Satisfiability Checking - WS 2023/2024

Series 13

teaching@ths.rwth-aachen.de
https://ths.rwth-aachen.de/teaching/

Exercise 1

The Cylindrical Algebraic Decomposition aims at decomposing the whole solution space into *sign-invariant regions*. Each such region is represented by a single sample point.

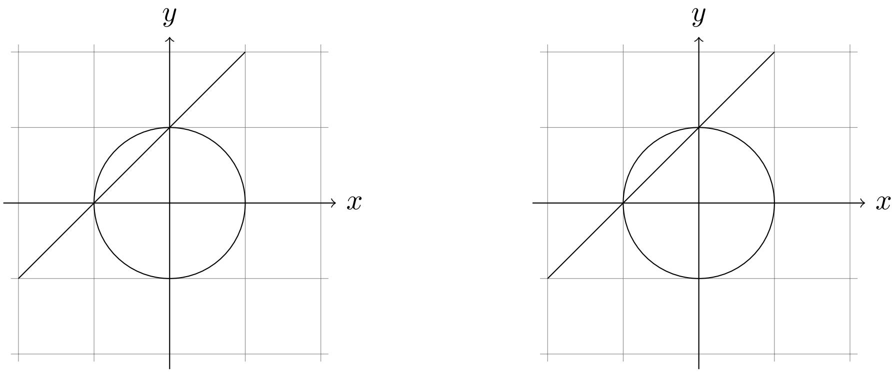
Why can you decide satisfiability using only a few sample points, although the solution space is infinitely large?

Exercise 2

1. For the polynomials $p_1(x, y) = x^2 + y^2 - 1$ and $p_2(x, y) = x - y + 1$, whose real zeros are depicted below, please give a minimal selection $S \subset \mathbb{R}^2$ of sample points such that for all real-arithmetic satisfiability problems φ that put sign conditions on these polynomials only, for example $(p_1 \le 0 \wedge p_2 \le 0) \vee (p_1 \ge 0 \wedge p_2 \ge 0)$, it holds that

$$\exists x.\exists y.\varphi \quad \Longleftrightarrow \quad \bigvee_{(v,u)\in S} \varphi[v/x][u/y] \,.$$

   You can draw the sample points as dots in the diagram.



2. Due to the way how the CAD algorithm determines the sample points, the set of sample points that will actually be used is much larger. Give a minimal set of sample points that the CAD method could generate when projecting y first for the above example, and argue why the additional sample points are included.

Exercise 3

How many cells are in the coarsest CAD, i.e. the one with the minimal number of cells, for

$$P = \{\underbrace{x - y}_{p_1},\ \underbrace{x + y}_{p_2}\}$$

when projecting y first (i.e. selecting samples for x first)?