电磁场与电磁波（英文版）

224

5.0

试题编号：P378

年级：　　专业：　　班级：　　姓名：　　学号：

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# 重庆邮电大学2013-2014学年第一学期

## 电磁场与电磁波（英）课程试卷（期末）（A卷）（闭卷）

| 题　号 | 一 | 二 | 三 | 四 | 五 | 六 | 七 | 八 | 总 分 |
|---|---|---|---|---|---|---|---|---|---|
| 得　分 | | | | | | | | | |
| 评卷人 | | | | | | | | | |

一、判断题（本大题共5小题，每小题2分，共10分）

(1) An irrotational field can be expressed as the gradient of a continuously differentiable scalar function. (　　)

(2) The tangential components of the electric field intensity are always continuous at the interface. (　　)

(3) In a simple medium, the magnetostatics vector potential $\vec{A}$ satisfy the equation: $\nabla^2\vec{A}=-\mu\vec{J}$. (　　)

(4) $\vec{J}=\sigma\vec{E}$ is hold in every medium. (　　)

(5) A material may be a good conductor at low frequencies but may have the properties of a lossy dielectric at very high frequencies. (　　)

二、选择题（本大题共10小题，每小题2分，共20分）

(1) Gradient of a Scalar Field $v=x^2+2y^2+5z^2$ at the point $p(-1,2,1)$ is (　　).

(A) $-\vec{a}_x2+\vec{a}_y8+\vec{a}_z10$　(B) $-\vec{a}_x8$　(C) $\sqrt{168}$　(D) $-\vec{a}_x8$

(2) In simple medium, if $B_x=x, B_y=y$ .then $B_z=$ (　　).

(A) 2　(B) $-2z$　(C) -2　(D) $2z$

(3) the curl of a vector field $\vec{F}=\vec{a}_x xy-\vec{a}_y 2x$ is (　　).

(A) $y$　　(B) $-\vec{a}_x(2+x)$　　(C) $-y$　　(D) 2

(4) A parallel-plate capacitor is filled with a dielectric with permittivity $\varepsilon$, then the capacitance is $C$. If remove the dielectric material, the capacitance will (　　).

(A) increase　　(B) fall　　(C) no change　　(D) unknown

(5) The physical bases for the method of images is (　　).

(A) Gauss's Law　　(B) Helmholtz's Theorem

(C) Uniqueness Theorem　　(D) Poynting's Theorem

(6) The mathematical expression for magnetostatic energy density in terms of $\vec{B}$ and $\vec{H}$ is (　　).

(A) $\frac{1}{2}\vec{B}\cdot\vec{H}$　　(B) $\frac{\vec{B}^2}{2\mu}$　　(C) $\frac{\mu\vec{B}^2}{2}$　　(D) $\frac{1}{2}\vec{B}\times\vec{H}$

(7) The Lorentz's force acting on a point charge $q$ due to electromagnetics fields is (　　).

(A) $\vec{F}=q\vec{E}$　　(B) $\vec{F}=q\vec{E}+q\vec{u}\cdot\vec{B}$

(C) $\vec{F}=q\vec{E}+q\vec{u}\times\vec{B}$　　(D) $\vec{F}=q\vec{u}\times\vec{B}$

(8) Given the boundary conditions of time-varying electromagnetic fields between two lossless simple media, which of the following is not true (　　).

(A) $\vec{a}_n\cdot(\vec{D}_1-\vec{D}_2)=0$　　(B) $\vec{a}_n\times(\vec{E}_1-\vec{E}_2)=0$

(C) $\vec{a}_n\times(\vec{H}_1-\vec{H}_2)=\vec{J}_s$　　(D) $\vec{a}_n\cdot(\vec{B}_1-\vec{B}_2)=0$

(9) The uniform plane wave is $\vec{E}=\vec{a}_x E_m e^{jkz}-j\vec{a}_y E_m e^{jkz}$,It's polarization is (　　).

(A) elliptically　　(B) right hand circularly

(C) left hand circularly　　(D) linearly

(10) In free space, the incident wave is $\vec{E}=\vec{a}_x E_m\cos(\omega t-kz)$, then the displacement current density is ( ).

(A) $-\vec{a}_x\varepsilon_0\omega E_m\sin(\omega t-kz)$ (B) $-\vec{a}_y\varepsilon_0\omega E_m\sin(\omega t-kz)/\eta$

(C) $\vec{a}_x\varepsilon_0 E_m\cos(\omega t-kz)$ (D) $\vec{a}_y\varepsilon_0 E_m\cos(\omega t-kz)/\eta$

三、计算题（本大题共 5 小题，共 70 分）

1.(10ps) Writing the Maxwell Equation in phasor form and show the two divergence equations can be derived from the two curl equations.

2. (15ps) The electric field intensity of a uniform plane wave traveling in free space is given by $\vec{E}(r) = (-j\vec{a}_x - 2\vec{a}_y + j\sqrt{3}\vec{a}_z)e^{-j0.05\pi(\sqrt{3}x+z)}$ (V/m), Find the following:

(1) The instantaneous value of the electric field intensity $\vec{E}(r,t)$

(2) The phasor representation of magnetic field intensity. $\vec{H}(r)$

(3) The instantaneous value of the electric field intensity. $\vec{H}(r,t)$

(4) the average power density in the medium. $\vec{S}_{av}$

(5) Poynting vector. $\vec{S}(r,t)$

3. (15ps) A wave propagates in a lossless medium characterized by $\mu_r = 1 \quad \varepsilon_r = 4$.The electric field intensity in the region is given by

$$\vec{E} = \vec{a}_x 3\cos\left(10^7 t - ky\right) - \vec{a}_z 3\cos\left(10^7 t - ky + \frac{\pi}{2}\right) \text{ (V/m)}$$

(1) The frequency of the wave $f$.

(2) The phase constant of the wave $k$.

(3)The phase velocity of the wave $\vec{v}_p$.

(4)The intrinsic impedance of the wave $\eta$.

(5)The polarization of the wave.

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4.(15ps) A uniform sinusoidal plane wave in air with the following phasor expression for electric intensity $\vec{E}^i(x,z)=\vec{a}_y 10e^{-j(6x+8z)}$ is incident on a perfectly conducting plane at $z=0$

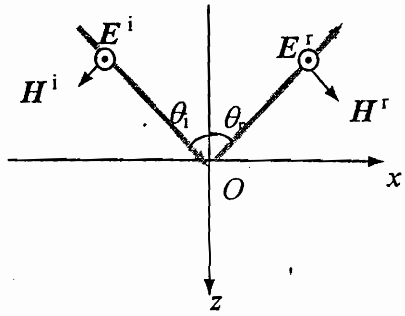
(1) Find the frequency and wavelength of the wave;

(2) Determine the angle of incidence;

(3) Find $\vec{E}^r(x,z)$ of the reflected wave;

(4) Find $\vec{H}^r(x,z)$ of the reflected wave;

(5) Find the surface current.



5. (15ps) A TE10 wave at 10GHz propagates in a rectangular waveguide with inner dimensions a=1.5cm and b=0.6cm, which is filled with a dielectric medium characterized by $\mu_r = 1$ $\varepsilon_r = 2.25$ Determine

(1) The cutoff frequency;

(2) the phase constant;

(3)the guide wavelength;

(4) the phase velocity;

(5) the wave impedance;

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# 重庆邮电大学 2010-2011 学年第 一 学期

## 《FIELD AND WAVE ELECTROMAGNETICS》 考试题

## （A卷）

| 题号 | 一 | 二 | 三 | 四 | 五 | 六 | 七 | 八 | 九 | 十 | 总分 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 分数 | | | | | | | | | | | |
| 评卷人 | | | | | | | | | | | |

考生必读（如果不读，后果自负）：

(1) 考生务必遵守考试纪律。

(2) 判断题答案写在题目括号内；其它题目答案全部写到答题纸上，并标明相应题号。

(3) 试卷中字母上方带箭头代表矢量，例如 $\bar{A}$；试卷中 $\hat{a}_x,\hat{a}_y,\hat{a}_z,\hat{a}_r,\hat{a}_\theta,\hat{a}_\varphi$ 为坐标系的单位矢量；$\bar{\bar{A}}$ 表示相量。

(4) 试卷正卷共 2 页，附带白纸 4 张（答题纸、草稿纸）。

(5) 交卷时，请将正卷和答卷一同交上，且保持试卷整洁。

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**Problem 1 (10 points)**

In the following descriptions, some are correct while others are incorrect. Please filling the correct mark "√" or incorrect mark "×" in the bracket ending each paragraph. (1 point per item)

(1) If the divergence of a vector field is zero, the vector field is said to be irrotational or conservative. (　　)

(2) The boundary conditions for time-varying electromagnetic fields between two lossless simple media interface are:

$$\hat{a}_{n2}\times\left(\bar{E}_1-\bar{E}_2\right)=0;\hat{a}_{n2}\times\left(\bar{H}_1-\bar{H}_2\right)=\bar{J}_s;$$

$$\hat{a}_{n2}\cdot\left(\bar{D}_1-\bar{D}_2\right)=0;\hat{a}_{n2}\cdot\left(\bar{B}_1-\bar{B}_2\right)=0;\qquad(\quad)$$

$\hat{a}_{n2}$ : from dielectric 2 to dielectric 1

(3) If a vector field is solenoidal(or divergenceless ),then it ca[illegible] expressed as the curl of another vector field. (    )

(4) In a charge-free region and simple medium, if $E_x = x; E_y = y$, then $E_z = -2z$ .(    )

(5) $\bar{J} = \sigma \bar{E}$ is hold in every medium . (    )

(6) The instantaneous magnetic energy density is $w_m = \frac{1}{2}\bar{B}(t)\cdot\bar{H}(t)$ (    )

(7) In a simple medium, the magnetostatics vector potential $\bar{A}$ satisfy the equation: $\nabla^2\bar{A} = -\mu\bar{J}$ (    )

(8) A uniform plane wave with the complex electric field intensity in a medium is $\tilde{\bar{H}} = \hat{a}_x 3 e^{-\sqrt{\frac{\omega\mu\sigma}{2}}z} e^{-j\sqrt{\frac{\omega\mu\sigma}{2}}z}$ ( $\omega$:angular frenquecy ).then the medium is dispersive.(    )

(9) in a conductor medium, the skin depth is independent of the dimensions of the conductor. (    )

(10)Rectangular wave-guide can support TE mode . (    )

**Problem 2 (15 points)**

The magnetic field intensity in a lossless, source free dielectric medium is given as $\bar{H} = \hat{a}_y H_0 \cos(\omega t + \beta x)$ (A/m), where $\omega, \beta$, and $H_0$ all are constants. Find:

(1) The phasor representation of magnetic field intensity $\tilde{\bar{H}}(r)$ .

(2) The electric field intensity $\bar{E}(r,t)$ in time domain.

(3) The phasor representation of electric field intensity $\tilde{\bar{E}}(r)$ .

(4) The instantaneous power density,or Poynting vector $\bar{S}$ .

(5) The average power density $< \bar{S} >$ or represented as $\bar{S}_{av}$ .

**Problem 3 (10 points)**

A uniform plane electromagnetic wave in a region have the following component of electric field intensity:

$$\bar{E} = \left(3j\hat{a}_z - 8\hat{a}_y\right)e^{-x}e^{-j0.2x} \ \text{V/m}$$

Determine:

(1) The propagation direction of the wave.

(2) The polarization of the wave.

(3) The propagation constant $\gamma$.

**Problem 4 (15 points)**

A wave propagates in a dielectric medium characterized by $\mu_r = 1, \varepsilon_r = 2.25$. The electric field intensity in the region is given by

$$\bar{E} = \hat{a}_y 50\cos\left(10^7 t - \beta x\right) \ \text{(V/m)}$$

Find:

(1) The frequency of the wave $f$.

(2) The phase constant $\beta$.

(3) The phase velocity $\bar{u}_p$.

(4) The intrinsic impedance $\eta$.

(5) The wavelength $\lambda$.

**Problem 5 (20 points)**

A uniform plane wave propagating in free space impinges at the plane surface of a perfect conductor. If the interface is at $z=0$, and the magnetic field intensity of the incident wave is given by $\bar{H}_i = \hat{a}_x H_0 e^{-j8(\sqrt{3}y+z)}$, where $H_0$ is a constant.

Find:

(1) the reflected angle $\vartheta_r$.

(2) $\bar{\bar{E}}_r(y,z)$ and $\bar{\bar{H}}_r(y,z)$ of the reflected wave.

**Problem 6(15 points)**

A rectangular wave-guide with $b$=**1cm** is filled with a dielectric of $\varepsilon_r = 4$, and is operated at **12GHz**. If the only mode of propagation is **$TE_{10}$**, and the phase constant of the $TE_{10}$ mode is **450rad/m**, calculate the length $a$ of the wave-guide.

**Problem 7 (15 points)**

The electric field intensity in the far zone from an antenna is given in terms of its maximum input current $I_0$ as $\bar{E}_\theta = \frac{15}{r} I_0 \sin\theta$.

(1) Obtain the corresponding expression for the magnetic field $\bar{H}_\varphi$.

(2) What is the total power radiated by the antenna $P_r$.

(3) What is the radiation resistance $R_r$.

# 重庆邮电大学 2010-2011学年第 一 学期

## 《FIELD AND WAVE ELECTROMAGNETICS》 考试题（A 卷）参考答案

**Problem 1 ( Each question with 1 point, total: 10 points )**

Decide whether the following statements are true or false. Write "T" for true and "F" for false.

(1) F  (2)F  (3)T  (4)T  (5)F  (6)T  (7)T  (8)T  (9)T  (10)T

**Problem 2 (15 points)**

The magnetic field intensity in lossless, source free dielectric medium is given as $\bar{H} = \hat{a}_y H_0 \cos(\omega t + \beta x)$ (A/m),and $\omega, \beta, H_0$ are consiant. Find

(1) the phasor representation of magnetic field intensity $\bar{\tilde{H}}(r)$

(2) the electric field intensity $\bar{E}(r,t)$ in time domain

(3) the phasor representation of electric field intensity $\bar{\tilde{E}}(r)$

(4) the instantaneous power density,or Poynting vector, $\bar{S}$

(5) the average power density, $< \bar{S} >$ or represented as $\bar{S}_{av}$.

Solution:

(1) the phasor representation of magnetic field intensity $\bar{\tilde{H}}(r)$

$$\bar{\tilde{H}}(r) = \hat{a}_y H_0 e^{j\beta x} \quad \text{(A/m)} \qquad 3分$$

(2) the electric field intensity $\bar{E}(r,t)$ in time domain

$$\nabla \times \bar{H} = \frac{\partial \bar{D}}{\partial t} \qquad 2分$$

$$\bar{E} = \hat{a}_z \frac{\beta}{\omega\varepsilon} H_0 \cos(\omega t + \beta x) \quad \text{(V/m)} \qquad 1分$$

Let us check to see if the given electric field intensity can exist in the dielectric medium.

$$\rho_v = \nabla \square \bar{D} = \frac{\partial}{\partial z}\left[\frac{\beta}{\omega} H_0 \cos(\omega t + \beta x)\right] = 0$$

So $\bar{D}$ can exist

$$\nabla \cdot \bar{B} = \frac{\partial}{\partial y}\left[\mu H_0 \cos(\omega t + \beta x)\right] = 0$$

So $\bar{B}$ also can exist

From $\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$, we get

$\beta^2 = \omega^2 \mu\varepsilon$ , this is the condition that the field exist.

(3) the phasor representation of electric field intensity $\bar{\bar{E}}(r)$

$$\bar{E}(r) = \hat{a}_z \frac{\beta}{\omega\varepsilon} H_0 e^{j\beta x} \quad \text{(V/m)} \qquad 3 分$$

(4) the instantaneous power density,or Poynting vector, $\bar{S}$

$$\bar{S} = \bar{E} \times \bar{H} = \hat{a}_z \frac{\beta}{\omega\varepsilon} H_0 \cos(\omega t + \beta x) \qquad 2 分$$

$$\times \hat{a}_y H_0 \cos(\omega t + \beta x)$$

$$= -\hat{a}_x \frac{\beta}{\omega\varepsilon} H_0^2 \cos^2(\omega t + \beta x) \ (W/m^2) \qquad 1 分$$

) the average power density, $< \bar{S} >$ or represented as $\bar{S}_{av}$

$$\bar{S}_{av} = \langle \bar{S} \rangle = \frac{1}{2}\mathrm{Re}\left(\bar{E} \times \bar{H}^*\right) \qquad 2分$$

$$= -\bar{a}_x \frac{\beta H_0^2}{2\omega\varepsilon}\left(\mathrm{W/m^2}\right) \qquad 1分$$

**Problem 3 (15 points)**

A uniform electric field intensity in a region have the following component of electric field intensity:

$$\bar{E} = \left(3j\hat{a}_z - 8\hat{a}_y\right)e^{-x}e^{-j0.2x} \ \text{V/m}$$

Determine:

(1) The propagation direction of the wave.

(2) The polarization of the wave.

(3) The propagation constant $\gamma$.

Solution:

(1) The propagation direction of the wave is in the $x$ direction. 5 分

(2) The polarization of the wave is a right-handed elliptically polarized wave. 5 分

(3) The propagation constant $\gamma = \alpha + j\beta = 1 + j0.2$. 5 分

**Problem 4 (15 points)**

A wave propagates in a dielectric medium characterized by $\mu_r = 1$, $\varepsilon_r = 2.25$. The electric field intensity in the region is given by

$$\bar{E} = \hat{a}_y 50\cos\left(10^7 t - \beta x\right) \text{ (V/m)}$$

Find the following:

(1) the wave frequency f.

(2) the phase constant ß

(3) the phase velocity of propagation $\bar{u}_p$

(4) the intrinsic impedance $\eta$

(5) the wavelength $\lambda$

slution:

(1) $f = \dfrac{\omega}{2\pi} = \dfrac{10^7}{2\pi} \ (Hz)$ 3 分

(2) $\beta = \omega\sqrt{\mu\varepsilon} = \omega\sqrt{\mu_r\varepsilon_r}\sqrt{\mu_0\varepsilon_0} = \dfrac{10^7\sqrt{2.25}}{3\times10^8} = 0.05 \ (rad/m)$ 3 分

(3) $u_p = \dfrac{\omega}{\beta} = \dfrac{10^7}{0.05} = 2\times10^8\bar{a}_x \ (m/s)$ 3 分

(4) $\eta = \sqrt{\dfrac{\mu}{\varepsilon}} = \eta_0\sqrt{\dfrac{\mu_r}{\varepsilon_r}} = \dfrac{120\pi}{1.5} = 80\pi \ (\Omega)$ 3 分

(5) $\lambda = \dfrac{2\pi}{\beta} = \dfrac{2\pi}{0.05} = 40\pi \ (m)$ 3 分

**Problem 5 (15 points)**

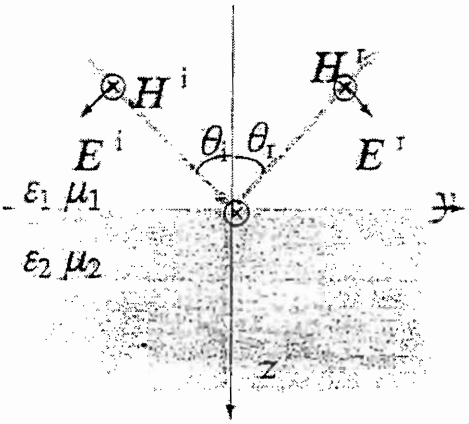
A uniform plane wave propagating in free space impinges at the plane surface of a perfect conductor. If the interface is at $z$=0, and the magnetic field intensity of the incident wave is given by $\bar{H}_i = \hat{a}_x H_0 e^{-j8(\sqrt{3}y+z)}$, where $H_0$ is a constant.

Find:

(1) the reflected angle $\vartheta_r$.

(2) $\bar{E}_r(y,z)$ and $\bar{H}_r(y,z)$ of the reflected wave.

Solution:




$$\bar{H}_i = \hat{a}_x H_0 e^{-\bar{\gamma}_1 z'} \tag{1}$$

$$= \hat{a}_x H_0 e^{-j\beta(z\cos\vartheta_i + y\sin\vartheta_i)}$$

$$= \hat{a}_x H_0 e^{-j16(\frac{\sqrt{3}}{2}y+\frac{1}{2}z)}$$

So the incidence angle $\vartheta_i = 60^0$ 5 分

form Snell's law of reflection, we obtain

$\vartheta_r = \vartheta_i = 60^0$

(2) as the medium 2 is a perfect conductorm, the fields exist only in free space. The reflection coefficient is $\rho = 1$. The magnetic field intensity of the reflected wave

$$\bar{H}_r = \hat{a}_x \rho H_0 e^{-j8(\sqrt{3}y-z)}$$

$$= \hat{a}_x H_0 e^{-j8(\sqrt{3}y-z)}$$ 5 分

The electric field intensity of the reflected wave $\bar{E}_r$

$$\bar{\bar{E}}_r = \eta_0 \bar{\bar{H}}_r \times \hat{a}_n$$

$$= \eta_0 H_0 e^{-j8(\sqrt{3}y-z)} \hat{a}_x \times \left(\bar{a}_y \sin\vartheta_r - \bar{a}_z \cos\vartheta_r\right)$$

$$= \eta_0 H_0 e^{-j8(\sqrt{3}y-z)} \left(\bar{a}_z \sin\vartheta_r + \bar{a}_y \cos\vartheta_r\right)$$

$$= \eta_0 H_0 e^{-j8(\sqrt{3}y-z)} \left(\bar{a}_z \frac{\sqrt{3}}{2} - \bar{a}_y \frac{1}{2}\right)$$ 5 分

**Problem 5 (15 points)**

A rectangular wave-guide with $b$=1cm is filled with a dielectric of $\varepsilon_r = 4$, and is operated at 12GHz. If the only mode of propagation is $TE_{10}$, and the phase constant of the $TE_{10}$ mode is 102.65rad/m, calculate the length $a$ of the wave-guide.

Solution: Form $\beta_{10} = \frac{\omega}{u_p}\sqrt{1-\left(\frac{f_{c10}}{f}\right)^2}$ the cutoff frequency is

$$f_{c10} = \sqrt{f^2 - \left(\frac{\beta_{10} u_p}{2\pi}\right)^2}$$

At the same time $f_{c10} = \frac{u_p}{2a}$

where $b = 0.01m, \beta_{10} = 450rad/m, \mu = \mu_0, \varepsilon = 4\varepsilon_0, f = 12\times10^9 GHz$

From above two equation ,wo get $a \approx 1.4(cm)$ 15 分

**Problem 6 (15 points)**

The electric field intensity in the far zone form an antenna is given in terms of its maximum input current $I_0$ as $\bar{E}_\theta = \frac{15}{r} I_0 \sin\theta$, Find the following:

(1) obtain the corresponding expression for the magnetic field $\bar{H}_\varphi$

(2) what is the total power radiated by the antenna $P_r$

(3) what is the radiation resistance $R_r$

solution:

(1) expression for the electric field field

$$\bar{\bar{E}}_\theta = \frac{15}{r} I_0 \sin\theta e^{-j\beta r} \bar{a}_\theta$$

expression for the magnetic field

$$\bar{\bar{H}}_\varphi = \bar{\bar{E}}_\theta / \eta = \frac{15}{r\eta} I_0 \sin\theta e^{-j\beta r} \bar{a}_\varphi \quad \left(\eta = \sqrt{\frac{\mu}{\varepsilon}}\right)$$ 5分

(2) the power density radiated by the antenna

$$\bar{S} = \frac{1}{2}\left[\bar{\bar{E}} \times \bar{\bar{H}}^*\right] = \frac{\bar{\bar{E}}_\theta^2}{2\eta} \bar{a}_r = \frac{15^2}{2\eta r^2} I_0^2 \sin^2\vartheta \bar{a}_r$$

the total power radiated by the antenna

$$\begin{aligned} P_r &= \oiint_S \langle \bar{S} \rangle \cdot d\bar{s} \\ &= \int_0^\pi \int_0^{2\pi} \frac{225 I_0^2 \sin^2\vartheta}{2\eta r^2} r^2 \sin\vartheta d\vartheta d\varphi \\ &= \frac{225 I_0^2}{2\eta} \int_0^\pi \sin^3\vartheta d\vartheta \int_0^{2\pi} d\varphi \\ &= \frac{300\pi I_0^2}{\eta} \end{aligned}$$ 5分

(3) the radiation resistance

$$R_r = \frac{2P_r}{I_0^2} = \frac{600\pi}{\eta}$$ 5分

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# 重庆邮电大学 2007—2008 学年第 一 学期

## 《FIELD AND WAVE ELECTROMAGNETICS》考试题

（B 卷）

| 题号 | 一 | 二 | 三 | 四 | 五 | 六 | 七 | 八 | 九 | 十 | 总分 |
|---|---|---|---|---|---|---|---|---|---|---|---|
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(4) 试卷正卷共 4 页，附带草稿纸 2 张

(5) 交卷时，请将正卷和答卷一同交上，且保持试卷整洁。

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**Problem 1 (10 points )**

In the following descriptions, some are correct while others are incorrect. Please filling the correct mark "√" or incorrect mark "×" in the bracket ending each paragraph. (1 point per item)

(1) If the curl of a vector field is zero, the vector field is said to be irrotational of conservative. (　　)

(2) The boundary conditions for time-varying electromagnetic fields between a dielectric and a perfect conductor interface are:

$\hat{a}_{n2}\times\bar{E}_1=0;\hat{a}_{n2}\bar{H}_1\times=\bar{J}_s;$

$\hat{a}_{n2}\cdot\bar{D}_1=0;\hat{a}_{n2}\cdot\bar{B}_1=0;$ (　　)

$\hat{a}_{n2}$ : from perfect conductor to dielectric

(3) The curl of the gradient of any scalar field is everywhere equal to zero. (  )

(4) In simple medium, if $B_x = x; B_y = y$, .then $B_z = 2z$ (  )

(5) $\bar{J} = \sigma \bar{E}$ is hold in every medium (  )

(6) The instantaneous energy density of electric field is $w_e = \frac{1}{2}\varepsilon \bar{D}(r,t)\cdot \bar{E}(r,t)$

(  )

(7) In a simple medium, the scalar potential V satisfy the equation in electrostatics: $\nabla^2 V = -\frac{\rho}{\varepsilon}$ (  )

(8) A uniform plane wave with the complex electric field intensity in a medium is $\bar{\tilde{E}} = \hat{a}_x 3e^{-\sqrt{\frac{\omega\mu\sigma}{2}}z} e^{-j\sqrt{\frac{\omega\mu\sigma}{2}}z}$ ($\omega$:angular frenquecy) , then the medium is dispersive (  )

(9) In a conductor medium, the skin depth is the half of the depth of the conductor

(  )

(10) Rectangular wave-guide can support TEM mode(  )

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**Problem 2 (15 points)**

The electric field intensity in air is given as $\bar{E} = \hat{a}_x E_0 \cos(\omega t - \beta z)$ (V/m),and $\omega, \beta, E_0$ all are constants.

Find:

(1) The phasor representation of electric field intensity. $\bar{\tilde{E}}(r)$ ?

(2) The magnetic field intensity $\bar{H}(r,t)$ in time domain?

(3) The phasor representation of magnetic field intensity, $\bar{\tilde{H}}(r)$ ?

(4) The instantaneous power density,or Poynting vector, $\bar{S}$ ?

(5) The average power density, $< \bar{S} >$ or represented as $\bar{S}_{av}$ ?

**Problem 3 (15 points)**

A uniform plane electromagnetic wave in a region have the following component of electric field intensity:

$$\bar{E} = \left(5\ \hat{a}_x + 5j\ \hat{a}_y\right) e^{-0.2z} e^{-j0.2z}\ \text{V/m}$$

Determine:

(1) The propagation direction of the wave

(2) The polarization of the wave

(3) The propagation constant $\gamma$

**Problem 4 (15 points)**

A wave propagates in a dielectric medium characterized by $\mu_r = 1, \varepsilon_r = 9$. The electric field intensity in the region is given by

$$\vec{E} = \hat{a}_z 377\cos\left(10^9 t + \beta x\right) \text{ (V/m)}$$

Find:

(1) The wave frequency f.

(2) The phase constant ß

(3) The phase velocity of propagation $\vec{u}_p$

(4) The intrinsic impedance $\eta$

(5) The wavelength $\lambda$

**Problem 5 (15 points)**

A uniform plane wave propagating in free space impinges at the plane surface of a perfect conductor. If the interface is at $z$=0, and the electric field intensity of the incident wave is given by $\bar{E}_i = \hat{a}_x E_0 e^{-j10(y+z)}$ where $E_0$ is a constant

Find:

(1) The incidence angle $\vartheta_i$

(2) The reflected angle $\vartheta_r$

(3) The electric field intensity of the reflected wave $\bar{\bar{E}}_r$ (phasor form)

(4) The magnetic field intensity of the reflected wave $\bar{\bar{H}}_r$ (phasor form)

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**Problem 6 (15 points)**

The phase constant of the $TE_{10}$ mode of an air-filled wave-guide with **b=1cm** is **102.65rad/m**. If the operating frequency of the wave-guide is **12GHz**, and the only mode of propagation is $TE_{10}$, calculate the length a of the wave-guide

**Problem 7 (15 points)**

The magnetic field intensity in the far zone from an antenna is given in terms of its maximum input current $I_0$ as $\tilde{H}_\varphi = \frac{1}{120\pi}\frac{15}{r}I_0$,

(1) Obtain the corresponding expression for the electric field $\tilde{E}_\theta$

(2) What is the total power radiated by the antenna $P_r$

(3) What is the radiation resistance $R_r$

第4页，共6页

# 重庆邮电大学 2007—2008 学年第 一 学期

# 《FIELD AND WAVE ELECTROMAGNETICS》考试题（B 卷答案）

**Problem 1 (10 points )**

In the following descriptions, some are correct while others are incorrect. Please filling the correct mark " √ " or incorrect mark " × " in the bracket ending each paragraph. (1 point per item)

(1) If the curl of a vector field is zero, the vector field is said to be irrotational of conservative. ( √ )

(2) The boundary conditions for time-varying electromagnetic fields between a dielectric and a perfect conductor interface are:

$$\hat{a}_{n2}\times\bar{E}_1=0;\hat{a}_{n2}\bar{H}_1\times=\bar{J}_s;$$
$$\hat{a}_{n2}\cdot\bar{D}_1=0;\hat{a}_{n2}\cdot\bar{B}_1=0; \qquad (\times)$$

$\hat{a}_{n2}$ : from perfect conductor to dielectric

(3) The curl of the gradient of any scalar field is everywhere equal to zero. ( √ )

(4) In simple medium, if $B_x=x;B_y=y$, then $B_z=2z$ ( × )

(5) $\bar{J}=\sigma\bar{E}$ is hold in every medium ( × )

(6) The instantaneous energy density of electric field is $w_e=\frac{1}{2}\varepsilon\bar{D}(r,t)\cdot\bar{E}(r,t)$ ( √ )

(7) In a simple medium, the scalar potential V satisfy the equation in electrostatics: $\nabla^2 V=-\frac{\rho}{\varepsilon}$ ( √ )

(8) A uniform plane wave with the complex electric field intensity in a medium is $\bar{\bar{E}}=\hat{a}_x 3e^{-\sqrt{\frac{\omega\mu\sigma}{2}}z}e^{-j\sqrt{\frac{\omega\mu\sigma}{2}}z}$ ( $\omega$:angular frenquecy ) , then the medium is dispersive ( √ )

(9) In a conductor medium, the skin depth is the half of the depth of the conductor ( × )

(10) Rectangular wave-guide can support TEM mode( × )

**Problem 2 (15 points)**

The electric field intensity in air is given as $\bar{E}=\hat{a}_x E_0\cos(\omega t-\beta z)$ (V/m),and $\omega,\beta,E_0$ all are constants. Find:

(1) The phasor representation of electric field intensity, $\bar{\bar{E}}(r)$ ; $\bar{\bar{E}}(r)=\hat{a}_x E_0 e^{-j\beta z}$ (3 points)

1

(2) The magnetic field intensity $\bar{H}(r,t)$ in time domain?

$$\bar{H}(z,t) = \mathrm{Re}\,\tilde{\bar{H}}(r)e^{j\omega t} = \mathrm{Re}\,\hat{a}_y \frac{E_0}{\eta} e^{-j\beta z} e^{j\omega t} = \hat{a}_y \frac{E_0}{\eta} \cos(\omega t - \beta z) \text{ (3 points)}$$

(3) The phasor representation of magnetic field intensity, $\tilde{\bar{H}}(r)$?

$$\tilde{\bar{H}}(r) = \frac{1}{\eta}\left[\hat{a}_z \times \tilde{\bar{E}}(r)\right] = \frac{1}{\eta}\left[\hat{a}_z \times \hat{a}_x E_0 e^{-j\beta z}\right] = \hat{a}_y \frac{E_0}{\eta} e^{-j\beta z} \text{ (3 points)}$$

(4) the instantaneous power density,or Poynting vector, $\bar{S}$?

$$\bar{S}(z,t) = \bar{E}(z,t) \times \bar{H}(z,t)$$

$$= \left[\hat{a}_x E_0 \cos(\omega t - \beta z)\right] \times \left[\hat{a}_y \frac{1}{\eta} E_0 \cos(\omega t - \beta z)\right] \text{ (3 points)}$$

$$= \hat{a}_z \frac{E_0^2}{\eta} \cos^2(\omega t - \beta z)$$

(5) The average power density, $<\bar{S}>$ or represented as $\bar{S}_{av}$?

$$\bar{S}_{av} = \frac{1}{2}\mathrm{Re}(\tilde{\bar{E}} \times \tilde{\bar{H}}^*) = \frac{1}{2}\mathrm{Re}(\hat{a}_x E_0 e^{-j\beta z} \times \hat{a}_y \frac{E_0}{\eta} e^{j\beta z}) = \hat{a}_z \frac{E_0^2}{2\eta} \text{ (3 points)}$$

**Problem 3 (15 points)**

A uniform plane electromagnetic wave in a region have the following component of electric field intensity:

$$\bar{E} = \left(5\,\hat{a}_x + 5j\,\hat{a}_y\right)e^{-0.2z}e^{-j0.2z} \text{ V/m}$$

Determine:

(1) The propagation direction of the wave: **z direction (5points)**

(2) The polarization of the wave

$$E_y(z,t) = 5e^{-0.2z}\cos(\omega t - 0.2z + \frac{\pi}{2}) = -5e^{-0.2z}\sin(\omega t - 0.2z)$$

$$E_x(0,t) = 5\cos\omega t\ ;\ \ E_y(0,t) = -5\sin\omega t$$

$$\frac{E_x^2(0,t)}{25} + \frac{E_y^2(0,t)}{25} = 1$$

$$t = 0,\ \bar{E} = \bar{a}_x 5\ ;\ \ t = \frac{T}{4},\ \bar{E} = -\bar{a}_y 5$$

∴ The polarization of the wave is **left-handed circularly polarized** (5 points)

(3) The propagation constant $\gamma$ : $\gamma = 0.2 + 0.2j$ (5 points)

**Problem 4 (15 points)**

A wave propagates in a dielectric medium characterized by $\mu_r = 1, \varepsilon_r = 9$. The electric field intensity in the region is given by $\bar{E} = \hat{a}_z 377\cos\left(10^9 t + \beta x\right)$ (V/m)

Find:

(1) The wave frequency f. $\omega = 10^9; \omega = 2\pi f; f = \frac{\omega}{2\pi} = \frac{10^9}{2\pi}$ Hz (3 points)

(2) The phase constant ß: $\beta = \omega\sqrt{\mu\varepsilon} = \omega\sqrt{\mu_0\varepsilon_0}\sqrt{\mu_r\varepsilon_r} = \frac{\omega}{c}\sqrt{\mu_r\varepsilon_r} = 10$ (rad/m) (3 points)

(3) The phase velocity of propagation $\bar{u}_p : \bar{u}_p = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{c}{\sqrt{\mu_r\varepsilon_r}} = \frac{c}{3} = 10^8\left(-a_x\right)$ (3 points)

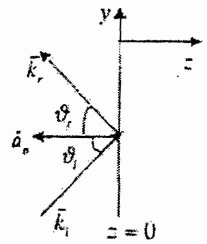
(4) The intrinsic impedance $\eta : \eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0}{\varepsilon_0}}\sqrt{\frac{\mu_r}{\varepsilon_r}} = \frac{\eta_0}{3} = 40\pi$ (3 points)

(5) The wavelength $\lambda = \frac{2\pi}{\beta} = \frac{\pi}{5}$ (3 points)

**Problem 5 (15 points)**

A uniform plane wave propagating in free space impinges at the plane surface of a perfect conductor. If the interface is at $z=0$, and the electric field intensity of the incident wave is given by

$\bar{E}_i = \hat{a}_x E_0 e^{-j10(y+z)}$ where $E_0$ is a constant




Find:

(1) The incidence angle $\vartheta_i$

$$\bar{E}_i = \hat{a}_x E_0 e^{-j10(y+z)} = \hat{a}_x E_0 e^{-j\left(k_{ix}x + k_{iy}y + k_{iz}z\right)}$$

$$\therefore k_{ix} = 0; k_{iy} = 10; k_{iz} = 10$$

$$\bar{k}_i = k_{ix}\hat{a}_x + k_{iy}\hat{a}_y + k_{iz}\hat{a}_z = 10\hat{a}_y + 10\hat{a}_z$$

$$\bar{k}_i = 10\hat{a}_y + 10\hat{a}_z = |\bar{k}_i|\left(\sin\vartheta_i\hat{a}_y + \cos\vartheta_i\hat{a}_z\right)$$

$$= 10\sqrt{2}\left(\sin\vartheta_i\hat{a}_y + \cos\vartheta_i\hat{a}_z\right) \therefore \sin\vartheta_i = \frac{\sqrt{2}}{2}; \cos\vartheta_i = \frac{\sqrt{2}}{2}$$ **(4 points)**

$$\therefore \vartheta_i = 45^0$$

(2)The reflected angle $\vartheta_r : \vartheta_r = \vartheta_i = 45^0$ **(3 points)**

(3)The electric field intensity of the reflected wave $\bar{\bar{E}}_r$ (phasor form)

$$\because \bar{k}_r = k_{rx}\hat{a}_x + k_{ry}\hat{a}_y + k_{rz}\hat{a}_z = 10\hat{a}_y - 10\hat{a}_z;$$

$$\rho = -1;$$

**(4 points)**

$$\bar{\bar{E}}_r = \hat{a}_x\left(-E_0\right)e^{-j10(y-z)};$$

(4)The magnetic field intensity of the reflected wave $\bar{\bar{H}}_r$ (phasor form)

$$\bar{\bar{H}}_r = \frac{1}{\eta_0}\hat{a}_r \times \bar{\bar{E}}_r$$

$$= \frac{1}{\eta_0}\left(\sin\vartheta_r\hat{a}_y - \cos\vartheta_r\hat{a}_z\right) \times \left(\hat{a}_x\left(-E_0\right)e^{-j10(y-z)}\right)$$

$$= \frac{1}{120\pi}\left(\frac{\sqrt{2}}{2}\hat{a}_y - \frac{\sqrt{2}}{2}\hat{a}_z\right) \times \left(\hat{a}_x\left(-E_0\right)e^{-j10(y-z)}\right)$$ **(4points)**

$$= \frac{\sqrt{2}}{204\pi}\left(\hat{a}_z + \hat{a}_y\right)E_0 e^{-j10(y-z)}$$

**Problem 6 (15 points)**

The phase constant of the $TE_{10}$ mode of an air-filled wave-guide with b=1cm is 102.65rad/m. If the operating frequency of the wave-guide is 12GHz, and the only mode of propagation is $TE_{10}$, calculate the length a of the wave-guide

$$\beta_{10} = \beta\sqrt{1-\left(\frac{f_{c10}}{f}\right)^2} = \omega\sqrt{\mu_0\varepsilon_0} = 251.44\left(rad/m\right)$$ **(5 points)**

$$102.65 = 251.44\sqrt{1-\left(\frac{f_{c10}}{12\times10^9}\right)^2}; f_{c10} = 10.95 \quad \text{GHz}$$

$$f_{c10} = \frac{u_p}{2a}; \text{ ( 5 points)}$$

$$a = \frac{u_p}{2f_{c10}} = 0.0136 \text{ m}; \quad a = 1.36 \text{ cm} \quad \text{( 5points)}$$

**Problem 7 (15 points)**

The magnetic field intensity in the far zone from an antenna is given in terms of its maximum input current $I_0$ as $\tilde{H}_\varphi = \frac{1}{120\pi}\frac{15}{r}I_0$,

(1) Obtain the corresponding expression for the electric field

$$\tilde{H}_\varphi = \frac{1}{120\pi}\frac{15}{r}I_0; \because \tilde{E}_\theta / \tilde{H}_\varphi = 120\pi$$

$$\therefore \tilde{E}_\theta = \frac{15}{r}I_0$$

**( 5 points)**

(2) What is the total power radiated by the antenna $P_r$

$$\tilde{S}_{av} = \hat{a}_r \frac{E_\theta^2}{2\eta_0} = \hat{a}_r \frac{E_\theta^2}{240\pi} = \hat{a}_r \frac{15*15I_0^2}{240\pi r^2} = \hat{a}_r \frac{15I_0^2}{16\pi r^2}$$

$$P_r = 4\pi r^2 * |\tilde{S}_{av}| = \frac{15}{4}I_0^2$$

**( 5 points)**

(3) What is the radiation resistance $R_r$

$$P_r = \frac{15}{4}I_0^2 = \frac{1}{2}R_r I_0^2$$

$$R_r = \frac{15}{2}$$

**( 5 points)**

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# 重庆邮电大学 2005—2006 学年第 一 学期

## 《FIELD AND WAVE ELECTROMAGNETICS》考试题

（A 卷）

| 题号 | 一 | 二 | 三 | 四 | 五 | 六 | 七 | 八 | 九 | 十 | 总分 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 分数 | | | | | | | | | | | |
| 评卷人 | | | | | | | | | | | |

**考生须知：**

**$\mathbf{a}_x$, $\mathbf{a}_y$, $\mathbf{a}_z$表示笛卡尔系的基矢量，黑斜体字符表示的物理量为矢量。**

**Problem 1** ( Each question with 1 point, total: 10 points )

Decide whether the following statements are true or false.

Write "T" for true and "F" for false

(1) "$\nabla\cdot$" and "$\nabla\times$" should be regarded as the del operator followed with a dot or cross product. _____

(2) The curl of the gradient of any scalar field is identically zero. _____

(3) If **D** is the electric flux density, $\nabla\cdot\mathbf{D}$ will always represent the free charge density in any medium. _____

(4) The tangential components of the magnetic field **H** at any boundary are always discontinuous. _____

(5) The relative permittivity of simple medium is a constant. _____

(6) The time –varying magnetic field will supply energy to the charged particle. _____

(7) Single-conductor waveguides are dispersive transmission systems.

______

(8) The normal component of current densities $J$ has to be discontinuous across the boundary between two different media under electrostatic conditions. ______

(9) The phase velocity and the wave impedance for TEM waves are independent of the frequency of the waves. ______

(10) $TM_{11}$ has the lowest cutoff frequency of all TM modes in a rectangular waveguide. ______

**Problem 2 (15 points)**

The upper and lower conducting plates of a large parallel-plate capacitor are separated by a distance $d$ and maintained at potentials $V_0$ and 0 respectively, as shown in Fig.1. A dielectric slab of dielectric constant $\varepsilon_r$ and uniform thickness placed 0.8$d$ is over the lower plate. Assuming negligible fringing effect, determine (a) the potential and electric field distribution in the dielectric slab, (b) the potential and electric field distribution in the air space between the dielectric slab and the upper plate. (c) the surface charge densities on the upper and lower plates.

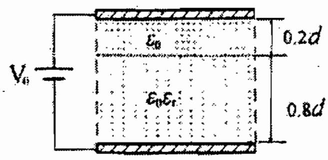


Fig.1

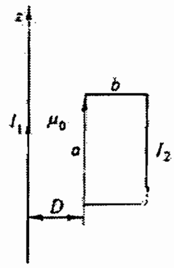


Fig.2

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**Problem 3 (15 points)**

Determine the mutual inductance between a very long straight wire and a conducting rectangular loop, as shown in Fig.2.

**Problem 4 (15 points)**

The electric field intensity of a uniform plane wave in free space is given by

$$\mathbf{E} = \mathbf{a}_x 94.25\cos(\omega t + 6z) \quad \text{(V/m)}$$

Determine (a) the velocity of propagation, (b) the wave frequency, (c) the wavelength, (d) the magnetic field intensity, and (e) the average power density in the medium.

**Problem 5 (15 points)**

A plane wave represented by the phasor

$$\mathbf{E}_i(z) = 4(\mathbf{a}_x + j\mathbf{a}_y)e^{-j\beta z} \quad \text{(V/m)}$$

impinges normally on a perfectly conducting wall at z = 0. Determine (a) the polarization of the reflected wave. (b) the induced current on the conducting wall. (c) the instantaneous expression of the total electric field intensity based on a cosine time reference.

**Problem 6 (15 points)**

An air-filled waveguide operates at 7GHz. The dimensions of the waveguide are a = 3 cm and b = 2 cm. Calculate (a) the cutoff frequency of $TE_{10}$, (b) the phase constant of $TE_{10}$, (c) the wave impedance of $TE_{10}$, (d) the maximum average power transmitted along the waveguide without causing any breakdown inside the waveguide at the $TE_{10}$ mode. The dielectric strength of air is 30 kV /cm. Use a safety factor of 10.

**Problem 7 (15 points)**

An electric dipole of length 50 cm is situated in free space. If the maximum value of the current is 25 A and its frequency is 10 MHz, determine (a) the electric and magnetic fields in the far zone, (b) the average power density, and (c) the radiation resistance.

Problem 4

Solution (a) The wave propagates in free space with the speed of light. Because the wave is traveling in the negative $z$ direction, the phase velocity is

$$\vec{u}_p = -3\times10^8\ \vec{a}_z\ \text{m/s} \qquad (3\ \text{points})$$

(b) $\beta_0 = 6$ rad/m, so the angular frequency of the wave is

$$\omega = \beta_0 u_p = 6\times3\times10^8 = 1.8\times10^9\ \text{rad/s} \quad (3\ \text{points})$$

(c) The wavelength of the wave in free space is

$$\lambda_0 = \frac{2\pi}{\beta_0} = \frac{2\pi}{6} = 1.047\ \text{m} \qquad (3\ \text{points})$$

(d) The electric field intensity, in phasor form, is

$$\vec{E} = 94.25\, e^{j\beta z}\ \vec{a}_x\ \text{V/m}$$

The corresponding $\vec{H}$ field for the backward-traveling wave, is

$$\vec{H} = -\frac{94.25}{377}\, e^{j6z}\, \vec{a}_x\ \text{A/m} \quad (3\ \text{points})$$

or

$$\vec{H}(z,t) = -0.25\cos(1.8\times10^9 t + 6z)\ \vec{a}_y\ \text{A/m}$$

(e) The average power density in the medium is

$$\vec{\mathcal{P}}_{av} = \frac{1}{2}\,\mathrm{Re}[\vec{E}\times\vec{H}^*]$$

$$= -\frac{1}{2}\times94.25\times0.25\ \vec{a}_z = -11.78\ \vec{a}_z\ \text{W/m}^2.$$

(3 points)

Problem 5

Solution: (a) $\vec{E}_i(z) = 4(\vec{a}_x + j\vec{a}_y)e^{-j\beta z}$ (V/m)

$\vec{E}_r(z) = \Gamma \vec{E}_i(z) = (-1)\,4(\vec{a}_x + j\vec{a}_y)e^{j\beta z}$ (V/m) 3 points

(1 points)

the reflected wave is a Right-hand circularly polarized wave. (1 points) (RHC 5 points).

(b) $\vec{J}_s = (-\vec{a}_z)\times \vec{H}_1\Big|_{z=0}$ 3 points

due to $\vec{H}_i(z) = \vec{a}_z \times \frac{1}{\eta_1}\vec{E}_i = \frac{1}{\eta_1}\vec{a}_z \times 4(\vec{a}_x + j\vec{a}_y)e^{-j\beta z}$

$$= \frac{4}{\eta_1}(\vec{a}_y - j\vec{a}_x)e^{-j\beta z}$$

$$\vec{H}_r(z) = (-\vec{a}_z)\times\frac{1}{\eta_1}\vec{E}_r$$

$$= \frac{4}{\eta_1}(\vec{a}_y - j\vec{a}_x)e^{j\beta z}$$

$$\vec{H}_1(z) = \vec{H}_i(z) + \vec{H}_r(z) = \frac{8}{\eta_1}(\vec{a}_y - j\vec{a}_x)\cos\beta z.$$

Hence $\vec{J}_s(z)\Big|_{z=0} = (-\hat{a}_z)\times\vec{H}_1(z)\Big|_{z=0} = \frac{8}{\eta_1}(\vec{a}_x + j\vec{a}_y)$

$$\vec{J}_s(z,t)\Big|_{z=0} = Re\left[\vec{J}_s(z)e^{j\omega t}\right]\Big|_{z=0} = Re\left[\frac{8}{\eta_1}(\vec{a}_x + j\vec{a}_y)e^{j\omega t}\right]$$

$$= \frac{8}{\eta_1}(\vec{a}_x\cos\omega t - \vec{a}_y\sin\omega t)$$ 2 points

(c) $\vec{E}_1(z) = \underbrace{\vec{E}_i + \vec{E}_r}_{\text{(2 points)}} = 8(\vec{a}_y - j\vec{a}_x)\sin\beta z$

$\vec{E}_1(z,t) = \underbrace{Re\left[\vec{E}_1(z)e^{j\omega t}\right]}_{\text{(4 points)}} = 8\underbrace{(\sin\beta z)(\vec{a}_y\cos\omega t + \vec{a}_x\sin\omega t)}_{\text{(1 points)}}$ V/m

where $\eta_1 = \sqrt{\frac{\mu_1}{\varepsilon_1}}$ is the intrinsic impedance in medium 1.

Problem 6

Solution (a) The cutoff frequency of $TE_{10}$ is

$$f_{c10} = \frac{u_p}{2a} = \frac{3\times10^8\ m/s}{2\times0.03\,m} = 5\times10^9\ Hz \quad (6\ \text{points})$$

(b) The phase constant of $TE_{10}$ is

$$\beta = \frac{\omega}{u_p}\sqrt{1-(f_c/f)^2} = \frac{2\pi\, 7\times10^9 Hz}{3\times10^8\ m/s}\sqrt{1-(5\times10^9/7\times10^9)^2}$$

$$= 102.67\ rad/m = 32.66\pi\ rad/m. \quad (3\ \text{points})$$

(c) The wave impedance / intrinsic impedance of $TE_{10}$ is

$$Z_{TE_{10}} = \frac{\sqrt{\frac{\mu}{\varepsilon}}}{\sqrt{1-(f_c/f)^2}} = \frac{120\pi}{\sqrt{1-(5\times10^9/7\times10^9)^2}} = \frac{120\pi}{\sqrt{1-(5/7)^2}}$$

$$= 538.68\ (\Omega) = 171.43\pi\ (\Omega) \quad (3\ \text{points})$$

(d) The existing component of the electric field for $TE_{10}$, is

$$E_y = E_{ym}\sin\left(\frac{\pi}{a}x\right)e^{-j\beta_{10}z}$$

The dielectric strength of air is given as 30kV/cm or 3MV/m, so the maximum value of $E_y$ is 0.3MV/m with a safety factor of 10. Thus

$$E_y = 3\times10^5\sin\left(\frac{\pi}{a}x\right)e^{-j\beta_{10}z}$$

the maximum average power density is

$$\vec{\mathcal{P}}_{10} = \frac{1}{2}\left[\frac{(3\times10^5)^2\sin^2\left(\frac{\pi}{a}x\right)}{536.68}\right]\vec{a}_z$$

$$= 83.54\sin^2\left(\frac{\pi}{a}x\right)\vec{a}_z \quad MW/m^2.$$

the maximum power that can be safely transmitted along the waveguide is

$$P_{10} = \int_0^b \int_0^a 83.54 \times 10^6 \sin^2\left(\frac{\pi}{a}x\right) dx\, dy = 25.66\ kW. \quad (3\ \text{point.})$$

Problem 7

Solution Since the dipole is radiating in free space, the fields propagate with the speed of light, $c = 3\times10^8$ m/s.

$$\omega = 2\pi f = 6.283\times10^7\ \text{rad/s}$$

The phase constant: $\beta = \frac{\omega}{c} = 0.209$ rad/m. 3 points

From the given data, we have $I = 25\angle 0°\ A$ and $L = 0.5m$

Substituting in the far-field components, we obtain

$$\vec{H} = \frac{j0.208}{r}\sin\theta\, e^{-j0.209r}\,\vec{a}_\phi\ \ A/m \qquad 3\ \text{points}$$

$$\vec{E} = \frac{j78.416}{r}\sin\theta\, e^{-j0.209r}\,\vec{a}_\theta\ \ V/m. \qquad 3\ \text{points}$$

Thus, the average power density in the radial direction, is

$$\vec{\mathcal{P}}_r = \frac{8.15}{r^2}\sin^2\theta\,\vec{a}_r\ \ W/m^2 \qquad 3\ \text{points}$$

and the total power crossing a spherical surface at r, is

$$P_{rad} = 68.25\ W/m^2$$

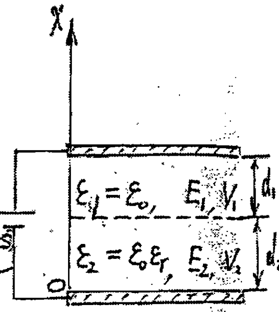
Finally, the radiation resistance, is

$$R_{rad} = \frac{2}{25^2}\times 68.25 = 0.22\ \Omega \qquad 3\ \text{points}$$

Problem 2

Solution (a) $V_0 = V_1 + V_2$, $V_1 = E_1 d_1$, $V_2 = E_2 d_2$

at the interface between two media, we have




$D_1 = D_2$, $\Rightarrow \varepsilon_1 E_1 = \varepsilon_2 E_2$. (1 points)

Hence

$$\vec{E}_2 = \frac{-\vec{a}_x V_0}{\left(\frac{\varepsilon_0\varepsilon_r}{\varepsilon_0} d_1 + d_2\right)} = \frac{-\vec{a}_x 5V_0}{(4+\varepsilon_r)d} \quad 2.5 \text{ points}$$

$$\vec{E}_1 = -\vec{a}_x \frac{\varepsilon_2}{\varepsilon_1} E_2 = \frac{-\vec{a}_x 5\varepsilon_r V_0}{(4+\varepsilon_r)d} \quad 2.5 \text{ points}$$

(b) due to $\vec{E} = -\nabla V = -\left(\vec{a}_x\frac{\partial V}{\partial x} + \vec{a}_y\frac{\partial V}{\partial y} + \vec{a}_z\frac{\partial V}{\partial z}\right) = -\vec{a}_x\frac{dV}{dx}$.

$$\vec{a}_x \cdot \vec{E} = -\frac{dV}{dx}.$$

thus $dV_2 = -\vec{a}_x \cdot \vec{E}_2\, dx = \frac{5V_0}{(4+\varepsilon_r)d}\, dx$

Hence $V_2 = \int_0^x \frac{5V_0}{(4+\varepsilon_r)}\, dx = \frac{5V_0 x}{(4+\varepsilon_r)d}$ $\quad 0 \le x \le d_2$

2.5 points

$$dV_1 = -\vec{a}_x \cdot \vec{E}\, dx.$$

$$V_1 = -\int_0^{0.8d} \vec{E}_2 \cdot d\vec{x} - \int_{0.8d}^{x} \vec{E}_1 \cdot d\vec{x}$$

$$= -\int_0^{0.8d} \frac{-\vec{a}_x 5V_0}{(4+\varepsilon_r)d}\, \vec{a}_x dx - \int_{0.8d}^{x} \frac{-\vec{a}_x 5\varepsilon_r}{(4+\varepsilon_r)d}\, \vec{a}_x dx$$

$$= \frac{5\varepsilon_r x - 4(\varepsilon_r - 1)d}{(4+\varepsilon_r)d} \cdot V_0 \quad 2.5 \text{ points}$$

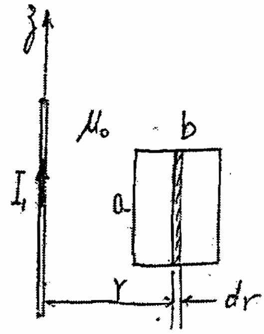
(c) $\rho_s\big|_{x=d} \overset{1\text{ point}}{=} \underline{\vec{a}_n \cdot \vec{D}} = (-\vec{a}_x)\cdot \varepsilon_1 \vec{E}_1 = \frac{5\varepsilon_0\varepsilon_r V_0}{(4+\varepsilon_r)d}$. $\quad$ 2.5 points

$\rho_s\big|_{x=0} = \underline{\vec{a}_n \cdot \vec{D}} = \vec{a}_x \cdot \varepsilon_2 \vec{E}_2 = -\frac{5\varepsilon_0\varepsilon_r V_0}{(4+\varepsilon_r)d}$, $\quad$ 2.5 points

Problem 3

Solution By cylindrical coordinate, we have

$$\vec{B}_1 = \frac{\mu_0 I_1}{2\pi r} \vec{a}_\phi \quad \text{2 points}$$




$$\Psi_{21} = \int_S \vec{B}_1 \cdot d\vec{s}$$

$$= \frac{\mu_0 I_1 a}{2\pi} \int_0^{D+d} \frac{1}{r} dr$$

$$= \frac{\mu_0 I_1 a}{2\pi} \ln\left(\frac{D+b}{D}\right) \quad \text{6 points}$$

Hence the mutual inductance is

$$M_{21} = \frac{\Psi_{21}}{I_1} = \frac{\mu_0 a}{2\pi} \ln\left(\frac{D+d}{D}\right) \quad \text{7 points}$$

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# 重庆邮电学院 2004—2005 学年第 一 学期

## 《 Field and Wave of Electromagnetics 》考试题

| 题号 | 一 | 二 | 三 | 四 | 五 | 六 | 七 | 八 | 九 | 十 | 总分 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 分数 | | | | | | | | | | | |
| 评卷人 | | | | | | | | | | | |

**考生须知：**

- 表示矢量时，须在有关符号上加箭头。
- 离开考场时，须将试题及答卷一同交上，否则，试卷无效。

I. 选择填空题：（每题 1 分， 共 10 分）

1. A vector field (vector point function), A, is determined to within an additive constant if _____ is specified everywhere.

a) $\nabla\cdot A$  b) $\nabla\times A$  c) both $\nabla\cdot A$ and $\nabla\times A$

2. Under static conditions the E field on a perfect conductor surface is everywhere ________.

a) normal to the surface.

b) parallel to the surface.  c) zero.

3. A simple medium means _____

a) a linear medium. b) a linear, and homogeneous medium. c) a linear, homogeneous, and isotropic medium.

4. The effect of the magnetization vector, M, is equivalent

to both a volume current density $J_m$ and a surface current density $J_{ms}$, and the respective $J_m$ and $J_{ms}$ are ______ .

a) $\nabla\times M$, and $M\times a_n$; b) $M\times a_n$ and $\nabla\times M$ c) $\nabla\times H_t$ and $a_{n2}\times(H_1-H_2)$

5. The electric field intensity in a region of time-varing magnetic flux density is ______

a) conservative. b) nonconservative. c) vectorial.

6. The term ____ is called displacement current density.

a) $\rho a$ b) $\partial D/\partial t$ c) $\sigma E$

7. A good conductor is a medium for which ______.

a) $\sigma/\omega\varepsilon >> 1$. b) $\sigma/\omega\varepsilon << 1$. c) $\sigma = \infty$.

8. The distance $\delta$ through which the amplitude of a traveling plane wave decreases by a factor of _____ is called the skin depth or the depth of penetration of a conductor.

a) $\exp(-\alpha z)$ b) $\alpha z$ c) $\exp(-1)$

9. Waves that contain a nonzero $E_z$, but $H_z = 0$, are _____

a) TEM wave. b) TE wave c) TM wave.

10. The angle $\theta_i$ for successive total internal reflection, as indicated in Fig.1, satisfies

a) $\sin\theta_i \le (\varepsilon_r - 1)^{1/2}$. b) $\sin\theta_i = 1/\varepsilon_r^{1/2}$. c) $\sin\theta_i \ge \varepsilon_r^{1/2}$.

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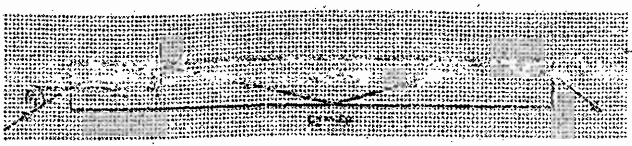

Fig. 1

II. 计算题：（每题 15 分，共 90 分）

1. An emf $V$ is applied across a cylindrical capacitor of length $L$, as shown in Fig.2. The radii of the inner and outer conductors are $a$ and $b$ respectively. The space between the conductors is filled with two different lossy dielectrics having, respectively, permittivity $\varepsilon_1$ and conductivity $\sigma_1$ in the region $a < r < c$, and permittivity $\varepsilon_2$ and conductivity $\sigma_2$ in the region $c < r < b$. Determine

   a) the current density in each region.

   b) the surface charge densities on the inner and outer conductors and at the interface between the two dielectrics.

2. Determine the mutual inductance between a very long straight wire and a conducting equilateral trangular loop, as shown in Fig.3

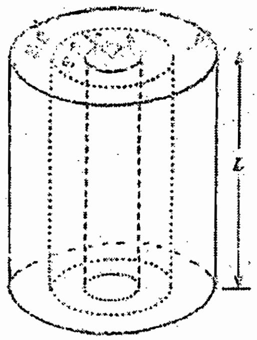


Fig. 2

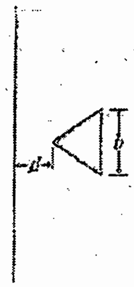


Fig. 3

3. The instantaneous expression for the magnetic field intensity of a uniform plane wave propagating in the $+y$ direction in air is given by

$$\mathbf{H}(y,t) = \mathbf{a}_x 4\times10^{-6} \cos(10^7\pi t - k_0 y + \pi/4) \quad (A/m)$$

a) Determine the $k_0$.

b) Write the complex vector (i.e. the vector phasor) $\mathbf{H}(y)$, $\mathbf{E}(y)$.

c) Write the instantaneous expression for $\mathbf{E}(y, t)$.

d) Find the time – average Poynting vector $\mathcal{P}_{av}$.

4. A right – hand circularly polarized plane wave represented by the phasor

$$\mathbf{E}(z) = E_0(\mathbf{a}_x - j\mathbf{a}_y)e^{-j\beta z} \qquad (V/m)$$

propagates in air and impinges normally on a perfectly conducting wall at $z = 0$.

a) Determine the polarization of the reflected wave.

b) Find the induced current on the conducting wall.

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c) Obtain the instantaneous expression of the total electric field intensity based on a cosine time reference.

5. A air–filled axb (b < a < 2b) rectangular waveguide is to be constructed to operate at 3 (GHz) in the dominant mode. We desire the operating frequency to be at least 20% higher than the cutoff frequency of the dominant mode and also at least 20% below the cutoff frequency of the next higher–order mode.

a) Given a typical design for the dimensions a and b.

b) Calculate for your design $\beta$, $u_p$, $\lambda_g$, and the wave impedance at the operating frequency.

6. The current distribution on a center - fed short dipole antenna of length 2h ( $h \ll \lambda$) can be approximated by a triangular function

$$I(z) = I_0\,(1 - |z|/h) \qquad (A)$$

Find (a) the far–zone electric and magnetic field intensities, (b) the radiation resistance, and (c) the directivity.

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c) Obtain the instantaneous expression of the total electric field intensity based on a cosine time reference.

5. A air–filled axb (b < a < 2b) rectangular waveguide is to be constructed to operate at 3 (GHz) in the dominant mode. We desire the operating frequency to be at least 20% higher than the cutoff frequency of the dominant mode and also at least 20% below the cutoff frequency of the next higher–order mode.

a) Given a typical design for the dimensions a and b.

b) Calculate for your design $\beta$, $u_p$, $\lambda_g$, and the wave impedance at the operating frequency.

6. The current distribution on a center - fed short dipole antenna of length 2h ( h << $\lambda$) can be approximated by a triangular function

$$I(z) = I_0 (1 - |z|/h) \qquad (A)$$

Find (a) the far–zone electric and magnetic field intensities, (b) the radiation resistance, and (c) the directivity.

Solution. 一、选择题: 1. c. 2. a. 3. c. 4. a. 5. b.
6. b. 7. a. 8. c. 9. c. 10. a

$\boxed{\text{II. 1}}$

Method (1)

a) due to $V = (R_1+R_2)I = (R_1+R_2)JS$

and $J_1 = J_2 = J \quad S = 2\pi r \cdot L$

where $R_1 = \int_a^c \frac{dr}{\sigma_1 (2\pi r \cdot L)}$

$$R_2 = \int_c^b \frac{dr}{\sigma_2 (2\pi r \cdot L)}$$

$$R_1+R_2 = \frac{1}{2\pi\sigma_1 \cdot L} \ln\frac{c}{a} + \frac{1}{2\pi\sigma_2 \cdot L} \ln\frac{b}{c}$$

Therefore $$J = \frac{V}{(R_1+R_2)S} = \frac{\sigma_1\sigma_2 V}{(\sigma_1 \ln\frac{b}{c} + \sigma_2 \ln\frac{c}{a})r} \quad (A/m^2) \quad (8\ \text{marks})$$

$$\vec{E}_1 = \hat{r}\frac{J_1}{\sigma_1} = \hat{r}\frac{\sigma_2 V}{(\sigma_1 \ln\frac{b}{c} + \sigma_2 \ln\frac{c}{a})r} \quad (V/m) \qquad (a<r<c)$$

$$\vec{E}_2 = \hat{r}\frac{J_2}{\sigma_2} = \hat{r}\frac{\sigma_1 V}{(\sigma_1 \ln\frac{b}{c} + \sigma_2 \ln\frac{c}{a})r} \quad (V/m) \qquad (c<r<b)$$

b) at $r=a$, $$\rho_{sa} = \hat{n}_1 \cdot \vec{D}_1\Big|_{r=a} = \hat{r}\cdot(\hat{r}\varepsilon_1 E_1)\Big|_{r=a} = \frac{\varepsilon_1\sigma_2 V}{(\sigma_1 \ln\frac{b}{c} + \sigma_2 \ln\frac{c}{a})a} \quad (C/m^2) \quad (2\ \text{marks})$$

at $r=b$ $$\rho_{sb} = \hat{n}_2 \cdot \vec{D}_2\Big|_{r=b} = -\hat{r}\cdot(\hat{r}\varepsilon_2 E_2)\Big|_{r=b} = -\frac{\varepsilon_2\sigma_1 V}{(\sigma_1 \ln\frac{b}{c} + \sigma_2 \ln\frac{c}{a})r} \quad (C/m^2) \quad (2\ \text{marks})$$

at $r=c$. $$\rho_{sc} = (D_2 - D_1)\Big|_{r=c} = \frac{\varepsilon_2\sigma_1 V}{(\sigma_1 \ln\frac{b}{c} + \sigma_2 \ln\frac{c}{a})c} - \frac{\varepsilon_1\sigma_2 V}{(\sigma_1 \ln\frac{b}{c} + \sigma_2 \ln\frac{c}{a})c} \quad (2\ \text{marks})$$

$$= \frac{(\varepsilon_2\sigma_1 - \varepsilon_1\sigma_2)V}{(\sigma_1 \ln\frac{b}{c} + \sigma_2 \ln\frac{c}{a})c} \quad (C/m^2)$$

Method (2)

a) due to $$\vec{J} = \hat{r}\, I/(2\pi r) \cdot L \quad (A/m^2) \quad (8\ \text{marks})$$

$$E_{1r} = J_r/\sigma_1 = I/(2\pi r) \cdot L \cdot \sigma_1 \quad (V/m) \qquad (a<r<c)$$

$$E_{2r} = J_r/\sigma_2 = I/(2\pi r) \cdot L \cdot \sigma_2 \quad (V/m) \qquad (c<r<b)$$

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The voltage between conductors.

$$V = -\int_b^c \vec{E}\cdot d\vec{l} - \int_c^a \vec{E}\cdot d\vec{l} = -\int_b^c \hat{r}E_{r2}\,(\hat{r}dr) - \int_c^a \hat{r}E_{r1}\,(\hat{r}dr)$$

$$= -\int_b^c E_{r2}\,dr - \int_c^a E_{r1}\,dr = -\int_b^c \frac{I}{(2\pi r)L\cdot\sigma_2}\,dr - \int_c^a \frac{I}{(2\pi r)L\cdot\sigma_1}\,dr$$

$$= \frac{I}{2\pi L\sigma_2}\ln\frac{b}{c} + \frac{I}{2\pi L\sigma_1}\ln\frac{c}{a} \qquad (V)$$

Hence

$$I = \frac{2\pi\sigma_1\sigma_2 V L}{\sigma_1\ln\frac{b}{c} + \sigma_2\ln\frac{c}{a}} \qquad (A)$$

$$\vec{E}_1 = \hat{r}\,\frac{I}{(2\pi r)\cdot L\cdot\sigma_1} = \frac{\sigma_2 V}{(\sigma_1\ln\frac{b}{c} + \sigma_2\ln\frac{c}{a})\,r} \quad (V/m) \quad (a<r<c)$$

$$\vec{E}_2 = \hat{r}\,\frac{I}{(2\pi r)\cdot L\sigma_2} = \frac{\sigma_1 V}{(\sigma_1\ln\frac{b}{c} + \sigma_2\ln\frac{c}{a})\,r} \quad (V/m) \quad (c<r<b)$$

b) at $r=a$ $\quad \rho_{sa} = \hat{n}_1\cdot\vec{D}_1\Big|_{r=a} = \hat{r}\cdot(\hat{r}\varepsilon_1 E_1)\Big|_{r=a} = \dfrac{\varepsilon_1\sigma_2 V}{(\sigma_1\ln\frac{b}{c} + \sigma_2\ln\frac{c}{a})\,a}\ (C/m^2)$ (2 marks)

at $r=b$ $\quad \rho_{sb} = \hat{n}_2\cdot\vec{D}_2\Big|_{r=b} = -\hat{r}\cdot(\hat{r}\varepsilon_2 E_2)\Big|_{r=b} = -\dfrac{\varepsilon_2\sigma_1 V}{(\sigma_1\ln\frac{b}{c} + \sigma_2\ln\frac{c}{a})\,b}\ (C/m^2)$ (2 marks)

at $r=c$. $\quad \rho_{sc} = (D_2 - D_1)\Big|_{r=c} = \dfrac{\varepsilon_2\sigma_1 V}{(\sigma_1\ln\frac{b}{c} + \sigma_2\ln\frac{c}{a})\,c} - \dfrac{\varepsilon_1\sigma_2 V}{(\sigma_1\ln\frac{b}{c} + \sigma_2\ln\frac{c}{a})\,c}$

$$= \frac{(\varepsilon_2\sigma_1 - \varepsilon_1\sigma_2)\,V}{(\sigma_1\ln\frac{b}{c} + \sigma_2\ln\frac{c}{a})\,c} \qquad (C/m^2) \quad \text{(3 marks)}$$

Sol. (3).

Assume the inner conductor, $r=a$ $(a<c<b)$ carry uniform density $\tau_1$ and the interface between mediums, $r=c$, carry charge density $\tau_2$, using Gauss's Law, we have

$$\vec{E}_1 = \hat{r}\frac{\tau_1}{2\pi\varepsilon_1 r} \qquad a \le r \le c \qquad (1)$$

$$\vec{E}_2 = \hat{r}\frac{\tau_1+\tau_2}{2\pi\varepsilon_2 r} \qquad c \le r \le b \qquad (2)$$

The voltage between inner conductor and outer conductor is

$$V = -\int_b^c \vec{E}\cdot d\vec{l} - \int_c^a \vec{E}\cdot d\vec{l} = -\int_b^c \hat{r}\frac{\tau_1+\tau_2}{2\pi\varepsilon_2 r}\hat{r}\,dr - \int_c^a \hat{r}\frac{\tau_1}{2\pi\varepsilon_1 r}\hat{r}\cdot dr$$

$$= \frac{\tau_1+\tau_2}{2\pi\varepsilon_2}\ln\frac{b}{c} + \frac{\tau_1}{2\pi\varepsilon_1}\cdot\ln\frac{c}{a} \qquad (V) \qquad (3)$$

at $r=c$, we have

$$J_1 = J_2 . \qquad (4)$$

or $$\sigma_1 E_1 = \sigma_2 E_2 \qquad (5)$$

Substituting Eqn (1) and Eqn (2) into Eqn (5), we have

$$\sigma_1\frac{\tau_1}{2\pi\varepsilon_1 c} = \sigma_2\frac{\tau_1+\tau_2}{2\pi\varepsilon_2 c}$$

or $$\tau_1+\tau_2 = \left(\frac{\sigma_1}{\sigma_2}\cdot\frac{\varepsilon_2}{\varepsilon_1}\right)\tau_1 \qquad (6)$$

Solving Eqn (3) and Eqn (6), we have

$$\tau_1 = \frac{2\pi\varepsilon_1\sigma_2 V}{\sigma_2\ln\frac{c}{a} + \sigma_1\ln\frac{b}{c}}$$

and $$\tau_2 = \frac{2\pi(\sigma_1\varepsilon_2 - \sigma_2\varepsilon_1)\cdot V}{\sigma_2\ln\frac{c}{a} + \sigma_1\ln\frac{b}{c}} \qquad (7)$$

Substituting Eqn (7) into Eqn (1) and Eqn (2) we obtain

$$\vec{E}_1 = \hat{r}\frac{\sigma_2 V}{(\sigma_2\ln\frac{c}{a} + \sigma_1\ln\frac{b}{c})r} \quad (V/m) \quad (a \le r \le c)$$

$$\vec{E}_2 = \hat{r}\frac{\sigma_1 V}{(\sigma_2\ln\frac{c}{a} + \sigma_1\ln\frac{b}{c})r} \quad (V/m) \quad (c \le r \le b)$$

(8 marks)

b) at $r=a$ $\rho_{sa} = \hat{n}\cdot\vec{D}_1 = D_1 = \varepsilon_1 E_1 = \frac{\varepsilon_1\sigma_2 V}{(\sigma_2\ln\frac{c}{a}+\sigma_1\ln\frac{b}{c})a}$ $(C/m^2)$ (2 marks)

at $r=b$ $\rho_{sb} = \hat{n}\cdot\vec{D}_2 = -D_2 = -\varepsilon_2 E_2 = -\varepsilon_2\sigma_1 V / (\sigma_2\ln\frac{c}{a}+\sigma_1\ln\frac{b}{c})\,b$ $(C/m^2)$ (2 marks)

at $r=c$ $\rho_{sc} = D_2 - D_1 = \varepsilon_2 E_2 - \varepsilon_1 E_1 = (\varepsilon_2\sigma_1 - \varepsilon_1\sigma_2)V / (\sigma_2\ln\frac{c}{a}+\sigma_1\ln\frac{b}{c})\cdot c$ (3 marks) $(C/m^2)$

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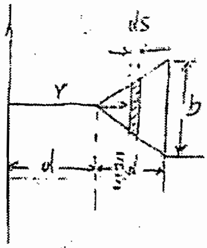
**[II. 2]**

due to $\vec{B} = \hat{\phi}\frac{\mu_0 I}{2\pi r}$, (T) (5 marks)




$$\Phi = \int_S \vec{B}\cdot d\vec{S} = \int_d^{d+\frac{\sqrt{3}}{2}b} \frac{\mu_0 I}{2\pi r}\cdot\frac{2(r-d)}{\sqrt{3}}\,dr$$

$$= \frac{\mu_0 I}{\pi\sqrt{3}}\left(r - d\ln r\right)\Big|_d^{d+\frac{\sqrt{3}}{2}b}$$

$$= \frac{\mu_0 I}{\pi\sqrt{3}}\left[\frac{\sqrt{3}}{2}b - d\ln\frac{d+\frac{\sqrt{3}}{2}b}{d}\right]$$

$$= \frac{\mu_0 I}{\pi}\left[\frac{b}{2} - \frac{d}{\sqrt{3}}\ln\left(1+\frac{\sqrt{3}}{2}\frac{b}{d}\right)\right] \quad \text{(Wb) (5 marks)}$$

Therefore

$$M = \frac{\Phi}{I} = \frac{\mu_0}{\pi}\left[\frac{b}{2} - \frac{d}{\sqrt{3}}\ln\left(1+\frac{\sqrt{3}}{2}\frac{b}{d}\right)\right] \quad \text{(H) (5 marks)}$$

**[II. 3.]**

(a) $k_0 = \omega\sqrt{\mu_0\epsilon_0} = \frac{\omega}{c} = \frac{10^7\pi}{3\times10^8} = \frac{\pi}{30}$ (rad/m) (3 marks)

(b) $\vec{H}(y) = \hat{a}_z 4\times10^{-6} e^{-jk_0 y + j\frac{\pi}{4}} = \hat{a}_z 4\times10^{-6} e^{-j\frac{\pi}{30}y + j\frac{\pi}{4}}$ (A/m) (3 marks)

$$\vec{E}(y) = -\hat{a}_y \times \eta\vec{H}(y) = -\hat{a}_x 120\pi\times4\times10^{-6} e^{-j\frac{\pi}{30}y + j\frac{\pi}{4}}$$

$$= -\hat{a}_x 480\pi\times10^{-6} e^{-j\frac{\pi}{30}y + j\frac{\pi}{4}} = -\hat{a}_x 1.508\times10^{-3} e^{-j\frac{\pi}{30}y + j\frac{\pi}{4}} \quad \text{(V/m) (3 marks)}$$

(c) $\vec{E}(y,t) = Re\left[\vec{E}(y)\cdot e^{j\omega t}\right] = Re\left[-\hat{a}_x 480\pi\times10^{-6} e^{-j\frac{\pi}{30}y + j\frac{\pi}{4} + j10^7\pi t}\right]$

$$= -\hat{a}_x 480\pi\times10^{-6}\cos\left(10^7\pi t - \frac{\pi}{30}y + \frac{\pi}{4}\right) \quad \text{(V/m)}$$

$$= -\hat{a}_x 1.508\times10^{-3}\cos\left(10^7\pi t - \frac{\pi}{30}y + \frac{\pi}{4}\right) \quad \text{(3 marks) (V/m)}$$

$$\vec{P}_{av} = \frac{1}{2}Re\left[\vec{E}(y)\times\vec{H}^*(y)\right]$$

$$= \frac{1}{2}Re\left[-\hat{a}_x 480\pi\times10^{-6} e^{-j\frac{\pi}{30}y + j\frac{\pi}{4}} \times \hat{a}_z 4\times10^{-6} e^{j\frac{\pi}{30}y - j\frac{\pi}{4}}\right]$$

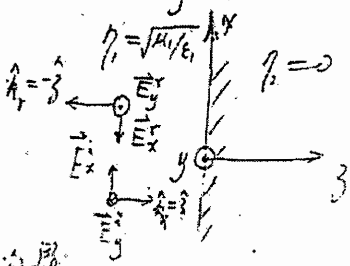
$$= \hat{a}_y \frac{1}{2}\times480\pi\times10^{-6}\times4\times10^{-6} \ \text{(W/m}^2\text{)} = \hat{a}_y 960\pi\times10^{-12} \ \text{(W/m}^2\text{)} \quad \text{(3 marks)}$$

$= \hat{a}_y\, 2.03 \times 10^{4} \quad (V/m)$

$\boxed{\text{II. } 4}$

a). $\vec{E}_r(z) = \Gamma \vec{E}_i(z) = (-1) E_0 (\hat{x} - j\hat{y})\, e^{j\beta z}$

the reflected wave is a left-hand circularly polarized wave. ( 5 marks )




b) $\vec{J}_s = (-\hat{z}) \times \vec{H}_1 \Big|_{z=0}$

due to $\vec{H}_i(z) = \hat{z} \times \frac{1}{\eta_1} E_i = \frac{E_0}{\eta_1}(\hat{y}+j\hat{x})\, e^{-j\beta z}$

$$\vec{H}_r(z) = (-\hat{z}) \times \frac{1}{\eta_1}\vec{E}_r = (-\hat{z})\times\frac{1}{\eta_1}(-1)E_0(\hat{x}-j\hat{y})e^{j\beta z}$$

$$= \frac{E_0}{\eta_1}(\hat{y}+j\hat{x})\, e^{j\beta z}$$

$$\vec{H}_1(z) = \vec{H}_i + \vec{H}_r = \frac{E_0}{\eta_1}(\hat{y}+j\hat{x})(e^{-j\beta z}+e^{j\beta z}) = \frac{2E_0}{\eta_1}(\hat{y}+j\hat{x})\cos\beta z$$

Hence $\vec{J}_s\Big|_{z=0} = (-\hat{z})\times \vec{H}_1\Big|_{z=0} = (-\hat{z})\times\frac{2E_0}{\eta_1}(\hat{y}+j\hat{x})\cos\beta z\Big|_{z=0}$

$$= (\hat{x}-j\hat{y})\frac{2E_0}{\eta_1}\cos\beta z\Big|_{z=0} = (\hat{x}-j\hat{y})\frac{2E_0}{\eta_1}$$

$$\vec{J}(z,t)\Big|_{z=0} = Re\left[\vec{J}_s(z)\, e^{j\omega t}\right] = Re\left[(\hat{x}-j\hat{y})\frac{2E_0}{\eta_1}e^{j\omega t}\right]$$

$$= \frac{2E_0}{\eta_1}\left(\hat{x}\cos\omega t + \hat{y}\sin\omega t\right)$$ (A/m) ( 5 marks )

( ) $\vec{E}_1(z) = \vec{E}_i + \vec{E}_r = E_0(\hat{x}-j\hat{y})e^{-j\beta z} + (-1)E_0(\hat{x}-j\hat{y})e^{j\beta z}$

$$= E_0(\hat{x}-j\hat{y})(e^{-j\beta z}-e^{j\beta z}) = -2E_0(j\hat{x}+\hat{y})\sin\beta z$$

$$\vec{E}_1(z,t) = Re\left[\vec{E}_1(z)\, e^{j\omega t}\right] = 2E_0(\sin\beta z)(\hat{x}\sin\omega t - \hat{y}\cos\omega t) \quad (V/m)$$

( 5 marks )

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