Exercise 1

We define the famous *Pigeon Hole Problem (PHP)* over $\mathbb{N} = \mathbb{N} \setminus \{0\}$:

Given:	$n \in \mathbb{N}$.
Question:	Do $n + 1$ pigeons fit into n holes, if no two pigeons fit into one hole?

a) What is the solution to the pigeon hole problems for all $n \in \mathbb{N}$?

b) Formulate the pigeon hole problem for $n = 2$ holes (and thus 3 pigeons) in propositional logic.

c) If your formula is not already in CNF, convert it into CNF. Use resolution to deduce the empty clause.

d) Specify a preferably small unsatisfiable core of the problem, that is a subset of the clause set that is already unsatisfiable.

e) The pigeon hole problems are a "worst-case" for many SAT-solvers. Can you guess why? Substantiate your claims!

Exercise 2

a) Transfer the formula you created in Exercise 1 b) into the standard SAT input format (DIMACS[1]). Also store your result as a text file and check it for satisfiability by using MiniSat[2]

b) Download additional pigeon whole problems for $n = 6, 7, \dots$ from the Moodle room. They are already in the DIMACS format. Use MiniSat to check for satisfiability. Note the running times of each computation in a table. What do you think is the largest n whose corresponding formula can be solved within one hour? Give a reason!

c) Download the modified versions of the pigeon hole problems having n holes and $n + 2$ pigeons. How are the running times compared to the original problems?

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[1]See for example `http://www.satcompetition.org/2009/format-benchmarks2009.html`

[2]Website: `http://minisat.se/MiniSat.html`. Install it on *Debian/Ubuntu* via `sudo apt install minisat`. Verify the result and give the running time of the computation; you may use the *Windows Subsystem for Linux* to install Ubuntu on Windows.