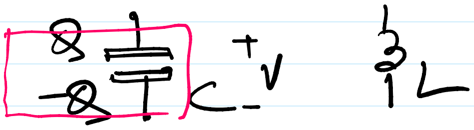
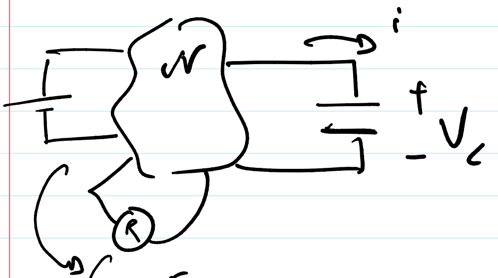
Lecture 20-21
09 September 2024 00:13

Fundamental Memory Elements



$$Q = CV \rightarrow \text{Capacitance}$$

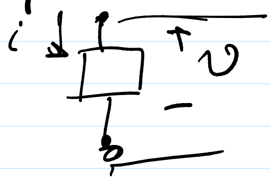
$$i = \frac{dQ}{dt} = \frac{d}{dt}(CV) = C\underbrace{\frac{dV}{dt}}$$ (If C is not a fn of V)



Find time independent sources.

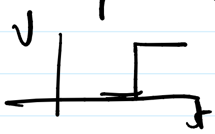
$V_c(t)$ is constant $\because$ there are no time dependent sources

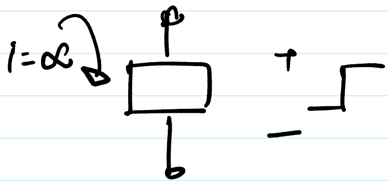
$i = 0$



A capacitor with no voltage change can be modeled as an open ckt.

$$i = C\frac{dV}{dt}$$

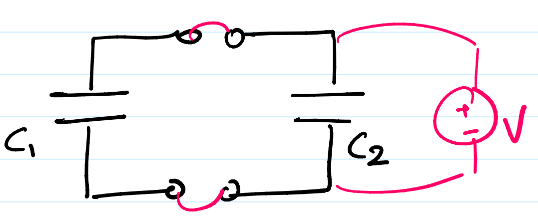




An abruptly changing voltage causes ∞ i.

$\Rightarrow$ C can be modeled as a short under this condition.

Equivalent Capacitance (Parallel / Series)



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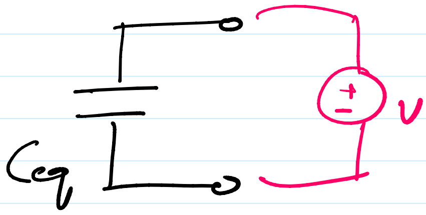
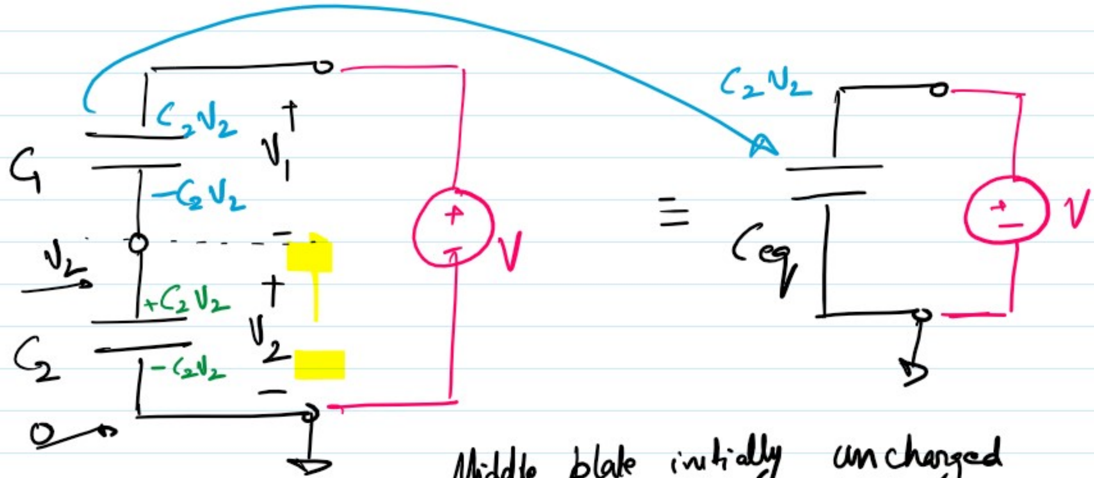


$$Q_{Total} = C_1V + C_2V \Longleftrightarrow Q_{Total} = C_{eq}V$$

C (parallel) = $(C_1 + C_2)$

$$C_{eq}\,(\text{parallel}) = (C_1 + C_2)$$



Middle plate initially uncharged

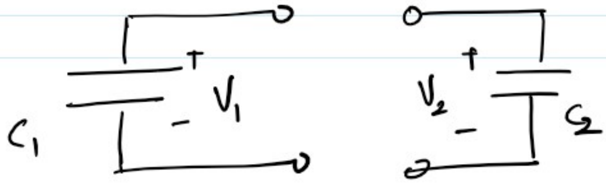
$$Q = C_2V_2 \qquad V_1 = \frac{Q_1}{C_1} = \frac{C_2V_2}{C_1}$$

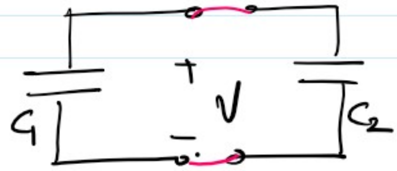
$$V = V_1 + V_2 = \left(\frac{C_2}{C_1} + 1\right)V_2 = \frac{C_2 + C_1}{C_1}\,V_2$$

$$\Rightarrow V = \left(\frac{C_1 + C_2}{C_1C_2}\right)Q$$

$$C_{eq}\,(\text{Series}) = \frac{C_1C_2}{C_1 + C_2}$$

Charge conservation (in isolated systems) in capacitors.



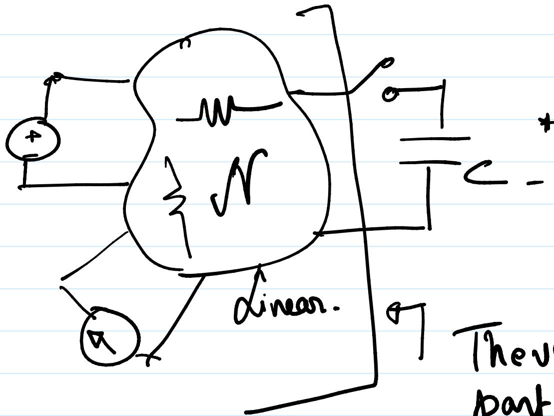


$$Q_{Total} = C_1V_1 + C_2V_2 \qquad Q_{Total} = C_{eq}\cdot V$$

$$= (C_1 + C_2)\,V$$

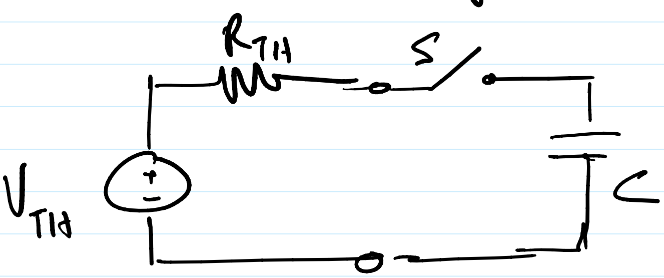
$$\Rightarrow V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

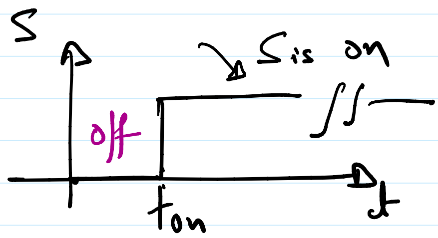
TRANSIENT ANALYSIS



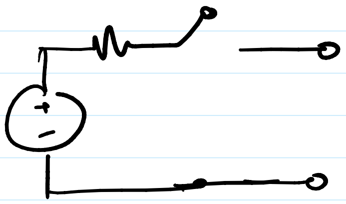
Assume C is initially uncharged.

Thevenize the non-Capacitive part.





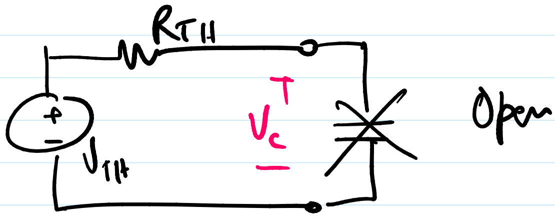
S is off



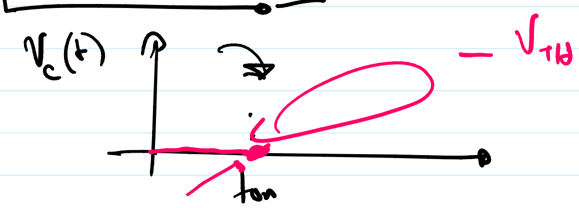
open

$+$
$v = 0$
$-$

at $t = \infty$

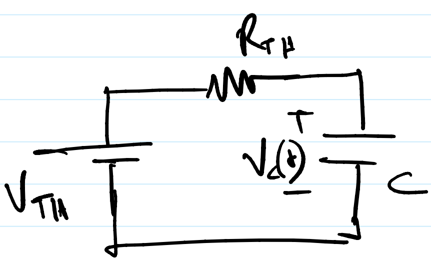


$V_c = V_{TH}$



@ $t = t_{on}^{+}$

$V_c(t)$

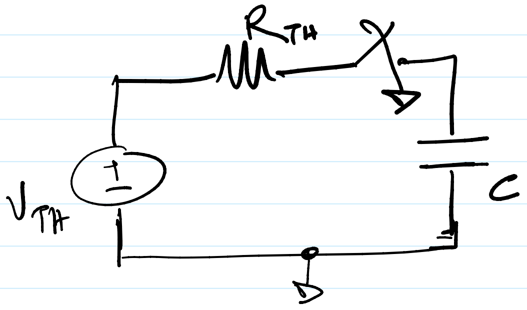


$$\frac{V_{TH} - V_c(t)}{R_{TH}} = C\,\frac{dV_c}{dt}$$

$$\Rightarrow \frac{dV_c}{dt} + \frac{1}{R_{TH}C}V_c(t) = \frac{V_{TH}}{R_{TH}}$$

$$V_c(t) = V_{TH}\left(1 - e^{-t/R_{TH}C}\right)$$

$$v_c(t) = V_{TH}\left(1 - e^{-t/R_{TH}C}\right)$$

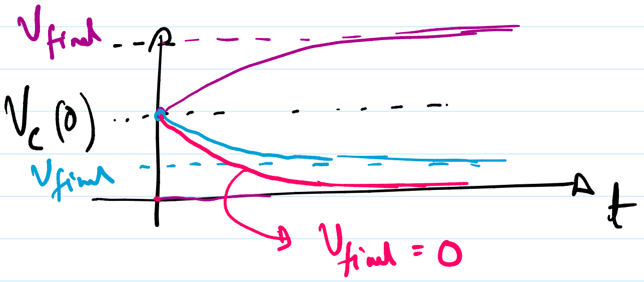


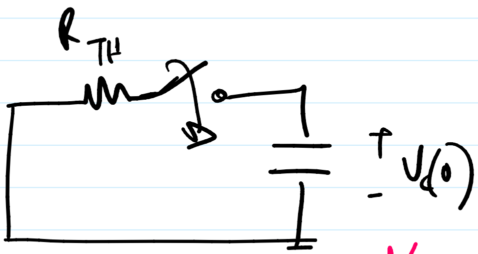
C has an initial charge of $Q(0)$.

$$V_c(0) = \frac{Q(0)}{C}$$

$$V_c(t) = V_c(0) + \left(V_{final} - V_c(0)\right)\left(1 - e^{-t/R_{TH}C}\right)$$

$R_{TH}C$ = Time Constant (τ)

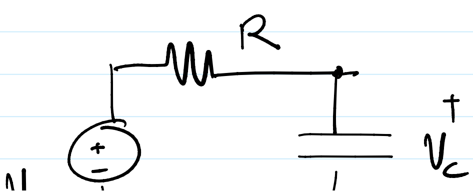


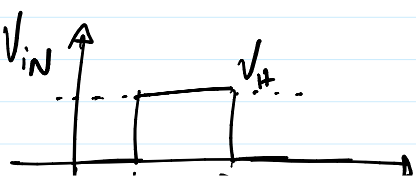


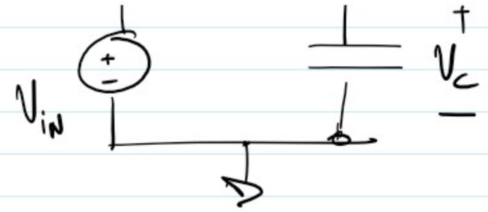
$$V_c(t) = \underbrace{V_c(0)\, e^{-t/R_{TH}C}}_{}$$

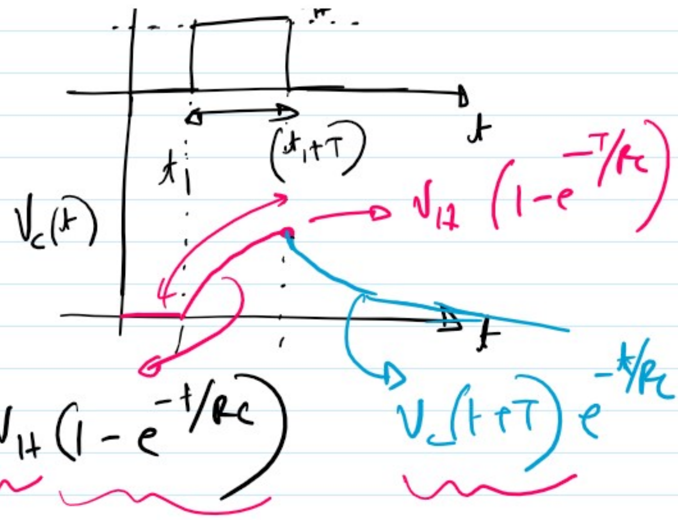
Natural Response of the ckt.

Natural Response: Response to initial condition.

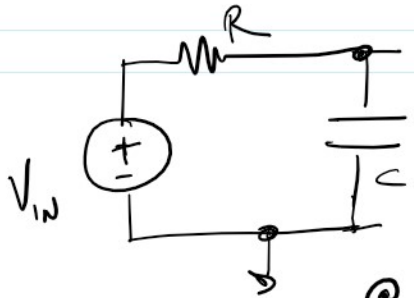


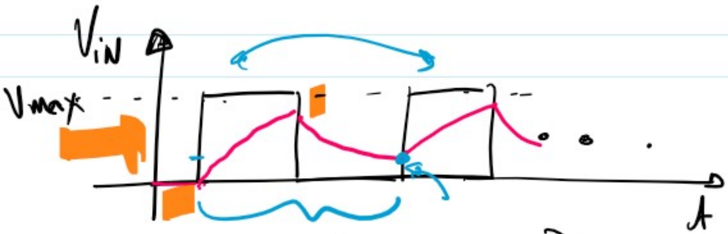






Initially C is uncharged.





@ steady state (ie $t \to \infty$)
(whatever happening in one cycle repeats)

$$V_L + (V_{max} - V_L)\left(1 - e^{-t/RC}\right)$$

@ t_2 $\quad V(t_2) = V_L + (V_{max} - V_L)\left(1 - e^{-T/RC}\right)$

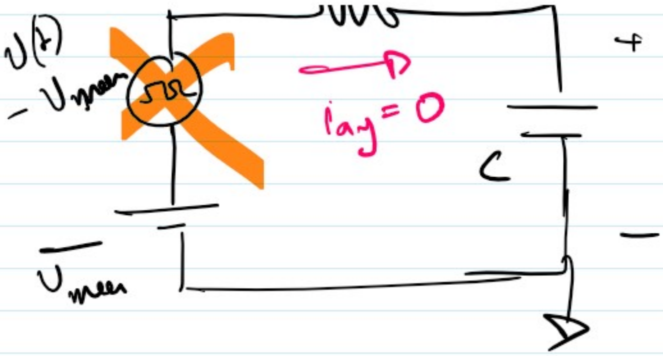
@ t_3 $\quad V(t_2)\, e^{-t/RC}\Big|_{t=T} = V_L$

$$v_i(t) = \overline{v_{mean}} + \underbrace{\left(v_i(t) - \bar{v}(t)\right)}_{0 \text{ mean}}$$

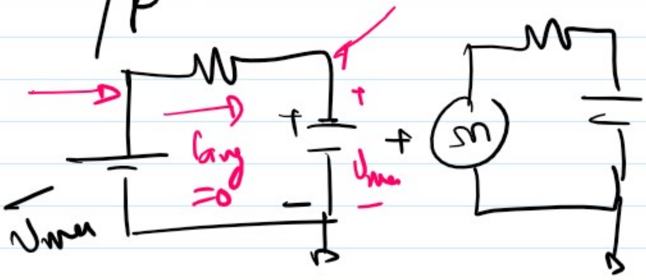
+

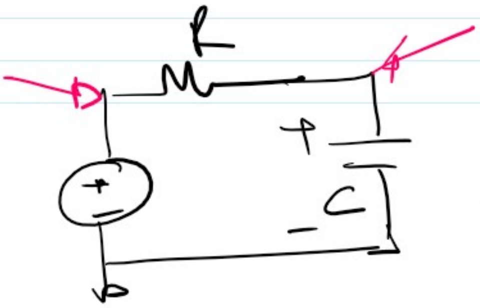
With only V_{mean}

o/p =



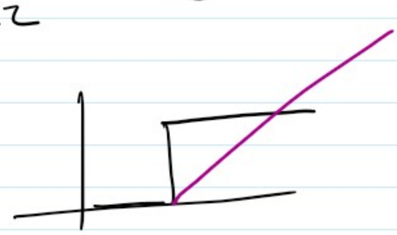
o/p =





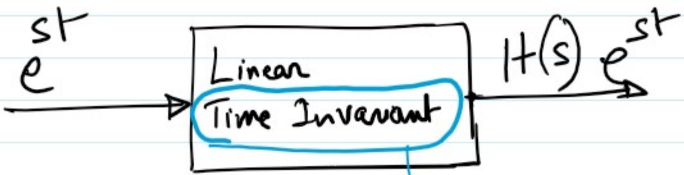
$V_{mean} = V_{max}/2$

$$\int_0^t v_i(\tau)\,d\tau \quad ?? \quad \int_0^t v_c(\tau)\,d\tau$$



Sinusoidal $\boxed{\text{Steady State}}$ Response

After a long time.



Time Invariant → System does not change with time, e.g. : No switching inside.

along time.

$$e^{j\omega t} = \cos(\omega t) + j\sin(\omega t)$$

$$\cos(\omega t) = \frac{1}{2}\left[e^{j\omega t} + e^{-j\omega t}\right]$$

$$\sin(\omega t) = \frac{1}{2}\left[e^{j\omega t} - e^{-j\omega t}\right]$$

Can be applied through signal gen.

Cannot be applied through sig. gen BUT convinient for analysis.

$$e^{j\omega t} \longrightarrow \boxed{\text{LTI}} \longrightarrow H(j\omega)\, e^{j\omega t}$$

$$e^{st} \longrightarrow \boxed{\text{LTI}} \longrightarrow H(s)\, e^{st}$$

$$e^{-j\omega t} \longrightarrow \boxed{\text{LTI}} \longrightarrow H(-j\omega)\, e^{-j\omega t}$$

$$H(j\omega) = |H(j\omega)|\, e^{j\phi(\omega)}$$

$$H(-j\omega) = |H(-j\omega)|\, e^{j\phi(-\omega)}$$

$$H(j\omega) = |H(j\omega)|\, e$$
$$H(-j\omega) = |H(-j\omega)|\, e^{j\phi(-\omega)}$$

$$V_{in} = 2\cos(\omega t) \longrightarrow \boxed{LTI} \longrightarrow V_o$$

$$e^{j\omega t} + e^{-j\omega t}$$

$$V_o = H(j\omega)e^{j\omega t} + H(-j\omega)\,e^{-j\omega t}$$
$$= |H(j\omega)|\, e^{j(\omega t+\phi(\omega))} + H(-j\omega)\,e^{-j\omega t+\phi(-\omega)}$$

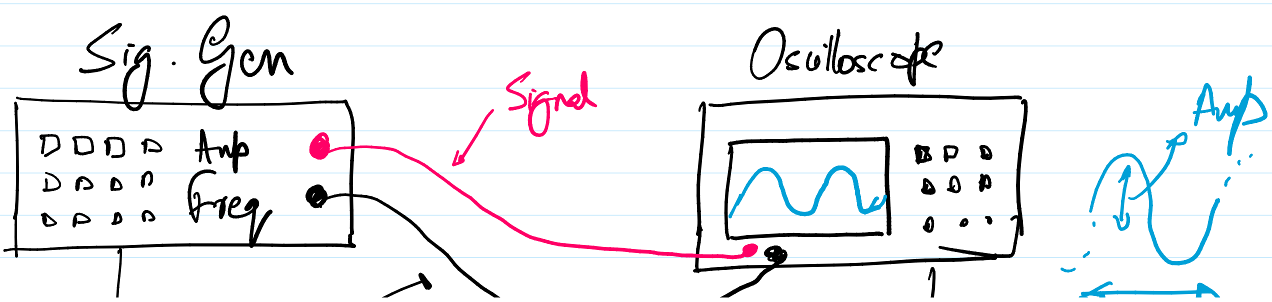
$$V_o(t) = |H(j\omega)|\cos(\omega t+\phi(\omega)) + \cancel{j|H(j\omega)|\sin(\omega t+\phi(\omega))}$$
$$+ |H(-j\omega)|\cos(\omega t-\phi(-\omega)) - \cancel{j|H(-j\omega)|\sin(\omega t-\phi(-\omega))} \rightarrow 0$$

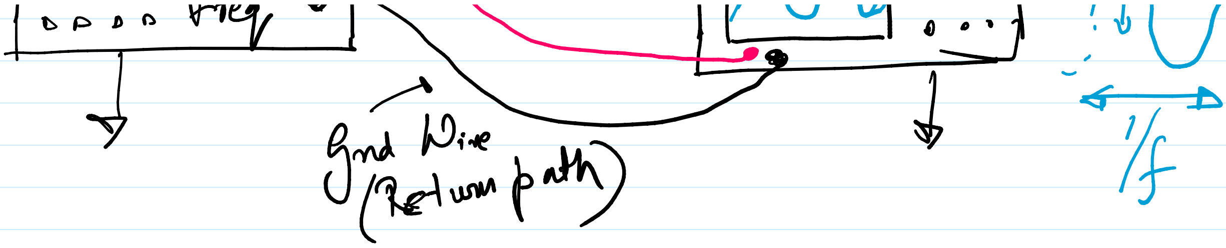
$$\left.\begin{aligned} H(j\omega) &= H(-j\omega) \\ \phi(\omega) &= -\phi(-\omega) \end{aligned}\right\} \text{Real Systems}$$

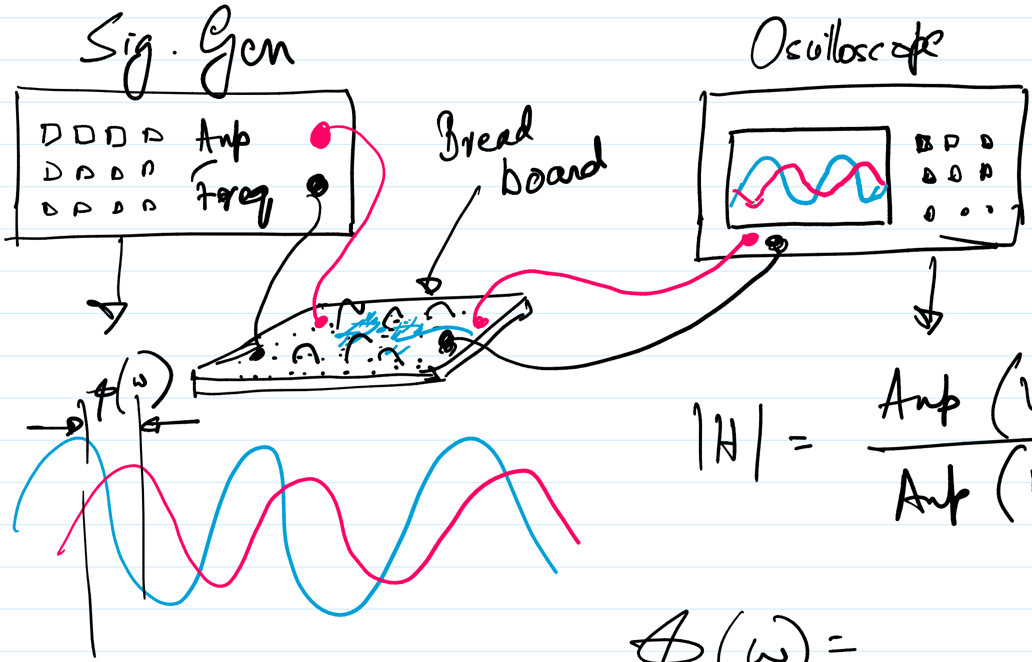
$$V_o(t) = 2|H(j\omega)|\cos(\omega t+\phi(\omega))$$

$$2\cos(\omega t) \longrightarrow \boxed{LTI} \longrightarrow 2|H(j\omega)|\cos(\omega t+\phi(\omega))$$

$$e^{j\omega t} \qquad \underbrace{|H(j\omega)|\, e^{j\phi}}\, e^{j\omega t}$$





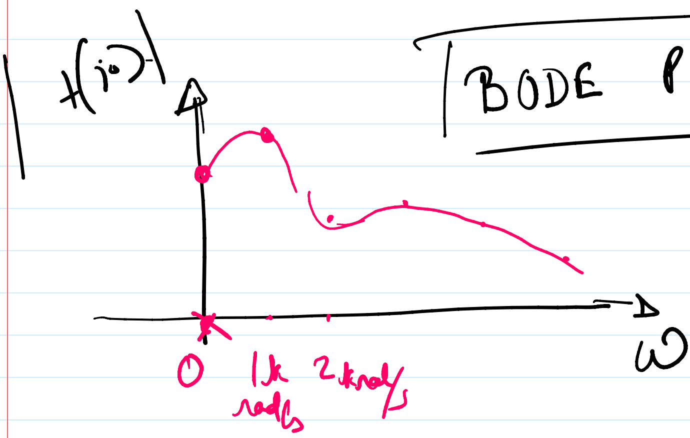


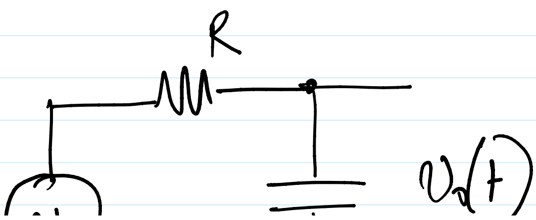
$$|H| = \frac{Amp\,(V_{out})}{Amp\,(V_{in})}$$

$$\phi(\omega) =$$

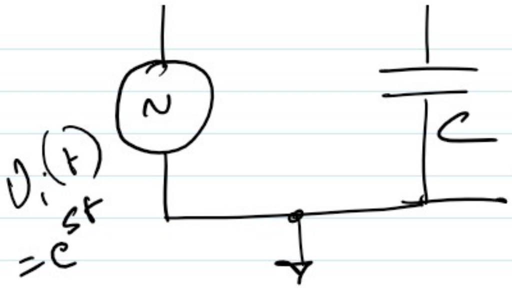
$H(j\omega)$ = Transfer Function (TF)

$\boxed{\text{BODE PLOT}}$





$$\frac{d\,v_o}{\ldots} + \frac{v_o}{\ldots} = \frac{v_i}{\ldots}$$

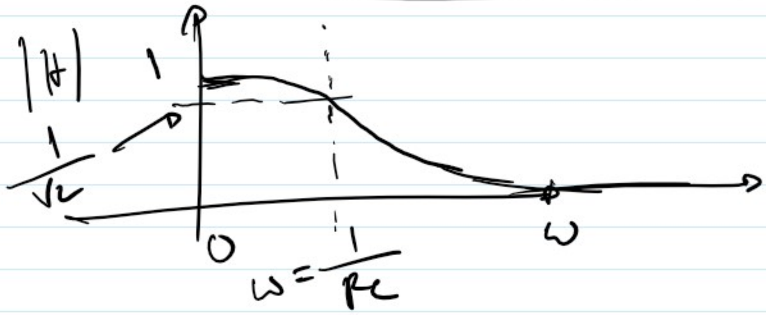


$$\frac{dv_o}{dt} + \frac{v_o}{RC} = \frac{v_i}{RC}$$

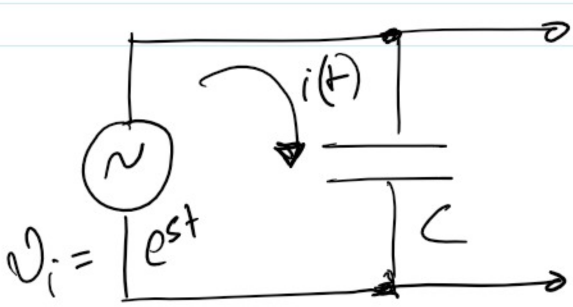
$$sH(s)\,\cancel{e^{st}} + \frac{1}{RC}H(s)\,\cancel{e^{st}} = \frac{\cancel{e^{st}}}{RC}$$

$$\Rightarrow \quad H(s) = \frac{1}{1+sRC}$$

$$\boxed{H(j\omega) = \frac{1}{1+j\omega RC}} \qquad |H(j\omega)| = \frac{1}{\sqrt{1+\omega^2R^2C^2}}$$

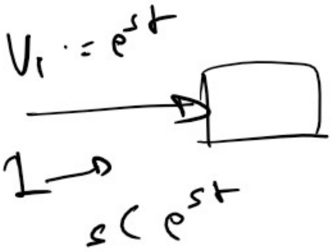


Impedance of a Capacitor



$$i(t) = C\frac{dV}{dt}$$

$$= sC\,e^{st}$$



$$Z(s) = \frac{V_C(s)}{I_C(s)} = \frac{1}{sC}$$

$$Z(j\omega) = \frac{1}{\dots}$$

$$\boxed{Z(j\omega) = \frac{1}{j\omega C}}$$

L