## Satisfiability Checking 06 SAT solving examples

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WS 22/23

## 06 SAT solving examples

1 Example 1

## SAT solving: Example1

- We use DPLL+CDCL SAT solving combining enumeration, propagation and resolution.
- We use watched literals (underlined in formulas) to speed up propagation.
- We use VSIDS as variable ordering heuristics and assign the value false to decision variables.
- In VSIDS, to order variables with the same activity value, we use the lexicographic order; in our example this will be  $x_1 < x_2 < x_3 < x_4$ .

$$c_{1}: (\underbrace{x_{1}} \lor \underbrace{x_{2}} \lor x_{4}) \land c_{2}: (\underbrace{x_{2}} \lor \underbrace{\neg x_{4}}) \land c_{3}: (\underbrace{x_{1}} \lor \underbrace{\neg x_{2}} \lor x_{4}) \land c_{4}: (\underbrace{x_{3}} \lor \underbrace{\neg x_{4}})$$
Watch lists:
$$c_{1}: c_{3}$$

$$c_{3}: c_{4}: c_{2}: c_{3}: c_{4}: c$$

$$c_1: (\underline{x_1} \vee \underline{x_2} \vee x_4) \wedge c_2: (\underline{x_2} \vee \underline{\neg x_4}) \wedge c_3: (\underline{x_1} \vee \underline{\neg x_2} \vee x_4) \wedge c_4: (\underline{x_3} \vee \underline{\neg x_4})$$

```
Watch lists: x_1 : c_1, c_3
```

 $\neg x_1$ :

 $x_2: c_1, c_2$ 

 $\neg x_2$ :  $c_3$ 

X3: C4

 $\neg x_3$ :

*X*<sub>4</sub> :

 $\neg x_4 : c_2, c_4$ 

Decide  $\neg x_1$ 

Activities Trail: (increment=1):

*x*<sub>1</sub> 0

 $x_2 = 0$ 

*x*<sub>3</sub> 0

~3 U

*x*<sub>4</sub> 0

DL1:  $\neg x_1$ : NULL

$$c_1: \left(\underline{x_1} \vee \underline{x_2} \vee x_4\right) \wedge c_2: \left(\underline{x_2} \vee \underline{\neg x_4}\right) \wedge c_3: \left(\underline{x_1} \vee \underline{\neg x_2} \vee x_4\right) \wedge c_4: \left(\underline{x_3} \vee \underline{\neg x_4}\right)$$

```
Watch lists: x_1 : c_1, c_3 \neg x_1 : x_2 : c_1, c_2 \neg x_2 : c_3 x_3 : c_4 \neg x_3 : x_4 : \neg x_4 : c_2, c_4
```

Propagate  $\neg x_1$  in

```
Activities Trail: (increment=1): DL1: \neg x_1 : \text{NULL}
\begin{array}{ccc} x_1 & 0 \\ x_2 & 0 \\ x_3 & 0 \\ x_4 & 0 \end{array}
```

$$c_1: \left(\underline{x_1} \vee \underline{x_2} \vee x_4\right) \wedge c_2: \left(\underline{x_2} \vee \underline{\neg x_4}\right) \wedge c_3: \left(\underline{x_1} \vee \underline{\neg x_2} \vee x_4\right) \wedge c_4: \left(\underline{x_3} \vee \underline{\neg x_4}\right)$$

```
Watch lists:
                                                            Trail:
                                      Activities
                                      (increment=1):
                                                                 DL1: \neg x_1: NULL
        x_1: c_1, c_3
       \neg x_1:
                                             x_1 0
        x_2: c_1, c_2
                                             x_2 0
       \neg x_2: c_3
                                             x_3 0
       X_3: C_4
                                                  0
                                             X_4
       \neg x_3:
        X<sub>4</sub> :
       \neg x_4: c_2, c_4
 Propagate \neg x_1 in c_1: (x_1 \lor x_2 \lor x_4)
```

```
c_1: (x_1 \lor x_2 \lor x_4) \land c_2: (x_2 \lor \neg x_4) \land c_3: (x_1 \lor \neg x_2 \lor x_4) \land c_4: (x_3 \lor \neg x_4)
Watch lists:
                                                                        Trail:
                                              Activities
                                              (increment=1):
                                                                              DL1: \neg x_1: NULL
          x_1: \mathcal{C}_1, \mathcal{C}_3
        \neg x_1: the watched literal x_1 = 0
          x_2: c_1, c_2
                                                      x_2 0
        \neg x_2: c_3
                                                      x_3 0
         X_3: C_4
                                                            0
                                                      X_{\Delta}
        \neg x_3:
         X<sub>4</sub> : C<sub>1</sub>
        \neg x_4: c_2, c_4
 Propagate \neg x_1 in c_1: (x_1 \lor x_2 \lor x_4) \rightarrow (x_1 \lor x_2 \lor x_4)
```

$$c_1: \left(x_1 \vee \underline{x_2} \vee \underline{x_4}\right) \wedge c_2: \left(\underline{x_2} \vee \underline{\neg x_4}\right) \wedge c_3: \left(\underline{x_1} \vee \underline{\neg x_2} \vee x_4\right) \wedge c_4: \left(\underline{x_3} \vee \underline{\neg x_4}\right)$$

```
Watch lists:
                                       Activities
                                                              Trail:
                                        (increment=1):
                                                                    DL1: \neg x_1: NULL
         x_1: \mathcal{C}_1, \mathcal{C}_3
       \neg x_1:
                                               x_1 0
         x_2: c_1, c_2
                                               x_2 0
       \neg x_2: c_3
                                               x_3 0
       X_3: C_4
                                                    0
                                               X_4
       \neg x_3:
        X_4: C_1
       \neg x_4: c_2, c_4
 Propagate \neg x_1 in c_3: (x_1 \lor \neg x_2 \lor x_4)
```

$$c_1: \left(x_1 \vee \underline{x_2} \vee \underline{x_4}\right) \wedge c_2: \left(\underline{x_2} \vee \underline{\neg x_4}\right) \wedge c_3: \left(x_1 \vee \underline{\neg x_2} \vee \underline{x_4}\right) \wedge c_4: \left(\underline{x_3} \vee \underline{\neg x_4}\right)$$

```
Watch lists:
                                                                   Trail:
                                           Activities
                                           (increment=1):
                                                                         DL1: \neg x_1: NULL
          x_1: \mathcal{G}_1, \mathcal{G}_3
        \neg x_1:
                                                   x_1 0
         x_2: c_1, c_2
                                                   x_2 0
        \neg x_2: c_3
                                                   x_3 0
        X_3: C_4
                                                        0
                                                   X_{\Delta}
        \neg x_3:
         X_4: C_1, C_3
        \neg x_4: c_2, c_4
  Propagate \neg x_1 in c_3: (x_1 \lor \neg x_2 \lor x_4) \rightarrow (x_1 \lor \neg x_2 \lor x_4)
```

$$c_1: (x_1 \vee \underline{x_2} \vee \underline{x_4}) \wedge c_2: (\underline{x_2} \vee \underline{\neg x_4}) \wedge c_3: (x_1 \vee \underline{\neg x_2} \vee \underline{x_4}) \wedge c_4: (\underline{x_3} \vee \underline{\neg x_4})$$

Watch lists:

 $X_1$ :  $\neg x_1$ :

 $x_2: c_1, c_2$ 

 $\neg x_2$ :  $c_3$ 

X3: C4  $\neg x_3$ :

 $x_4$ :  $c_1, c_3$ 

 $\neg x_4$ :  $c_2, c_4$ 

Decide  $\neg x_2$ 

Activities (increment=1):

 $x_1 = 0$ 

 $x_2 = 0$  $x_3$  0

0

*X*4

Trail:

DL1:  $\neg x_1$ : NULL

DL2:  $\neg x_2$ : NULL

$$c_1: (x_1 \vee \underline{x_2} \vee \underline{x_4}) \wedge c_2: (\underline{x_2} \vee \underline{\neg x_4}) \wedge c_3: (x_1 \vee \underline{\neg x_2} \vee \underline{x_4}) \wedge c_4: (\underline{x_3} \vee \underline{\neg x_4})$$

```
Watch lists:

x_1:

\neg x_1:

x_2: c_1, c_2

\neg x_2: c_3

x_3: c_4

\neg x_3:

x_4: x_4: x_5: x_6:
```

 $\neg x_4$ :  $c_2, c_4$ Propagate  $\neg x_2$  in

```
Activities (increment=1):  x_1 \quad 0 
 x_2 \quad 0 
 x_3 \quad 0 
 x_4 \quad 0
```

```
Trail:

DL1: \neg x_1: NULL

DL2: \neg x_2: NULL
```

$$c_1: (x_1 \vee \underline{x_2} \vee \underline{x_4}) \wedge c_2: (\underline{x_2} \vee \underline{\neg x_4}) \wedge c_3: (x_1 \vee \underline{\neg x_2} \vee \underline{x_4}) \wedge c_4: (\underline{x_3} \vee \underline{\neg x_4})$$

```
Watch lists:
                                                         Trail:
                                    Activities
                                    (increment=1):
                                                              DL1: \neg x_1: NULL
        X_1:
                                                              DL2: \neg x_2: NULL
       \neg x_1:
                                           x_1 0
        x_2: c_1, c_2
                                                0
                                           X2
      \neg x_2: c_3
                                           x_3 0
       X_3: C_4
                                                0
                                           X_4
       \neg x_3:
       x_4: c_1, c_3
      \neg x_4: c_2, c_4
 Propagate \neg x_2 in c_1: (x_1 \lor x_2 \lor x_4)
```

$$c_1: (x_1 \vee \underline{x_2} \vee \underline{x_4}) \wedge c_2: (\underline{x_2} \vee \underline{\neg x_4}) \wedge c_3: (x_1 \vee \underline{\neg x_2} \vee \underline{x_4}) \wedge c_4: (\underline{x_3} \vee \underline{\neg x_4})$$

```
Watch lists:
                                                                    Trail:
                                           Activities
                                           (increment=1):
                                                                          DL1: \neg x_1: NULL
          X_1:
                                                                          DL2: \neg x_2: NULL
        \neg x_1:
                                                   x_1 0
         x_2: c_1, c_2
                                                                                        X4 : C1
                                                   x_2 0
        \neg x_2: c_3
                                                   x_3 0
        X_3: C_4
                                                         0
                                                   XΔ
        \neg x_3:
         x_4: c_1, c_3
        \neg x_4 : c_2, c_4
 \neg x_4: c_2, c_4
Propagate \neg x_2 in c_1: (x_1 \lor \underline{x_2} \lor \underline{x_4}) \to \text{Assign } x_4 \times_{4} = \text{True}
```

$$c_1: \left(x_1 \vee \underline{x_2} \vee \underline{x_4}\right) \wedge c_2: \left(\underline{x_2} \vee \underline{\neg x_4}\right) \wedge c_3: \left(x_1 \vee \underline{\neg x_2} \vee \underline{x_4}\right) \wedge c_4: \left(\underline{x_3} \vee \underline{\neg x_4}\right)$$

```
Watch lists:
                                                              Trail:
                                                                                   化表不定约束
                                        Activities
                                        (increment=1):
                                                                    DL1: \neg x_1 : \text{NULL} \neq nil
         X_1:
                                                                    DL2: \neg x_2: NULL \overrightarrow{a} n \cdot \lambda
       \neg x_1:
                                               x_1 0
        x_2: c_1, c_2
                                                                               X4 : C1
                                                    0
                                               X2
       \neg x_2: c_3
                                               x_3 0
        X_3: C_4
                                                    0
                                               X_4
       \neg x_3:
        x_4: c_1, c_3
       \neg x_4: c_2, c_4
```

Propagate  $\neg x_2$  in  $c_2:(x_2 \vee \neg x_4)$ 

$$c_1: (x_1 \vee \underline{x_2} \vee \underline{x_4}) \wedge c_2: (\underline{x_2} \vee \underline{\neg x_4}) \wedge c_3: (x_1 \vee \underline{\neg x_2} \vee \underline{x_4}) \wedge c_4: (\underline{x_3} \vee \underline{\neg x_4})$$

```
Watch lists:
                                                            Trail:
                                      Activities
                                      (increment=1):
                                                                  DL1: \neg x_1: NULL
         X_1:
                                                                  DL2: \neg x_2: NULL
       \neg x_1:
                                             x_1 0
        x_2: c_1, c_2
                                                                            X4 : C1
                                                  0
                                             X2
       \neg x_2: c_3
                                             x_3 0
       X_3: C_4
                                                   0
                                             XΔ
       \neg x_3:
        x_4: c_1, c_3
       \neg x_4: c_2, c_4
 Propagate \neg x_2 in c_2: (x_2 \vee \neg x_4) \rightarrow \text{$\not$$conflict!}
```

$$c_1: (x_1 \vee \underline{x_2} \vee \underline{x_4}) \wedge c_2: (\underline{x_2} \vee \underline{\neg x_4}) \wedge c_3: (x_1 \vee \underline{\neg x_2} \vee \underline{x_4}) \wedge c_4: (\underline{x_3} \vee \underline{\neg x_4})$$

Watch lists:

 $X_1$ :  $\neg x_1$ :

 $x_2$ :  $c_1, c_2$ 

 $\neg x_2$ :  $c_3$ 

x<sub>3</sub>: c<sub>4</sub>

 $\neg x_3$ :

 $x_4: c_1, c_3$ 

 $\neg x_4$ :  $c_2, c_4$ 

Conflict resolution:

Activities (increment=1):

> $x_1$  0 0 X2

 $x_3$  0

0 *X*4

Trail:

DL1:  $\neg x_1$ : NULL

DL2:  $\neg x_2$ : NULL

 $X_4 : C_1$ 

$$c_1: (x_1 \vee \underline{x_2} \vee \underline{x_4}) \wedge c_2: (\underline{x_2} \vee \underline{\neg x_4}) \wedge c_3: (x_1 \vee \underline{\neg x_2} \vee \underline{x_4}) \wedge c_4: (\underline{x_3} \vee \underline{\neg x_4})$$

Watch lists:			Activities		Trail:	
$x_1$ :			(increme	nt=1):	DL1:	$\neg x_1 : NULL$
$\neg x_1$ :			$x_1$	0	DL2:	$\neg x_2 : NULL$
$x_2$ :	$c_1, c_2$		<i>x</i> <sub>2</sub>	0		$x_4 : c_1$
$\neg x_2$ :	<i>c</i> <sub>3</sub>		<i>X</i> 3	0		
<i>x</i> <sub>3</sub> :	C4		<i>X</i> <sub>4</sub>	0	M C	<u> </u>
$\neg x_3$ :					( v)	no Ci
<i>X</i> <sub>4</sub> :	$c_1, c_3$				\ <u>@</u> _	>B
¬ <i>x</i> <sub>4</sub> :	$c_2, c_4$	G	<b>C</b> 1			CI
Conflict res	solution:	$\frac{(x_2 \vee \neg x_4)}{(x_1)}$	$\underbrace{(x_1 \lor x_2 \lor x_4)}_{\lor x_2)}$ Cs			

Add conflict clause

```
c_1: (x_1 \lor x_2 \lor x_4) \land c_2: (x_2 \lor \neg x_4) \land c_3: (x_1 \lor \neg x_2 \lor x_4) \land c_4: (x_3 \lor \neg x_4)
c_5:(x_1\vee x_2)
                                                                                                                                                                                                                                                                                                                                                                                                            X1 : C5
                                                                    \neg x_1:
                                                                                  x_2: c_1, c_2, c_5
                                                                    \neg X_2: C_3
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              x_3 0
                                                                           X3 : C4
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              x_4 1
                                                                    \neg x_3:
                                                                             x_4: c_1, c_3
                                                                  \neg x_4: c_2, c_4
```

## 図到 Xi th assignment 所能

$$c_1: (x_1 \vee \underline{x_2} \vee \underline{x_4}) \wedge c_2: (\underline{x_2} \vee \underline{-x_4}) \wedge c_3: (x_1 \vee \underline{-x_2} \vee \underline{x_4}) \wedge c_4: (\underline{x_3} \vee \underline{-x_4}) \wedge c_5: (x_1 \vee x_2)$$

Watch lists:

$$x_1 : c_5$$

$$\neg x_1$$
:

$$x_2: c_1, c_2, c_5$$

$$\neg x_2$$
:  $c_3$ 

 $\neg x_3$ :

$$x_4: c_1, c_3$$

$$\neg x_4$$
:  $c_2, c_4$ 

Backtrack to DI 1

Activities Trail: (increment=2):

 $x_1$  1

$$x_2$$
 1

$$x_4$$
 1

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DL1:  $\neg x_1 : \text{NULL}$ 

```
c_1: (x_1 \lor x_2 \lor x_4) \land c_2: (x_2 \lor \neg x_4) \land c_3: (x_1 \lor \neg x_2 \lor x_4) \land c_4: (x_3 \lor \neg x_4) \land c_4: (x_3 \lor \neg x_4) \land (x_4 \lor x_2 \lor x_4) \land (x_4 \lor x_4) \lor (x_4 \lor x_4) \land (x_4 \lor x_4) \lor (x_4 \lor x_4)
c_5:(x_1\vee x_2)
```

Watch lists: Trail: Activities (increment=2):

C5 当watch的对象为unit clause时,不需要再更换watched litera对象,就是直接BCP

 $\neg x_1$ :

 $x_2$ :  $c_1, c_2, c_5$ 

 $\neg x_2$ :  $c_3$ 

 $X_3$ :  $C_4$ 

 $\neg x_3$ :

 $x_4$ :  $c_1, c_3$ 

 $\neg x_4$ :  $c_2, c_4$ 

Assign  $x_2$  at DL1 by  $c_5$ 

 $x_1$  1

 $x_2$  1

 $x_3$  0

 $x_4$  1

X2 : C5

$$c_1: (x_1 \vee \underline{x_2} \vee \underline{x_4}) \wedge c_2: (\underline{x_2} \vee \underline{-x_4}) \wedge c_3: (x_1 \vee \underline{-x_2} \vee \underline{x_4}) \wedge c_4: (\underline{x_3} \vee \underline{-x_4}) \wedge c_5: (x_1 \vee x_2)$$

```
Watch lists:
```

```
X1: C5
\neg x_1:
 x_2: c_1, c_2, c_5
\neg x_2: c_3
X3 : C4
\neg x_3:
x_4: c_1, c_3
```

 $\neg x_4$ :  $c_2, c_4$ Propagate  $x_2$  in

```
Activities
                Trail:
(increment=2):
```

 $x_1$  1  $x_2$  1  $x_3$  0 DL1:  $\neg x_1$ : NULL

X2: C5

 $x_4$  1

```
c_1: (x_1 \vee \underline{x_2} \vee \underline{x_4}) \wedge c_2: (\underline{x_2} \vee \underline{-x_4}) \wedge c_3: (x_1 \vee \underline{-x_2} \vee \underline{x_4}) \wedge c_4: (\underline{x_3} \vee \underline{-x_4}) \wedge c_5: (x_1 \vee x_2)
```

```
Watch lists:
                                                       Trail:
                                   Activities
                                   (increment=2):
                                                            DL1: \neg x_1: NULL
        X1 :
            C5
      \neg x_1:
                                                                     X2 : C5
                                         x_1 1
        x_2: c_1, c_2, c_5
                                         x_2 1
      \neg X_2: C_3
                                         x<sub>3</sub> 0
       X_3: C_4
                                         XΔ
      \neg x_3:
        x_4: c_1, c_3
                           当watch的对象为unit clause时,不需要再更换watched literal对象,而
                           是直接BCP
      \neg x_4: c_2, c_4
 Propagate x_2 in c_3: (x_1 \vee \neg x_2 \vee x_4)
```

```
c_1: (x_1 \vee \underline{x_2} \vee \underline{x_4}) \wedge c_2: (\underline{x_2} \vee \underline{-x_4}) \wedge c_3: (x_1 \vee \underline{-x_2} \vee \underline{x_4}) \wedge c_4: (\underline{x_3} \vee \underline{-x_4}) \wedge c_5: (x_1 \vee x_2)
```

```
Watch lists:
                                                           Trail:
                                     Activities
                                      (increment=2):
                                                                DL1: \neg x_1: NULL
        X1: C5
       \neg x_1:
                                                                          X2 : C5
                                            x_1 1
        x_2: c_1, c_2, c_5
                                                                          X4:C3
                                            x_2 1
       \neg x_2: c_3
                                            x_3 0
       X_3: C_4
                                            x_4 1
       \neg x_3:
       x_4: c_1, c_3
       \neg x_4: c_2, c_4
 Propagate x_2 in c_3: (x_1 \vee \neg x_2 \vee x_4) \rightarrow \text{Assign } x_4
```

$$c_1: (x_1 \vee \underline{x_2} \vee \underline{x_4}) \wedge c_2: (\underline{x_2} \vee \underline{-x_4}) \wedge c_3: (x_1 \vee \underline{-x_2} \vee \underline{x_4}) \wedge c_4: (\underline{x_3} \vee \underline{-x_4}) \wedge c_5: (x_1 \vee x_2)$$

Watch lists:

$$x_1$$
:  $c_5$ 

$$x_2: c_1, c_2, c_5$$

$$\neg x_2$$
:  $c_3$ 

$$\neg x_3$$
:

$$x_4: c_1, c_3$$

$$\neg x_4$$
:  $c_2, c_4$ 

Propagate  $x_4$  in

```
Activities
(increment=2):
     x_1 1
```

 $x_2$  1

 $x_3$  0

 $x_4$  1

Trail:

DL1:  $\neg x_1$ : NULL

 $X_2 : C_5$ 

 $X_4 : C_3$ 

```
c_1: (x_1 \vee \underline{x_2} \vee \underline{x_4}) \wedge c_2: (\underline{x_2} \vee \underline{-x_4}) \wedge c_3: (x_1 \vee \underline{-x_2} \vee \underline{x_4}) \wedge c_4: (\underline{x_3} \vee \underline{-x_4}) \wedge c_5: (x_1 \vee x_2)
```

```
Watch lists:
                                                       Trail:
                                   Activities
                                   (increment=2):
                                                            DL1: \neg x_1: NULL
        X1: C5
      \neg x_1:
                                                                      X2 : C5
                                         x_1 1
        x_2: c_1, c_2, c_5
                                                                      X4: C3
                                         x_2 1
      \neg x_2: c_3
                                         x_3 0
       X_3: C_4
                                          x_4 1
      \neg x_3:
       x_4: c_1, c_3
      \neg x_4: c_2, c_4
 Propagate x_4 in c_2:(x_2\vee \neg x_4)
```

```
c_1: (x_1 \vee \underline{x_2} \vee \underline{x_4}) \wedge c_2: (\underline{x_2} \vee \underline{-x_4}) \wedge c_3: (x_1 \vee \underline{-x_2} \vee \underline{x_4}) \wedge c_4: (\underline{x_3} \vee \underline{-x_4}) \wedge c_5: (x_1 \vee x_2)
```

```
Watch lists:
                                                         Trail:
                                    Activities
                                    (increment=2):
                                                              DL1: \neg x_1: NULL
        X1: C5
       \neg x_1:
                                                                        X2 : C5
                                           x_1 1
        x_2: c_1, c_2, c_5
                                                                        X4: C3
                                           x_2 1
      \neg x_2: c_3
                                           x_3 0
       X_3: C_4
                                           x_4 1
       \neg x_3:
       x_4: c_1, c_3
      \neg x_4: c_2, c_4
 Propagate x_4 in c_2: (x_2 \vee \neg x_4) \rightarrow O.K.
```

```
c_1: (x_1 \vee \underline{x_2} \vee \underline{x_4}) \wedge c_2: (\underline{x_2} \vee \underline{-x_4}) \wedge c_3: (x_1 \vee \underline{-x_2} \vee \underline{x_4}) \wedge c_4: (\underline{x_3} \vee \underline{-x_4}) \wedge c_5: (x_1 \vee x_2)
```

```
Watch lists:
                                                       Trail:
                                   Activities
                                   (increment=2):
                                                            DL1: \neg x_1: NULL
        X1: C5
      \neg x_1:
                                                                     X2 : C5
                                         x_1 1
       x_2: c_1, c_2, c_5
                                                                     X4: C3
                                         x_2 1
      \neg x_2: c_3
                                         x_3 0
       X_3: C_4
                                         x_4 1
      \neg x_3:
       x_4: c_1, c_3
      \neg x_4: c_2, c_4
 Propagate x_4 in c_4:(x_3\vee \neg x_4)
```

```
c_1: (x_1 \lor x_2 \lor x_4) \land c_2: (x_2 \lor \neg x_4) \land c_3: (x_1 \lor \neg x_2 \lor x_4) \land c_4: (x_3 \lor \neg x_4) \land c_4: (x_3 \lor \neg x_4) \land (x_4 \lor x_2 \lor x_4) \land (x_4 \lor x_4) \lor (x_4 \lor x_4)
             c_5:(x_1 \vee x_2)
Watch lists:
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 Trail:
                                                                                                                                                                                                                                                                                                                              Activities
                                                                                                                                                                                                                                                                                                                               (increment=2):
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              DL1: \neg x_1: NULL
                                                                           X1: C5
                                                             \neg x_1:
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   X2 : C5
                                                                                                                                                                                                                                                                                                                                                                                      x_1 1
                                                                       x_2: c_1, c_2, c_5
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   X_4:C_3
                                                                                                                                                                                                                                                                                                                                                                                     x_2 1
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 X3 : C4
                                                             \neg x_2: c_3
                                                                                                                                                                                                                                                                                                                                                                                     x_3 0
                                                                 X_3: C_4
                                                                                                                                                                                                                                                                                                                                                                                      x_4 1
                                                             \neg x_3:
                                                                    x_4: c_1, c_3
                                                            \neg x_4: c_2, c_4
             Propagate x_4 in c_4: (x_3 \vee \neg x_4) \rightarrow \text{Assign } x_3
```

$$c_1: (x_1 \vee \underline{x_2} \vee \underline{x_4}) \wedge c_2: (\underline{x_2} \vee \underline{-x_4}) \wedge c_3: (x_1 \vee \underline{-x_2} \vee \underline{x_4}) \wedge c_4: (\underline{x_3} \vee \underline{-x_4}) \wedge c_5: (x_1 \vee x_2)$$

Watch lists:

 $\rightarrow$  SAT

$$x_1: c_5$$
  
 $\neg x_1:$   
 $x_2: c_1, c_2, c_5$   
 $\neg x_2: c_3$   
 $x_3: c_4$   
 $\neg x_3:$ 

 $x_4$ :  $c_1, c_3$  $\neg x_4$ :  $c_2$ ,

```
Activities (increment=2): x_1 \quad 1x_2 \quad 1x_3 \quad 0x_4 \quad 1
```

```
DL1: \neg x_1 : \text{NULL}

x_2 : c_5

x_4 : c_3

x_3 : c_4
```

Trail:

# Bonus exercise 8

Assume the following propositional logic formula in CNF:  $(C \lor \neg D) \land (A \lor \neg B \lor \neg D) \land (\neg A \lor \neg B \lor \neg C \lor \neg D) \land (A \lor D)$ 

Apply the DPLL+CDCL algorithm until it detects either a conflict or a complete solution. For a decision, always take the smallest unassigned variable in the order A < B < C < D and assign false to it.

At the first conflict of full solution, how many variables are assigned the

value true?

DL1: 7A: NVLL

P: C3

C: D0

78: C1

华麗为 乙

Assume the following propositional logic formula in CNF:  $(A \lor B \lor \neg D) \land (B \lor D) \land (A \lor \neg D) \land (\neg A \lor C \lor \neg D)$  Apply the DPLL+CDCL algorithm until it detects either a conflict or all variables are assigned and the formula is satisfied. For a decision, always take the smallest unassigned variable in the order A < B < C < D and assign false to it. At the first conflict or full solution, how many variables are assigned the value true? Please answer by writing the number using digits without whitespaces.

Answer: | DLI: ¬A:NVIL ¬D: C

Finish attempt ...

## 06 SAT solving examples

1 Example 1

## SAT solving: Example 2

- We use DPLL+CDCL SAT solving combining enumeration, propagation and resolution.
- We use watched literals (underlined in formulas) to speed up propagation.
- We use VSIDS as variable ordering heuristics and assign the value false to decision variables.
- In VSIDS, to order variables with the same activity value, we use the lexicographic order; in our example this will be a < b < c < d.

$$c_1: (a \lor b \lor \neg c) \land c_2: (b \lor c) \land c_3: (\neg a \lor b \lor \neg c) \land c_4: (a \lor \neg b \lor c) \land c_5: (a \lor \neg c \lor d) \land c_6: (\neg c \lor \neg d)$$

## Example – Watchlists

$$c_1: (a \lor b \lor \neg c) \land c_2: (b \lor c) \land c_3: (\neg a \lor b \lor \neg c) \land c_4: (a \lor \neg b \lor c) \land c_5: (a \lor \neg c \lor d) \land c_6: (\neg c \lor \neg d)$$

#### Watch lists:

- a :
- $\neg a$  :
  - b :
- $\neg b$ :
  - **c** :
- ¬c:
  - d :
- ٠.,
- $\neg d$ :

$$c_1: (\underline{a} \vee \underline{b} \vee \neg c) \wedge c_2: (b \vee c) \wedge c_3: (\neg a \vee b \vee \neg c) \wedge c_4: (a \vee \neg b \vee c) \wedge c_5: (a \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d)$$

#### Watch lists:

- $a: c_1$
- ¬a:
  - $b: c_1$
- $\neg b$ :
  - **c** :
- $\neg c$ :
- d :
- $\neg d$ :

$$c_1: (\underline{a} \vee \underline{b} \vee \neg c) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\neg a \vee b \vee \neg c) \wedge c_4: (a \vee \neg b \vee c) \wedge c_5: (a \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d)$$

#### Watch lists:

 $a: c_1$   $\neg a:$   $b: c_1, c_2$   $\neg b:$   $c: c_2$   $\neg c:$  d:  $\neg d:$ 

$$c_1: (\underline{a} \vee \underline{b} \vee \neg c) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (\underline{a} \vee \neg b \vee c) \wedge c_5: (\underline{a} \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d)$$

#### Watch lists:

```
a: c_1
\neg a: c_3
b: c_1, c_2, c_3
\neg b:
c: c_2
\neg c:
d:
\neg d:
```

$$c_1: (\underline{a} \vee \underline{b} \vee \neg c) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (\underline{a} \vee \underline{\neg b} \vee c) \wedge c_5: (a \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d)$$

#### Watch lists:

 $a: c_1, c_4$   $\neg a: c_3$   $b: c_1, c_2, c_3$   $\neg b: c_4$   $c: c_2$   $\neg c:$  d:

$$c_1: (\underline{a} \vee \underline{b} \vee \neg c) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (\underline{a} \vee \underline{\neg b} \vee c) \wedge c_5: (\underline{a} \vee \underline{\neg c} \vee d) \wedge c_6: (\neg c \vee \neg d)$$

#### Watch lists:

```
a: c_1, c_4, c_5
\neg a: c_3
b: c_1, c_2, c_3
\neg b: c_4
c: c_2
\neg c: c_5
d:
```

$$c_1: (\underline{a} \vee \underline{b} \vee \neg c) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (\underline{a} \vee \underline{\neg b} \vee c) \wedge c_5: (\underline{a} \vee \underline{\neg c} \vee d) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d})$$

#### Watch lists:

 $a: c_1, c_4, c_5$   $\neg a: c_3$   $b: c_1, c_2, c_3$   $\neg b: c_4$   $c: c_2$   $\neg c: c_5, c_6$  d:  $\neg d: c_6$ 

### Example – Activities

$$c_1: (\underline{a} \vee \underline{b} \vee \neg c) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (\underline{a} \vee \underline{\neg b} \vee c) \wedge c_5: (\underline{a} \vee \underline{\neg c} \vee d) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d})$$

Watch lists:

$$a: c_1, c_4, c_5$$
  
 $\neg a: c_3$   
 $b: c_1, c_2, c_3$   
 $\neg b: c_4$   
 $c: c_2$ 

d:  $\neg d$ :  $c_6$ 

 $\neg c: c_5, c_6$ d: Activities (increment=1):

a 0b 0c 0

d 0

$$c_1: (\underline{a} \vee \underline{b} \vee \neg c) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (\underline{a} \vee \underline{\neg b} \vee c) \wedge c_5: (\underline{a} \vee \underline{\neg c} \vee d) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d})$$

#### Watch lists:

$$a: c_1, c_4, c_5$$
  
 $\neg a: c_3$   
 $b: c_1, c_2, c_3$   
 $\neg b: c_4$   
 $c: c_2$ 

d :  $\neg d$ :  $c_6$ 

 $\neg c$ :  $c_5, c_6$ 

Activities Trail: (increment=1):

0

d = 0

$$c_1: (\underline{a} \vee \underline{b} \vee \neg c) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (\underline{a} \vee \underline{\neg b} \vee c) \wedge c_5: (\underline{a} \vee \underline{\neg c} \vee d) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d})$$

c = 0

d = 0

#### Watch lists:

Decide  $\neg a$ 

```
a: c_1, c_4, c_5
\neg a: c_3
b: c_1, c_2, c_3
\neg b: c_4
c: c_2
\neg c: c_5, c_6
d:
\neg d: c_6
```

```
Activities Trail: (increment=1): DL1: \neg a: NULL a = 0 b = 0
```

$$c_1: (\underline{a} \vee \underline{b} \vee \neg c) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (\underline{a} \vee \underline{\neg b} \vee c) \wedge c_5: (\underline{a} \vee \underline{\neg c} \vee d) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d})$$

#### Watch lists:

```
a: c_1, c_4, c_5
\neg a: c_3
b: c_1, c_2, c_3
\neg b: c_4
c: c_2
\neg c: c_5, c_6
d:
\neg d: c_6
```

Propagate  $\neg a$  in

```
Activities Trail: (increment=1): DL1: \neg a: NULL a = 0
```

b 0c 0d 0

 $\neg c$ :  $c_5, c_6$ d:  $\neg d$ :  $c_6$ 

Propagate  $\neg a$  in  $c_1: (a \lor b \lor \neg c)$ 

```
c_1: (a \lor b \lor \neg c) \land c_2: (b \lor c) \land c_3: (\neg a \lor b \lor \neg c) \land c_4: (a \lor \neg b \lor c) \land c_4: (a \lor \neg
                  c_5: (a \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d)
Watch lists:
                                                                                                                                                                                                                                                                                                                                                                                                                                                                 Activities
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            Trail:
                                                                                                                                                                                                                                                                                                                                                                                                                                                                 (increment=1):
                                                                                                          a: c_1, c_4, c_5
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              DL1: \neg a: NULL
                                                                                       \neg a: c_3
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        0
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      a
                                                                                                        b: c_1, c_2, c_3
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    b 0
                                                                                     \neg b: c_4
                                                                                              c: c_2
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      d = 0
```

```
c_{1}: (a \lor \underline{b} \lor \neg c) \land c_{2}: (\underline{b} \lor \underline{c}) \land c_{3}: (\neg \underline{a} \lor \underline{b} \lor \neg c) \land c_{4}: (\underline{a} \lor \neg \underline{b} \lor c) \land c_{5}: (\underline{a} \lor \neg \underline{c} \lor d) \land c_{6}: (\neg \underline{c} \lor \neg \underline{d})
Watch lists:
Activities \qquad Trail: (increment=1): \\ a: & \not \sim 1, c_{4}, c_{5} \\ \neg a: & c_{3} \qquad \qquad a \quad 0
```

 $a: \mathcal{A}, c_4, c_5$   $\neg a: c_3$   $b: c_1, c_2, c_3$   $\neg b: c_4$   $c: c_2$   $\neg c: c_5, c_6, c_1$ 

c 0 d 0

d:

 $\neg d$ :  $c_6$ 

Propagate  $\neg a$  in  $c_1: (\underline{a} \lor \underline{b} \lor \neg c) \rightarrow (a \lor \underline{b} \lor \neg c)$ 

```
c_1: (a \lor b \lor \neg c) \land c_2: (b \lor c) \land c_3: (\neg a \lor b \lor \neg c) \land c_4: (a \lor \neg b \lor c) \land c_4: (a \lor \neg
                 c_5: (a \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d)
Watch lists:
                                                                                                                                                                                                                                                                                                                                                                                                                                Activities
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         Trail:
                                                                                                                                                                                                                                                                                                                                                                                                                                (increment=1):
                                                                                                  a: \mathcal{C}_1, \mathcal{C}_4, \mathcal{C}_5
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       DL1: \neg a: NULL
                                                                               \neg a: c_3
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              0
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               a
                                                                                                b: c_1, c_2, c_3
                                                                               \neg b: c_4
                                                                                         c: c_2
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               d = 0
                                                                               \neg c: c_5, c_6, c_1
                                                                                   d :
                                                                               \neg d: c_6
```

Propagate  $\neg a$  in  $c_4: (a \lor \neg b \lor c)$ 

```
c_1: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (\underline{a} \vee \underline{\neg c} \vee d) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d})
```

```
Watch lists:
                                         Activities
                                                               Trail:
                                         (increment=1):
         a: \mathcal{G}_1, \mathcal{G}_4, \mathcal{C}_5
                                                                     DL1: \neg a: NULL
       \neg a: c_3
                                                     0
                                                a
         b: c_1, c_2, c_3
       \neg b: c_4
        C: C_2, C_4
       \neg c: c_5, c_6, c_1
        d :
       \neg d: c_6
 Propagate \neg a in c_4: (a \lor \neg b \lor c) \to (a \lor \neg b \lor c)
```

```
c_1: (a \lor b \lor \neg c) \land c_2: (b \lor c) \land c_3: (\neg a \lor b \lor \neg c) \land c_4: (a \lor \neg b \lor c) \land c_4: (a \lor \neg
               c_5: (a \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d)
Watch lists:
                                                                                                                                                                                                                                                                                                                                                                             Activities
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        Trail:
                                                                                                                                                                                                                                                                                                                                                                             (increment=1):
                                                                                      a: \mathcal{G}_1, \mathcal{G}_4, \mathcal{C}_5
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              DL1: \neg a: NULL
                                                                       \neg a: c_3
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          0
                                                                                                                                                                                                                                                                                                                                                                                                                                                  a
                                                                                    b: c_1, c_2, c_3
                                                                                                                                                                                                                                                                                                                                                                                                                                                b 0
                                                                     \neg b: c_4
                                                                              C: C_2, C_4
                                                                     \neg c: c_5, c_6, c_1
                                                                         d :
                                                                     \neg d: c_6
                 Propagate \neg a in c_5: (a \lor \neg c \lor d)
```

```
c_1: (a \lor \underline{b} \lor \neg \underline{c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg \underline{a} \lor \underline{b} \lor \neg c) \land c_4: (a \lor \neg \underline{b} \lor \underline{c}) \land c_5: (a \lor \neg \underline{c} \lor \underline{d}) \land c_6: (\neg \underline{c} \lor \neg \underline{d})
Watch lists:

Activities

Trail:
```

```
(increment=1):

a: \mathcal{A}, \mathcal{A}, \mathcal{A}

\neg a: c_3

b: c_1, c_2, c_3

\neg b: c_4

c: c_2, c_4

d: c_5

\neg d: c_6
```

Propagate  $\neg a$  in  $c_5: (\underline{a} \lor \underline{\neg c} \lor d) \rightarrow (a \lor \underline{\neg c} \lor \underline{d})$ 

```
c_1: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d})
```

```
Watch lists:

a:
\neg a: c_3
b: c_1, c_2, c_3
\neg b: c_4
```

Decide  $\neg b$ 

```
\neg a: c_3

b: c_1, c_2, c_3

\neg b: c_4

c: c_2, c_4

\neg c: c_5, c_6, c_1

d: c_5

\neg d: c_6
```

```
Activities (increment=1):

a 0
b 0
c 0
d 0
```

```
Trail: \neg a : \text{NULL} DL2: \neg b : \text{NULL}
```

```
c_1: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d})
```

```
Watch lists:
a:
\neg a: c_3
```

```
    \neg a: c_3 

    b: c_1, c_2, c_3 

    \neg b: c_4
```

 $c: c_2, c_4$  $\neg c: c_5, c_6, c_1$ 

 $d: c_5$ 

 $\neg d$ :  $c_6$ 

Propagate  $\neg b$  in

```
Activities Trail: (increment=1):
```

a 0b 0

c 0 d 0 DL1:  $\neg a$ : NULL DL2:  $\neg b$ : NULL

DL2:  $\neg b$ : NULL

```
c_1: (a \lor b \lor \neg c) \land c_2: (b \lor c) \land c_3: (\neg a \lor b \lor \neg c) \land c_4: (a \lor \neg b \lor c) \land c_4: (a \lor \neg
              c_5: (a \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d)
Watch lists:
                                                                                                                                                                                                                                                                                                                                                       Activities
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      Trail:
                                                                                                                                                                                                                                                                                                                                                       (increment=1):
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         DL1: \neg a: NULL
                                                                                 a :
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         DL2: \neg b: NULL
                                                                 \neg a: c_3
                                                                                                                                                                                                                                                                                                                                                                                                                                                              0
                                                                                                                                                                                                                                                                                                                                                                                                                        a
                                                                             b: c_1, c_2, c_3
                                                                                                                                                                                                                                                                                                                                                                                                                      b 0
                                                                 \neg b: c_4
                                                                                                                                                                                                                                                                                                                                                                                                                        c = 0
                                                                        C: C_2, C_4
                                                                                                                                                                                                                                                                                                                                                                                                                        d = 0
                                                                 \neg c: c_5, c_6, c_1
                                                                  d: c_5
                                                                 \neg d: c_6
                Propagate \neg b in c_1:(a \lor b \lor \neg c)
```

```
c_1: (a \lor b \lor \neg c) \land c_2: (b \lor c) \land c_3: (\neg a \lor b \lor \neg c) \land c_4: (a \lor \neg b \lor c) \land c_4: (a \lor \neg
             c_5: (a \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d)
Watch lists:
                                                                                                                                                                                                                                                                                                                                          Activities
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  Trail:
                                                                                                                                                                                                                                                                                                                                          (increment=1):
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   DL1: \neg a: NULL
                                                                              a :
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   DL2: \neg b: NULL
                                                                \neg a: c_3
                                                                                                                                                                                                                                                                                                                                                                                                                                             0
                                                                                                                                                                                                                                                                                                                                                                                                         a
                                                                          b: c_1, c_2, c_3
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            \neg c : c_1
                                                                                                                                                                                                                                                                                                                                                                                                       b 0
                                                              \neg b: c_4
                                                                     c: c_2, c_4
                                                                                                                                                                                                                                                                                                                                                                                                         d = 0
                                                              \neg c: c_5, c_6, c_1
                                                                  d: c_5
                                                              \neg d: c_6
               Propagate \neg b in c_1: (a \lor b \lor \neg c) \rightarrow assign \neg c
```

 $\neg d$ :  $c_6$ 

Propagate  $\neg b$  in  $c_2:(b \lor c)$ 

```
c_1: (a \lor b \lor \neg c) \land c_2: (b \lor c) \land c_3: (\neg a \lor b \lor \neg c) \land c_4: (a \lor \neg b \lor c) \land c_4: (a \lor \neg
                c_5: (a \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d)
Watch lists:
                                                                                                                                                                                                                                                                                                                                                                                                  Activities
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           Trail:
                                                                                                                                                                                                                                                                                                                                                                                                    (increment=1):
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  DL1: \neg a: NULL
                                                                                           a :
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  DL2: \neg b: NULL
                                                                           \neg a: c_3
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      0
                                                                                                                                                                                                                                                                                                                                                                                                                                                                           a
                                                                                       b: c_1, c_2, c_3
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       \neg c : c_1
                                                                                                                                                                                                                                                                                                                                                                                                                                                                         b 0
                                                                         \neg b: c_4
                                                                                                                                                                                                                                                                                                                                                                                                                                                                           c = 0
                                                                                   c: c_2, c_4
                                                                                                                                                                                                                                                                                                                                                                                                                                                                           d = 0
                                                                         \neg c: c_5, c_6, c_1
                                                                             d: c_5
```

```
c_1: (a \lor b \lor \neg c) \land c_2: (b \lor c) \land c_3: (\neg a \lor b \lor \neg c) \land c_4: (a \lor \neg b \lor c) \land c_4: (a \lor \neg
               c_5: (a \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d)
Watch lists:
                                                                                                                                                                                                                                                                                                                                                                                          Activities
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              Trail:
                                                                                                                                                                                                                                                                                                                                                                                          (increment=1):
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      DL1: \neg a: NULL
                                                                                         a :
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      DL2: \neg b: NULL
                                                                       \neg a: c_3
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            0
                                                                                                                                                                                                                                                                                                                                                                                                                                                                  a
                                                                                     b: c_1, c_2, c_3
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         \neg c : c_1
                                                                                                                                                                                                                                                                                                                                                                                                                                                                b 0
                                                                       \neg b: c_4
                                                                                 C: C_2, C_4
                                                                                                                                                                                                                                                                                                                                                                                                                                                                  d = 0
                                                                       \neg c: c_5, c_6, c_1
                                                                         d: c_5
                                                                       \neg d: c_6
```

Propagate  $\neg b$  in  $c_2: (\underline{b} \lor \underline{c}) \to \{ \text{conflict!} \}$ 

```
c_1: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d})
```

Watch lists:

```
a:
\neg a: c_3
b: c_1, c_2, c_3
\neg b: c_4
c: c_2, c_4
\neg c: c_5, c_6, c_1
```

 $d: c_5$   $\neg d: c_6$ 

Conflict resolution:

```
Activities Trail:

(increment=1):

\begin{array}{cccc}
a & 0 & DL1: & \neg a : NULL \\
DL2: & \neg b : NULL \\
DL3: & \neg c : c_1
\end{array}

\begin{array}{cccc}
c & 0 & \sigma & \sigma & \sigma & \sigma \\
c & 0 & \sigma & \sigma & \sigma & \sigma \\
c & 0 & \sigma & \sigma & \sigma & \sigma \\
c & 0 & \sigma & \sigma & \sigma & \sigma \\
c & 0 & \sigma & \sigma & \sigma & \sigma \\
c & 0 & \sigma & \sigma & \sigma & \sigma \\
c & 0 & \sigma & \sigma & \sigma & \sigma \\
c & 0 & \sigma & \sigma & \sigma & \sigma \\
c & 0 & \sigma & \sigma & \sigma & \sigma \\
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c & 0 & \sigma & \sigma & \sigma & \sigma \\
c & 0 & \sigma & \sigma & \sigma & \sigma \\
c & 0 & \sigma & \sigma & \sigma & \sigma \\
```

```
c_1: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d})
```

```
Watch lists:
                                    Activities
                                                        Trail:
                                    (increment=2):
                                                             DL1: \neg a: NULL
        a :
                                                             DL2: \neg b: NULL
       \neg a: c_3
                                           a Ø 1
        b: c_1, c_2, c_3
                                                                     \neg c : c_1
                                          b Ø 1
      \neg b: c_4
                                          c \emptyset 1
        C: C_2, C_4
                                           d 0
      \neg c: c_5, c_6, c_1
       d: c_5
      \neg d: c_6
```

Conflict resolution:  $\frac{c_2:(b\lor c) \ c_1:(a\lor b\lor \neg c)}{c_7:(a\lor b)}$ 

```
c_1: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b})
```

#### Watch lists:

```
a: \quad c_7
\neg a: \quad c_3
b: \quad c_1, c_2, c_3, c_7
\neg b: \quad c_4
c: \quad c_2, c_4
\neg c: \quad c_5, c_6, c_1
d: \quad c_5
\neg d: \quad c_6
```

Add conflict clause

```
Activities Trail: (increment=2):

a 1
b 1
c 1
d 0
```

```
DL1: \neg a : \text{NULL}
DL2: \neg b : \text{NULL}
\neg c : c_1
```

```
c_1: (a \vee \underline{b} \vee \underline{\neg c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b})
```

#### Watch lists:

```
a: c_7
\neg a: c_3
b: c_1, c_2, c_3, c_7
\neg b: c_4
c: c_2, c_4
\neg c: c_5, c_6, c_1
d: c_5
```

 $\neg d$ :  $c_6$ Backtrack to DI 1

```
Activities (increment=2):

a
1

b
1

c
1

d
0
```

```
DL1: ¬a: NULL
DL2: ¬b: NULL
¬c: cī
```

Trail:

```
c_1: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b})
```

#### Watch lists:

```
a: c_7
\neg a: c_3
b: c_1, c_2, c_3, c_7
\neg b: c_4
c: c_2, c_4
\neg c: c_5, c_6, c_1
d: c_5
\neg d: c_6
```

Assign b at DL1 by c<sub>7</sub>

$$c_1: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b})$$

d = 0

#### Watch lists:

```
a: c_7
\neg a: c_3
b: c_1, c_2, c_3, c_7
\neg b: c_4
c: c_2, c_4
\neg c: c_5, c_6, c_1
d: c_5
\neg d: c_6
```

Propagate b in

```
Activities Trail: (increment=2): DL1: \neg a: NULL a 1 b: c_7 b 1 c 1
```

 $\neg d$ :  $c_6$ 

Propagate b in  $c_4:(a \vee \neg b \vee c)$ 

```
c_1: (a \lor b \lor \neg c) \land c_2: (b \lor c) \land c_3: (\neg a \lor b \lor \neg c) \land c_4: (a \lor \neg b \lor c) \land c_4: (a \lor \neg
              c_5: (a \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d) \wedge c_7: (a \vee b)
Watch lists:
                                                                                                                                                                                                                                                                                                                                                                                             Activities
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  Trail:
                                                                                                                                                                                                                                                                                                                                                                                             (increment=2):
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          DL1: \neg a: NULL
                                                                                          a: c7
                                                                        \neg a: c_3
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  b : C7
                                                                                                                                                                                                                                                                                                                                                                                                                                                                   a 1
                                                                                          b: c_1, c_2, c_3, c_7
                                                                                                                                                                                                                                                                                                                                                                                                                                                                   b 1
                                                                        \neg b: c_4
                                                                                                                                                                                                                                                                                                                                                                                                                                                                     c 1
                                                                                c: c_2, c_4
                                                                                                                                                                                                                                                                                                                                                                                                                                                                     d = 0
                                                                        \neg c: c_5, c_6, c_1
                                                                          d: c<sub>5</sub>
```

 $\neg d$ :  $c_6$ 

```
c_1: (a \lor b \lor \neg c) \land c_2: (b \lor c) \land c_3: (\neg a \lor b \lor \neg c) \land c_4: (a \lor \neg b \lor c) \land c_4: (a \lor \neg
             c_5: (a \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d) \wedge c_7: (a \vee b)
Watch lists:
                                                                                                                                                                                                                                                                                                                                                                                      Activities
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       Trail:
                                                                                                                                                                                                                                                                                                                                                                                      (increment=2):
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               DL1: \neg a: NULL
                                                                                        a: c7
                                                                       \neg a: c_3
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     b : C7
                                                                                                                                                                                                                                                                                                                                                                                                                                                           a 1
                                                                                        b: c_1, c_2, c_3, c_7
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     C: C_4
                                                                                                                                                                                                                                                                                                                                                                                                                                                           b 1
                                                                       \neg b: c_4
                                                                                                                                                                                                                                                                                                                                                                                                                                                             c 1
                                                                              c: c_2, c_4
                                                                                                                                                                                                                                                                                                                                                                                                                                                             d = 0
                                                                       \neg c: c_5, c_6, c_1
                                                                        d: c<sub>5</sub>
```

Propagate b in  $c_4: (a \lor \neg b \lor c) \to assign c$ 

$$c_1: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b})$$

#### Watch lists:

Propagate c in

```
a: c_7
\neg a: c_3
b: c_1, c_2, c_3, c_7
\neg b: c_4
c: c_2, c_4
\neg c: c_5, c_6, c_1
d: c_5
\neg d: c_6
```

```
c_1: (a \vee \underline{b} \vee \underline{\neg c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b})
```

```
Watch lists:
                                     Activities
                                                         Trail:
                                     (increment=2):
                                                               DL1: \neg a: NULL
        a: c7
       \neg a: c_3
                                                                         b : c<sub>7</sub>
                                            a 1
        b: c_1, c_2, c_3, c_7
                                                                         C: C_4
                                            b 1
       \neg b: c_4
                                            c 1
       c: c_2, c_4
                                            d = 0
       \neg c: c_5, c_6, c_1
       d: c<sub>5</sub>
       \neg d: c_6
 Propagate c in c_5: (a \lor \neg c \lor d)
```

```
c_{1}: (a \vee \underline{b} \vee \neg c) \wedge c_{2}: (\underline{b} \vee \underline{c}) \wedge c_{3}: (\neg \underline{a} \vee \underline{b} \vee \neg c) \wedge c_{4}: (a \vee \neg \underline{b} \vee \underline{c}) \wedge c_{5}: (a \vee \neg \underline{c} \vee \underline{d}) \wedge c_{6}: (\neg \underline{c} \vee \neg \underline{d}) \wedge c_{7}: (\underline{a} \vee \underline{b})
\text{Watch lists:} \qquad \text{Activities} \qquad \text{Trail:} \\ (\text{increment=2}): \qquad \text{DL1:} \quad \neg a: \text{NULL}
```

Propagate c in  $c_5: (a \lor \neg c \lor \underline{d}) \to assign d$ 

```
c_1: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b})
```

```
Watch lists:
                                   Activities
                                                       Trail:
                                   (increment=2):
                                                            DL1: \neg a: NULL
        a: c7
      \neg a: c_3
                                                                      b : c7
                                          a 1
        b: c_1, c_2, c_3, c_7
                                                                      C:C_4
                                          b 1
      \neg b: c_4
                                                                      d: c_5
                                          c 1
       c: c_2, c_4
                                          d = 0
      \neg c: C_5, C_6, C_1
      d: c<sub>5</sub>
      \neg d: c_6
 Propagate c in c_6: (\neg c \lor \neg d)
```

```
c_1: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b})
```

```
Watch lists:
                                    Activities
                                                         Trail:
                                    (increment=2):
                                                              DL1: \neg a: NULL
        a: c7
      \neg a: c_3
                                                                        b : c7
                                           a 1
        b: c_1, c_2, c_3, c_7
                                                                        C:C_4
                                           b 1
      \neg b: c_4
                                                                        d: c_5
                                           c 1
       c: c_2, c_4
                                           d = 0
      \neg c: C_5, C_6, C_1
       d: c<sub>5</sub>
      \neg d: c_6
 Propagate c in c_6: (\neg c \lor \neg d) \to \mbox{$f$ conflict!}
```

$$c_1: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b})$$

#### Watch lists:

```
a: c_7
\neg a: c_3
b: c_1, c_2, c_3, c_7
\neg b: c_4
c: c_2, c_4
\neg c: c_5, c_6, c_1
d: c_5
```

 $\neg d$ :  $c_6$  Conflict resolution:

Activities Trail: (increment=2):

a 1b 1c 1d 0

DL1:  $\neg a$ : NULL b:  $c_7$  c:  $c_4$ d:  $c_5$ 

Conflict resolution:

$$c_1: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b})$$

```
Watch lists:
                                  Activities
                                                      Trail:
                                  (increment=3):
                                                           DL1: \neg a: NULL
        a: c7
      \neg a: c_3
                                                                    b : c7
                                         a 1/3
        b: c_1, c_2, c_3, c_7
                                         b 1/3
                                                                    C:C_4
      \neg b: c_4
                                                                    d: c_5
                                         c 1/3
                                         d Ø 2
        C: C_2, C_4
      \neg c: C_5, C_6, C_1
      d: c<sub>5</sub>
      \neg d: c_6
```

 $c_6:(\neg c \lor \neg d) \ c_5:(a \lor \neg c \lor d)$ 

 $\frac{(a \vee \neg c) \ c_4 : (a \vee \neg b \vee c)}{(a \vee \neg b) \ c_7 : (a \vee b)}$ 

Satisfiability Checking — Prof. Dr. Erika Ábrahám (RWTH Aachen University)

$$c_1: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b}) \wedge c_8: (\underline{a})$$

#### Watch lists:

```
a: c_7
\neg a: c_3
b: c_1, c_2, c_3, c_7
\neg b: c_4
c: c_2, c_4
\neg c: c_5, c_6, c_1
d: c_5
```

 $\neg d$ :  $c_6$ Add conflict clause Activities Trail: (increment=3):

a 3 b 3 c 3 d 2 DL1:  $\neg a$ : NULL b:  $c_7$  c:  $c_4$ d:  $c_5$ 

$$c_1: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b}) \wedge c_8: (a)$$

#### Watch lists:

```
a: c_7
\neg a: c_3
b: c_1, c_2, c_3, c_7
\neg b: c_4
c: c_2, c_4
\neg c: c_5, c_6, c_1
d: c_5
```

(increment=3):

a 3
b 3
c 3

d 2

Activities

Trail:

DLT: Ja: NUTT

b: c7

c: e4

d: c5

$$c_1: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b}) \wedge c_8: (\underline{a})$$

#### Watch lists:

```
a: c_7
\neg a: c_3
b: c_1, c_2, c_3, c_7
\neg b: c_4
c: c_2, c_4
\neg c: c_5, c_6, c_1
d: c_5
\neg d: c_6
```

Assign a at DL0 by  $c_8$ 

```
Activities Trail: (increment=3):
```

a 3b 3c 3d 2

DL0:  $a: c_8$ 

$$c_1: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b}) \wedge c_8: (a)$$

#### Watch lists:

```
a: c_7
\neg a: c_3
b: c_1, c_2, c_3, c_7
\neg b: c_4
c: c_2, c_4
\neg c: c_5, c_6, c_1
d: c_5
```

 $\neg d$ :  $c_6$  Propagate a in

Activities Trail: (increment=3): DL0: a: c<sub>8</sub>

a 3b 3c 3d 2

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```
c_1: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b}) \wedge c_8: (a)
```

```
Watch lists:
                                     Activities
                                                         Trail:
                                     (increment=3):
        a: c<sub>7</sub>
                                                               DL0: a: c_8
       \neg a: c_3
                                            a 3
        b: c_1, c_2, c_3, c_7
                                            b 3
       \neg b: c_4
                                            c 3
       c: c_2, c_4
                                            d 2
       \neg c: c_5, c_6, c_1
       d: c<sub>5</sub>
       \neg d: c_6
 Propagate a in c_3: (\neg a \lor b \lor \neg c)
```

```
c_1: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\neg a \vee \underline{b} \vee \neg \underline{c}) \wedge c_4: (a \vee \neg \underline{b} \vee \underline{c}) \wedge c_5: (a \vee \neg \underline{c} \vee \underline{d}) \wedge c_6: (\neg \underline{c} \vee \neg \underline{d}) \wedge c_7: (\underline{a} \vee \underline{b}) \wedge c_8: (a)
```

```
Activities
Watch lists:
                                                       Trail:
                                   (increment=3):
        a: c<sub>7</sub>
                                                            DL0: a: c_8
      ¬a: 03
                                         a 3
        b: c_1, c_2, c_3, c_7
                                         b 3
      \neg b: c_4
                                          c 3
       C: C_2, C_4
                                          d 2
      \neg c: c_5, c_6, c_1, c_3
      d: c<sub>5</sub>
      \neg d: c_6
```

Propagate a in  $c_3: (\underline{\neg a} \lor \underline{b} \lor \neg c) \to (\neg a \lor \underline{b} \lor \underline{\neg c})$ 

```
c_1: (a \vee \underline{b} \vee \underline{\neg c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\neg a \vee \underline{b} \vee \underline{\neg c}) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b}) \wedge c_8: (a)
```

#### Watch lists:

Decide  $\neg b$ 

```
a: c_7
\neg a:
b: c_1, c_2, c_3, c_7
\neg b: c_4
c: c_2, c_4
\neg c: c_5, c_6, c_1, c_3
d: c_5
\neg d: c_6
```

```
Activities Trail: (increment=3):
```

b 3c 3d 2

a 3

```
DL0: a: c_8
DL1: \neg b: \text{NULL}
```

$$c_1: (a \vee \underline{b} \vee \underline{\neg c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\neg a \vee \underline{b} \vee \underline{\neg c}) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b}) \wedge c_8: (a)$$

#### Watch lists:

```
a: c_7
\neg a:
b: c_1, c_2, c_3, c_7
\neg b: c_4
c: c_2, c_4
\neg c: c_5, c_6, c_1, c_3
d: c_5
\neg d: c_6
```

Propagate  $\neg b$  in

Activities Trail: (increment=3):

a 3b 3c 3d 2

DL0:  $a: c_8$ DI1:  $\neg b: \text{NULL}$ 

```
c_1: (a \vee \underline{b} \vee \underline{\neg c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\neg a \vee \underline{b} \vee \underline{\neg c}) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b}) \wedge c_8: (a)
```

```
Activities
Watch lists:
                                                         Trail:
                                     (increment=3):
                                                               DL0: a: c_8
        a: c<sub>7</sub>
                                                               DI 1: \neg b: NULL
       \neg a:
                                            a 3
        b: c_1, c_2, c_3, c_7
                                            b 3
       \neg b: c_4
                                            c 3
        c: c_2, c_4
                                            d 2
       \neg c: c_5, c_6, c_1, c_3
       d: c<sub>5</sub>
       \neg d: c_6
 Propagate \neg b in c_1:(a \lor b \lor \neg c)
```

 $d: c_5$  $\neg d: c_6$ 

```
c_1: (a \lor b \lor \neg c) \land c_2: (b \lor c) \land c_3: (\neg a \lor b \lor \neg c) \land c_4: (a \lor \neg b \lor c) \lor c_4: (a \lor \neg b \lor c) \lor c_4: (a \lor \neg b \lor c) \lor c_4: (a \lor \neg
             c_5: (a \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d) \wedge c_7: (a \vee b) \wedge c_8: (a)
                                                                                                                                                                                                                                                                                                                                                                               Activities
Watch lists:
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            Trail:
                                                                                                                                                                                                                                                                                                                                                                               (increment=3):
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   DL0: a:c_8
                                                                                       a: c_7, c_1
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   DL1: \neg b: NULL
                                                                       ¬a:
                                                                                                                                                                                                                                                                                                                                                                                                                                                     a 3
                                                                                 b: \varphi_1, c_2, c_3, c_7
                                                                                                                                                                                                                                                                                                                                                                                                                                                     b 3
                                                                     \neg b: c_4
                                                                                                                                                                                                                                                                                                                                                                                                                                                     c 3
                                                                               C: C_2, C_4
                                                                                                                                                                                                                                                                                                                                                                                                                                                     d 2
                                                                     \neg c: c_5, c_6, c_1, c_3
```

Propagate  $\neg b$  in  $c_1: (a \lor b \lor \neg c) \to (a \lor b \lor \neg c)$ 

```
c_1: (\underline{a} \lor \underline{b} \lor \underline{\neg}\underline{c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg \underline{a} \lor \underline{b} \lor \underline{\neg}\underline{c}) \land c_4: (\underline{a} \lor \underline{\neg}\underline{b} \lor \underline{c}) \land c_5: (\underline{a} \lor \underline{\neg}\underline{c} \lor \underline{d}) \land c_6: (\underline{\neg}\underline{c} \lor \underline{\neg}\underline{d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (\underline{a})
```

```
Activities
Watch lists:
                                                     Trail:
                                  (increment=3):
                                                          DL0: a: c_8
        a: c_7, c_1
                                                          DL1: \neg b: NULL
      \neg a:
                                        a 3
       b: \varphi_1, c_2, c_3, c_7
                                        b 3
      \neg b: c_4
                                        c 3
       C: C_2, C_4
                                        d 2
      \neg c: c_5, c_6, c_1, c_3
      d: c_5
      \neg d: c_6
 Propagate \neg b in c_2:(b \lor c)
```

```
c_1: (\underline{a} \lor \underline{b} \lor \underline{\neg}\underline{c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg \underline{a} \lor \underline{b} \lor \underline{\neg}\underline{c}) \land c_4: (\underline{a} \lor \underline{\neg}\underline{b} \lor \underline{c}) \land c_5: (\underline{a} \lor \underline{\neg}\underline{c} \lor \underline{d}) \land c_6: (\underline{\neg}\underline{c} \lor \underline{\neg}\underline{d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (\underline{a})
```

```
Activities
Watch lists:
                                                    Trail:
                                 (increment=3):
                                                         DL0: a:c_8
       a: c_7, c_1
                                                         DL1: \neg b: NULL
      ¬a:
                                       a 3
       b: \varphi_1, c_2, c_3, c_7
                                       b 3
                                                                  C:C_2
      \neg b: c_4
                                       c 3
       C: C_2, C_4
                                       d 2
      \neg c: c_5, c_6, c_1, c_3
      d: c_5
      \neg d: c_6
 Propagate \neg b in c_2:(b \lor c) \to assign c
```

```
c_1: (\underline{a} \lor \underline{b} \lor \underline{\neg}\underline{c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg \underline{a} \lor \underline{b} \lor \underline{\neg}\underline{c}) \land c_4: (\underline{a} \lor \underline{\neg}\underline{b} \lor \underline{c}) \land c_5: (\underline{a} \lor \underline{\neg}\underline{c} \lor \underline{d}) \land c_6: (\underline{\neg}\underline{c} \lor \underline{\neg}\underline{d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (\underline{a})
```

```
Activities
Watch lists:
                                                        Trail:
                                    (increment=3):
                                                              DL0: a:c_8
        a: c_7, c_1
                                                              DL1: \neg b: NULL
       \neg a:
                                           a 3
        b: \varphi_1, c_2, c_3, c_7
                                                                        C: C2
                                           b 3
      \neg b: c_4
                                           c 3
       c: c_2, c_4
                                           d 2
      \neg c: c_5, c_6, c_1, c_3
       d: c<sub>5</sub>
      \neg d: c_6
 Propagate \neg b in c_3: (\neg a \lor b \lor \neg c)
```

```
c_1: (\underline{a} \lor b \lor \underline{\neg c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg a \lor \underline{b} \lor \underline{\neg c}) \land c_4: (a \lor \underline{\neg b} \lor \underline{c}) \land c_5: (a \lor \underline{\neg c} \lor \underline{d}) \land c_6: (\underline{\neg c} \lor \underline{\neg d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (a)
```

```
Activities
Watch lists:
                                                            Trail:
                                      (increment=3):
                                                                  DL0: a:c_8
         a: c_7, c_1
                                                                  DL1: \neg b: NULL
       \neg a:
                                              a 3
        b: \varphi_1, c_2, c_3, c_7
                                                                            C: C2
                                             b 3
       \neg b: c_4
                                              c 3
        c: c_2, c_4
                                              d 2
       \neg c: c_5, c_6, c_1, c_3
       d: c<sub>5</sub>
       \neg d: c_6
 Propagate \neg b in c_3: (\neg a \lor b \lor \neg c) \to \frac{1}{2} \text{ conflict!}
```

$$c_1: (\underline{a} \lor b \lor \underline{\neg c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg a \lor \underline{b} \lor \underline{\neg c}) \land c_4: (a \lor \underline{\neg b} \lor \underline{c}) \land c_5: (a \lor \underline{\neg c} \lor \underline{d}) \land c_6: (\underline{\neg c} \lor \underline{\neg d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (a)$$

Activities

Watch lists:

$$a: c_7, c_1$$
 $\neg a:$ 
 $b: g_1, c_2, c_3, c_7$ 
 $\neg b: c_4$ 
 $c: c_2, c_4$ 
 $\neg c: c_5, c_6, c_1, c_3$ 
 $d: c_5$ 

 $\neg d$ :  $c_6$  Conflict resolution:

(increment=3):

a 3
b 3
c 3
d 2

DL0:  $a: c_8$ DL1:  $\neg b: \text{NULL}$  $c: c_2$ 

Trail:

```
c_1: (\underline{a} \lor \underline{b} \lor \underline{\neg}\underline{c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg \underline{a} \lor \underline{b} \lor \underline{\neg}\underline{c}) \land c_4: (\underline{a} \lor \underline{\neg}\underline{b} \lor \underline{c}) \land c_5: (\underline{a} \lor \underline{\neg}\underline{c} \lor \underline{d}) \land c_6: (\underline{\neg}\underline{c} \lor \underline{\neg}\underline{d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (\underline{a})
```

```
Activities
Watch lists:
                                                        Trail:
                                    (increment=4):
                                                             DL0: a:c_8
        a: c_7, c_1
                                                             DL1: \neg b: NULL
       \neg a:
                                          a 36
        b: c_1, c_2, c_3, c_7
                                          b 36
                                                                       C: C2
      \neg b: c_4
                                          c 36
        C: C_2, C_4
                                          d 2
      \neg c: c_5, c_6, c_1, c_3
       d: c_5
      \neg d: c_6
                          c_3:(\neg a \lor b \lor \neg c) c_2:(b \lor c)
 Conflict resolution:
```

 $c_0:(\neg a \lor b)$ 

$$c_1: (\underline{a} \lor b \lor \underline{\neg c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg a \lor \underline{b} \lor \underline{\neg c}) \land c_4: (a \lor \underline{\neg b} \lor \underline{c}) \land c_5: (a \lor \underline{\neg c} \lor \underline{d}) \land c_6: (\underline{\neg c} \lor \underline{\neg d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (a) \land \underline{c_9}: (\underline{\neg a} \lor \underline{b})$$

#### Watch lists:

```
a: c_7, c_1
\neg a: c_9
b: \varphi_1, c_2, c_3, c_7, c_9
\neg b: c_4
c: c_2, c_4
\neg c: c_5, c_6, c_1, c_3
d: c_5
\neg d: c_6
```

Add conflict clause

$$c_1: (\underline{a} \lor b \lor \underline{\neg c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg a \lor \underline{b} \lor \underline{\neg c}) \land c_4: (a \lor \underline{\neg b} \lor \underline{c}) \land c_5: (a \lor \underline{\neg c} \lor \underline{d}) \land c_6: (\underline{\neg c} \lor \underline{\neg d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (a) \land \underline{c_9}: (\underline{\neg a} \lor \underline{b})$$

#### Watch lists:

```
a: c_7, c_1
\neg a: c_9
b: c_4, c_2, c_3, c_7, c_9
\neg b: c_4
c: c_2, c_4
\neg c: c_5, c_6, c_1, c_3
d: c_5
\neg d: c_6
```

Backtrack to DL0

```
Activities Trail: (increment=4):

a 6
b 6
c 6
d 2
```

```
DLO: a: c<sub>8</sub>
DLT: ¬b: NULL

c:e<sub>2</sub>
```

$$c_1: (\underline{a} \lor b \lor \underline{\neg c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg a \lor \underline{b} \lor \underline{\neg c}) \land c_4: (a \lor \underline{\neg b} \lor \underline{c}) \land c_5: (a \lor \underline{\neg c} \lor \underline{d}) \land c_6: (\underline{\neg c} \lor \underline{\neg d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (a) \land \underline{c_9}: (\underline{\neg a} \lor \underline{b})$$

#### Watch lists:

```
a: c_7, c_1
\neg a: c_9
b: \varphi_1, c_2, c_3, c_7, c_9
\neg b: c_4
c: c_2, c_4
\neg c: c_5, c_6, c_1, c_3
d: c_5
```

 $\neg d : c_6$  Assign **b** by  $c_9$ 

Activities Trail: (increment=4):

a 6b 6c 6

d 2

DL0: a: c<sub>8</sub>

*b* : *c*<sub>9</sub>

$$c_1: (\underline{a} \lor b \lor \underline{\neg c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg a \lor \underline{b} \lor \underline{\neg c}) \land c_4: (a \lor \underline{\neg b} \lor \underline{c}) \land c_5: (a \lor \underline{\neg c} \lor \underline{d}) \land c_6: (\underline{\neg c} \lor \underline{\neg d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (a) \land c_9: (\underline{\neg a} \lor \underline{b})$$

#### Watch lists:

Propagate b in

```
a: c_7, c_1
\neg a: c_0
 b: c_2, c_3, c_7, c_9
\neg b: c_4
 c: c_2, c_4
\neg c: c_5, c_6, c_1, c_3
d: c_5
\neg d: c_6
```

Activities Trail: (increment=4): DL0:  $a:c_8$ a 6 b 6

 $b: c_0$ 

```
c_1: (\underline{a} \lor \underline{b} \lor \underline{\neg}\underline{c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg \underline{a} \lor \underline{b} \lor \underline{\neg}\underline{c}) \land c_4: (\underline{a} \lor \underline{\neg}\underline{b} \lor \underline{c}) \land c_5: (\underline{a} \lor \underline{\neg}\underline{c} \lor \underline{d}) \land c_6: (\underline{\neg}\underline{c} \lor \underline{\neg}\underline{d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (\underline{a}) \land c_9: (\underline{\neg}\underline{a} \lor \underline{b})
```

```
Watch lists:
                                  Activities
                                                     Trail:
                                  (increment=4):
                                                          DL0: a:c_8
        a: c_7, c_1
      \neg a: c_0
                                                                   b: c_0
                                        a 6
        b: c_2, c_3, c_7, c_9
                                        b 6
      \neg b: c_4
                                        c 6
       c: c_2, c_4
                                        d 2
      \neg c: c_5, c_6, c_1, c_3
      d: c_5
      \neg d: c_6
 Propagate b in c_4:(a \vee \neg b \vee c)
```

```
c_1: (\underline{a} \lor \underline{b} \lor \underline{\neg}\underline{c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg \underline{a} \lor \underline{b} \lor \underline{\neg}\underline{c}) \land c_4: (\underline{a} \lor \neg \underline{b} \lor \underline{c}) \land c_5: (\underline{a} \lor \underline{\neg}\underline{c} \lor \underline{d}) \land c_6: (\underline{\neg}\underline{c} \lor \underline{\neg}\underline{d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (\underline{a}) \land c_9: (\underline{\neg}\underline{a} \lor \underline{b})
```

```
Activities
Watch lists:
                                                           Trail:
                                      (increment=4):
                                                                DL0: a:c_8
         a: c_7, c_1, c_4
                                                                           b : c9
       \neg a: c_0
                                             a 6
         b: c_2, c_3, c_7, c_9
                                             b 6
       \neg b: \mathcal{G}_4
                                             c = 6
        C: C_2, C_4
                                             d 2
       \neg c: c_5, c_6, c_1, c_3
       d: c_5
       \neg d: c_6
 Propagate b in c_4: (a \vee \neg b \vee c) \rightarrow (a \vee \neg b \vee c)
```

$$c_1: (\underline{a} \lor b \lor \underline{\neg c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg a \lor \underline{b} \lor \underline{\neg c}) \land c_4: (\underline{a} \lor \neg b \lor \underline{c}) \land c_5: (a \lor \underline{\neg c} \lor \underline{d}) \land c_6: (\underline{\neg c} \lor \underline{\neg d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (a) \land c_9: (\underline{\neg a} \lor \underline{b})$$

#### Watch lists:

Decide  $\neg c$ 

```
a: c_7, c_1, c_4
\neg a: c_9
b: c_2, c_3, c_7, c_9
\neg b:
c: c_2, c_4
\neg c: c_5, c_6, c_1, c_3
d: c_5
\neg d: c_6
```

```
Activities Trail: (increment=4): DL0: a: c_8
a \quad 6 \qquad b: c_9
b \quad 6 \qquad DL1: \neg c: NULL
c \quad 6
d \quad 2
```

$$c_1: (\underline{a} \lor b \lor \underline{\neg c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg a \lor \underline{b} \lor \underline{\neg c}) \land c_4: (\underline{a} \lor \neg b \lor \underline{c}) \land c_5: (a \lor \underline{\neg c} \lor \underline{d}) \land c_6: (\underline{\neg c} \lor \underline{\neg d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (a) \land c_9: (\underline{\neg a} \lor \underline{b})$$

#### Watch lists:

$$a: c_7, c_1, c_4$$
 $\neg a: c_9$ 
 $b: c_2, c_3, c_7, c_9$ 
 $\neg b:$ 
 $c: c_2, c_4$ 
 $\neg c: c_5, c_6, c_1, c_3$ 
 $d: c_5$ 
 $\neg d: c_6$ 

Propagate  $\neg c$  in

$$c_1: (\underline{a} \lor b \lor \underline{\neg c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg a \lor \underline{b} \lor \underline{\neg c}) \land c_4: (\underline{a} \lor \neg b \lor \underline{c}) \land c_5: (a \lor \underline{\neg c} \lor \underline{d}) \land c_6: (\underline{\neg c} \lor \underline{\neg d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (a) \land c_9: (\underline{\neg a} \lor \underline{b})$$

```
Activities
Watch lists:
                                                    Trail:
                                  (increment=4):
                                                         DL0: a:c_8
        a: c_7, c_1, c_4
      \neg a: c_0
                                                                   b : c9
                                        a 6
                                                         DL1: \neg c : NULL
        b: c_2, c_3, c_7, c_9
                                        b 6
      \neg b:
                                        c 6
       C: C_2, C_4
                                        d 2
      \neg c: c_5, c_6, c_1, c_3
      d: c_5
      \neg d: c_6
 Propagate \neg c in c_2:(b \lor c)
```

```
c_1: (\underline{a} \lor \underline{b} \lor \underline{\neg}\underline{c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg \underline{a} \lor \underline{b} \lor \underline{\neg}\underline{c}) \land c_4: (\underline{a} \lor \neg \underline{b} \lor \underline{c}) \land c_5: (\underline{a} \lor \underline{\neg}\underline{c} \lor \underline{d}) \land c_6: (\underline{\neg}\underline{c} \lor \underline{\neg}\underline{d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (\underline{a}) \land c_9: (\underline{\neg}\underline{a} \lor \underline{b})
```

```
Activities
Watch lists:
                                                      Trail:
                                   (increment=4):
                                                           DL0: a:c_8
        a: c_7, c_1, c_4
      \neg a: c_0
                                                                     b : c9
                                         a 6
                                                           DL1: \neg c : NULL
        b: c_2, c_3, c_7, c_9
                                         b 6
      \neg b:
                                         c = 6
       C: C_2, C_4
                                         d 2
      \neg c: c_5, c_6, c_1, c_3
       d: c_5
      \neg d: c_6
 Propagate \neg c in c_4: (a \lor \neg b \lor c)
```

$$c_1: (\underline{a} \lor \underline{b} \lor \underline{\neg}\underline{c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg \underline{a} \lor \underline{b} \lor \underline{\neg}\underline{c}) \land c_4: (\underline{a} \lor \neg \underline{b} \lor \underline{c}) \land c_5: (\underline{a} \lor \underline{\neg}\underline{c} \lor \underline{d}) \land c_6: (\underline{\neg}\underline{c} \lor \underline{\neg}\underline{d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (\underline{a}) \land c_9: (\underline{\neg}\underline{a} \lor \underline{b})$$

#### Watch lists:

Decide  $\neg d$ 

```
a: c_7, c_1, c_4
\neg a: c_9
b: c_2, c_3, c_7, c_9
\neg b:
c: c_2, c_4
\neg c: c_5, c_6, c_1, c_3
d: c_5
\neg d: c_6
```

b 6 c 6 d 2 DL0:  $a: c_8$   $b: c_9$ DL1:  $\neg c: NULL$ DL2:  $\neg d: NULL$ 

$$c_1: (\underline{a} \lor b \lor \underline{\neg c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg a \lor \underline{b} \lor \underline{\neg c}) \land c_4: (\underline{a} \lor \neg b \lor \underline{c}) \land c_5: (a \lor \underline{\neg c} \lor \underline{d}) \land c_6: (\underline{\neg c} \lor \underline{\neg d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (a) \land c_9: (\underline{\neg a} \lor \underline{b})$$

#### Watch lists:

```
a: c_7, c_1, c_4
\neg a: c_9
b: c_2, c_3, c_7, c_9
\neg b:
c: c_2, c_4
\neg c: c_5, c_6, c_1, c_3
d: c_5
\neg d: c_6
```

Propagate  $\neg d$  in

Activities Trail: (increment=4):

a 6b 6c 6d 2

DL0:  $a: c_8$   $b: c_9$ DL1:  $\neg c: NULL$ DL2:  $\neg d: NULL$ 

```
c_1: (\underline{a} \lor b \lor \underline{\neg c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg a \lor \underline{b} \lor \underline{\neg c}) \land c_4: (\underline{a} \lor \neg b \lor \underline{c}) \land c_5: (a \lor \underline{\neg c} \lor \underline{d}) \land c_6: (\underline{\neg c} \lor \underline{\neg d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (a) \land c_9: (\underline{\neg a} \lor \underline{b})
```

```
Activities
Watch lists:
                                                     Trail:
                                  (increment=4):
                                                           DL0: a:c_8
        a: c_7, c_1, c_4
      \neg a: c_0
                                                                    b : c9
                                         a 6
        b: c_2, c_3, c_7, c_9
                                                           DL1: \neg c : NULL
                                        b 6
                                                           DI 2: \neg d: NULL
      \neg b:
                                         c 6
       C: C_2, C_4
                                         d 2
      \neg c: c_5, c_6, c_1, c_3
      d: c_5
      \neg d: c_6
 Propagate \neg d in c_5: (a \lor \neg c \lor d)
```

$$c_1: (\underline{a} \lor \underline{b} \lor \underline{\neg}\underline{c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg \underline{a} \lor \underline{b} \lor \underline{\neg}\underline{c}) \land c_4: (\underline{a} \lor \neg \underline{b} \lor \underline{c}) \land c_5: (\underline{a} \lor \underline{\neg}\underline{c} \lor \underline{d}) \land c_6: (\underline{\neg}\underline{c} \lor \underline{\neg}\underline{d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (\underline{a}) \land c_9: (\underline{\neg}\underline{a} \lor \underline{b})$$

Watch lists:

$$a: c_7, c_1, c_4$$
¬a: c<sub>9</sub>

$$b: c_2, c_3, c_7, c_9$$
¬b:
$$c: c_2, c_4$$
¬c: c<sub>5</sub>, c<sub>6</sub>, c<sub>1</sub>, c<sub>3</sub>

$$d: c_5$$
¬d: c<sub>6</sub>
⇒ SAT a b ¬c ¬d

Activities (increment=4):

a 6
b 6
c 6
d 2

DL0:  $a: c_8$   $b: c_9$ DL1:  $\neg c: NULL$ DL2:  $\neg d: NULL$ 

Trail:

#### Learning target

■ How to compute with the DPLL+CDCL-based SAT solving algorithm?