Satisfiability Checking - WS 2020/2021

Written Exam II

Tuesday, April 06, 2021

Sample solution

1.) SAT Checking **(6+4)+4+6 Points**

i) Consider the following propositional logic formula:

$$\begin{array}{lll} c_1 : (\neg a \vee e) & \wedge \;\; c_2 : (\neg a \vee d \vee \neg e) & \wedge \;\; c_3 : (\neg b \vee \neg c) \\ \wedge \;\; c_4 : (\neg a \vee c \vee f) & \wedge \;\; c_5 : (\neg a \vee \neg b \vee \neg f) & \end{array}$$

(a) **Apply the CDCL SAT-checking algorithm** to the given formula until the first conflict appears. When making a decision, take the lexicographically smallest unassigned variable first and assign **true** to it. Show the (partial) assignments after each decision level.

(b) Apply **conflict resolution** as presented in the lecture.

Solution:

(a) Updated assignment: ()
Decide $a = 1@1$
Propagate $e = 1@1$ using c_1
Propagate $d = 1@1$ using c_2
Updated assignment: $(a = 1@1, e = 1@1, d = 1@1)$
Decide $b = 1@2$
Propagate $c = 0@2$ using c_3
Propagate $f = 1@2$ using c_4 (or $f = 0@2$ using c_5)
Updated assignment: $(a = 1@1, e = 1@1, d = 1@1, b = 1@2, c = 0@2, f = 1@2)$ (respectively $f = 0@2$)
Clause $c_5 : (\neg a \vee \neg b \vee \neg f)$ is conflicting (or c_4)

(b) Resolve using variable f:

$$\frac{c_5 : (\neg a \vee \neg b \vee \neg f) \quad c_4 : (\neg a \vee c \vee f)}{c_6 : (\neg a \vee \neg b \vee c)}$$

Resolve using variable c:

$$\frac{c_6 : (\neg a \vee \neg b \vee c) \quad c_3 : (\neg b \vee \neg c)}{c_7 : (\neg a \vee \neg b)}$$

Resulting clause $c_7 : (\neg a \vee \neg b)$ is asserting, backtrack to decision level 1
Propagate $b = 0@1$ using c_7

ii) Apply Tseitin transformation to the following formula:

$$a \vee (b \wedge (c \vee d))$$

Solution:

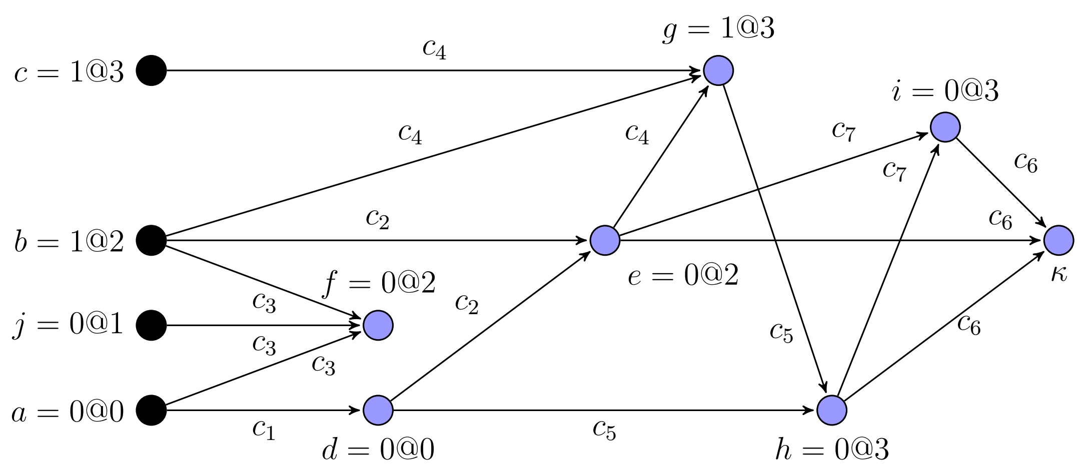
$(c \lor d) \leftrightarrow t_1$

$$\begin{aligned}&(a \lor (b \land t_1))\\ \land&(\neg t_1 \lor c \lor d) \land (t_1 \lor \neg c) \land (t_1 \lor \neg d)\end{aligned}$$

$(b \land t_1) \leftrightarrow t_2$

$$\begin{aligned}&(a \lor t_2)\\ \land&(\neg t_1 \lor c \lor d) \land (t_1 \lor \neg c) \land (t_1 \lor \neg d)\\ \land&(\neg t_2 \lor b) \land (\neg t_2 \lor t_1) \land (t_2 \lor \neg b \lor \neg t_1)\end{aligned}$$

iii) Consider the following implication graph:



a) What is the conflicting clause? Give its label **and** its literals.

b) Which clause is generated as result of conflict resoltion in CDCL? Which is the first UIP?

c) Give the node labels of all further unique implication points.

d) Which decisions from this implication graph were sufficient to cause this conflict? Give a **minimal** set of decisions.

Solution:

a) $c_6 : (i \lor e \lor h)$

b) Conflict clause: $e \lor h$, first UIP: $h = 0@3$

c) $g = 1@3$ and $c = 1@3$

d) $\{a = 0, b = 1, c = 1\}$

2.) Equality Logic and Uninterpreted Functions 5+10

Points

i) Assume the following formula of equality logic with uninterpreted functions:

$$\varphi^{\mathbf{UF}} \quad := \quad x_1 = x_2 \wedge (F(x_1, x_3) \neq F(x_1, x_2) \vee F(x_1, x_3) = F(x_1, x_1))$$

Use Ackermann's reduction to construct an equality logic formula φ^E **without** uninterpreted functions that is satisfiability-equivalent to $\varphi^{\mathbf{UF}}$.

Solution:

We first assign indices to the F-instances:

$$\varphi^{\mathbf{UF}} \quad := \quad x_1 = x_2 \wedge (\underbrace{F(x_1, x_3)}_{f_1} \neq \underbrace{F(x_1, x_2)}_{f_2} \vee \underbrace{F(x_1, x_3)}_{f_1} = \underbrace{F(x_1, x_1)}_{f_3})$$

$$\begin{aligned}
\varphi_{\mathbf{flat}} &:= x_1 = x_2 \wedge (f_1 \neq f_2 \vee f_1 = f_3) \\
\varphi_{\mathbf{cong}} &:= \big((x_1 = x_1 \wedge x_3 = x_2) \rightarrow f_1 = f_2\big) \wedge \\
&= \big((x_1 = x_1 \wedge x_3 = x_1) \rightarrow f_1 = f_3\big) \wedge \\
&= \big((x_1 = x_1 \wedge x_2 = x_1) \rightarrow f_2 = f_3\big) \\
\varphi^E &:= \varphi_{\mathbf{flat}} \wedge \varphi_{\mathbf{cong}}
\end{aligned}$$

ii) Consider the following formula in equality logic:

$$\varphi^E \quad := \quad (\neg(\underbrace{x_1 = x_2}_{e_1}) \vee \underbrace{x_1 = x_3}_{e_2}) \wedge (\underbrace{x_3 = x_4}_{e_3} \vee \underbrace{x_4 = x_2}_{e_4}) \wedge (\neg(\underbrace{x_4 = x_5}_{e_5}) \vee \underbrace{x_1 = x_4}_{e_6})$$

a) Construct the equality graph **with** polarity for φ^E.

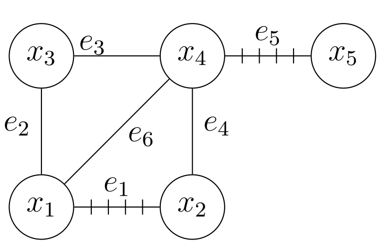
b) Simplify the constructed equality graph and the formula using the method presented in the lecture.

c) Make the simplified equality graph without polarity chordal.

d) Construct the satisfiability-equivalent propositional logic formula for φ using the previous results from c) (without considering polarities).

Solution:

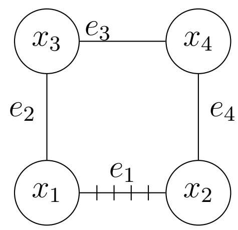
a) We construct the following E-graph with polarity:



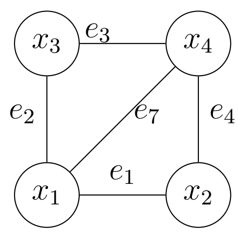
b) The inequality edge e_5 is not part of any contradictory cycle, thus we can assign false to e_5. The formula simplifies to

$$\varphi^E \quad = \quad \big(\neg(\underbrace{x_1 = x_2}_{e_1}) \vee \underbrace{x_1 = x_3}_{e_2}\big) \wedge \big(\underbrace{x_3 = x_4}_{e_3} \vee \underbrace{x_4 = x_2}_{e_4}\big)$$

with E-graph



c) We make the above E-graph non-polar and chordal:



d) We construct the following satisfiability-equivalent propositional logic formula φ for φ^E:

$$\begin{aligned}
\varphi_{\mathbf{sk}} &:= (\neg e_1 \vee e_2) \wedge (e_3 \vee e_4) \\
\varphi_{\mathbf{trans}} &:= ((e_1 \wedge e_4) \rightarrow e_7) \wedge ((e_4 \wedge e_7) \rightarrow e_1) \wedge ((e_7 \wedge e_1) \rightarrow e_4) \wedge \\
& ((e_2 \wedge e_3) \rightarrow e_7) \wedge ((e_3 \wedge e_7) \rightarrow e_2) \wedge ((e_7 \wedge e_2) \rightarrow e_3) \\
\varphi &:= \varphi_{\mathbf{sk}} \wedge \varphi_{\mathbf{trans}}
\end{aligned}$$

3.) Fourier-Motzkin Variable Elimination **5+9 Points**

i) Assume that we apply Fourier-Motzkin variable elimination to eliminate **all** variables $x_1, ..., x_n$ from a set of non-strict inequalities from linear real arithmetic. Assume that the variables are eliminated in the order $x_n, ..., x_1$.

How can we generate a **satisfying assignment** for the variables if the constraint set turned out to be satisfiable? Start with the base case and continue by extending a partial assignment for a single variable at a time.

Solution: Fourier-Motzkin constructs a sequence of constraint sets $C_n, ..., C_0$ such that C_k contains only variables $x_1, ...x_k$. The last set C_0 finally contains no variables and thus all constraints are trivially true (if the set is satisfiable).

We start with the empty assignment α_0, which trivially satisfies C_0.

Given a partial assignment α_k that satisfies C_k, we now construct α_{k+1} which shall satisfy C_{k+1}. We substitute α_k into all constraints from C_{k+1} and obtain a list of **constant** lower and upper bounds for x_{k+1}. We select the **largest lower** and **smallest upper** bound to obtain a solution interval $[l, u]$ (where $l = -\infty$ if no lower bound exists and $u = \infty$ if no upper bound exists). We know that $l \leq u$ because the constraint set was found to be satisfiable. Finally, we extend α_k by $x_{k+1} = \beta$ for some arbitrary $\beta \in [l, u]$ and obtain α_{k+1}.

We iteratively apply this procedure until we obtain α_n.

ii) Consider the following set of linear real-arithmetic constraints:

$$\begin{array}{lrcrcr} c_1: & x & - & 2y & \leq & -5 \\ c_2: & x & + & y & \leq & 2 \\ c_3: & -x & + & y & \leq & 2 \\ c_4: & -2x & - & 3y & \leq & 3 \end{array}$$

a) **Eliminate** y by applying the Fourier-Motzkin variable elimination.

b) Is the constraint set satisfiable? Either give a **satisfying assignment** or determine a **minimal infeasible subset**.

Solution:

a) We first identify the lower and the upper bounds on y:

$$\begin{array}{lrclcl} c_1: & {}^1\!/_2 x + {}^5\!/_2 & \leq & y & & \\ c_2: & & & y & \leq & -x + 2 \\ c_3: & & & y & \leq & x + 2 \\ c_4: & -{}^2\!/_3 x - 1 & \leq & y & & \end{array}$$

We now state that each lower bound is less or equal each upper bound:

$$\begin{array}{lrcl}
(c_1, c_2): & {}^1\!/_2 x + {}^5\!/_2 & \leq & -x + 2 \\
\Rightarrow c_5: & x & \leq & -{}^1\!/_3 \\
\hline
(c_1, c_3): & {}^1\!/_2 x + {}^5\!/_2 & \leq & x + 2 \\
\Rightarrow c_6: & 1 & \leq & x \\
\hline
(c_4, c_2): & -{}^2\!/_3 x - 1 & \leq & -x + 2 \\
\Rightarrow c_7: & x & \leq & 9 \\
\hline
(c_4, c_3): & -{}^2\!/_3 x - 1 & \leq & x + 2 \\
\Rightarrow c_8: & -{}^9\!/_5 & \leq & x
\end{array}$$

b) When eliminating x, from the lower bound by c_6 and the upper bound by c_5 we get $1 \leq -{}^1\!/_3$, which is a contradiction. The constraint set is therefore not satisfiable. We get the minimal infeasible subset $\{c_5, c_6\}$. Considering the origin of c_5 and c_6, we get $\{c_1, c_2, c_3\}$.

4.) Simplex **9+3 Points**

i) Apply one **pivoting step** and update the current assignment, where the variable order is $x_1 < x_2 < s_1 < s_2 < s_3 < s_4$ and where the current tableau, the bounds and the current assignment are given by:

	s_1	x_2
x_1	1	-1
s_2	1	-2
s_3	1	1
s_4	0	1

$$\begin{array}{rcl} s_1 & \geq & 3 \\ s_2 & \leq & -1 \\ s_3 & \geq & 2 \\ s_4 & \leq & 3/2 \end{array}$$

$$\begin{array}{ll} \alpha(x_1) & = 3 \\ \alpha(x_2) & = 0 \\ \alpha(s_1) & = 3 \\ \alpha(s_2) & = 3 \\ \alpha(s_3) & = 3 \\ \alpha(s_4) & = 0 \end{array}$$

Solution: We pivot the second row and second column with

$$s_2 = 1 \cdot s_1 + -2 \cdot x_2 \Leftrightarrow x_2 = 1/2 \cdot s_1 + -1/2 \cdot s_2$$

and obtain

	s_1	s_2
x_1	$1/2$	$1/2$
x_2	$1/2$	$-1/2$
s_3	$3/2$	$-1/2$
s_4	$1/2$	$-1/2$

$$\begin{array}{rcl} s_1 & \geq & 3 \\ s_2 & \leq & -1 \\ s_3 & \geq & 2 \\ s_4 & \leq & 3/2 \end{array}$$

$$\begin{array}{ll} \alpha(x_1) & = 1 \\ \alpha(x_2) & = 2 \\ \alpha(s_1) & = 3 \\ \alpha(s_2) & = -1 \\ \alpha(s_3) & = 5 \\ \alpha(s_4) & = 2 \end{array}$$

ii) The simplex method terminates on the resulting tableau. Is the tableau satisfied or conflicting? Why? Give the satisfying assignment for the original variables or the slack variables corresponding to the infeasible subset.

Solution: The tableau is conflicting as there is not suitable variable for s_4. The slack variables corresponding to the infeasible subset are s_1, s_2, s_4.

5.) Integer Arithmetic **2+6 Points**

i) Is the Branch and Bound method complete on input problems, whose relaxation has a bounded solution set? Please argue why.

Solution:

Yes, because finite sets can be splitted in each dimension only finitely many times at integer values.

ii) We want to check the satisfiability of the linear integer arithmetic constraint set

$$P = \{y \leq x + 0.5, \quad y \leq -x + 1.5, \quad y \geq x - 0.5, \quad y \geq -x + 0.5\}$$

using the Branch and Bound method as presented in the lecture, with the heuristics to prefer x for splitting (if its value is non-integer) and handle lower branches first. Starting with the initial call with parameter P, please define the parameter sequence of all recursive calls until termination.

Solution:

- $P \wedge x \leq 0$
- $P \wedge x \leq 0 \wedge y \leq 0$
- $P \wedge x \leq 0 \wedge y \geq 1$
- $P \wedge x \geq 1$
- $P \wedge x \geq 1 \wedge y \leq 0$
- $P \wedge x \geq 1 \wedge y \geq 1$

6.) Interval Constraint Propagation **4+10 Points**

i) Specify the formal rule for **dividing** an interval $A = [\underline{A}, \overline{A}] \subseteq \mathbb{R}^{>0}$, i.e. with $0 < \underline{A}$, by an arbitrary interval B with $0 \in B$ as presented in the lecture.

Solution:

If $A = [1; 0]$ the result is $[1; 0]$, otherwise

for	$B = [0; 0]$	$\underline{B} < \overline{B} = 0$	$\underline{B} < 0 < \overline{B}$	$0 = \underline{B} < \overline{B}$
$A \div B$	$[1; 0]$	$[-\infty; \underline{A}/\underline{B}]$	$[-\infty; \underline{A}/\underline{B}] \cup [\underline{A}/\overline{B}; +\infty]$	$[\underline{A}/\overline{B}; +\infty]$

ii) Consider the constraints $c_1 : x^2 - 2 = y$ and $c_2 : 2 \cdot x \geq y + 2$ and initial intervals $x \in [-1, 1], y \in [-2, 2]$. Give **all contraction candidates**, **perform** contractions with all of them and argue which caused the largest relative contraction.

Solution:

$$
\begin{array}{rlllll}
(c_1, x): & x & = \pm\sqrt{y+2} & = \pm\sqrt{[-2;2]+2} & = [-2;2] & \\
\Rightarrow & x & \in [-1;1] \cap [-2;2] & = [-1;1] & & \text{rel. contraction: } 0 \\
(c_1, y): & y & = x^2 - 2 & = [-1;1]^2 - 2 & = [-2;-1] & \\
\Rightarrow & y & \in [-2;2] \cap [-2;-1] & = [-2;-1] & & \text{rel. contraction: } 0.75 \\
(c_2, x): & x & \geq \frac{1}{2}y + 1 & = \frac{[-2;2]}{2} + 1 & = [0;2] & \\
\Rightarrow & x & \in [-1;1] \cap [0;\infty] & = [0;1] & & \text{rel. contraction: } 0.5 \\
(c_2, y): & y & \leq 2 \cdot x - 2 & = 2 \cdot [-1;1] - 2 & = [-4;0] & \\
\Rightarrow & y & \in [-2;2] \cap [-\infty;0] & = [-2;0] & & \text{rel. contraction: } 0.5
\end{array}
$$

The contraction candidate (c_1, y) achieved the highest relative contraction and is therefore the most valuable contraction candidate.

7.) Subtropical Satisfiability **6+6 Points**

i) Let $p(x, y)$ be the polynomial $-x^2 - xy + 1$. Which solutions can be found for the equation $p(x, y) = 0$ by the subtropical satisfiability method using the sample points $p(1, 1) < 0$ and $p(0, 0) > 0$?

Solution:

- We reduce the two-dimensional problem to a one-dimensional problem by searching for solutions on the line segment between $(0, 0)$ and $(1, 1)$. Such solutions have the form $(x_0, y_0) = (0, 0) + t\big((1, 1) - (0, 0)\big) = t \cdot (1, 1) = (t, t)$ for some $t \in \mathbb{R}$, $0 < t < 1$.
- From $x_0 = t$ and $y_0 = t$ we get the univariate form $-t^2 - t \cdot t + 1 = -2t^2 + 1 = 0$.
- We have $-2t^2 + 1 = 0$ iff $2t^2 = 1$ iff $t^2 = \frac{1}{2}$ iff $t \in \{+\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\}$. However, the requirement $0 < t < 1$ excludes the negative solution. Thus the only valid solution is $t = \frac{1}{\sqrt{2}}$.
- Transforming back to the two-dimensional problem, we get as the only possible solution, $x_0 = y_0 = \frac{1}{\sqrt{2}}$.

ii) For the polynomial $p(x, y) = -x^2y - 3x + y$ and the separating hyperplane specified by $(n_x, n_y) = (-1, 1)$ and $b = 0$, use the subtropical satisfiability method to compute a solution for $p(x, y) > 0$. As in the lecture, please start with $a = 2$.

Solution: We start with $a = 2$ and check whether $p(a^{-1}, a^1) > 0$. If yes, we have found a solution, otherwise we repeat the test iteratively after doubling a.

- $a = 2$: $p(a^{-1}, a^1) = p(\frac{1}{2}, 2) = -\left(\frac{1}{2}\right)^2 \cdot 2 - 3 \cdot \frac{1}{2} + 2 = 0 \rightsquigarrow$ double a
- $a = 4$: $p(a^{-1}, a^1) = p(\frac{1}{4}, 4) = -\left(\frac{1}{4}\right)^2 \cdot 4 - 3 \cdot \frac{1}{4} + 4 = 3$

Thus we get $p(x, y) > 0$ for $x = \frac{1}{4}$ and $y = 4$.

8.) Virtual Substitution **(5+3)+5 Points**

i) Assume we want to virtually substitute the test candidate $t = -\infty$ **for the variable** y in the constraint $x \cdot y^2 + 2 \cdot y^2 + x^2 + y + z^2 \cdot y < 0$ using one of the following rules:

- Substitute $-\infty$ for x in $b \cdot x + c < 0$:

$$\begin{array}{lllll} & (& b > 0 & &) \\ \vee & (& b = 0 & \wedge \quad (c < 0 &) \end{array}$$

- Substitute $-\infty$ for x in $a \cdot x^2 + b \cdot x + c < 0$:

$$\begin{array}{lllllll} & (& a < 0 & & & &) \\ \vee & (& a = 0 & \wedge & b > 0 & &) \\ \vee & (& a = 0 & \wedge & b = 0 & \wedge \quad c < 0 &) \end{array}$$

a) Identify the appropriate substitution rule from above and apply virtual substitution. Normalize all polynomials by transforming them to sums of products.

b) Further simplify the result as far as possible. Use the result to state whether the formula is satisfiable.

Solution:

a) Note that we substitute for y and not for x. We need to compute $(a \cdot y^2 + b \cdot y + c < 0)[-\infty//y]$, $a = x + 2$, $b = z^2 + 1$ and $c = x^2$. I.e., we need to instantiate the second rule.

Substitute $-\infty$ into $a \cdot y^2 + b \cdot y + c < 0$:

$$\begin{array}{lllllll} & (& a < 0 & & & &) \\ \vee & (& a = 0 & \wedge & b > 0 & &) \\ \vee & (& a = 0 & \wedge & b = 0 & \wedge \quad c < 0 &) \end{array}$$

$$\begin{array}{lllllll} & (& x + 2 < 0 & & & &) \\ \vee & (& x + 2 = 0 & \wedge & z^2 + 1 > 0 & &) \\ \vee & (& x + 2 = 0 & \wedge & z^2 + 1 = 0 & \wedge \quad x^2 < 0 &) \end{array}$$

b) Simplification:

$$\begin{array}{lllllll} & (& x + 2 < 0 & & & &) \\ \vee & (& x + 2 = 0 & \wedge & true & &) \\ \vee & (& x + 2 = 0 & \wedge & false & \wedge \quad false &) \end{array}$$

$$x + 2 \leq 0$$

Yes, the formula is satisfiable.

ii) Give all test candidates of the following polynomial in variable x and their side conditions:

$$zx^2 + x^2y^2 + 2xy + z < 0$$

Simplify all expressions as far as possible by multiplying out all brackets.

Solution: $(z + y^2)x^2 + (2y)x + z < 0$

Zeros of $ax^2 + bx + c$:

Real root		Side condition
$\xi_0 = -\frac{c}{b}$	, if	$a = 0 \wedge b \neq 0$
$\xi_1 = \frac{-b+\sqrt{b^2-4ac}}{2a}$	, if	$a \neq 0 \wedge b^2 - 4ac \geq 0$
$\xi_2 = \frac{-b-\sqrt{b^2-4ac}}{2a}$	, if	$a \neq 0 \wedge b^2 - 4ac > 0$

... with $a = z + y^2, b = 2y, c = z$:

Real root		Side condition
$\xi_0 = -\frac{z}{2y}$	, if	$z + y^2 = 0 \wedge 2y \neq 0$
$\xi_1 = \frac{-2y+\sqrt{4y^2-4z^2-4zy^2}}{2z+2y^2}$	, if	$z + y^2 \neq 0 \wedge 4y^2 - 4z^2 - 4zy^2 \geq 0$
$\xi_2 = \frac{-2y-\sqrt{4y^2-4z^2-4zy^2}}{2z+2y^2}$	, if	$z + y^2 \neq 0 \wedge 4y^2 - 4z^2 - 4zy^2 > 0$

Resulting test candidates:

Real root		Side condition
$-\infty$	, if	true
$-\frac{z}{2y} + \varepsilon$	, if	$z + y^2 = 0 \wedge 2y \neq 0$
$\frac{-2y+\sqrt{4y^2-4z^2-4zy^2}}{2z+2y^2} + \varepsilon$	, if	$z + y^2 \neq 0 \wedge 4y^2 - 4z^2 - 4zy^2 \geq 0$
$\frac{-2y-\sqrt{4y^2-4z^2-4zy^2}}{2z+2y^2} + \varepsilon$	, if	$z + y^2 \neq 0 \wedge 4y^2 - 4z^2 - 4zy^2 > 0$

9.) Cylindrical Algebraic Decomposition **12 Points**

Consider the polynomial $p = x^3 - 4x^2 + 3 \cdot x$.

Use the Cauchy bound $C = 5$ and the Sturm sequence of p:

$$\begin{aligned} p_0 &= x^3 - 4x^2 + 3 \cdot x \\ p_1 &= 3 \cdot x^2 - 8 \cdot x + 3 \\ p_2 &= {}^{14}\!/_{9} \cdot x - {}^{4}\!/_{3} \\ p_3 &= {}^{81}\!/_{49} \end{aligned}$$

to isolate all real roots of p with the method presented in the lecture, using interval midpoints for splitting. Give the table of sign changes for all relevant interval bounds. Describe and explain every split and give the resulting interval representations of the real roots.

Solution: All real roots of p are in the interval $[-5; 5]$ according to the lecture.
Since $p(-5) \neq 0$ and $p(5) \neq 0$ all real roots are in the interval $(-5; 5)$.
We use the Sturm sequence to compute the number of real roots in $(-5; 5)$:

Sturm sequence	values at			
	-5	0	2.5	5
$p_0 = x^3 - 4x^2 + 3 \cdot x$	-	0	-	+
$p_1 = 3 \cdot x^2 - 8 \cdot x + 3$	+	+	+	+
$p_2 = {}^{14}\!/_{9} \cdot x - {}^{4}\!/_{3}$	-	-	+	+
$p_3 = {}^{81}\!/_{49}$	+	+	+	+
# sign changes $\sigma(\cdot)$	3	2	1	0

There are $\sigma(-5) - \sigma(5) = 3 - 0 = 3$ real roots of p in $(-5; 5)$. To isolate the real roots, we split the interval at its midpoint 0 into $(-5; 0)$, $[0; 0]$ and $(0; 5)$.
Number of real roots of p in $(-5; 0)$: 0; in $[0; 0]$: 1; in $(0; 5)$: 2

In the interval $(0; 5)$ are still two real roots, therefore, we split the interval at its midpoint 2.5 into $(0; 2.5)$, $[2.5; 2.5]$ and $(2.5; 5)$.
Number of real roots of p in $(0; 2.5)$: 1; in $[2.5; 2.5]$: 0 ; in $(2.5; 5)$: 1

In every interval is at most one root, therefore, all real roots are isolated. The interval representations of the real roots are $(p, [0; 0])$, $(p, (0; 2.5))$ and $(p, (2.5; 5))$.