Satisfiability Checking

01 Overview

Prof. Dr. Erika Ábrahám

RWTH Aachen University
Informatik 2
LuFG Theory of Hybrid Systems

WS 23/24

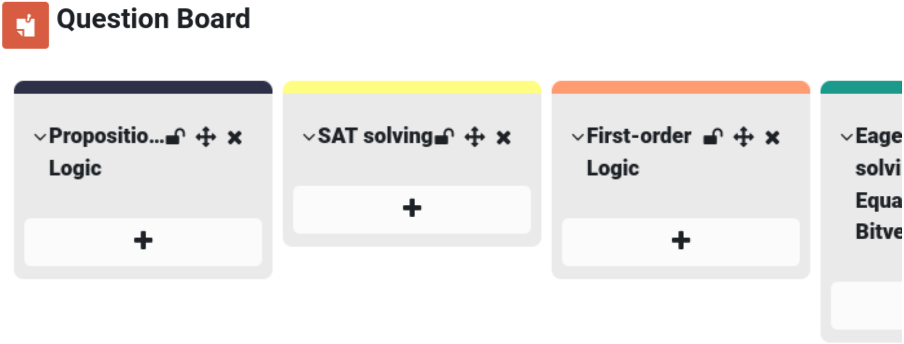
Organizational

- Language: English
- Lecture (V3+Ü1):
  Monday 08:30-10:00 (AH III) and
  Friday 08:30-10:00 (AH II)
- Exercise (Ü1):
  Tuesday 10:30-11:15 (AH VI)
- Room or schedule changes:
  communicated via Moodle
- Assistants:
  Jasper Nalbach
  Valentin Promies
- Contact:
  `teaching@ths.rwth-aachen.de`

Organizational

- Weekly exercise sheets. Not mandatory (no submission) but strongly recommended.
- 3 mandatory eTests in Moodle: at least 8 out of $3x5 = 15$ eTest points are needed for exam admission.
- Exam: written, 120 minutes, 120 exam points
- During lectures, you can earn up to 12 bonus exam points! We pose 36 questions during the lectures over the whole semester, which you can answer in Moodle. Each correct answer brings you 1/3 exam point.
  Submitting answers is possible only during the lecture!

Question board in Moodle



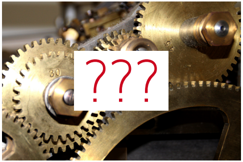
Learning materials

- Daniel Kroening and Ofer Strichman.
  Decision Procedures: An Algorithmic Point of View.
  Springer-Verlag, Berlin, 2008.
- Slides (grateful for parts from
  `www.decision-procedures.org/slides/`)
- Selected papers and other materials (especially recordings from a previous year's lecture) in Moodle

What is this lecture about?

???

Quantifier-free logical formula



Solver

???

Satisfiability of the input formula

The Boolean satisfiability problem...

Satisfiability problem for propositional logic

Given a formula combining some atomic propositions
using the Boolean operators "and" ($\wedge$), "or" ($\vee$) and "not" ($\neg$),
decide whether we can substitute truth values for the propositions
such that the formula evaluates to true.

Example

Formula:	$(a \vee \neg b) \wedge (\neg a \vee b \vee c)$
Satisfying assignment:	$a = \mathit{true}$, $b = \mathit{false}$, $c = \mathit{true}$

It is the perhaps most well-known NP-complete problem [Cook, 1971] [Levin, 1973].

...and its extension to theories

Satisfiability modulo theories problem (informal)

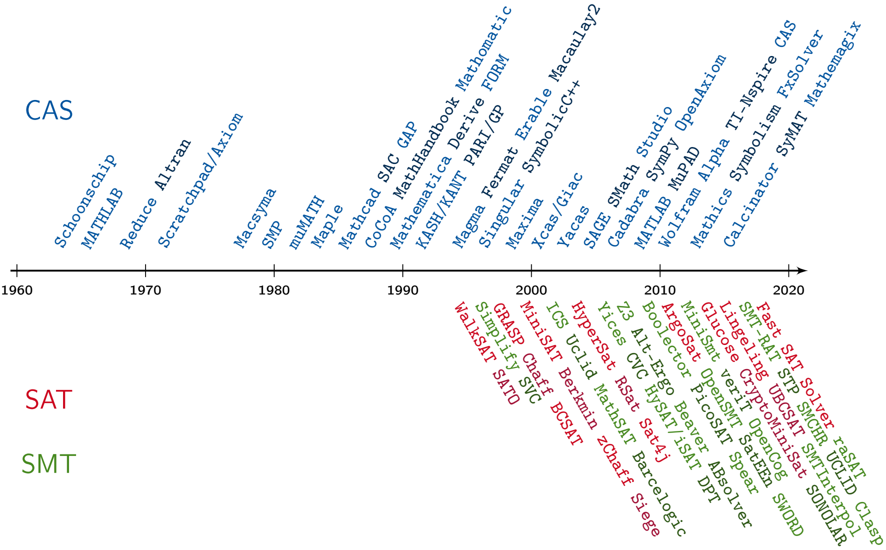
Given a Boolean combination of constraints from some theories, decide whether we can substitute (type-correct) values for the (theory) variables such that the formula evaluates to true.

A non-linear real arithmetic example

Formula: $(x - 2y > 0 \vee x^2 - 2 = 0) \wedge x^4y + 2x^2 - 4 > 0$

Satisfying assignment: $x = \sqrt{2}, \quad y = 2$

Hard problems... non-linear integer arithmetic is even undecidable.


CAS
SAT
SMT
1960
1970
1980
1990
2000
2010
2020
Schoonschip
MATHLAB
Reduce Altran
Scratchpad/Axiom
Macsyma
SMP
muMATH
Maple
Mathcad SAC GAP
CoCoA MathHandbook Mathomatic
Mathematica Derive FORM
KASH/KANT PARI/GP
Magma Fermat Erable Macaulay2
Singular SymbolicC++
Maxima
Xcas/Giac
Yacas
SAGE SMath Studio
Cadabra SymPy OpenAxiom
MATLAB MuPAD
Wolfram Alpha TI-Nspire CAS
Mathics Symbolism FxSolver
Calcinator SyMAT Mathemagix
WalkSAT SATO
Simplify SVC
GRASP Chaff BCSAT
MiniSAT Berkmin zChaff Siege
ICS Uclid MathSAT Barcelogic
HyperSat RSat Sat4j
Yices CVC HySAT/iSAT DPT
Z3 Alt-Ergo Beaver ABsolver
Boolector PicoSAT Spear
ArgoSat OpenSMT SatEEn SWORD
MiniSmt veriT OpenCog
Glucose CryptoMiniSat
Lingeling UBCSAT SMTInterpol SONOLAR
SMT-RAT STP SMCHR UCLID Clasp
Fast SAT Solver raSAT

Success story: SAT-solving

- Practical problems with millions of variables are solvable.
- Frequently used in different research areas for, e.g., analysis, synthesis and optimisation.
- Also massively used in industry for, e.g., digital circuit design and verification.

Community support:

- Standardised input language, lots of benchmarks available.
- Competitions since 2002.

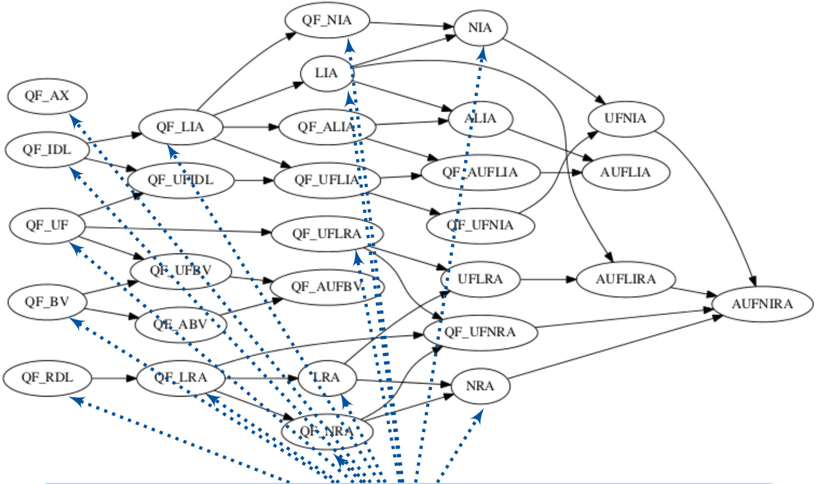
  2021: 4 tracks, 45 versions of 18 solvers in main track

  SAT Live! forum as community platform, dedicated conferences, journals, etc.

Satisfiability modulo theories solving

- Propositional logic is sometimes too weak for modelling.
- We need more expressive logics and decision procedures for them.
- Logics:
  quantifier-free fragments of first-order logic over various theories.
- Our focus: SAT-modulo-theories (SMT) solving.
- SMT-LIB as standard input language since 2004.
- Competitions since 2005.
- 2021 SMT-COMP competition: 25 solvers

SMT-LIB theories

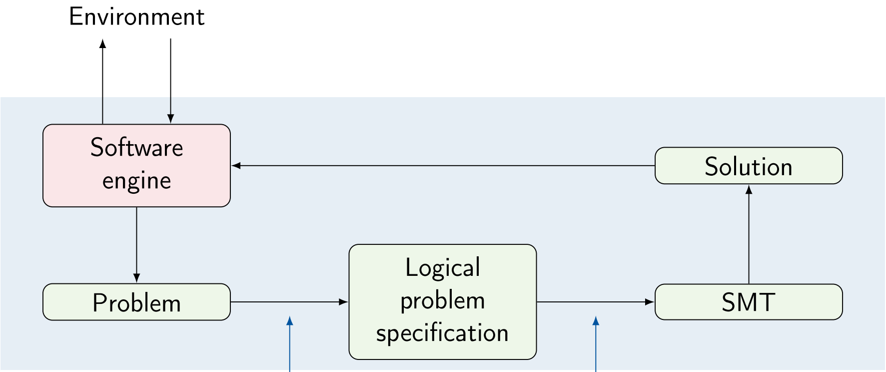


Combined theories

$$2f(x) + 5y > 0 \land \neg(f(x) = y \lor x + 2y = 0)$$

Source: `http://smtlib.cs.uiowa.edu/logics.shtml`

SAT/SMT embedding structure



Encoding: SAT/SMT-LIB standard
elaborate encoding is extremely important!

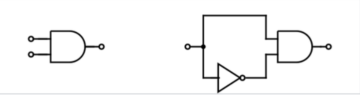
standard input language → free solver choice

Application example: Circuit satisfiability

Circuit satisfiability problem

From Wikipedia, the free encyclopedia

In theoretical computer science, the **circuit satisfiability problem** (also known as **CIRCUIT-SAT**, **CircuitSAT**, **CSAT**, etc.) is the decision problem of determining whether a given Boolean circuit has an assignment of its inputs that makes the output true.[1] In other words, it asks whether the inputs to a given Boolean circuit can be consistently set to **1** or **0** such that the circuit outputs **1**. If that is the case, the circuit is called *satisfiable*. Otherwise, the circuit is called *unsatisfiable.* In the figure to the right, the left circuit can be satisfied by setting both inputs to be **1**, but the right circuit is unsatisfiable.



The circuit on the left is satisfiable but the circuit on the right is not.

CircuitSAT is closely related to Boolean satisfiability problem (SAT), and likewise, has been proven to be NP-complete.[2] It is a prototypical NP-complete problem; the Cook–Levin theorem is sometimes proved on CircuitSAT instead of on the SAT and then reduced to the other satisfiability problems to prove their NP-completeness.[1][3] The satisfiability of a circuit containing m arbitrary binary gates can be decided in time $O(2^{0.4058m})$.[4]

Source: Wikipedia.

Application example: Symbolic execution

Program 1.2.1 A recursion-free program with bounded loops and an SSA unfolding.

```
int Main(int x, int y)
{
  if (x < y)
     x = x + y;
  for (int i = 0; i < 3; ++i) {
      y = x + Next(y);
  }
  return x + y;
}

int Next(int x) {
  return x + 1;
}
```

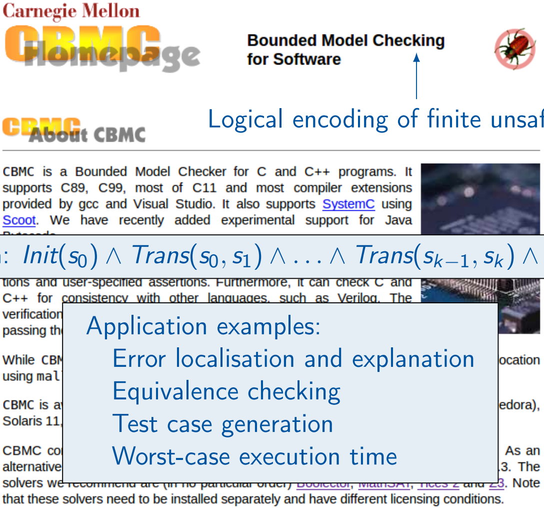
```
int Main(int x0, int y0)
{
  int x1;
  if (x0 < y0)
     x1 = x0 + y0;
  else
     x1 = x0;
  int y1 = x1 + y0 + 1;
  int y2 = x1 + y1 + 1;
  int y3 = x1 + y2 + 1;
  return x1 + y3;
}
```

$$\exists x_1, y_1, y_2, y_3 \begin{pmatrix} (x_0 < y_0 \implies x_1 = x_0 + y_0) \wedge (\neg(x_0 < y_0) \implies x_1 = x_0) \wedge \\ y_1 = x_1 + y_0 + 1 \wedge y_2 = x_1 + y_1 + 1 \wedge y_3 = x_1 + y_2 + 1 \wedge \\ result = x_1 + y_3 \end{pmatrix}$$

Source: Nikolaj Bjørner and Leonardo de Moura. *Applications of SMT solvers to Program Verification*.

Rough notes for SSFT 2014.

Application example: Bounded model checking

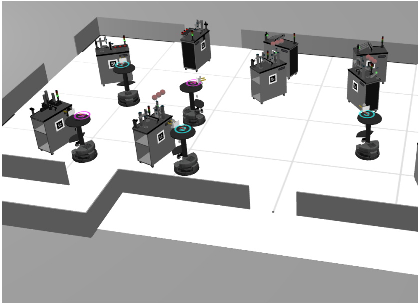


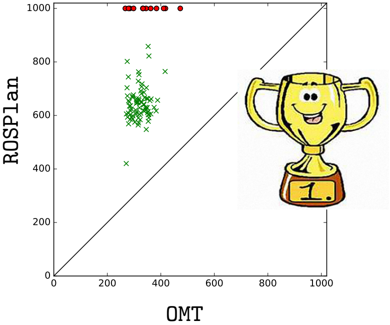
Logical encoding of finite unsafe paths

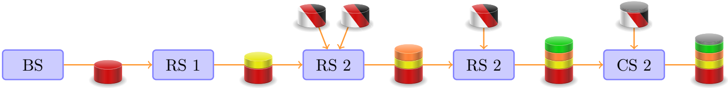
Encoding idea: $Init(s_0) \wedge Trans(s_0, s_1) \wedge \ldots \wedge Trans(s_{k-1}, s_k) \wedge Bad(s_0, \ldots, s_k)$

Application examples:
- Error localisation and explanation
- Equivalence checking
- Test case generation
- Worst-case execution time

Source: D. Kroening. **CBMC home page.** `http://www.cprover.org/cbmc/`







Source: F. Leofante, E. Giunchiglia, E. Ábrahám, A. Tacchella. **Optimal Planning Modulo Theories.** Proc. of IJCAI'20.

Application example: Security at Amazon

https://www.amazon.science/blog/a-billion-smt-queries-a-day

:arted 7 Simple Ways to Free … Career Paths in Comp…

amazon | science

Research areas Blog News and features Publications Conferences Collaborations Careers

AUTOMATED REASONING

A billion SMT queries a day

CAV keynote lecture by the director of applied science for AWS Identity explains how AWS is making the power of automated reasoning available to all customers.

By Neha Rungta

August 18, 2022

Share

Conference

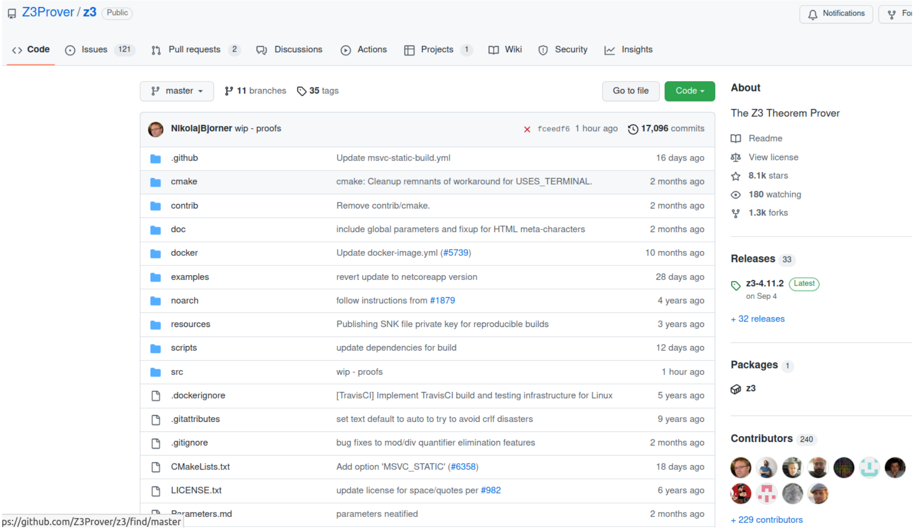
FLoC 2022

At this year's Computer-Aided Verification (CAV) conference — a leading automated-reasoning conference collocated with the Federated Logic Conferences (FLoC) — Amazon's Neha Rungta delivered a keynote talk in which she suggested that innovations at Amazon have "ushered in the golden age of automated reasoning".

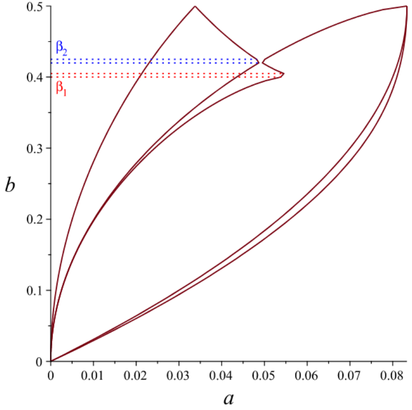
Amazon scientists and engineers are using automated reasoning to prove the

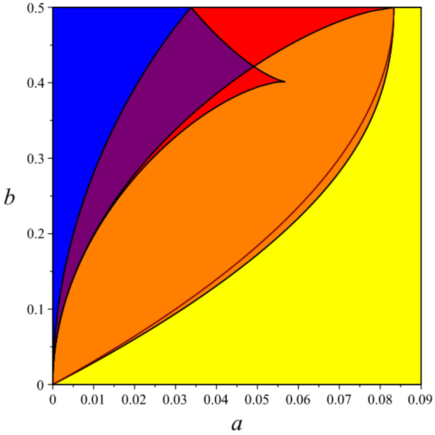
CAV 2022 Keynote - A Billion SMT …

Application example: Z3 from Microsoft



Application example: Biology





Numeric computation

Symbolic computation

Source: Röst, Gergely, and AmirHosein Sadeghimanesh. 'Exotic Bifurcations in Three Connected Populations with Allee Effect'. International Journal of Bifurcation and Chaos 31, no. 13 (October 2021): 2150202. https://doi.org/10.1142/S0218127421502023.

Learning target

- What is the satisfiability checking problem?
- How can SAT/SMT solvers be used in applications?

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