Chapter 3 Two –Dimensional Transformations

The orientation, size, and shape of the output primitives are accomplished with geometric transformations that alter the coordinate descriptions of objects. The basic geometric transformations are translation, rotation, and scaling. Other transformations that are often applied to objects include reflection and shear. In these all cases we consider the reference point is origin so if we have to do these transformations about any point then we have to shift these point to the origin first and then perform required operation and then again shift to that position.

Basic Transformations:

1. Translation
2. Rotation
3. Scaling
4. Reflection
5. Shearing

Homogenous Form:

We may have to perform more than one transformation in same object like scaling the object, then rotate the same object, and finally translation. For this, first co-ordinate positions are scaled, then this scaled co-ordinates are rotated and finally translated. A more efficient approach would be to combine the transformation so that final positions are obtained directly from initial co-ordinates thereby eliminating the calculation of intermediate co-ordinates. This allows us to represent all geometric transformation as matrix multiplication.

Expressing position in homogeneous coordinates allows us to represent all geometric transformations equation as matrix multiplications.

We represent each Cartesian co-ordinate position (x, y) with homogenous triple co-ordinate (x_h, y_h, h)

(x, y) -------------------- (x_h, y_h, h)

$x = x_h / h \qquad y = y_h / h$

where h is any non zero value

For convenient $h=1$

(x,y)------------------------$(x, y, 1)$.

Chapter 3 Two –Dimensional Transformations

Translation:

A translation is applied to an object by repositioning it along a straight-line path from one coordinate location to another. We translate a two-dimensional point by adding translation distances, t_x, and t_y, to the original coordinate position (x, y) to move the point to a new position (x ' , y').

Every point on the object is translated by the same amount

$$x' = x + t_x, \qquad y' = y + t_y$$

$$P = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}, \qquad p' = \begin{bmatrix} x_1' \\ y_1' \end{bmatrix}, \qquad T = \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

In homogeneous representation if position P = (x, y) is translated to new position P’= (x’, y’) then:

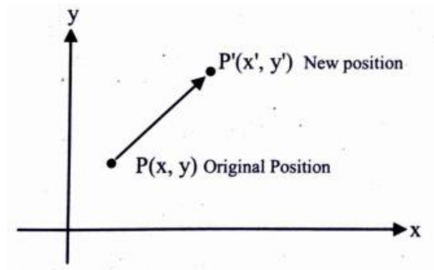
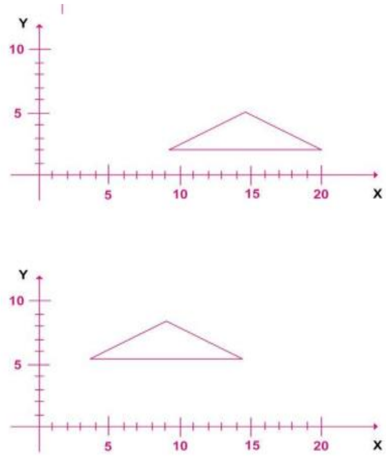


Figure 3.1 Translation of a point



$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \Rightarrow P' = T(t_x, t_y).P$$

Figure 3.2 Translation of a object

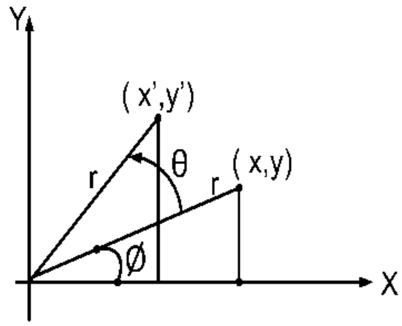
Rotation:

A two-dimensional rotation is applied to an object by repositioning it along a circular path in the xy plane. To generate a rotation, we specify a rotation angle θ and the position (x_r, y_r) of the rotation point (or pivot point) about which the object is to be rotated.

- + Value for ‘θ’ define counter-clockwise rotation about a point
- -Value for ‘θ’ defines clockwise rotation about a point

REQUIREMENT:
Rotation angle θ
&
pivot point

- Clockwise rotation (Negative)
- Anticlockwise rotation (positive)

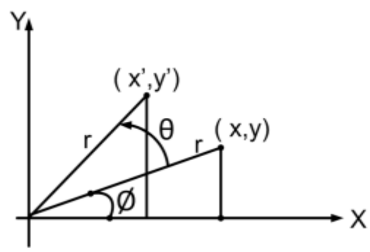


At origin

Coordinates of point (x,y) in polar form

$$x = r\cos\phi, \qquad y = r\sin\phi$$

$$x' = r\cos(\phi+\theta) = r\cos\phi.\cos\theta - r\sin\phi.\sin\theta$$
$$y' = r\sin(\phi+\theta) = r\cos\phi.\sin\theta + r\sin\phi.\cos\theta$$



$$x' = x\cos\theta - y\sin\theta$$
$$y' = x\sin\theta + y\cos\theta$$

$$P' = R.P$$

$$R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

ROTATION MATRX

If Co-ordinates represented as row vector, Then:

$$P'^{T} = (R.P)^{T} = P^{T}.R^{T}$$

In homogeneous co-ordinate

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \Rightarrow P' = R(\theta).P$$

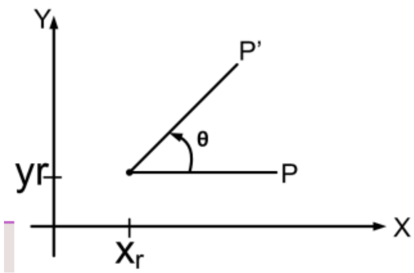
Clockwise direction:

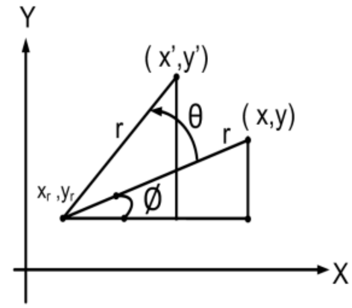
$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \Rightarrow P' = R(\theta).P$$

Rotation of a point (x, y) about any point (x_r, y_r) / Fixed Point Rotation:

Steps

1. Translate object so as to coincide pivot to origin
2. Rotate object about the origin
3. Translate object back so as to return pivot to original position





Composite Transformations

$$\begin{bmatrix} 1 & 0 & x_r \\ 0 & 1 & y_r \\ 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} 1 & 0 & -x_r \\ 0 & 1 & -y_r \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta & -\sin\theta & x_r(1-\cos\theta)+y_r\sin\theta \\ \sin\theta & \cos\theta & y_r(1-\cos\theta)-x_r\sin\theta \\ 0 & 0 & 1 \end{bmatrix}$$

$$T(x_r, y_r).R(\theta).T(-x_r, -y_r) = R(x_r, y_r, \theta)$$

$$T(-x_r, -y_r) = T^{-1}(x_r, y_r)$$

Q1. Rotate the triangle (5, 5), (7, 3), (3, 3) in counter clockwise (CCW) by 90 degree.

Answer:

$P' = R.\ P$

$$= \begin{pmatrix} \text{Cos}90 & \text{-Sin}90 & 0 \\ \text{Sin}90 & \text{Cos}90 & 0 \\ 0 & 0 & 1 \end{pmatrix} . \begin{pmatrix} 5 & 7 & 3 \\ 5 & 3 & 3 \\ 1 & 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} . \begin{pmatrix} 5 & 7 & 3 \\ 5 & 3 & 3 \\ 1 & 1 & 1 \end{pmatrix}$$

$$= \begin{bmatrix} -5 & -3 & -3 \\ 5 & 7 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

Q2. Rotate the triangle (5, 5), (7, 3), (3, 3) about fixed point (5, 4) in counter clockwise (CCW) by 90 degree.

Solution:

Here, the required steps are:

1. Translate the fixed point to origin.
2. Rotate about the origin by specified angle θ.
3. Reverse the translation as performed earlier.

Thus, the composite matrix is given by

Com = T(xf, yf) . R_θ .T(-xf, -yf)

$$= \begin{pmatrix} 1 & 0 & 5 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \text{Cos}90 & -\text{Sin}90 & 0 \\ \text{Sin}90 & \text{Cos}90 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & -5 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 5 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & -5 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 5 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & -1 & 4 \\ 1 & 0 & -5 \\ 0 & 0 & 1 \end{pmatrix}$$

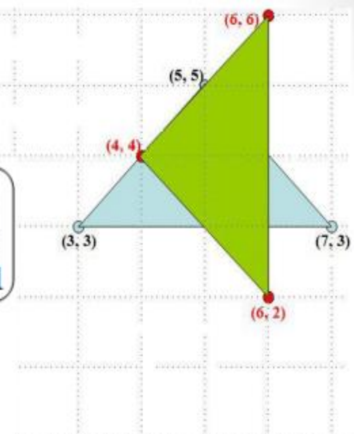
$$= \begin{pmatrix} 0 & -1 & 9 \\ 1 & 0 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

Now, the required co-ordinate can be calculated as:

P' = Com . P

$$= \begin{pmatrix} 0 & -1 & 9 \\ 1 & 0 & -1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 5 & 7 & 3 \\ 5 & 3 & 3 \\ 1 & 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & 6 & 6 \\ 4 & 6 & 2 \\ 1 & 1 & 1 \end{pmatrix}$$



Hence, the new required coordinate points are **(4, 4), (6, 6) & (6, 2).**

Scaling:

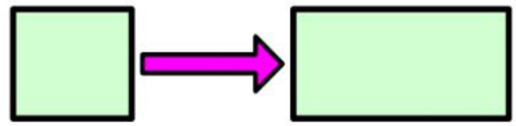
A scaling transformation alters the size of an object. This operation can be carried out for polygons by multiplying the coordinate values (x, y) of each vertex by scaling factors s_x and s_y to produce the transformed coordinates (x', y').

- s_x scales object in 'x' direction
- s_y scales object in 'y' direction

$$x' = x.s_x, \qquad y' = y.s_y$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \underbrace{\begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix}}_{\text{2 x 2 Scaling Matrix}} . \begin{bmatrix} x \\ y \end{bmatrix}$$

$$P' = S.P$$



Square to Rectangle (Sx = 2 , Sy =1)

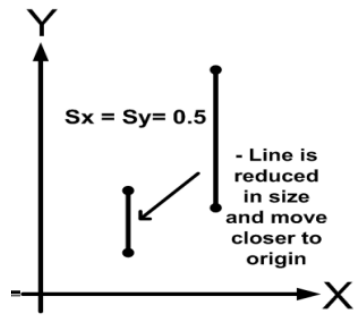
(Sx, Sy)→ Scaling factors

Sx=Sy→ Uniform Scaling

Sx≠ Sy → Differential Scaling

In homogeneous co-ordinate

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \Rightarrow P' = S(s_x, s_y).P$$

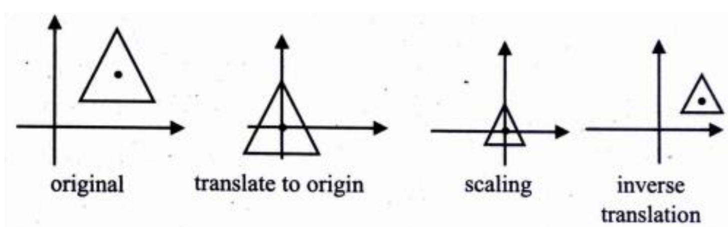


Fixed point scaling:

The location of the scaled object can be controlled by choosing a position called fixed point that is to remain unchanged after the scaling transformation. Fixed point (x_f, y_f) can be chosen as one of the vertices, centroid of the object, or any other position.

Steps:

- Translate object so that the fixed point coincides with the co-ordinate origin.
- Scale the object with respect to the co-ordinate origin.
- Use the inverse translation of steps 1 to return the object to its original position.



$$T(x_f, y_f) \cdot S(s_x, s_y) \cdot T(-x_f, -y_f) = S(x_f, y_f, s_x, s_y)$$

$$\begin{bmatrix} 1 & 0 & x_f \\ 0 & 1 & y_f \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & -x_f \\ 0 & 1 & -y_f \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & x_f(1-s_x) \\ 0 & s_y & y_f(1-s_y) \\ 0 & 0 & 1 \end{bmatrix}$$

Reflection:

A reflection is a transformation that produces a mirror image of an object. The mirror image for a 2D reflection is generated relative to an axis of reflection by rotating the object 180" about the reflection axis. We can choose an axis of reflection in the xy-plane or perpendicular to the xy plane. When the reflection axis is a line in the xy plane, the rotation path about this axis is in a plane perpendicular to the xy-plane. For reflection axes that are perpendicular to the xy-plane, the rotation path is in the xy plane.

i. **Reflection about x axis or about line y = 0**

Keeps **X** value same but flips **Y** value of coordinate points

$x' = x$
$y' = -y$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

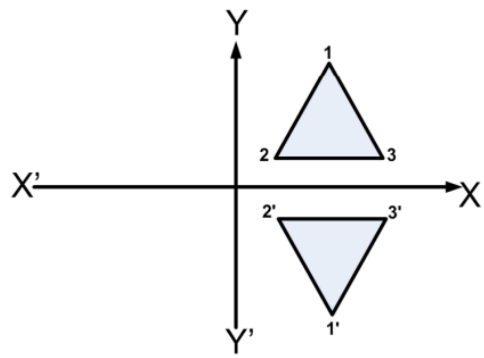


Figure 3.3 Reflection of object about x-axis.

ii. **Reflection about y axis or about line x = 0**

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

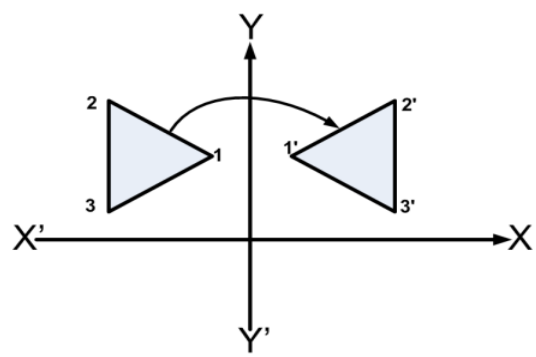


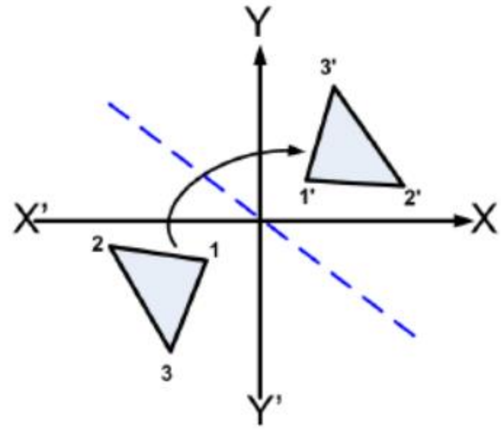
Figure 3.4 Reflection of object about y-axis.

iii. Reflection about origin

Flip both 'x' and 'y' coordinates of a point

$x' = -x$
$y' = -y$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$



Homogeneous co-ordinate

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

iv. Reflection about line y = x

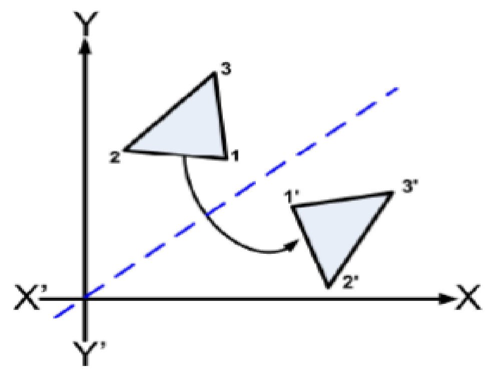
$x' = y$
$y' = x$

Thus, reflection against x=y-axis (i.e. θ =45)

$$R_{x=y} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Hence

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$



Homogeneous co-ordinate

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Equivalent to:

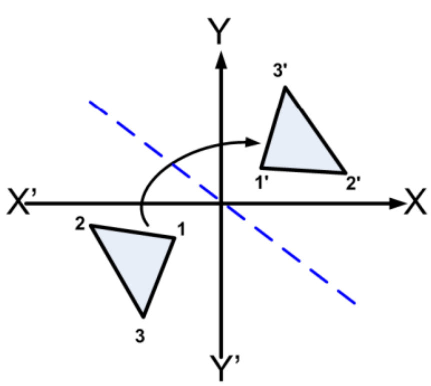
- Reflection about x-axis
- Rotate anticlockwise **90°**

OR

- Clockwise rotation 45 °
- Reflection with x-axis
- anticlockwise rotation 45 °

v. Reflection about line y = - x

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

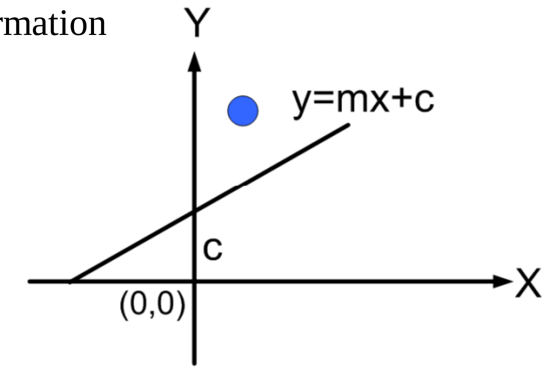


Equivalent to:

- wise rotation 45 °
- tion with y-axis
- anticlockwise rotation 45 °

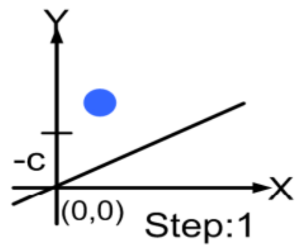
vi. Reflection about y=m*x + c

Combination of translate-rotate-reflect transformation



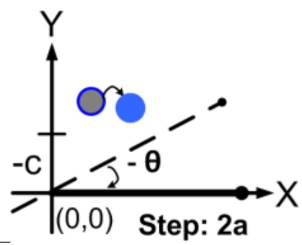
1. First translate the line so that it passes through the origin

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -c \\ 0 & 0 & 1 \end{bmatrix}}_{translation}$$

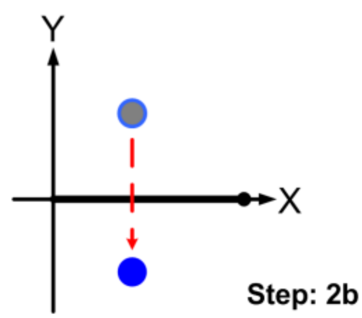


2. Rotate the line onto one of the coordinate axes(say x-axis) and reflect about that axis (x-axis)

$$\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} --- \textit{rotation}$$



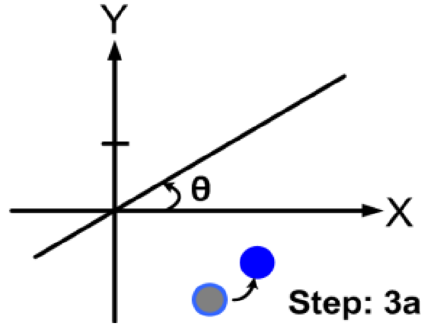
$$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{reflection}$$



3. Finally, restore the line to its original position with the inverse rotation and translation transformation.

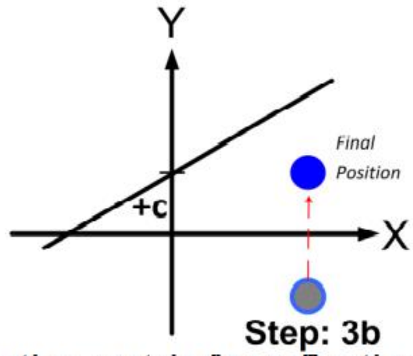
$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

rotation



$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & c \\ 0 & 0 & 1 \end{bmatrix}$$

translation



- Thus the composite transformation matrix for reflection about y=m*x+c is

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & c \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -c \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Substituting value of tanθ , sinθ & cosθ you will get the reflection matrix

$$\begin{bmatrix} \frac{1-m^2}{1+m^2} & \frac{2m}{1+m^2} & \frac{-2cm}{1+m^2} \\ \frac{2m}{1+m^2} & \frac{m^2-1}{1+m^2} & \frac{2c}{1+m^2} \\ 0 & 0 & 1 \end{bmatrix}$$

Q1. Reflect an object (2, 3), (4, 3), (4, 5) about line y = x +1.

Solution

Here,

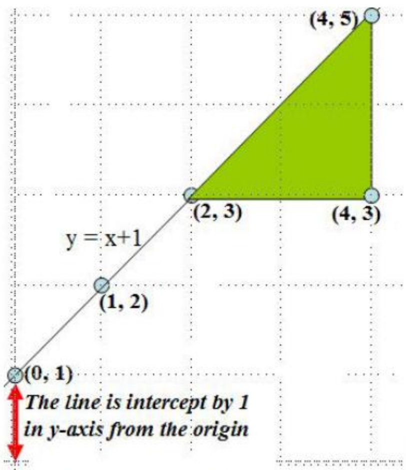
The given line is *y = x +1.*

Thus,

When x = 0, y=1

When x = 1, y=2

When x = 2, y=3



Also,

The slope of the line (m) = 1

Thus, the rotation angle (θ) = $Tan^{-1}(m) = Tan^{-1}(1) = 45^0$

Here, the required steps are:

- Translate the line to origin by decreasing the y-intercept with one.
- Rotate the line by angle 45^0 in clockwise direction so that the given line must overlap x-axis.
- Reflect the object about the x-axis.
- Reverse rotate the line by angle -45^0 in counter-clockwise direction.
- Reverse translate the line to original position by adding the y-intercept with one.

Thus, the composite matrix is given by:

$$CM = T'_{(0,C)} \cdot R'_{\theta} \cdot R_{refl} \cdot R_{\theta} \cdot T_{(0,-C)}$$

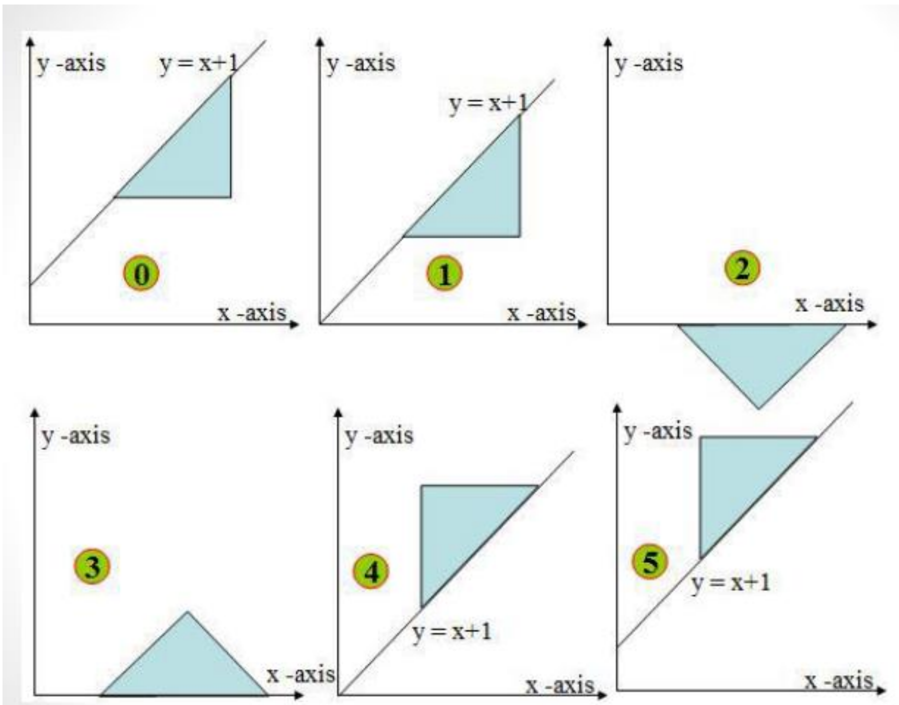
$$= \overset{\textit{Addition y-intercept}}{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}} \overset{\textit{CCW Rotation}}{\begin{pmatrix} Cos45 & -Sin45 & 0 \\ Sin45 & Cos45 & 0 \\ 0 & 0 & 1 \end{pmatrix}} \overset{\textit{Reflection about x-axis}}{\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}} \overset{\textit{CW Rotation}}{\begin{pmatrix} Cos45 & Sin45 & 0 \\ -Sin45 & Cos45 & 0 \\ 0 & 0 & 1 \end{pmatrix}} \overset{\textit{Reduce y-intercept}}{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & -1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} & -1/\sqrt{2} \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & -1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 & -1 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$



Now, the required co-ordinate can be calculated as:

P' = Com x P

$$= \begin{pmatrix} 0 & 1 & -1 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & 4 & 4 \\ 3 & 3 & 5 \\ 1 & 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 2 & 4 \\ 3 & 5 & 5 \\ 1 & 1 & 1 \end{pmatrix}$$

Hence, the final coordinates are (2, 3), (2, 5) & (4, 5).

Q2. A mirror is placed such that it passes through (0, 10), (10, 0). Fin the mirror image of an object (6, 7), (7, 6), (6, 9).

Solution

Here,

The given mirror or line is passing through the points *(0, 10) & (10, 0).*

Now, the slope of the line

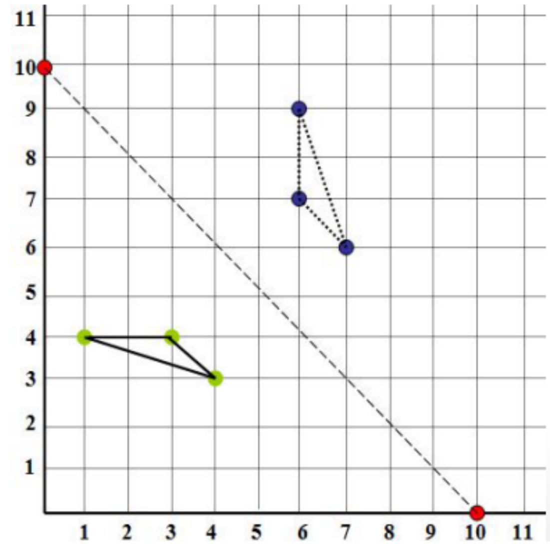
(m) = (y2-y1) / (x2-x1)

= (0 - 10) / (10 - 0) = -1

Thus, the rotation angle (**θ**)

= Tan $^{-1}$ (m) = Tan $^{-1}$(-1)

= - 45^0



The composite matrix is given by:

Com

$$= T_{(0,\ 10)\ or\ (10,\ 0)} . R_{\theta\ in\ CW} . R_{fx} . R_{\theta\ in\ CCW} . T_{(0,\ -10)\ or\ (-10,\ 0)}$$

Addition x-intercept — *CW Rotation* — *Reflection about x-axis* — *CCW Rotation* — *Reduce x-intercept*

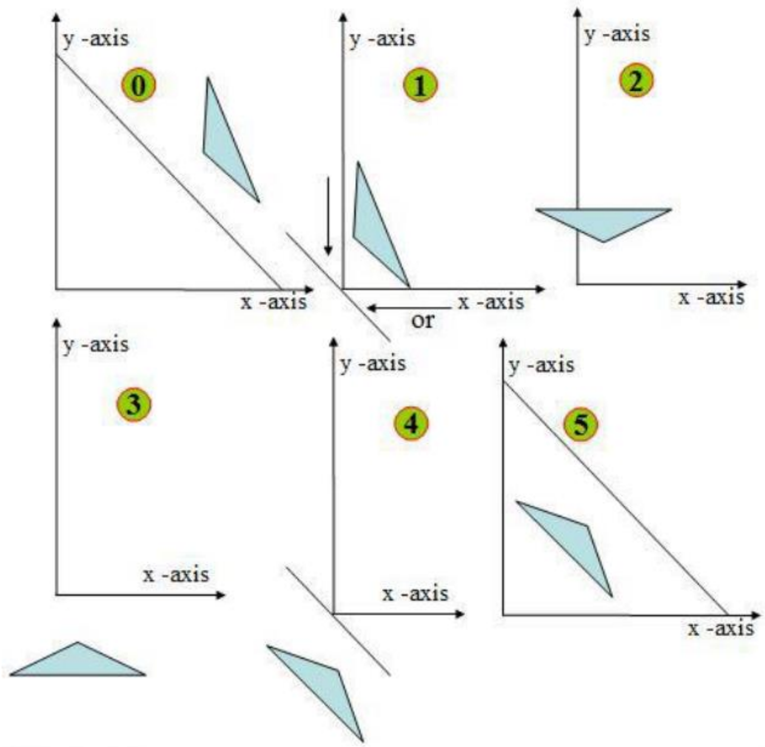
$$= \begin{pmatrix} 1 & 0 & 10 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} Cos45 & Sin45 & 0 \\ -Sin45 & Cos45 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} Cos45 & -Sin45 & 0 \\ Sin45 & Cos45 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & -10 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 10 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & -10 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 10 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} & -10/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} & -10/\sqrt{2} \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 10 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} & -10/\sqrt{2} \\ -1/\sqrt{2} & -1/\sqrt{2} & 10/\sqrt{2} \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 10 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & 10 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & -1 & 10 \\ -1 & 0 & 10 \\ 0 & 0 & 1 \end{pmatrix}$$



Now, the required co-ordinate can be calculated as:

P' = Com . P

$$= \begin{pmatrix} 0 & -1 & 10 \\ -1 & 0 & 10 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 6 & 7 & 6 \\ 7 & 6 & 9 \\ 1 & 1 & 1 \end{pmatrix}$$

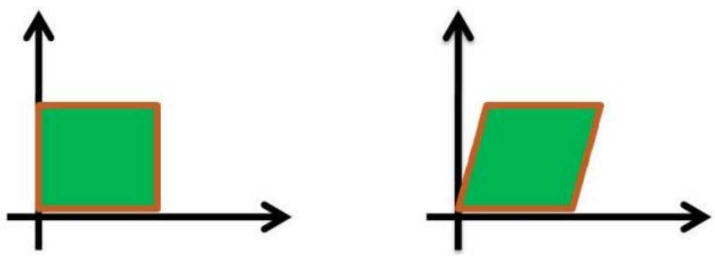
$$= \begin{pmatrix} 3 & 4 & 1 \\ 4 & 3 & 4 \\ 1 & 1 & 1 \end{pmatrix}$$

Hence, the final coordinates are (3, 4), (4, 3) & (1, 4).

Shearing:

It distorts the shape of object in either 'x' or 'y' or both direction. In case of single directional shearing (e.g. in 'x' direction can be viewed as an object made up of very thin layer and slid over each other with the base remaining where it is). Shearing is a non-rigid-body transformation that moves objects with deformation.

Shearing factor (Sh_x, Sh_y)



x- direction shear relative to x-axis

x'=x+y*Shx

y'=y

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & Shx & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

y- direction shear relative to y-axis

x'=x

y'=y +x *Shy

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ Shy & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

x- direction shear relative to other reference line $y=y_{ref}$

x'=x +Shx(y-y_{ref})

y'=y

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & Shx & -Shx \cdot y_{ref} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

y- direction shear relative to other reference line $x=x_{ref}$

x'=x

y'=y+Shy(x-x_{ref})

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ Shy & 1 & -Shy \cdot X_{ref} \\ 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Composite Transformations:

Translation:

For Successive Translation vectors (tx1,ty1) and (tx2,ty2)

$$P' = T(t_{x2}, t_{y2}).\{T(t_{x1}, t_{y1}).P\}$$
$$= \{T(t_{x2}, t_{y2}).T(t_{x1}, t_{y1})\}.P$$

Successive Translations are additive

$$\begin{bmatrix} 1 & 0 & t_{x2} \\ 0 & 1 & t_{y2} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & t_{x1} \\ 0 & 1 & t_{y1} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_{x1}+t_{x2} \\ 0 & 1 & t_{y1}+t_{y2} \\ 0 & 0 & 1 \end{bmatrix}$$

$$T(t_{x2}, t_{y2}).T(t_{x1}, t_{y1}) = T(t_{x1}+t_{x2}, t_{y1}+t_{y2})$$

Rotation:

For Successive Rotations θ_1 and θ_2

$$P' = R(\theta_2).\{R(\theta_1).P\}$$
$$= \{R(\theta_2).R(\theta_1)\}.P$$

Successive Rotations are additive.

$$R(\theta_2).R(\theta_1) = R(\theta_1 + \theta_2)$$

Chapter 3 Two –Dimensional Transformations

Scaling:

Successive Scaling are multiplicative

$$S(s_{x2}, s_{y2}).S(s_{x1}, s_{y1}) = S(s_{x1}.s_{x2}, s_{y1}.s_{y2})$$

$$\begin{bmatrix} s_{x2} & 0 & 0 \\ 0 & s_{y2} & 0 \\ 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} s_{x1} & 0 & 0 \\ 0 & s_{y1} & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} s_{x1}.s_{x2} & 0 & 0 \\ 0 & S_{y1}.s_{y2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Fixed Point Scaling

$$\begin{bmatrix} 1 & 0 & x_f \\ 0 & 1 & y_f \\ 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} 1 & 0 & -x_f \\ 0 & 1 & -y_f \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & x_f(1-s_x) \\ 0 & s_y & y_f(1-s_y) \\ 0 & 0 & 1 \end{bmatrix}$$

$$T(x_f, y_f).S(s_x, s_y).T(-x_f, -y_f) = S(x_f, y_f, s_x, s_y)$$

Two-dimensional viewing:

, world to screen viewing transformations and clipping (Cohen-Sutherland Line Clipping, Liang-Barsky Line Clipping)

Three-Dimensional Graphics

Three –dimensional translation, rotation, scaling, reflection, shear transforms

Three-dimensional composite transformation

Three-dimensional viewing pipeline, world to screen viewing transformation, projection concepts (orthographic, parallel, perspective projections)