Satisfiability Checking - WS 2023/2024

Series 12

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Exercise 1

The solution domain of formulas in propositional logic is always finite, hence we can check the formula for satisfiability by testing all assignments. How about formulas in linear or non-linear real arithmetic? Would a procedure testing all assignments always terminate, if the formula is satisfiable?

Exercise 2

For the polynomial

$$p(x, y, z) = x^2 + 2yx - z + 1$$

please list the symbolic descriptions of its real roots in x with side conditions in all those cases, where the side conditions are not trivially false.

Exercise 3

Consider the following non-linear real arithmetic formula:

$$\varphi \quad = \quad \exists x, y.\big((\, xy - 1 = 0 \vee y - x \geq 0 \,) \wedge (\, y^2 - 1 < 0 \vee x + y + 1 > 0 \,)\big)$$

a) List the test candidates you obtain for y by the constraints of φ.

b) Apply the virtual substitution[1] of y by all test candidates of the constraint $y - x \geq 0$.

c) List all test candidates you obtain for x by the constraints of the result of part b).

d) Choose one of these test candidates, not containing a square root but an infinitesimal, and apply it to one of the resulting constraints.

e) Why can the virtual substitution method as presented in the lecture not solve all non-linear real arithmetic formulas? Could this procedure check formulas for satisfiability, where each variable occurs at most quadratic?

[1] You find the virtual substitution rules in the learning room besides the lecture slides.