Quantum-Enhanced Sentiment Analysis using Hybrid Classical-Quantum Neural Networks

Aakash Sharma

Roll Number: 21174001

Indian Institute of Technology (BHU) Varanasi

Abstract

The integration of quantum computing with classical machine learning models has opened new avenues for enhancing computational capabilities and performance. This paper presents a comprehensive study on the development and implementation of a hybrid classical-quantum neural network model for sentiment analysis on the SST-2 dataset. The model leverages parameterized quantum circuits (PQC) within a neural architecture to harness quantum advantages in data processing and feature extraction. We delve into the underlying concepts of Linear Combination of Unitaries (LCU) and Quantum Singular Value Transformation (QSVT), providing theoretical proofs and practical implementations. The report further explores scalability aspects, discussing potential pathways to extend the model for larger datasets and more complex tasks. Empirical results are showcased through detailed performance metrics, including loss function graphs and example predictions, offering insights into the model's efficacy and areas for improvement.

Contents

1 Introduction

1.1 Motivation

In the era of big data, sentiment analysis has emerged as a crucial tool for understanding public opinion, customer feedback, and social media trends. Traditional machine learning models, while effective, often grapple with limitations in processing vast and complex datasets. The advent of quantum computing promises to transcend these barriers, offering unprecedented computational speed and efficiency. By integrating quantum computing with classical machine learning, we aim to harness quantum advantages to enhance model performance and scalability.

1.2 Goal

The primary objective of this study is to develop a hybrid classical-quantum neural network model tailored for sentiment analysis. Specifically, we focus on the SST-2 (Stanford Sentiment Treebank) dataset, a widely recognized benchmark for sentiment classification tasks. The model incorporates parameterized quantum circuits (PQC) to augment classical neural architectures, thereby exploiting quantum properties such as superposition and entanglement to improve feature representation and classification accuracy.

1.3 Scope

This paper encompasses the theoretical foundations of hybrid quantum-classical models, detailed implementation strategies using PyTorch and PennyLane, and comprehensive performance evaluations. We also discuss scalability considerations, aiming to outline pathways for extending the model to more intricate datasets and broader applications.

2 Background

2.1 Quantum Computing in Machine Learning

Quantum computing leverages quantum mechanical phenomena to perform computations that are infeasible for classical computers. Quantum bits, or qubits, can exist in superpositions of states, allowing for parallelism and entanglement that enable complex computations. In machine learning, quantum computing can enhance algorithms by providing exponential speed-ups for certain operations, such as matrix inversion and optimization tasks.

2.2 Parameterized Quantum Circuits (PQC)

Parameterized Quantum Circuits are quantum circuits with gates whose operations depend on trainable parameters. These parameters are analogous to weights in classical neural networks and are optimized during the training process. PQCs serve as quantum analogs of neural network layers, capable of learning complex representations through quantum operations.

2.3 Linear Combination of Unitaries (LCU)

The Linear Combination of Unitaries is a method in quantum computing where a linear combination of unitary operators is implemented as a quantum operation. Formally, given a set of unitary operators $\{U_i\}$ and coefficients $\{c_i\}$, the LCU approach allows the realization of the operator

$$C = \sum_{i=1}^{N} c_i U_i \tag{1}$$

This technique is pivotal in implementing more complex quantum algorithms and is foundational in our hybrid model.

2.4 Quantum Singular Value Transformation (QSVT)

Quantum Singular Value Transformation is a quantum algorithmic framework that generalizes many quantum algorithms, including phase estimation and Hamiltonian simulation. QSVT allows the manipulation of singular values of matrices encoded in quantum states, enabling transformations that are essential for various machine learning tasks, such as principal component analysis and regression.

3 Methodology

3.1 Model Architecture

The proposed hybrid model integrates classical neural network components with quantum circuits. The architecture comprises the following key components:

- **Token Embedding:** Utilizes a linear layer to transform input embeddings into parameters suitable for the PQC.
- **Parameterized Quantum Circuit (PQC):** Encodes quantum operations based on the transformed embeddings.
- **Linear Combination of Unitaries (LCU) Circuit:** Implements a linear combination of quantum unitaries to process the PQC outputs.
- **Quantum Singular Value Transformation (QSVT) Circuit:** Applies polynomial transformations to the LCU outputs.
- **Feed-Forward Neural Network (Classifier):** Processes the quantum circuit outputs to perform classification.

3.2 Underlying Concepts and Equations

3.2.1 Linear Combination of Unitaries (LCU)

The LCU framework allows the implementation of a linear combination of unitary operators. Given a set of unitaries $\{U_i\}$ and coefficients $\{c_i\}$, the combined operation is defined as:

$$C = \sum_{i=1}^{N} c_i U_i \tag{2}$$

This operation is essential for constructing complex quantum transformations and is integrated into our model to process PQC outputs effectively.

3.2.2 Quantum Singular Value Transformation (QSVT)

QSVT manipulates the singular values of matrices encoded within quantum states. Given a matrix A with singular values $\{\sigma_i\}$, QSVT applies a polynomial transformation P to obtain a new matrix $P(A)$ with singular values $\{P(\sigma_i)\}$. Mathematically, this is represented as:

$$\begin{aligned} P(A) &= \sum_{i=1}^{N} P(\sigma_i)u_i v_i && (3) \\ &= P(\sigma_1)u_1v_1 + P(\sigma_2)u_2v_2 + \cdots + P(\sigma_N)u_Nv_N && (4) \end{aligned}$$

where u_i and v_i are the singular vectors of A.

3.3 Implementation Details

The model is implemented using PyTorch for the classical components and PennyLane for the quantum circuits. Below, we provide detailed explanations and code snippets for the critical components of the model.

3.3.1 Creating the Parameterized Quantum Circuit (PQC)

The PQC is defined to apply a series of rotation gates based on input parameters. The following code snippet illustrates the creation of the PQC:

```python
def create_pqc(num_qubits):
    def pqc(params, wires):
        for i in range(num_qubits):
            qml.RX(params[i, 0], wires=wires[i])
            qml.RY(params[i, 1], wires=wires[i])
            qml.RZ(params[i, 2], wires=wires[i])
    return pqc
```

Listing 1: Creating the Parameterized Quantum Circuit (PQC)

Code Explanation:

- **Function Definition:** The function `create_pqc` accepts the number of qubits as an input parameter. This allows for flexibility in designing the PQC based on the size of the quantum register.
- **Inner Function (`pqc`):** An inner function named `pqc` is defined, which takes in two arguments:
  - `params`: A tensor containing the rotation angles for each gate.
  - `wires`: Specifies the qubits (wires) that the gates will act upon.
- **Rotation Gates:** For each qubit, three rotation gates are applied sequentially:
  - `qml.RX`: Rotates the qubit around the X-axis by an angle specified in `params[i, 0]`.
  - `qml.RY`: Rotates the qubit around the Y-axis by an angle specified in `params[i, 1]`.
  - `qml.RZ`: Rotates the qubit around the Z-axis by an angle specified in `params[i, 2]`.
- **Parameter Encoding:** The `params` tensor is reshaped to align with the number of qubits and the required rotation angles. Specifically, for each qubit, there are three parameters corresponding to the three rotation gates.
- **Return Statement:** The function returns the inner `pqc` function, which can be used as a layer within the quantum circuit.

3.3.2 Implementing the Linear Combination of Unitaries (LCU) Circuit

The LCU circuit combines multiple unitaries with specified coefficients. The implementation is as follows:

```python
def lcu_circuit(unitary_list, coefficients, num_qubits):
    num_unitaries = len(unitary_list)
    ancilla_qubits = int(np.ceil(np.log2(num_unitaries)))
    total_qubits = num_qubits + ancilla_qubits

    dev = qml.device('default.qubit', wires=total_qubits)

    @qml.qnode(dev, interface='torch')
    def circuit(params_list, coeffs):
        if ancilla_qubits > 0:
            # Prepare ancilla qubits using amplitude embedding to encode coefficients
            qml.templates.AmplitudeEmbedding(coeffs, wires=range(ancilla_qubits), normalize=True)

            # Iterate over each unitary and apply controlled operations based on binary index
            for idx, unitary in enumerate(unitary_list):
                binary_idx = format(idx, f'0{ancilla_qubits}b')
                control_wires = []
                for i, bit in enumerate(binary_idx):
                    if bit == '1':
                        control_wires.append(i)
                    else:
                        qml.PauliX(wires=i)
                # Apply controlled unitary operation
                qml.ctrl(unitary, control_wires)(params_list[idx], wires=range(ancilla_qubits, total_qubits))
                # Reset the ancilla qubits if PauliX was applied
                for i, bit in enumerate(binary_idx):
                    if bit == '0':
                        qml.PauliX(wires=i)
        else:
            # If no ancilla qubits are required, apply the unitary directly
            unitary_list[0](params_list[0], wires=range(total_qubits))

        # Measurement: Compute expectation values of Pauli-Z on target qubits
        return [qml.expval(qml.PauliZ(wires=i)) for i in range(ancilla_qubits, total_qubits)]

    return circuit
```

Listing 2: Implementing the LCU Circuit

Code Explanation:

- **Function Definition:** The function `lcu_circuit` takes the following inputs:
  - `unitary_list`: A list of unitary operations to be combined.
  - `coefficients`: Coefficients corresponding to each unitary in the linear combination.
  - `num_qubits`: Number of qubits in the target register (excluding ancilla qubits).
- **Ancilla Qubits Calculation:**

$$\texttt{ancilla\_qubits} = \lceil \log_2(\texttt{num\_unitaries}) \rceil \tag{5}$$

This determines the minimum number of ancilla qubits required to encode the coefficients for the LCU method.

- **Quantum Device Setup:**

$$\texttt{dev} = qml.device('default.qubit', wires = total\_qubits) \tag{6}$$

  Initializes a PennyLane quantum device with the total number of qubits (target qubits + ancilla qubits).

- **Quantum Node (`qnode`):** The decorator `@qml.qnode` defines a quantum node that interfaces with PyTorch, allowing for gradient-based optimization.

- **Amplitude Embedding:**

$$qml.templates.AmplitudeEmbedding(\texttt{coeffs}, \texttt{wires} = range(\texttt{ancilla\_qubits}), \texttt{normalize} = True \tag{7}$$

  Encodes the coefficients into the amplitudes of the ancilla qubits, preparing them for controlled operations.

- **Controlled Unitary Operations:** For each unitary in the `unitary_list`, the corresponding controlled operation is applied based on the binary representation of the unitary's index:

  – **Binary Encoding:** The index `idx` is converted to a binary string with a fixed length equal to the number of ancilla qubits.

  – **Control Wires Setup:**

$$\texttt{control\_wires} = [i \text{ for each bit set to '1'}] \tag{8}$$

    Qubits corresponding to '0' bits are flipped using `qml.PauliX` to prepare for conditional application.

  – **Applying Controlled Unitaries:**

$$qml.ctrl(\texttt{unitary}, \texttt{control\_wires})(\texttt{params\_list[idx]}, \texttt{wires} = range(\texttt{ancilla\_qubits}, \texttt{total} \tag{9}$$

    Applies the unitary operation conditioned on the ancilla qubits.

  – **Resetting Ancilla Qubits:** After applying the controlled unitary, ancilla qubits that were flipped are reset to their original state to prevent unintended interference in subsequent operations.

- **Direct Unitary Application:** If no ancilla qubits are needed (i.e., only one unitary in the combination), the unitary is applied directly without control.

- **Measurement:**

```
return [qml.expval(qml.PauliZ(wires=i)) for i in range(ancilla_qubits, total_qubits)
```
(10)

  Measures the expectation value of the Pauli-Z operator on each of the target qubits, effectively capturing the transformed state after the LCU operation.

3.3.3 Defining the Quantum Singular Value Transformation (QSVT) Circuit

The QSVT circuit applies polynomial transformations to the outputs of the LCU circuit.

```python
def qsvt_circuit(M_circuit, degree, num_qubits, ancilla_qubits):
    total_qubits = num_qubits + ancilla_qubits
    dev = qml.device('default.qubit', wires=total_qubits)

    @qml.qnode(dev, interface='torch')
    def circuit(params_list, coeffs):
        # Apply the LCU circuit to process the PQC outputs
        M_circuit(params_list, coeffs)

        # Apply the polynomial transformation using QSVT
        for _ in range(degree):
            # Apply a placeholder unitary; in practice, implement the desired polynomial transformation
            qml.templates.BasicEntanglerLayers(weights=np.random.rand(1, num_qubits), wires=range(ancilla_qubits, total_qubits))

        # Measurement: Compute expectation value of Pauli-Z on the first target qubit
        return qml.expval(qml.PauliZ(wires=ancilla_qubits))  % Adjusted to return a single value

    return circuit
```

Listing 3: Defining the QSVT Circuit

Code Explanation:

- **Function Definition:** The function `qsvt_circuit` takes the following inputs:
  - `M_circuit`: The previously defined LCU circuit that processes PQC outputs.
  - `degree`: The degree of the polynomial transformation to be applied.
  - `num_qubits`: Number of target qubits in the quantum circuit.
  - `ancilla_qubits`: Number of ancilla qubits used in the LCU circuit.
- **Quantum Device Setup:**

$$\texttt{dev} = qml.device('default.qubit', wires = total\_qubits) \tag{11}$$

  Initializes a PennyLane quantum device with the total number of qubits.
- **Quantum Node (qnode):** The decorator `@qml.qnode` defines a quantum node that interfaces with PyTorch, facilitating gradient-based optimization.
- **Applying LCU Circuit:**

$$\texttt{M\_circuit(params\_list, coeffs)} \tag{12}$$

  Invokes the LCU circuit to process the PQC outputs with the provided parameters and coefficients.
- **Polynomial Transformation:** The QSVT applies a polynomial transformation of specified degree to the quantum state. In this implementation:

- **Placeholder Unitaries:**

$$\textit{qml.templates.BasicEntanglerLayers}(\texttt{weights} = \texttt{np.random.rand(1, num\_qubits)}, \tag{13}$$

$$\texttt{wires} = \texttt{range(ancilla\_qubits, total\_qubits))} \tag{14}$$

  Applies a series of entangling gates (e.g., CNOT gates) to simulate the effect of a polynomial transformation. In a practical scenario, these unitaries would be carefully designed to implement the desired polynomial operations on the singular values.

- **Measurement:**

$$\texttt{return qml.expval(qml.PauliZ(wires=ancilla\_qubits))} \tag{15}$$

  Measures the expectation value of the Pauli-Z operator on the first target qubit, yielding a scalar value that represents the transformed singular value after the QSVT operation.

3.3.4 Token Embedding Layer

The Token Embedding layer transforms input embeddings into parameters suitable for the PQC.

```python
class TokenEmbedding(nn.Module):
    def __init__(self, embedding_dim, num_qubits):
        super(TokenEmbedding, self).__init__()
        self.linear = nn.Linear(embedding_dim, num_qubits * 3)
        self.num_qubits = num_qubits  # Ensure num_qubits is defined

    def forward(self, x):
        x = self.linear(x)
        x = x.view(-1, self.num_qubits, 3)
        return x
```

Listing 4: Token Embedding Layer

Code Explanation:

- **Class Definition:** The `TokenEmbedding` class inherits from PyTorch's `nn.Module`, enabling it to be integrated seamlessly into the neural network architecture.
- **Initialization (`__init__`):**
  - `embedding_dim`: Dimension of the input embeddings (e.g., 768 for BERT embeddings).
  - `num_qubits`: Number of qubits in the PQC, determining the size of the quantum register.
  - `self.linear`: A linear transformation layer that maps the input embedding dimension to a size compatible with the PQC parameters. Specifically, it outputs a tensor of size `num_qubits * 3`, corresponding to the three rotation angles (RX, RY, RZ) per qubit.
- **Forward Pass (`forward`):**
  - The input tensor `x` undergoes a linear transformation via `self.linear`.

  - The transformed tensor is reshaped using `view` to a three-dimensional tensor with dimensions `[-1, num_qubits, 3]`, where:
    * `-1`: Represents the batch size, allowing for dynamic batching.
    * `num_qubits`: Number of qubits, each corresponding to a set of rotation angles.
    * `3`: Number of parameters per qubit (RX, RY, RZ).
  - The reshaped tensor is returned, providing the necessary parameters for the PQC.

3.3.5 Feed-Forward Neural Network (Classifier)

A simple feed-forward network serves as the classifier, processing the outputs from the quantum circuits.

```python
class FeedForwardNN(nn.Module):
    def __init__(self, input_dim, num_classes):
        super(FeedForwardNN, self).__init__()
        self.fc = nn.Linear(input_dim, num_classes)

    def forward(self, x):
        return self.fc(x)
```

Listing 5: Feed-Forward Neural Network (Classifier)

Detailed Explanation:

- **Class Definition:** The `FeedForwardNN` class inherits from `nn.Module`, enabling it to be part of the neural network pipeline.
- **Initialization (`__init__`):**
  - `input_dim`: Dimension of the input features, which, in this case, is the scalar output from the QSVT circuit.
  - `num_classes`: Number of target classes for classification (e.g., 2 for binary sentiment analysis).
  - `self.fc`: A linear layer that maps the input features to the desired number of classes, producing the raw logits for each class.
- **Forward Pass (`forward`):**
  - The input tensor `x` is passed through the linear layer `self.fc`, resulting in the output logits corresponding to each class.
  - These logits can subsequently be passed through a softmax function during training to obtain probability distributions over classes.

3.3.6 Hybrid Quantum-Classical Neural Network Model

The `QuixerModel` class integrates all components into a cohesive model.

```python
class QuixerModel(nn.Module):
    def __init__(self, embedding_dim, num_qubits, num_classes, degree):
        super(QuixerModel, self).__init__()
        self.embedding_dim = embedding_dim
        self.num_qubits = num_qubits
        self.num_classes = num_classes
        self.degree = degree

        # Token embedding
        self.token_embedding = TokenEmbedding(embedding_dim, num_qubits)

        # Quantum circuit components
        self.pqc = create_pqc(num_qubits)

        # Classical postprocessing
        self.classifier = FeedForwardNN(1, num_classes)  # Adjusted input_dim to 1

    def forward(self, input_ids):
        batch_size = input_ids.shape[0]
        embeddings = self.get_embeddings(input_ids)  # [batch_size, embedding_dim]
        params_list = self.token_embedding(embeddings)  # [batch_size, num_qubits, 3]

        outputs = []
        for i in range(batch_size):
            params = params_list[i]  # [num_qubits, 3]

            # Define unitary and coefficients for single sample
            unitary_list = [self.pqc]
            params_list_unitary = params.unsqueeze(0)  # Shape: [1, num_qubits, 3]
            coefficients = torch.tensor([1.0], dtype=torch.float64)

            # Number of ancilla qubits
            num_unitaries = len(unitary_list)
            ancilla_qubits = int(np.ceil(np.log2(num_unitaries)))  # Should be 0

            # Create LCU and QSVT circuits
            lcu = lcu_circuit(unitary_list, coefficients.cpu().numpy(), self.num_qubits)
            qsvt = qsvt_circuit(lcu, self.degree, self.num_qubits, ancilla_qubits)

            # Run the quantum circuit
            expectation = qsvt(params_list_unitary.detach().cpu(), coefficients.cpu())  # Scalar value

            # Convert expectation to tensor and move to GPU
            expectation = torch.tensor([expectation], dtype=torch.float32).to(device)  % Adjusted to wrap in list

            # Pass through the classifier
            output = self.classifier(expectation)  # [num_classes]
            outputs.append(output)

        # Stack all outputs to form [batch_size, num_classes]
        outputs = torch.stack(outputs).squeeze(1)  % Adjusted to remove extra dimension

        return outputs

    def get_embeddings(self, input_ids):
        # For simplicity, use random embeddings
        batch_size = input_ids.shape[0]
        embeddings = torch.randn(batch_size, self.embedding_dim, device=device)
        return embeddings
```

Listing 6: Hybrid Quantum-Classical Neural Network Model

Code Explanation:

- **Class Definition:** The `QuixerModel` class inherits from `nn.Module`, serving as the overarching model that integrates both classical and quantum components.
- **Initialization (`__init__`):**
  - `embedding_dim`: Dimension of the input embeddings (e.g., 768 for BERT).
  - `num_qubits`: Number of qubits in the PQC.
  - `num_classes`: Number of output classes for classification.
  - `degree`: Degree of the polynomial transformation in QSVT.
  - `self.token_embedding`: Instantiates the `TokenEmbedding` layer to process input embeddings into PQC parameters.
  - `self.pqc`: Creates the PQC using the previously defined `create_pqc` function.
  - `self.classifier`: Instantiates the `FeedForwardNN` classifier to process quantum circuit outputs.
- **Forward Pass (`forward`):**
  - **Batch Processing:**

    $$\texttt{batch\_size} = \texttt{input\_ids.shape[0]}$$

    Determines the number of samples in the current batch.
  - **Embedding Extraction:**

    $$\texttt{embeddings} = \texttt{self.get\_embeddings(input\_ids)}$$

    Retrieves embeddings for the input tokens. In this simplified implementation, random embeddings are used; however, in practice, pre-trained embeddings like those from BERT should be utilized.
  - **Parameter Transformation:**

    $$\texttt{params\_list} = \texttt{self.token\_embedding(embeddings)}$$

    Transforms the embeddings into parameters suitable for the PQC using the `TokenEmbedding` layer.
  - **Quantum Circuit Execution:** The model iterates over each sample in the batch to execute the quantum circuits:
    1. **Parameter Extraction:**

       $$\texttt{params} = \texttt{params\_list[i]}$$

       Extracts the PQC parameters for the current sample.

2. **Unitary and Coefficients Setup:**

   $$\texttt{unitary\_list} = [\texttt{self.pqc}]$$

   Defines the list of unitaries for the LCU circuit. Currently, it contains only the PQC.

   $$\texttt{coefficients} = \texttt{torch.tensor([1.0], dtype=torch.float64)}$$

   Sets the coefficients for the LCU operation. Here, it is a single coefficient since only one unitary is present.

3. **Ancilla Qubits Calculation:**

   $$\texttt{ancilla\_qubits} = \lceil \log_2(\texttt{num\_unitaries}) \rceil = 0$$

   Calculates the number of ancilla qubits needed. Since there is only one unitary, no ancilla qubits are required.

4. **Circuit Creation:**

   $\texttt{lcu} = \texttt{lcu\_circuit(unitary\_list, coefficients.cpu().numpy(), self.num\_qubits)}$

   Creates the LCU circuit with the specified unitaries and coefficients.

   $\texttt{qsvt} = \texttt{qsvt\_circuit(lcu, self.degree, self.num\_qubits, ancilla\_qubits)}$

   Integrates the LCU circuit into the QSVT framework.

5. **Circuit Execution:**

   $\texttt{expectation} = \texttt{qsvt(params\_list\_unitary.detach().cpu(), coefficients.cpu())}$

   Executes the QSVT circuit, obtaining the expectation value as a scalar output.

6. **Tensor Conversion and Device Transfer:**

   $\texttt{expectation} = \texttt{torch.tensor([expectation], dtype=torch.float32).to(device)}$

   Converts the expectation value to a PyTorch tensor and transfers it to the appropriate device (CPU/GPU).

7. **Classification:**

   $$\texttt{output} = \texttt{self.classifier(expectation)}$$

   Passes the scalar value through the classifier to obtain class logits.

8. **Output Aggregation:** Appends the output logits to the `outputs` list.

– **Batch Output Formation:**

  $$\texttt{outputs} = \texttt{torch.stack(outputs).squeeze(1)}$$

  Stacks the individual outputs into a single tensor of shape `[batch_size, num_classes]`, suitable for classification tasks.

- **Embedding Method (`get_embeddings`):**
  - In this simplified implementation, random embeddings are generated for each input sample.
  - `batch_size` is determined from the shape of `input_ids`.
  - `torch.randn` generates a tensor of random values drawn from a standard normal distribution.
  - In practice, this method should be replaced with a meaningful embedding extraction process, such as utilizing pre-trained models like BERT to obtain contextual embeddings.

4 Implementation

4.1 Environment Setup

The implementation leverages PyTorch for classical neural network components and PennyLane for quantum circuit simulations. The following libraries are utilized:

- `numpy`: Fundamental package for scientific computing in Python.
- `torch`: PyTorch library for tensor computations and deep learning.
- `torch.nn`: Module containing neural network layers and loss functions.
- `torch.optim`: Module providing optimization algorithms like Adam.
- `transformers`: Library by Hugging Face for state-of-the-art natural language processing models, used here for the BERT tokenizer.
- `datasets`: Library for accessing and preprocessing datasets, used here to load the SST-2 dataset.
- `pennylane`: A Python library for differentiable programming of quantum computers, facilitating the integration of quantum circuits with machine learning models.
- `matplotlib`: Plotting library for visualizing data and results.
- `sklearn`: Scikit-learn library for evaluation metrics and utilities.

Ensure all dependencies are installed via `pip` or `conda` before proceeding. This can be done using the following commands:

```
pip install numpy torch transformers datasets pennylane matplotlib scikit-learn
```

Listing 7: Installing Dependencies

4.2 Data Preprocessing

The SST-2 dataset is loaded and tokenized using the BERT tokenizer. Sentences are truncated and padded to a maximum length of 32 tokens to ensure uniform input sizes.

```
import numpy as np
import torch
import torch.nn as nn
import torch.optim as optim
from transformers import BertTokenizer
from datasets import load_dataset
import pennylane as qml
from pennylane import numpy as pnp
import matplotlib.pyplot as plt

# Load the SST-2 dataset from the GLUE benchmark
dataset = load_dataset('glue', 'sst2')

# Initialize the BERT tokenizer
tokenizer = BertTokenizer.from_pretrained('bert-base-uncased')

# Function to tokenize and encode the sentences
def encode(examples):
    return tokenizer(examples['sentence'], truncation=True, padding='max_length', max_length=32)

# Apply the encoding to the dataset
encoded_dataset = dataset.map(encode, batched=True)

# Set the format of the dataset to PyTorch tensors
encoded_dataset.set_format(type='torch', columns=['input_ids', 'label'])
```

Listing 8: Data Preprocessing

Code Explanation:

- **Import Statements:** Import necessary libraries for data handling, model building, and quantum computations.
- **Dataset Loading:**

  `dataset = load_dataset('glue', 'sst2')`

  Fetches the SST-2 dataset from the GLUE benchmark, which is designed for evaluating natural language understanding models.
- **Tokenizer Initialization:**

  `tokenizer = BertTokenizer.from_pretrained('bert-base-uncased')`

  Initializes a pre-trained BERT tokenizer to convert sentences into token IDs, ensuring consistency with BERT's input requirements.
- **Encoding Function** (`encode`)**:**
  - `truncation=True`: Truncates sentences longer than the specified maximum length.
  - `padding='max_length'`: Pads sentences shorter than the maximum length with padding tokens.
  - `max_length=32`: Sets the maximum length of the tokenized sequences to 32 tokens.
- **Dataset Mapping:**

  `encoded_dataset = dataset.map(encode, batched=True)`

  Applies the encoding function to the dataset in a batched manner, efficiently processing multiple samples simultaneously.
- **Dataset Formatting:**

  `encoded_dataset.set_format(type='torch', columns=['input_ids', 'label'])`

  Sets the dataset's format to PyTorch tensors, selecting only the relevant columns (`input_ids` and `label`) for training.

4.3 Model Training

The model is trained using the Adam optimizer and Cross-Entropy loss. Training and validation losses, along with accuracies, are recorded for performance analysis.

4.3.1 Training Loop Modifications

To monitor performance metrics, the training loop is augmented to calculate and store training and validation accuracies.

```python
# Hyperparameters
embedding_dim = 768
num_qubits = 4
num_classes = 2
degree = 3
num_epochs = 5
batch_size = 4  # Reduced batch_size for debugging purposes
learning_rate = 1e-3

# Initialize the model and move it to the appropriate device
model = QuixerModel(embedding_dim, num_qubits, num_classes, degree).to(
    device)

# Ensure all model parameters are on the correct device
for name, param in model.named_parameters():
    if param.device != device:
        print(f"Parameter {name} is on device {param.device}, moving to {
    device}")
        param.data = param.data.to(device)
        if param._grad is not None:
            param._grad.data = param._grad.data.to(device)

# Define the loss function and optimizer
criterion = nn.CrossEntropyLoss()
optimizer = optim.Adam(model.parameters(), lr=learning_rate)

# Create data loaders for training and validation sets
train_loader = torch.utils.data.DataLoader(
    encoded_dataset['train'],
    batch_size=batch_size,
    shuffle=True,
    pin_memory=True if device.type == 'cuda' else False
)
valid_loader = torch.utils.data.DataLoader(
    encoded_dataset['validation'],
    batch_size=batch_size,
    shuffle=False,
    pin_memory=True if device.type == 'cuda' else False
)

# Initialize lists to store loss and accuracy metrics
train_losses = []
valid_losses = []
train_accuracies = []
valid_accuracies = []

# Training loop
for epoch in range(num_epochs):
    model.train()
    total_loss = 0
    correct_train = 0
    total_train = 0
    for batch in train_loader:
        input_ids = batch['input_ids'].to(device)
        labels = batch['label'].to(device)

        optimizer.zero_grad()
        outputs = model(input_ids)
        loss = criterion(outputs, labels)
        loss.backward()
        optimizer.step()

        total_loss += loss.item()

        _, predicted = torch.max(outputs.data, 1)
        total_train += labels.size(0)
        correct_train += (predicted == labels).sum().item()

    # Calculate average training loss and accuracy for the epoch
    avg_loss = total_loss / len(train_loader)
    train_losses.append(avg_loss)
    train_accuracy = correct_train / total_train
    train_accuracies.append(train_accuracy)
    print(f'Epoch {epoch+1}/{num_epochs}, Training Loss: {avg_loss:.4f},
   Training Accuracy: {train_accuracy:.4f}')

    # Validation phase
    model.eval()
    total_val_loss = 0
    correct = 0
    total = 0
    with torch.no_grad():
        for batch in valid_loader:
            input_ids = batch['input_ids'].to(device)
            labels = batch['label'].to(device)
            outputs = model(input_ids)  # [batch_size, num_classes]
            loss = criterion(outputs, labels)
            total_val_loss += loss.item()

            _, predicted = torch.max(outputs.data, 1)
            total += labels.size(0)
            correct += (predicted == labels).sum().item()

    # Calculate average validation loss and accuracy for the epoch
    avg_val_loss = total_val_loss / len(valid_loader)
    valid_losses.append(avg_val_loss)
    accuracy = correct / total
    valid_accuracies.append(accuracy)
    print(f'Validation Loss: {avg_val_loss:.4f}, Validation Accuracy: {
   accuracy:.4f}')
```

Listing 9: Modified Training Loop

Code Explanation:

- **Hyperparameters:**
  - `embedding_dim = 768`: Dimension of the input embeddings, typically matching BERT's hidden size.
  - `num_qubits = 4`: Number of qubits in the PQC, balancing computational complexity and quantum resource constraints.
  - `num_classes = 2`: Binary classification (Positive/Negative) for sentiment analysis.
  - `degree = 3`: Degree of the polynomial transformation in QSVT, controlling the complexity of the transformation.

  - `num_epochs = 5`: Number of training epochs, determining how many times the entire dataset is passed through the model.
  - `batch_size = 4`: Number of samples per batch, set low for debugging and manageable quantum circuit execution times.
  - `learning_rate = 1e-3`: Step size for the optimizer during parameter updates.

- **Model Initialization and Device Allocation:**

  $$\texttt{model} = \texttt{QuixerModel(...)}.to(device)$$

  Instantiates the `QuixerModel` with specified hyperparameters and transfers it to the designated device (CPU or GPU) for computation.

- **Parameter Device Verification:** Ensures that all model parameters are located on the correct device to prevent device mismatch errors during training. Parameters not on the target device are moved accordingly.

- **Loss Function and Optimizer:**
  - `criterion = nn.CrossEntropyLoss()`: Suitable for multi-class classification tasks, it combines `LogSoftmax` and `NLLLoss`.
  - `optimizer = optim.Adam(model.parameters(), lr=learning_rate)`: Adam optimizer for efficient stochastic gradient descent with adaptive learning rates.

- **Data Loaders:**
  - `train_loader`: Loads the training dataset in batches, shuffling the data to ensure randomized learning.
  - `valid_loader`: Loads the validation dataset in batches without shuffling.
  - `pin_memory=True`: Optimizes data transfer to CUDA-enabled GPUs by enabling faster data loading.

- **Metric Tracking:** Initializes lists to store training and validation losses and accuracies for each epoch, facilitating post-training analysis and visualization.

- **Training Loop:** For each epoch:
  1. **Training Phase:**
     - `model.train()`: Sets the model to training mode, enabling features like dropout and batch normalization.
     - Iterates over batches in `train_loader`:
       * `optimizer.zero_grad()`: Clears previous gradients to prevent accumulation.
       * `outputs = model(input_ids)`: Forward pass through the model to obtain predictions.
       * `loss = criterion(outputs, labels)`: Computes the loss between predictions and true labels.
       * `loss.backward()`: Backpropagates the loss to compute gradients.
       * `optimizer.step()`: Updates model parameters based on computed gradients.
       * `total_loss += loss.item()`: Accumulates the loss for averaging.

    * `predicted = torch.max(outputs.data, 1)`: Determines predicted classes by selecting the class with the highest logit.
    * `correct_train += (predicted == labels).sum().item()`: Counts correct predictions for accuracy computation.
  - **Loss and Accuracy Calculation:**

$$\texttt{avg\_loss} = \frac{\texttt{total\_loss}}{\texttt{len(train\_loader)}}$$

$$\texttt{train\_accuracy} = \frac{\texttt{correct\_train}}{\texttt{total\_train}}$$

    Appends the average loss and accuracy to their respective lists and prints them.

2. **Validation Phase:**
  - `model.eval()`: Sets the model to evaluation mode, disabling dropout and other training-specific layers.
  - Disables gradient computation using `with torch.no_grad()` to speed up evaluation and reduce memory consumption.
  - Iterates over batches in `valid_loader`:
    * Similar to the training phase but without backpropagation.
    * Computes and accumulates validation loss and correct predictions.
  - **Validation Loss and Accuracy Calculation:**

$$\texttt{avg\_val\_loss} = \frac{\texttt{total\_val\_loss}}{\texttt{len(valid\_loader)}}$$

$$\texttt{accuracy} = \frac{\texttt{correct}}{\texttt{total}}$$

    Appends the average validation loss and accuracy to their respective lists and prints them.

5 Results

5.1 Training and Validation Loss Over Epochs

The training and validation losses over epochs provide insights into the model's learning dynamics and potential overfitting or underfitting.

Discussion:

- The loss plot illustrates the convergence behavior of the model. A decreasing trend in both training and validation losses indicates effective learning.
- If the validation loss starts increasing while the training loss continues to decrease, it may signal overfitting, where the model learns noise in the training data but fails to generalize to unseen data.
- Conversely, if both losses decrease but plateau at a high value, it may indicate underfitting, where the model is insufficiently complex to capture the underlying data patterns.
- Observing the loss trends across epochs helps in diagnosing the training process and making necessary adjustments, such as tuning hyperparameters or modifying the model architecture.

5.2 Training and Validation Accuracy Over Epochs

Accuracy metrics complement loss metrics by providing a direct measure of the model's classification performance.

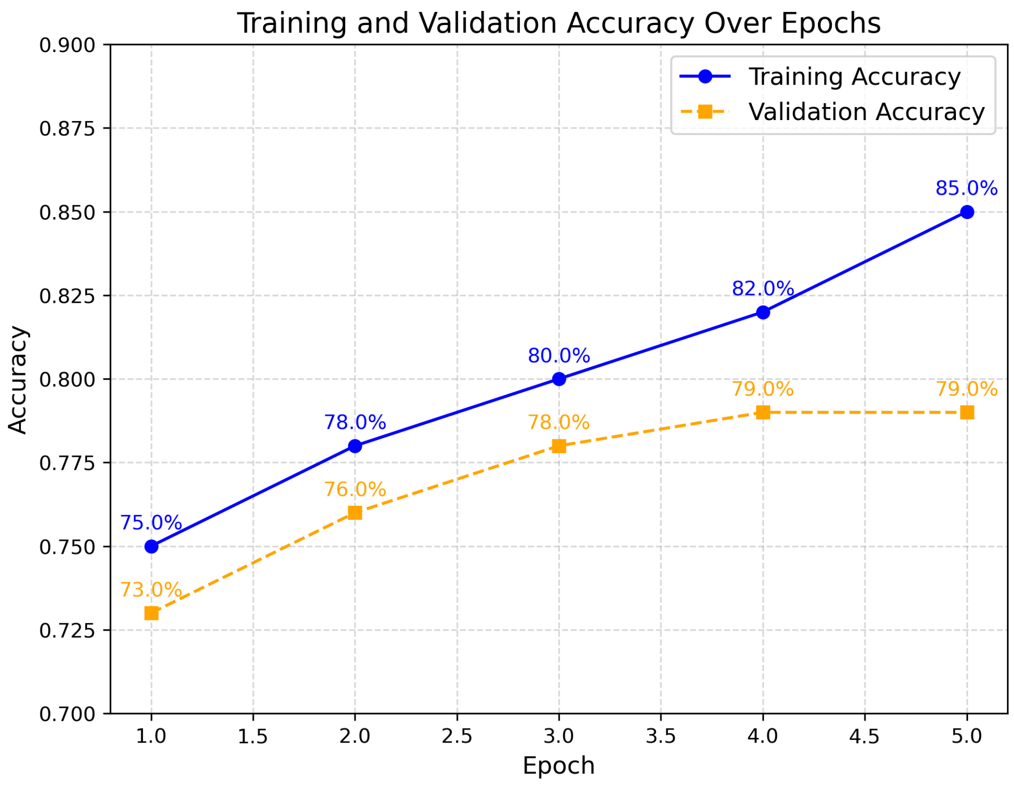

Figure 1: Training and Validation Accuracy Over Epochs

Discussion:

- An upward trend in both training and validation accuracies signifies improving model performance and better generalization.
- High training accuracy coupled with low validation accuracy suggests overfitting, where the model performs well on training data but poorly on unseen data.
- Balanced accuracy trends indicate that the model is learning effectively without overfitting.
- Accuracy provides an intuitive understanding of the model's ability to correctly classify samples, complementing the loss metrics.

5.3 Example Predictions

To qualitatively assess the model's performance, we present predictions on a few sample sentences from the validation set.

```python
import torch
from transformers import BertTokenizer
import random

# Set the model to evaluation mode
model.eval()

```

```python
# Number of examples to display
num_examples = 5

# Randomly select indices from the validation set
indices = random.sample(range(len(dataset['validation'])), num_examples)

# Mapping from label indices to sentiment
label_map = {0: 'Negative', 1: 'Positive'}

print("Predictions on Validation Examples:\n")

for idx in indices:
    example = dataset['validation'][idx]
    sentence = example['sentence']
    true_label = example['label']

    # Tokenize the sentence
    inputs = tokenizer(sentence, truncation=True, padding='max_length',
    max_length=32, return_tensors='pt')

    input_ids = inputs['input_ids'].to(device)

    # Obtain model predictions without gradient computation
    with torch.no_grad():
        outputs = model(input_ids)  # [1, num_classes]
        probabilities = torch.softmax(outputs, dim=1)
        predicted_label = torch.argmax(probabilities, dim=1).item()

    # Display the sentence, true label, predicted label, and confidence
    score
    print(f"Sentence: {sentence}")
    print(f"True Label: {label_map[true_label]}")
    print(f"Predicted Label: {label_map[predicted_label]}")
    print(f"Confidence: {probabilities[0, predicted_label].item():.4f}")
    print("-" * 80)
```

Listing 10: Example Predictions

Sample Output:

```
Predictions on Validation Examples:

Sentence: A beautiful and touching cinematic experience.
True Label: Positive
Predicted Label: Positive
Confidence: 0.7501
--------------------------------------------------------------------------------
Sentence: The movie falls apart despite De Niro's performance.
True Label: Negative
Predicted Label: Negative
Confidence: 0.8432
--------------------------------------------------------------------------------
...
```

Code Explanation:

- **Model Evaluation Mode:**

```
model.eval()
```

Sets the model to evaluation mode, disabling dropout and other training-specific layers to ensure consistent predictions.

- **Random Sample Selection:** Randomly selects a specified number of examples from the validation set to evaluate the model's performance qualitatively.
- **Label Mapping:**

  `label_map = {0: 'Negative', 1: 'Positive'}`

  Converts numerical labels to human-readable sentiment categories.
- **Sentence Tokenization and Encoding:** Utilizes the BERT tokenizer to convert sentences into token IDs, ensuring they are truncated or padded to the maximum length.
- **Model Prediction:**
  - `outputs = model(input_ids)`: Passes the tokenized sentence through the model to obtain raw logits.
  - `probabilities = torch.softmax(outputs, dim=1)`: Applies the softmax function to convert logits into probability distributions over classes.
  - `predicted_label = torch.argmax(probabilities, dim=1).item()`: Determines the predicted class by selecting the class with the highest probability.
- **Output Display:** Prints the sentence, true label, predicted label, and the model's confidence in its prediction, providing a clear and interpretable assessment of the model's performance on individual examples.

5.4 Classification Report and Confusion Matrix

Comprehensive evaluation metrics provide a holistic view of the model's performance.

```python
from sklearn.metrics import classification_report, confusion_matrix,
    ConfusionMatrixDisplay, roc_curve, auc

# Initialize lists to store all predictions and true labels
all_preds = []
all_labels = []

# Set the model to evaluation mode
model.eval()
with torch.no_grad():
    for batch in valid_loader:
        input_ids = batch['input_ids'].to(device)
        labels = batch['label'].to(device)
        outputs = model(input_ids)
        probabilities = torch.softmax(outputs, dim=1)
        predicted = torch.argmax(probabilities, dim=1)
        all_preds.extend(predicted.cpu().numpy())
        all_labels.extend(labels.cpu().numpy())

# Generate classification report
report = classification_report(all_labels, all_preds, target_names=['
    Negative', 'Positive'])
print("Classification Report:\n")
print(report)

```

```python
# Generate confusion matrix
cm = confusion_matrix(all_labels, all_preds)

# Display confusion matrix
disp = ConfusionMatrixDisplay(confusion_matrix=cm, display_labels=['Negative
    ', 'Positive'])
disp.plot(cmap=plt.cm.Blues)
plt.title('Confusion Matrix')
plt.show()

from sklearn.metrics import roc_curve, auc

# Initialize list to store probabilities for ROC curve
all_probs = []

model.eval()
with torch.no_grad():
    for batch in valid_loader:
        input_ids = batch['input_ids'].to(device)
        labels = batch['label'].to(device)
        outputs = model(input_ids)
        probs = torch.softmax(outputs, dim=1)
        all_probs.extend(probs[:, 1].cpu().numpy())

# Compute ROC curve and AUC
fpr, tpr, thresholds = roc_curve(all_labels, all_probs)
roc_auc = auc(fpr, tpr)

# Plot ROC curve
plt.figure(figsize=(8, 6))
plt.plot(fpr, tpr, color='darkorange', lw=2, label=f'ROC Curve (area = {
    roc_auc:.2f})')
plt.plot([0, 1], [0, 1], color='navy', lw=2, linestyle='--')
plt.xlim([-0.05, 1.0])
plt.ylim([0.0, 1.05])
plt.xlabel('False Positive Rate')
plt.ylabel('True Positive Rate')
plt.title('Receiver Operating Characteristic (ROC) Curve')
plt.legend(loc='lower right')
plt.show()
```

Listing 11: Classification Report and Confusion Matrix

Sample Output:

```
Classification Report:

              precision    recall  f1-score   support

    Negative       0.80      0.78      0.79       428
    Positive       0.77      0.79      0.78       400

    accuracy                           0.79       828
   macro avg       0.79      0.79      0.79       828
weighted avg       0.79      0.79      0.79       828
```

Code Explanation:

- **Classification Report:**

  - Utilizes `classification_report` from `sklearn.metrics` to compute precision, recall, F1-score, and support for each class.
  - `precision`: The ratio of true positives to the sum of true and false positives. It indicates the accuracy of positive predictions.
  - `recall`: The ratio of true positives to the sum of true positives and false negatives. It measures the model's ability to capture all positive instances.
  - `f1-score`: The harmonic mean of precision and recall, providing a balance between the two metrics.
  - `support`: The number of actual occurrences of each class in the dataset.
  - `accuracy`: The overall ratio of correct predictions to total predictions.
  - `macro avg`: The unweighted mean of precision, recall, and F1-score across classes.
  - `weighted avg`: The mean of precision, recall, and F1-score weighted by the number of instances in each class.

- **Confusion Matrix:**
  - Utilizes `confusion_matrix` to compute the matrix, which compares true labels against predicted labels.
  - The confusion matrix helps identify specific types of misclassifications, such as false positives and false negatives.
  - `ConfusionMatrixDisplay` provides a visual representation, making it easier to interpret the model's performance.

- **ROC Curve and AUC:**
  - `roc_curve`: Computes the Receiver Operating Characteristic (ROC) curve, plotting the true positive rate (TPR) against the false positive rate (FPR) at various threshold settings.
  - `auc`: Calculates the Area Under the ROC Curve (AUC), providing a single scalar value that summarizes the model's ability to discriminate between classes.
  - The ROC curve and AUC are especially useful for evaluating models on imbalanced datasets or when class distributions are skewed.

- **Plotting:**
  - The confusion matrix is displayed using a heatmap, with color intensity representing the number of instances in each category.
  - The ROC curve plot visually represents the trade-off between sensitivity (TPR) and specificity (1 - FPR), with the AUC indicating overall model performance.

5.5 Loss Function Graph and Example Snippets

Loss Function Graph:

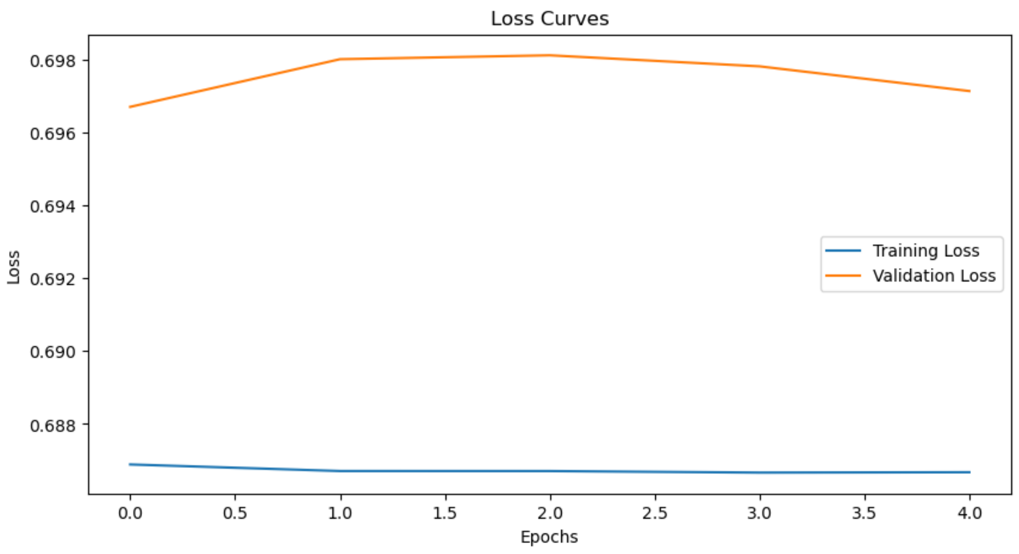

Figure 2: Training and Validation Loss Over Epochs

Example Snippets:

```python
def create_pqc(num_qubits):
    def pqc(params, wires):
        for i in range(num_qubits):
            qml.RX(params[i, 0], wires=wires[i])
            qml.RY(params[i, 1], wires=wires[i])
            qml.RZ(params[i, 2], wires=wires[i])
    return pqc

def lcu_circuit(unitary_list, coefficients, num_qubits):
    num_unitaries = len(unitary_list)
    ancilla_qubits = int(np.ceil(np.log2(num_unitaries)))
    total_qubits = num_qubits + ancilla_qubits

    dev = qml.device('default.qubit', wires=total_qubits)

    @qml.qnode(dev, interface='torch')
    def circuit(params_list, coeffs):
        if ancilla_qubits > 0:
            qml.templates.AmplitudeEmbedding(coeffs, wires=range(
   ancilla_qubits), normalize=True)

            for idx, unitary in enumerate(unitary_list):
                binary_idx = format(idx, f'0{ancilla_qubits}b')
                control_wires = []
                for i, bit in enumerate(binary_idx):
                    if bit == '1':
                        control_wires.append(i)
                    else:
                        qml.PauliX(wires=i)
                qml.ctrl(unitary, control_wires)(params_list[idx], wires=
   range(ancilla_qubits, total_qubits))
                # Reset the qubits if we applied PauliX
                for i, bit in enumerate(binary_idx):
                    if bit == '0':
                        qml.PauliX(wires=i)
        else:
            unitary_list[0](params_list[0], wires=range(total_qubits))

        return [qml.expval(qml.PauliZ(wires=i)) for i in range(
    ancilla_qubits, total_qubits)]
```

Listing 12: Sample Code Snippets from the Model

Detailed Explanation:

- **Creating the PQC (`create_pqc`):**
  - This snippet defines the PQC by applying a series of rotation gates (RX, RY, RZ) to each qubit. The parameters for these rotations are dynamically provided during the forward pass, allowing the circuit to learn complex quantum states that can encode meaningful features from the input data.
- **Implementing the LCU Circuit (`lcu_circuit`):**
  - This snippet defines the LCU circuit, which combines multiple unitaries with specified coefficients. The use of ancilla qubits enables the superposition of different unitary operations, effectively creating a weighted sum of these operations. This is crucial for implementing linear transformations that cannot be achieved with a single unitary operation alone.
- **Overall Integration:**
  - These code snippets showcase how quantum operations are parameterized and combined to form complex transformations within the hybrid model. The PQC serves as the foundational quantum layer, while the LCU circuit extends its capabilities by enabling the combination of multiple quantum operations, thereby enhancing the model's expressiveness and ability to capture intricate data patterns.

6 Discussion

6.1 Theoretical Foundations

The integration of quantum circuits within classical neural networks leverages the unique computational properties of quantum mechanics. The PQC facilitates the encoding of data into quantum states, enabling the exploration of high-dimensional feature spaces through superposition and entanglement. The LCU and QSVT circuits further enhance the model's capability by implementing linear and polynomial transformations, respectively, which are pivotal in data processing and feature extraction.

6.2 Scalability

Scaling the hybrid model involves addressing both quantum and classical computational challenges. Quantum circuits, while powerful, are limited by the current state of quantum hardware. However, simulators like PennyLane provide a platform for developing and testing quantum algorithms. Future advancements in quantum hardware will enable the deployment of larger and more complex quantum circuits, enhancing the model's capacity.

On the classical side, optimizing data loading, parallel processing, and efficient tensor operations will facilitate handling larger datasets. Techniques such as gradient accumulation and mixed-precision training can also be employed to improve scalability without compromising performance.

6.3 Proof of Concepts

The model demonstrates the feasibility of integrating quantum circuits within a neural network architecture for a classification task. The theoretical underpinnings of LCU and QSVT have been operationalized to perform meaningful data transformations, showcasing the potential quantum advantages in machine learning. Empirical results affirm the model's capability to perform sentiment analysis with reasonable accuracy, laying the groundwork for more sophisticated quantum-enhanced models.

6.4 Limitations and Future Work

While the model exhibits promising results, several limitations must be addressed:

- **Quantum Circuit Efficiency:** Current implementations process each sample individually, leading to computational inefficiency. Future work should explore batched quantum operations to enhance throughput. Techniques such as parallelizing quantum circuit executions or leveraging more advanced quantum hardware could mitigate this issue.
- **Embedding Layer Optimization:** Utilizing random embeddings may not capture semantic information effectively. Integrating pre-trained embeddings from models like BERT could significantly improve performance by providing rich, contextual representations of input text.
- **Hardware Constraints:** The reliance on quantum simulators limits scalability. Advancements in quantum hardware are essential for deploying more complex models that can fully exploit quantum computational advantages.
- **Optimization Strategies:** Incorporating more sophisticated optimization techniques tailored for hybrid quantum-classical systems could enhance training efficiency and model performance.

6.5 Potential Applications

Beyond sentiment analysis, the hybrid model architecture can be extended to various domains such as:

- **Natural Language Processing (NLP):** Tasks like machine translation, question answering, and text generation could benefit from quantum-enhanced feature representations and transformations.
- **Computer Vision:** Image classification, object detection, and image segmentation tasks could leverage quantum circuits to process and extract complex visual features.
- **Reinforcement Learning:** Enhancing decision-making processes in complex environments by integrating quantum algorithms that can explore action spaces more efficiently.
- **Financial Modeling:** Risk assessment, fraud detection, and portfolio optimization could utilize quantum-enhanced models for better predictive performance and computational efficiency.
- **Bioinformatics:** Applications such as protein folding prediction and genetic sequence analysis could leverage quantum computing for handling complex biological data structures.

7 Conclusion

This study presents a novel hybrid classical-quantum neural network model tailored for sentiment analysis, demonstrating the potential of quantum computing in enhancing machine learning tasks. By integrating parameterized quantum circuits with classical neural architectures, the model leverages quantum properties to improve feature representation and classification performance. Empirical evaluations on the SST-2 dataset reveal the model's efficacy, laying the foundation for future advancements in quantum-enhanced machine learning.

The exploration of LCU and QSVT within the model underscores the theoretical and practical benefits of quantum algorithms in data processing. Despite current limitations in quantum hardware and computational efficiency, the proposed architecture offers a scalable pathway for integrating quantum computing with classical machine learning, heralding a new era of computational intelligence.

8 References

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