

Satisfiability Checking

04 Propositional logic III

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RWTH Aachen University
Informatik 2
LuFG Theory of Hybrid Systems

WS 23/24

1 Modeling with propositional logic

Lecture example:

$(A \vee \neg B \vee C) \wedge (\neg B) \wedge (B \vee \neg C \vee D) \wedge (B \vee D)$

after elim. A : $(\neg B) \wedge (B \vee \neg C \vee D) \wedge (B \vee D)$

after elim. B : $(\neg C \vee D) \wedge (D)$

after elim. C : (D)

after elim. D : true

1. 优先使用resolution消去变量

2. 如果clause中只有a/非a, 无法使用

resolution, 则通过给变量赋值的方式消去变量

A无法使用resolution, 则赋值a为true
使用resolution消去B

赋值非c为true, c为false

赋值D为true



(VU) Satisfiability Checking / Bonus test 04

Bonus test 04**Started on** Friday, 20 October 2023, 9:00 AM**State** Finished**Completed on** Friday, 20 October 2023, 9:05 AM**Time taken** 4 mins 51 secs**Grade** 0.00 out of 0.33 (0%)**Question 1**

Incorrect

Mark 0.00 out of 0.33

Flag question

Apply resolution to eliminate the propositions in the order A, B, C and D from the following propositional logic formula in CNF:

$(\neg A \vee B \vee C) \wedge (A \vee \neg B \vee C \vee D) \wedge (A \vee B \vee \neg D) \wedge (\neg A \vee \neg B \vee C \vee D)$

How many clauses are generated during this process that differ in their literal sets from all input clauses and are not trivial (i.e., neither trivially true nor trivially false)? Please answer by writing the number using digits without whitespaces.

Answer: I'll no m) 2

The correct answer is: 2

resolution 如果式子中不是同时有a 和非a, 则不可约去a

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1. 使用resolution 除去a

 $(C \vee D) \wedge C$

2. 除去b, 没有效果

3. 除去c

使c为True

 $(C \vee D) \wedge C \Rightarrow \text{True}$

4. 除去 D, 没有效果

步骤中产生的新 non-trivial clause:

 $(C \vee D), C$

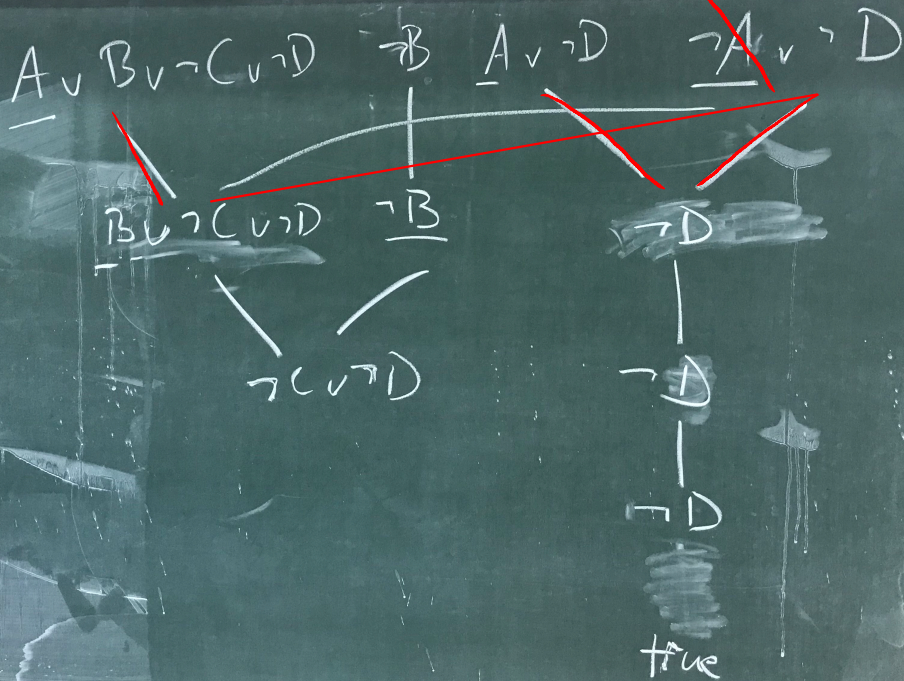
故答案为2

trivial 意为
clause 为 0 或
 $(\neg A \vee A)$
true

[Finish review](#)

resolution 不要求a和非a数量一样, 可以重复使用

Resolution 可以重复使用一个 clause



- Suppose we can solve the satisfiability problem... how can this help us?
- There are numerous problems in the industry that are solved via the satisfiability problem of propositional logic
 - Logistics
 - Planning
 - Electronic Design Automation industry
 - Cryptography
 - ...

Example 1: Placement of wedding guests

- Three chairs in a row: 1, 2, 3
- We need to place Aunt, Sister and Father.
- Constraints:
 - Aunt doesn't want to sit near Father
 - Aunt doesn't want to sit in the left chair
 - Sister doesn't want to sit to the right of Father
- Q: Can we satisfy these constraints?

Example 1 (continued)

- **Notation:** Aunt = 1, Sister = 2, Father = 3

Left chair = 1, Middle chair = 2, Right chair = 3

Introduce a propositional variable for each pair (person, chair):

$x_{p,c}$ = "person p is sited in chair c " for $1 \leq p, c \leq 3$

- **Constraints:**

Aunt doesn't want to sit near Father:

$$((x_{1,1} \vee x_{1,3}) \rightarrow \neg x_{3,2}) \wedge (x_{1,2} \rightarrow (\neg x_{3,1} \wedge \neg x_{3,3}))$$

Aunt 在 1, 3 位 \rightarrow father 不能在 2

Aunt 在 2 \rightarrow father 不能在 1, 3 位

Aunt doesn't want to sit in the left chair:

$$\neg x_{1,1}$$

Sister doesn't want to sit to the right of Father:

$$(x_{3,1} \rightarrow \neg x_{2,2}) \wedge (x_{3,2} \rightarrow \neg x_{2,3})$$

Example 1 (continued)

Each person is placed:

$$(x_{1,1} \vee x_{1,2} \vee x_{1,3}) \wedge (x_{2,1} \vee x_{2,2} \vee x_{2,3}) \wedge (x_{3,1} \vee x_{3,2} \vee x_{3,3})$$

$$\bigwedge_{p=1}^3 \bigvee_{c=1}^3 x_{p,c}$$

At most one person per chair:

$$\bigwedge_{p1=1}^{\cancel{2}^3} \bigwedge_{p2=p1+1}^3 \bigwedge_{c=1}^3 (\neg x_{p1,c} \vee \neg x_{p2,c})$$

No person is seated twice:

$$\bigwedge_{p=1}^3 \bigwedge_{c1=1}^3 \bigwedge_{c2=c1+1}^3 (\neg x_{p,c1} \vee \neg x_{p,c2})$$

Example 2: Assignment of frequencies

- n radio stations
- For each station assign one of k transmission frequencies, $k < n$.
- E – set of pairs of stations, that are too close to have the same frequency.
- **Q:** Can we assign to each station a frequency, such that no station pairs from E have the same frequency?

Example 2 (continued)

■ Notation:

$x_{s,f}$ = "station s is assigned frequency f " for $1 \leq s \leq n$, $1 \leq f \leq k$

■ Constraints:

Every station is assigned **at least** one frequency:

$$\bigwedge_{s=1}^n \left(\bigvee_{f=1}^k x_{s,f} \right)$$

at least: 最内层会有一个V
每个frequency都至少对应一个station

Every station is assigned **at most** one frequency:

$$\bigwedge_{s=1}^n \bigwedge_{f1=1}^{k-1} \bigwedge_{f2=f1+1}^k (\neg x_{s,f1} \vee \neg x_{s,f2})$$

at most: 只有^
每个station都只能有一个frequency

Close stations are not assigned the same frequency:

For each $(s1, s2) \in E$,

$$\bigwedge_{f=1}^k (\neg x_{s1,f} \vee \neg x_{s2,f})$$

Example 3: Seminar topic assignment

- n participants
- n topics
- Set of preferences $E \subseteq \{1, \dots, n\} \times \{1, \dots, n\}$
 $(p, t) \in E$ means: participant p would take topic t
- **Q:** Can we assign to each participant a topic which he/she is willing to take?

Example 3 (continued)

■ **Notation:** $x_{p,t}$ = “participant p is assigned topic t ”

■ **Constraints:**

Each participant is assigned **at least one topic:**

$$\bigwedge_{p=1}^n \left(\bigvee_{t=1}^n x_{p,t} \right)$$

at least: 最内层会有一个 \vee
每个人至少有一个 topic

Each participant is assigned **at most one topic:**

$$\bigwedge_{p=1}^n \bigwedge_{t1=1}^{n-1} \bigwedge_{t2=t1+1}^n (\neg x_{p,t1} \vee \neg x_{p,t2})$$

at most: 只有 \wedge

} redundant with first and last constraint (next slide)

Each participant is **willing to take his/her assigned topic:**

$$\bigwedge_{p=1}^n \bigwedge_{(p,t) \notin E} \neg x_{p,t}$$

Example 3 (continued)

Each topic is assigned to at most one participant:

$$\bigwedge_{t=1}^n \bigwedge_{p1=1}^{\cancel{n}^{n-1}} \bigwedge_{p2=p1+1}^n (\neg x_{p1,t} \vee \neg x_{p2,t})$$

- How to encode real world problems in propositional logic?

Bonus exercise 5

Assume three persons A, B, C and three sequentially ordered seats (1-3 from left to right).

1	2	3
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left

In the following formula, let $x_{i,j}$ denote that person i is seated in seat j :

$$\begin{aligned} & \bigwedge_{i=1}^3 (x_{A,i} \vee x_{B,i} \vee x_{C,i}) \\ & \wedge (\bigvee_{i=1}^2 x_{A,i}) \wedge (\bigvee_{i=1}^2 x_{B,i}) \wedge (\bigvee_{i=1}^2 x_{C,i}) \\ & \wedge \bigwedge_{i=1}^3 (\neg(x_{A,i} \wedge x_{B,i}) \wedge \neg(x_{A,i} \wedge x_{C,i}) \wedge \neg(x_{B,i} \wedge x_{C,i})) \\ & \wedge x_{B,1} \end{aligned}$$

Which of the following statements hold for all solutions of the above formula?

- ☒ B sits in either seat 1 or seat 2.
- ☒ A and C sit next to each other.
- ☒ C does not sit on the right of A.
- ☒ B has two neighbours.

a) 1 b) 1,2 c) 1,2,3 d) 2,3,4



大小

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守望者-电影-



下棋 Stockfis...



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sat solving...



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Bonus test 0...

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Started on Friday, 20 October 2023, 9:34 AM

State Finished

Completed on Friday, 20 October 2023, 9:40 AM

Time taken 5 mins 16 secs

Grade 0.33 out of 0.33 (100%)

Question 1

Correct

Mark 0.33 out
of 0.33Flag
question

Assume three persons A, B, C and three sequentially ordered seats (1-3 from left to right).

In the following formula, let $x_{i,j}$ denote that person i is seated in seat j :

$$\bigwedge_{i=1}^3 (x_{A,i} \vee x_{B,i} \vee x_{C,i}) \\ \wedge (\bigvee_{i=1}^3 x_{A,i}) \wedge (\bigvee_{i=1}^3 x_{B,i}) \wedge (\bigvee_{i=1}^3 x_{C,i}) \\ \wedge x_{A,1}$$

Which of the following statements hold for all solutions of the above formula?

Select one or more:

- ☒ C is seated in at least one seat. ✓
- ☒ A is seated in seat 1. ✓
- ☐ C is seated in either seat 1 or seat 3.
- ☐ A is seated in seat 2.

The correct answers are: C is seated in at least one seat., A is seated in seat 1.

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