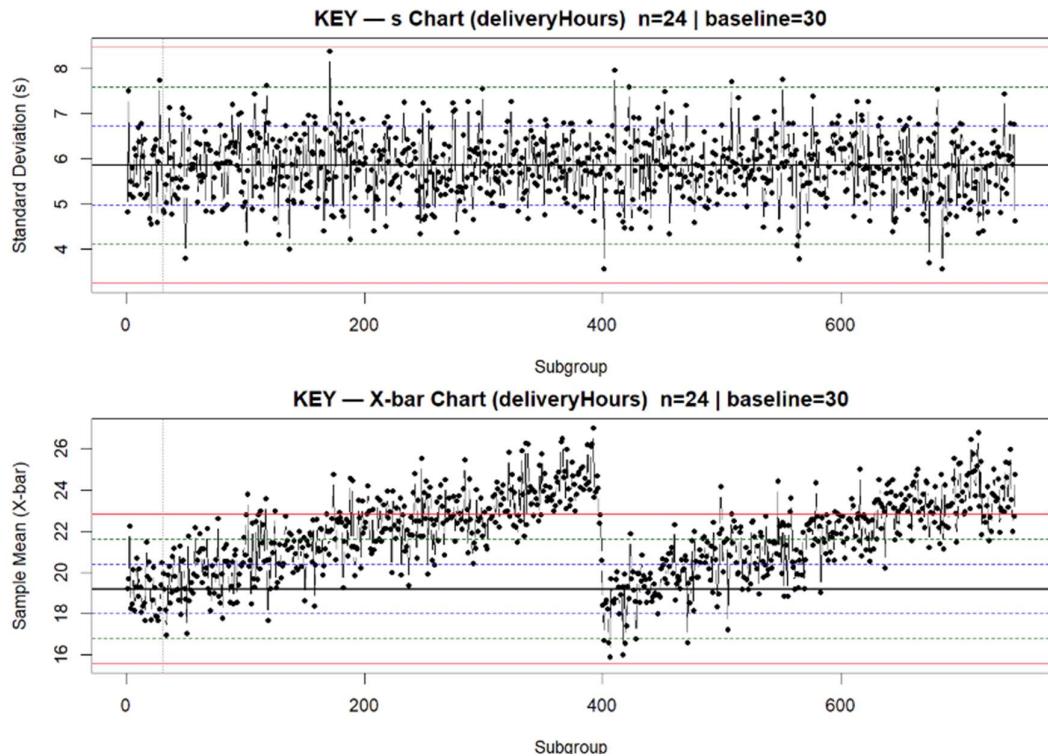


Figure 22: Keyboard X-bar Chart and S-Chart

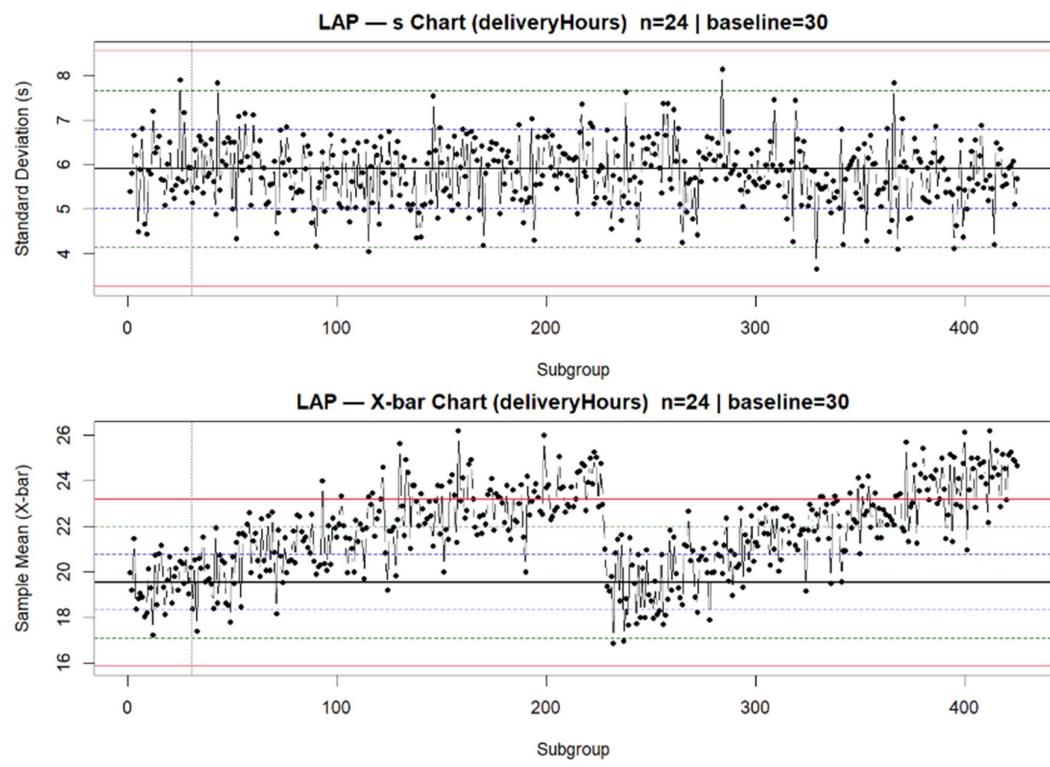


Figure 23: Laptop X-bar Chart and S-Chart

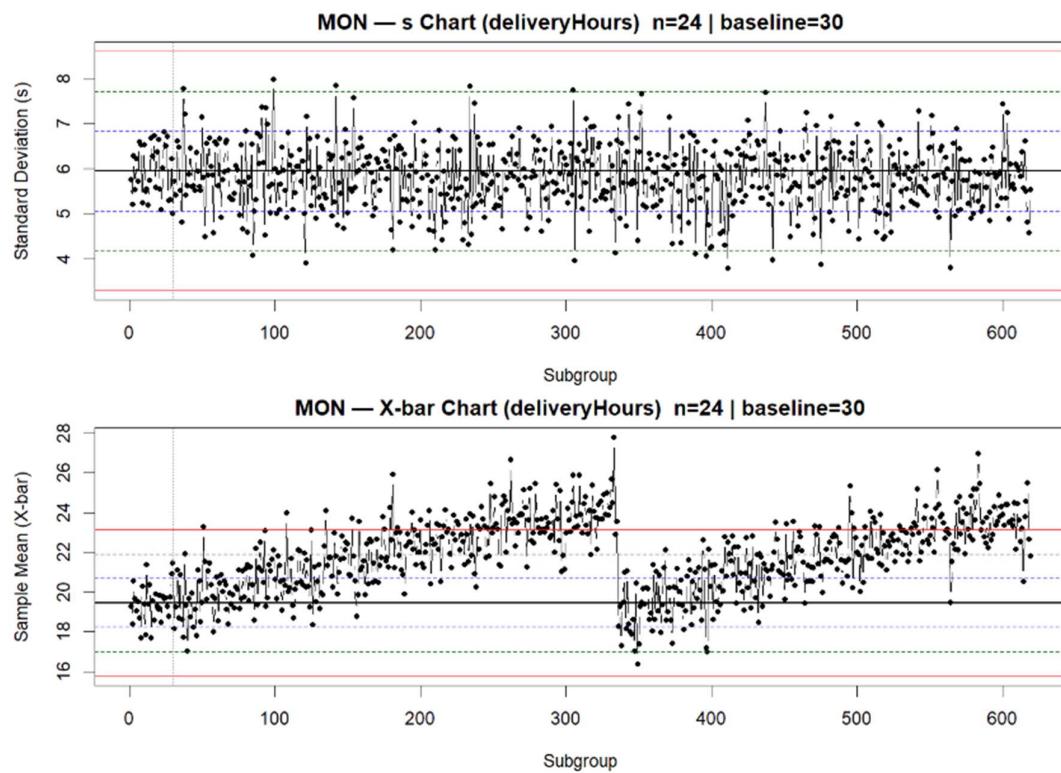


Figure 24: Monitor X-bar Chart and S-Chart

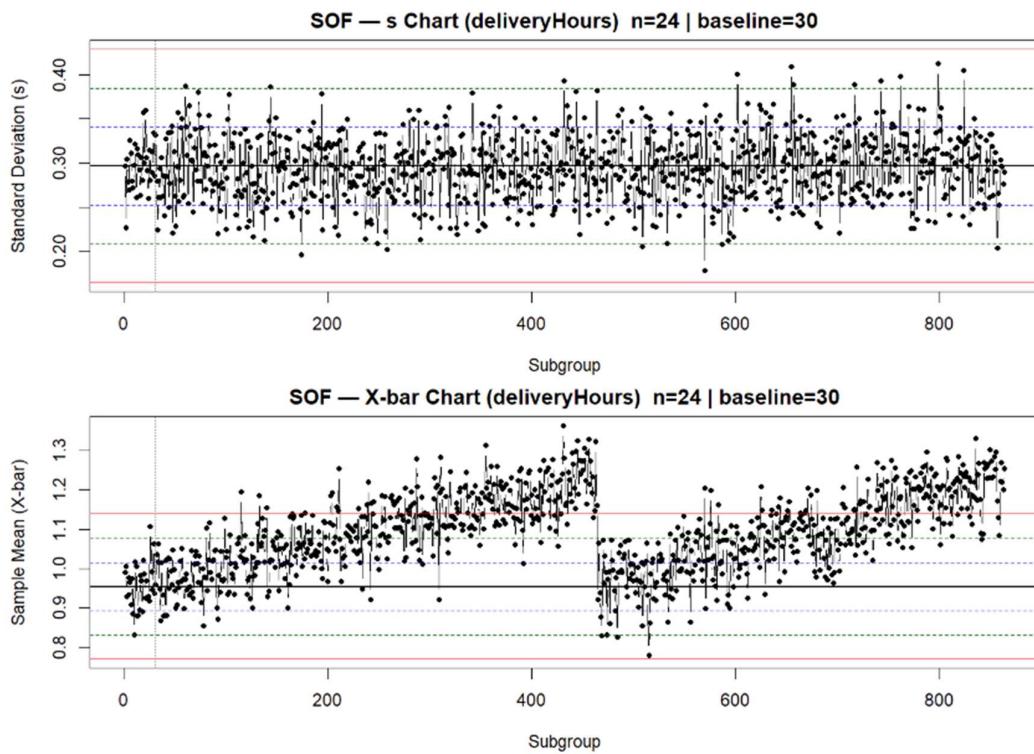


Figure 26: Software X-bar Chart and S-Chart

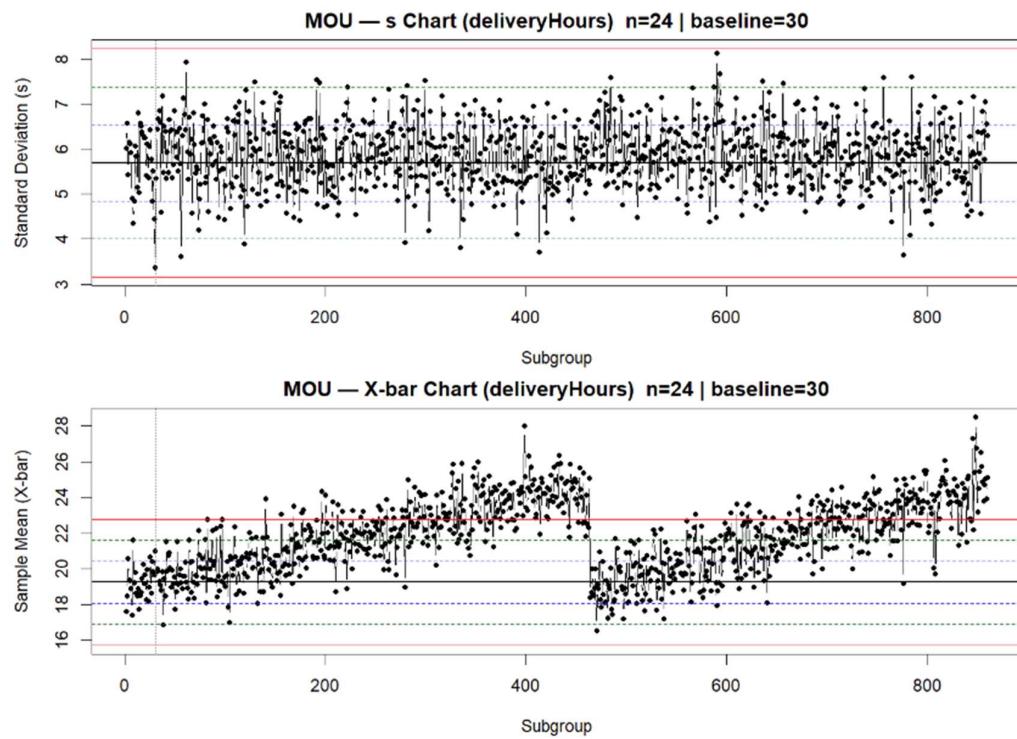


Figure 25: Mouse X-bar Chart and S-Chart

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