Practical No. 1

1. $$\lim_{x \to a} \left[\frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \right]$$

$$\longrightarrow \lim_{x \to a} \left[\frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \times \frac{\sqrt{a+2x} + \sqrt{3x}}{\sqrt{a+2x} + \sqrt{3x}} \times \frac{\sqrt{3a+x} + 2\sqrt{x}}{\sqrt{3a+x} + 2\sqrt{x}} \right]$$

$$\lim_{x \to a} \left[\frac{(a+2x-3x)(\sqrt{3a+x} + 2\sqrt{x})}{(3a+x-4x)(\sqrt{a+2x} + \sqrt{3x})} \right]$$

$$\frac{1}{3} \lim_{x \to a} \frac{(a-x)(\sqrt{3a+x} + 2\sqrt{x})}{(a-x)(\sqrt{a+2x} + \sqrt{3x})}$$

$$\frac{1}{3} \frac{\sqrt{3a+a} + 2\sqrt{a}}{\sqrt{a+2a} + 3\sqrt{a}}$$

$$\frac{1}{3} \times \frac{\sqrt{4a} + 2\sqrt{a}}{\sqrt{3a} + \sqrt{3a}}$$

$$\frac{1}{3} \times \frac{2\sqrt{a} + 2\sqrt{a}}{\sqrt{3a} + \sqrt{3a}}$$

$$\frac{1}{3} \times \frac{\cancel{4}^{2} \cancel{\sqrt{a}}}{\cancel{2}\sqrt{3}\cancel{\sqrt{a}}}$$

$$= \frac{2}{}$$

2. $$\lim_{y \to 0} \left[\frac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \right]$$

$$\lim_{y \to 0} \left[\frac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \times \frac{\sqrt{a+y} + \sqrt{a}}{\sqrt{a+y} + \sqrt{a}} \right]$$

$$\lim_{y \to 0} \frac{a+y-a}{y\sqrt{a+y}(\sqrt{a+y} + \sqrt{a})}$$

$$\lim_{y \to 0} \frac{y}{y\sqrt{a+y}(\sqrt{a+y} + \sqrt{a})}$$

$$= \frac{1}{\sqrt{a+0}(\sqrt{a+0} + \sqrt{a})}$$

$$= \frac{1}{\sqrt{a}(\sqrt{a} + \sqrt{a})}$$

$$= \frac{1}{\sqrt{a}(2\sqrt{a})}$$

$$= \frac{1}{2a}$$

3 $\lim_{x \to \pi/6} \dfrac{\cos x - \sqrt{3}\sin x}{\pi - 6x}$

By Substituting $x - \frac{\pi}{6} = h$

$x = h + \frac{\pi}{6}$

where $h = 0$

$$\lim_{h \to 0} \frac{\cos(h + \pi/6) - \sqrt{3}\sin(h + \pi/6)}{\pi - 6(h + \pi/6)}$$

$$\lim_{h \to 0} \frac{\cos h \cdot \cos\frac{\pi}{6} - \sin h \sin\frac{\pi}{6} - \sqrt{3}\sin h \cos\frac{\pi}{6} + \cos h \sin\frac{\pi}{6}}{\pi - \cancel{6}\left(\frac{6h + \pi}{\cancel{6}}\right)}$$

$$\lim_{h \to 0} \frac{\cos h \cdot \sqrt{3}/2 - \sin h/2 - \sqrt{3}\left(\sin h \sqrt{3}/2 + \cos h \cdot 1/2\right)}{\pi - 6h + \pi}$$

$$\lim_{h \to 0} \frac{\cos\frac{\sqrt{3}h}{2} - \sin\frac{h}{2} - \sin\frac{3h}{2} - \cos\frac{\sqrt{3}h}{2}}{-6h}$$

$$\lim_{h \to 0} \frac{\cancel{-}\sin 4h/2}{\cancel{-}6h}$$

$$\lim_{h \to 0} \frac{\sin 4h}{3 \times 2h}$$

$$\frac{1}{3} \lim_{h \to 0} \frac{\sin h}{h} = \frac{1}{3} \times 1 = \frac{1}{3}$$

4. $\lim_{x \to \infty} \left[\dfrac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2-1}}\right]$

By rationalizing Numerator and Denominator

$$\lim_{x \to \infty} \left[\frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}}\right] \times \frac{\sqrt{x^2+5} + \sqrt{x^2-3}}{\sqrt{x^2+5} + \sqrt{x^2-3}} \times \frac{\sqrt{x^2+3} + \sqrt{\ }}{\sqrt{x^2+3} + \ }$$

$$\lim_{x \to \infty} \left[\frac{(x^2+5-x^2+3)(\sqrt{x^2+3} + \sqrt{x^2+1})}{(x^2+3-x^2-1)(\sqrt{x^2+5} + \sqrt{x^2-3})}\right]$$

$$\lim_{x \to \infty} \frac{8(\sqrt{x^2+3} + \sqrt{x^2+1})}{2(\sqrt{x^2+5} + \sqrt{x^2-3})}$$

$$4 \lim_{x \to \infty} \frac{\sqrt{x^2(1+3/x^2)} + \sqrt{x^2(1+1/x^2)}}{\sqrt{x^2(1+5/x^2)} + \sqrt{x^2(1-3/x^2)}}$$

After applying limit

we get,

$= 4$

5. $f(x) = \dfrac{\sin 2x}{\sqrt{1-\cos 2x}}$, for $0 < x \le \pi/2$

$= \dfrac{\cos x}{\pi - 2x}$, for $\dfrac{\pi}{2} < x < \pi$ } at $x = \dfrac{\pi}{2}$

$\rightarrow f(\pi/2) = \dfrac{\sin^2(\pi/2)}{\sqrt{1-\cos^2(\pi/2)}}$ $\therefore f(\pi/2) = 0$

f at $x = \dfrac{\pi}{2}$ define

ii $\lim\limits_{x \to \pi/2^+} f(x) = \lim\limits_{x \to \pi/2^+} \dfrac{\cos x}{\pi - 2x}$

By substituting method

$x - \dfrac{\pi}{2} = h$

$x = h + \pi/2$

where $h \to 0$

$\lim\limits_{h \to 0} \dfrac{\cos(h + \pi/2)}{\pi - 2(h + \pi/2)}$

$\lim\limits_{h \to 0} \dfrac{\cos(h + \pi/2)}{\pi - \cancel{2}\left(\frac{2h + \pi}{\cancel{2}}\right)}$

$\lim\limits_{h \to 0} \dfrac{\cos(h + \pi/2)}{-2h}$

Using $\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\lim\limits_{h \to 0} \dfrac{\cos h \cdot \cos \pi/2 - \sin h \cdot \sin \pi/2}{-2h}$

$\lim\limits_{h \to 0} \dfrac{\cos h \cdot 0 - \sin h}{-2h}$

$\lim\limits_{h \to 0} \dfrac{\cancel{-}\sin h}{\cancel{-}2h}$

$\dfrac{1}{2} \lim\limits_{h \to 0} \dfrac{\sin h}{h}$

$= \dfrac{1}{2}$

6. $\lim\limits_{x \to \pi/2^-} f(x) = \lim\limits_{x \to \pi/2^-} \dfrac{\sin 2x}{\sqrt{1-\cos 2x}}$

$\lim\limits_{x \to \pi/2^-} \dfrac{2 \sin x \cdot \cos x}{\sqrt{2 \sin^2 x}}$

$\lim\limits_{x \to \pi/2^-} \dfrac{2 \cancel{\sin x} \cos x}{\sqrt{2}\, \cancel{\sin x}}$

$\lim\limits_{x \to \pi/2^-} \dfrac{2 \cos x}{\sqrt{2}}$

$\dfrac{2}{\sqrt{2}} \lim\limits_{x \to \pi/2^-} \cos x$

L.H.L $\neq$ R.H.L

f is not continous at $x = \pi/2$

5. ii $f(x) = \begin{cases} \dfrac{x^2-9}{x-3} & 0<x<3 \\ x+3 & 3 \le x \le 6 \\ \dfrac{x^2-9}{x+3} & 6 \le x < 9 \end{cases}$ at $x=3$ & $x=6$

at $x=3$

i. $f(3) = \dfrac{x^2-9}{x-3} = 0$

f at $x=3$ define

ii $\lim\limits_{x\to 3^+} f(x) = \lim\limits_{x\to 3^+} x+3$

$f(3) = x+3 = 3+3 = 6$

f is defined at $x=3$

$\lim\limits_{x\to 3^+} f(x) = \lim\limits_{x\to 3^+} (x+3) = 6$

$\lim\limits_{x\to 3^-} f(x) = \lim\limits_{x\to 3^-} \dfrac{x^2-9}{x-3} = \dfrac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}}$

$\therefore$ L.H.L = R.H.L

$\therefore$ f is continous at $x=3$

for $x=6$

$f(6) = \dfrac{x^2-9}{x+3} = \dfrac{36-9}{6+3} = \dfrac{\cancel{27}\,3}{\cancel{9}\,1}$

$\lim\limits_{x\to 6^+} \dfrac{x^2-9}{x+3}$

$\lim\limits_{x\to 6^+} \dfrac{(x-3)\cancel{(x+3)}}{\cancel{(x+3)}}$

$\lim\limits_{x\to 6^-} (x-3) = 6-3 = 3$

$\lim\limits_{x\to 6^-} x+3 = 3+6 = 9$

L.H.L $\neq$ R.H.L

function is not continuous.

6. i $f(x) = \begin{cases} \dfrac{1-\cos 4x}{x^2} & x<0 \\ k & x=0 \end{cases}$ at $x=0$

$\rightarrow$ f is continuous at $x=0$

$\lim\limits_{x\to 0} f(x) = f(0)$

$\lim\limits_{x\to 0} \dfrac{1-\cos 4x}{x^2} = k$

$$\lim_{x\to 0} \frac{2\sin^2 2x}{x^2} = k$$

$$2\lim_{x\to 0} \frac{\sin^2 2x}{x^2} = k$$

$$2\lim_{x\to 0} \left(\frac{\sin 2x}{x}\right)^2 = k$$

$$2(2)^2 = k$$

$$k = 8$$

ii. $f(x) = (\sec^2 x)^{\cot^2 x}$ $\quad x \neq 0$ $\quad$ at $x = 0$

$= k$ $\quad x = 0$

$\rightarrow f(x) = (\sec^2 x)^{\cot^2 x}$

$$\lim_{x\to 0} (\sec^2 x)^{\cot^2 x}$$

$$\lim_{x\to 0} (1+\tan^2 x)^{1/\tan^2 x}$$

we know that

$$\lim_{x\to 0} (1+px)^{1/px} = e$$

$= e$

$\therefore k = e$

iii. $f(x) = \dfrac{\sqrt{3} - \tan x}{\pi - 3x}$ $\quad x \neq \pi/3$, $x = \pi/3$ $\Big\}$ at $x = \pi/3$

$= k$

$$x - \frac{\pi}{3} = h$$

$$x = h + \pi/3$$

where $h \to 0$

$$f(\pi/3 + h) = \frac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3(\pi/3 + h)}$$

$$\lim_{h\to 0} \frac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3(\pi/3 + h)}$$

$$\lim_{h\to 0} \frac{\dfrac{\sqrt{3} - \tan\pi/3 + \tan h}{1 - \tan\pi/3 \cdot \tan h}}{\cancel{\pi} - \cancel{\pi} - 3h}$$

$$\lim_{h\to 0} \frac{\dfrac{\sqrt{3}(1 - \tan\pi/3 \cdot \tan h) - (\sqrt{3} + \tan h)}{1 - \tan\pi/3 \cdot \tan h}}{-3h}$$

$$\lim_{h\to 0} \frac{\dfrac{(\sqrt{3} - 3\cdot\tan h) - (\sqrt{3} + \tan h)}{1 - \sqrt{3}\cdot\tan h}}{-3h}$$

$$\lim_{h\to 0} \frac{\dfrac{(\cancel{\sqrt{3}} - 3\tan h - \cancel{\sqrt{3}} - \tan h)}{1 - \sqrt{3}\cdot\tan h}}{-3h}$$

$$\lim_{h\to 0} \frac{\cancel{-}4\tan h}{\cancel{-}3h(1)3\tan h}$$

$$\lim_{h\to 0} \frac{4\tan h}{3h(1-\sqrt{3}\tan h)}$$

$$\frac{4}{3}\lim_{h\to 0}\frac{\tan h}{h}\lim_{h\to 0}\frac{1}{(1-\sqrt{3}\tan h)}$$

$$= \frac{4}{3}\,\frac{1}{1-\sqrt{3}(0)}$$

$$= \frac{4}{3}\left(\frac{1}{1}\right) = 4/3$$

7. $f(x) = \begin{cases} \dfrac{1-\cos 3x}{x\tan x} & x \neq 0 \\ 9 & x = 0 \end{cases}$ at $x = 0$

$$f(x) = \frac{1-\cos 3x}{x\tan x}$$

$$\lim_{x\to 0} \frac{2\sin^2 3/2\, x}{x\tan x}$$

$$\lim_{x\to 0} = \frac{\dfrac{2\sin^2 \frac{3x}{2}}{x^2} \times x^2}{x - \dfrac{\tan x}{x^2}\times x^2}$$

$$= 2\lim_{x\to 0}\frac{(3/2)^2}{1}$$

$$= \cancel{2}\times\frac{9}{\cancel{4}\,2}$$

$$= \frac{9}{2}$$

$$\lim_{x\to 0} f(x) = \frac{9}{2} \quad g = f(0)$$

$\therefore f$ is not continuous at $x = 0$

Redefine function

$$f(x) = \begin{cases} \dfrac{1-\cos 3x}{x\tan x} & x \neq 0 \\ \dfrac{9}{2} & x = 0 \end{cases}$$

Now $\lim_{x\to 0} f(x) = f(0)$

f has removable discontinuity at $x = 0$.

7. ii. $f(x) = \begin{cases} \dfrac{(e^{3x}-1)\sin x}{x^2} & x \neq 0 \\ \dfrac{\pi}{6} & x = 0 \end{cases}$ at $x = 0$

$$\lim_{x\to 0}\frac{(e^{3x}-1)\sin(\pi x/180)}{x^2}$$

$$\lim_{x\to 0}\frac{e^{3x}-1}{x}\quad \lim_{x\to 0}\frac{\sin(\pi x/180)}{x}$$

$$\lim_{x\to 0}\frac{3\cdot e^{3x}-1}{3x}\quad \lim_{x\to 0}\frac{\sin(\pi x/180)}{x}$$

$$3\lim_{x\to 0}\frac{e^{3x}-1}{3x}\quad \lim_{x\to 0}\frac{\sin(\pi x/180)}{x}$$

$3 \log_e \frac{\pi}{\cancel{180}\,60} = \frac{\pi}{60} = f(0)$

f is continuous at $x=0$

8. $f(x) = \frac{e^{x^2} - \cos x}{x^2}$ $x=0$

is continous at $x=0$

∴ Given,

f is continous at $x=0$

$$\lim_{x\to 0} f(x) = f(0)$$

$$= \lim_{x\to 0} \frac{e^{x^2} - \cos x}{x^2} = f(0)$$

$$= \lim_{x\to 0} \frac{e^{x^2} - \cos x - 1 + 1}{x^2}$$

$$= \lim_{x\to 0} \frac{(e^{x^2} - 1) + (1 - \cos x)}{x^2}$$

$$\lim_{x\to 0} \frac{e^{x^2} - 1}{x^2} + \lim_{x\to 0} \frac{1 - \cos x}{x^2}$$

$$= \frac{1}{2(\sqrt{2}+\sqrt{2})}$$

$$= \frac{1}{2(2\sqrt{2})}$$

$$= \frac{1}{4\sqrt{2}}$$

$$\therefore f(\pi/2) = \frac{1}{4\sqrt{2}}$$

$$\log e + \lim_{x \to 0} \frac{2\sin^2 x/2}{x^2}$$

$$\log e + 2 \lim_{x \to 0} \left(\frac{\sin x/2}{x}\right)^2$$

Multiply with 2 on Num & Denominator

$$= 1 - \cancel{2}^{1} \times \frac{1}{\cancel{4}_{2}} = \frac{3}{2} = f(0)$$

9. $f(x) = \dfrac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x}$ $\quad x \neq \pi/2$

$f(0)$ is continuous at $x = \pi/2$

$$\lim_{x \to \pi/2} \frac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x} \times \frac{\sqrt{2} + \sqrt{1+\sin x}}{\sqrt{2} + \sqrt{1+\sin x}}$$

$$\lim_{x \to \pi/2} \frac{2 - 1 + \sin x}{\cos^2 x (\sqrt{2} + \sqrt{1+\sin x})}$$

$$= \lim_{x \to \pi/2} \frac{1 + \sin x}{(1-\sin x)(1+\sin x)(\sqrt{2}+\sqrt{1+\sin x})}$$

$$= \lim_{x \to \pi/2} \frac{1}{(1-\sin x)(\sqrt{2}+\sqrt{1+\sin x})}$$

05/12/19

Practical No. 2

Derivative

2. $f(x) = 4x+1 \quad , x \le 2$
$\quad = x^2+5 \quad , x > 2$ at $x = 2$

At $x = 2$

$f(2) = 4(2)+1 = 9$

$Df(2^+) = \lim_{x \to 2^+} \frac{f(x)-f(2)}{x-2}$

$\lim_{x \to 2^+} \frac{x^2+5-9}{x-2}$

$\lim_{x \to 2^+} \frac{x^2-4}{x-2}$

$\lim_{x \to 2^+} \frac{(x+2)\cancel{(x-2)}}{\cancel{x-2}}$

$f(2^-)$

$\lim_{x \to 2^-} \frac{f(x)-f(2)}{x-2}$

$\lim_{x \to 2^-} \frac{4x+1-9}{x-2}$

$\lim_{x \to 2^-} \frac{4x-8}{x-2}$

$\lim_{x \to 2^-} \frac{4\cancel{(x-2)}}{\cancel{x-2}}$

$= 4$

f is diff at $x = 2$

3. $f(x) = 4x+7 \quad x < 3$
$\quad = x^2+3x+1 \quad x \ge 3$

$f(3) = 3^2 + 3(3)+1$
$= 9+9+1$
$= 19$

$f(3^+) = \lim_{x \to 3^+} \frac{f(x)-f(3)}{x-3}$

$\lim_{x \to 3^+} \frac{x^2+3x+1-19}{x-3}$

$\lim_{x \to 3^+} \frac{x^2+3x-18}{x-3}$

$\lim_{x \to 3^+} \frac{x^2+6x-3x-18}{x-3}$

$$\lim_{x \to 3^+} \frac{x(x+6)-3(x+6)}{x-3}$$

$$\lim_{x \to 3^+} \frac{\cancel{(x-3)}(x+6)}{\cancel{(x-3)}}$$

$$= x + 6$$

$$= 9$$

$f(3^-)$

$$\lim_{x \to 3^-} \frac{f(x)-f(3)}{x-3}$$

$$\lim_{x \to 3^-} \frac{4x+7-19}{x-3}$$

$$\lim_{x \to 3^-} \frac{4x-12}{x-3}$$

$$\lim_{x \to 3^-} \frac{4\cancel{(x-3)}}{\cancel{x-3}}$$

$$= 4$$

4. $f(x) = 8x-5 \quad , x \leq 2$

$\quad\quad = 3x^2-4x+7 \; , \; x > 2$

$$f(2) = 8(2)-5$$
$$= 16-5$$
$$= 11$$

$$f(2^+) = \frac{f(x)-f(2)}{x-2}$$

$$\lim_{x \to 2^+} = \frac{3x^2-4x+7-11}{x-2}$$

$$\lim_{x \to 2^+} = \frac{3x^2-4x-4}{x-2}$$

$$\lim_{x \to 2^+} = \frac{3x^2-6x+2x-4}{x-2}$$

$$\lim_{x \to 2^+} = \frac{3x(x-2)+2(x-2)}{(x-2)}$$

$$\lim_{x \to 2^+} = \frac{(3x+2)(x-2)}{(x-2)}$$

$$\lim_{x \to 2^+} = 3(2)+2$$

$$= 8$$

$$f(2^-) = \frac{f(x)-f(2)}{x-2}$$

$$\lim_{x\to 2^-} = \frac{8x-5-11}{x-2}$$

$$\lim_{x\to 2^-} = \frac{8x-16}{x-2}$$

$$\lim_{x\to 2^-} = \frac{8\cancel{(x-2)}}{\cancel{(x-2)}}$$

1. $$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$\lim_{h\to 0} \frac{\cot(x+h)-\cot x}{h}$$

$$\lim_{h\to 0} \frac{\frac{\cos(x+h)}{\sin(x+h)} - \frac{\cos x}{\sin x}}{h}$$

$$\lim_{h\to 0} \frac{\cancel{\sin(x)\cos(x+h)} - \cos(x)\sin(x+h)}{h\cdot\sin(x)\sin(x+h)}$$

$$\lim_{h\to 0} \frac{\sin(x-(x+h))}{h\cdot\sin x\sin(x+h)}$$

$$\lim_{h\to 0} \frac{\sin h}{h\sin x\sin(x+h)}$$

$$\lim_{h\to 0} \frac{\sin h}{h} \lim_{h\to 0} \frac{1}{\sin x\cdot\sin(x+h)}$$

$$= -1\cdot\frac{1}{\sin x\cdot\sin x}$$

$$= -\operatorname{cosec}^2 x$$

(ii) $\operatorname{cosec} x$

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$\lim_{h\to 0} \frac{\operatorname{cosec}(x+h)-\operatorname{cosec}(x)}{h}$$

$$\lim_{h\to 0} \frac{\frac{1}{\sin(x+h)} - \frac{1}{\sin x}}{h}$$

$$\lim_{h\to 0} \frac{\sin(x)-\sin(x+h)}{h\cdot\sin x\cancel{\sin(x+h)}}$$

$$\lim_{h\to 0} \frac{2\cos\left(\frac{x+x+h}{2}\right)\sin\left(\frac{x-x+h}{2}\right)}{h\cdot\sin x\sin(x+h)}$$

$$\lim_{h\to 0} 2\cos(x+h) \lim_{h\to 0} \frac{\sin(-h/2)\,1/2}{h/2} \lim_{h\to 0} \frac{1}{\sin x\sin(x+h)}$$

$=$

$$= 2\cos x \cdot \frac{1}{2} \cdot \frac{1}{\sin x \sin x}$$

$$= \frac{-1}{\sin x} \quad \frac{\cos x}{\sin x}$$

$$= \frac{-1}{\sin x} \cdot \cot x$$

$$= -\operatorname{cosec} x \cdot \cot x$$

A1
19/12/19

Practical no. 3

Topic : Application of derivatives

Q. Find the intervals in which function is increasing or decreasing

(a) $f(x) = x^3 - 5x - 11$

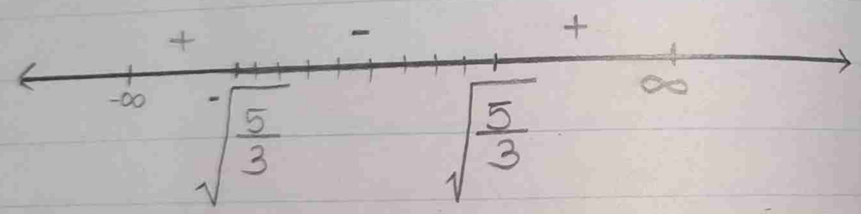
$\longrightarrow$ f is increasing if and only if $f'(x) > 0$

$\therefore f(x) = x^3 - 5x - 11$

$\therefore f'(x) = 3x^2 - 5$

$\therefore 3x^2 - 5 > 0$

$\therefore x = \pm\sqrt{\frac{5}{3}}$



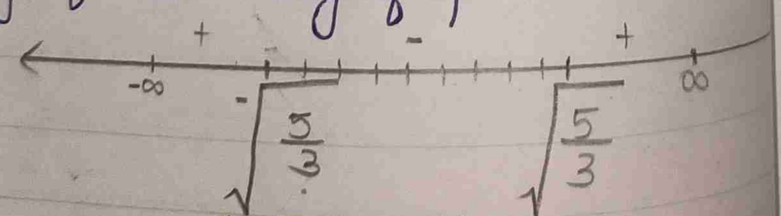
$\therefore x \in \left(-\infty, -\sqrt{\frac{5}{3}}\right) \cup \left(\sqrt{\frac{5}{3}}, \infty\right)$

Now f is decreasing if and only if $f'(x) < 0$

$f'(x) < 0$

$\therefore 3x^2 - 5 < 0$

$\therefore x = \pm\sqrt{\frac{5}{3}}$



$\therefore x \in \left(-\sqrt{\frac{5}{3}}, \sqrt{\frac{5}{3}}\right)$

b $f(x) = x^2 - 4x$

$\longrightarrow$ f is increasing if and only if $f'(x) > 0$

$\therefore f(x) = x^2 - 4x$

$\therefore f'(x) = 2x - 4$

$\therefore 2x - 4 > 0$

$\therefore 2(x-2) > 0$

$\therefore x - 2 > 0$

$\therefore x = 2$

$\therefore x \in (2, \infty)$

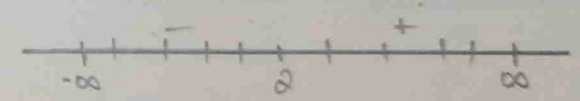
Now f is decreasing if and only if $f'(x) < 0$

$\therefore 2x - 4 < 0$

$\therefore 2(x-2) < 0$

$\therefore x - 2 < 0$

$\therefore x = 2$



$\therefore x \in (-\infty, 2)$

c $f(x) = 2x^3 + x^2 - 20x + 4$

$\longrightarrow$ f is increasing if and only if $f'(x) > 0$

$\therefore f(x) = 2x^3 + x^2 - 20x + 4$

$\therefore f'(x) = 6x^2 + 2x - 20$

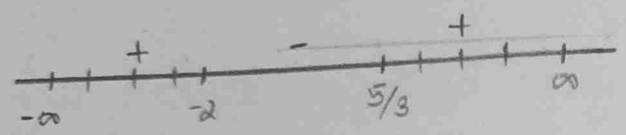
$\therefore 6x^2 + 2x - 20 > 0$

$\therefore 6x^2 + 12x - 10x - 20 > 0$

$\therefore 6x(x+2) - 10(x+2) > 0$

$\therefore (x+2)(6x-10) > 0$

$x = -2, \frac{5}{3}$



$x \in (-\infty, -2) \cup (5/3, \infty)$

Now f is decreasing if and only if $f'(x) < 0$

$\therefore 6x^2 + 2x - 20 < 0$

$\therefore (x+2)(6x-10) < 0$

$\therefore x = -2, 5/3$

$\therefore x \in \left(-2, \frac{5}{3}\right)$

d. $f(x) = x^3 - 27x + 5$

$\rightarrow$ f is increasing if and only if $f'(x) > 0$

$\therefore f(x) = x^3 - 27x + 5$

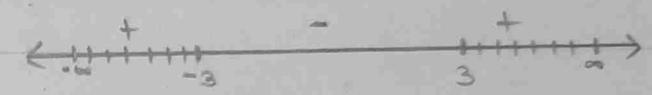
$\therefore f'(x) = 3x^2 - 27$

$\therefore 3x^2 - 27 > 0$

$\therefore 3(x^2 - 9) > 0$

$\therefore x^2 - 9 > 0$

$\therefore x = 3, -3$



$\therefore x \in (-\infty, -3) \cup (3, \infty)$

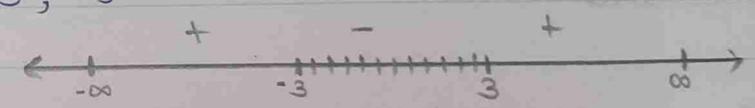
Now f is decreasing if and only if $f'(x) < 0$

$\therefore 3x^2 - 27 < 0$

$\therefore 3(x^2 - 9) < 0$

$\therefore x^2 - 9 < 0$

$\therefore x = 3, -3$



$\therefore x \in (-3, 3)$

e. $f(x) = 69 - 24x - 9x^2 + 2x^3$

$\rightarrow$ f is increasing if and only if $f'(x) > 0$

$\therefore f(x) = 69 - 24x - 9x^2 + 2x^3$

$\therefore f'(x) = -24 - 18x + 6x^2$

$\therefore -24 - 18x + 6x^2 > 0$

$\therefore 6(-4, -3x + x^2) > 0$

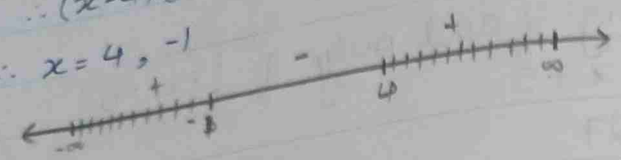
$x^2 - 3x - 4 > 0$

$x^2 - 4x + x - 4 > 0$

$x(x-4) + (x-4) > 0$

$\therefore (x-4)(x+1)>0$

$\therefore x = 4, -1$



$\therefore x \in (-\infty, -1) \cup (4, \infty)$

Now f is decreasing if and only if $f'(x)<0$

$\therefore -24 - 18x + 6x^2 < 0$

$\therefore 6(-4 - 3x + x^2) < 0$

$(x-4)(x+1) < 0$

$x = 4, -1$

$\therefore x \in (-1, 4)$

Q2. Find the interval in which function is concave upwa concave downward.

a. $y = 3x^2 - 2x^3$

Solution :- $f(x) = 3x^2 - 2x^3$

$\therefore f'(x) = 6x - 6x^2$

$\therefore f''(x) = 6 - 12x$

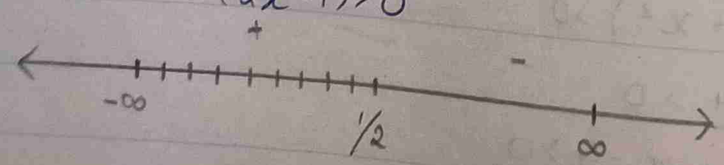
$\therefore f$ is concave upward if and only if $f''(x) > 0$

$\therefore 6 - 12x > 0$

$\therefore 6(1-2x) > 0$

$\therefore 1 - 2x > 0$

$\therefore -(2x-1) > 0$

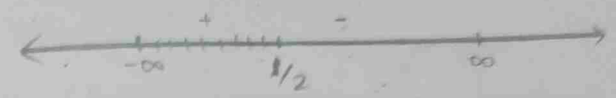


$x \in \left(-\infty, \frac{1}{2}\right)$

$\therefore f$ is concave downward if and only if $f''(x) < 0$

$\therefore 6(1-2x) < 0$

$\therefore -(2x-1) < 0$

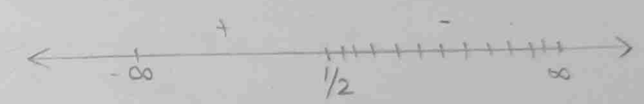


$x \in \left(\frac{1}{2}, \infty\right)$

f is concave downward if and only if $f''(x) < 0$

$\therefore 6(1-2x) < 0$

$\therefore -(2x-1) < 0$



$\therefore x \in \left(\frac{1}{2}, \infty\right)$

b. $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

→ Solution :- $\therefore y = f(x)$

$\therefore f(x) = x^4 - 6x^3 + 12x^2 + 5x + 7$

$\therefore f'(x) = 4x^3 - 18x^2 + 24x + 5$

$\therefore f''(x) = 12x^2 - 36x + 24$

f is concave upward if and only if $f''(x) > 0$

$\therefore 12x^2 - 36x + 24 > 0$

$\therefore 12(x^2 - 3x + 2) > 0$

$\therefore x^2 - 3x + 2 > 0$

$\therefore (x-2)(x-1) > 0$

$\therefore x = 2, 1$

$x \in (-\infty, 1) \cup (2, \infty)$

$\therefore f$ is concave downward if and only if $f''(x) < 0$

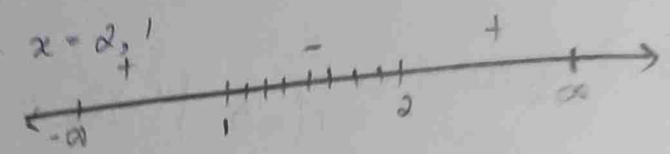
$12x^2 - 36x + 24 < 0$

$12(x^2 - 3x + 2) < 0$

$\therefore x^2 - 3x + 2 < 0$

$(x-2)(x-1) < 0$

$x = 2, 1$



$x \in (1, 2)$

c. $y = x^3 - 27x + 5$

$\longrightarrow$

$y = f(x)$

$f(x) = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

$f''(x) = 6x$

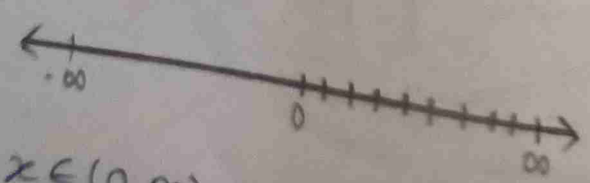
$\therefore f$ is concave if and only if

$f''(x) > 0$

$6x > 0$

$\therefore x > 0$

$x = 0$



$x \in (0, \infty)$

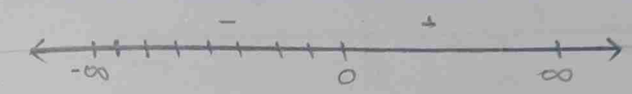
f is concave downward if and only if

$f''(x) < 0$

$6x < 0$

$x < 0$

$\therefore x = 0$



$\therefore x \in (-\infty, 0)$

d. $y = 69 - 24x - 9x^2 + 2x^3$

$\longrightarrow \therefore y = f(x)$

$\therefore f(x) = 69 - 24x - 9x^2 + 2x^3$

$\therefore f'(x) = -24 - 18x + 6x^2$

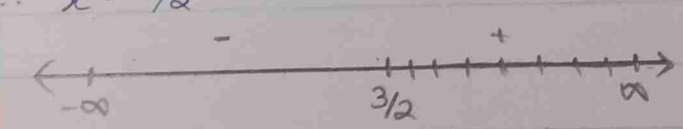
$\therefore f''(x) = -18 + 12x$

$\therefore f$ is concave upward if and only if $f''(x) > 0$

$\therefore -18 + 12x > 0$

$\therefore 6(2x - 3) > 0$

$\therefore x = 3/2$



$\therefore x \in \left(\frac{3}{2}, \infty\right)$

Practical No 4

Application of derivatives and newton's method

1. Find maximum and minimum value of following
   1. $f(x) = x^2 + \frac{16}{x^2}$
   2. $f(x) = 3 - 5x^3 + 3x^5$
   3. $f(x) = x^3 - 3x^2 + 1$ $[-1/2, 4]$
   4. $f(x) = 2x^3 - 3x^2 - 12x + 1$ $[-2, 3]$

2. Find the root of the following equation by newton (Take 4 iteration only corred upto 4 decimal)
   1. $f(x) = x^3 - 3x^2 - 55x + 95$ (take $x_0 = 0$)
   2. $f(x) = x^3 - 4x - 9$ in $[2, 3]$
   3. $f(x) = x^3 - 18x^2 - 10x + 17$ in $[1, 2]$

Solution

1. $f(x) = x^2 + \frac{16}{x^2}$

$f'(x) = 2x - 32/x^3$

Now consider, $f'(x) = 0$

$2x - 32/x^3 = 0$

$2x = 32/x^3$

$2^4 = 32/x$

$x^4 = 16$

$x = \pm 2$

$f'(x) = 2 + 96/x^4$

$f''(x) = 2 + 96/2^4$

$= 2 + 96/16$

$= 2 + 6$

$= 8 > 0$

$\therefore f$ has maximum value at $x = 2$

$f(2) = 2^2 + 16/2^2$

$= 4 + 16/4$

$= 4 + 4$

$= 8$

$f''(-2) = 2 + 96/(-2)^4$

$= 2 + 96/16$

$= 2 + 6$

$= 8 > 0$

$\therefore f$ has maximum value at $x = 2$

Function reaches minimum value at $x = 2$ and $x = -2$

2. $f(x) = 3 - 5x^3 + 3x^5$

$f'(x) = -15x^2 - 15x^4$

consider, $f'(x) = 0$

$\therefore 15x^2 + 15x^4 = 0$

$\therefore 15x^4 = 15x^2$

$\therefore x^2 = 1$

$\therefore x = \pm 1$

$f''(x) = -30x + 60x^3$

$f(1) = -30 + 60$

$= 30 > 0$ $\therefore f$ is maximum

$f(1) = 3 - 5(1)^3 + 3(1)^5$

$= \cancel{3 + 5 - 3 = 5}$ $6 - 5 = 1$

$f^n(-1) = 3 - 5(-1)^3 + 3(-1)^5$

$= 3 + 5 - 3 = 5$

$\therefore f$ has maximum value 5 at $x = -1$ and has minimum value 1 at $x = 1$

3. $f(x) = x^3 - 3x^2 + 1$

$f'(x) = 3x^2 - 6x$

consider, $f'(x) = 0$

$3x^2 - 6x = 0$

$3(x - 2) = 0$

$3x = 0$ or $x - 2 = 0$

$x = 0$ or $x = 2$

$\therefore f''(x) = 6x - 6$

$f''(0) = 6(0) - 6$

$= -6 < 0$ $\therefore f$ has miximum value of $x = 0$

$f(0) = (0)^3 - 3(0)^2 + 1 = 1$

$f''(2) = 6(2) - 6$

$= 12 - 6$

$= 6 > 0$

$\therefore f$ has minimum value at $x = 2$

$f(2) = (2)^3 - 3(2)^2 + 1$

$= 8 - 12 + 1$

$= 9 - 12 = -3$

$\therefore f$ has maximum va

at $x = 0$ & f has -3 at x

4. $f(x) = 2x^3 - 3x^2 - 12x + 1$

$\therefore f'(x) = 6x^2 - 6x - 12$

Consider, $f'(x) = 0$

$\therefore 6x^2 - 6x - 12 = 0$

$\therefore 6(x^2 - x - 12) = 0$

$x^2 - x - 2 = 0$

$x^2 + x - 2x - 2 = 0$

$\therefore x = 2$ or $x = -1$

$\therefore f''(x) = 12x - 6$

$f''(2) = 12(2) = 6$

$= 24 = 6 = 1870$

$\therefore f$ has minimum value at $x = +2$

$\therefore f(2) = 2(2)^3 - 3(2)^2 - 12(2) + 1$

$= 2(8) - 3(4) - 24 + 1$

$= 16 - 12 - 24 + 1$

$= -19$

$\therefore f''(-1) = 12(-1) - 6$

$= -12 - 6$

$= -1820$

$\therefore f$ has maximum value at $x = -1$

$\therefore f(-1) = 2(-1)^3 - 3(-1)^2 - 12(1) + 1$

$= -2 - 3 + 12 + 1$

$= 8$

$\therefore f$ has maximum value 8 at $x = -1$ f

$\therefore f$ has minimum value -19 at $x = 2$

Q2 1. $f(x) = x^3 - 3x^2 - 55x + 9.5$, $x_0 = 0$

$f'(x) = 3x^2 - 6x - 55$

By Newton method

$x_{n+1} = x_n - f(x_n) / f'(x_n)$

$x_1 = x_0 - f(x_0) / f'(x_0)$

$x_1 = 0 + 9.5/55$

$x_1 = 0.1727$

$f(x_1) = (0.1727)^3 - 3(0.1727)^2 - 55(0.1725) + 9.5$

$= 0.0051 - 0.0895 - 9.4985 + 9.5$

$= -0.0829$

$f'(x_1) = 3(0.1727)^2 - 6(0.1727) - 55$

$= 0.0895 - 1.0362 - 55$

$= -55.9467$

$\therefore x_2 = x_1 - f(x_1) / f'(x_1)$

$= 0.1727$

$f(x_2) = (0.1712)^3 - 3(0.1712)^2 - 55(0.1712) + 9.5$

$= 0.0050 - 0.0899 - 9.416 + 9.5$

$= 0.0011$

$f'(x_2) = 3(0.1712)^2 - 6(0.1712) - 55$

$= 0.0879 - 1.0272 - 55$

$= -55.9393$

$x_3 = x_2 \cancel{f(x_2)} / f'(x_2)$

$= 0.1712 + 0.0011 / 55.9393$

$= 0.1712$

$\therefore$ The root of the equation is 0.1712

2. $f(x) = x^3 - 4x - 9$

$f'(x) = 3x^2 - 4$

$f(2) = 23 - 4(2) - 9$

$= 27 - 12 - 19$

$= 6$

let $x_0 = 3$ be the initial approximation by newton method

$x_{n+1} = x_n - f(x_n)/f'(x_n)$

$x_1 = x_0 - f(x_0)/f'(x_0)$

$= 3 - 6/23$

$= 2.7392$

$f(x_1) = (2.7392)^3 - 4(2.7392) - 9$

$= 20.5528 - 10.9568 - 9$

$= 0.596$

$f'(x_1) = 3(2.7392)^2 - 4$

$= 22.5096$

$x_2 = x_1 - f(x_1)/f'(x_1)$

$= 2.7392 - 6.546/18.5096$

$= 2.7071$

$f(x_2) = (2.7071)^3 - 4(2.7071)$

$= 19.8386 - 10.8284$

$= 0.0102$

$(x_2) = 3(2.7071)^2 - 4$

$= 21.9851 - 4$

$= 17.9851$

$2.7071 - \frac{0.0102}{17.9851}$

$= 2.7071 - 0.0056 = 2.7015$

$f(x_3) = (2.7015)^3 - 4(2.7015) - 9$

$= (9.7)58 - 10.806 - 9 = -0.0901$

$f(3) = 3(2.7015)^2 - 4 = 2.18943 - 4 = 17.8943$

$x_4 = 2.7015 + 0.0901/17.8943 = 2.7015 + 0.0080$

$= 2.7065$

3. $f(x) = x^3 - 1.8x^2 - 10x + 17$

$f'(x) = 3x^2 - 3.6x - 10$

$f(1) = (1)3 - 1.8(7)^2 - 10(1) + 17$

$= -1.8 - 10 + 17$

$= 6.2$

$f(2) = (2)3 - 1.8(2)^2 - 10(2) + 17$

$= 8 - 7.2 - 20 + 17 = -2.2$

let $x_0 = 2$ be initial app. by neuton method

$x_{n+1} = x_n - f(x_n)/f'(x_n)$

$x_1 = x_0 - f(x_2)/f'(x_2)$

$= 2 - 2.2/5.2$

$= 2 - 0.4230 = 1.597$

$f(x_1) = (1.577)3 - 1.8(1.577)^2 - 10(1.577) + 17$

$= 3.9219 - 4.4784 - 15.77 + 17$

$= 0.6755$

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$f'(x) = 3(1.577)^2 - 36(1.577) - 10$

$= 2.4608 - 5.6772 - 10$

$= -8.2164$

$x_2 = x_1 - f(x_1) / f'(x_1)$

$= 1.577 + 0.022$

$= 1.6592$

$f(x_2) = 4.5677 - 4.9553 - 16.592 + 17$

$= 0.0204$

$f'(x_2) = 8.2588 - 5.97312 - 10$

$= -7.7143$

$x_3 = 1.6592 / 0.0204 / 7.7143$

$= 1.6592 + 0.0026$

$= 1.6618$

$f(x_3) = (1.6618)^3 - 1.8(1.6618)^2 - 10(1.6618) + 17$

$= 0.0004$

$f'(x_3) = 8.2847 - 5.9824 - 10$

$= -7.6977$

$x_4 = x_3 - \cancel{f(x_3)} / f'(x_3)$

$= 1.\cancel{8}618 + \frac{0.0004}{7.6977}$

$= 1.6618$

02/01/2020

Practical No. 5.

Aim: Integration

a) Solve the following integration

1. $\int \frac{1}{x^2+2x-3} dx$

$\rightarrow = \int \frac{1}{\sqrt{x^2+2x-3}} dx$

$= \int \frac{1}{\sqrt{x^2+2x+1-4}}$

$= a^2+2ab+b^2 = (a+b)^2$

$= \int \frac{1}{\sqrt{(x+1)^2-4}} dx$

using

$\# \int \frac{1}{\sqrt{x^2-a^2}} dx = \ln(|x+\sqrt{x^2-a^2}|)$

$= \ln(|t+\sqrt{t^2-4}|)$

$= \ln(|x+1\sqrt{(x+1)^2-4}|)$

$= \ln(|x+1+\sqrt{x^2+2x-3}|)$

$= \ln(|x+1+\sqrt{\quad}$

2. $\int (4e^{3x+1}) dx$

$= \int 4e^{3x} dx + \int 1 dx$

$= 4\int e^{3x} dx + \int 1 dx \qquad \# \int e^{dx} dx = 1/x\, e^{dx}$

$= \frac{4e^{3x}}{3} + x$

$= \frac{4e^{3x}}{3} + x + c$

3. $\int 2x^2 - 3\sin(x) + 5\sqrt{x}\, dx$

$= \int 2x^2 - 3\sin(x) + 5x^{1/2} dx \qquad \# \sqrt[n]{a^m} = a^{m/n}$

$= \int 2x^2 dx - \int 3\sin(x) dx + \int 5x^{1/2} dx$

$= \frac{2x^3}{3} + 3\cos x + \frac{10x\sqrt{x}}{3} + c$

$= \frac{2x^3 + \cos\sqrt{x}}{3} + 3\cos x + c$

4. $\int \frac{x^3+3x+4}{\sqrt{x}} dx$

$= \int \frac{x^3+3x+4}{x^{1/2}} dx$

$= \int \frac{x^3}{x^{1/2}} + \frac{3x}{x^{1/2}} + 4/x^{1/2} dx$

$= \int x^{5/2} + 3x^{1/2} + 4/x^{1/2} dx$

$= \int x^{5/2} dx + \int 3x^{1/2} dx + \int 4/x^{1/2} dx$

$$= \frac{x^{5/2+1}}{5/2+1}$$

$$= \frac{2x^3\sqrt{x}}{7} + 2x\sqrt{x} + 8\sqrt{x} + c$$

5. $\int t^7 \times \sin(2t^4)\,dt$

put $u = 2t^4$

$du = 2 \times 4t^3$

$$= \int t^7 \times \sin(2t^4) \times \frac{1}{2 \times 4t^3}\,du$$

$$= \int t^4 \sin(2t^4) \times \frac{1}{2 \times 4}\,du$$

$$= \int t^4 \sin(2t^4) \times \frac{1}{8}\,du = \frac{t^4 \times \sin(2t^4)}{8}\,du$$

Q2 Substitute t^4 with $u/2$

$$= \int \frac{4/2 \times \sin(u)}{8}\,du$$

a

$$= \int \frac{4 \times \sin(u)}{2} / 8\,du$$

$$= \int \frac{4 \times \sin(u)}{16}\,du$$

$$= \frac{1}{16} \int 4 \times \sin(u)\,du$$

$$= \frac{1}{16}\left(4 \times (-\cos(u)) - \int -\cos(u)\,dx\right)$$

$$= \frac{1}{16} \times \left(4 \times (-\cos u)\right) + \int \cos(u)\,dx$$

$$\int \cos x\,dx = \sin(x)$$

$$= \frac{1}{16} \times \left(4 \times (-\cos(u)) + \sin(u)\right)$$

Return the subsitution $u = 2t^4$

$$= \frac{1}{16} \times \left(2t^4 \times (-\cos(2+u)) + \sin(2t^4)\right)$$

$$= \frac{-t^4 \times \cos(2t^4)}{8} + \frac{\sin(2+u)}{16} + c$$

6. $\int \sqrt{x}\,(x^2 - 1)\,dx$

$$= \int \sqrt{x}\; x^2 - \sqrt{x}\,dx$$

$$= \int x^{1/2} \times x^2 - x^{1/2}\,dx$$

$$= \int x^{5/2} - x^{1/2}\,dx$$

$$= \int x^{5/2}\,dx - \int x^{1/5}\,dx$$

$$= I_1 = \frac{x^{5/2+1}}{5/2+1} = \frac{x^{7/2}}{7/2} = \frac{2x^{7/2}}{7} = \frac{2\sqrt{x^7}}{7} = \frac{2x^3\sqrt{x}}{7}$$

$$= I_2 = \frac{x^{1/2+1}}{1/2+1} = \frac{x^{3/2}}{3/2} = \frac{2x^{3/2}}{3/2} = \frac{2\sqrt{x^3}}{3}$$

$$= \frac{2x^3\sqrt{x}}{7} + \frac{2\sqrt{x^3}}{3} + c$$

$$\int \frac{\cos x}{3\sqrt{\sin(x)^2}}\, dx$$

$$= \int \frac{\cos x}{\sin x^{2/3}}\, dx$$

put

$t = \sin(x)$

$t = \cos x$

$$= \int \frac{\cos(x)}{\sin(x)^{3/2}} \times \frac{1}{\cos(x)}\, dt$$

$$= \frac{1}{\sin x^{3/2}}\, dt$$

$$= \frac{1}{t^{2/3}}\, dt$$

$$I = \int \frac{1}{t^{2/3}}\, dt = \frac{-1}{(2/3^{-1})\, t^{2/3-1}} = \frac{-1}{-1/3\, t^{2/3-1}}$$

$$= \frac{1}{1/3\, t^{-1/3}} = \frac{t^{1/3}}{1/3} = 3t^{1/3}$$

$$= 3\sqrt[3]{\sin(x)} + c$$

$$\int \frac{x^2 - 2x}{x^3 - 3x^2 + 1}\, dx$$

put $x^3 - 3x^2 + 1 = dt$

$$I = \int \frac{x^2 - 2x}{x^3 - 3x^2 + 1} \times \frac{1}{3x^2 - 3 \times 2x}\, dt$$

$$= \int \frac{x^2 - 2x}{x^3 - 3x^2 + 1} \times \frac{1}{3x^2 - 6x}\, dt$$

$$= \int \frac{x^2 - 2x}{x^3 - 3x^2 + 1} \times \frac{1}{3(x^2 - 2x)}\, dt$$

$$= \int \frac{1}{x^3 - 3x^2 + 1} \times 1/3\, dt$$

$$= \int \frac{1}{x^2(x^3 - 3x^2 + 1)}\, dt = \int 1/3\, dt$$

$$= 1/3 \int 1/t\, dt \qquad \int 1/x\, dx = \ln(x)$$

$$= 1/3 \times \ln|t| + c$$

$$= 1/3 \times \ln(|x^3 - 3x^2 + 1|) + c$$

Practical No. 6

1. $V = \sqrt{4 - n^2}$

$$\therefore \frac{dy}{dx} = \frac{1}{2\sqrt{4-n^2}} \times (-2n)$$

$$= \frac{-x}{\sqrt{4-x^2}}$$

$$l = \int_{0}^{2} \sqrt{1 + \left(\frac{dy}{dx}\right)^{x}}\, dx$$

$$= \int \sqrt{1 + \frac{x^2}{4-x^2}}\, dx$$

$$= \int_{-2}^{2} \sqrt{\frac{4}{4-x^2}}\, dx$$

Q2

$$= \int_{2}^{2} \sqrt{\frac{1}{\sqrt{2^2 - x^2}}}\, dx$$

a.

$$= 2\left[\sin^{-1}\left(\frac{x}{2}\right)\right]^{2}$$

$$= 2\left[\sin^{-1}(1) \cdot \sin^{-1}(-1)\right]$$

$$= 2\left[\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right]$$

$$= L = 2\pi$$

2. $y = n^{3/2} \quad x \in [0,4]$

$$\frac{dy}{dx} = \frac{3}{2} x^{1/2}$$

$$2 = \int_{0}^{1} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$= \int_{0}^{4} \sqrt{1 + \frac{9}{4}x}\, dx$$

$$= \frac{1}{2}\int_{0}^{4} \sqrt{4 + 9x}\, dx$$

$$= \frac{1}{2}\left[\frac{(4+9x)^{3/2}}{3/2} \times \frac{1}{9}\right]$$

$$= \frac{1}{27}\left[(4+ax)^{3/2}\right]$$

$$= \frac{-1}{27}\left[(4+0)^{3/2} - (4+36)^{3/2}\right]$$

$$= \frac{1}{27}\left[40^{3/8} - 8\right] \text{ unit}$$

3. $x = 3n \sin t \quad y = 3\cos t$

$\frac{dx}{dt} \quad 3\cos t \quad \frac{dy}{dt} = -3\sin t$

$$L = \int_0^{2n} \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dn}{dt}\right)^2}\, dt$$

$$= \int_0^{2n} \sqrt{(3\cos t)^2 + (3\sin t)^2}\, dt$$

$$= \int_0^{2} \sqrt{9\sin^2 + 9\cos^2 t}\, dt$$

$$= \int_0^{2} 3\sqrt{1}\, dt$$

$$= 3\int_0^{2n} dt$$

$$= 3\,[x]^{2n}$$

$$= 3\,(2x - 0)$$

$L = 6n$ units.

$$n = \frac{1}{6}y^3 + \frac{1}{2y}$$

$$\therefore \frac{dx}{dy} = \frac{y^2}{2} = \frac{-1}{2y^2}$$

$$\frac{dx}{dy} = \frac{y^4 - 1}{2y^2}$$

$$L = \int_0^{2} \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$= \int_0^{2} \sqrt{1 + \left(\frac{y^3 - 1}{4y^2}\right)^2}\, dy$$

$$= \int_0^{2} \sqrt{\frac{(y^6 - 1) + 4 \times y^3 \times 1}{4y}}\, dy$$

$$= \int_1^{2} \sqrt{\frac{(y^4 + 1)}{(2y)^2}}\, dy$$

$$= \int_1^{2} \frac{y^4 + 1}{2y^2}\, dy$$

$$= \frac{1}{2}\int_1^{2} y^2\, dy + \frac{1}{2}\int_1^{2} y^{-2}\, dy$$

$$= \frac{1}{2}\left[\frac{y}{3} - \frac{y^{-2}}{1}\right]^2$$

$$= \frac{1}{2}\left[\frac{8}{3} - \frac{1}{2} - \frac{1}{3} + 1\right]$$

$$= \frac{1}{2}\left[\frac{7}{3} + \frac{1}{2}\right]$$

$$= \frac{1}{2}\left[\frac{17}{6}\right]$$

$$L = \frac{17}{12} \text{ units}$$

Q2. i. $\int_0^2 e^x dx$ with $n=4$

$$L = \frac{b-a}{n} = \frac{2-0}{4} = 0.5$$

x	0	0.5	1	1.5	2
y	1	1.284	2.7183	9.5877	54.5982
	y	y_1	y_2	y_3	y_4

$$= \frac{0.5}{3}\left[(1+(54.5482)+5(1.284+9.4877)+2\times 2.7883)\right]$$

$$= \frac{0.5}{3}\left[55.5982 + 43.868 + 5.436\right]$$

$$= \int_0^2 e^{x^2} dx = 17.3535$$

ii. $\int_0^4 x^2 dx$ $\quad a=4 \quad h = \frac{4-0}{4} = 1$

x	0	1	2	3	4
y	0	1	4	9	16
	y_0	y_1	y_2	y_3	y_4

$$\int_0^4 x^2 dx = \frac{h}{3}\left[(y_0+y_0) + 4(y_1+y_3) + 2^{1/2}\right]$$

$$= \frac{1}{3}\left[16 + 4(10) + 8\right]$$

$$= \frac{64}{3}$$

$$\int_0^4 x^2 dx = 21.3333$$

(iii) $\int_0^{2/3} \sqrt{\sin dx}$

$$= 0$$

$$h = \frac{\pi/3 - 0}{6} = \frac{\pi}{18}$$

0	$\frac{\pi}{18}$	$\frac{2\pi}{18}$	$\frac{3\pi}{18}$	$\frac{4\pi}{18}$	$\frac{5\pi}{18}$
0	0.4167	0.5848	0.7071	0.8753	0.4306
40	y_1	y_2	y_3	y_4	y_5

$$\int_0^2 \sqrt{\sin x\, dx} = \frac{L}{3}\left[y_0 + y_6 + y(x_1 + y_3 + y_5) + 2(y_2 + y_6)\right]$$

$$= \pi/54\left[1.34737 + 7.9967 + 2.773\right]$$

$$= \pi/34 \times 12.1163$$

$$= \int_0^{\pi/3} \sqrt{\sin dx} = 0.7049$$

Practical No :- 7

Topic : Differential Equation

1. $x\frac{dy}{dx} + y = e^x$

$\frac{dy}{dx} + \frac{1}{x}y = \frac{e^x}{x}$

$P(x) = \frac{1}{x}$ $\quad Q(x) = \frac{e^x}{x}$

$I.F. = e^{\int 1/(x)\, dx}$

$= e^{\int 1/(x)\, dx}$

$= e^{\ln x}$

$I.F = x$

$y(I.F) = \int Q(x)(I.F)\, dx + c$

$= \int \frac{e^x}{x} \cdot x \cdot dx + c$

$= \int \int e^x dx + 1$

$xy = e^x + c$

Q2. $e^x \frac{dy}{dx} + 2e^x y = 1$

$\frac{dy}{dx} + \frac{2e^x}{e^x} y = \frac{1}{e^x}$ $\quad (\div \text{ by } e^x)$

$\frac{dy}{dx} + 2y = \frac{1}{e^x}$

$\frac{dy}{dx} + 2y = e^{-x}$

$P(x) = 2$ $\quad Q(x) = e^{-x}$

$\int P(x)\, dx$

$I.F = e^{\int 2\, dx}$

$= e^{2x}$

$y(I.F) = \int Q(x)(I.F)\, dx + c$

$y \cdot e^{2x} \int e^{-x} + 2x\, dx + c$

$= \int e^x dx + c$

$y \cdot e^{2x} = e^x + c$

3. $x \frac{dy}{dx} = \frac{\cos x}{x} - 2y$

$x \frac{dy}{dx} = \frac{\cos x}{x} - 2y$

$\therefore \frac{dy}{dx} + \frac{2y}{x} = \frac{\cos x}{x^2}$

$P(x) = 2(x) \quad Q(x) = \frac{\cos x}{x^2}$

$I.F = e^{\int P(x)\,dx}$

$= e^{\int 2/x\,dx}$

$= e^{2/x} \; x$

$= \ln x^2$

$I.F = x^2$

$y(I.F) = \int Q(x)\,(I.F)\,dx + c$

$= \int \frac{\cos x}{x^2} \cdot x^2\,dx + c$

$= \int \cos x + c$

$x^2 y = \sin x + c$

4. $x \frac{dy}{dx} + 3y = \frac{\sin x}{x^2}$

$\frac{dy}{dx} + \frac{3y}{x} = \frac{\sin x}{x^3}$

$P(x) = 3/x \quad Q(x) = \sin x / x^3$

$= e^{\int P(x)\,dx}$

$= e^{\int B(x)\,dx}$

$e^{3/x} \; x$

$= e^{\ln x^3}$

$I.F = x^3$

$y(I.F) = \int Q(x)\,(I.F)\,dx + c$

$= \int \frac{\sin x}{x^3} \cdot x^3\,dx + c$

$= \int \sin x\,dx + c$

$x^3 y = -\cos x + c$

✓ $e^{2x} \frac{dy}{dx} + 2e^{2x} y = 2x$

$\frac{dy}{dx} + 2y = \frac{2x}{e^{2x}}$

$P(x) = 2 \quad Q(x) = 2x/e^{2x} = 2x\,e^{-2x}$

$I.F = e^{\int P(x)\,dx}$

$= e^{\int 2\,dx + c}$

$= ye^{2x} = x^2 + c$

$\sec^2 x \cdot \tan y\,dx + \sec^2 y \tan x\,dy = 0$

$\sec^2 x \cdot \tan y\,dx = -\sec^2 y \cdot \tan x\,dy$

$= \frac{\sec^2 x\,dx}{\tan x} = \frac{-\sec^2 y\,dy}{\tan y}$

$\int \frac{\sec^2 x\,dx}{\tan x} = -\int \frac{\sec^2 y\,dy}{\tan y}$

$\therefore \log|\tan x| = -\log|\tan y| + c$

$\log|\tan x - \tan y| = c$

$\tan x \cdot \tan y = e^c$

6 $\frac{dy}{dx} = \sin^2(x-y+1)$

Put $x-y+1 = v$

Differentiating on both side

$x = y+1-v$

$1 - \frac{dy}{dx} = \frac{dv}{dx}$

$\frac{1-dv}{dx} = \frac{dy}{dx}$

$1 - \frac{dv}{dx} = \sin^2 x$

$\frac{dv}{dx} = 1 - \sin^2 v$

$\frac{dv}{dx} = \cos^2 v$

$\frac{dv}{\cos^2 v} = dx$

$\int \sec^2 v\,dv = \int dx$

$\tan v = x + c$

$\tan(x+y-1) = x + c$

7 $\frac{dy}{dx} = \frac{2x+3y-1}{6x+9y+6}$

Put $2x+3y = v$

$2 + 3\frac{dy}{dx} = \frac{dv}{dx}$

$\frac{dy}{dx} = \frac{1}{3}\left(\frac{dv}{dx} - 2\right)$

$\frac{1}{3}\left(\frac{dv}{dx} - 2\right) = \frac{1}{3}\frac{(v-1)}{(v+2)}$

$\frac{dv}{dx} = \frac{v-1}{v+2} + 2$

$\frac{dv}{dx} = \frac{v-1+2v+4}{v+2}$

$= \frac{3v+3}{v+2}$

$= \frac{3(v+1)}{v+2}$

$= \int\left(\frac{v+2}{v+1}\right)dv = 3dx$

$= \int \frac{v+1}{v}\,dx + \int \frac{1}{v+1}\,dv = 3x$

$v + \log|x| = 3x + c$

$2x + 3y + \log|2x+3y+1| = 3x + c$

$3y = x - \log|2x+3y+1| + c$

Practical No. 8

Topic : Euler's Method

$\frac{dy}{dx} = y + e^x - 2$ $y(0) = 2$ $h = 0.5$ find $y(2) = ?$

→ $f(x) = y + e^x - 2$ $x_0 = 0$ $y(0) = 2$ $h = 0.5$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	2	1	2.5
1	0.5	2.5	2.1487	3.5743
2	1	3.5743	4.2925	5.7205
3	1.5	5.7205	8.2021	9.8215
4	2	9.8215		

$\therefore y(2) = 9.8215$

2. $\frac{dy}{dx} = 1 + y^2$, $y(0) = 1$, $h = 0.2$ find $y(1) = ?$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	0	1	0.2
1	0.2	0.2	1.04	0.408
2	0.4	0.408	1.1664	0.6412
3	0.6	0.6412	1.4111	0.9234
4	0.8	0.9234	1.8526	1.2939
5	1	1.2939		

$y(1) = 1.2939$

Q3. $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$ $y(0)$ $h = 0.2$ find $y(1) = ?$

$x_0 = 0$ $y(0) = 1$ $h = 0.2$

n	xn	yn	f(xn, yn)	yn+1
0	0	1	0	1
1	0.2	1	0.4472	1.0894
2	0.4	1.0894	0.6059	1.2105
3	0.6	1.2105	0.7040	1.3513
4	0.8	1.3513	0.7696	1.5051
5	1	1.5051		

$y(1) = 1.5051$

Q4. $\frac{dy}{dx} = 3x^2 + 1$ $y(1) = 2$ find $y(2)$ $h = 0.5$

$h = 0.25$

$y_0 = 2$ $x_0 = 1$ $h = 0.5$

n	xn	yn	f(xn, yn)	yn+1
0	1	2	4	4
1	1.5	4	7.75	7.875
2	2	7.875		

$y(2) = 7.875$

$y_0 = 2$ $x_0 = 1$ $h = 0.25$

n	xn	yn	f(xn, yn)	yn+1
0	1	2	4	3
1	1.25	3	5.6875	4.4218
2	1.5	4.4218	59.6569	19.3360
3	1.75	19.3360	1122.6426	299.9960
4	2	299.9960		

$y(2) = 299.9960$

5. $\frac{dy}{dx} = \sqrt{xy} + 2$ $y(1) = 1$ $h = 0.2$

$x_0 = 1$ $y_0 = 1$ $h = 0.2$

x	xn	yn	f(xn, yn)	yn+1
0	1	1	3	3.6
1	1.2	3.6		

$y(1.2) = 3.6$

23/01/2020

Practical No. 9.

limits and Partial order Derivative

1. Evaluate the following limits.

i. $\lim_{(x,y)\to(-4,-1)} \frac{x^3-3y+y^2-1}{xy+5}$

Applying limits.

$$\frac{(-4)^3-3(-1)+(-1)^2-1}{(-4)(-1)+5}$$

$$\frac{-64+3+1-1}{4+5} = \frac{-61}{9}$$

ii $\lim_{(x,y)\to(2,0)} \frac{(y+1)(x^2+y^2-4x)}{x+3y}$

Applying limits

$$\frac{(0+1)(2)^2+(0)^2-4(2)}{2+3(0)}$$

$$\frac{1(4-8)}{2} = \frac{-4}{2} = -2$$

iii. $\lim_{(x,y,z)\to(1,1,1)} \frac{x^2-y^2z^2}{x^3-x^2yz}$

$$\frac{(x)^2-(yz)^2}{x^2(x-yz)}$$

$$\frac{1+(1)(1)}{1} = 2$$

Find f_x, f_y for each of the following

i. $f(x,y) = xy\, e^{x^2+y^2}$

$$f(x) = \frac{\partial f}{\partial x}$$

$$= \frac{\partial(xy\, e^{x^2+y^2})}{\partial x}$$

$$= \frac{y\, \partial(x \cdot e^{x^2+y^2})}{\partial x}$$

$$= y\left[x \cdot \frac{d}{dy}(e^{x^2+y^2}) + e^{x^2+y^2} \cdot \frac{d}{dx}(x)\right]$$

$$= y\left[x \cdot e^{x^2+y^2} \cdot 2x + e^{x^2+y^2}(1)\right]$$

$$= y \cdot e^{x^2+y^2}[2x+1]$$

Now, $f(y) = \frac{\partial f}{\partial y}$

$$= \frac{\partial(xy\, e^{x^2+y^2})}{\partial y}$$

$$= x \cdot \frac{\partial}{\partial y}(y \cdot e^{x^2+y^2})$$

$$= x\left[y \cdot \frac{d}{dy}(e^{x^2+y^2}) + e^{x^2+y^2} \cdot \frac{d}{dy}(y)\right]$$

$$= x \cdot \left[2y^2 \cdot e^{x^2+y^2} + e^{x^2+y^2}\right]$$

$$= x \cdot e^{x^2+y^2}[2y^2+1]$$

ii. $f(x,y) = e^x \cos y$

$\therefore f(x) = e^x \cos y$

$f(y) = e^x \dfrac{\cos y}{\partial y}$

$= e^x(-\sin y)$

$= -e^x \sin y$

iii. $f(x,y) = x^3y^2 - 3x^2y + y^3 + 1$

$f(x) = \dfrac{f}{\partial x} = \dfrac{\partial(x^3y^2 - 3x^2y + y^3 + 1)}{\partial x}$

$= 3x^2y^2 - 3(2x)y$

$= 3x^2y^2 - 6xy$

$f(y) = \dfrac{\partial f}{\partial y} = \dfrac{\partial(x^3y^2 - 3x^2y + y^3 + 1)}{\partial y}$

$= x^3(2y) - 3(1)x^2 + 3y^2$

$= 2x^3y - 3x^2 + 3y^2$

Q3. Using defination find values of f_x, f_y at $(0,0)$ for $f(x,y) = \dfrac{2x}{1+y^2}$

$f_x(a,b) = \lim_{h \to 0} \dfrac{f(a+h,b) - f(a,b)}{h}$

where $(a,b) = (0,0)$

$\therefore f(x)(0,0) = \lim_{h \to 0} \dfrac{f(h,0) - f(0,0)}{h}$

$= \dfrac{2h - 0}{2}$

$= 2$

Similarly

$f_y(a,b) = \lim_{h \to 0} \dfrac{f(h,0) - f(0,0)}{h}$

$\dfrac{0 - 0}{h} = 0$

$\therefore f_x = 2, \quad f_y = 0$

Q4 Find all second order partial derivates of f also verify whether $f_{xy} = f_{yx}$

1. $f(x,y) = \frac{y^2 - xy}{x^2}$

$\therefore f(x) = \frac{\partial f}{\partial x} = \frac{\partial (y^2 - xy)}{x^2}$

$= \frac{x^2 \cdot \frac{d}{dx}(y^2 - xy) - (y^2 - xy)\frac{d}{dx}(x^2)}{(x^2)^2}$

$\frac{-x^2 y - 2xy^2 + 2x^2 y}{x^4}$

$= \frac{x^2 y - 2xy^2}{x^3}$

$f(y) = \frac{\partial f}{\partial y} = \frac{\partial \left(\frac{y^2 - xy}{x^2}\right)}{}$

$= \frac{\partial \left(y^2/x^2 - xy/x^2\right)}{\partial y}$

$= \partial \left(y^2/x^2 - y/x\right) / \partial y$

$= \frac{1}{x^2} 2y - \frac{1}{x}$

$f(y) = \frac{2y - x}{}$

$f(x,x) = \frac{\partial \left(\frac{xy - 2y^2}{x^3}\right)}{\partial x}$

$= \frac{x^3 \cdot \frac{d}{dx}(xy - 2y^2) - (xy - 2y^2)\frac{d}{dx}(x^3)}{(x^3)^2}$

$= \frac{x^3(y) - (xy - 2y^2)(3x^2)}{6}$

$= \frac{x^3 y - 3x^3 y + 6x^2 y^2}{6}$

$= \frac{6x^2 y^2 - 2x^3 y}{x^6}$

$= \frac{6y^2 - 2xy}{x^4}$

$f(yy) = \frac{\partial \left(\frac{2y - x}{x^2}\right)}{\partial y}$

$= \frac{\frac{1}{x^2} \partial (2y - x)}{\partial y} = \frac{1}{x}(2) = \frac{2}{x^2}$

$f(x,y) = \frac{\partial \frac{(xy - 2y^2)}{x^3}}{\partial y} = \frac{\partial \left(\frac{xy}{x^3} - \frac{2y^2}{x^3}\right)}{\partial y}$

$= \frac{\partial \left(y/x^2 - 2y^2/x^3\right)}{\partial y}$

Q

$$\frac{1}{x^2} - \frac{1}{x^3} \quad 2(2y)$$

$$= \frac{1}{x^2} - \frac{4y}{x^3} \qquad = \frac{x^3 - 4y^2}{x^6}$$

$$= \frac{x^2(x-4y)}{x^6}$$

$$= \frac{x-4y}{x^4}$$

$$f(yx) = \frac{\partial\left(\frac{2y-x}{x^2}\right)}{\partial x}$$

$$= \frac{\partial\left(\frac{2y}{x^2} - \frac{x}{x^2}\right)}{\partial x} \qquad = \partial\left(\frac{2y}{x^2} - \frac{1}{x}\right)$$

$$= 2y\left(\frac{-2}{x^3}\right) - \left(\frac{-1}{x^2}\right)$$

$$= \frac{-4y}{x^3} + \frac{1}{x^2}$$

$$= \frac{-4yx^2 + x^3}{x^6}$$

$$= \frac{x^2(x-4y)}{x^6}$$

$$f(x,y) = \sin(xy) + e^{x+y}$$

$$f(x) = \frac{\partial f}{\partial x} = \frac{\partial(\sin(xy) + e^{x+y})}{\partial y}$$

$$= \cos(xy)(y) + e^{x+y}(1)$$

$$= y\cos xy + e^{x+y}$$

$$f(y) = \frac{\partial f}{\partial y} = \frac{\partial(\sin(xy) + e^{x+y})}{\partial y}$$

$$= \cos(xy)(x) + e^{x+y}(1)$$

$$= x\cos xy + e^{x+y}$$

$$= y\cos xy(y) + e^{x+y}(1)$$

$$= y^2\cos xy + e^{x+y}$$

$$f(y,y)$$

$$= x\cos xy(x) + e^{x+y}(1)$$

$$= x^2\cos xy + e^{x+y}$$

$$f(x,y) = \frac{\partial fx}{\partial y} = \frac{\partial(y\cos xy + e^{x+y})}{\partial y}$$

$$= y[-\sin(xy)(x) + \cos(x,y)(1)] + e^{x+y}(1)$$

$$= -xy\sin(xy) + \cos xy + e^{x+y}$$

$$f(y,x) = \frac{\partial f y}{\partial x} = \frac{\partial (x\cos xy + e^{x+y})}{\partial x}$$

$$= \cos xy\,(1) + x(-\sin(xy))(y) + e^{x+y}$$

$$= -xy\sin(xy) + \cos(xy) + e^{x+y}$$

$$\therefore f(x,y) = f(y,x) = -xy\sin(xy) + \cos(xy) + e^{x+y}$$

Q5. Find the linerization of $f(x,y)$ at given point

$f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$

$$f(1,1) = \sqrt{(1)^2+(1)^2}$$
$$= \sqrt{2}$$

$$f(x) = \frac{1}{2\sqrt{x^2+y^2}} \cdot 2x = \frac{x}{\sqrt{x^2+y^2}}$$

$$f(y) = \frac{1}{2\sqrt{x^2+y^2}} \cdot 2y = \frac{y}{\sqrt{x^2+y^2}}$$

$$f(x)\,(1,1) = \frac{1}{\sqrt{(1)^2+(1)^2}} = \frac{1}{\sqrt{2}}$$

$$f_y(1,1) = \frac{1}{\sqrt{(1)^2+(1)^2}} = \frac{1}{\sqrt{2}}$$

$$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$$
$$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1) + \frac{1}{\sqrt{2}}(y-1)$$

$$= \frac{2+x+y-2}{\sqrt{2}}$$

$$= \frac{x+y}{\sqrt{2}}$$

2. $f(x,y) = 1 - x + y\sin x$ at $(\pi/2, 0)$

$$\rightarrow f(\pi/2, 0) = 1 - \frac{\pi}{2} + 0\,(\sin(\pi/2))$$

$$= 1 - \frac{\pi}{2}$$

$$f(x) = -1 + y\cos x$$

$$f(y) = 1$$

$$f_x(\pi/2, 0) = -1 + 0\cos(\pi/2)$$
$$= -1$$

$$f_y(\pi/2, 0) = 1$$

$$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$$
$$= 1 - \frac{\pi}{2} + (-1)\left(x - \frac{\pi}{2}\right) + 1(y-0)$$

$$= 1 - \frac{\pi}{2} - x + \frac{\pi}{2} + y$$

$= y - x + 1$

3. $f(x,y) = \log x + \log y$ at $(1,1)$

$\rightarrow f(1,1) = \log(1) + \log(1)$
$= 0 + 0$
$= 0$

$f(x) = \frac{1}{x}$ $\qquad$ $f(y) = \frac{1}{y}$

$fx\ (1,1) = 1$ $\qquad$ $fy(1,1) = 1$

$L(x,y) = f(a,b) + fx(a,b)(x-a) + fy(a-b)(y-b)$

$= 0 + 1(x-1) + 1(y-1)$
$= x - 1 + y - 1$
$= x + y - 2$

Practical No:- 10

Q1. Find the directional derivates of the following function at given points and in the direction of given vector

1. $f(x,y) = x + 2y - 3$ $\quad a(1,-1) \quad u = 3i - j$

Hence $u \cdot 3i - j$ is not a unit vector

$$|\bar{u}| = \sqrt{(3)^2 + (-1)^2} = \sqrt{9+1} = \sqrt{10}$$

unit vector along u is $\frac{u}{|u|} = \frac{1}{\sqrt{10}}(3,-1)$

$$= \left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$$

$$= f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$$

$$f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$$

$$1 + \frac{3}{\sqrt{10}} - 2 - \frac{2h}{\sqrt{10}} - 3$$

$$f(a + hu) = -4 + \frac{h}{\sqrt{10}}$$

$$\lim_{h \to 0} \frac{f(a+hu) - f(a)}{h} = \frac{-\cancel{4} + \cancel{h}/\sqrt{10} + \cancel{4}}{\cancel{h}} = \frac{1}{\sqrt{10}}$$

2. $f(x) = y^2 - 4x + 1$ $\quad a = (3,4) \quad u = i + 5j$

$|\bar{u}| = \sqrt{(1)^2 + (5)^2} = \sqrt{26}$

Unit vector along u is $\frac{u}{|u|} = \frac{1}{\sqrt{26}} (1,5)$

$= \left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$

$f(a) = f(3,4) = (4)^2 - 4(3) + 1 = 5$

$f(a+hu) = f(3,4) + h\left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$

$= \left(4 + \frac{5h}{\sqrt{26}}\right)^2 - 4\left(3 + \frac{h}{\sqrt{26}}\right) + 1$

$= 16 + \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - 12 - \frac{4h}{\sqrt{26}} + 1$

$= \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - \frac{4h}{\sqrt{20}} + 5$

$= \frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5$

$\lim_{h \to 0} \frac{\frac{25h^2}{26} + \frac{36h}{\sqrt{26}}}{h}$

$\frac{\cancel{h}\left(\frac{25h}{26} + \frac{36}{\sqrt{26}}\right)}{\cancel{h}}$

Q2. Find gradient vector

1. $f(x,y) = x^y + y^x \quad = a = (1,1)$

$dy = xy \log x + xy^{x-1}$

$f(x,y) = (f_x, f_y)$

$= (yx^{y-1} + y^2 \log y, x^y \log x + xy^{x-1})$

$f(1,1) = (1+0, 1+0)$

$= (1,1)$

2. $f(x,y) = (\tan^{-1} x) \cdot y^2 \quad a = (1,-1)$

$f_x = 1/1+x^2 \cdot y^2$

$f_y = 2y \tan^{-1} x$

$f(x,y) = (f_x, f_y)$

$= (y^2/1+x^2, 2y \tan^{-1} x)$

$f(1,-1) = (1/2, \tan^{-1}(1)(-2))$

$= (1/2, \pi/\cancel{4}_2 (-2))$

$= (1/2, -\pi/2)$

3. $f(x,y,z) = xyz - e^{x+y+z}$, $a = (1,-1,0)$

$f_x = yz - e^{x+y+z}$

$f_y = xz - e^{x+y+z}$

$f_z = xy - e^{x+y+z}$

$\nabla f(x,y,z) = f_x, f_y, f_z$

$= yz - e^{x+y+z}, xz - e^{x+y+z}, xy - e^{x+y+z}$

$f(1,-1,0) = ((-1)(0) - e^{(1+(-1)+0)}, (1)(0) - e^{x(1+(-1)+0}, (1)(-1) - e^{1+(-1)}$

$= (0 - e^0, 0 - e^0, -1 - e^0)$

$= (-1, -1, -2)$

Q3.

1. $x^2 \cos y + e^{xy} = 2$ at $(1,0)$

$f_x = \cos y\, 2x + e^{xy} y$

$d_y = x^2(-\sin y) + e^{xy}.x$

$(x_0, y_0) = (1,0) = \therefore x_0 = 1, y_0 = 0$

eqn of tangent

$f_x(x_0,y_0) = \cos 0\, 2(1) + e^{0} 0$

$= 1(2) + 0$

$= 2$

$f_y(x_0,y_0) = (1)^2(-\sin 0) + e^0 \cdot 1$

$= 0 + 1 \cdot 1$

$= 1$

$2(x-1) + 1(y-0) = 0$

$2x - 2 + y = 0$

$2x + y - 2 = 0$

equation of Normal

$= ax + by + c = 0$

$= bx + ay + d = 0$

$1(1) + 2(y) + d = 0$

$1 + 2y + d = 0$ at $(1,0)$

$1 + 2(0) + d = 0$

$d + 1 = 0$

$\therefore d = -1$

i. $x^2 + y^2 = 2x + 3y + 2 = 0$ at $(2,-2)$

$f_x = 2x + 0 - 2 + 0 + 0$

$= 2x - 2$

$f_y = 0 + 2y - 0 + 3 + 0$

$= 2y + 3$

$(x_0, y_0) = (2,-2) \therefore x_0 = 2, y_0 = -2$

$f_x(x_0,y_0) = 2(2) - 2 = 2$

$f_y(x_0,y_0) = 2(-2) + 3 = 1$

equation of tangent

$f_x(x-x_0) + f_y(y-y_0) = 0$

$2(x-2) + (-1(y+2)) = 0$

$2x - 2 - y - 2 = 0$

$2x - y - 4 = 0 \rightarrow$ It is required eqn of tangent

eqn of Normal

$= ax + by + c = 0$

$= bx + ay + d = 0$

$= -1(x) + 2(y) + d = 0$

$-x + 2y + d = 0$ at $(2,-2)$

$-2 + 2(-2) + d = 0$

$-2 - 4 + d = 0$

$-6 + d = 0$

$\therefore d = 6$

Q4. Find the equation

1. $x^2 - 2yz + 3y + xz = 7$ at $(2,1,0)$

$f_x = 2x - 0 + 0 + z$

$f_x = 2x + z$

$f_y = 0 - 2z + 3 + 0$

$= 2z + 3$

$x_0 = 2,\ y_0 = 1,\ z_0 = 0$

$f_z = 0 - 2y - 0 + x$

$= -2y + x$

$f_x(x_0, y_0, z_0) = 2(2) + 0 = 4$

$f_y(x_0, y_0, z_0) = 2(0) + 3 = 3$

$f_z(x_0, y_0, z_0) = -2(1) + 2 = 0$

eqn of tangent

$= f_x(x_0 - x_0) + f_y(y_0 - y_0) + f_z(z_0 - z_0) = 0$

$= 4(x-2) + 3(y-1) + 0(z-0) = 0$

$= 4x - 8 + 3y - 3 = 0$

$= 4x + 3y - 11 = 0$

Eqn of normal at $(4, 3, -11)$

$$\frac{x - x_0}{f_x} = \frac{y - y_0}{f_y} = \frac{z - z_0}{f_z}$$

$$= \frac{x-2}{4} = \frac{y-1}{3} = \frac{z+11}{0}$$

Q5. Find the local maxima & minima for the follow function

1. $f(x,y) = 3x^2 + y^2 - 3xy + 6x - 4y$

$f_x = 6x + 0 - 3y + 6 - 0$

$= 6x - 3y + 6$

$f_y = 0 + 2y - 3x + 0 - 4$
$= 2y - 3x - 4$

$f_x = 0$
$6x - 3y + 6 = 0$
$3(2x + 2 - y) = 0$
$2x - y + 2 = 0$
$2x - y = -2$ — (1)

$f_y = 0$
$2y - 3x - 4 = 0$
$2y - 3x = 4$ — (2)

Multiply eq^n 1 with 2
$4x - 2y = -4$
$2y - 3x = 4$
$x = 0$

Substitute value of x in eq^n (1)
$2(0) - y = -2$
$-y = -2$
$\therefore y = 2$

Critical points are $(0, 2)$
$r = f_{xx} = 6$
$t = f_{yy} = 2$
$s = f_{xy} = -3$

Here $r > 0$
$= rt - s^2$
$= 6(2) - (-3)^2$
$= 12 - 9$
$= 3 > 0$

$3x + y^2 - 3xy + 6x - 4y$ at $(0,2)$
$3(0)^2 + (2)^2 - 3(0)(2) + 6(0) - 4(2)$
$0 + 4 - 0 + 0 - 8$
$= -4$

iii. $f(x, y) = x^2 - y^2 + 2x + 8y - 70$
$f_x = 2x + 2$
$f_y = -2y + 8$

$f_x = 0 \quad \therefore 2x + 2 = 0$
$x = \frac{-2}{2}$

$\therefore x = -1$

$f_y = 0 \quad = -2y + 8 = 0$
$y = \frac{-8}{2}$

$\therefore y = 4$

$\therefore$ Critical point is $(-1, 4)$

$r = f_{xx} = 2$
$t = f_{yy} = -2$
$s = f_{xy} = 0$

$r > 0$

$rt - s^2 = 2(-2) - (0)^2$
$= -4 - 0$
$= -4 < 0$

$f(x,y)$ at $(-1,4)$

$(-1)^2 - (4)^2 + 2(-1) + 8(4) - 70$

$= 1 + 16 - 2 + 32 - 70$
$= 17 + 30 - 70$
$= 37 - 70$

$= -33$

Ans
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