MATH 142: MATHEMATICAL MODELING FINAL PROJECT

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1. Summary

The goal of this project is for you to get some hands on experience with mathematical modeling. Your group will pick one of the provided MCM problems and develop your own model. If you would like to propose a different project, you must speak with me by the end of Week 6.

Expectations. While working on your project you should start thinking about the following things.

(1) The MCM problems are quite open ended. You must identify **what is the modeling question you wish to answer** and build your model from there. This question does not have to be exactly what the MCM problem is asking, but it should be inspired by it.
(2) The MCM problems provide way more detail and complexity than you can reasonably incorporate. It is **necessary** to make simplifying assumptions. Clearly state what assumptions you are making and justify them. A justification of "incorporating this information made the model too challenging to work with" or "we didn't know how to incorporate this information" is acceptable.
(3) You do NOT have to build a model from scratch. Think about models we've seen in class or on homework assignments. Can you modify/generalize them to work in your framework? You cannot simply use an existing model (for example, the logistic equation), but you can use it as inspiration and build on it (for example, how we started with the a linear growth model, then added the carrying capacity, then the Allee effect, and lastly harvesting).
(4) Once you write a model, you must analyze it. **Discuss qualitative and quantitative properties of the model; that is, the equilibrium points, stability analysis, long-term behavior, existence and uniqueness of solutions.** This can be done analytically in some cases, or it may require using Python/Matlab to generate phase portraits and solutions from some initial conditions that demonstrate the system's qualitative behaviors.
(5) Discuss limitations of your model. What factors aren't accounted for that could have some effect in the real world? If you had more time, what would you like to add to your model or what additional analysis would you do?

2. Timeline

Project proposal due ..... Friday November 15
Project checkpoint due ..... Friday November 22
Presentation due ..... Monday December 2
Write up due ..... Friday December 6

NOTE: If your group is having difficulty or wants to discuss anything with me at any point then please let me know!

3. Project Components and Rubrics

Each group must complete the following parts of the project.

3.1. **Project Proposal (10 Points).** By the end of Week 7, you must submit an approximately 1-2 page project proposal. It should be cohesive and consist of full sentences. It must summarize your proposed project and detail how the work will be divided fairly among group members. Remember each group member must make a mathematical/technical contribution. This can include some portion of the analysis, generating examples using MATLAB/python, reading related papers, and summarizing results of your model. Things like writing the conclusion for the report or making a summary slide for the presentation do **not** count as technical contributions.

You should briefly summarize the MCM problem you chose and discuss what aspect of the problem you plan to work on. For example, what is the main question you're trying to investigate? What mathematical approach will you take (discrete time? continuous time?) What simplifying assumptions will you make to make the problem more feasible? What challenges do you foresee?

Rubric.

- 5 Points: Project topic is reasonable and includes sufficient detail.
- 2 Points: Proposal is cohesive, written in full sentences.
- 2 Points: Proposal outlines a plan for dividing the work fairly (including a technical contribution for each group member).
- 1 Point: Includes relevant references.

3.2. **Project Checkpoint (5 Points).** By the end of Week 8, each **individual group member** should submit a paragraph about where they are at with the project. This should reflect progress your group has made, anything you're stuck on, and your next steps. This is also a place to let me know if you are having any issues with the dynamic of your group.

This portion of the grade is individual and contributes only to YOUR individual project grade. Thus, your grade will not be harmed if one of your group members does not submit in time.

Rubric.

- 5 points: Checkpoint indicates some progress has been made and that the group is on schedule to complete their project. Is not a direct copy of what other group members submitted.

3.3. **Project Presentation (30 Points).** During Week 10, your group will give an approximately 7-8 minute presentation on your project (exact amount of time for each group is TBD). Your presentation should be slides made using Google slides, powerpoint, or Beamer, whichever your group prefers. Regardless, your presentation slides must be submitted as a PDF on BruinLearn before class on Monday of week 10. Your presentation must have a title slide (with the title and names of group members) at the beginning and a references slide at the end. The structure of your presentation between these two things is up to you, but it should essentially summarize your writeup.

Each member of the group must speak in the presentation. Remember that your audience for your presentation is your classmates. You can assume the know the material presented in class, but if you used any additional tools or techniques you should present them to the class.

After the presentation, there will be time for the audience to ask questions. You should be prepared to answer any questions about your presentation.

Rubric.

- 10 points: Group clearly explained their model and analysis.
- 10 points: Presentation slides are well organized, easy to read, and include the appropriate information.
- 5 points: Group was able to answer questions after the presentation
- 3 points: Each group member participated in the presentation
- 2 points: Presentation stays within given time-frame

3.4. **Write Ups (50+5 Points).** You will have two written documents: your mathematical writeup (50 pts) and your non-mathematical summary (5pts).

Your mathematical writeup should be written in the form of a research article where your intended audience is your classmates in other groups. You should write it as if you are talking to other students in the class; you can expect readers know the topics we covered in lecture, but will need a more thorough description of methods/concepts you learned for this project outside of class. A specific LaTeXtemplate will be provided for you on BruinLearn. I recommend using Overleaf as it's free to make an account and all group members can collaborate together on the same document. Your non-mathematical write up should be a 1 page document/letter/summary written for an audience as specified in your MCM problem (if your MCM problem doesn't have one specified, come talk to me). This does not have to be written in latex.

Mathematical Writeup. Your writeup should include the following components

(1) Title and author list
(2) Abstract: This gives a brief synopsis of what is in your report. It should be understandable to students in Math 142 at a broad level.
(3) Introduction: This describes, in big-picture terms, what the goal of your project is.
(4) Background: More precise mathematical details.
(5) Your Model: Carefully describe your model. This should be understandable to all Math 142 students.
(6) Results: Carefully describe the results of your project.
(7) Conclusions and Discussion: Summarize your results and discuss their implications. It may be relevant to bring up improvements you would make if you had time. For example, if there is a limitation of your project like a step that doesn't work as well as you wanted, how would you go about fixing it if you had time?
(8) Acknowledgments: Acknowledge all help and resources from others. This could include the instructor or TA if they provided specific advice, other classmates who gave you ideas or proof read your report, etc.
(9) References: You must include proper citations with in-text references and full citations in the references section of all sources that you use. This includes (but is not limited to) sources for data sets, methods used, similar work that exists or that you used for inspiration, background work that helps with the big picture, and so on. If someone in your group has read this far you get two points added to your project for free. Email me to let me know you found it. You can (and probably will) use the textbook as a citation. I have included an example .bib file. You do not have to use a .bib file, but you will almost certainly find that to be the easiest way to do things. Make sure you cite everything.

(10) Author contributions: State who did what. Note all group members should make *technical* contributions, not simply writing or putting together slides.

Requirements.

- Writeup length: approx 4 pages but no more than $N + 2$ where N is the number of students in your group. Note this page limit does not including references.
- Figures: There should be figures to illustrate your work. This typically includes figures of results, and it often includes figures to help describe the problem (or, when relevant, how a mathematical model or algorithm works).
- Equations: You will have equations. Number the important ones (for example, the one that specifies your model) and refer to them by number. Explicitly define all notation in them.
- References: You must have them!!
- Non-Mathematical Document: 1 page, written for the appropriate audience.

Formatting. Formatting can be tricky and takes time. Do not wait until the last minute to put together the report. Feel free to e-mail or talk to me with any specific questions regarding formatting or the report contents. If this is your first time working with LaTeX, it has a learning curve but you will get used to it. It is a great skill for math majors to have!

Rubric – Mathematical Write-Up.

- 5 points: Abstract – Concisely summarizes the motivations for the study and your main findings.
- 7.5 points: Introduction – Clearly summarizes the topic. It references previous works that are relevant to your study, and it describes their main conclusions (and, as necessary, methods) and their relevance to your work.
- 12.5 points: Methods – Clear description of your model and what you did (any tools or algorithms used, code that you wrote, etc.). It should include enough detail that an interested reader can replicate the work that you did. Include a hyperlink to a code repository.
- 12 points: Results and Analysis – Present the results of your model using appropriate text and visual representations (figures, tables, etc.). Use varied methods of visualization, and make sure that all axes are appropriately labeled. Every figure should explained, with an appropriately descriptive caption, and referenced explicitly in the main text. Arguments flow together, and figures are well integrated with the text.
- 7.5 points: Conclusions and Discussion – Summarize your write-up, as well as the extent to which your model meets the stated aims. Describe issues, limitations, and possible future directions for the work.
- 2.5 points: References – The references are appropriate for the paper, written correctly and precisely, and in a consistent format.
- 2.5 points: Author Contributions – Clearly explains what each person contributed to the project and to its write-up.
- 2.5 Points: Formatting – Properly formatted using Latex.

Rubric – Non-Mathematical Write-Up.

- 2 points: Is written with the appropriate audience in mind
- 3 points: Is a clear interpretation of the mathematical results. Accurately summarizes with was found using the model without being technical.