# Satisfiability Checking

## 04 Propositional logic III

Prof. Dr. Erika Ábrahám

RWTH Aachen University
Informatik 2
LuFG Theory of Hybrid Systems

WS 23/24

## 1 Modeling with propositional logic

1. 优先使用resolution消去变量
2. 如果clause中只有a/非a, 无法使用resolution, 则通过给变量赋值的方式消去变量

**Lecture example:**

$(A \vee \neg B \vee C) \wedge (\neg B) \wedge (B \vee \neg C \vee D) \wedge (B \vee D)$

after elim. A : $(\neg B) \wedge (B \vee \neg C \vee D) \wedge (B \vee D)$ A无法使用resolution, 则赋值a为true

after elim. B : $(\neg C \vee D) \wedge (D)$ 使用resolution消去B

after elim. C : $(D)$ 赋值非c为true, c为false

after elim. D : true 赋值D为true

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(VU) Satisfiability Checking / Bonus test 04

# Bonus test 04

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|---|---|
| **Started on** | Friday, 20 October 2023, 9:00 AM |
| **State** | Finished |
| **Completed on** | Friday, 20 October 2023, 9:05 AM |
| **Time taken** | 4 mins 51 secs |
| **Grade** | **0.00** out of 0.33 (**0**%) |

**Question 1**

Incorrect

Mark 0.00 out of 0.33

Flag question

Apply resolution to eliminate the propositions in the order $A, B, C$ and $D$ from the following propositional logic formula in CNF:

$(\neg A \vee B \vee C) \wedge (A \vee \neg B \vee C \vee D) \wedge (A \vee B \vee \neg D) \wedge (\neg A \vee \neg B \vee C$ $\vee D)$

How many clauses are generated during this process that differ in their literal sets from all input clauses and are not trivial (i.e., neither trivially true nor trivially false)? Please answer by writing the number using digits without whitespaces.

$(C \vee D)$ $(C)$

trivial 意为 clause 为 ( ) 或 $(\neg A \vee A)$ true

Answer: I'll no m} 2

The correct answer is: 2

Finish review

## Quiz navigation

1

Finish review

resolution如果式子中不是同时有a 和非a, 则不可约去a

1.使用resolution 除去a

(C v D) ^C

2. 除去b, 没有效果

3. 除去c

使c为True

(C v D) ^C=>True

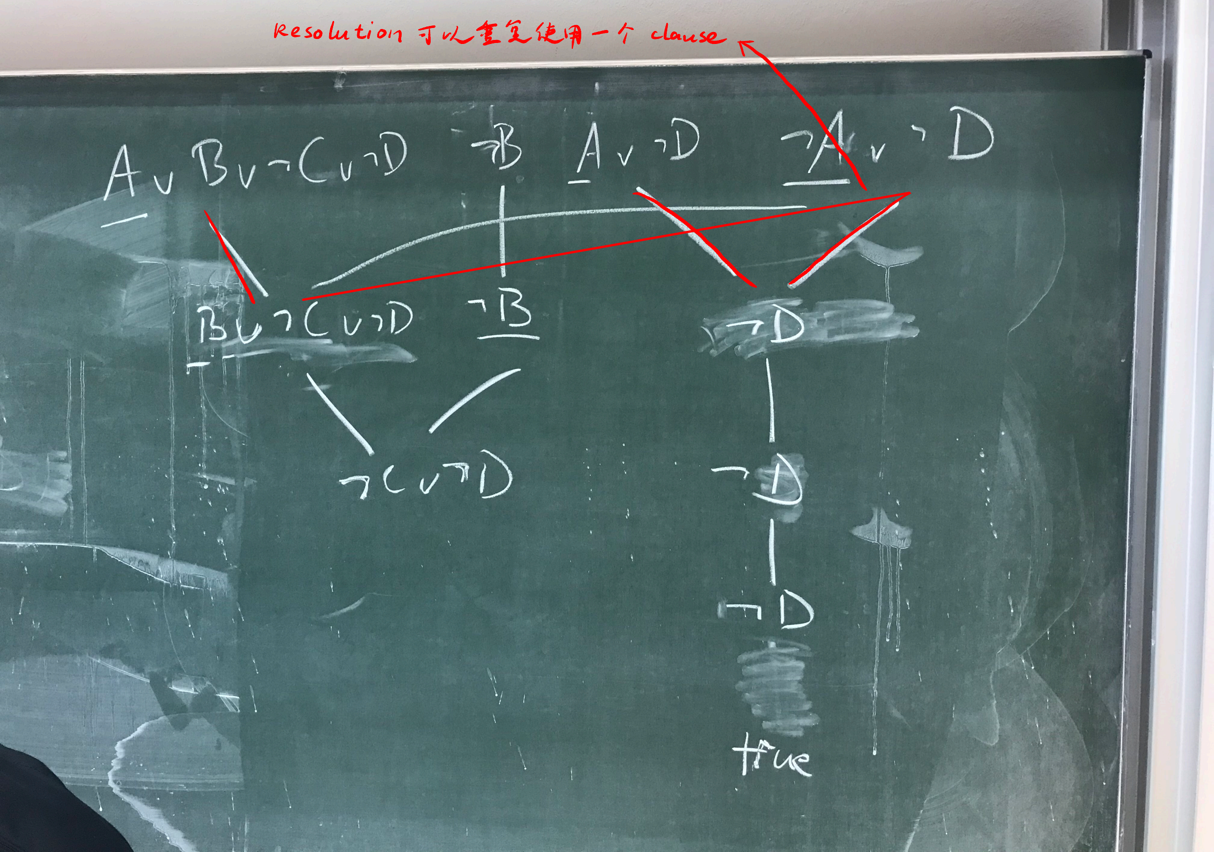
4. 除去 D, 没有效果

步骤中产生的新 non-trivial clause:

(C v D), C

故答案为2

resolution 不要求a和非a数量一样, 可以重复使用



# Before we go on...

- Suppose we can solve the satisfiability problem... how can this help us?
- There are numerous problems in the industry that are solved via the satisfiability problem of propositional logic
  - Logistics
  - Planning
  - Electronic Design Automation industry
  - Cryptography
  - ...

# Example 1: Placement of wedding guests

- Three chairs in a row: $1, 2, 3$
- We need to place Aunt, Sister and Father.
- Constraints:
  - Aunt doesn't want to sit near Father
  - Aunt doesn't want to sit in the left chair
  - Sister doesn't want to sit to the right of Father

- Q: Can we satisfy these constraints?

# Example 1 (continued)

- Notation: Aunt = 1, Sister = 2, Father = 3
  Left chair = 1, Middle chair = 2, Right chair = 3

  Introduce a propositional variable for each pair (person, chair):

  $x_{p,c}$ = "person $p$ is sited in chair $c$" for $1 \leq p, c \leq 3$
- Constraints:

  Aunt doesn't want to sit near Father:

  $$((x_{1,1} \vee x_{1,3}) \rightarrow \neg x_{3,2}) \wedge (x_{1,2} \rightarrow (\neg x_{3,1} \wedge \neg x_{3,3}))$$

  Aunt 在1,3位 → father 不能在 2   Aunt 在2 → father 不能在 1,3位

  Aunt doesn't want to sit in the left chair:

  $$\neg x_{1,1}$$

  Sister doesn't want to sit to the right of Father:

  $$(x_{3,1} \rightarrow \neg x_{2,2}) \wedge (x_{3,2} \rightarrow \neg x_{2,3})$$

# Example 1 (continued)

Each person is placed:

$$(x_{1,1} \vee x_{1,2} \vee x_{1,3}) \wedge (x_{2,1} \vee x_{2,2} \vee x_{2,3}) \wedge (x_{3,1} \vee x_{3,2} \vee x_{3,3})$$

$$\bigwedge_{p=1}^{3} \bigvee_{c=1}^{3} x_{p,c}$$

At most one person per chair:

$$\bigwedge_{p1=1}^{3} \bigwedge_{p2=p1+1}^{3} \bigwedge_{c=1}^{3} (\neg x_{p1,c} \vee \neg x_{p2,c})$$

No person is seated twice:

$$\bigwedge_{p=1}^{3} \bigwedge_{c1=1}^{3} \bigwedge_{c2=c_1+1}^{3} (\neg x_{p,c_1} \vee \neg x_{p,c_2})$$

# Example 2: Assignment of frequencies

- $n$ radio stations
- For each station assign one of $k$ transmission frequencies, $k < n$.
- $E$ – set of pairs of stations, that are too close to have the same frequency.

- Q: Can we assign to each station a frequency, such that no station pairs from $E$ have the same frequency?

# Example 2 (continued)

- Notation:
  $x_{s,f}$ = "station $s$ is assigned frequency $f$" for $1 \leq s \leq n$, $1 \leq f \leq k$
- Constraints:
  Every station is assigned at least one frequency:

$$\bigwedge_{s=1}^{n} \left( \bigvee_{f=1}^{k} x_{s,f} \right)$$

  Every station is assigned at most one frequency:

$$\bigwedge_{s=1}^{n} \bigwedge_{f1=1}^{k-1} \bigwedge_{f2=f1+1}^{k} (\neg x_{s,f1} \vee \neg x_{s,f2})$$

  Close stations are not assigned the same frequency:
  For each $(s1, s2) \in E$,

$$\bigwedge_{f=1}^{k} (\neg x_{s1,f} \vee \neg x_{s2,f})$$

# Example 3: Seminar topic assignment

- $n$ participants
- $n$ topics
- Set of preferences $E \subseteq \{1, \ldots, n\} \times \{1, \ldots, n\}$
  $(p, t) \in E$ means: participant $p$ would take topic $t$

- Q: Can we assign to each participant a topic which he/she is willing to take?

# Example 3 (continued)

- Notation: $x_{p,t}$ = "participant $p$ is assigned topic $t$"
- Constraints:

  Each participant is assigned at least one topic:

$$\bigwedge_{p=1}^{n}\left(\bigvee_{t=1}^{n} x_{p,t}\right)$$

  Each participant is assigned at most one topic:

$$\bigwedge_{p=1}^{n}\ \bigwedge_{t1=1}^{n-1}\ \bigwedge_{t2=t1+1}^{n} (\neg x_{p,t1} \vee \neg x_{p,t2})$$

redundant with first and last constraint (next slide)

  Each participant is willing to take his/her assigned topic:

$$\bigwedge_{p=1}^{n}\ \bigwedge_{(p,t)\notin E} \neg x_{p,t}$$

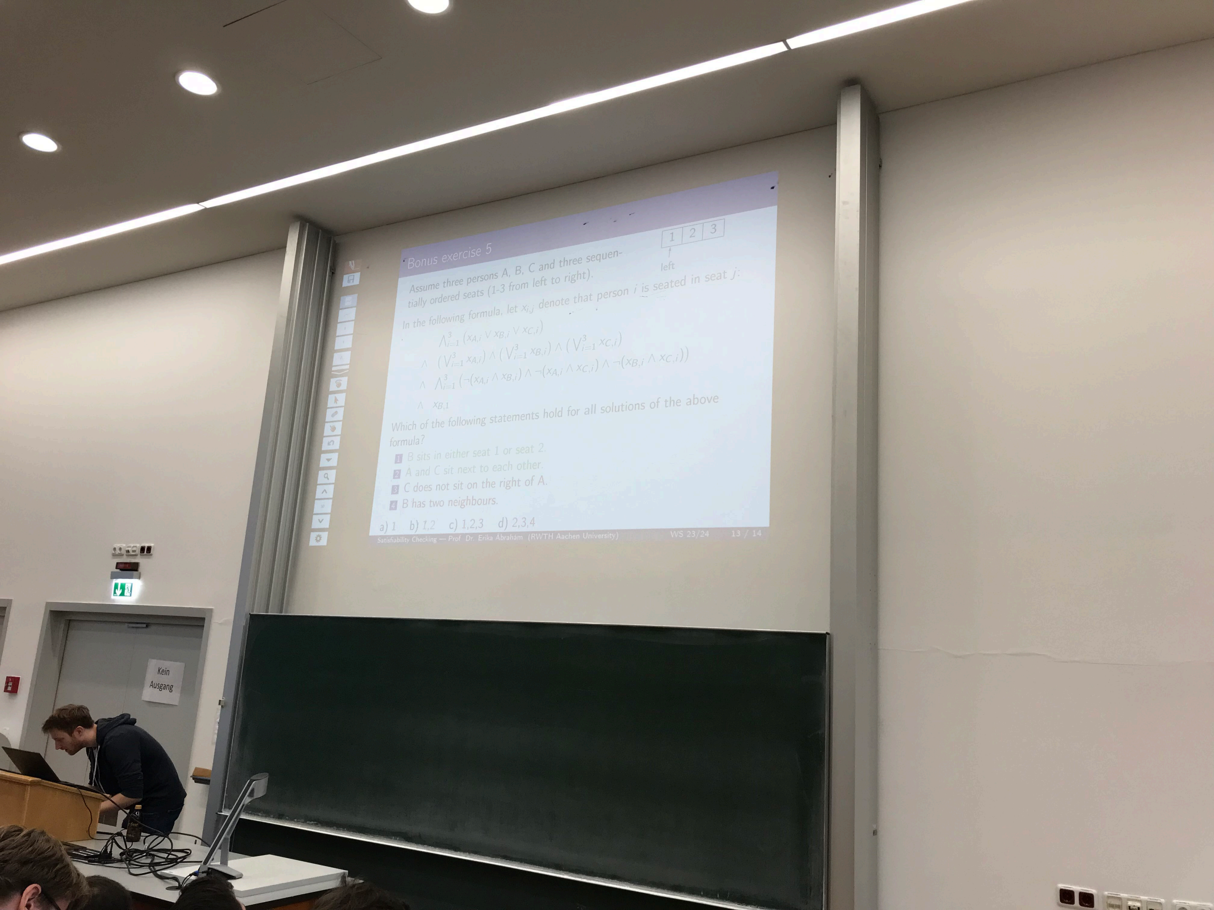
# Example 3 (continued)

Each topic is assigned to at most one participant:

$$\bigwedge_{t=1}^{n} \bigwedge_{p1=1}^{n} \bigwedge_{p2=p1+1}^{n} (\neg x_{p1,t} \vee \neg x_{p2,t})$$

# Learning target

- How to encode real world problems in propositional logic?


Bonus exercise 5
1 2 3
left
Assume three persons A, B, C and three sequentially ordered seats (1-3 from left to right).
In the following formula, let $x_{i,j}$ denote that person $i$ is seated in seat $j$:
$\bigwedge_{i=1}^{3} (x_{A,i} \vee x_{B,i} \vee x_{C,i})$
$\wedge \ (\bigvee_{i=1}^{3} x_{A,i}) \wedge (\bigvee_{i=1}^{3} x_{B,i}) \wedge (\bigvee_{i=1}^{3} x_{C,i})$
$\wedge \ \bigwedge_{i=1}^{3} (\neg(x_{A,i} \wedge x_{B,i}) \wedge \neg(x_{A,i} \wedge x_{C,i}) \wedge \neg(x_{B,i} \wedge x_{C,i}))$
$\wedge \ x_{B,1}$
Which of the following statements hold for all solutions of the above formula?
1 B sits in either seat 1 or seat 2.
2 A and C sit next to each other.
3 C does not sit on the right of A.
4 B has two neighbours.
a) 1 b) 1,2 c) 1,2,3 d) 2,3,4
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Kein Ausgang

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|---|---|
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| Grade | **0.33** out of 0.33 (**100**%) |

**Question 1**

Correct

Mark 0.33 out of 0.33

Flag question

Assume three persons A, B, C and three sequentially ordered seats (1-3 from left to right).
In the following formula, let $x_{i,j}$ denote that person $i$ is seated in seat $j$:

$$\bigwedge_{i=1}^{3} (x_{A,i} \vee x_{B,i} \vee x_{C,i})$$
$$\wedge \left(\bigvee_{i=1}^{3} x_{A,i}\right) \wedge \left(\bigvee_{i=1}^{3} x_{B,i}\right) \wedge \left(\bigvee_{i=1}^{3} x_{C,i}\right)$$
$$\wedge x_{A,1}$$

Which of the following statements hold for all solutions of the above formula?

Select one or more:

- [x] C is seated in at least one seat. ✔
- [x] A is seated in seat 1. ✔
- [ ] C is seated in either seat 1 or seat 3.
- [ ] A is seated in seat 2.

The correct answers are: C is seated in at least one seat., A is seated in seat 1.

## Quiz navigation

1 ✔

Finish review

Chat

Finish review