N1, p 85, ex 4a

$$LU = U\begin{pmatrix} 3 & 1 & 2 \\ 6 & 3 & 4 \\ 3 & 1 & 5 \end{pmatrix} = \begin{pmatrix} 1 & -3 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 1 & 2 \\ 6 & 3 & 4 \\ 0 & 0 & 3 \end{pmatrix} \begin{matrix} \\ j - 2 \cdot \\ \end{matrix} = \begin{pmatrix} 3 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix} = U$$

$$L = \begin{pmatrix} 3 & 0 & 0 \\ 6 & 1 & 0 \\ 3 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 3 & 1 & 2 \\ 6 & 3 & 4 \\ 3 & 0 & 1 \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}}_{L} \underbrace{\underbrace{\begin{pmatrix} 3 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix}}_{U} \underbrace{\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}}_{x}} = \underbrace{\begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix}}_{b}$$

1) $Ly = b$

$$\begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix}$$

$y_1 = 0$

$y_2 = 1$

$y_3 = 3$

$$\begin{pmatrix} 3 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix}$$

ответ

$x_1 = -1$

$x_2 = 1$

$x_3 = 1$

$x_3 = 1$

$x_2 = 1$

$3x_1 + x_2 + 2x_3 =$

$3x_1 = -x_2 - 2x_3 = -3$

$x_1 = -1$

W2 p85 ex 7

full Gassin elimenaton
upper triangular matrix $\Big\} \frac{2n^3}{3}$

$Ux = b$

number of

$u_{nn}\, x = b$

$x_{nn} = \frac{b}{u_{nn}}$

$u_{n-1\,n-1} \cdot x_{n-1} + u_{n-1\,n} \cdot x_n = b_{n-1}$

$x_{n-1} = \frac{b_{n-1} - u_{n-1\,n} \cdot x_n}{u_{n-1\,n-1}}$

$$x_i = \frac{b_i - \sum_{i=0}^{n-1} u_{ij} \cdot x_j}{u_{ii}}$$

U has $\frac{n(n-1)}{2}$ elements

we should make n divisions

$\frac{(n-1)n}{2}$ - multiplications

$\frac{(n-1)n}{2}$ - substractions

total complexity $= (n-1)n + n$

$n = 500$

$499 \cdot 500 + 500 = 250000$

total operations $250000 \cdot 10^3 =$
$= 25 \cdot 10^7$

$n = 5000$

$4999 \cdot 5000 + 5000 = 25000000$

total operations $25 \cdot 10^6 \cdot 10^3 =$
$= 25 \cdot 10^9$

$$\frac{25\cdot 10^9}{25\cdot 10^7} = 10^2$$

$$10^2 \cdot 1\,\text{sec} = 100\,\text{sec} = 1\text{m}\;40\text{s}$$

ответ

N5 p 93 ex 6 a c e

$$\begin{pmatrix} 1 & 2 \\ 2 & 4.01 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 3 \\ 6.01 \end{pmatrix}$$

$$\begin{cases} x_1 + 2x_2 = 3 \\ 2x_1 + 4.01x_2 = 6.01 \end{cases}$$

$0.01 x_2 = 0.01$

$x_2 = 1$

$x_1 = 1$

Точное решение

4

$$x_4 = \begin{pmatrix} -10 \\ 6 \end{pmatrix}$$

backward

$$\|b - A \cdot x_4\|_\infty = \left\| \begin{pmatrix} 3 \\ 6.01 \end{pmatrix} - \begin{pmatrix} 1 & 2 \\ 2 & 4.01 \end{pmatrix} \cdot \begin{pmatrix} -10 \\ 6 \end{pmatrix} \right\|_\infty$$

$$= \left\| \begin{matrix} 3 - (-10 + 12) \\ 6.01 - (-20 + 24.06) \end{matrix} \right\|_\infty =$$

$$= \left\| \begin{pmatrix} 1 \\ 1.95 \end{pmatrix} \right\|_\infty = 1.95$$

forward

$$\left\| \begin{pmatrix} 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 10 \\ 6 \end{pmatrix} \right\|_\infty = \left\| \begin{pmatrix} -9 \\ -5 \end{pmatrix} \right\|_\infty = 9$$

Backward

error magnification

$$r = \begin{pmatrix} 1 \\ 1.95 \end{pmatrix} \quad \|r\|_\infty = 1.95$$

$$\|b\|_\infty = 6.01$$

$$\|x\|_\infty = 1$$

$$\frac{\frac{\|x - x_a\|_\infty}{\|x\|_\infty}}{\frac{\|r\|_\infty}{\|B\|_\infty}} = \frac{\frac{9}{1}}{\frac{1.95}{6.01}} = 27,738$$

error magnif.

c)

$$x_a = \begin{pmatrix} -600 \\ 301 \end{pmatrix}$$

Backward

$$\|b - Ax_a\|_\infty = \left\| \begin{pmatrix} 3 \\ 6.01 \end{pmatrix} - \begin{pmatrix} 1 & 2 \\ 2 & 4.01 \end{pmatrix} \cdot \begin{pmatrix} -600 \\ 301 \end{pmatrix} \right\|_\infty$$

$$\begin{pmatrix} 1 & 2 \\ 2 & 4.01 \end{pmatrix} \begin{pmatrix} -600 \\ 301 \end{pmatrix} = \begin{pmatrix} 2 \\ 7.01 \end{pmatrix}$$

$$\|b - Ax_a\|_\infty = \left\| \begin{pmatrix} 3 - 2 \\ 6.01 - 7.01 \end{pmatrix} \right\|_\infty =$$

$$= \left\| \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right\|_\infty = 1 \leftarrow \text{Backward}$$

Forward

$$\left\| \begin{matrix} 1 - (-600) \\ 1 - 301 \end{matrix} \right\|_\infty = \left\| \begin{matrix} 599 \\ -300 \end{matrix} \right\|_\infty = 599$$

$$\|x\|_\infty = 1$$

error magnification

$$\frac{\dfrac{\|x - x_a\|_\infty}{\|x\|_\infty}}{\dfrac{\|r\|_\infty}{\|b\|_\infty}} = \frac{\dfrac{5.99}{1}}{\dfrac{1.95}{6.01}} = 1846.1487$$

e)

$$A = \begin{pmatrix} 1 & 2 \\ 2 & 4.01 \end{pmatrix} =$$

$$K = \|A\| \cdot \|A^{-1}\|$$

$$A^{-1} = \frac{1}{4.01 - 4} \begin{pmatrix} 4.01 & -2 \\ -2 & 1 \end{pmatrix} =$$

$$A^{-1} = \frac{1}{10^{-2}} \begin{pmatrix} 4.01 & -2 \\ -2 & 1 \end{pmatrix} =$$

$$= 100 \cdot \begin{pmatrix} 4.01 & -2 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} 401 & -200 \\ -200 & 100 \end{pmatrix}$$

κ_∞

$$\|A\|_\infty = \max_i \sum_j |a_{ij}| = 6.01$$

$$\|A^{-1}\|_\infty = 601$$

$$\kappa_\infty(A) = 6.01 \cdot 601 = 3612.01$$

κ_2

$$\|A\|_2 = \sigma_{max}(A) = 5.0080032$$

$$\|A^{-1}\|_2 = 500.80032$$

$$\|A\|_2 \cdot \|A^{-1}\|_2 = 2508{,}009605$$

NF p116 N 2.5.2 (c)

$$u - 8v - 2w = 1$$

$$u + v + 5w = 4$$

$$3u - v + w = -2$$

$$\begin{array}{ccc} v & w & u \\ \hline \end{array}$$

$$\begin{pmatrix} -8 & -2 & 1 \\ 1 & 5 & 1 \\ -1 & 1 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix}$$

$$A = D + L + U =$$

$$= \begin{pmatrix} -8 & & 0 \\ & 5 & \\ 0 & & 3 \end{pmatrix} + \begin{pmatrix} 0 & -2 & 1 \\ 1 & 0 & 1 \\ -1 & 1 & 0 \end{pmatrix}$$

Jacobi

$$x_0 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

X_1

$$X_1 = D^{-1}\left(b - (L+U)\,x_0\right)$$

$$x_1 = \begin{pmatrix} \frac{1}{8} & 0 & \\ & \frac{1}{5} & \\ 0 & & \frac{1}{3} \end{pmatrix} \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix} = \underbrace{\begin{pmatrix} -\frac{1}{8} \\ \frac{4}{5} \\ -\frac{2}{3} \end{pmatrix}}_{x_1}$$

X_2

$$(L+U) \cdot \begin{pmatrix} -\frac{1}{8} \\ \frac{4}{5} \\ -\frac{2}{3} \end{pmatrix} =$$

$$\begin{pmatrix} 0 & -2 & 1 \\ 1 & 0 & 1 \\ -1 & 1 & 0 \end{pmatrix}$$

$$-8/5 - \frac{2}{3} = -2.2(6)$$

$$-\frac{1}{8} - \frac{2}{3} = -0.791(6)$$

$$1/8 + \frac{4}{5} = 0.925$$

$$(L+U)x_1 = \begin{pmatrix} -2.2(6) \\ -0.791(6) \\ 0.925 \end{pmatrix}$$

$$b - (L+U)x_1 = \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix} - \begin{pmatrix} 2.2 \\ 0.791 \\ 0.95 \end{pmatrix} \approx$$

$$\approx \begin{pmatrix} -1.2 \\ 3.209 \\ -1.05 \end{pmatrix}$$

$$x_2 \approx D^{-1} \begin{pmatrix} -1.2 \\ 3.21 \\ -1.05 \end{pmatrix} \approx \begin{pmatrix} -0.15 \\ 0.642 \\ -0.35 \end{pmatrix}$$

x_2

Jacobi

Gauss Seidel

$$x_{k+1} = (L+D)^{-1}(B - Ux_k)$$

$$(L+D)^{-1} = \begin{pmatrix} -8 & 0 & 0 \\ 1 & 5 & 0 \\ -1 & 1 & 3 \end{pmatrix}^{-1} = \begin{pmatrix} -1/8 & 0 & 0 \\ \frac{1}{40} & \frac{1}{5} & 0 \\ -\frac{1}{20} & -\frac{1}{15} & 1/3 \end{pmatrix}$$

$$U \cdot x_0 = 0$$

$$(L+D)^{-1} \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix} = \begin{pmatrix} -\frac{1}{8} \\ \frac{7}{8} \\ \approx 0.98(3) \end{pmatrix}$$

$$x_1 = \begin{pmatrix} -1/8 \\ 7/8 \\ -0.98 \end{pmatrix}$$

$\leftarrow x_1$

$$Ux_1 = \begin{pmatrix} -2.6(3) \\ -0.98(3) \\ 0 \end{pmatrix}$$

$$b - Ux_1 \approx \begin{pmatrix} 3.6 \\ 4.98 \\ -2 \end{pmatrix}$$

$$(L+D)^{-1} \cdot \begin{pmatrix} 3.6 \\ 4.98 \\ -2 \end{pmatrix} \approx \begin{pmatrix} -0.4541 \\ 1{,}0875 \\ -1{,}1805 \end{pmatrix}$$

x_2

N11 p. 149 3.11 c 3.12 c

3.11 c

$$\begin{pmatrix} 0, & -2 \\ 2, & 1 \\ 4, & 4 \end{pmatrix}$$

$$P_2(x) = y_1 \frac{(x-x_2)(x-x_3)}{(x_1-x_2)(x_1-x_3)} + y_2 \frac{(x-x_1)(x-x_3)}{(x_2-x_1)(x_2-x_3)} + y_3 \frac{(x-x_1)(x-x_2)}{(x_3-x_1)(x_3-x_2)}$$

$$P_2(x) = -2 \frac{(x-2)(x-4)}{4 \cdot 2} + 1 \cdot \frac{(x-0)(x-4)}{(2-0)\cdot(-2)} + 4 \cdot \frac{(x-0)(x-2)}{(4-0)(4-2)} =$$

$$-\frac{1}{4}(x^2-2x-4x+8)-$$

$$-\frac{1}{4}(x^2-4x)+$$

$$+\frac{2}{4}(x^2-2x)=$$

$$=-\frac{1}{4}\left(\cancel{x^2}-6x+8\cancel{+x^2}\cancel{-4x}\right.$$

$$\left.\cancel{-2x^2}+\cancel{4x}\right)=-\frac{1}{4}(8-6x)$$

dots	poly
0, −2	-2
2, 1	1
4, −1	$-\frac{1}{4}\cdot -16 = 4$

Lagrange

Newton

$$P(x) = f[x_1] + f[x_1, x_2](x - x_1)$$
$$+ f[x_1, x_2, x_3](x - x_1)(x - x_2)$$

$$f[x_1] = -2$$

$$f[x_1, x_2] = \frac{f[x_2] - f[x_1]}{x_2 - x_1} =$$

$$= \frac{1 - (-2)}{2} = \frac{3}{2}$$

$$f[x_1, x_2, x_3] = \frac{3/2 - 3/2}{4 - 0}$$

dots

$$\begin{pmatrix} 0, & -2 \\ 2, & 1 \\ 4, & 4 \end{pmatrix}$$

$$f[x_1, x_2] = \frac{3}{2}$$

$$f[x_2, x_3] = \frac{f[x_3] - f[x_2]}{x_3 - x_2}$$

$$= \frac{4-1}{4-2} = \frac{3}{2}$$

$$f[x_1, x_2, x_3] = 0$$

$$= -2 + \frac{3}{2}(x - \cancel{x_1}^{\,0}) =$$

$$P(x) = \frac{3}{2}x - 2$$

$$\begin{matrix} 0 & -2 \\ 2 & 1 \\ 4 & 4 \end{matrix}$$

Newton

Lagrange =

$$-\frac{1}{4}(8-6x) - 2 + \frac{3}{2}x$$

Newton =

$$\frac{3}{2}x - 2$$

N14, p 156 ex 3.2.4

$$f(x) - P(x) = \frac{(x-x_1) \ldots (x-x_n)}{n!} f^n(c)$$

c is betwen smallest and largest

$x = 0, 2, 4, 6, 8, 10$

$$f(x) - P(x) \qquad \frac{\quad}{5!}$$

$$f'(x) = -\frac{1}{(x+5)^2}$$

$$f''(x) = \frac{2}{(x+5)^3}$$

$$f'''(x) = -\frac{1 \cdot 2 \cdot 3}{(x+5)^4}$$

$$f''''(x) = \frac{1 \cdot 2 \cdot 3 \cdot 4}{(x+5)^5}$$

$$f^{(5)}(x) = -\frac{5!}{(x+5)^6}$$

$$f^{(6)} = \frac{6!}{(x+5)^7}$$

$$f(x) - P(x) = \frac{(x-0)(x-2)(x-4)(x-6)(x-8)(x-10)}{\cancel{6!}} \cdot \frac{\cancel{6!}}{(1+5)^7}$$

$c = c$

$x = 1$

$$\frac{1\cdot(-1)\cdot(-3)\cdot(-5)\cdot(-7)\cdot(-9)\,\cancel{6!}}{\cancel{6!}\,(c+5)^7} =$$

$$= -\frac{9!!}{(c+5)^7}$$ — ответ

$x = 5$

$c = c$

$$\frac{5\cdot 3\cdot 1\cdot(-1)(-3)\cdot(-5)\,6!}{6!\,(c+5)^7}$$

$$= \frac{-(5!!)^2}{(c+5)^7}$$ ← ответ