

1) Is $(P \rightarrow Q) \rightarrow [(P \rightarrow Q) \rightarrow Q]$ a tautology? Why or why not?

we design the truth table.

P	Q	$P \rightarrow Q$	$(P \rightarrow Q) \rightarrow Q$	Question
T	T	T	T	T
T	F	F	T	T
F	T	T	T	T
F	F	T	F	F

It is not a tautology since all the value in the last columns are not true, one value is false hence the given statement is not tautology.

2) Show that $[(P \vee Q) \wedge (R \vee \neg Q)] \rightarrow (P \vee R)$ is a tautology without using truth table.

$$= [\neg(P \vee q) \vee \neg(r \vee \neg q)] \vee (P \vee r)$$

$$= [(\neg p \wedge \neg q) \vee (\neg r \wedge q)] \vee (P \vee r)$$

$$= [(\neg p \wedge \neg q)] \vee [(\neg r \wedge q)] \vee P \vee r$$

$$= [(\neg p \vee p) \wedge (p \vee \neg q)] \vee [(\neg r \vee r) \wedge (q \vee r)]$$

$$= [T \wedge (P \vee \neg q)] \vee [T \wedge (q \vee r)]$$

$$= T \wedge (P \vee \neg q) \vee (q \vee r)$$

$$= T \wedge (P \vee r \vee T)$$

$$= T \wedge T$$

$$=$$

$$T$$