# Satisfiability Checking 06 SAT solving examples

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## 06 SAT solving examples

Example 1

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## SAT solving: Example1

- We use DPLL+CDCL SAT solving combining enumeration, propagation and resolution.
- We use watched literals (underlined in formulas) to speed up propagation.
- We use VSIDS as variable ordering heuristics and assign the value false to decision variables.
- In VSIDS, to order variables with the same activity value, we use the lexicographic order; in our example this will be  $x_1 < x_2 < x_3 < x_4$ .

$$c_1: \left(\underline{x_1} \vee \underline{x_2} \vee x_4\right) \wedge c_2: \left(\underline{x_2} \vee \underline{\neg x_4}\right) \wedge c_3: \left(\underline{x_1} \vee \underline{\neg x_2} \vee x_4\right) \wedge c_4: \left(\underline{x_3} \vee \underline{\neg x_4}\right)$$

```
Watch lists: x_1: c_1, c_3 \neg x_1: x_2: c_1, c_2 \neg x_2: c_3 x_3: c_4 \neg x_3: x_4:
```

 $\neg x_4$ :  $c_2, c_4$ 

$$c_1: \left(\underline{x_1} \vee \underline{x_2} \vee x_4\right) \wedge c_2: \left(\underline{x_2} \vee \underline{\neg x_4}\right) \wedge c_3: \left(\underline{x_1} \vee \underline{\neg x_2} \vee x_4\right) \wedge c_4: \left(\underline{x_3} \vee \underline{\neg x_4}\right)$$

```
Watch lists: x_1 : c_1, c_3 \neg x_1 : x_2 : c_1, c_2 \neg x_2 : c_3 x_3 : c_4 \neg x_3 : x_4 : \neg x_4 : c_2, c_4
```

Decide  $\neg x_1$ 

```
Activities Trail: (increment=1): DL1: \neg x_1 : \text{NULL}
\begin{array}{ccc} x_1 & 0 \\ x_2 & 0 \\ x_3 & 0 \\ x_4 & 0 \end{array}
```

$$c_1: \left(\underline{x_1} \vee \underline{x_2} \vee x_4\right) \wedge c_2: \left(\underline{x_2} \vee \underline{\neg x_4}\right) \wedge c_3: \left(\underline{x_1} \vee \underline{\neg x_2} \vee x_4\right) \wedge c_4: \left(\underline{x_3} \vee \underline{\neg x_4}\right)$$

```
Watch lists: x_1 : c_1, c_3 \neg x_1 : x_2 : c_1, c_2 \neg x_2 : c_3 x_3 : c_4 \neg x_3 : x_4 :
```

 $\neg x_4$ :  $c_2, c_4$ Propagate  $\neg x_1$  in

```
Activities Trail: (increment=1): DL1: \neg x_1 : \text{NULL}
\begin{array}{ccc} x_1 & 0 \\ x_2 & 0 \\ x_3 & 0 \\ x_4 & 0 \end{array}
```

$$c_1: \left(\underline{x_1} \vee \underline{x_2} \vee x_4\right) \wedge c_2: \left(\underline{x_2} \vee \underline{\neg x_4}\right) \wedge c_3: \left(\underline{x_1} \vee \underline{\neg x_2} \vee x_4\right) \wedge c_4: \left(\underline{x_3} \vee \underline{\neg x_4}\right)$$

```
Watch lists:
                                    Activities
                                                         Trail:
                                    (increment=1):
                                                              DL1: \neg x_1: NULL
        x_1: c_1, c_3
       \neg x_1:
                                           x_1 0
        x_2: c_1, c_2
                                           x_2 0
      \neg x_2: c_3
                                           x_3 0
       X_3: C_4
                                                0
                                           X_4
       \neg x_3:
        X4 :
      \neg x_4: c_2, c_4
 Propagate \neg x_1 in c_1: (x_1 \lor x_2 \lor x_4)
```

$$c_1: \left(x_1 \vee \underline{x_2} \vee \underline{x_4}\right) \wedge c_2: \left(\underline{x_2} \vee \underline{\neg x_4}\right) \wedge c_3: \left(\underline{x_1} \vee \underline{\neg x_2} \vee x_4\right) \wedge c_4: \left(\underline{x_3} \vee \underline{\neg x_4}\right)$$

```
Watch lists:
                                          Activities
                                                                  Trail:
                                          (increment=1):
                                                                        DL1: \neg x_1: NULL
         x_1: \mathcal{C}_1, \mathcal{C}_3
        \neg x_1:
                                                 x_1 0
         x_2: c_1, c_2
                                                 x_2 0
        \neg x_2: c_3
                                                 x_3 0
        X_3: C_4
                                                       0
                                                  X_{\Delta}
        \neg x_3:
        x_4: c_1
        \neg x_4: c_2, c_4
 Propagate \neg x_1 in c_1: (x_1 \lor x_2 \lor x_4) \rightarrow (x_1 \lor x_2 \lor x_4)
```

$$c_1: \left(x_1 \vee \underline{x_2} \vee \underline{x_4}\right) \wedge c_2: \left(\underline{x_2} \vee \underline{\neg x_4}\right) \wedge c_3: \left(\underline{x_1} \vee \underline{\neg x_2} \vee x_4\right) \wedge c_4: \left(\underline{x_3} \vee \underline{\neg x_4}\right)$$

```
Watch lists:
                                       Activities
                                                              Trail:
                                        (increment=1):
                                                                    DL1: \neg x_1: NULL
         x_1: \mathcal{C}_1, \mathcal{C}_3
       \neg x_1:
                                               x_1 0
         x_2: c_1, c_2
                                               x_2 0
       \neg x_2: c_3
                                               x_3 0
       X_3: C_4
                                                    0
                                               X_4
       \neg x_3:
        X_4: C_1
       \neg x_4: c_2, c_4
 Propagate \neg x_1 in c_3: (x_1 \lor \neg x_2 \lor x_4)
```

$$c_1: \left(x_1 \vee \underline{x_2} \vee \underline{x_4}\right) \wedge c_2: \left(\underline{x_2} \vee \underline{\neg x_4}\right) \wedge c_3: \left(x_1 \vee \underline{\neg x_2} \vee \underline{x_4}\right) \wedge c_4: \left(\underline{x_3} \vee \underline{\neg x_4}\right)$$

```
Watch lists:
                                                                   Trail:
                                           Activities
                                           (increment=1):
                                                                         DL1: \neg x_1: NULL
          x_1: \mathcal{G}_1, \mathcal{G}_3
        \neg x_1:
                                                   x_1 0
         x_2: c_1, c_2
                                                   x_2 0
        \neg x_2: c_3
                                                   x_3 0
        X_3: C_4
                                                        0
                                                   X_{\Delta}
        \neg x_3:
         X_4: C_1, C_3
        \neg x_4: c_2, c_4
  Propagate \neg x_1 in c_3: (x_1 \lor \neg x_2 \lor x_4) \rightarrow (x_1 \lor \neg x_2 \lor x_4)
```

$$c_1: (x_1 \vee \underline{x_2} \vee \underline{x_4}) \wedge c_2: (\underline{x_2} \vee \underline{\neg x_4}) \wedge c_3: (x_1 \vee \underline{\neg x_2} \vee \underline{x_4}) \wedge c_4: (\underline{x_3} \vee \underline{\neg x_4})$$

```
Watch lists: x_1: \neg x_1: x_2: c_1, c_2 \neg x_2: c_3 x_3: c_4 \neg x_3: x_4: x_4: x_5: x_5: x_6: x_6:
```

 $\neg x_4$ :  $c_2$ ,  $c_4$ 

Decide  $\neg x_2$ 

```
Activities (increment=1):  x_1 \quad 0 \\ x_2 \quad 0 \\ x_3 \quad 0 \\ x_4 \quad 0
```

```
Trail:

DL1: \neg x_1: NULL

DL2: \neg x_2: NULL
```

$$c_1: (x_1 \vee \underline{x_2} \vee \underline{x_4}) \wedge c_2: (\underline{x_2} \vee \underline{\neg x_4}) \wedge c_3: (x_1 \vee \underline{\neg x_2} \vee \underline{x_4}) \wedge c_4: (\underline{x_3} \vee \underline{\neg x_4})$$

```
Watch lists:

x_1:

\neg x_1:

x_2: c_1, c_2

\neg x_2: c_3

x_3: c_4

\neg x_3:
```

$$c_1: (x_1 \vee \underline{x_2} \vee \underline{x_4}) \wedge c_2: (\underline{x_2} \vee \underline{\neg x_4}) \wedge c_3: (x_1 \vee \underline{\neg x_2} \vee \underline{x_4}) \wedge c_4: (\underline{x_3} \vee \underline{\neg x_4})$$

```
Watch lists:
                                                         Trail:
                                    Activities
                                    (increment=1):
                                                              DL1: \neg x_1: NULL
        X_1:
                                                              DL2: \neg x_2: NULL
       \neg x_1:
                                           x_1 0
        x_2: c_1, c_2
                                                0
                                           X2
      \neg x_2: c_3
                                           x_3 0
       X3 : C4
                                                0
                                           X_4
       \neg x_3:
        x_4: c_1, c_3
      \neg x_4: c_2, c_4
 Propagate \neg x_2 in c_1:(x_1 \lor x_2 \lor x_4)
```

$$c_1: (x_1 \vee \underline{x_2} \vee \underline{x_4}) \wedge c_2: (\underline{x_2} \vee \underline{\neg x_4}) \wedge c_3: (x_1 \vee \underline{\neg x_2} \vee \underline{x_4}) \wedge c_4: (\underline{x_3} \vee \underline{\neg x_4})$$

```
Watch lists:
                                                            Trail:
                                      Activities
                                      (increment=1):
                                                                 DL1: \neg x_1: NULL
         X_1:
                                                                 DL2: \neg x_2: NULL
       \neg x_1:
                                             x_1 0
        x_2: c_1, c_2
                                                                             X4 : C1
                                                  0
                                             X2
       \neg x_2: c_3
                                             x_3 0
       X_3: C_4
                                                  0
                                             X_4
       \neg x_3:
        x_4: c_1, c_3
       \neg x_4: c_2, c_4
 Propagate \neg x_2 in c_1: (x_1 \lor x_2 \lor x_4) \rightarrow \text{Assign } x_4
```

$$c_1: (x_1 \vee \underline{x_2} \vee \underline{x_4}) \wedge c_2: (\underline{x_2} \vee \underline{\neg x_4}) \wedge c_3: (x_1 \vee \underline{\neg x_2} \vee \underline{x_4}) \wedge c_4: (\underline{x_3} \vee \underline{\neg x_4})$$

```
Watch lists:
                                                        Trail:
                                   Activities
                                    (increment=1):
                                                             DL1: \neg x_1: NULL
        X1:
                                                             DL2: \neg x_2: NULL
      \neg x_1:
                                          x_1 0
        x_2: c_1, c_2
                                                                      X4 : C1
                                               0
                                          X2
      \neg x_2: c_3
                                               0
                                          X3
       X3 : C4
                                               0
                                          X_4
      \neg x_3:
       x_4: c_1, c_3
      \neg x_4: c_2, c_4
 Propagate \neg x_2 in c_2:(x_2 \vee \neg x_4)
```

$$c_1: (x_1 \vee \underline{x_2} \vee \underline{x_4}) \wedge c_2: (\underline{x_2} \vee \underline{\neg x_4}) \wedge c_3: (x_1 \vee \underline{\neg x_2} \vee \underline{x_4}) \wedge c_4: (\underline{x_3} \vee \underline{\neg x_4})$$

```
Watch lists:
                                                            Trail:
                                      Activities
                                       (increment=1):
                                                                  DL1: \neg x_1: NULL
         X_1:
                                                                  DL2: \neg x_2: NULL
       \neg x_1:
                                             x_1 0
        x_2: c_1, c_2
                                                                            X4 : C1
                                                   0
                                             X2
       \neg x_2: c_3
                                             x_3 0
       X_3: C_4
                                                   0
                                             XΔ
       \neg x_3:
        x_4: c_1, c_3
       \neg x_4: c_2, c_4
 Propagate \neg x_2 in c_2: (x_2 \vee \neg x_4) \rightarrow \text{$\not$$conflict!}
```

$$c_1: (x_1 \vee \underline{x_2} \vee \underline{x_4}) \wedge c_2: (\underline{x_2} \vee \underline{\neg x_4}) \wedge c_3: (x_1 \vee \underline{\neg x_2} \vee \underline{x_4}) \wedge c_4: (\underline{x_3} \vee \underline{\neg x_4})$$

Watch lists:

 $X_1$ :  $\neg X_1$ :

 $x_2: c_1, c_2$ 

 $\neg x_2$ :  $c_3$ 

 $X_3$ :  $C_4$ 

 $\neg x_3$ :

 $x_4: c_1, c_3$ 

 $\neg x_4$ :  $c_2, c_4$ 

Conflict resolution:

Activities (increment=1):

 $x_1 0$  $x_2 0$ 

 $x_3 = 0$ 

x<sub>4</sub> 0

Trail:

DL1:  $\neg x_1$ : NULL DL2:  $\neg x_2$ : NULL

X₄ : C₁

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$$c_1: (x_1 \vee \underline{x_2} \vee \underline{x_4}) \wedge c_2: (\underline{x_2} \vee \underline{\neg x_4}) \wedge c_3: (x_1 \vee \underline{\neg x_2} \vee \underline{x_4}) \wedge c_4: (\underline{x_3} \vee \underline{\neg x_4})$$

```
Watch lists:
                                         Activities
                                                                 Trail:
                                          (increment=1):
                                                                       DL1: \neg x_1: NULL
         X1:
                                                                       DL2: \neg x_2: NULL
        \neg x_1:
                                                 x_1 = 0
        x_2: c_1, c_2
                                                                                   X_4 : C_1
                                                 X2
                                                       0
       \neg x_2: c_3
                                                 x_3 0
         X3: C4
                                                 X4
                                                       0
       \neg x_3:
         X_4: C_1, C_3
        \neg x_4: c_2, c_4
 Conflict resolution: (x_2 \lor x_4) (x_1 \lor x_2 \lor x_4) (x_1 \lor x_2 \lor x_4)
```

$$c_1: (x_1 \vee \underline{x_2} \vee \underline{x_4}) \wedge c_2: (\underline{x_2} \vee \underline{-x_4}) \wedge c_3: (x_1 \vee \underline{-x_2} \vee \underline{x_4}) \wedge c_4: (\underline{x_3} \vee \underline{-x_4}) \wedge c_5: (x_1 \vee x_2)$$

Watch lists:

$$X_1$$
:  $C_5$ 

$$x_2: c_1, c_2, c_5$$

$$\neg x_2$$
:  $c_3$ 

 $\neg x_3$ :

$$x_4: c_1, c_3$$

$$\neg x_4$$
:  $c_2, c_4$ 

Add conflict clause

Activities (increment=2):

> $x_1$  1  $x_2$  1

 $x_3$  0

 $x_4$  1

Trail:

DL1:  $\neg x_1$ : NULL DL2:  $\neg x_2$ : NULL

 $X_4 : C_1$ 

$$c_1: (x_1 \vee \underline{x_2} \vee \underline{x_4}) \wedge c_2: (\underline{x_2} \vee \underline{-x_4}) \wedge c_3: (x_1 \vee \underline{-x_2} \vee \underline{x_4}) \wedge c_4: (\underline{x_3} \vee \underline{-x_4}) \wedge c_5: (x_1 \vee x_2)$$

Watch lists:

$$x_1$$
:  $c_5$ 

$$x_2: c_1, c_2, c_5$$

$$\neg x_2$$
:  $c_3$ 

 $\neg x_3$ :

$$x_4: c_1, c_3$$

$$\neg x_4$$
:  $c_2, c_4$ 

Backtrack to DL1

Activities Trail: (increment=2):

 $x_1$  1

 $x_2$  1

*x*<sub>3</sub> 0

 $x_4$  1

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DL1:  $\neg x_1$ : NULL

$$c_1: (x_1 \vee \underline{x_2} \vee \underline{x_4}) \wedge c_2: (\underline{x_2} \vee \underline{-x_4}) \wedge c_3: (x_1 \vee \underline{-x_2} \vee \underline{x_4}) \wedge c_4: (\underline{x_3} \vee \underline{-x_4}) \wedge c_5: (x_1 \vee x_2)$$

Watch lists:

X1: C5  $\neg x_1$ :  $x_2: c_1, c_2, c_5$ 

 $\neg x_2$ :  $c_3$  $X_3$ :  $C_4$ 

 $\neg x_3$ :

 $x_4: c_1, c_3$  $\neg x_4$ :  $c_2, c_4$ 

Assign  $x_2$  at DL1 by  $c_5$ 

(increment=2):

 $x_1$  1

 $x_2$  1  $x_3$  0

 $x_4$  1

Trail: Activities

DL1:  $\neg x_1$ : NULL

X2 : C5

$$c_1: (x_1 \vee \underline{x_2} \vee \underline{x_4}) \wedge c_2: (\underline{x_2} \vee \underline{-x_4}) \wedge c_3: (x_1 \vee \underline{-x_2} \vee \underline{x_4}) \wedge c_4: (\underline{x_3} \vee \underline{-x_4}) \wedge c_5: (x_1 \vee x_2)$$

```
Watch lists:
```

```
X1: C5
\neg x_1:
 x_2: c_1, c_2, c_5
\neg x_2: c_3
X3 : C4
\neg x_3:
```

 $\neg x_4$ :  $c_2, c_4$ Propagate  $x_2$  in

 $x_4$ :  $c_1, c_3$ 

```
Activities
                Trail:
(increment=2):
     x_1 1
```

 $x_2$  1  $x_3$  0  $x_4$  1

DL1:  $\neg x_1$ : NULL

X2: C5

```
c_1: (x_1 \vee \underline{x_2} \vee \underline{x_4}) \wedge c_2: (\underline{x_2} \vee \underline{-x_4}) \wedge c_3: (x_1 \vee \underline{-x_2} \vee \underline{x_4}) \wedge c_4: (\underline{x_3} \vee \underline{-x_4}) \wedge c_5: (x_1 \vee x_2)
```

```
Watch lists:
                                    Activities
                                                        Trail:
                                    (increment=2):
                                                              DL1: \neg x_1: NULL
        X1: C5
       \neg x_1:
                                                                        X2 : C5
                                          x_1 1
        x_2: c_1, c_2, c_5
                                          x_2 1
      \neg X_2: C_3
                                          x_3 0
       X_3: C_4
                                          x_4 1
       \neg x_3:
       x_4: c_1, c_3
      \neg x_4: c_2, c_4
 Propagate x_2 in c_3: (x_1 \vee \neg x_2 \vee x_4)
```

```
c_1: (x_1 \vee \underline{x_2} \vee \underline{x_4}) \wedge c_2: (\underline{x_2} \vee \underline{-x_4}) \wedge c_3: (x_1 \vee \underline{-x_2} \vee \underline{x_4}) \wedge c_4: (\underline{x_3} \vee \underline{-x_4}) \wedge c_5: (x_1 \vee x_2)
```

```
Watch lists:
                                                           Trail:
                                     Activities
                                      (increment=2):
                                                                DL1: \neg x_1: NULL
        X1: C5
       \neg x_1:
                                                                          X2 : C5
                                            x_1 1
        x_2: c_1, c_2, c_5
                                                                          X4:C3
                                            x_2 1
       \neg x_2: c_3
                                            x_3 0
       X_3: C_4
                                            x_4 1
       \neg x_3:
       x_4: c_1, c_3
       \neg x_4: c_2, c_4
 Propagate x_2 in c_3: (x_1 \vee \neg x_2 \vee x_4) \rightarrow \text{Assign } x_4
```

$$c_1: (x_1 \vee \underline{x_2} \vee \underline{x_4}) \wedge c_2: (\underline{x_2} \vee \underline{-x_4}) \wedge c_3: (x_1 \vee \underline{-x_2} \vee \underline{x_4}) \wedge c_4: (\underline{x_3} \vee \underline{-x_4}) \wedge c_5: (x_1 \vee x_2)$$

```
Watch lists:
```

$$x_1 : c_5$$
  
 $\neg x_1 : x_2 : c_1, c_2, c_5$   
 $\neg x_2 : c_3$   
 $x_3 : c_4$ 

 $x_4$ :  $c_1, c_3$  $\neg x_4$ :  $c_2, c_4$ 

 $\neg x_3$ :

Propagate  $x_4$  in

```
Activities (increment=2): x_1 \quad 1 x_2 \quad 1 x_3 \quad 0
```

 $x_4$  1

Trail:

DL1:  $\neg x_1 : \text{NULL}$   $x_2 : c_5$   $x_4 : c_3$ 

```
c_1: (x_1 \vee \underline{x_2} \vee \underline{x_4}) \wedge c_2: (\underline{x_2} \vee \underline{-x_4}) \wedge c_3: (x_1 \vee \underline{-x_2} \vee \underline{x_4}) \wedge c_4: (\underline{x_3} \vee \underline{-x_4}) \wedge c_5: (x_1 \vee x_2)
```

```
Watch lists:
                                   Activities
                                                       Trail:
                                   (increment=2):
                                                            DL1: \neg x_1: NULL
        X1: C5
      \neg x_1:
                                                                      X2 : C5
                                         x_1 1
        x_2: c_1, c_2, c_5
                                                                      X4: C3
                                         x_2 1
      \neg x_2: c_3
                                         x_3 0
       X_3: C_4
                                          x_4 1
      \neg x_3:
       x_4: c_1, c_3
      \neg x_4: c_2, c_4
 Propagate x_4 in c_2:(x_2\vee \neg x_4)
```

```
c_1: (x_1 \vee \underline{x_2} \vee \underline{x_4}) \wedge c_2: (\underline{x_2} \vee \underline{-x_4}) \wedge c_3: (x_1 \vee \underline{-x_2} \vee \underline{x_4}) \wedge c_4: (\underline{x_3} \vee \underline{-x_4}) \wedge c_5: (x_1 \vee x_2)
```

```
Watch lists:
                                    Activities
                                                         Trail:
                                    (increment=2):
                                                              DL1: \neg x_1: NULL
        X1: C5
       \neg x_1:
                                                                        X2 : C5
                                           x_1 1
        x_2: c_1, c_2, c_5
                                                                        X4: C3
                                           x_2 1
      \neg x_2: c_3
                                           x_3 0
       X_3: C_4
                                           x_4 1
       \neg x_3:
       x_4: c_1, c_3
      \neg x_4: c_2, c_4
 Propagate x_4 in c_2: (x_2 \vee \neg x_4) \rightarrow O.K.
```

```
c_1: (x_1 \vee \underline{x_2} \vee \underline{x_4}) \wedge c_2: (\underline{x_2} \vee \underline{-x_4}) \wedge c_3: (x_1 \vee \underline{-x_2} \vee \underline{x_4}) \wedge c_4: (\underline{x_3} \vee \underline{-x_4}) \wedge c_5: (x_1 \vee x_2)
```

```
Watch lists:
                                   Activities
                                                       Trail:
                                   (increment=2):
                                                            DL1: \neg x_1: NULL
        X1: C5
      \neg x_1:
                                                                     X2 : C5
                                         x_1 1
       x_2: c_1, c_2, c_5
                                                                     X4: C3
                                         x_2 1
      \neg x_2: c_3
                                         x_3 0
       X_3: C_4
                                         x_4 1
      \neg x_3:
       x_4: c_1, c_3
      \neg x_4: c_2, c_4
 Propagate x_4 in c_4:(x_3\vee \neg x_4)
```

```
c_1: (x_1 \lor x_2 \lor x_4) \land c_2: (x_2 \lor \neg x_4) \land c_3: (x_1 \lor \neg x_2 \lor x_4) \land c_4: (x_3 \lor \neg x_4)
             c_5:(x_1 \vee x_2)
Watch lists:
                                                                                                                                                                                                                                                                                                                            Activities
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               Trail:
                                                                                                                                                                                                                                                                                                                              (increment=2):
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            DL1: \neg x_1: NULL
                                                                           X1: C5
                                                             \neg x_1:
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 X2 : C5
                                                                                                                                                                                                                                                                                                                                                                                     x_1 1
                                                                       x_2: c_1, c_2, c_5
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 X_4:C_3
                                                                                                                                                                                                                                                                                                                                                                                   x_2 1
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 X3 : C4
                                                             \neg x_2: c_3
                                                                                                                                                                                                                                                                                                                                                                                   x_3 0
                                                                 X_3: C_4
                                                                                                                                                                                                                                                                                                                                                                                     x_4 1
                                                             \neg x_3:
                                                                    x_4: c_1, c_3
                                                            \neg x_4: c_2, c_4
             Propagate x_4 in c_4: (x_3 \vee \neg x_4) \rightarrow \text{Assign } x_3
```

$$c_1: (x_1 \vee \underline{x_2} \vee \underline{x_4}) \wedge c_2: (\underline{x_2} \vee \underline{-x_4}) \wedge c_3: (x_1 \vee \underline{-x_2} \vee \underline{x_4}) \wedge c_4: (\underline{x_3} \vee \underline{-x_4}) \wedge c_5: (x_1 \vee x_2)$$

Activities

Watch lists:

 $\rightarrow$  SAT

$$x_1: c_5$$
  
 $\neg x_1:$   
 $x_2: c_1, c_2, c_5$   
 $\neg x_2: c_3$   
 $x_3: c_4$   
 $\neg x_3:$ 

 $x_4 : c_1, c_3$  $\neg x_4 : c_2,$  (increment=2):  $x_1 \quad 1$   $x_2 \quad 1$   $x_3 \quad 0$   $x_4 \quad 1$ 

DL1:  $\neg x_1 : \text{NULL}$   $x_2 : c_5$   $x_4 : c_3$  $x_3 : c_4$ 

Trail:

#### Bonus exercise 8

Assume the following propositional logic formula in CNF:

$$\underbrace{(C \vee \neg D)}_{c_0} \wedge \underbrace{(A \vee \neg B \vee \neg D)}_{c_1} \wedge \underbrace{(\neg A \vee \neg B \vee \neg C \vee \neg D)}_{c_2} \wedge \underbrace{(A \vee D)}_{c_3}$$

Apply the DPLL+CDCL algorithm until it detects either a conflict or a complete solution. For a decision, always take the smallest unassigned variable in the order A < B < C < D and assign false to it.

At the first conflict or full solution, how many variables are assigned the value true?

#### Bonus exercise 8

Assume the following propositional logic formula in CNF:

$$\underbrace{(C \vee \neg D)}_{c_0} \wedge \underbrace{(A \vee \neg B \vee \neg D)}_{c_1} \wedge \underbrace{(\neg A \vee \neg B \vee \neg C \vee \neg D)}_{c_2} \wedge \underbrace{(A \vee D)}_{c_3}$$

Apply the DPLL+CDCL algorithm until it detects either a conflict or a complete solution. For a decision, always take the smallest unassigned variable in the order A < B < C < D and assign false to it.

At the first conflict or full solution, how many variables are assigned the value true?

Correct answer: 2

DL0: -

DL1:  $\neg A: nil \quad D: c_3 \quad C: c_0 \quad \neg B: c_1$ 

## 06 SAT solving examples

1 Example 1

## SAT solving: Example 2

- We use DPLL+CDCL SAT solving combining enumeration, propagation and resolution.
- We use watched literals (underlined in formulas) to speed up propagation.
- We use VSIDS as variable ordering heuristics and assign the value false to decision variables.
- In VSIDS, to order variables with the same activity value, we use the lexicographic order; in our example this will be a < b < c < d.

$$c_1: (a \lor b \lor \neg c) \land c_2: (b \lor c) \land c_3: (\neg a \lor b \lor \neg c) \land c_4: (a \lor \neg b \lor c) \land c_5: (a \lor \neg c \lor d) \land c_6: (\neg c \lor \neg d)$$

#### Example – Watchlists

$$c_1: (a \lor b \lor \neg c) \land c_2: (b \lor c) \land c_3: (\neg a \lor b \lor \neg c) \land c_4: (a \lor \neg b \lor c) \land c_5: (a \lor \neg c \lor d) \land c_6: (\neg c \lor \neg d)$$

#### Watch lists:

- a :
- $\neg a$  :
  - b :
- $\neg b$ :
- **c** :
- ¬c:
  - d :
  - ٠.
- $\neg d$ :

$$c_1: (\underline{a} \vee \underline{b} \vee \neg c) \wedge c_2: (b \vee c) \wedge c_3: (\neg a \vee b \vee \neg c) \wedge c_4: (a \vee \neg b \vee c) \wedge c_5: (a \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d)$$

#### Watch lists:

- $a: c_1$
- $\neg a$  :
  - $b: c_1$
- $\neg b$ :
  - **c** :
- $\neg c$  :
- d :
- $\neg d$ :

$$c_1: (\underline{a} \vee \underline{b} \vee \neg c) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\neg a \vee b \vee \neg c) \wedge c_4: (a \vee \neg b \vee c) \wedge c_5: (a \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d)$$

#### Watch lists:

 $a: c_1$   $\neg a:$   $b: c_1, c_2$   $\neg b:$   $c: c_2$   $\neg c:$  d:  $\neg d:$ 

$$c_1: (\underline{a} \vee \underline{b} \vee \neg c) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (\underline{a} \vee \neg b \vee c) \wedge c_5: (\underline{a} \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d)$$

#### Watch lists:

```
a: c_1
\neg a: c_3
b: c_1, c_2, c_3
\neg b:
c: c_2
\neg c:
d:
\neg d:
```

$$c_1: (\underline{a} \vee \underline{b} \vee \neg c) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (\underline{a} \vee \underline{\neg b} \vee c) \wedge c_5: (a \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d)$$

#### Watch lists:

 $a: c_1, c_4$   $\neg a: c_3$   $b: c_1, c_2, c_3$   $\neg b: c_4$   $c: c_2$   $\neg c:$  d:

$$c_1: (\underline{a} \vee \underline{b} \vee \neg c) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (\underline{a} \vee \underline{\neg b} \vee c) \wedge c_5: (\underline{a} \vee \underline{\neg c} \vee d) \wedge c_6: (\neg c \vee \neg d)$$

#### Watch lists:

```
a: c_1, c_4, c_5
\neg a: c_3
b: c_1, c_2, c_3
\neg b: c_4
c: c_2
\neg c: c_5
d:
```

$$c_1: (\underline{a} \vee \underline{b} \vee \neg c) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (\underline{a} \vee \underline{\neg b} \vee c) \wedge c_5: (\underline{a} \vee \underline{\neg c} \vee d) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d})$$

#### Watch lists:

 $a: c_1, c_4, c_5$   $\neg a: c_3$   $b: c_1, c_2, c_3$   $\neg b: c_4$   $c: c_2$   $\neg c: c_5, c_6$  d:  $\neg d: c_6$ 

#### Example – Activities

$$c_1: (\underline{a} \vee \underline{b} \vee \neg c) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (\underline{a} \vee \underline{\neg b} \vee c) \wedge c_5: (\underline{a} \vee \underline{\neg c} \vee d) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d})$$

Watch lists:

$$a: c_1, c_4, c_5$$
  
 $\neg a: c_3$   
 $b: c_1, c_2, c_3$   
 $\neg b: c_4$   
 $c: c_2$ 

 $\neg c$ :  $c_5, c_6$ d :

 $\neg d$ :  $c_6$ 

Activities (increment=1):

0

d = 0

$$c_1: (\underline{a} \vee \underline{b} \vee \neg c) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (\underline{a} \vee \underline{\neg b} \vee c) \wedge c_5: (\underline{a} \vee \underline{\neg c} \vee d) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d})$$

#### Watch lists:

$$a: c_1, c_4, c_5$$
  
 $\neg a: c_3$   
 $b: c_1, c_2, c_3$   
 $\neg b: c_4$   
 $c: c_2$ 

d : ¬d :

 $\neg d$ :  $c_6$ 

 $\neg c$ :  $c_5, c_6$ 

Activities Trail: (increment=1):

a 0b 0c 0

d = 0

$$c_1: (\underline{a} \vee \underline{b} \vee \neg c) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (\underline{a} \vee \underline{\neg b} \vee c) \wedge c_5: (\underline{a} \vee \underline{\neg c} \vee d) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d})$$

#### Watch lists:

Decide  $\neg a$ 

```
a: c_1, c_4, c_5
\neg a: c_3
b: c_1, c_2, c_3
\neg b: c_4
c: c_2
\neg c: c_5, c_6
d:
\neg d: c_6
```

```
Activities Trail: (increment=1):
```

a 0b 0c 0

d = 0

DL1: ¬a: NULL

$$c_1: (\underline{a} \vee \underline{b} \vee \neg c) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (\underline{a} \vee \underline{\neg b} \vee c) \wedge c_5: (\underline{a} \vee \underline{\neg c} \vee d) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d})$$

#### Watch lists:

```
a: c_1, c_4, c_5
\neg a: c_3
b: c_1, c_2, c_3
\neg b: c_4
c: c_2
\neg c: c_5, c_6
d:
\neg d: c_6
```

Propagate  $\neg a$  in

```
Activities Trail: (increment=1): DL1: \neg a: NULL
```

a 0b 0c 0d 0

```
c_1: (a \lor b \lor \neg c) \land c_2: (b \lor c) \land c_3: (\neg a \lor b \lor \neg c) \land c_4: (a \lor \neg b \lor c) \land c_4: (a \lor \neg
               c_5: (a \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d)
Watch lists:
                                                                                                                                                                                                                                                                                                                                                                              Activities
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          Trail:
                                                                                                                                                                                                                                                                                                                                                                              (increment=1):
                                                                                      a: c_1, c_4, c_5
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                DL1: \neg a: NULL
                                                                       \neg a: c_3
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            0
                                                                                                                                                                                                                                                                                                                                                                                                                                                   a
                                                                                    b: c_1, c_2, c_3
                                                                                                                                                                                                                                                                                                                                                                                                                                                 b 0
                                                                     \neg b: c_4
                                                                             c: c_2
                                                                                                                                                                                                                                                                                                                                                                                                                                                   d = 0
                                                                     \neg c: c_5, c_6
                                                                         d :
                                                                     \neg d: c_6
                 Propagate \neg a in c_1: (a \lor b \lor \neg c)
```

 $\neg d$ :  $c_6$ 

```
c_1: (a \lor b \lor \neg c) \land c_2: (b \lor c) \land c_3: (\neg a \lor b \lor \neg c) \land c_4: (a \lor \neg b \lor c) \land c_4: (a \lor \neg
                  c_5: (a \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d)
Watch lists:
                                                                                                                                                                                                                                                                                                                                                                                                                                              Activities
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              Trail:
                                                                                                                                                                                                                                                                                                                                                                                                                                              (increment=1):
                                                                                                     a: \mathcal{C}_1, \mathcal{C}_4, \mathcal{C}_5
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             DL1: \neg a: NULL
                                                                                 \neg a: c_3
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             0
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               a
                                                                                                   b: c_1, c_2, c_3
                                                                                 \neg b: c_4
                                                                                            c: c_2
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               d = 0
                                                                                 \neg c: c_5, c_6, c_1
                                                                                      d :
```

Propagate  $\neg a$  in  $c_1: (a \lor b \lor \neg c) \rightarrow (a \lor b \lor \neg c)$ 

```
c_1: (a \lor b \lor \neg c) \land c_2: (b \lor c) \land c_3: (\neg a \lor b \lor \neg c) \land c_4: (a \lor \neg b \lor c) \land c_4: (a \lor \neg
                 c_5: (a \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d)
Watch lists:
                                                                                                                                                                                                                                                                                                                                                                                                                                 Activities
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         Trail:
                                                                                                                                                                                                                                                                                                                                                                                                                                 (increment=1):
                                                                                                  a: \mathcal{C}_1, \mathcal{C}_4, \mathcal{C}_5
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       DL1: \neg a: NULL
                                                                                 \neg a: c_3
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              0
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                a
                                                                                                b: c_1, c_2, c_3
                                                                               \neg b: c_4
                                                                                          c: c_2
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                d = 0
                                                                               \neg c: c_5, c_6, c_1
                                                                                   d :
                                                                               \neg d: c_6
```

Propagate  $\neg a$  in  $c_4: (a \lor \neg b \lor c)$ 

```
c_1: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (\underline{a} \vee \underline{\neg c} \vee d) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d})
```

```
Watch lists:
                                         Activities
                                                                Trail:
                                         (increment=1):
         a: \mathcal{G}_1, \mathcal{G}_4, \mathcal{C}_5
                                                                     DL1: \neg a: NULL
       \neg a: c_3
                                                     0
                                                a
         b: c_1, c_2, c_3
       \neg b: c_4
        C: C_2, C_4
       \neg c: c_5, c_6, c_1
        d :
       \neg d: c_6
 Propagate \neg a in c_4: (a \lor \neg b \lor c) \to (a \lor \neg b \lor c)
```

```
c_1: (a \lor b \lor \neg c) \land c_2: (b \lor c) \land c_3: (\neg a \lor b \lor \neg c) \land c_4: (a \lor \neg b \lor c) \land c_4: (a \lor \neg
               c_5: (a \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d)
Watch lists:
                                                                                                                                                                                                                                                                                                                                                                             Activities
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         Trail:
                                                                                                                                                                                                                                                                                                                                                                             (increment=1):
                                                                                      a: \mathcal{G}_1, \mathcal{G}_4, \mathcal{C}_5
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              DL1: \neg a: NULL
                                                                       \neg a: c_3
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          0
                                                                                                                                                                                                                                                                                                                                                                                                                                                  a
                                                                                    b: c_1, c_2, c_3
                                                                                                                                                                                                                                                                                                                                                                                                                                                b 0
                                                                     \neg b: c_4
                                                                              C: C_2, C_4
                                                                     \neg c: c_5, c_6, c_1
                                                                          d :
                                                                     \neg d: c_6
                 Propagate \neg a in c_5: (a \lor \neg c \lor d)
```

```
c_1: (a \vee \underline{b} \vee \underline{\neg c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d})
```

```
Watch lists:
                                      Activities
                                                           Trail:
                                      (increment=1):
         a: 91,94,95
                                                                DL1: \neg a: NULL
       \neg a: c_3
                                                 0
                                             a
        b: c_1, c_2, c_3
       \neg b: c_4
        C: C_2, C_4
       \neg c: c_5, c_6, c_1
       d: c<sub>5</sub>
       \neg d: c_6
 Propagate \neg a in c_5: (a \lor \neg c \lor d) \rightarrow (a \lor \neg c \lor d)
```

```
c_1: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d})
```

```
Watch lists:

a:
\neg a: c_3
b: c_1, c_2, c_3
\neg b: c_4
c: c_2, c_4
\neg c: c_5, c_6, c_1
```

 $d: c_5$  $\neg d: c_6$ 

Decide  $\neg b$ 

```
Activities
(increment=1):

a 0
b 0
c 0
d 0
```

```
Trail: \neg a: NULL DL2: \neg b: NULL
```

```
c_1: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d})
```

```
Watch lists:

a:
\neg a: c_3
b: c_1, c_2, c_3
\neg b: c_4
c: c_2, c_4
\neg c: c_5, c_6, c_1
```

 $d: c_5$   $\neg d: c_6$ Propagate  $\neg b$  in

```
Activities
(increment=1):
        0
     a
     b 0
     c = 0
     d = 0
```

```
Trail: \neg a : \text{NULL} DL2: \neg b : \text{NULL}
```

```
c_1: (a \lor b \lor \neg c) \land c_2: (b \lor c) \land c_3: (\neg a \lor b \lor \neg c) \land c_4: (a \lor \neg b \lor c) \land c_4: (a \lor \neg
              c_5: (a \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d)
Watch lists:
                                                                                                                                                                                                                                                                                                                                                       Activities
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         Trail:
                                                                                                                                                                                                                                                                                                                                                       (increment=1):
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          DL1: \neg a: NULL
                                                                                 a :
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          DL2: \neg b: NULL
                                                                  \neg a: c_3
                                                                                                                                                                                                                                                                                                                                                                                                                                                              0
                                                                                                                                                                                                                                                                                                                                                                                                                        a
                                                                             b: c_1, c_2, c_3
                                                                                                                                                                                                                                                                                                                                                                                                                       b 0
                                                                 \neg b: c_4
                                                                                                                                                                                                                                                                                                                                                                                                                        c = 0
                                                                          C: C_2, C_4
                                                                                                                                                                                                                                                                                                                                                                                                                        d = 0
                                                                 \neg c: c_5, c_6, c_1
                                                                  d: c_5
                                                                 \neg d: c_6
                Propagate \neg b in c_1:(a \lor b \lor \neg c)
```

```
c_1: (a \lor b \lor \neg c) \land c_2: (b \lor c) \land c_3: (\neg a \lor b \lor \neg c) \land c_4: (a \lor \neg b \lor c) \land c_4: (a \lor \neg
             c_5: (a \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d)
Watch lists:
                                                                                                                                                                                                                                                                                                                                          Activities
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     Trail:
                                                                                                                                                                                                                                                                                                                                            (increment=1):
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    DL1: \neg a: NULL
                                                                              a :
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    DL2: \neg b: NULL
                                                                \neg a: c_3
                                                                                                                                                                                                                                                                                                                                                                                                                                              0
                                                                                                                                                                                                                                                                                                                                                                                                         a
                                                                          b: c_1, c_2, c_3
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             \neg c : c_1
                                                                                                                                                                                                                                                                                                                                                                                                       b 0
                                                              \neg b: c_4
                                                                       C: C_2, C_4
                                                                                                                                                                                                                                                                                                                                                                                                         d = 0
                                                              \neg c: c_5, c_6, c_1
                                                                  d: c_5
                                                              \neg d: c_6
               Propagate \neg b in c_1: (a \lor b \lor \neg c) \rightarrow assign \neg c
```

```
c_{1}: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_{2}: (\underline{b} \vee \underline{c}) \wedge c_{3}: (\neg \underline{a} \vee \underline{b} \vee \neg c) \wedge c_{4}: (a \vee \neg \underline{b} \vee \underline{c}) \wedge c_{5}: (a \vee \neg \underline{c} \vee \underline{d}) \wedge c_{6}: (\neg \underline{c} \vee \neg \underline{d})
\text{Watch lists:} \qquad \text{Activities} \qquad \text{Trail:} \\ (\text{increment} = 1): \qquad \qquad \text{DL1:} \quad \neg a: \text{NULL}
```

```
(increment=1): DL

a: \quad a: \quad b: \quad c_3 \quad a \quad 0 \quad DL

b: \quad c_1, c_2, c_3 \quad b \quad 0

\neg b: \quad c_4 \quad c \quad 0

c: \quad c_2, c_4 \quad d \quad 0

\neg c: \quad c_5, c_6, c_1

d: \quad c_5

\neg d: \quad c_6

Propagate \neg b in c_2: (b \lor c)
```

DL2:  $\neg b$ : NULL  $\neg c$ :  $c_1$ 

```
c_1: (a \lor b \lor \neg c) \land c_2: (b \lor c) \land c_3: (\neg a \lor b \lor \neg c) \land c_4: (a \lor \neg b \lor c) \land c_4: (a \lor \neg
               c_5: (a \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d)
Watch lists:
                                                                                                                                                                                                                                                                                                                                                                                           Activities
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              Trail:
                                                                                                                                                                                                                                                                                                                                                                                           (increment=1):
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      DL1: \neg a: NULL
                                                                                         a :
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      DL2: \neg b: NULL
                                                                       \neg a: c_3
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            0
                                                                                                                                                                                                                                                                                                                                                                                                                                                                  a
                                                                                     b: c_1, c_2, c_3
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          \neg c : c_1
                                                                                                                                                                                                                                                                                                                                                                                                                                                                b 0
                                                                       \neg b: c_4
                                                                                 C: C_2, C_4
                                                                                                                                                                                                                                                                                                                                                                                                                                                                  d = 0
                                                                       \neg c: c_5, c_6, c_1
                                                                         d: c_5
                                                                       \neg d: c_6
```

Propagate  $\neg b$  in  $c_2: (\underline{b} \lor \underline{c}) \to \{ \text{conflict!} \}$ 

$$c_1: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d})$$

Watch lists:

```
a:
\neg a: c_3
b: c_1, c_2, c_3
\neg b: c_4
c: c_2, c_4
\neg c: c_5, c_6, c_1
```

 $d: c_5$  $\neg d: c_6$ 

Conflict resolution:

```
Activities Trail:
(increment=1):

\begin{array}{cccc}
a & 0 & DL1: & \neg a : NULL \\
DL2: & \neg b : NULL \\
DL3: & \neg c : c_1
\end{array}

\begin{array}{cccc}
c & 0 & \sigma & \sigma & \sigma & \sigma & \sigma \\
d & 0 & & \sigma & \sigma & \sigma & \sigma \\
\end{array}
```

```
c_1: (a \lor b \lor \neg c) \land c_2: (b \lor c) \land c_3: (\neg a \lor b \lor \neg c) \land c_4: (a \lor \neg b \lor c) \land c_4: (a \lor \neg
     c_5: (a \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d)
```

```
Watch lists:
                                      Activities
                                                           Trail:
                                      (increment=2):
                                                                DL1: \neg a: NULL
         a :
                                                                DL2: \neg b: NULL
       \neg a: c_3
                                             a Ø 1
        b: c_1, c_2, c_3
                                                                         \neg c : c_1
                                             b Ø 1
       \neg b: c_4
                                             c \emptyset 1
        C: C_2, C_4
                                             d 0
       \neg c: c_5, c_6, c_1
       d: c_5
       \neg d: c_6
 Conflict resolution: c_2:(b\lor c) c_1:(a\lor b\lor \neg c)
```

c7:(a\b)

```
c_1: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b})
```

#### Watch lists:

```
a: c_7
\neg a: c_3
b: c_1, c_2, c_3, c_7
\neg b: c_4
c: c_2, c_4
\neg c: c_5, c_6, c_1
d: c_5
\neg d: c_6
```

Add conflict clause

```
Activities
(increment=2):

a 1
b 1
c 1
d 0
```

```
DL1: \neg a : \text{NULL}
DL2: \neg b : \text{NULL}
\neg c : c_1
```

Trail:

```
c_1: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b})
```

# Watch lists:

```
a: c_7
\neg a: c_3
b: c_1, c_2, c_3, c_7
\neg b: c_4
c: c_2, c_4
\neg c: c_5, c_6, c_1
d: c_5
```

 $\neg d: c_5$ 

Backtrack to DI 1

Activities Trail: (increment=2):

a 1
b 1

c 1 d 0

DL1: ¬a: NULL
DL2: ¬b: NULL

```
c_1: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b})
```

#### Watch lists:

```
a: c_7
\neg a: c_3
b: c_1, c_2, c_3, c_7
\neg b: c_4
c: c_2, c_4
\neg c: c_5, c_6, c_1
d: c_5
\neg d: c_6
```

Assign b at DL1 by c<sub>7</sub>

$$c_1: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b})$$

#### Watch lists:

```
a: c_7
\neg a: c_3
b: c_1, c_2, c_3, c_7
\neg b: c_4
c: c_2, c_4
\neg c: c_5, c_6, c_1
d: c_5
\neg d: c_6
```

Propagate b in

Activities Trail: (increment=2):

b 1c 1d 0

a 1

DL1: ¬a: NULL b: c<sub>7</sub>

```
c_1: (a \lor b \lor \neg c) \land c_2: (b \lor c) \land c_3: (\neg a \lor b \lor \neg c) \land c_4: (a \lor \neg b \lor c) \land c_4: (a \lor \neg
            c_5: (a \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d) \wedge c_7: (a \vee b)
                                                                                                                                                                                                                                                                                                                                                Activities
Watch lists:
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              Trail:
                                                                                                                                                                                                                                                                                                                                                 (increment=2):
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                DL1: \neg a: NULL
                                                                                a: c7
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            b : c<sub>7</sub>
                                                                \neg a: c_3
                                                                                                                                                                                                                                                                                                                                                                                                                a 1
                                                                                b: c_1, c_2, c_3, c_7
                                                                                                                                                                                                                                                                                                                                                                                                                b 1
                                                                \neg b: c_4
                                                                                                                                                                                                                                                                                                                                                                                                                 c 1
                                                                       c: c_2, c_4
                                                                                                                                                                                                                                                                                                                                                                                                                 d = 0
                                                                \neg c: c_5, c_6, c_1
                                                                 d: c<sub>5</sub>
                                                                \neg d: c_6
                Propagate b in c_4:(a \vee \neg b \vee c)
```

```
c_1: (a \lor b \lor \neg c) \land c_2: (b \lor c) \land c_3: (\neg a \lor b \lor \neg c) \land c_4: (a \lor \neg b \lor c) \land c_4: (a \lor \neg
             c_5: (a \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d) \wedge c_7: (a \vee b)
                                                                                                                                                                                                                                                                                                                                                                          Activities
Watch lists:
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        Trail:
                                                                                                                                                                                                                                                                                                                                                                            (increment=2):
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              DL1: \neg a: NULL
                                                                                      a: c7
                                                                     \neg a: c_3
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 b : C7
                                                                                                                                                                                                                                                                                                                                                                                                                                                a 1
                                                                                      b: c_1, c_2, c_3, c_7
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 C: C_4
                                                                                                                                                                                                                                                                                                                                                                                                                                                b 1
                                                                     \neg b: c_4
                                                                                                                                                                                                                                                                                                                                                                                                                                                 c 1
                                                                            c: c_2, c_4
                                                                                                                                                                                                                                                                                                                                                                                                                                                 d = 0
                                                                     \neg c: c_5, c_6, c_1
                                                                       d: c<sub>5</sub>
                                                                     \neg d: c_6
```

Propagate b in  $c_4: (a \lor \neg b \lor c) \to assign c$ 

$$c_1: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b})$$

#### Watch lists:

Propagate c in

```
a: c_7
\neg a: c_3
b: c_1, c_2, c_3, c_7
\neg b: c_4
c: c_2, c_4
\neg c: c_5, c_6, c_1
d: c_5
\neg d: c_6
```

Activities Trail: (increment=2):

a 1b 1c 1d 0

DL1:  $\neg a$ : NULL b:  $c_7$ 

```
c_1: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b})
```

```
Watch lists:
                                     Activities
                                                          Trail:
                                     (increment=2):
                                                               DL1: \neg a: NULL
        a: c7
       \neg a: c_3
                                                                         b : c<sub>7</sub>
                                            a 1
        b: c_1, c_2, c_3, c_7
                                                                         C: C_4
                                            b 1
       \neg b: c_4
                                            c 1
       c: c_2, c_4
                                            d = 0
       \neg c: c_5, c_6, c_1
       d: c<sub>5</sub>
       \neg d: c_6
 Propagate c in c_5: (a \lor \neg c \lor d)
```

```
c_1: (a \lor b \lor \neg c) \land c_2: (b \lor c) \land c_3: (\neg a \lor b \lor \neg c) \land c_4: (a \lor \neg b \lor c) \land c_4: (a \lor \neg
           c_5: (a \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d) \wedge c_7: (a \vee b)
                                                                                                                                                                                                                                                                                                                          Activities
Watch lists:
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            Trail:
                                                                                                                                                                                                                                                                                                                            (increment=2):
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           DL1: \neg a: NULL
                                                                          a: c7
                                                           \neg a: c_3
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  b : c7
                                                                                                                                                                                                                                                                                                                                                                                      a 1
                                                                         b: c_1, c_2, c_3, c_7
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  C:C_4
                                                                                                                                                                                                                                                                                                                                                                                      b 1
                                                           \neg b: c_4
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  d: c_5
                                                                                                                                                                                                                                                                                                                                                                                        c 1
                                                                  c: c_2, c_4
                                                                                                                                                                                                                                                                                                                                                                                        d = 0
                                                           \neg c: c_5, c_6, c_1
                                                             d: c<sub>5</sub>
                                                           \neg d: c_6
              Propagate c in c_5: (a \lor \neg c \lor d) \rightarrow assign d
```

```
c_1: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b})
```

```
Watch lists:
                                   Activities
                                                       Trail:
                                   (increment=2):
                                                            DL1: \neg a: NULL
        a: c7
      \neg a: c_3
                                                                      b : c7
                                          a 1
        b: c_1, c_2, c_3, c_7
                                                                      C:C_4
                                          b 1
      \neg b: c_4
                                                                      d: c_5
                                          c 1
       c: c_2, c_4
                                          d = 0
      \neg c: C_5, C_6, C_1
      d: c<sub>5</sub>
      \neg d: c_6
 Propagate c in c_6: (\neg c \lor \neg d)
```

```
c_1: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b})
```

```
Watch lists:
                                    Activities
                                                         Trail:
                                    (increment=2):
                                                              DL1: \neg a: NULL
        a: c7
      \neg a: c_3
                                                                        b : c7
                                           a 1
        b: c_1, c_2, c_3, c_7
                                                                        C:C_4
                                           b 1
      \neg b: c_4
                                                                        d: c_5
                                           c 1
       c: c_2, c_4
                                           d = 0
      \neg c: C_5, C_6, C_1
       d: c<sub>5</sub>
      \neg d: c_6
 Propagate c in c_6: (\neg c \lor \neg d) \to \mbox{$f$ conflict!}
```

$$c_1: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b})$$

Watch lists:

```
a: c_7
\neg a: c_3
b: c_1, c_2, c_3, c_7
\neg b: c_4
c: c_2, c_4
\neg c: c_5, c_6, c_1
d: c_5
```

 $\neg d$ :  $c_6$  Conflict resolution:

Activities Trail:
(increment=2):

a 1
b 1
c 1
d 0

DL1:  $\neg a : \text{NULL}$   $b : c_7$   $c : c_4$   $d : c_5$ 

$$c_1: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b})$$

```
Watch lists:
                                  Activities
                                                      Trail:
                                  (increment=3):
                                                           DL1: \neg a: NULL
        a: c7
      \neg a: c_3
                                                                    b : c7
                                         a 1/3
        b: c_1, c_2, c_3, c_7
                                         b 1/3
                                                                    C:C_4
      \neg b: c_4
                                                                    d: c_5
                                         c 1/3
                                         d Ø 2
        C: C_2, C_4
      \neg c: C_5, C_6, C_1
      d: c<sub>5</sub>
      \neg d: c_6
```

Conflict resolution:

$$\frac{c_6{:}(\neg c \lor \neg d) \ c_5{:}(a \lor \neg c \lor d)}{\frac{(a \lor \neg c) \ c_4{:}(a \lor \neg b \lor c)}{(a \lor \neg b) \ c_7{:}(a \lor b)}}{\frac{(a \lor \neg b) \ c_7{:}(a \lor b)}{c_8{:}(a)}}$$

$$c_1: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b}) \wedge c_8: (\underline{a})$$

#### Watch lists:

```
a: c_7
\neg a: c_3
b: c_1, c_2, c_3, c_7
\neg b: c_4
c: c_2, c_4
\neg c: c_5, c_6, c_1
d: c_5
```

 $\neg d$ :  $c_6$ Add conflict clause Activities Trail: (increment=3):

a 3 b 3 c 3 d 2 DL1:  $\neg a$ : NULL b:  $c_7$  c:  $c_4$ d:  $c_5$ 

$$c_1: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b}) \wedge c_8: (\underline{a})$$

#### Watch lists:

```
a: c_7
\neg a: c_3
b: c_1, c_2, c_3, c_7
\neg b: c_4
c: c_2, c_4
\neg c: c_5, c_6, c_1
d: c_5
```

 $\neg d$ :  $c_6$ Backtrack to DL0 Activities Trail: (increment=3):

a 3 b 3 c 3 d 2 DLT: Ja: NUTT
b: c7
c:e4
d:c5

```
c_1: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b}) \wedge c_8: (a)
```

# Watch lists:

```
a: c<sub>7</sub>
¬a: c<sub>3</sub>
b: c<sub>1</sub>, c<sub>2</sub>, c<sub>3</sub>, c<sub>7</sub>
¬b: c<sub>4</sub>
c: c<sub>2</sub>, c<sub>4</sub>
¬c: c<sub>5</sub>, c<sub>6</sub>, c<sub>1</sub>
d: c<sub>5</sub>
¬d: c<sub>6</sub>
```

Assign a at DL0 by  $c_8$ 

Activities Trail: (increment=3):

a 3b 3c 3d 2

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DL0:  $a:c_8$ 

$$c_1: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b}) \wedge c_8: (a)$$

#### Watch lists:

```
a: c_7
\neg a: c_3
b: c_1, c_2, c_3, c_7
\neg b: c_4
c: c_2, c_4
\neg c: c_5, c_6, c_1
d: c_5
```

 $\neg d$ :  $c_6$  Propagate a in

Activities Trail: (increment=3):

a 3b 3c 3d 2

DL0:  $a:c_8$ 

```
c_1: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b}) \wedge c_8: (a)
```

```
Watch lists:
                                     Activities
                                                         Trail:
                                     (increment=3):
        a: c<sub>7</sub>
                                                               DL0: a:c_8
       \neg a: c_3
                                            a 3
        b: c_1, c_2, c_3, c_7
                                            b 3
       \neg b: c_4
                                            c 3
       c: c_2, c_4
                                            d 2
       \neg c: c_5, c_6, c_1
       d: c<sub>5</sub>
       \neg d: c_6
 Propagate a in c_3: (\neg a \lor b \lor \neg c)
```

 $\neg d$ :  $c_6$ 

```
c_1: (a \lor b \lor \neg c) \land c_2: (b \lor c) \land c_3: (\neg a \lor b \lor \neg c) \land c_4: (a \lor \neg b \lor c) \land c_4: (a \lor \neg
             c_5: (a \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d) \wedge c_7: (a \vee b) \wedge c_8: (a)
Watch lists:
                                                                                                                                                                                                                                                                                                                                                                                         Activities
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           Trail:
                                                                                                                                                                                                                                                                                                                                                                                         (increment=3):
                                                                                         a: c<sub>7</sub>
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   DL0: a: c_8
                                                                       ¬a: 03
                                                                                                                                                                                                                                                                                                                                                                                                                                                              a 3
                                                                                       b: c_1, c_2, c_3, c_7
                                                                                                                                                                                                                                                                                                                                                                                                                                                              b 3
                                                                       \neg b: c_4
                                                                                                                                                                                                                                                                                                                                                                                                                                                                c 3
                                                                               C: C_2, C_4
                                                                                                                                                                                                                                                                                                                                                                                                                                                                d 2
                                                                       \neg c: c_5, c_6, c_1, c_3
                                                                         d: c<sub>5</sub>
```

Propagate a in  $c_3: (\neg a \lor b \lor \neg c) \to (\neg a \lor b \lor \neg c)$ 

```
c_1: (a \vee \underline{b} \vee \underline{\neg c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\neg a \vee \underline{b} \vee \underline{\neg c}) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b}) \wedge c_8: (a)
```

#### Watch lists:

```
a: c_7
\neg a:
b: c_1, c_2, c_3, c_7
\neg b: c_4
c: c_2, c_4
\neg c: c_5, c_6, c_1, c_3
d: c_5
\neg d: c_6

Decide \neg b
```

```
Activities Trail: (increment=3):
```

b 3c 3d 2

```
DL0: a: c_8
DL1: \neg b: \text{NULL}
```

$$c_1: (a \vee \underline{b} \vee \underline{\neg c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\neg a \vee \underline{b} \vee \underline{\neg c}) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b}) \wedge c_8: (a)$$

#### Watch lists:

```
a: c_7
\neg a:
b: c_1, c_2, c_3, c_7
\neg b: c_4
c: c_2, c_4
\neg c: c_5, c_6, c_1, c_3
d: c_5
\neg d: c_6
```

Propagate  $\neg b$  in

```
Activities Trail: (increment=3):
```

a 3b 3c 3d 2

DL0:  $a: c_8$ DL1:  $\neg b: \text{NULL}$ 

```
c_1: (a \vee \underline{b} \vee \underline{\neg c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\neg a \vee \underline{b} \vee \underline{\neg c}) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b}) \wedge c_8: (a)
```

```
Activities
Watch lists:
                                                         Trail:
                                     (increment=3):
                                                               DL0: a: c_8
        a: c<sub>7</sub>
                                                               DL1: \neg b: NULL
       \neg a:
                                            a 3
        b: c_1, c_2, c_3, c_7
                                            b 3
       \neg b: c_4
                                            c 3
       c: c_2, c_4
                                            d 2
       \neg c: c_5, c_6, c_1, c_3
       d: c<sub>5</sub>
       \neg d: c_6
 Propagate \neg b in c_1:(a \lor b \lor \neg c)
```

```
c_1: (a \lor b \lor \neg c) \land c_2: (b \lor c) \land c_3: (\neg a \lor b \lor \neg c) \land c_4: (a \lor \neg b \lor c) \land c_4: (a \lor \neg
            c_5: (a \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d) \wedge c_7: (a \vee b) \wedge c_8: (a)
Watch lists:
                                                                                                                                                                                                                                                                                                                                                             Activities
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 Trail:
                                                                                                                                                                                                                                                                                                                                                             (increment=3):
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    DL0: a: c_8
                                                                                  a: c_7, c_1
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    DL1: \neg b: NULL
                                                                    ¬a:
                                                                                                                                                                                                                                                                                                                                                                                                                                a 3
                                                                             b: \varphi_1, c_2, c_3, c_7
                                                                                                                                                                                                                                                                                                                                                                                                                                b 3
                                                                  \neg b: c_4
                                                                                                                                                                                                                                                                                                                                                                                                                                c 3
                                                                           C: C_2, C_4
                                                                                                                                                                                                                                                                                                                                                                                                                                d 2
                                                                  \neg c: c_5, c_6, c_1, c_3
                                                                     d: c_5
                                                                  \neg d: c_6
```

Propagate  $\neg b$  in  $c_1: (a \lor b \lor \neg c) \to (a \lor b \lor \neg c)$ 

```
c_1: (\underline{a} \lor \underline{b} \lor \underline{\neg}\underline{c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg \underline{a} \lor \underline{b} \lor \underline{\neg}\underline{c}) \land c_4: (\underline{a} \lor \underline{\neg}\underline{b} \lor \underline{c}) \land c_5: (\underline{a} \lor \underline{\neg}\underline{c} \lor \underline{d}) \land c_6: (\underline{\neg}\underline{c} \lor \underline{\neg}\underline{d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (\underline{a})
```

```
Activities
Watch lists:
                                                     Trail:
                                  (increment=3):
                                                          DL0: a: c_8
        a: c_7, c_1
                                                          DL1: \neg b: NULL
      \neg a:
                                        a 3
       b: \varphi_1, c_2, c_3, c_7
                                        b 3
      \neg b: c_4
                                        c 3
       C: C_2, C_4
                                        d 2
      \neg c: c_5, c_6, c_1, c_3
      d: c_5
      \neg d: c_6
 Propagate \neg b in c_2:(b \lor c)
```

```
c_1: (\underline{a} \lor b \lor \neg \underline{c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg a \lor \underline{b} \lor \neg \underline{c}) \land c_4: (a \lor \neg \underline{b} \lor \underline{c}) \land c_5: (a \lor \neg \underline{c} \lor \underline{d}) \land c_6: (\neg \underline{c} \lor \neg \underline{d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (a)
```

```
Activities
Watch lists:
                                                    Trail:
                                 (increment=3):
                                                         DL0: a: c_8
       a: c_7, c_1
                                                         DL1: \neg b: NULL
      ¬a:
                                        a 3
       b: \varphi_1, c_2, c_3, c_7
                                        b 3
                                                                  C:C_2
      \neg b: c_4
                                        c 3
       C: C_2, C_4
                                        d 2
      \neg c: c_5, c_6, c_1, c_3
      d: c_5
      \neg d: c_6
 Propagate \neg b in c_2:(b \lor c) \to assign c
```

```
c_1: (\underline{a} \lor \underline{b} \lor \underline{\neg}\underline{c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg \underline{a} \lor \underline{b} \lor \underline{\neg}\underline{c}) \land c_4: (\underline{a} \lor \underline{\neg}\underline{b} \lor \underline{c}) \land c_5: (\underline{a} \lor \underline{\neg}\underline{c} \lor \underline{d}) \land c_6: (\underline{\neg}\underline{c} \lor \underline{\neg}\underline{d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (\underline{a})
```

```
Activities
Watch lists:
                                                         Trail:
                                    (increment=3):
                                                              DL0: a: c_8
        a: c_7, c_1
                                                              DL1: \neg b: NULL
       \neg a:
                                           a 3
        b: \varphi_1, c_2, c_3, c_7
                                                                        C:C_2
                                           b 3
      \neg b: c_4
                                           c 3
       c: c_2, c_4
                                           d 2
      \neg c: c_5, c_6, c_1, c_3
       d: c<sub>5</sub>
      \neg d: c_6
 Propagate \neg b in c_3: (\neg a \lor b \lor \neg c)
```

```
c_1: (\underline{a} \lor b \lor \underline{\neg c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg a \lor \underline{b} \lor \underline{\neg c}) \land c_4: (a \lor \underline{\neg b} \lor \underline{c}) \land c_5: (a \lor \underline{\neg c} \lor \underline{d}) \land c_6: (\underline{\neg c} \lor \underline{\neg d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (a)
```

```
Watch lists:
                                      Activities
                                                            Trail:
                                      (increment=3):
                                                                  DL0: a: c_8
         a: c_7, c_1
                                                                  DL1: \neg b: NULL
       \neg a:
                                              a 3
        b: \varphi_1, c_2, c_3, c_7
                                                                             C:C_2
                                              b 3
       \neg b: c_4
                                              c 3
        c: c_2, c_4
                                              d 2
       \neg c: c_5, c_6, c_1, c_3
       d: c<sub>5</sub>
       \neg d: c_6
 Propagate \neg b in c_3: (\neg a \lor b \lor \neg c) \to \frac{1}{2} \text{ conflict!}
```

$$c_1: (\underline{a} \lor b \lor \underline{\neg c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg a \lor \underline{b} \lor \underline{\neg c}) \land c_4: (a \lor \underline{\neg b} \lor \underline{c}) \land c_5: (a \lor \underline{\neg c} \lor \underline{d}) \land c_6: (\underline{\neg c} \lor \underline{\neg d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (a)$$

#### Watch lists:

```
a: c_7, c_1
\neg a:
b: c_4, c_2, c_3, c_7
\neg b: c_4
c: c_2, c_4
\neg c: c_5, c_6, c_1, c_3
d: c_5
\neg d: c_6
```

Activities Trail: (increment=3):

a 3b 3c 3

d 2

DL0:  $a: c_8$ DL1:  $\neg b: \text{NULL}$  $c: c_2$ 

```
c_1: (\underline{a} \lor \underline{b} \lor \underline{\neg}\underline{c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg \underline{a} \lor \underline{b} \lor \underline{\neg}\underline{c}) \land c_4: (\underline{a} \lor \underline{\neg}\underline{b} \lor \underline{c}) \land c_5: (\underline{a} \lor \underline{\neg}\underline{c} \lor \underline{d}) \land c_6: (\underline{\neg}\underline{c} \lor \underline{\neg}\underline{d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (\underline{a})
```

```
Watch lists:
                                    Activities
                                                        Trail:
                                    (increment=4):
                                                             DL0: a:c_8
        a: c_7, c_1
                                                             DL1: \neg b: NULL
       \neg a:
                                          a 36
        b: c_1, c_2, c_3, c_7
                                          b 36
                                                                       C:C_2
      \neg b: c_4
                                          c 36
        C: C_2, C_4
                                          d 2
      \neg c: c_5, c_6, c_1, c_3
       d: c_5
      \neg d: c_6
                          c_3:(\neg a \lor b \lor \neg c) c_2:(b \lor c)
 Conflict resolution:
```

 $c_0:(\neg a \lor b)$ 

$$c_1: (\underline{a} \lor b \lor \underline{\neg c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg a \lor \underline{b} \lor \underline{\neg c}) \land c_4: (a \lor \underline{\neg b} \lor \underline{c}) \land c_5: (a \lor \underline{\neg c} \lor \underline{d}) \land c_6: (\underline{\neg c} \lor \underline{\neg d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (a) \land \underline{c_9}: (\underline{\neg a} \lor \underline{b})$$

#### Watch lists:

```
a: c_7, c_1
\neg a: c_9
b: g_1, c_2, c_3, c_7, c_9
\neg b: c_4
c: c_2, c_4
\neg c: c_5, c_6, c_1, c_3
d: c_5
\neg d: c_6
```

Add conflict clause

Activities Trail: (increment=4):

a 6b 6c 6

d 2

DL0:  $a: c_8$ DL1:  $\neg b: \text{NULL}$  $c: c_2$ 

$$c_1: (\underline{a} \lor b \lor \underline{\neg c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg a \lor \underline{b} \lor \underline{\neg c}) \land c_4: (a \lor \underline{\neg b} \lor \underline{c}) \land c_5: (a \lor \underline{\neg c} \lor \underline{d}) \land c_6: (\underline{\neg c} \lor \underline{\neg d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (a) \land \underline{c_9}: (\underline{\neg a} \lor \underline{b})$$

#### Watch lists:

```
a: c_7, c_1
\neg a: c_9
b: \mathscr{A}, c_2, c_3, c_7, c_9
\neg b: c_4
c: c_2, c_4
\neg c: c_5, c_6, c_1, c_3
d: c_5
\neg d: c_6
```

Activities (increment=4):

a 6 b 6 c 6 d 2 DL0: *a* : *c*<sub>8</sub>

DL1: ¬*b*: NULL

Trail:

sies

Backtrack to DL0

$$c_1: (\underline{a} \lor b \lor \underline{\neg c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg a \lor \underline{b} \lor \underline{\neg c}) \land c_4: (a \lor \underline{\neg b} \lor \underline{c}) \land c_5: (a \lor \underline{\neg c} \lor \underline{d}) \land c_6: (\underline{\neg c} \lor \underline{\neg d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (a) \land \underline{c_9}: (\underline{\neg a} \lor \underline{b})$$

#### Watch lists:

```
a: c_7, c_1
\neg a: c_9
b: \varphi_1, c_2, c_3, c_7, c_9
\neg b: c_4
c: c_2, c_4
\neg c: c_5, c_6, c_1, c_3
d: c_5
\neg d: c_6
Assign b by c_9
```

$$c_1: (\underline{a} \lor b \lor \underline{\neg c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg a \lor \underline{b} \lor \underline{\neg c}) \land c_4: (a \lor \underline{\neg b} \lor \underline{c}) \land c_5: (a \lor \underline{\neg c} \lor \underline{d}) \land c_6: (\underline{\neg c} \lor \underline{\neg d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (a) \land c_9: (\underline{\neg a} \lor \underline{b})$$

#### Watch lists:

Propagate b in

```
a: c_7, c_1
\neg a: c_9
b: c_2, c_3, c_7, c_9
\neg b: c_4
c: c_2, c_4
\neg c: c_5, c_6, c_1, c_3
d: c_5
\neg d: c_6
```

Activities Trail: (increment=4):

a 6 b 6 c 6 d 2 DL0: a: c<sub>8</sub> b: c<sub>9</sub>

*D* . eg

```
c_1: (\underline{a} \lor b \lor \underline{\neg c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg a \lor \underline{b} \lor \underline{\neg c}) \land c_4: (\underline{a} \lor \underline{\neg b} \lor \underline{c}) \land c_5: (\underline{a} \lor \underline{\neg c} \lor \underline{d}) \land c_6: (\underline{\neg c} \lor \underline{\neg d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (\underline{a}) \land c_9: (\underline{\neg a} \lor \underline{b})
```

```
Watch lists:
                                  Activities
                                                     Trail:
                                  (increment=4):
                                                          DL0: a: c_8
        a: c_7, c_1
                                                                   b : c9
      \neg a: c_0
                                        a 6
        b: c_2, c_3, c_7, c_9
                                        b 6
      \neg b: c_4
                                        c 6
       c: c_2, c_4
                                        d 2
      \neg c: c_5, c_6, c_1, c_3
      d: c_5
      \neg d: c_6
 Propagate b in c_4:(a \vee \neg b \vee c)
```

```
c_1: (\underline{a} \lor \underline{b} \lor \underline{\neg}\underline{c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg \underline{a} \lor \underline{b} \lor \underline{\neg}\underline{c}) \land c_4: (\underline{a} \lor \neg \underline{b} \lor \underline{c}) \land c_5: (\underline{a} \lor \underline{\neg}\underline{c} \lor \underline{d}) \land c_6: (\underline{\neg}\underline{c} \lor \underline{\neg}\underline{d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (\underline{a}) \land c_9: (\underline{\neg}\underline{a} \lor \underline{b})
```

```
Activities
Watch lists:
                                                         Trail:
                                     (increment=4):
                                                               DL0: a: c_8
        a: c_7, c_1, c_4
       \neg a: c_0
                                                                         b: c_0
                                            a 6
        b: c_2, c_3, c_7, c_9
                                            b 6
       \neg b: \mathcal{G}_4
                                            c = 6
       C: C_2, C_4
                                            d 2
       \neg c: c_5, c_6, c_1, c_3
       d: c_5
       \neg d: c_6
 Propagate b in c_4: (a \lor \neg b \lor c) \to (a \lor \neg b \lor c)
```

$$c_1: (\underline{a} \lor \underline{b} \lor \underline{\neg}\underline{c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg \underline{a} \lor \underline{b} \lor \underline{\neg}\underline{c}) \land c_4: (\underline{a} \lor \neg \underline{b} \lor \underline{c}) \land c_5: (\underline{a} \lor \underline{\neg}\underline{c} \lor \underline{d}) \land c_6: (\underline{\neg}\underline{c} \lor \underline{\neg}\underline{d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (\underline{a}) \land c_9: (\underline{\neg}\underline{a} \lor \underline{b})$$

#### Watch lists:

Decide  $\neg c$ 

```
a: c_7, c_1, c_4
\neg a: c_9
b: c_2, c_3, c_7, c_9
\neg b:
c: c_2, c_4
\neg c: c_5, c_6, c_1, c_3
d: c_5
\neg d: c_6
```

```
Activities Trail: (increment=4): DL0: a: c_8
a \quad 6 \qquad b: c_9
b \quad 6 \qquad DL1: \neg c: NULL
c \quad 6
d \quad 2
```

$$c_1: (\underline{a} \lor b \lor \underline{\neg c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg a \lor \underline{b} \lor \underline{\neg c}) \land c_4: (\underline{a} \lor \neg b \lor \underline{c}) \land c_5: (a \lor \underline{\neg c} \lor \underline{d}) \land c_6: (\underline{\neg c} \lor \underline{\neg d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (a) \land c_9: (\underline{\neg a} \lor \underline{b})$$

#### Watch lists:

$$a: c_7, c_1, c_4$$
 $\neg a: c_9$ 
 $b: c_2, c_3, c_7, c_9$ 
 $\neg b:$ 
 $c: c_2, c_4$ 
 $\neg c: c_5, c_6, c_1, c_3$ 
 $d: c_5$ 
 $\neg d: c_6$ 

Propagate  $\neg c$  in

$$c_1: (\underline{a} \lor \underline{b} \lor \underline{\neg}\underline{c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg \underline{a} \lor \underline{b} \lor \underline{\neg}\underline{c}) \land c_4: (\underline{a} \lor \neg \underline{b} \lor \underline{c}) \land c_5: (\underline{a} \lor \underline{\neg}\underline{c} \lor \underline{d}) \land c_6: (\underline{\neg}\underline{c} \lor \underline{\neg}\underline{d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (\underline{a}) \land c_9: (\underline{\neg}\underline{a} \lor \underline{b})$$

```
Activities
Watch lists:
                                                     Trail:
                                  (increment=4):
                                                          DL0: a: c_8
        a: c_7, c_1, c_4
      \neg a: c_0
                                                                   b : c9
                                        a 6
                                                          DL1: \neg c : NULL
        b: c_2, c_3, c_7, c_9
                                        b 6
      \neg b:
                                        c 6
       C: C_2, C_4
                                        d 2
      \neg c: c_5, c_6, c_1, c_3
      d: c_5
      \neg d: c_6
 Propagate \neg c in c_2:(b \lor c)
```

```
c_1: (\underline{a} \lor \underline{b} \lor \underline{\neg}\underline{c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg \underline{a} \lor \underline{b} \lor \underline{\neg}\underline{c}) \land c_4: (\underline{a} \lor \neg \underline{b} \lor \underline{c}) \land c_5: (\underline{a} \lor \underline{\neg}\underline{c} \lor \underline{d}) \land c_6: (\underline{\neg}\underline{c} \lor \underline{\neg}\underline{d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (\underline{a}) \land c_9: (\underline{\neg}\underline{a} \lor \underline{b})
```

```
Activities
Watch lists:
                                                      Trail:
                                   (increment=4):
                                                            DL0: a: c_8
        a: c_7, c_1, c_4
      \neg a: c_0
                                                                     b : c9
                                         a 6
                                                            DL1: \neg c : NULL
        b: c_2, c_3, c_7, c_9
                                         b 6
      \neg b:
                                         c = 6
       C: C_2, C_4
                                         d 2
      \neg c: c_5, c_6, c_1, c_3
       d: c_5
      \neg d: c_6
 Propagate \neg c in c_4: (a \lor \neg b \lor c)
```

$$c_1: (\underline{a} \lor \underline{b} \lor \underline{\neg}\underline{c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg \underline{a} \lor \underline{b} \lor \underline{\neg}\underline{c}) \land c_4: (\underline{a} \lor \neg \underline{b} \lor \underline{c}) \land c_5: (\underline{a} \lor \underline{\neg}\underline{c} \lor \underline{d}) \land c_6: (\underline{\neg}\underline{c} \lor \underline{\neg}\underline{d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (\underline{a}) \land c_9: (\underline{\neg}\underline{a} \lor \underline{b})$$

#### Watch lists:

Decide  $\neg d$ 

```
a: c_7, c_1, c_4
\neg a: c_9
b: c_2, c_3, c_7, c_9
\neg b:
c: c_2, c_4
\neg c: c_5, c_6, c_1, c_3
d: c_5
\neg d: c_6
```

Activities Trail: (increment=4):

a 6
b 6
c 6
d 2

DL0:  $a: c_8$   $b: c_9$ DL1:  $\neg c: NULL$ DL2:  $\neg d: NULL$ 

$$c_1: (\underline{a} \lor b \lor \underline{\neg c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg a \lor \underline{b} \lor \underline{\neg c}) \land c_4: (\underline{a} \lor \neg b \lor \underline{c}) \land c_5: (a \lor \underline{\neg c} \lor \underline{d}) \land c_6: (\underline{\neg c} \lor \underline{\neg d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (a) \land c_9: (\underline{\neg a} \lor \underline{b})$$

#### Watch lists:

$$a: c_7, c_1, c_4$$
 $\neg a: c_9$ 
 $b: c_2, c_3, c_7, c_9$ 
 $\neg b:$ 
 $c: c_2, c_4$ 
 $\neg c: c_5, c_6, c_1, c_3$ 
 $d: c_5$ 
 $\neg d: c_6$ 

Propagate  $\neg d$  in

Activities Trail: (increment=4):

a 6
b 6
c 6
d 2

DL0:  $a: c_8$   $b: c_9$ DL1:  $\neg c: NULL$ DL2:  $\neg d: NULL$ 

```
c_1: (\underline{a} \lor \underline{b} \lor \underline{\neg}\underline{c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg \underline{a} \lor \underline{b} \lor \underline{\neg}\underline{c}) \land c_4: (\underline{a} \lor \neg \underline{b} \lor \underline{c}) \land c_5: (\underline{a} \lor \underline{\neg}\underline{c} \lor \underline{d}) \land c_6: (\underline{\neg}\underline{c} \lor \underline{\neg}\underline{d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (\underline{a}) \land c_9: (\underline{\neg}\underline{a} \lor \underline{b})
```

```
Activities
Watch lists:
                                                      Trail:
                                  (increment=4):
                                                           DL0: a: c_8
        a: c_7, c_1, c_4
      \neg a: c_0
                                                                    b : c9
                                         a 6
        b: c_2, c_3, c_7, c_9
                                                           DL1: \neg c : NULL
                                         b 6
                                                           DI 2: \neg d: NULL
      \neg b:
                                         c 6
       C: C_2, C_4
                                         d 2
      \neg c: c_5, c_6, c_1, c_3
      d: c_5
      \neg d: c_6
 Propagate \neg d in c_5: (a \lor \neg c \lor d)
```

$$c_1: (\underline{a} \lor \underline{b} \lor \underline{\neg}\underline{c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg \underline{a} \lor \underline{b} \lor \underline{\neg}\underline{c}) \land c_4: (\underline{a} \lor \neg \underline{b} \lor \underline{c}) \land c_5: (\underline{a} \lor \underline{\neg}\underline{c} \lor \underline{d}) \land c_6: (\underline{\neg}\underline{c} \lor \underline{\neg}\underline{d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (\underline{a}) \land c_9: (\underline{\neg}\underline{a} \lor \underline{b})$$

Watch lists:

$$a: c_7, c_1, c_4$$
¬a: c<sub>9</sub>

$$b: c_2, c_3, c_7, c_9$$
¬b:
$$c: c_2, c_4$$
¬c: c<sub>5</sub>, c<sub>6</sub>, c<sub>1</sub>, c<sub>3</sub>

$$d: c_5$$
¬d: c<sub>6</sub>
⇒ SAT a b ¬c ¬d

```
Activities
               Trail:
(increment=4):
                   DL0: a: c_8
     a 6
                   DL1: \neg c : NULL
     b 6
                   DL2: \neg d: NULL
     c 6
     d 2
```

b : c9

### Learning target

■ How to compute with the DPLL+CDCL-based SAT solving algorithm?