Consistently Estimating the Selectivity of Conjuncts of Predicates

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Abstract

Cost-based query optimizers need to estimate the selectivity of conjunctive predicates when comparing alternative query execution plans. To this end, advanced optimizers use multivariate statistics (MVS) to improve information about the joint distribution of attribute values in a table. The joint distribution for all columns is almost always too large to store completely, and the resulting use of partial distribution information raises the possibility that multiple, non-equivalent selectivity estimates may be available for a given predicate. Current optimizers use ad hoc methods to ensure that selectivities are estimated in a consistent manner. These methods ignore valuable information and tend to bias the optimizer toward query plans for which the least information is available, often yielding poor results. In this paper we present a novel method for consistent selectivity estimation based on the principle of maximum entropy (ME). Our method efficiently exploits all available information and avoids the bias problem. In the absence of detailed knowledge, the ME approach reduces to standard uniformity and independence assumptions. Our implementation using a prototype version of DB2 UDB shows that ME improves the optimizer's cardinality estimates by orders of magnitude, resulting in better plan quality and significantly reduced query execution times.

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1. Introduction

Estimating the selectivity of predicates has always been a challenging task for a query optimizer in a relational database management system. A classic problem has been the lack of detailed information about the joint frequency distribution of attribute values in the table of interest. Perhaps ironically, the additional information now available to modern optimizers has in a certain sense made the selectivity-estimation problem even harder.

Specifically, consider the problem of estimating the selectivity $s_{1,2,\ldots,n}$ of a *conjunctive predicate* of the form $p_1 \wedge p_2 \wedge \ldots \wedge p_n$, where each p_i is a *simple predicate* (also called a *Boolean Factor,* or BF) of the form "*column op literal*". Here *column* is a column name, *op* is a relational comparison operator such as "=", ">", or "LIKE", and *literal* is a literal in the domain of the column; some examples of simple predicates are 'make = "Honda"' and 'year > 1984'. By the *selectivity* of a predicate p, we mean, as usual, the fraction of rows in the table that satisfy p.[1] In older optimizers, statistics are maintained on each individual column, so that the individual selectivities $s_1, s_2, \ldots, s_n$ of $p_1, p_2, \ldots, p_n$ are available. Such a query optimizer would then impose an *independence assumption* and estimate the desired selectivity as $s_{1,2,\ldots,n} = s_1 * s_2 * \ldots * s_n$. Such estimates ignore correlations between attribute values, and consequently can be wildly inaccurate, often underestimating the true selectivity by orders of magnitude and leading to a poor choice of query execution plan (QEP).

Ideally, to overcome the problems caused by the independence assumption, the optimizer should store the multidimensional joint frequency distribution for all of the columns in the database. In practice, the amount of

[1] Note that without loss of generality each p_i can also be a disjunction of simple predicates or any other kind of predicate (e.g., subquery, IN-list). For this work we only require that the optimizer has some way to estimate the selectivity s_i of p_i.

storage required for the full distribution is exponentially large, making this approach infeasible. Researchers therefore have proposed storage of selected *multivariate statistics* (MVS) that summarize important partial information about the joint distribution. Proposals have ranged from multidimensional histograms [PI97] on selected columns to other, simpler forms of column-group statistics [IMH+04]. Thus, for predicates $p_1, p_2, \ldots, p_n$, the optimizer typically has access to the individual selectivities $s_1, s_2, \ldots, s_n$ as well as a limited collection of joint selectivities, such as $s_{1,2}$, $s_{3,5}$, and $s_{2,3,4}$. The independence assumption is then used to "fill in the gaps" in the incomplete information, e.g., we can estimate the unknown selectivity $s_{1,2,3}$ by $s_{1,2} * s_3$.

A new and serious problem now arises, however. There may be multiple, non-equivalent ways of estimating the selectivity for a given predicate. Figure 1, for example, shows possible QEPs for a query consisting of the conjunctive predicate $p_1 \wedge p_2 \wedge p_3$. The QEP in Figure 1(a) uses an index-ANDing operation ($\wedge$) to apply $p_1 \wedge p_2$ and afterwards applies predicate p_3 by a FETCH operator, which retrieves rows from a base table according to the row identifiers returned from the index-ANDing operator.

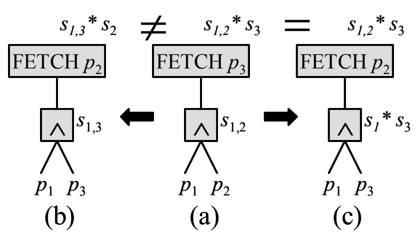

Figure 1: QEPs and Selectivity Estimation

Suppose that the optimizer knows the selectivities s_1, s_2, s_3 of the BFs p_1, p_2, p_3. Also suppose that it knows about a correlation between p_1 and p_2 via knowledge of the selectivity $s_{1,2}$ of $p_1 \wedge p_2$. Using independence, the optimizer might then estimate the selectivity of $p_1 \wedge p_2 \wedge p_3$ as $s^a_{1,2,3} = s_{1,2} * s_3$.

Figure 1(b) shows an alternative QEP that first applies $p_1 \wedge p_3$ and then applies p_2. If the optimizer also knows the selectivity $s_{1,3}$ of $p_1 \wedge p_3$, use of the independence assumption might yield a selectivity estimate $s^b_{1,2,3} = s_{1,3} * s_2$. However, this would result in an inconsistency if, as is likely, $s^a_{1,2,3} \neq s^b_{1,2,3}$. There are potentially other choices, such as $s_1 * s_2 * s_3$ or, if $s_{2,3}$ is known, $s_{1,2}*s_{2,3}/s_2$; the latter estimate amounts to a conditional independence assumption. Any choice of estimate will be arbitrary, since there is no supporting knowledge to justify ignoring a correlation or assuming conditional independence; such a choice will then arbitrarily bias the optimizer toward choosing one plan over the other. Even worse, if the optimizer does not use the same choice of estimate every time that it is required, then different plans will be costed inconsistently, leading to "apples and oranges" comparisons and unreliable plan choices

Assuming that the QEP in Figure 1(a) is the first to be evaluated, a modern optimizer would avoid the foregoing *consistency* problem by recording the fact that $s_{1,2}$ was applied and then avoiding future application of any other MVS that contain either p_1 or p_2, but not both. In our example, the selectivities for the QEP in Figure 1(c) would be used and the ones in Figure 1(b) would not. The optimizer would therefore compute the selectivity of $p_1 \wedge p_3$ to be $s_1 * s_3$ using independence, instead of using the MVS $s_{1,3}$. Thus the selectivity $s_{1,2,3}$ would be estimated in a manner consistent with Figure 1(a). Note that, when evaluating the QEP in Figure 1(a), the optimizer used the estimate $s^a_{1,2,3} = s_{1,2} * s_3$ rather than $s_1 * s_2 * s_3$, since, intuitively, the former estimate better exploits the available correlation information. In general, there may be many possible choices; the complicated (ad hoc) decision algorithm used by DB2 UDB is described in more detail in the Appendix.

Although the ad hoc method described above ensures consistency, it ignores valuable knowledge, e.g., of the correlation between p_1 and p_3. Moreover, this method complicates the logic of the optimizer, because cumbersome bookkeeping is required to keep track of how an estimate was derived initially and to ensure that it will always be computed in the same way when costing other plans. Even worse, ignoring the known correlation between p_1 and p_3 also introduces *bias* towards certain QEPs: if, as is often the case with correlation, $s_{1,3} >> s_1 * s_3$, and $s_{1,2} >> s_1 * s_2$, and if $s_{1,2}$ and $s_{1,3}$ have comparable values, then the optimizer will be biased towards the plan in Figure 1(c), even though the plan in Figure 1(a) might be cheaper, i.e., the optimizer thinks that the plan in Figure 1(c) will produce fewer rows during index-ANDing, but this might not actually be the case. In general, an optimizer will often be drawn towards those QEPs about which it knows the least, because use of the independence assumption makes these plans seem cheaper due to underestimation. We call this problem "fleeing from knowledge to ignorance".

In this paper, we provide a novel method for estimating the selectivity of a conjunctive predicate; the method exploits and combines all of the available MVS in a principled, consistent, and unbiased manner. Our technique rests on the principle of maximum entropy (ME) [GS85], which is a mathematical embodiment of Occam's Razor and provides the "simplest" possible selectivity estimate that is consistent with all of the available information. (In the absence of detailed knowledge, the ME approach reduces to standard uniformity and independence assumptions.) Our new approach avoids the problems of inconsistent QEP comparisons and the flight from knowledge to ignorance.

We emphasize that, unlike DB2's ad hoc method or the method proposed in [BC02] (which tries to choose the "best" of the available MVS for estimating a selectivity) the ME method is the first to exploit *all* of the available MVS and actually refine the optimizer's cardinality model beyond the information explicitly given by the statistics. Also, as discussed in Section 6, our results differ from virtually all current and previous work in this area, which deals only with constructing [BC04, IMH+04, BC03, SHM+05], storing [PIH+96], and maintaining [SLM+01, BCG01, AC99] multivariate statistics. Indeed our method can be used in conjunction with any of the foregoing techniques.

Thus the contributions of our paper are: (1) enunciating and formalizing the problem of consistency and bias during QEP evaluation in the presence of partial knowledge about the joint frequency distribution (as embodied in the available MVS), (2) proposing a new method for cardinality estimation in this setting that exploits all available distributional information, (3) applying the iterative scaling algorithm to compute consistent and unbiased selectivity estimates based on our problem formulation using the ME principle, (4) providing a detailed experimental evaluation of our approach with respect to quality and computation time, as well as a comparison to the DB2 UDB optimizer. Our work appears to be the first to apply information-theoretic ideas to the problem of producing consistent selectivity estimates.

The paper is organized as follows. Section 2 gives some background and formalizes the selectivity-estimation problem. In Section 3, we describe the ME approach to unbiased, efficient, and consistent selectivity estimation. We show how the iterative scaling algorithm can be applied in our setting; this well-known algorithm uses a Lagrange-multiplier approach to numerically compute an approximate ME solution. Section 4 provides an experimental evaluation, and in Section 5 we discuss some practical considerations. After surveying related work in Section 6, we conclude in Section 7. The Appendix describes the current state of the art in using MVS for cardinality estimation in a commercial DBMS.

2. Background

Commercial query optimizers [ATL+03, IBM02, IBM04, Mic04] use statistical information on the number of rows in a table and the number of distinct values in a column to compute the selectivity of a simple predicate p. Assuming ten distinct values in the MAKE column and using the *uniformity assumption*, the selectivity of the predicate p_1: 'MAKE = "Honda"' is estimated as $s_1 = 1/10$. Similarly, with 100 distinct values in the MODEL column and 10 distinct values in the COLOR column, we obtain $s_2 = 1/100$ for p_2: MODEL = "Accord" and $s_3 = 1/10$ for p_3: COLOR = "red". Advanced commercial optimizers can improve upon these basic estimates by maintaining frequency histograms on the values in individual columns.

As indicated previously, in the absence of other information, current optimizers compute the selectivity of a conjunctive predicate using the independence assumption. For instance, $p_{1,2,3} = p_1 \wedge p_2 \wedge p_3$ is the predicate restricting a query to retrieve all red Honda Accords, and the selectivity of $p_{1,2,3}$ is computed as $s_{1,2,3} = s_1 * s_2 * s_3$. In our example, the optimizer would estimate the selectivity of red Honda Accords to be 1/10000. As only Honda makes Accords, there is a strong correlation between these two columns, actually a functional dependency in this case. The actual selectivity of $p_{1,2}$ must be 1/100. Thus a more appropriate estimate of the selectivity of $p_{1,2,3}$ is 1/1000, one order of magnitude greater than the estimate using the independence assumption.

2.1 Formalizing the Selectivity Estimation Problem

We now formalize the problem of selectivity estimation for conjunctive predicates, given partial MVS, and define some useful terminology. Let $P = \{p_1, \dots, p_n\}$ be a set of BFs. For any $X \subseteq N = \{1, \dots, n\}$, denote by p_X the conjunctive predicate $\wedge_{i \in X} p_i$. Let s be a probability measure over 2^N, the powerset of N, with the interpretation that s_X is the selectivity of the predicate p_X. Usually, for $|X| = 1$, the histograms and column statistics from the system catalog determine s_X and are all known. For $|X| > 1$, the MVS may be stored in the database system catalog either as multidimensional histograms, index statistics, or some other form of column-group statistics or statistics on intermediate tables. In practice, s_X is not known for all possible predicate combinations due to the exponential number of combinations of columns that can be used to define MVS. Suppose that s_X is known for every X in some collection[2] $T \subset 2^N$. Then the selectivity estimation problem is to compute s_X for $X \in 2^N \setminus T$.

It is intuitively clear that the query optimizer should avoid any extraneous assumptions about the unknown selectivities while simultaneously exploiting all existing knowledge in order to avoid unjustified bias towards any particular solution. In the Appendix, we survey the method that DB2 uses to compute missing selectivities and illustrate why this approach cannot use all existing knowledge without producing an inconsistent model. In the following section, we present the ME principle, which formalizes the notion of avoiding bias.

2.2 The Maximum-Entropy Principle

The maximum-entropy principle [GS85] models all that is known and assumes nothing about the unknown. It is a method for analyzing the available information in order to determine a unique epistemic probability distribution. Information theory [Sha48] defines for a probability distribution $\boldsymbol{q} = (q_1, q_2, \dots)$ a measure of uncertainty called entropy: $H(q) = -\sum_i q_i \log q_i$. The ME principle prescribes selection of the unique probability distribution that maximizes the entropy function $H(\boldsymbol{q})$ and is consistent with respect to the known information.

[2] Note that the empty set $\emptyset$ is part of T, as $s_\emptyset = 1$ when applying no predicates.