Que Explain different types of relation with proper examples?

Sol^n:- There are following types of relation.

① Empty Relation :- An empty relation (or void) is one in which there is no relation between any element of a set.

Ex → if set $A = \{1,2,3\}$

& $R = \{x, y\}$, where

$R\ \phi \subset A \times A$.

② Universal relation → A universal relation is a type of relation in which every element of set is related to each other - consider set $A = \{a,b,c\}$

Now $R = \{x, y\}$ where $|x-y| \geq 0$

for universal relation

$R = A \times A$.

③ Identity relation - In such relation, every element of set A is related to itself only,

$I = \{(a,a), (A\}$

Ex If we throw two dice we get 36 outcomes If we define identify relation then it will be $R = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$

(4) Reflexive relation - In reflexive relation every element maps to itself.

Ex → Considered set $A = \{1,2\}$

Reflexive relation $R = \{(1,1), (2,2), (1,2), (2,1)\}$

$(a,a) \in R$

(5) Symmetric Relation - A relation 'R' on set 'A' is said to be symmetric relation if and only if $(a,b) \in R$ then $(b,a) \in R$.

Ex → $R = \{(1,2), (2,1)\}$ for a set $A = \{1,2\}$

(6) Transitive Relation :- A relation R a set A is transitive if $(a,b) \in R$, $(b,c) \in R$ then $(a,c) \in R$ for all $a,b,c \in R$.

Ex → let us define relation R on set $A = \{1,2,3\}$

$R = \{(1,1), (2,2), (3,3), (1,2), (2,3), (1,3)\}$

so such relation is transitive in nature.

(7) Equivalance Relation → A Relation R on set A is said to be equivalence relation if & only if the relation is.

i) Reflexive relation

ii) Symmetric relation

iii) it is transitive.

Ex → set $A = \{1,2,3,4\}$

Relation $R = \{(1,1),(1,3),(2,2),(2,4),(3,1),(3,3),(4,2),(4,4)\}$

Above relation is reflexive, symmetric, & transitive so it is equivalance relation.

Kind of functions

(1) One - one function (Injective)

Let $f : A \to B$ be a function & it is called One - one if $x \neq y \Rightarrow f(x) \neq f(y)$

for $x, y \in R$

if $f(x) = f(y) \Rightarrow x = y$.

Ex ÷ Let $f : R \to R$ & s.t. $f(x) = ax + b$; $a \neq 0$

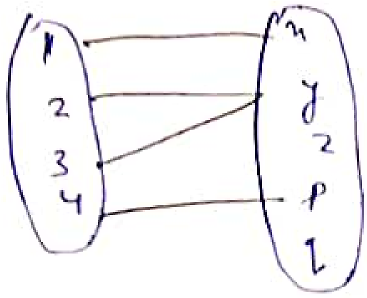
$$f(x) = f(y)$$
$$ax + b = ay + b$$
$$x = y.$$

Note : If f is not one-one then it is called many one..

image.

② Onto function ÷ A function $f: A \to B$ is (surjective function) called onto if Range $f = B$.

Ex ÷ Let $A = \{1, 2, 3, 4\}$ & $B = \{x, y, z, p, q\}$



Here Range $f = \{x, y, p\}$
which is not onto

③ Bijective function ÷ Let $f: A \to B$ be a function & it's called bijective.
if f is one-one & onto.

Ex ÷ The function $f: R \to R$ $f(x) = 2x - 3$ is bijective

for one - one

$$f(x) = f(y)$$
$$2x - 3 = 2y - 3$$
$$x = y$$

f is one - one.

for onto

suppose $y \in$ Range (f), states that $y = 2x - 3$

$$x = \frac{y+3}{2} \in R$$

$\Rightarrow$ Range $F = R$

$\Rightarrow$ F is onto function

So, F is bijective function.

(4) Many one function :- If two or more element of set A have same mapping in set then, the function is said to many one function.

Ex → $A = \{1, 2, 3, 4, 5\}$ $B = (x, y, z)$

$f : A \to B$

$f : \{(1, x), (2, x), (3, x), (4, y), (5, z)\}$