Northwestern University
Department of Electrical and Computer Engineering

ELEC ENG 422 Winter 2020
Problem set 6: Date due: March 11, 2020.

Reading: Sections 3.1-3.6 and 10.1 -10.2 in Gallager.

1. Exercise 3.1 in Gallager.

2. Exercise 3.5 in Gallager. (In part (b) - you can use the fact that a single random vector is Gaussian if and only if its MGF has the form of a Gaussian MGF. The point of this part is to use this to then prove the same property for a random vector using the result in part (a).)

3. Exercise 3.9 in Gallager.

4. This problem provides an argument as to why the normalized covariance, ρ_{XY}, for two random variables X and Y satisfies $|\rho_{XY}| \leq 1$. Let X and Y be two random variables with finite first and second moments.

   a.) Let $Z = aX - bY$ for some non-zero choice of a and b and calculate $E(Z^2)$ in terms of the first and second moments of X and Y and their correlation, $E(XY)$. (*Note:if X and Y are not zero mean, then $E(XY)$ is not their covariance.*)

   b.) Use your answer to part (a) to argue that $E(XY)^2 \leq E(X^2)E(Y^2)$. This is known as the Cauchy-Schwartz inequality for random variables.

   c. Use the Cauchy-Schwartz inequality to show that $|\rho_{XY}| \leq 1$.

5. Problem 3.11 in Gallager.

6. Problem 3.17 in Gallager.

7. Let X and Y have the joint pdf given by

$$f_{X,Y}(x,y) = \begin{cases} 2, & \text{if } x+y \leq 1,\ x \geq 0, \text{ and } y \geq 0, \\ 0, & \text{otherwise.} \end{cases}$$

   a.) Find the MMSE estimate $\hat{X}$ of X given Y.

   b.) Will the estimation error $Z = X - \hat{X}$ be independent of Y? Justify your answer (*Hint: a calculation is not required*).

8. a.) Consider finding an estimate $\hat{X}$ of the random variable X given an observation Y, but instead of using the squared error cost function as in class, use the absolute error cost given by $C(\hat{X}, X) = |\hat{X} - X|$. Give the optimal Bayesian estimator using this cost function. (*Hint: one of the problems from an earlier homework essentially solves this.*)

b.) Give this estimator for the case where X and Y are jointly Gaussian with 0 means and the covariance matrix

$$\mathbf{K} = \begin{bmatrix} 1 & \rho \\ \rho & 1. \end{bmatrix}$$

Hint: think first - this requires no calculation!

9. Problem 10.3 part (a) in Gallager.