hybr2d Theory of Hybrid Systems Informatik 2
RWTH AACHEN UNIVERSITY

Satisfiability Checking - WS 2023/2024

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Series 1

Exercise 1

Let $AP = \{a, b\}$ be a set of propositions and let

$$\begin{aligned} \varphi_1 &:= ((a \oplus \neg b) \rightarrow b) \lor (\neg a \leftrightarrow \neg b) \\ \varphi_2 &:= (((b \rightarrow \neg a) \oplus \neg b) \\ \varphi_3 &:= (\varphi_2 \land (a \lor \neg b)) \end{aligned}$$

be formulas over AP.

a) What are the truth tables for the above formulas?

b) What are $sat(\varphi_1)$, $sat(\varphi_2)$ and $sat(\varphi_3)$?

c) Which of the above formulas are satisfiable, which are unsatisfiable, and which are tautologies?

Exercise 2

Let $AP = \{a, b\}$ be a set of propositions and let $\alpha, \beta \in Assign$ with $\alpha(a) = 1$, $\alpha(b) = 1$ and $\beta(a) = 0$, $\beta(b) = 1$. Do the following hold?

1. $\alpha \models a \lor \neg b$
2. $\beta \not\models \neg a \land \neg b$
3. $\{\alpha, \beta\} \models a \land b$
4. $\{\alpha, \beta\} \models a \rightarrow b$
5. $a \lor b \models a \oplus b$
6. $sat(a \leftrightarrow b) \subseteq sat(a \rightarrow b)$

Exercise 3

Let $AP := \{a, b\}$ be a set of propositions and let $\varphi := (a \leftrightarrow b)$ be a formula over AP. Give a formula equivalent to φ that contains only propositions from AP and

1. the operators $\neg$ and $\land$,
2. the operators $\neg$ and $\lor$,
3. or the operator $\uparrow$ (called NAND).

(The binary operator $\uparrow$ has the following semantics: $\alpha \models (a \uparrow b) \leftrightarrow \alpha \models (\neg(a \land b))$ for all $a, b \in AP$ and $\alpha \in Assigns$.)